POLITECNICO
MILANO 1863

Spacecraft Attitude Dynamics

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Quaternions and the Gravity Gradient perturbation

Task 1: Simulate the dynamics and kinematics w.r.t the inertial frame using the quaternions

Dynamics

$$\dot{\omega}_1 = \frac{I_2 - I_3}{I_1}\omega_2\omega_3$$
$$\dot{\omega}_2 = \frac{I_3 - I_1}{I_2}\omega_1\omega_3$$
$$\dot{\omega}_3 = \frac{I_1 - I_2}{I_3}\omega_2\omega_1$$

Kinematics

$$\begin{bmatrix}\dot{q}_1\\ \dot{q}_2\\ \dot{q}_3\\ \dot{q}_4\end{bmatrix} = \frac{1}{2}\begin{pmatrix} 0 & \omega_3 & -\omega_2 & \omega_1 \\ -\omega_3 & 0 & \omega_1 & \omega_2 \\ \omega_2 & -\omega_1 & 0 & \omega_3 \\ -\omega_1 & -\omega_2 & -\omega_3 & 0 \end{pmatrix}\begin{bmatrix} q_1\\ q_2\\ q_3\\ q_4\end{bmatrix}$$

$$\begin{bmatrix} q_1(0)\\ q_2(0)\\ q_3(0)\\ q_4(0)\end{bmatrix} = \begin{bmatrix}0\\0\\0\\1\end{bmatrix}$$

$$A_{B/N} = \begin{bmatrix} q_1^2 - q_2^2 - q_3^2 + q_4^2 & 2(q_1q_2 + q_3q_4) & 2(q_1q_3 - q_2q_4) \\ 2(q_1q_2 - q_3q_4) & -q_1^2 + q_2^2 - q_3^2 + q_4^2 & 2(q_2q_3 + q_1q_4) \\ 2(q_1q_3 + q_2q_4) & 2(q_2q_3 - q_1q_4) & -q_1^2 - q_2^2 + q_3^2 + q_4^2 \end{bmatrix}$$

Task 2 – Compute the relative attitude between the spacecraft body frame and a moving reference frame

$$I_x\dot{\omega}_x + (I_z - I_y)\omega_z\omega_y = 0$$
$$I_y\dot{\omega}_y + (I_x - I_z)\omega_x\omega_z = 0$$
$$I_z\dot{\omega}_z + (I_y - I_x)\omega_y\omega_x = 0$$

$$\frac{dA_{B/N}}{dt} = -[\omega_{B/N}]^{\wedge} A_{B/N}$$

$$\omega_x(0) = 0, \omega_y(0) = 0, \omega_z(0) = n \;\; rad/\sec$$

$$\omega_x(0) = 1e-06 rad/\sec, \omega_y(0) = 1e-06 rad/\sec, \omega_z(0) = n rad/\sec$$

where

$$I_x = 0.01 kgm^2, I_y = 0.05 kgm^2, I_z = 0.09 kgm^2$$

$$I_x = 0.05 kgm^2, I_y = 0.09 kgm^2, I_z = 0.01 kgm^2$$

Reference attitude

$$A_{L/N} = \begin{bmatrix} \cos n\,t & \sin n\,t & 0 \\ -\sin n\,t & \cos n\,t & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \omega_{L/N} = \begin{bmatrix} 0 \\ 0 \\ n \end{bmatrix}$$

$$n^2 = \frac{Gm_t}{R^3} \qquad \left[\frac{N\,\frac{m^{\cancel{2}}}{kg^{\cancel{2}}}\cdot \cancel{kg}}{m^{\cancel{3}}}\right]$$

$$= \left[\frac{N}{kg\cdot m} = \frac{kg\,\frac{\cancel{m}}{s^2}}{\cancel{kg\cdot m}} = \frac{1}{s^2}\right]$$

Attitude error

$$A_{B/L} = A_{B/N}A_{L/N}^T$$

$$\underline{\omega}_{B/L} = \underline{\omega}_B - A_{B/L}\underline{\omega}_L$$

Task 3: Repeat Task 2 with gravity gradient included

$$I_x\dot{\omega}_x + \left(I_z - I_y\right)\omega_z\omega_y = \frac{3Gm_t}{R^3}\left(I_z - I_y\right)c_3c_2$$

$$I_y\dot{\omega}_y + (I_x - I_z)\omega_x\omega_z = \frac{3Gm_t}{R^3}(I_x - I_z)c_1c_3$$

$$I_z\dot{\omega}_z + \left(I_y - I_x\right)\omega_y\omega_x = \frac{3Gm_t}{R^3}\left(I_y - I_x\right)c_2c_1$$

$$\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = A_{B/L}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Time permitting: Validate some points in the stability diagram in the notes