Assignment 01 - Guidance

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Spacecraft Guidance and Navigation

AY 2021-2022



Laboratory sessions - Assignments

Goals



- Contribute towards the final grade
- Relief the workload of the project by spreading it
- Assets for MSc thesis: Matlab, SPICE, and Latex

Weights Fail (\mathbf{x}) **Excellent Poor** Good Minor mismatches w.r.t. Answers concise and Major mismatches w.r.t. Answers lengthy, but the assignment clear the assignment correct Report Figures and tables not • Figures and table clear Figures and tables clear clear and meaningful Good English Assignment Report awfully written English is poor Good English evaluation Code does not run Minor algorithmic Code runs smoothly Code runs smoothly Code is fairly Code is well errors Code Major algorithmic errors Code not documented documented documented Code takes unnecessary
 Computational Care is taken to account Code not complete efficiency improvable computational efficiency long to run

Laboratory sessions

- ➤ Laboratory sessions will not be recorded
 - we are here to give you answers while you are working
- When writing emails:
 - Prepend the following to the subject:
 [SGN Assignment1]
 - Do not attach/send code asking for debugging

Timetable		Room
20 Oct	16.15-18.15	BL28.12 (+ Morselli virtual room)
22 Oct	10.15-13.15	L11 (+ Morselli virtual room)
26 Oct	14.15-16.15	BL27.14 (+ Morselli virtual room)
27 Oct	16.15-18.15	BL28.12 (+ Morselli virtual room)
29 Oct	10.15-13.15	L11 (+ Morselli virtual room)
02 Nov	14.15-16.15	BL27.15 (Lecture)
03 Nov	16.15-18.15	BL28.12 (Lecture)
05 Nov	10.15-13.15	L11 (Lecture)
11 Nov	23.30 CET	Assignment 1 deadline

Delivery of assignments

Assigment will be delivered through **WeBeep**:

1) Click on the link to **load Assignment 1** in your Overleaf

https://bit.ly/SGN Assignment1 21



- 2) Fill the report and be sure it is compiled properly
- 3) Download the PDF and merge it in a zipped file with MATLAB code. Rename it lastname123456 Assign1.zip
- 4) Submit the compressed file by uploading it on Webeep

Assignment 01 - Topics

4 Exercises

- Impulsive guidance
- Ex 2: coming soon
- Continuous guidance
- Ex 4: coming soon

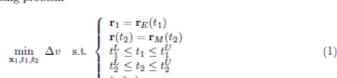
Suggested MATLAB Version: R2021b

Report

1 Impulsive guidance

Exercise 1

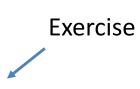
Let $\mathbf{x}(t) = \varphi(\mathbf{x}_0, t_0; t)$ be the flow of the geocentric two-body model. 1) Using one of Matlab's built-in integrators, implement and validate a propagator that returns $\mathbf{x}(t)$ for given \mathbf{x}_0 , t_0 , t, and μ . 2) Given the pairs $\{\mathbf{r}_1, \mathbf{r}_2\}$ and $\{t_1, t_2\}$, develop a solver that finds \mathbf{v}_1 such that $\mathbf{r}(t_2) = \mathbf{r}_2$, where $(\mathbf{r}(t), \mathbf{v}(t))^{\top} = \varphi((\mathbf{r}_1, \mathbf{v}_1)^{\top}, t_1; t_2)$ (Lambert's problem). To compute the derivatives of the shooting function, use either a) finite differences or b) the state transition matrix $\Phi = d\varphi/dx_0$. Validate the algorithms against the classic Lambert solver. 3) Using the propagator of point 1) in the heliocentric case, and reading the motion of the Earth and Mars from SPICE, solve the shooting problem

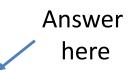


where $\Delta v = \Delta v_1 + \Delta v_2$, $\Delta \mathbf{v}_1 = \mathbf{v}_1 - \mathbf{v}_E(t_1)$, $\Delta \mathbf{v}_2 = \mathbf{v}(t_2) - \mathbf{v}_M(t_2)$. $\mathbf{x}_1 = (\mathbf{r}_1, \mathbf{v}_1)^T$, and $(\mathbf{r}(t), \mathbf{v}(t))^{\top} = \varphi(\mathbf{x}_1, t_1; t_2)$. Define lower and upper bounds, and make sure to solve the problem stated in Eq. (1) for different initial guesses.

