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Orbital Mechanics

Module 2: Orbit representation

Academic year 2020/21

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ORBIT REPRESENTATION

Cartesian and Keplerian elements

Basic orbit representation

- The most common representations for the state of an orbiting body are the **Cartesian coordinates** and the **Keplerian elements** [1]:
 - **Cartesian coordinates** $\mathbf{s} = [\mathbf{r}, \mathbf{v}]$. Position \mathbf{r} and velocity \mathbf{v} at a given time t .
 - **Keplerian elements** $\boldsymbol{\alpha} = [a, e, i, \Omega, \omega, M]$. Based on a geometrical description of the conic:
 - a and e give the geometry of the planar conic.
 - i , Ω , and ω give the orientation of the conic in space (Euler angles).
 - M (or true anomaly f) gives the object position in the orbit. M and f are related through Kepler's equation.
- It is possible to convert between both [1]:
 - You will need `kep2car` and `car2kep` functions for the labs.
 - Be careful with singularities at $e = 0$ and $i = 0$ deg.
 - Test your functions converting a set of element back and forth and checking if you recover the original value: $\mathbf{s} \rightarrow \boldsymbol{\alpha} \rightarrow \tilde{\mathbf{s}} = \mathbf{s}$

[1] Curtis, H. D.. *Orbital mechanics for engineering students*, Butterworth-Heinemann , 2014

Cartesian and Keplerian elements

Osculating orbit and singularities

- Strictly, Keplerian elements are defined for Keplerian orbits (2BP)
 - $[a, e, i, \Omega, \omega]$ constant, M changes in time. That is, the orbit remains constant and the body moves along it.
 - In the presence of perturbations the orbit changes slowly in time.
Osculating orbit: Keplerian orbit that the object would have around the central body at a particular time instant if perturbations were absent.
Osculating elements: Keplerian elements of the osculating orbit. They evolve in time.
- Keplerian elements have 2 singularities:
 - **Zero inclination:** The line of nodes is not defined. Ω can take an arbitrary value, the angle of the apse line with the X axis is $\Omega + \omega$. $\rightarrow \omega = \Omega + \omega$
 - **Circular orbit:** The line of apses is not defined. ω can take an arbitrary value, the angular position of the body with the line of nodes is $\omega + f$.

$$\omega = \emptyset \rightarrow$$

Ephemeris

Locating objects in space

- A table of the coordinates of celestial bodies as a function of time is called an **ephemeris** [1].
 - Can be provided in the form of a look-up table, polynomial fittings, TLEs, etc.
 - Computed from astronomical observations and precise orbit propagation.
 - Importance of the **reference time** (epoch of the ephemeris).
- We will make an overview of one of the most common sources of ephemeris: the **Simplified General Perturbation (SGP)** models and the **Two-Line Elements (TLE)**.
- Other ephemeris will be provided in future labs when needed.

[1] Curtis, H. D.. *Orbital mechanics for engineering students*, Butterworth-Heinemann , 2014

Simplified General Perturbation models

Overview

- **Simplified General Perturbation (SGP) models:**
 - Analytical methods for propagating the orbital state of a satellite or debris, **from some ad-hoc mean elements**.
 - They include expressions for the **secular, long-periodic and short-periodic evolutions**, depending on the force models included.
- The US Air Force began development of their SGP model series in the 1960s:
 - Five different models: SGP, SGP4/SDP4, and SGP8/SDP8.
 - SGP/SGP4/SPG8 for close-Earth objects (period <225 minutes), SDP4/SDP8 for deep-space objects (period \geq 225 minutes).
 - Most used one is SGP4/SDP4.
 - Mean elements distributed as **Two-Line Element (TLE) sets**.
 - SGP4/SDP4 was documented in Spacetrack Report #3, to promote compatibility in the operational community.

Simplified General Perturbation models

Two-line elements

- NORAD (North American Aerospace Defense Command) provides the TLEs for SGP4/SDP4.
 - TLEs for SGP/SGP8/SDP8 can be derived from them (some fields are included only for this purpose).
- TLEs are computed from observations and orbit determination procedures.
 - Exact data and models used are classified information.
 - A single TLE allows to propagate the orbital state for a **limited time range** around the TLE epoch (accuracy degradation).
- Data is stored in two different lines (hence the name):
 - Originally in 80-column punch cards.
 - Currently in 70-column ASCII files.
 - They are made publicly available through Space-Track, but not all objects can be queried (e.g. classified satellites are excluded).

Simplified General Perturbation models

Two-line elements – Line 1

1 20580U 90037B 19341.79883884 .00000350 00000-0 10277-4 0 9996
2 20580 28.4681 153.8207 0002675 174.2378 205.9686 15.09309002426911

Column	Description
01	Line number
03-07	Satellite number
08	Classification (U=Unclassified)
10-11	International Designator (last two digits of launch year)
12-14	International Designator (launch number of the year)
15-17	International Designator (piece of the launch)
19-20	Epoch year (last two digits of year)
21-32	Epoch (day of the year and fraction portion of the day)
34-43	First time derivative of the mean motion
45-52	Second Time Derivative of Mean Motion (decimal point assumed)
54-61	BSTAR drag term (decimal point assumed)
63	Ephemeris type
65-68	Element number
69	Checksum (Modulo 10)

Simplified General Perturbation models

Two-line elements – Line 2

1	20580U	90037B	19341.79883884	.00000350	00000-0	10277-4	0	9996
2	20580	28.4681	153.8207	0002675	174.2378	205.9686	15.09309002	426911

Column	Description
01	Line Number of Element Data
03-07	Satellite Number
09-16	Inclination [deg]
18-25	Right Ascension of the Ascending Node [deg]
27-33	Eccentricity (decimal point assumed)
35-42	Argument of Perigee [deg]
44-51	Mean Anomaly [deg]
53-63	Mean Motion [rev/day]
64-68	Revolution number at epoch [rev]
69	Checksum (Modulo 10)

CelesTrak

<https://celesttrak.com/>



New ways to get GP data now available! Up until now, we've used the TLE format to ingest data into SGP4. With the growth of the catalog soon to exceed the current range of 5-digit catalog numbers, you may be wondering how we will handle that. If so, be sure to check out our [prototype Current Data page](#) and read the article titled "A New Way to Obtain GP Data (aka TLEs)." More to follow on Twitter @TSKelso.

NEW CelesTrak Orbit Visualization!
Now even more capable! Perpetually propagate the full public catalog on the fly at up to 30 fps!
[Launch Orbit Visualization](#)

[Follow CelesTrak on Twitter @TSKelso](#)

NORAD Two-Line Element Sets		
Current Data	Historical Archives	Documentation
Software Repository		
Computers & Satellites Columns		
"Frequently Asked Questions: Two-Line Element Set Format"		
"More Frequently Asked Questions"		
Special Event Coverage		
ORS-3 Launch of 2013 Nov 20		
Dnepr Launch of 2013 Nov 21		
Breeze-M Rocket Body (38746) Breakup		
Indium 33/Cosmos 2251 Collision		
USA 193 Post-Shutdown Analysis		
Chinese ASAT Test		
2007 Orbis Events		
Online Satellite Catalog (SATCAT)		
Search the SATCAT	SATCAT Boxscore	
SOCRATES		
Satellite Orbital Conjunction Reports		
Assessing threatening Encounters in Space		
GPS Data		
Status Messages	NANUS	
SEM Almanacs	Yuma Almanacs	
Space Data		
Earth Orientation Parameters	Space Weather	
About the Webmaster		

Dr. T.S. Kelso [TS.Kelso@celesttrak.com]
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Current system time: 2020 Oct 09 10:08:58 UTC
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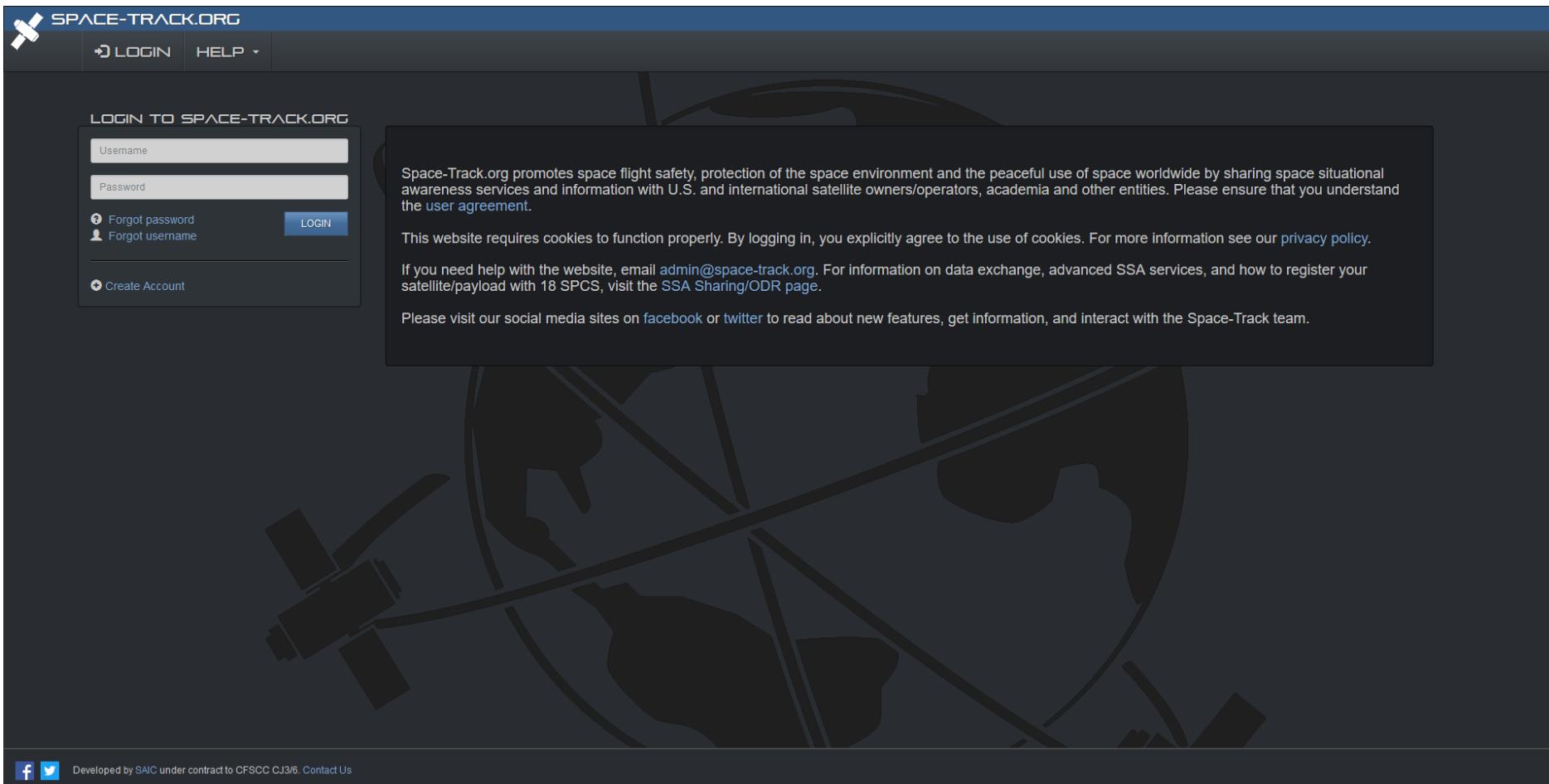
CelesTrak

