Introduction to Financial Engineering (HW6)

Organizer: Albert Ferreiro-Castilla
room: PC email: `aferreiro@mat.uab.cat`

- Please follow the guidelines for assignments given in the Module Handbook.
- All programs should be written in R (compilable without errors or warnings).
- You should submit a write-up (.pdf) of the program as well as the source code (.r).
- File names should be as `yoursurname_yourname_HW6.extension`
- You should submit via moddle.
- Deadline: 1st December 2023 at 10am.

The objective of the laboratory is to plot the efficient frontier for a set of 4 stocks.

1. The first task consists in plotting a large set of possible portfolios.

   (a) **Download data**: Use `quantmod` library to download the adjusted prices for FaceBook (FB), Apple (AAPL), Microsoft (MSFT) and Google (GOOG) between 01/01/2015 and 31/12/2015.

   (b) **Returns and Covariances**: Compute the mean of daily returns and the covariance matrix of the daily returns. Let us denote the vector of mean returns, μ, and the matrix of covariance return, C. Therefore μ is a vector of dimension 4×1 and C has dimension 4×4. Remember that the download data from `quantmod` are prices, hence you need to transform them to returns.

   (c) **Portfolio weights simulation**: Draw four uniform random numbers from $[0,1]$ and normalize them. That means, drawing $W_i \sim U[0,1]$ for $i=1,\ldots,4$, and computing the normalized weights as

   $$\omega_i = \frac{W_i}{\sum_{j=1}^{4} W_j}$$

   In other words, we have $\sum_{i=1}^{4}\omega_i = 1$. Therefore, ω represents the amount of money invested in each of the 4 stocks. Compute the mean and variance of the portfolio associated with the weights ω:

   $$\begin{aligned}\mu_P &= \omega^t \mu \\ \sigma_P^2 &= \omega^t C \omega\end{aligned}$$

   where ^{t} represents the transpose.

   (d) **Feasible set**: Repeat the previous computation drawing 1000 random vectors ω and plot the pairs (σ_P, μ_P) in a plot.

2. Now we are going to plot the efficient frontier using the Lagrangian multipliers derived in the lecture notes.

   (a) **Construct the following vector**:

   ```
   rbase <- seq(min(mu),max(mu),length=N)
   ```

   where N is some large number, say 500.

(b) **Building up the linear model**: For each element in $r \in$`rbase`, build up the following matrix Q and vector b:

$$Q = \begin{pmatrix} 2C & \mu & \mathbb{1} \\ \mu^t & 0 & 0 \\ \mathbb{1}^t & 0 & 0 \end{pmatrix}$$

$$b = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ r \\ 1 \end{pmatrix}$$

where $\mathbb{1} = (1, 1, 1, 1)^t$, the unitary vector of length 4. (Hint: use the functions `rbind` and `cbind` to form a matrix from a collection of matrices).

(c) **Solve the linear model**:Use the following function to solve the linear model of the Lagrange multipliers:

```
y <- solve(Q,b)
```

where

$$y = \begin{pmatrix} \omega_r \\ \lambda_1 \\ \lambda_2 \end{pmatrix}$$

and ω_r is a 1×4 vector representing the weights of the optimal portfolio with a given level of return r. The parameters λ_1 and λ_2 are the Lagrange multipliers and of no use for us right now.

(d) **Plot the efficient frontier**: Use ω_r for each $r \in$`rbase` to compute the return and volatility as done in section c of the first exercise. Plot the resulting pairs in a plot.

3. **Portfolio Optimization**: Plot in a single figure, the resulting plots from last section of exercise 1 and last section of exercise 2. Show the feasible set of portfolios, the efficient portfolio and the minimum variance portfolio in the figure.