Introduction to Financial Engineering (W10)

Probability: Risk Management

1 Context

Financial institutions have the necessity to monitor the risk for a given portfolio or group of assets. The task of risk departments are mostly summarized as overseeing the financial activity of the institution and making sure that the individual or aggregated risk stands within the risk appetite of the management.

The Basel Committee is a committee that brings together the global bank regulators. One of the first activities of the committee was to set the terms on which banks need to compute capital reserve for credit risk. Capital reserve will enable to mitigate losses on periods of crises. One of the measures used to compute capital would be Value at Risk (VaR). VaR is a metric that summarizes the total risk of a group of assets. The first rule books from the committee have evolve including the need to take into consideration market risk, operational risk and it is becoming more and more complex.

The aim of this chapter is to present the measure VaR and show a couple of different approaches to compute it.

The risk Map is the illustration or identification of all risks affecting a particular activity, in the context of financial investments one could identify without limitation the following main risks:

- **Market risk**: Reflected in the evolution of market prices of underlying operations in equity, FX, bonds, ... This would be the most important risk in our framework.
- **Credit risk**: It relates to the ability of a counterpart to face its obligations in a financial contract.
- **Balance sheet risk**: For credit institutions, such as banks, this risk arises from the asymmetry between the short term interest rate on deposits against long term interest rate on loans or mortgages.
- **Capital risk**: This risk comes from the potential effect on solvency metrics that a regulatory, economic or capital allocation may have.
- **Operational risk**: Present in every activity this refers to human or technological errors in the process.

Refer to Chapter 22 in [1] for further reading.

2 The VaR measure

When using VaR we aim to make a statement of the form:

> There is α percent certainty there will be not a loss of more V units of money in the next N days.

Needless to say that the term certainty above is given on a statistically meaning framework. The variable V is the VaR and depends upon on the confidence level α and the time horizon N, and hence the VaR is the $(100-\alpha)$-th percentile of the distribution of the loss of the portfolio over the next N days. Therefore the VaR function is focused on the left tail of the gains and losses distribution.

Even though VaR accomplish to summarize the risk of a portfolio, it might be too simple as it may hinder the true risk of certain investments. Below two distributions with the same VaR but one distribution of losses is much riskier than the other. To put it in statistical terms, one thing is to compute the x-th percentile of a distribution and other would be to compute the expected conditional loss, i.e. $\mathbb{E}[X \mid X < x]$ and usually known as tail VaR.

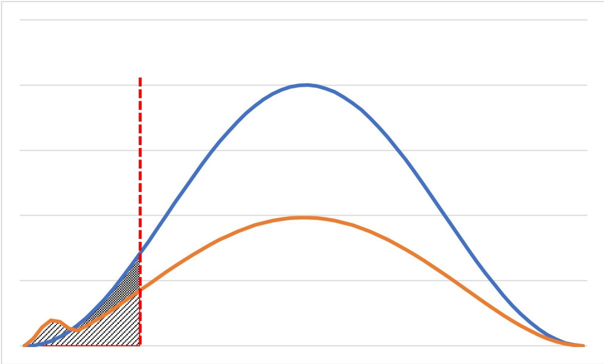

Figure 1. Even though two distributions might have the same VaR, one might be more risky than the other.

The second parameter that affects the VaR calculation is N. The time horizon should fir the purpose of the risk measurement, for instance trading market activity risk is often measured with $N = 1$, while portfolio management activities focus on larger time windows such as $N = 10$. A common approximation to compare VaR between different time frames is:

$$\text{VaR}_{N\text{-day}} = \text{VaR}_{1\text{-day}} \times \sqrt{N}\,, \tag{1}$$

equation (1) holds exact when value of the portfolio in successive days have independent and identically normal distribution.

Before attempting to compute any risk metric one needs to identify the sources of riskiness in their portfolio. For instance, if we are monitoring a portfolio of equity shares it is clear that equity prices will affect the value of the portfolio, but there might be other sources of risk related to prices such as macroeconomic data, geopolitical risk, ...

To fix ideas on the following derivation we will be focusing on monitoring the risk of a portfolio consisting of a set of equity shares which the only source of risk comes from direct prices of the underlying.

3 Historical simulation

Assume the task of simulating the VaR of a portfolio of risky financial instruments at 99% confidence and one-day time horizon, and we have an historical data base of 500 days market data. The first step in any risk monitoring process is to identify the sources of risk, but in this scenario we have agreed to be the prices of the underlying assets. Therefor we need the price evolution of every asset in out portfolio for the last 500 days.

For every day we can compute the value of our portfolio and the return it would had for one-day horizon, ie on the next day. This return computation will generate a distribution of on-day returns and in order to compute VaR at 99% confidence we need to calculate the 99-th percentile of the empirical distribution.

