Function approximation using finite element

$$f = 2xy - x^2 \tag{1}$$

We want to check how to reproduce this function in a discrete space. The function u represents an approximation of f:

$$u = f \tag{2}$$

After the approximation using finite element, the following error (L2 norm) should be small enough:

$$\text{error} = \sum_{i=1}^{n} (u_i - f_i)^2 \tag{3}$$

where n is the number of degrees of freedom (e.g. the number of nodes).
We multiply each term of the Eq. 2 by an arbitrary test function v:

$$uv = fv \tag{4}$$

Integrating over the whole domain (e.g. a square) yields:

$$\int_\Omega uvd\omega = \int_\Omega fvd\omega \tag{5}$$

which can be rearranged to:

$$\int_\Omega uvd\omega - \int_\Omega fvd\omega = 0 \tag{6}$$

This form can be directly implemented in FreeFEM, with Ω being the finite element mesh, after which the error can be calculated based on Eq 3 to check if u is close enough to f.