<https://celestak.com/>

- Created and maintained by **Dr. T.S. Kelso**.
 - Originally introduced in 1985 to make TLEs electronically available (NASA was providing them as printed bulletins).
- Contains a lot of **useful information**:
 - Current TLEs for non-classified satellites, organized by types of satellites/missions.
 - Historical TLEs archive for selected objects.
 - Satellite catalogue (SATCAT).
 - Documentation and Q&A (including a detailed description of TLEs).
 - Software.
 - Others: GPS data, space weather data, etc.
 - Orbit visualization tool.

Space-Track

<https://www.space-track.org>



The image shows the Space-Track.org login page. At the top left is the logo 'SPACE-TRACK.ORG'. Below it are 'LOGIN' and 'HELP' buttons. A large central box contains the 'LOGIN TO SPACE-TRACK.ORG' form with fields for 'Username' and 'Password', and links for 'Forgot password', 'Forgot username', and 'Create Account'. To the right of the form is a text box containing information about the website's purpose, user agreement, cookie policy, help email, and social media links. The background features a dark image of Earth with a satellite in orbit.

SPACE-TRACK.ORG

LOGIN HELP

LOGIN TO SPACE-TRACK.ORG

Username

Password

Forgot password
Forgot username

Create Account

LOGIN

Space-Track.org promotes space flight safety, protection of the space environment and the peaceful use of space worldwide by sharing space situational awareness services and information with U.S. and international satellite owners/operators, academia and other entities. Please ensure that you understand the user agreement.

This website requires cookies to function properly. By logging in, you explicitly agree to the use of cookies. For more information see our [privacy policy](#).

If you need help with the website, email admin@space-track.org. For information on data exchange, advanced SSA services, and how to register your satellite/payload with 18 SPCS, visit the [SSA Sharing/ODR page](#).

Please visit our social media sites on [facebook](#) or [twitter](#) to read about new features, get information, and interact with the Space-Track team.

f t Developed by SAIC under contract to CFSCC CJ3/6. Contact Us

Space-Track

<https://www.space-track.org>

- **TLE information published by USSPACECOM/NORAD:**
 - Objective: *promoting a safe, stable, sustainable, and secure space environment through Space Situational Awareness information sharing.*
 - Includes a Windows/Linux implementation of SGP4.
 - API for automatic queries.
 - Information about decay/reentry of satellites.
 - **Requires an account.**

Space-Track

SATCAT

- Satellite catalog can be filtered, ordered and queried for different fields.

The screenshot shows the Space-Track.org website's SATCAT page. The top navigation bar includes links for HOME, HELP, and various search functions like Box Score, Decay/Reentry, Query Builder, Favorites, ELSET Search, Recent ELSETS, SSR, and Conjunctions. The main content area is titled "SATELLITE CATALOG". It displays a table of 10 entries from a total of 46,534. The columns represent satellite identifiers, names, launch years, types, countries, launch sites, decay dates, orbital periods, inclinations, apogees, perigees, RCS values, and latest ELSETs. The data includes historical satellites like SPUTNIK 1 and 2, EXPLORER 1, and VANGUARD 1. The table has a header row with sorting icons and a footer row with column headers. Navigation buttons at the bottom allow users to page through the results.

NORAD CAT ID	SATNAME	INTLDES	TYPE	COUNTRY	LAUNCH	SITE	DECAY	PERIOD	INCL	APOGEE	PERIGEE	RCS	LATEST ELSET
1	SL-1 R/B	1957-001A	ROCKET BODY	CIS	1957-10-04	TTMTR	1957-12-01	96.19	65.10	938	214	LARGE	TLE OMM
2	SPUTNIK 1	1957-001B	PAYOUT	CIS	1957-10-04	TTMTR	1958-01-03	96.10	65.00	1080	64		TLE OMM
3	SPUTNIK 2	1957-002A	PAYOUT	CIS	1957-11-03	TTMTR	1958-04-14	103.74	65.33	1659	211	SMALL	TLE OMM
4	EXPLORER 1	1958-001A	PAYOUT	US	1958-02-01	AFETR	1970-03-31	88.48	33.15	215	183		TLE OMM
5	VANGUARD 1	1958-002B	PAYOUT	US	1958-03-17	AFETR		132.73	34.24	3832	651	MEDIUM	TLE OMM
6	EXPLORER 3	1958-003A	PAYOUT	US	1958-03-26	AFETR	1958-06-28	103.60	33.50	1739	117		TLE OMM
7	SL-1 R/B	1958-004A	ROCKET BODY	CIS	1958-05-15	TTMTR	1958-12-03	102.74	65.14	1571	206		TLE OMM
8	SPUTNIK 3	1958-004B	PAYOUT	CIS	1958-05-15	TTMTR	1960-04-06	88.43	65.06	255	139	LARGE	TLE OMM
9	EXPLORER 4	1958-005A	PAYOUT	US	1958-07-26	AFETR	1959-10-23	92.81	50.25	585	239		TLE OMM
10	SCORE	1958-006A	PAYOUT	US	1958-12-18	AFETR	1959-01-21	98.21	32.29	1187	159		TLE OMM

Space-Track

ELSET Search

- Search of historical element sets (TLEs, 3LEs, Keplerian, ...) for given satellites and epoch ranges.

The screenshot shows the Space-Track.org website's ELSET Search feature. The top navigation bar includes links for HOME, HELP, and various search functions like Welcome, Box Score, SATCAT, Decay/Reentry, Query Builder, Favorites, and Recent ELSETS. The 'ELSET Search' button is highlighted. The main search interface is titled 'HISTORICAL ELSET SEARCH'. It features a text input for 'Entries' containing '25544'. Below this are sorting options ('SORT BY') set to 'NORAD_CAT_ID' and 'Descending', and an epoch selection section ('EPOCH') with 'Latest' selected. There are also date range inputs ('From: 2020-09-27' and 'To: 2020-10-03'). A green 'LOAD DATA' button is present. At the bottom, an API endpoint is shown: https://www.space-track.org/basicspacedata/query/class/gp_history/NORAD_CAT_ID/25544/orderby/TLE_LINE1%20ASC/EPOCH/2020-09-27--2020-10-03/format/tle, followed by download links for AS TLE, AS 3LE, AS XML, AS KVN, and AS JSON. The data table below lists several TLE entries for satellite 25544U, NORAD ID 25544, from September 27, 2020, to October 3, 2020. The footer includes social media links (Facebook, Twitter), a copyright notice for SAIC, and a 'Back to top' link.

Line 1	Line 2	Line 3	Line 4	Line 5	Line 6	Line 7	Line 8	Line 9	Line 10	Line 11	Line 12
1 25544U 98067A 20271.08021030 .00000996 00000-0 26394-4 0 9998	2 25544 51.6438 204.2810 0001378 100.4432 330.8917 15.48788627247815	1 25544U 98067A 20271.22546113 .00001307 00000-0 32075-4 0 9998	2 25544 51.6438 203.5589 0001578 104.6445 57.0970 15.48789479247830	1 25544U 98067A 20271.54999066 .00002229 00000-0 48911-4 0 9992	2 25544 51.6438 201.9584 0001464 101.9257 70.4734 15.48793666247887	1 25544U 98067A 20271.54999066 +.00002229 +00000-0 +48911-4 0 9992	2 25544 051.6438 201.9584 0001464 101.9257 070.4734 15.48793666247887	1 25544U 98067A 20271.72562047 .00002484 00000-0 53573-4 0 9990	2 25544 51.6438 201.0894 0001420 101.3124 330.9868 15.48795481247914	1 25544U 98067A 20271.86033066 .00002484 00000-0 53573-4 0 9999	2 25544 51.6438 200.4230 0001420 101.8102 2.0837 15.48796272247936
1 25544U 98067A 20271.11000000 000000-0 45645-4 0 9996	1 25544U 98067A 20271.11000000 000000-0 45645-4 0 9996	1 25544U 98067A 20271.11000000 000000-0 45645-4 0 9996	1 25544U 98067A 20271.11000000 000000-0 45645-4 0 9996	1 25544U 98067A 20271.11000000 000000-0 45645-4 0 9996	1 25544U 98067A 20271.11000000 000000-0 45645-4 0 9996	1 25544U 98067A 20271.11000000 000000-0 45645-4 0 9996	1 25544U 98067A 20271.11000000 000000-0 45645-4 0 9996	1 25544U 98067A 20271.11000000 000000-0 45645-4 0 9996	1 25544U 98067A 20271.11000000 000000-0 45645-4 0 9996	1 25544U 98067A 20271.11000000 000000-0 45645-4 0 9996	1 25544U 98067A 20271.11000000 000000-0 45645-4 0 9996

NASA/JPL HORIZONS

<https://ssd.jpl.nasa.gov/horizons.cgi>

JPL HOME EARTH SOLAR SYSTEM STARS & GALAXIES TECHNOLOGY

Solar System Dynamics

BODIES ORBITS EPHEMERIDES TOOLS PHYSICAL DATA DISCOVERY FAQ SITE MAP

HORIZONS Web-Interface

This tool provides a web-based *limited* interface to [JPL's HORIZONS system](#) which can be used to generate ephemerides for solar-system bodies. Full access to [HORIZONS](#) features is available via the primary [telnet interface](#). [HORIZONS](#) system news shows recent changes and improvements. A [web-interface tutorial](#) is available to assist new users.

