Entregar al Campus Virtual abans del dia: 10/04/2024.

Exercise 1 (6 points) *Let* $x = (x_1, x_2, ...x_n)$*,* $n > 2$*, be observations of i.i.d. random variables with probability function*

$$\mathbb{P}\left[X_i = x_i\right] = (1-p)^{(x_i - 1)}p,$$

where $x_i \in \mathbb{N},\ p \in (0,1)$*.*

(a) *Calculate the expected value of* $S(x) := \sum_{i=1}^{n} x_i$*.* [1 point]

(b) *Find a sufficient statistic.* [1 point]

(c) *Find an unbiased estimator of* $\frac{1}{p}$ *based on* S*.* [2 points]

(d) *Find an unbiased estimator of* p *based on* S*.* [2 points]

$\left(\textit{Hint: Consider that } S \sim NB(n,p) \ \textit{(Negative Binomial)} \textit{ and consider } \mathbb{E}\left[\frac{1}{S-1}\right]\right)$.

Exercise 2 (4 points) *A sample* $X_1, \ldots, X_n$ *is drawn from the normal distribution* $N(\theta, \theta^2)$*.*

(a) *Find a 90% confidence interval for the population mean* θ*.* [2 points]

(b) *We aim to perform a hypothesis test to determine if there has been any change in the population mean. What sample size would be required to detect a change of* 0.1 *with a significance level of 95% and a power of 90%, given that* $\bar{X} = 12$*?* [2 points]