# Politecnico di Milano

Master of Science in
Space Engineering

# Orbital Mechanics Project


Group 33

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A.Y. 2020-2021

# Nomenclature

| | |
|---|---|
| $a$ | semi-major axis |
| $e$ | eccentricity |
| $f$ | true anomaly |
| $i$ | inclination |
| $\Omega$ | Right ascension of the ascending node |
| $\omega$ | periapsis anomaly |

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# Assignment 2: Planetary Explorer Mission

# Mission requirements

# Mission analysis outputs

## 4.1 Nominal orbit

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## 4.3 Orbit Perturbations

In our model we included perturbations due to two effects:

- **Second Zonal Harmonic *J2***

  Models the Earth Oblateness using the spherical geopotentials model, truncated to the second term.

- **Solar Radiation Pressure *SRP***

  A force given by the impact of momentum-carrying photons on the surfaces of the spacecraft. For this perturbation we have written a function that, based on the initial date gets the position vector of the Earth from the ephemerides function and uses that together with the position vector of the spacecraft with respect to the Earth to calculate the direction of the disturbing acceleration. A simple algorithm for the eclipse condition found in [4] has been implemented.

$$\mathbf{a_{SRP}} = -P_{SR@1AU}\frac{AU^2}{||\mathbf{r}_{sc-Sun}||^3}c_R\frac{A_{Sun}}{m}\mathbf{r}_{sc-Sun} \tag{4.1}$$

These forces give an acceleration to the spacecraft so that the total acceleration it perceives in orbit is

$$\mathbf{a} = -\frac{\mu_\oplus}{r^3}\mathbf{r} + \mathbf{a}_{SRP} + \mathbf{a}_{J2} \tag{4.2}$$

## 4.4 Orbit Propagation

### 4.4.1 Methods

To propagate the orbit we used two different methods:

- **Gauss Planetary Equations**

$$
\begin{aligned}
\frac{da}{dt} &= \frac{2a^2}{h}\Big(e\sin f\ a_r + \frac{p}{r}a_s\Big)\\
\frac{de}{dt} &= \frac{1}{h}\Big(p\sin(f)\ a_r + \Big((p+r)\cos f + re\Big)a_s\Big)\\
\frac{di}{dt} &= \frac{r\cos(f+\omega)}{h}a_w\\
\frac{d\Omega}{dt} &= \frac{r\sin(f+\omega)}{h\sin i}a_w\\
\frac{d\omega}{dt} &= \frac{1}{he}\Big(\cos f\ a_r + (p+r)\sin f\ a_s\Big) - \frac{r\sin(f+\omega)\cos i}{h\sin i}a_w\\
\frac{df}{dt} &= \frac{h}{r^2} + \frac{1}{eh}\Big(p\cos f\ a_r - (p+r)\sin f\ a_s
\end{aligned}
$$

  We numerically integrate these equations with the ode113 solver, by using them to set the derivatives of the state. The reference frame for this equations is the RSW frame so $a_r, a_s, a_w$ are respectively the radial, transversal and out-of-plane components. [1] [2] [3]

- **Numeric integration of cartesian equations** This method consists in directly integrating the Cartesian equations of motion

$$
\ddot{\mathbf{r}} = -\frac{\mu_\oplus}{r^3}\mathbf{r} + \sum a_p \tag{4.3}
$$

### 4.4.2 Comparison

There is a marked difference in the computational time between the two methods as the Gaussian elements propagation for a timespan of 50 $T_{orbit}$ has a computation time $t_{GAUSS} = 5.4660 * 10^{-4}s$ that is much faster than the Cartesian $t_{CART} = 62.2s$ one.

There are numerical error introducing differences between the two methods, but the error only grows up to an order of $10^{-5}$ for a propagation of 200 periods.