Current Settings

Ephemeris Type [change] : **OBSERVER**
Target Body [change] : **Mars** [499]
Observer Location [change] : **Geocentric** [500]
Time Span [change] : Start=2020-10-09, Stop=2020-11-08, Step=1 d
Table Settings [change] : *defaults*
Display/Output [change] : *default* (formatted HTML)

Special Options:

- set default ephemeris settings (preserves only the selected target body and ephemeris type)
- reset *all* settings to their *defaults* (caution: all previously stored/selected settings will be lost)
- show "batch-file" data (for use by the E-mail interface)

ABOUT SSD	CREDITS/AWARDS	PRIVACY/COPYRIGHT	GLOSSARY	LINKS
	2020-Oct-09 10:23 UT (server date/time)		Site Manager: Ryan S. Park Webmaster: Alan B. Chamberlin	

NASA/JPL HORIZONS

<https://ssd.jpl.nasa.gov/horizons.cgi>

- NASA/JPL HORIZONS is a **solar system data and ephemeris computation service**:
 - Maintained by the Solar System Dynamics Group of the Jet Propulsion Laboratory (JPL).
 - Provides access to key solar system data.
 - Computes highly accurate ephemeris for solar system objects (856810 asteroids, 3598 comets, 210 planetary satellites, 8 planets, the Sun, L1, L2, select spacecraft, and system barycenters).
 - Few Earth-orbiting objects are included in the database, but users can provide their own set of TLEs (i.e. we can use it as a SGP4 implementation).
 - Full access through telnet connection or email (submitting batch-style input files).
 - A web interface is available (limited capabilities).

NASA/JPL HORIZONS

<https://ssd.jpl.nasa.gov/horizons.cgi>

- Settings for the web interface (remember to confirm all changes before proceeding to the next setting):
 - **Ephemeris type:** Orbital Elements.
 - **Target body:** Select a satellite from the list, or provide up to 750 sets of SGP4/SDP4 TLEs in standard form.
 - **Observer Location:** Geocentric (code 500) for Earth-orbiting objects.
 - **Time span:** Initial time, final time, and step size.
 - **Table settings:** Available options depend on the Ephemeris type. Remember to choose adequate units (km and km/s), reference plane (Earth mean equator), and set output to CSV format (easier to import into Matlab or other tools).
 - **Display/Output:** Select ‘download/save’.

References

- **CelesTrak** (TLEs and other resources):
<https://celestak.com/>
- **Space-Track** (satellite/debris catalogue and TLEs):
<https://www.space-track.org/>
- **NASA/JPL's HORIZONS web-interface** (ephemeris generation):
<https://ssd.jpl.nasa.gov/horizons.cgi>
- References about SGP4/SDP4 and TLEs (accessible in CelesTrak):
 - F.R. Hoots, "Spacetrack report no. 3, models for propagation of NORAD element sets", *Department of Commerce, National Technical Information Service*, 1980.
 - D. Vallado, P. Crawford, Hujasak, R., and Kelso, T.S., "Revisiting Spacetrack Report #3," *AIAA/AAS Astrodynamics Specialist Conference*, Keystone, CO, 21-24 August 2006.



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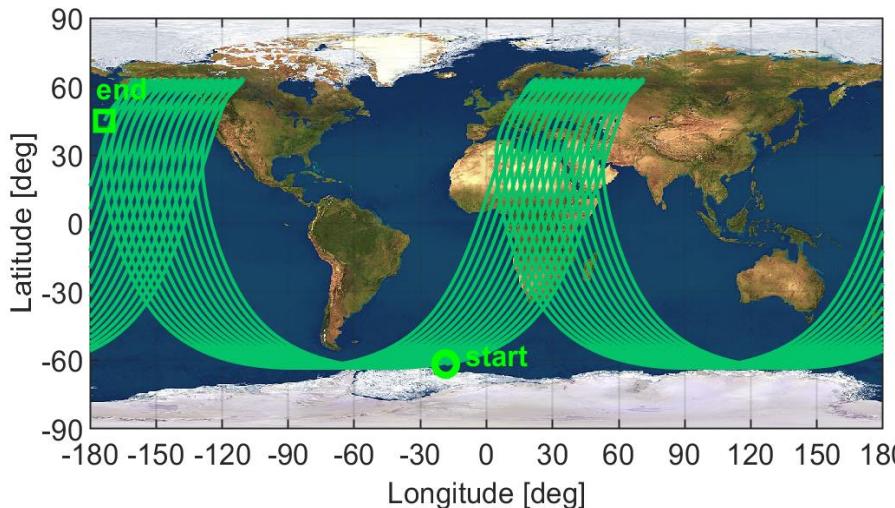
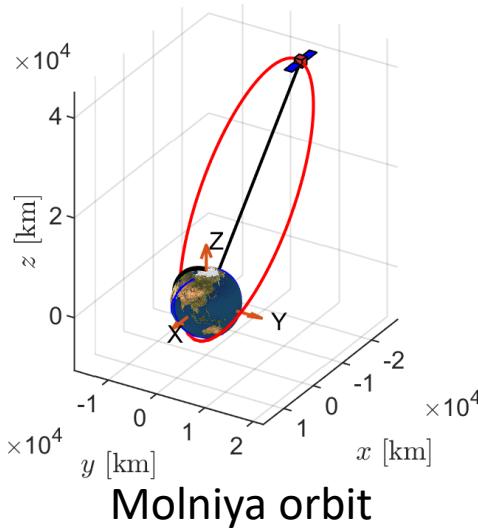
GROUND TRACK

Ground Track

Definition

Ground track: *Projection of a satellite's orbit onto the Earth's surface [1].*

- Neglecting Earth's oblateness, it can be plotted as the trace left on the planet's surface by the line connecting the centre of the Earth and the satellite as it travels its orbit.
- At each time t , the corresponding ground track point is located by its **latitude** ϕ and **longitude** λ relative to the **rotating Earth**.

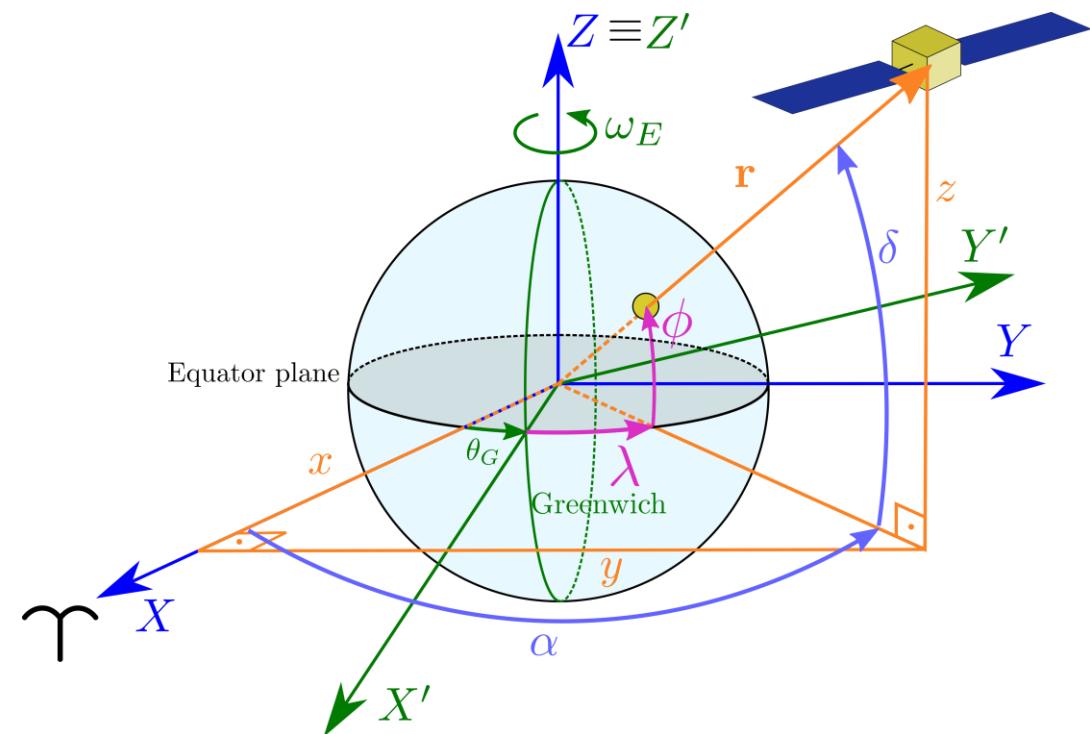


$a = 26600$ km
 $e = 0.74$
 $i = 63.4$ deg
 $\Omega = 50$ deg
 $\omega = 280$ deg
 $f_0 = 0$ deg [‡]
30 orbits
[‡] f denotes true anomaly

[1] Curtis, H. D.. *Orbital mechanics for engineering students*, Butterworth-Heinemann , 2014

Ground Track

Angles for the ground track



$$\theta_G(t) = \theta_G(t_0) + \omega_E (t - t_0)$$

Declination

$$\delta = \arcsin \frac{z}{r}$$

Right ascension

$$\alpha = \begin{cases} \arccos \frac{x/r}{\cos \delta} & \frac{y}{r} > 0 \\ 2\pi - \arccos \frac{x/r}{\cos \delta} & \frac{y}{r} \leq 0 \end{cases}$$