In real life applications often the identification of the risk factors and their dependence to the portfolio value is not that straight forward. One often need to establish a formula for certain parameters that affect asset value. For instance, when performing a risk monitoring of a fixed income portfolio or a portfolio of derivatives or forwards, one would need to incorporate interest rate curves and establish the relationship between forward prices and rates for example.

The principal drawback on this approach is that no one can assure that the future behavior of assets will resemble the past.

4 Model based approach

The main alternative to historical simulation is the model building approach. Depending on the complexity of the model to be used the one may need to calibrate different data, but for the most simple approach one could assume that the returns of the given portfolio follow a multivariate normal distribution and volatilizes and correlations are available or can be computed.

In this situation we would estimate from the preceding data base the variance-covariance matrix and the expected returns and derive from it a multivariate normal distribution which then could be turn into an analytical distribution for the returns. By means of the inverse function one could compute the 99-th percentile. Nevertheless, the approach is far from simple as we already state it is difficult to give a good estimate of:

$$C = [\sigma_{ij}]_{1<\leq i,j\leq N} \text{ where } \sigma_{ij} = \mathrm{Cov}(a_i, a_j)\ .$$

Refer to Chapter 23 in [1] for further reading on estimating volatilities and correlations for financial data.

Clearly, this approach has to consider that certain assumptions on the behavior of the assets are plausible such as the normal distribution of returns. As we already show in previous chapters, this is just an approximation and there are some stylized characteristics in real data that show it. Nevertheless, in real life applications we can used more complex models and the general framework applies similarly.

5 Monte Carlo Simulation

An alternative to the close form solution for the model based approach, one could draw several times from the analytical distribution and get an approximation of the percentile. Clearly, for the simple example of a multivariate normal distribution presented before this approach is not very advantageous, but the real applications of this approach are in highly complex portfolios. For instance, think about path dependent options such as a binary option. In this derivative one would get one unit of money if the stock hits a pre-define level of price in the course of the option life or zero otherwise. In such cases often one would need to simulate paths to compute the profit and loss distribution as analytical forms for the distribution of the maximum may not be available.

Figure 2. Barrier options have a payoff depending upon the path of the underlying asset up to maturity.

In a similar way as the previous approach, Monte Carlo simulation need certain assumptions on the evolution of assets and the same conclusion on the model applies here, as well as the fact that in applied situations highly complex dynamic models are used to get better approximations.

6 Principal Component Analysis

Often portfolios or groups of assets to be motorize are very complex and not from the same nature, i.e. usually one has a portfolio which is a mixture of derivatives, bonds, futures, equities, fx positions, ... in such cases it might be difficult to identify true sources of risk factors as some positions in your portfolio might be acting as hedges of part of those risk for other positions. For instance, think about a bond position and a interest rate swap hedging the variations of the risk-free rate but not on the credit risk. In those cases we would first want to reduce the sources of risk to the main or principal factors affecting our portfolio. Principal component analysis just does that and it is a standard statistical tool.

The importance of a given factor is measures by how much the variance of that factor impacts the variance of the portfolio value and different factors are ranked by this score. The outcome of the process is an orthogonal set of factors and often the first 4 or 5 are able to explain 99% of the variability of the portfolio.

Principal component is used to compute VaR the variation score of the first factors to give an estimate of the variation of the portfolio value.

7 Recap

This chapter have shown one of the most used risk metrics, but many companies perform additional metrics to test the robustness and risk of their investments. One of the most common additional test is what is known a stress test, which consists in simulating the behavior of a given portfolio under the most severe financial and macroeconomic scenarios of the past 20 or 30 years.

Regardless of the risk metric used, one of the key features to ensure a comprehensive methodology would be backtesting the metrics. The process of backtesting consists in using a set of historical data for which the computation of the risk metric is done within a rolling window to asses whether the metric is in fact able to monitor true risk.

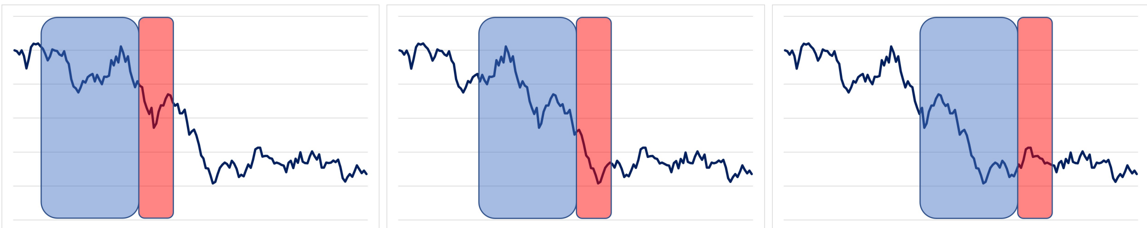

Figure 3. Schematic layout for a backtest analysis. For a set of historical data a calibration of the risk metric is perform only with the information of the blue set and test its significance on the red set in a rolling window.

References

[1] John C. Hull (2014) *Options, Futures and Other Derivatives*, Pearson.