Assignment 01 - Guidance

Spacecraft Guidance and Navigation

AY 2021-2022

POLITECNICO MILANO 1863

Laboratory sessions - Assignments

Goals ⇨

- Contribute towards the final grade
- Relief the workload of the project by spreading it
- Assets for MSc thesis: Matlab, SPICE, and Latex

Assignment evaluation

Weights	Fail	Poor	Good	Excellent
Report	• Major mismatches w.r.t. the assignment • Report awfully written	• Minor mismatches w.r.t. the assignment • Figures and tables not clear • English is poor	• Answers lengthy, but correct • Figures and tables clear • Good English	• Answers concise and clear • Figures and table clear and meaningful • Good English
Code	• Code does not run • Major algorithmic errors • Code not complete	• Minor algorithmic errors • Code not documented • Code takes unnecessary long to run	• Code runs smoothly • Code is fairly documented • Computational efficiency improvable	• Code runs smoothly • Code is well documented • Care is taken to account computational efficiency

Laboratory sessions

- Laboratory sessions **will not be recorded**
  - we are here to give you answers while you are working
- When writing emails:
  - Prepend the following to the subject: **[SGN – Assignment1]**
  - Do **not** attach/send code asking for debugging

Timetable		Room
20 Oct	16.15-18.15	BL28.12 (+ Morselli virtual room)
22 Oct	10.15-13.15	L11 (+ Morselli virtual room)
26 Oct	14.15-16.15	BL27.14 (+ Morselli virtual room)
27 Oct	16.15-18.15	BL28.12 (+ Morselli virtual room)
29 Oct	10.15-13.15	L11 (+ Morselli virtual room)
02 Nov	14.15-16.15	BL27.15 (Lecture)
03 Nov	16.15-18.15	BL28.12 (Lecture)
05 Nov	10.15-13.15	L11 (Lecture)
11 Nov	**23.30 CET**	**Assignment 1 deadline**

Delivery of assignments

Assigment will be delivered through **WeBeep**:

DEADLINE ⇨ Nov 11, 2021, 23:30:00 CET

1) Click on the link to **load Assignment 1** in your Overleaf

   https://bit.ly/SGN_Assignment1_21

   NOT ACTIVE YET

2) **Fill the report** and be sure it is compiled properly
3) **Download the PDF** and merge it in a **zipped file** with MATLAB code.
   Rename it `lastname123456_Assign1.zip`
4) **Submit the compressed file** by uploading it on Webeep

Assignment 01 - Topics

4 Exercises

- Impulsive guidance
- Ex 2: coming soon
- Continuous guidance
- Ex 4: coming soon

Suggested MATLAB Version: R2021b

Report

1 Impulsive guidance

Exercise 1

Let $\mathbf{x}(t) = \varphi(\mathbf{x}_0, t_0; t)$ be the flow of the geocentric two-body model. 1) Using one of Matlab's built-in integrators, implement and validate[*] a propagator that returns $\mathbf{x}(t)$ for given $\mathbf{x}_0$, t_0, t, and μ. 2) Given the pairs $\{\mathbf{r}_1, \mathbf{r}_2\}$ and $\{t_1, t_2\}$, develop a solver that finds $\mathbf{v}_1$ such that $\mathbf{r}(t_2) = \mathbf{r}_2$, where $(\mathbf{r}(t), \mathbf{v}(t))^\top = \varphi((\mathbf{r}_1, \mathbf{v}_1)^\top, t_1; t_2)$ (Lambert's problem). To compute the derivatives of the shooting function, use either a) finite differences or b) the state transition matrix $\Phi = \mathrm{d}\varphi/\mathrm{d}\mathbf{x}_0$. Validate the algorithms against the classic Lambert solver. 3) Using the propagator of point 1) in the heliocentric case, and reading the motion of the Earth and Mars from SPICE, solve the shooting problem

$$\min_{\mathbf{x}_1, t_1, t_2} \Delta v \quad \text{s.t.} \begin{cases} \mathbf{r}_1 = \mathbf{r}_E(t_1) \\ \mathbf{r}(t_2) = \mathbf{r}_M(t_2) \\ t_1^L \le t_1 \le t_1^U \\ t_2^L \le t_2 \le t_2^U \\ t_2 \ge t_1 \end{cases} \tag{1}$$

where $\Delta v = \Delta v_1 + \Delta v_2$, $\Delta \mathbf{v}_1 = \mathbf{v}_1 - \mathbf{v}_E(t_1)$, $\Delta \mathbf{v}_2 = \mathbf{v}(t_2) - \mathbf{v}_M(t_2)$. $\mathbf{x}_1 = (\mathbf{r}_1, \mathbf{v}_1)^\top$, and $(\mathbf{r}(t), \mathbf{v}(t))^\top = \varphi(\mathbf{x}_1, t_1; t_2)$. Define lower and upper bounds, and make sure to solve the problem stated in Eq. (1) for different initial guesses.