Longitude

$$\lambda = \alpha - \theta_G$$

Latitude

$$\phi = \delta$$

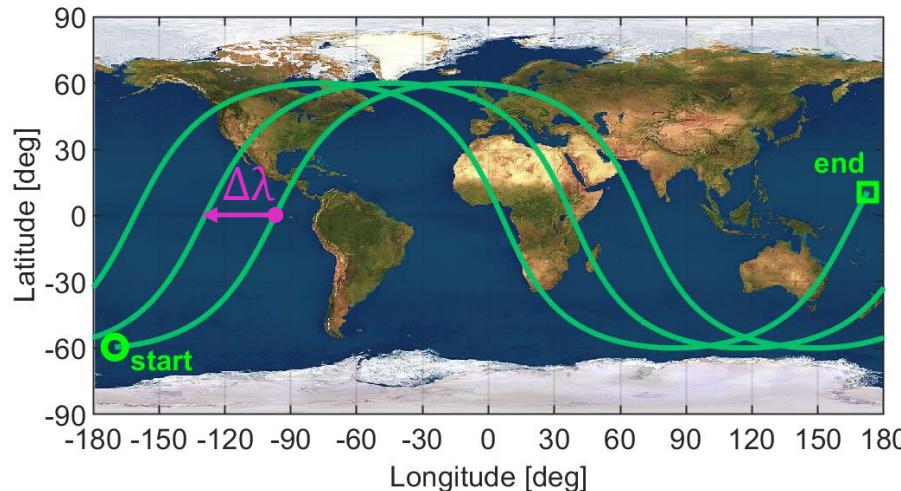
Ground Track

Geometrical properties

- For each orbital revolution, the ground track presents *2 equator crossings, one maximum in latitude, and one minimum in latitude.*
- For the unperturbed two-body problem, the orbit remains constant. However, **the ground track advances westward** by an angle $\Delta\lambda$ equal to Earth's rotation during one orbital period T of the satellite:

$$\Delta\lambda = T \omega_E$$

Earth's rotation velocity (eastwards): $\omega_E = 15.04 \text{ deg/h}$



$a = 8350 \text{ km}$
 $e = 0.19760$
 $i = 60 \text{ deg}$
 $\Omega = 270 \text{ deg}$
 $\omega = 45 \text{ deg}$
 $f_0 = 230 \text{ deg}$
3.25 orbits

Ground Track

Repeating ground tracks

- A ground track will repeat itself after k revolutions of the satellite and m rotations of the planet if the total ground track drift $k\Delta\lambda$ is equal to the corresponding planet rotation $m2\pi$:

$$k \Delta\lambda = m 2\pi \Rightarrow \frac{\Delta\lambda}{2\pi} = \frac{m}{k} \quad \text{with } k, m \in \mathbb{N}$$

- Substituting $\Delta\lambda = T\omega_E$ and operating, we see that this is equivalent to imposing that *the ratio of the satellite's orbital period T and Earth's rotational period $T_E = 2\pi/\omega_E$ is a rational number*:

$$\frac{T}{T_E} = \frac{m}{k} \quad \text{with } k, m \in \mathbb{N}$$

- Or expressed in terms of the satellite's mean motion $n = 2\pi/T$ and Earth's rotation velocity ω_E :

$$\frac{\omega_E}{n} = \frac{m}{k} \quad \text{with } k, m \in \mathbb{N}$$

Ground Track

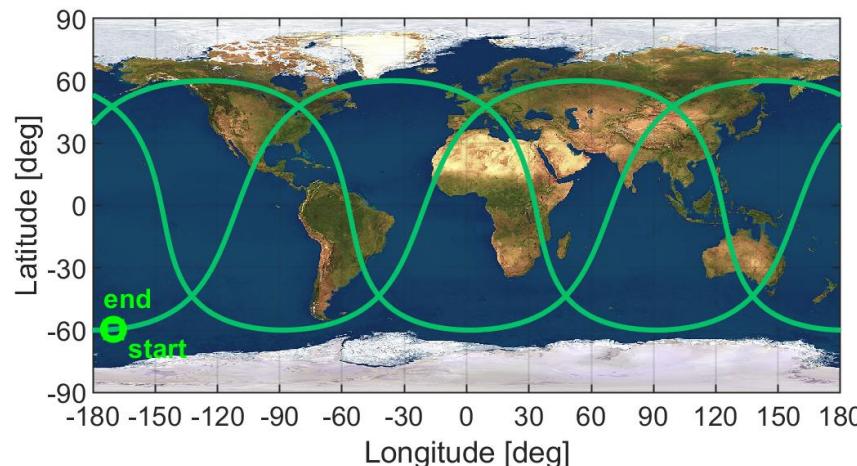
Repeating ground tracks

- Therefore, **repeating ground tracks can be obtained by choosing an orbit with a period T such that:**
- Or equivalently, with a mean motion n such that:
- Keep in mind that period T and mean motion n only depend on the semimajor axis a :

$$T = T_E \frac{m}{k} = \frac{2\pi m}{\omega_E k}$$

$$n = \omega_E \frac{k}{m}$$

$$n = \frac{2\pi}{T} = \sqrt{\frac{\mu}{a^3}}$$



$$a = 16733.65 \text{ km}$$

$$e = 0.19760$$

$$i = 60 \text{ deg}$$

$$\Omega = 270 \text{ deg}$$

$$\omega = 45 \text{ deg}$$

$$f_0 = 230 \text{ deg}$$

$$k = 4$$

$$m = 1$$

Exercise 1: Computation of ground tracks

Exercise 1: Computation of ground tracks

1. Implement a function `groundTrack` that computes the ground track of an orbit

- **Inputs:**

- State of the orbit at the initial time (either in Cartesian or Keplerian elements)
- Longitude of Greenwich meridian at initial time
- Vector of times at which the ground track will be computed
- Other inputs that you consider useful (e.g. ω_E , μ , t_0)

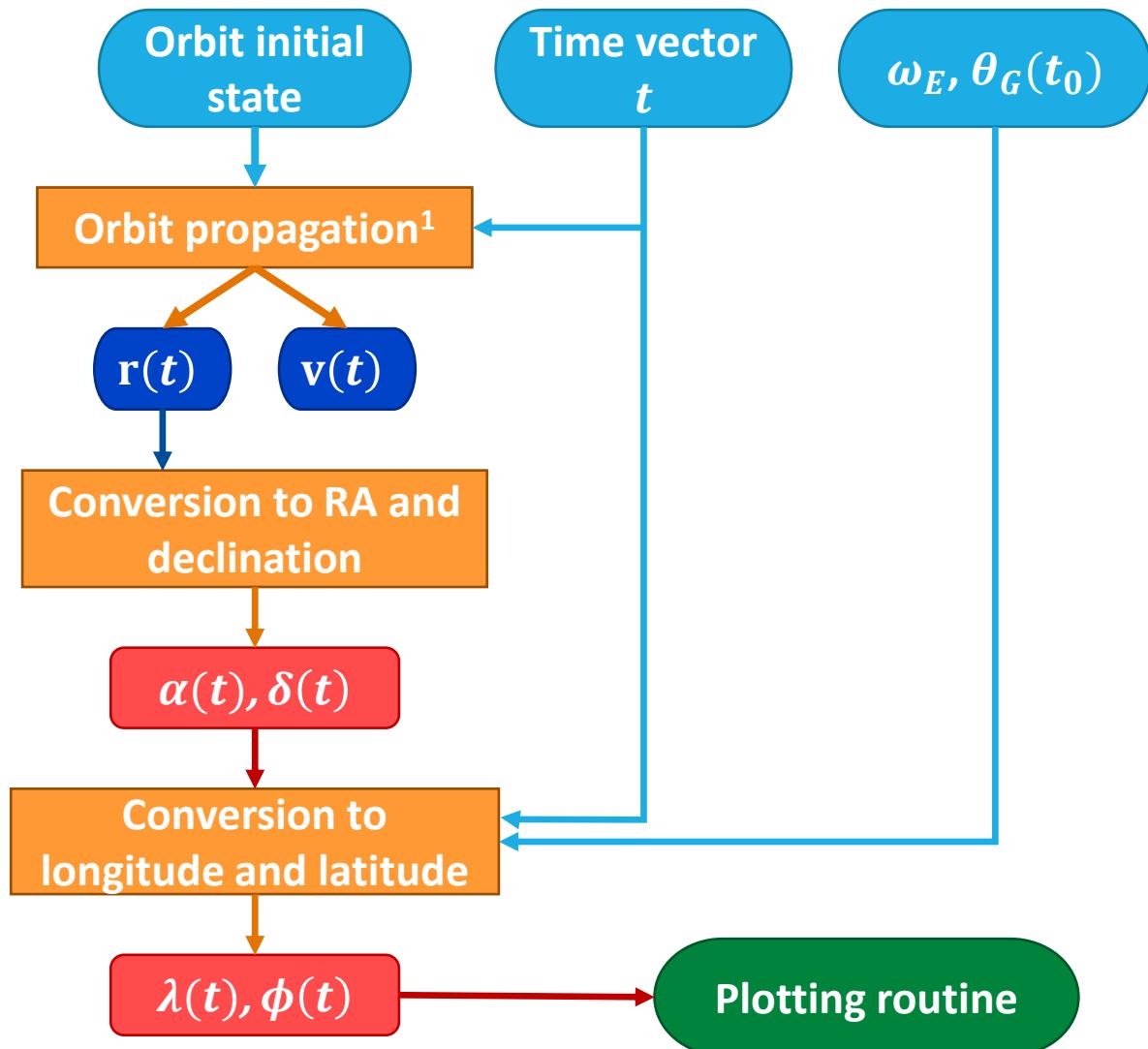
- **Outputs:**

- `alpha`: right ascension in Earth Centered Equatorial Inertial frame
- `delta`: declination in Earth Centered Equatorial Inertial frame
- `lon`: longitude with respect to rotating Earth (0 deg at Greenwich meridian)
- `lat`: latitude with respect to rotating Earth

Exercise 1: Computation of ground tracks

Flow diagram

Hint: For numerical orbit propagation¹, you can reuse the code from [Module 1](#).



¹ **Orbit propagation:** prediction of a body's orbital characteristics at a future date given the current orbital characteristics.

