DATA.STAT.610 Financial Mathematics and Statistics

Project work

April 7, 2025

- The maximum points is 12 (the half of the total, 24).
- The first task is to calibrate the model, i.e. estimate the model parameters of Heston model, $\Psi = \{\kappa^Q, \theta^Q, \eta, \rho, V_0\}$. The risk-neutral dynamics of Heston model is

$$\begin{aligned} dS_t &= rS_t dt + \sqrt{V_t} S_t dW^Q_{1,t}, \\ dV_t &= \kappa^Q \left(\theta^Q - V_t\right) dt + \eta\sqrt{V_t} dW^Q_{2,t}, \end{aligned}$$

  with two correlated W^Q_1, W^Q_2 Wiener processes, $\mathrm{Corr}_t[W^Q_1, W^Q_2] = \rho$.
- The calibration will be based on the use of a snapshot of volatility surface (empVolatilitySurfaceData.mat)
  - The file contains a data structure
    - data.K: 1×42 strike prices
    - data.T: 8×1 maturities
    - data.IVolSurf: 8×41 Implied volatilities (that is, option prices are provided in terms of B-S implied volatilities)
    - data.r: interest rate (0.0466)
    - data.S0: current price of the underlying stock (1.00)
  - The loss function to be minimized with respect to Ψ is

$$\sum_{i=1}^{N} \sum_{j=1}^{M} \left(\mathrm{IV}_{\mathrm{Market}}(K_i, T_j) - \mathrm{IV}_{\mathrm{Model}}(K_i, T_j; \Psi)\right)^2,$$

    where $N = 42$ and $M = 8$, which is to be minimized with respect to model parameters $\Psi = \{\kappa^Q, \theta^Q, \eta, \rho, V_0\}$. Here $\mathrm{IV}_{\mathrm{Market}}(K_i, T_j)$ is the implied volatility from the market and $\mathrm{IV}_{\mathrm{Model}}(K_i, T_j; \Psi)$ the implied volatility from the model with the given values of parameters.
  - Use some numerical optimization methods, such as Nelder-Mead. Include constrains with the parameter. Their lower and upper bounds will be given in a settings file delivered along this assignment. *Do not use Feller's condition.*

- Use the provided codes with FFT-approach to price options with Heston model on given parameter values and convert the dollar prices to Implied Volatilities.
- You may visualize the market and volatility surfaces in every optimization iteration.
- *Report the parameter estimates along with their standard errors and the loss function value at the optima.*

- Use the parameter estimates to price the following exotic options with Heston model:
  - A down-and-in arithmetic Asian average strike call option, which pays

$$\max(S_T - S_{\text{avg}}, 0)$$

    if, at any time $0 < t \leq T$, the underlying asset price S_t falls below the threshold H, i.e., $S_t < H$ for some t. Here, S_{avg} is the arithmetic average of the daily stock prices over the period $[0, T]$, given by

$$S_{\text{avg}} = \frac{1}{T} \sum_{t=0}^{T} S_t.$$

  - Use Monte Carlo simulation to price the option using the risk-free asset (money market account) as the numeraire.
  - To eliminate negative variances, apply Milstein and truncation schemes:

$$V_{t+\Delta t} = \max\left(V_t + \kappa^Q(\theta^Q - V_t)\Delta t + \eta\sqrt{V_t \Delta t}\epsilon_{2,t} + \frac{1}{4}\eta^2 \Delta t(\epsilon_{2,t}^2 - 1), 0\right).$$

  - Implement variance reduction using antithetic variates.
    * For stock price and volatility processes, generate correlated variables ε_1 and ε_2 with correlation coefficient ρ_{12} as

$$\varepsilon_2 = \rho_{12}\varepsilon_1 + \sqrt{1 - \rho_{12}^2}e_{12},$$

      where $e_{12} \sim N(0, 1)$ is independent of ε_1.
    * To generate antithetic versions of the random variables for both stock price and volatility, set $\varepsilon_1^{(a)} = -\varepsilon_1$ and $e_{12}^{(a)} = -e_{12}$ and solve

$$\varepsilon_2^{(a)} = \rho_{12}\varepsilon_1^{(a)} + \sqrt{1 - \rho_{12}^2}e_{12}^{(a)},$$

      which can be applied to generate antithetic versions for stock price and volatility
    * For the antithetic stock price, use the antithetic volatility for consistency.
  - Parameter Values:

    * Current time $t = 0$, maturity time $T = 1$
    * Down barrier $H = 0.85$
    * Number of simulations $M = 100000$
    * Daily time-steps $\Delta t = 1/252$

- Provide option price on estimated parameters.

Provide a well-commented solution in MATLAB, Python, R, or Julia with clear comments and all necessary files, including the data file, to execute the main file.

On top of codes, please provide .txt or .pdf for parameter estimates, parameter standard errors, MSE, and option price.

Please include all the files, including the data file, so that the code can be run directly. Rename the main file to 'Main'. Make sure to zip all the files!