POLITECNICO
MILANO 1863

Spacecraft Attitude Dynamics

Prof. Franco Bernelli

Lab 3 - Attitude Kinematics

Task 1: Simulate the pointing error of a spin-stabilized spacecraft

$$I_x = 0.07kgm^2, I_y = 0.0504kgm^2, I_z = 0.0109kgm^2$$

$$\dot{\omega}_x = \frac{\left(I_y - I_z\right)}{I_x}\omega_z\omega_y$$
$$\dot{\omega}_y = \frac{\left(I_z - I_x\right)}{I_y}\omega_x\omega_z$$
$$\dot{\omega}_z = \frac{\left(I_x - I_y\right)}{I_z}\omega_y\omega_x$$

$$\omega_x(0) = Crad/\sec, \omega_y(0) = 1e - 2rad/\sec, \omega_z(0) = 1\text{e}{-}2rad/\text{sec}$$
$$\Gamma(0) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T$$
$$2\pi \geq C \geq 0.1$$

Task 2: Simulate the dynamics and kinematics using the DCM using standard integration

Dynamics

$$\dot{\omega}_1 = \frac{I_2 - I_3}{I_1}\omega_2\omega_3$$
$$\dot{\omega}_2 = \frac{I_3 - I_1}{I_2}\omega_1\omega_3$$
$$\dot{\omega}_3 = \frac{I_1 - I_2}{I_3}\omega_2\omega_1$$

Kinematics

$$\frac{dA(t)}{dt} = -[\omega]^{\wedge}A(t)$$

$$[\omega^{\wedge}] = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}$$

Dynamics → Kinematics

Test the orthonormaility of the solution

Task 3: Orthonormalize the matrix at each sampling period

Iterative formulas for orthonormalization

$A_{k+1}(t) = A_k(t)*3/2 - A_k(t)*A_k^T(t)*A_k(t)/2$

converges rapidly, with increasing k, to the exact value of A. In a first order approximation it is possible to adopt a single step iteration

$A(t) = A_0(t)*3/2 - A_0(t)*A_0^T(t)*A_0(t)/2$

Test the orthonormaility of the solution