Exercise 1: Computation of ground tracks

Exercise 1: Computation of ground tracks

2. Plot the ground track for the following orbits:
 1. $a = 8350 \text{ km}$, $e = 0.1976$, $i = 60 \text{ deg}$, $\Omega = 270 \text{ deg}$,
 $\omega = 45 \text{ deg}$, $f_0 = 230 \text{ deg}$ (taken from [1], Example 4.12)
 2. A Molniya orbit with $a = 26600 \text{ km}$, $e = 0.74$, $i = 63.4 \text{ deg}$,
 $\Omega = 50 \text{ deg}$, $\omega = 280 \text{ deg}$, $f_0 = 0 \text{ deg}$
 3. Three circular LEO orbits with altitude 800 km, $\Omega = 0 \text{ deg}$,
 $\omega = 40 \text{ deg}$, $f_0 = 0 \text{ deg}$, and different inclinations: 0 deg ,
98 deg , and 30 deg
 4. A geostationary orbit (GEO)

Data:

$$\mu_{\oplus} = 398600 \text{ km}^3/\text{s}^2$$

$$\omega_E = 15.04 \text{ deg/h}$$

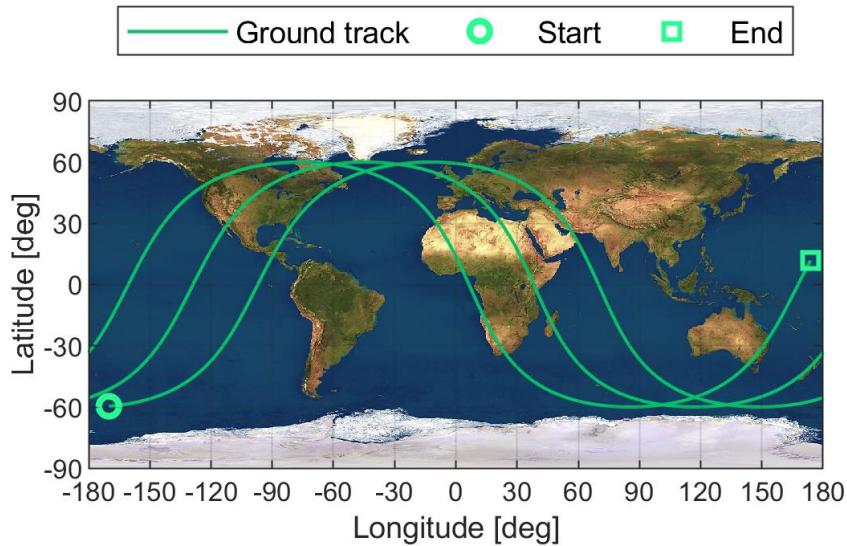
$$R_{\oplus} = 6378.137 \text{ km}$$

$$\theta_G(t_0) = 0 \text{ deg}$$

[1] Curtis, H. D.. *Orbital mechanics for engineering students*, Butterworth-Heinemann , 2014

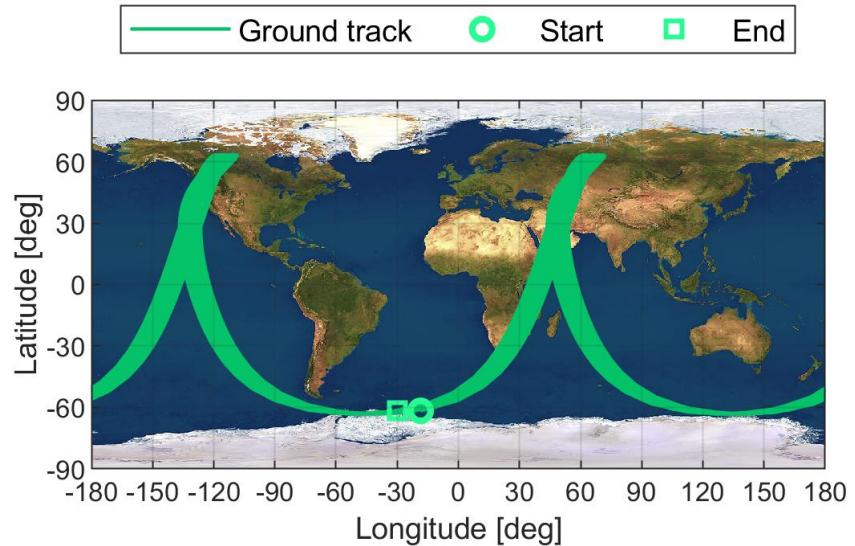
Exercise 1: Computation of ground tracks

Sample solutions



Case 1

$a = 8350 \text{ km}$, $e = 0.19760$
 $i = 60 \text{ deg}$, $\Omega = 270 \text{ deg}$
 $\omega = 45 \text{ deg}$, $f_0 = 230 \text{ deg}$
 $\theta_G(t_0) = 0 \text{ deg}$
3.25 orbits



Case 2

$a = 26600 \text{ km}$, $e = 0.74$
 $i = 63.4 \text{ deg}$, $\Omega = 50 \text{ deg}$
 $\omega = 280 \text{ deg}$, $f_0 = 0 \text{ deg}$
 $\theta_G(t_0) = 0 \text{ deg}$
30 orbits

Exercise 1: Computation of ground tracks

Sample solutions

Case 3

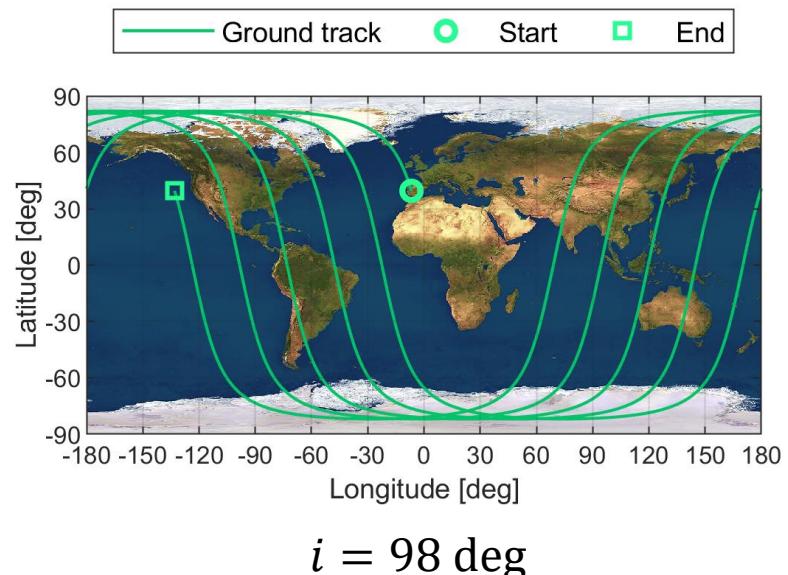
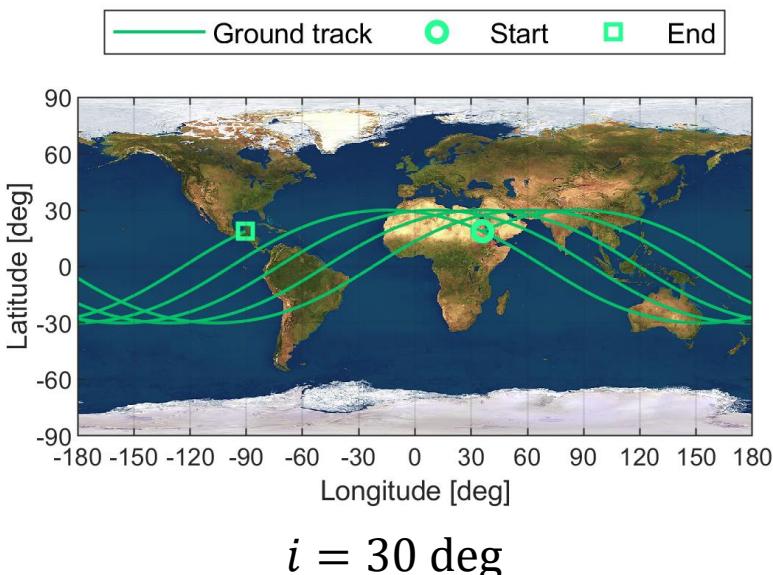
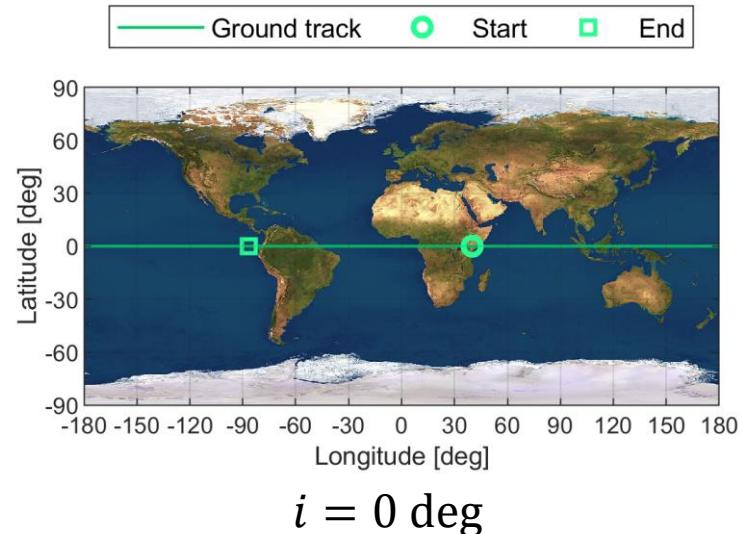
$$a = 7178.137 \text{ km}, e = 0$$

$$\Omega = 0 \text{ deg}, \omega = 40 \text{ deg},$$

$$f_0 = 0 \text{ deg}$$

$$\theta_G(t_0) = 0 \text{ deg}$$

5 orbits



Exercise 2: Design of repeating ground tracks

Exercise 2: Design of repeating ground tracks

1. Implement a function that computes the required a for a repeating ground track with k satellite revolutions and m Earth revolutions.
2. Modify the semimajor axis of the orbits in [Exercise 1](#) (cases 1 to 3) to get repeating ground tracks with:
 - [Case 1](#): $k = 12, m = 1$
 - [Case 2 \(Molniya orbit\)](#): $k = 2, m = 1$
 - [Case 3 \(circular LEOs\)](#):
 - For $i = 0$ deg: $k = 20, m = 2$
 - For $i = 30$ deg: $k = 29, m = 2$
 - For $i = 98$ deg: $k = 15, m = 1$
3. Plot each modified ground track together with the original one and compare them

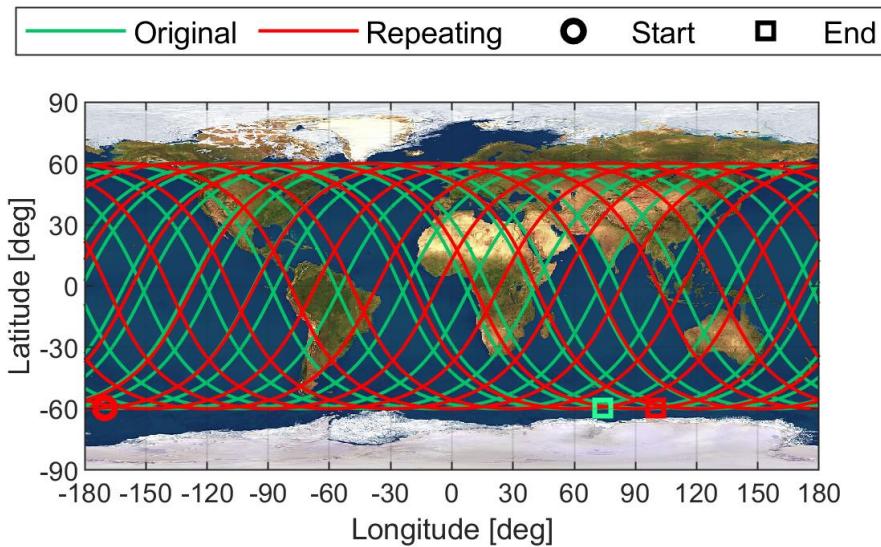
Exercise 2: Design of repeating ground tracks

Hints

- Because **longitude λ** is defined in [0, 360] deg, the ground track plot will be **discontinuous** at the boundaries of the angular range in λ :
 - If you include a NaN in the data arrays for a plot, the resulting line will have a discontinuity at that point.
- **Radians** are more convenient for calculations, while **degrees** are better for graphical representation.
- If you want, you can **add Earth's surface as background**, using function `imread` to load the image as a matrix of colored pixels, and function `image` to plot it.
 - Check the **documentation center** for detailed information on how to use them.
- Don't forget to *adjust the limits for the plotting regions*, to *label the axes*, and to *add a legend* if more than one line is represented.