Write your answer here

- Develop the exercises in one Matlab script; name the file `lastname123456_Assign1.m`
- Organize the script in sections, one for each exercise; use local functions if needed.
- Download the PDF from the Main menu.
- Create a single .zip file containing both the report in PDF and the MATLAB file. The name shall be `lastname123456_Assign1.zip`.
- Red text indicates where answers are needed; be sure there is no red stuff in your report.
- In your answers, be concise: to the point.
- **Deadline for the submission: Nov 11 2021, 23:30.**
- **Load the compressed file to the Assignments folder on Webeep.**

Exercise

Answer here

Script

LastName123456_Assign1.m

```matlab
% Modeling and Simulation of Aerospace Systems (2020/2021)
% Assignment # 1
% Author: XXX YYY

%% Ex 1
clearvars; close all; clc;

% Develop the exercise here

%% Ex 2
clearvars; close all; clc;

% Develop the exercise here

%% Ex N
clearvars; close all; clc;

% Develop the exercise here

%% Functions
% Define your functions at the end of the script

function y = MyFunction(x)

% Content...
y = 2*x;

end
```

Name of the script

One section per exercise

Functions at the end of the script

Exercise 1

Let $\mathbf{x}(t) = \varphi(\mathbf{x}_0, t_0; t)$ be the flow of the geocentric two-body model. 1) Using one of Matlab's built-in integrators, implement and validate* a propagator that returns $\mathbf{x}(t)$ for given $\mathbf{x}_0$, t_0, t, and μ. 2) Given the pairs $\{\mathbf{r}_1, \mathbf{r}_2\}$ and $\{t_1, t_2\}$, develop a solver that finds $\mathbf{v}_1$ such that $\mathbf{r}(t_2) = \mathbf{r}_2$, where $(\mathbf{r}(t), \mathbf{v}(t))^\top = \varphi((\mathbf{r}_1, \mathbf{v}_1)^\top, t_1; t_2)$ (Lambert's problem). To compute the derivatives of the shooting function, use either a) finite differences or b) the state transition matrix $\Phi = \mathrm{d}\varphi/\mathrm{d}\mathbf{x}_0$. Validate the algorithms against the classic Lambert solver. 3) Using the propagator of point 1) in the heliocentric case, and reading the motion of the Earth and Mars from SPICE, solve the shooting problem

$$\min_{\mathbf{x}_1, t_1, t_2} \Delta v \quad \text{s.t.} \begin{cases} \mathbf{r}_1 = \mathbf{r}_E(t_1) \\ \mathbf{r}(t_2) = \mathbf{r}_M(t_2) \\ t_1^L \le t_1 \le t_1^U \\ t_2^L \le t_2 \le t_2^U \\ t_2 \ge t_1 \end{cases} \tag{1}$$

where $\Delta v = \Delta v_1 + \Delta v_2$, $\boldsymbol{\Delta}\mathbf{v}_1 = \mathbf{v}_1 - \mathbf{v}_E(t_1)$, $\boldsymbol{\Delta}\mathbf{v}_2 = \mathbf{v}(t_2) - \mathbf{v}_M(t_2)$. $\mathbf{x}_1 = (\mathbf{r}_1, \mathbf{v}_1)^\top$, and $(\mathbf{r}(t), \mathbf{v}(t))^\top = \varphi(\mathbf{x}_1, t_1; t_2)$. Define lower and upper bounds, and make sure to solve the problem stated in Eq. (1) for different initial guesses.

*It can be done by taking $\mathbf{x}_0$ at the periapsis of elliptic orbits and t_f equal to their period; get μ from SPICE.

Exercise 3

A spacecraft equipped with low-thrust propulsion moves in the geocentric two-body problem. The spacecraft has to accomplish a *time-optimal* transfer from the initial point $(\mathbf{r}_0, \mathbf{v}_0, m_0)^\top$ to the final point $(\mathbf{r}_f, \mathbf{v}_f)^\top$. 1) Write down the spacecraft equations of motion, the costate dynamics, and the zero-finding problem for the unknowns $\{\boldsymbol{\lambda}_0, t_f\}$. 2) Solve the problem with the following data: $\mathbf{r}_0 = (0, -29597.43, 0)^\top$ km, $\mathbf{v}_0 = (1.8349, 0.0002, 3.1783)^\top$ km/s, $m_0 =$ 735 kg; $\mathbf{r}_f = (0, -29617.43, 0)^\top$ km, $\mathbf{v}_f = (1.8371, 0.0002, 3.1755)^\top$ km/s; $T_{\max} = 100$ mN, $I_{sp} = 3000$ s. 3) Optional: solve the problem in 2) for several values of $T_{\max}$ and plot $t_f(T_{\max})$.