Spacecraft Attitude Dynamics

Prof. Franco Bernelli

Magnetic disturbance torque

Task 1: Implement approximate orbit models

Define your spacecraft orbit with respect to the Earth in the body frame

$$r_N = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = R \begin{bmatrix} \cos\theta \\ \sin\theta\cos i \\ \sin\theta\sin i \end{bmatrix}$$

$$R = \frac{a(1-e^2)}{1+e\cos\theta}$$

$$\dot{\theta} = \frac{n(1+e\cos\theta)^2}{(1-e^2)^{3/2}},$$

$n - Earth$

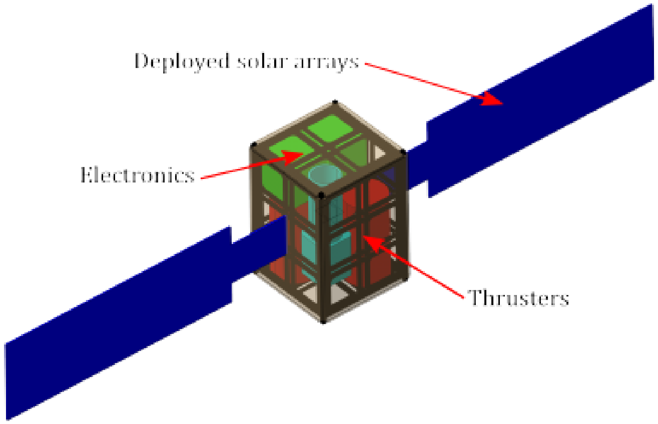
$$\hat{r}_B = A_{B/N}\hat{r}_N$$

Task 2: Update the gravity gradient torque with your new elliptic orbit for 12 U Cubesat with magnetic torque

$$I_x\dot{\omega}_x + (I_z - I_y)\omega_z\omega_y = \frac{3Gm_t}{R^3}(I_z - I_y)c_3c_2$$
$$I_y\dot{\omega}_y + (I_x - I_z)\omega_x\omega_z = \frac{3Gm_t}{R^3}(I_x - I_z)c_1c_3$$
$$I_z\dot{\omega}_z + (I_y - I_x)\omega_y\omega_x = \frac{3Gm_t}{R^3}(I_y - I_x)c_2c_1$$

$$J_{depl} = \begin{bmatrix} 100.9 & 0 & 0 \\ 0 & 25.1 & 0 \\ 0 & 0 & 91.6 \end{bmatrix} \cdot 10^{-2} [Kgm^2].$$

$$\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = A_{B/L} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$



$$A_{L/N} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos i & \sin i \\ 0 & -\sin i & \cos i \end{bmatrix}$$

Task 2: Simple magnetic disturbance model

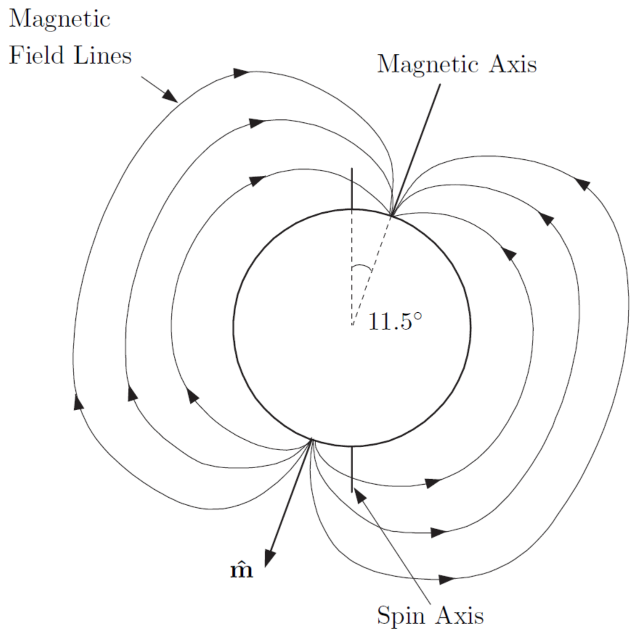
1. Compute magnetic field in the inertial frame

$$\underline{b}_N = \frac{R^3 H_0}{r^3}\left[3(\underline{\hat{m}} \cdot \underline{\hat{r}})\underline{\hat{r}} - \underline{\hat{m}}\right]$$

$$H_0 = \left((g_1^0)^2 + (g_1^1)^2 + (h_1^1)^2\right)^{1/2}$$

$$\hat{m} = \begin{bmatrix} \sin 11.5^\circ \cos \omega_\oplus t \\ \sin 11.5^\circ \sin \omega_\oplus t \\ \cos 11.5^\circ \end{bmatrix}$$

		IGRF 1995		IGRF 2000	
n	m	g_n^m	h_n^m	g_n^m	h_n^m
1	0	-29682	-	-29615	-
1	1	-1789	5318	-1728	5186
2	0	-2197	-	-2267	-
2	1	3074	-2356	3072	-2478
2	2	1685	-425	1672	-458
3	0	1329	-	1341	-
3	1	-2268	-263	-2290	-227
3	2	1249	302	1253	296
3	3	769	-406	715	-492
4	0	941	-	935	-
4	1	782	262	787	272
4	2	291	-232	251	-232
4	3	-421	98	-405	119
4	4	116	-301	110	-304



2. Compute magnetic field in the body fixed frame

$$\underline{b}_B = A_{B/N}\underline{b}_N$$

3. Compute the parasitic magnetic torque

$$\underline{M} = \underline{m} \times \underline{b}$$

$$\underline{m} = [0.01 \quad 0.05 \quad 0.01]^T Am^2$$

Note IGRF data is in nT