Exercise 2: Design of repeating ground tracks

Sample solutions



Case 1

$$a_0 = 8350 \text{ km}, e = 0.19760$$

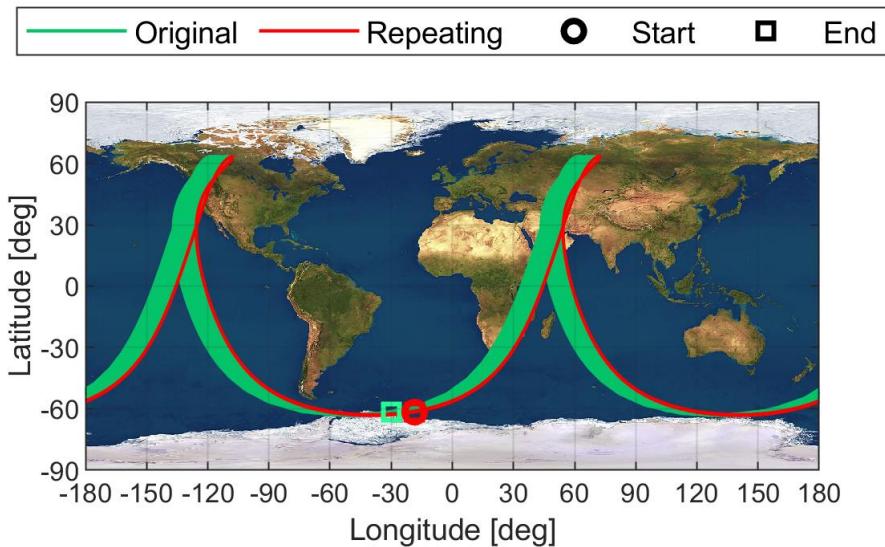
$$i = 60 \text{ deg}, \Omega = 270 \text{ deg}$$

$$\omega = 45 \text{ deg}, f_0 = 230 \text{ deg}$$

$$\theta_G(t_0) = 0 \text{ deg}, 15 \text{ orbits}$$

$$a_{\text{new}} = 8044.699 \text{ km}$$

$$k = 12, m = 1$$



Case 2

$$a_0 = 26600 \text{ km}, e = 0.74$$

$$i = 63.4 \text{ deg}, \Omega = 50 \text{ deg}$$

$$\omega = 280 \text{ deg}, f_0 = 0 \text{ deg}$$

$$\theta_G(t_0) = 0 \text{ deg}, 30 \text{ orbits}$$

$$a_{\text{new}} = 26563.011 \text{ km}$$

$$k = 2, m = 1$$

Exercise 2: Design of repeating ground tracks

Sample solutions

Case 3

$$a_0 = 7178.137 \text{ km}$$

$$e = 0$$

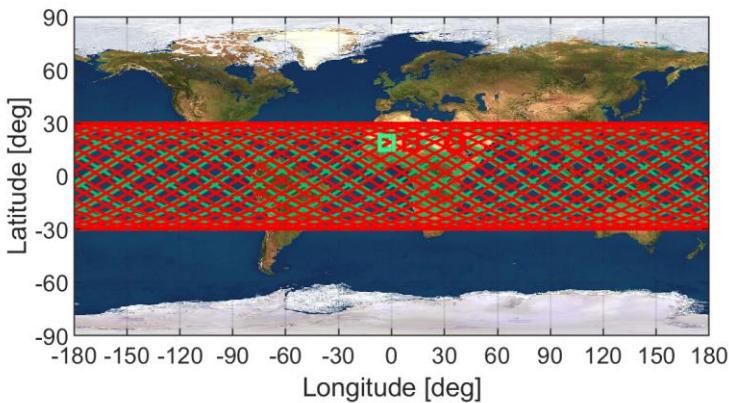
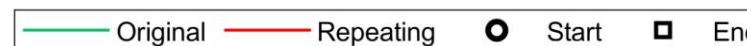
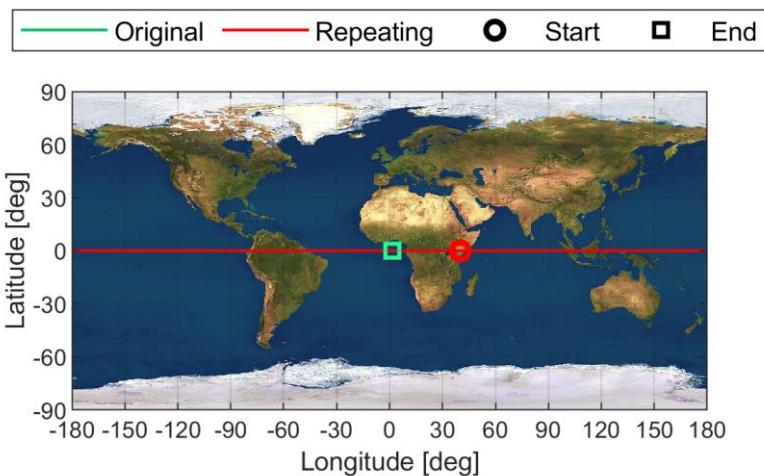
$$\Omega = 0 \text{ deg}$$

$$\omega = 40 \text{ deg}$$

$$f_0 = 0 \text{ deg}$$

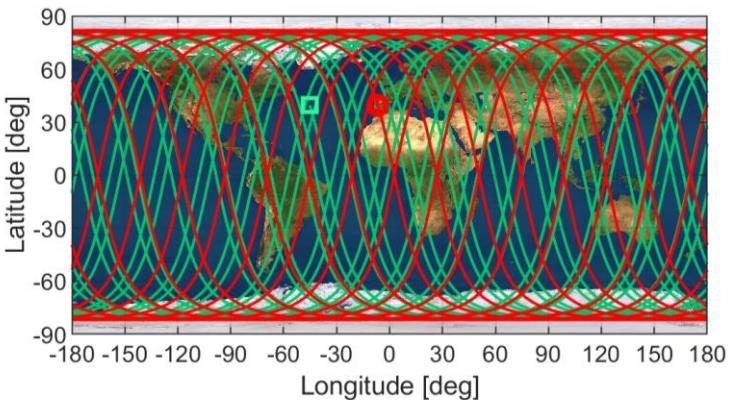
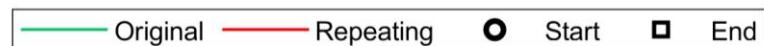
$$\theta_G(t_0) = 0 \text{ deg}, 30 \text{ orbits}$$

$$\begin{aligned} i &= 0 \text{ deg} \\ a_{\text{new}} &= 9084.422 \text{ km} \\ k &= 20, m = 2 \end{aligned}$$



$$i = 30 \text{ deg}$$

$$a_{\text{new}} = 7091.182 \text{ km}, k = 29, m = 2$$



$$i = 98 \text{ deg}$$

$$a_{\text{new}} = 6932.711 \text{ km}, k = 15, m = 1$$

Repeating ground track with perturbations

Nodal periods

- Previous results correspond to the **unperturbed 2BP**.
- The repeating ground track condition can be generalized introducing the **nodal periods for satellite and Earth** [2]:
 - Satellite nodal period \tilde{T} : Time between two successive equator crossings.
 - Greenwich nodal period \tilde{T}_E : Earth's rotation period with respect to the ascending node.
- \tilde{T} and \tilde{T}_E are constant and equal to T and T_E in the 2BP, but may change under other perturbations (J_2 , atmospheric drag, etc.)
- Same as before, *the condition for repeating ground track is that the ratio of the periods must be a rational number:*

as initial given

$$\frac{\tilde{T}}{\tilde{T}_E} = \frac{m}{k} \quad \text{with } k, m \in \mathbb{N}$$

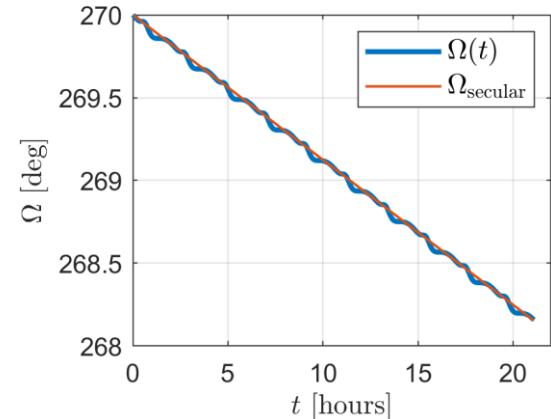
[2] Vallado, D.A. *Fundamental of Astrodynamics and Applications*, 4th Ed, Microcosm Press, 2013. Section 11.4.2.

Repeating ground track with perturbations

Effect of J_2

- The Earth bulges out at the equator due to centrifugal forces, taking the form of an **oblate spheroid**.
 - Oblateness** is defined as: $f = \frac{R_e - R_p}{R_e}$ R_e : equatorial radius R_p : polar radius
 - Geocentric latitude**: Oblateness modifies the latitude with respect to the spherical case (the normal to Earth's surface below the satellite is no longer parallel to \mathbf{r}). We will not consider this effect for the computer labs.
- J_2 introduces time variations in the Keplerian elements of the orbit [1]
 - Short-term variations in all elements.
 - Long-term variations** only for Ω , ω , and M
 - The **secular (average long-term)** evolution for Ω , ω , and M are straight lines. Analytical approximation are available for the **average change rates**.
 - For a , e , and i long-term variations average out.