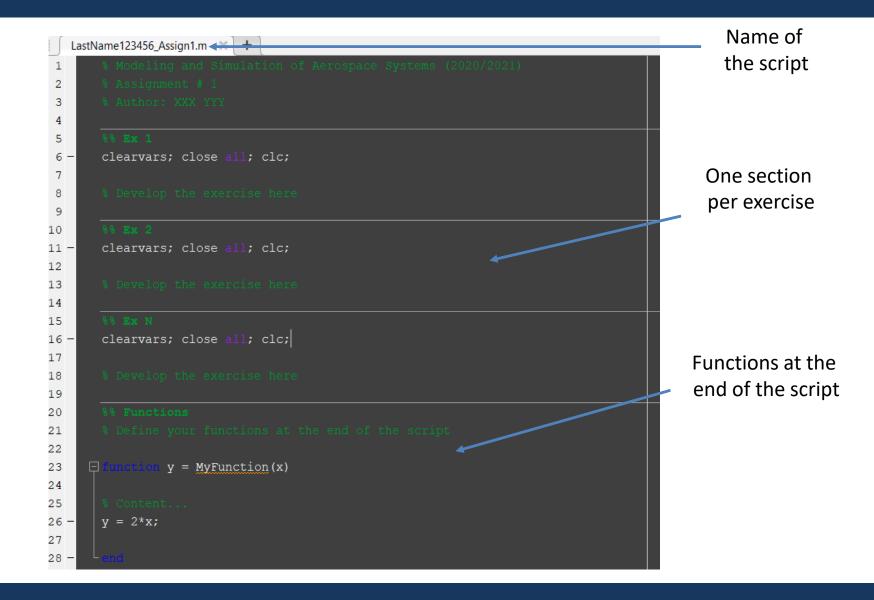
Write your answer here

- . Develop the exercises in one Matlab script; name the file lastname123456 Assign1.m
- Organize the script in sections, one for each exercise; use local functions if needed.
- Download the PDF from the Main menu.
- Create a single .zip file containing both the report in PDF and the MATLAB file. The name shall be lastname123456_Assign1.zip.
- Red text indicates where answers are needed; be sure there is no red stuff in your report.
- In your answers, <u>be concise</u>: to the point.
- Deadline for the submission: Nov 11 2021, 23:30.
- Load the compressed file to the Assignments folder on Webeep.





Script



Exercise 1 – Impulsive guidance

Exercise 1

Let $\mathbf{x}(t) = \varphi(\mathbf{x}_0, t_0; t)$ be the flow of the geocentric two-body model. 1) Using one of Matlab's built-in integrators, implement and validate* a propagator that returns $\mathbf{x}(t)$ for given \mathbf{x}_0 , t_0 , t, and μ . 2) Given the pairs $\{\mathbf{r}_1, \mathbf{r}_2\}$ and $\{t_1, t_2\}$, develop a solver that finds \mathbf{v}_1 such that $\mathbf{r}(t_2) = \mathbf{r}_2$, where $(\mathbf{r}(t), \mathbf{v}(t))^{\top} = \varphi((\mathbf{r}_1, \mathbf{v}_1)^{\top}, t_1; t_2)$ (Lambert's problem). To compute the derivatives of the shooting function, use either a) finite differences or b) the state transition matrix $\Phi = \mathrm{d}\varphi/\mathrm{d}\mathbf{x}_0$. Validate the algorithms against the classic Lambert solver. 3) Using the propagator of point 1) in the heliocentric case, and reading the motion of the Earth and Mars from SPICE, solve the shooting problem

$$\min_{\mathbf{x}_{1},t_{1},t_{2}} \Delta v \quad \text{s.t.} \begin{cases}
\mathbf{r}_{1} = \mathbf{r}_{E}(t_{1}) \\
\mathbf{r}(t_{2}) = \mathbf{r}_{M}(t_{2}) \\
t_{1}^{L} \leq t_{1} \leq t_{1}^{U} \\
t_{2}^{L} \leq t_{2} \leq t_{2}^{U} \\
t_{2} \geq t_{1}
\end{cases} \tag{1}$$

where $\Delta v = \Delta v_1 + \Delta v_2$, $\Delta \mathbf{v}_1 = \mathbf{v}_1 - \mathbf{v}_E(t_1)$, $\Delta \mathbf{v}_2 = \mathbf{v}(t_2) - \mathbf{v}_M(t_2)$. $\mathbf{x}_1 = (\mathbf{r}_1, \mathbf{v}_1)^{\top}$, and $(\mathbf{r}(t), \mathbf{v}(t))^{\top} = \varphi(\mathbf{x}_1, t_1; t_2)$. Define lower and upper bounds, and make sure to solve the problem stated in Eq. (1) for different initial guesses.

^{*}It can be done by taking \mathbf{x}_0 at the periapsis of elliptic orbits and t_f equal to their period; get μ from SPICE.

Exercise 3 – Continuous guidance

Exercise 3

A spacecraft equipped with low-thrust propulsion moves in the geocentric two-body problem. The spacecraft has to accomplish a time-optimal transfer from the initial point $(\mathbf{r}_0, \mathbf{v}_0, m_0)^{\top}$ to the final point $(\mathbf{r}_f, \mathbf{v}_f)^{\top}$. 1) Write down the spacecraft equations of motion, the costate dynamics, and the zero-finding problem for the unknowns $\{\lambda_0, t_f\}$. 2) Solve the problem with the following data: $\mathbf{r}_0 = (0, -29597.43, 0)^{\top}$ km, $\mathbf{v}_0 = (1.8349, 0.0002, 3.1783)^{\top}$ km/s, $m_0 = 735$ kg; $\mathbf{r}_f = (0, -29617.43, 0)^{\top}$ km, $\mathbf{v}_f = (1.8371, 0.0002, 3.1755)^{\top}$ km/s; $T_{\text{max}} = 100$ mN, $I_{sp} = 3000$ s. 3) Optional: solve the problem in 2) for several values of T_{max} and plot $t_f(T_{\text{max}})$.