[1] Curtis, H. D.. *Orbital mechanics for engineering students*, Butterworth-Heinemann , 2014



Evolution of Ω for the initial conditions of Exercise 1, case 1

Repeating ground track with perturbations

Secular effects of J_2

- The **secular evolution** (average evolution for long times) of Ω , ω , and $M_0 = M(t_0)$ are straight lines with **average change rates** [1]:

$$\dot{\Omega} = - \left[\frac{3}{2} \frac{\sqrt{\mu} J_2 R_e^2}{(1-e^2)^2 a^{7/2}} \right] \cos i$$

$$\dot{\omega} = - \left[\frac{3}{2} \frac{\sqrt{\mu} J_2 R_e^2}{(1-e^2)^2 a^{7/2}} \right] \left(\frac{5}{2} \sin^2 i - 2 \right)$$

$$\dot{M}_0 = - \left[\frac{3}{2} \frac{\sqrt{\mu} J_2 R_e^2}{(1-e^2)^2 a^{7/2}} \right] \left(1 - \frac{3}{2} \sin^2 i \right)$$

- These effects are known as:
 - Regression of the node:** The line of nodes drifts westward for prograde orbits, and eastward for retrograde orbits.
 - Advance of the perigee:** The perigee advances in the direction of the satellite motion for $0 \text{ deg} \leq i < 63.4 \text{ deg}$ or $116.6 \text{ deg} < i \leq 180 \text{ deg}$, and opposite to it for $63.4 \text{ deg} < i < 116.6 \text{ deg}$. It does not move for $i = 63.4 \text{ deg}$ or 116.6 deg .

[1] Curtis, H. D.. *Orbital mechanics for engineering students*, Butterworth-Heinemann , 2014

Repeating ground track with perturbations

Repeating ground track with J_2

- The **secular effects of J_2** modify the nodal periods of satellite and Earth:

- Satellite nodal period \tilde{T} is modified by $\dot{\omega}$ and \dot{M}_0

$$\tilde{T} = \frac{2\pi}{n + \dot{M}_0 + \dot{\omega}}$$

- Greenwich nodal period \tilde{T}_E is modified due to the regression of the node (because the ascending node is changing in time)

$$\tilde{T}_E = \frac{2\pi}{\omega_E - \dot{\Omega}}$$

- The **repeating ground track with secular effects of J_2** becomes an **implicit equation** involving a , e , and i [2]

as initial guess a_0
use the calculations
for UNPERTURBED

$$\frac{m}{k} = \frac{\tilde{T}}{\tilde{T}_E} = \frac{\omega_E - \dot{\Omega}}{n + \dot{\omega} + \dot{M}_0}$$

- Keep in mind that this solution only accounts for the **secular effects**.

[2] Vallado, D.A. *Fundamental of Astrodynamics and Applications*, 4th Ed, Microcosm Press, 2013. Section 11.4.2.

Exercise 3: Ground tracks with J_2

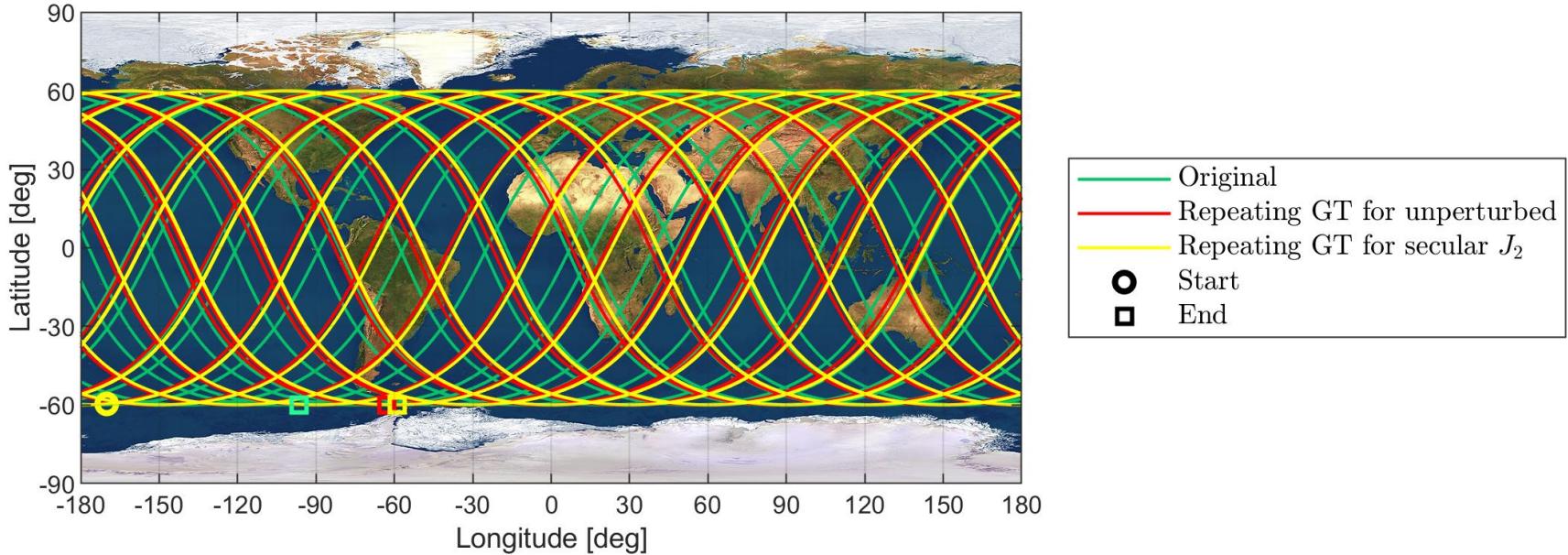
Exercise 3: Ground tracks with J_2

1. Modify function `groundTrack` from [Exercise 1](#) to include J_2 effects
 - Use values $R_e = 6378.137$ km and $J_2 = 0.00108263$ (taken from [3])
 - **Hint:** Recall the exercise on orbit propagation with J_2 from the previous module
2. Implement a new function to compute the required semimajor axis a for a repeating ground track with secular J_2 effects, with k satellite revolutions and m Earth revolutions
 - **Hint:** Solve the implicit equation in a using `fzero` or `fsolve`
3. Plot again the test cases in [Exercises 1 and 2](#)
 - Include the modified ground tracks obtained imposing the repeating GT conditions both for unperturbed orbit and with secular J_2 effects
 - Check that the repeating GT for unperturbed orbit no longer closes
 - Does the repeating GT with secular J_2 perfectly close? Why?
 - Is the GT of the GEO still a point under the effect of J_2 ?

[3] <https://nssdc.gsfc.nasa.gov/planetary/factsheet/earthfact.html>

Exercise 3: Ground tracks with J_2

Sample solutions



Case 1

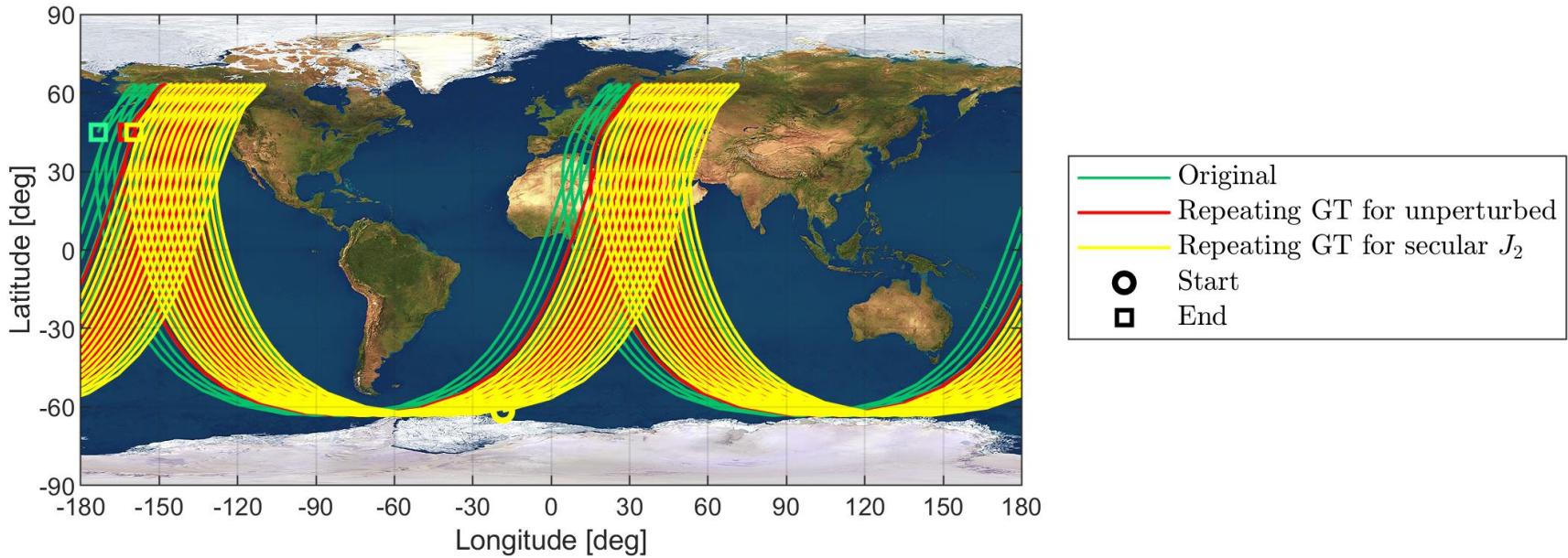
$a_0 = 8350 \text{ km}$, $e = 0.19760$, $i = 60 \text{ deg}$, $\Omega = 270 \text{ deg}$, $\omega = 45 \text{ deg}$, $f_0 = 230 \text{ deg}$
 $\theta_G(t_0) = 0 \text{ deg}$, 20 orbits

Repeating ground tracks for $k = 12$, $m = 1$:

$$a_{\text{unpert}} = 8044.699 \text{ km}, a_{\text{secular } J_2} = 8008.782 \text{ km}$$

Exercise 3: Ground tracks with J_2

Sample solutions



Case 2

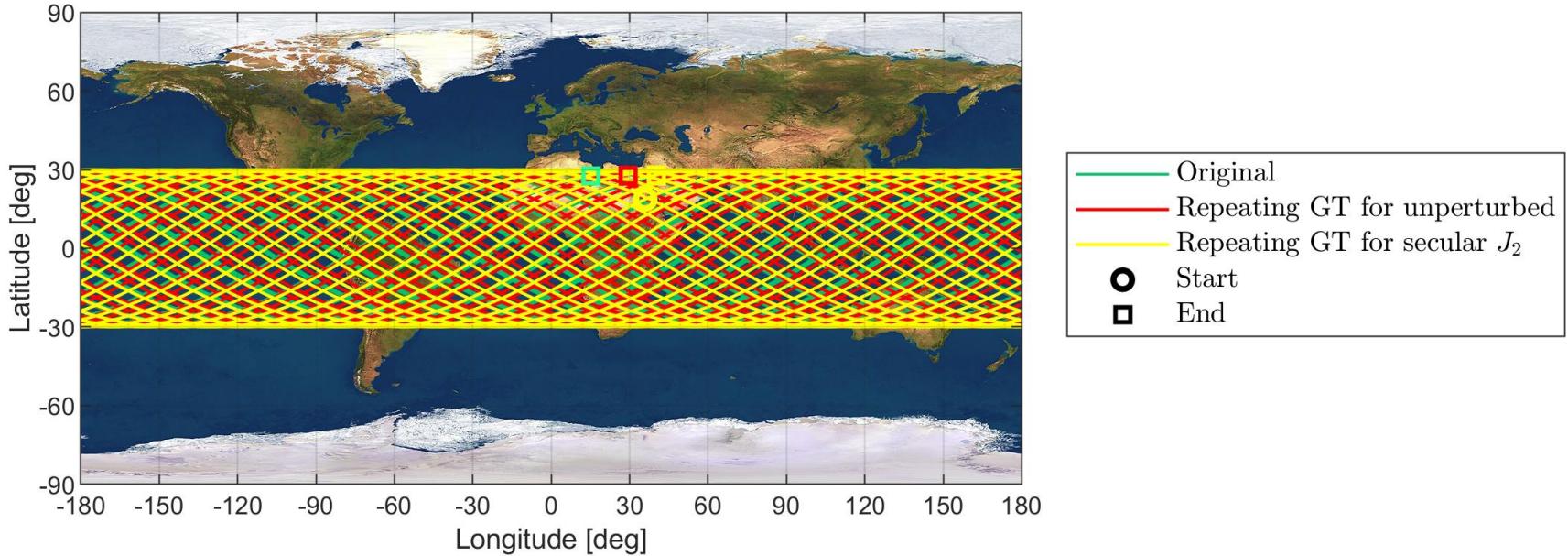
$a_0 = 26600 \text{ km}$, $e = 0.74$, $i = 63.4 \text{ deg}$, $\Omega = 50 \text{ deg}$, $\omega = 280 \text{ deg}$, $f_0 = 0 \text{ deg}$
 $\theta_G(t_0) = 0 \text{ deg}$, 30 orbits

Repeating ground tracks for $k = 2$, $m = 1$:

$$a_{\text{unpert}} = 26563.011 \text{ km}, a_{\text{secular } J_2} = 26554.674 \text{ km}$$

Exercise 3: Ground tracks with J_2

Sample solutions



Case 3

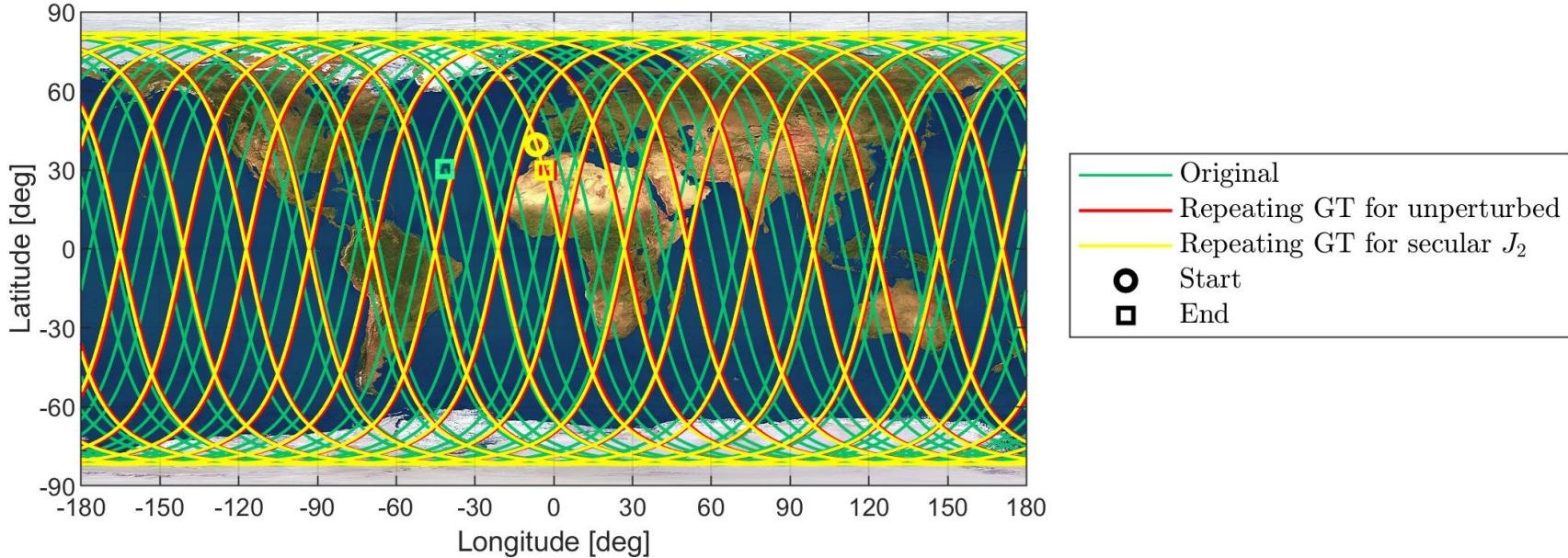
$a_0 = 7178.137 \text{ km}$, $e = 0$, $i = 30 \text{ deg}$, $\Omega = 0 \text{ deg}$, $\omega = 40 \text{ deg}$, $f_0 = 0 \text{ deg}$
 $\theta_G(t_0) = 0 \text{ deg}$, 30 orbits

Repeating ground tracks for $k = 29$, $m = 2$:

$$a_{\text{unpert}} = 7091.182 \text{ km}, a_{\text{secular } J_2} = 7024.197 \text{ km}$$

Exercise 3: Ground tracks with J_2

Sample solutions



Case 3

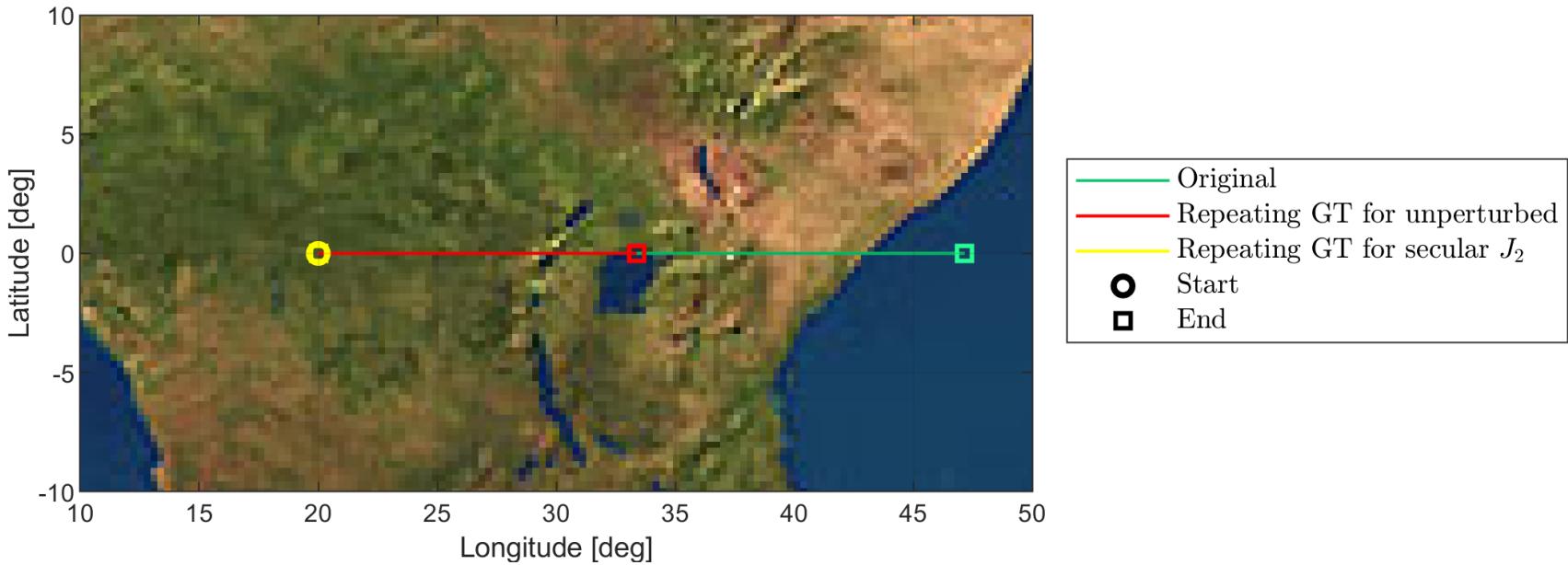
$a_0 = 7178.137 \text{ km}$, $e = 0$, $i = 98 \text{ deg}$, $\Omega = 0 \text{ deg}$, $\omega = 40 \text{ deg}$, $f_0 = 0 \text{ deg}$
 $\theta_G(t_0) = 0 \text{ deg}$, 30 orbits

Repeating ground tracks for $k = 15$, $m = 1$:

$$a_{\text{unpert}} = 6932.711 \text{ km}, a_{\text{secular } J_2} = 6940.095 \text{ km}$$

Exercise 3: Ground tracks with J_2

Sample solutions



Case 4

$a_0 = 42164 \text{ km}$, $e = 0$, $i = 0 \text{ deg}$, $\Omega = 0 \text{ deg}$, $\omega = 0 \text{ deg}$, $f_0 = 20 \text{ deg}$
 $\theta_G(t_0) = 0 \text{ deg}$, 500 orbits

Repeating ground tracks for $k = 1$, $m = 1$:

$$a_{\text{unpert}} = 42166.151 \text{ km}, a_{\text{secular } J_2} = 42168.240 \text{ km}$$