## 4.5 History of the Keplerian elements

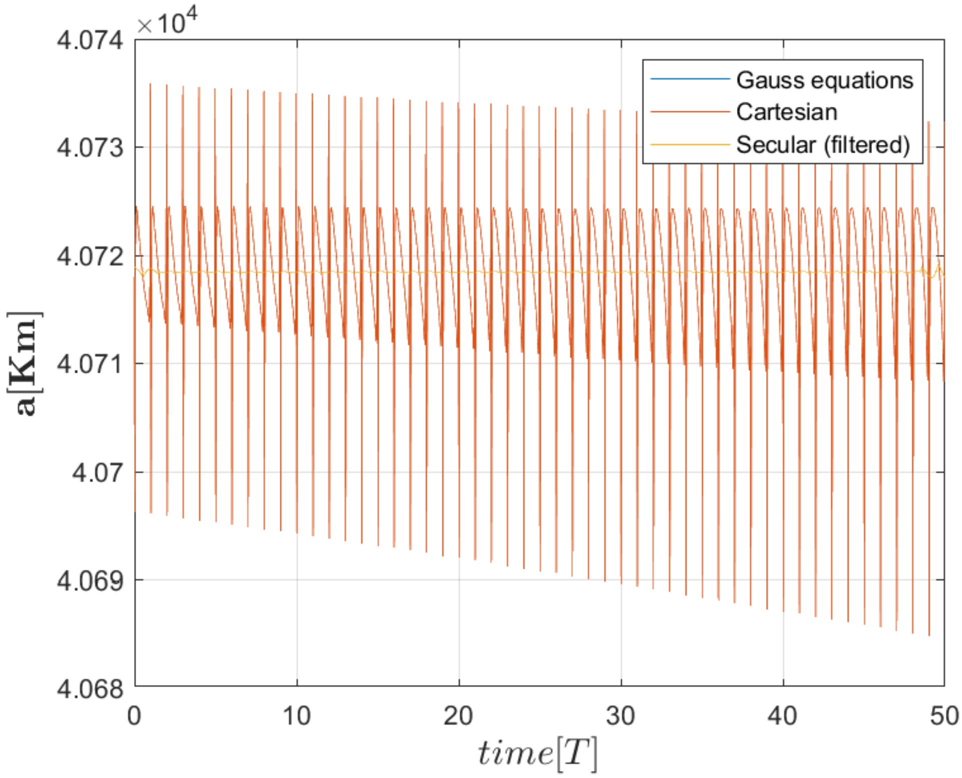


Figure 4.1: Evolution of the semi-major axis

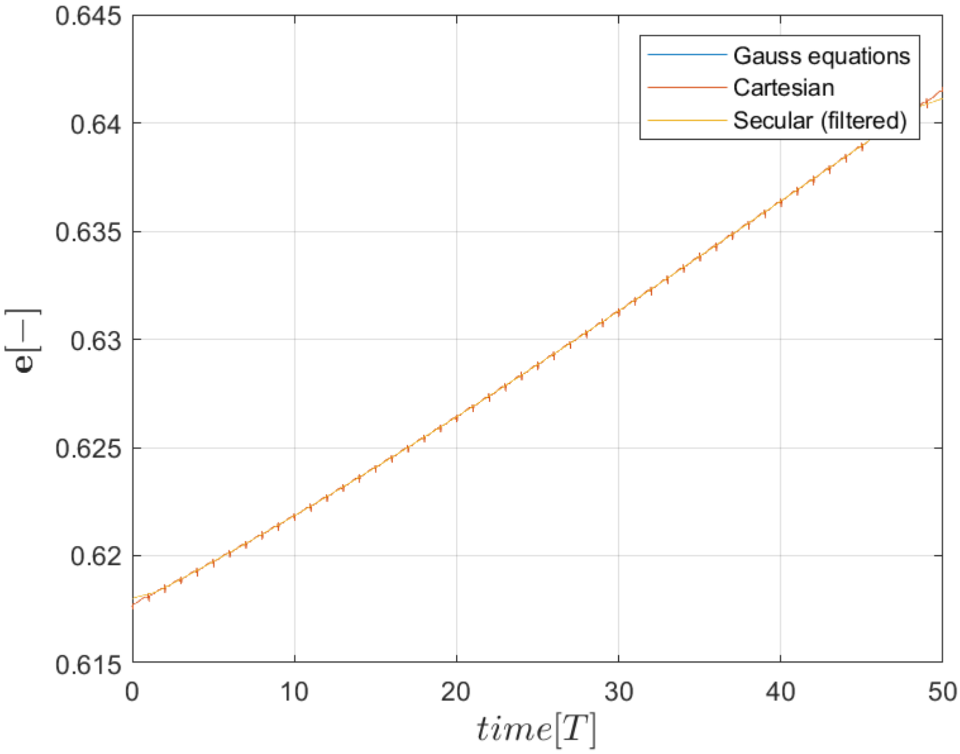


Figure 4.2: Evolution of the eccentricity

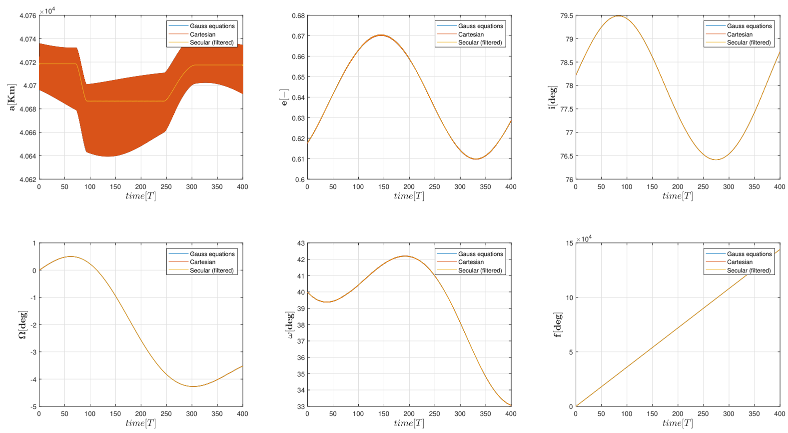

Figure 4.3: Evolution of keplerian elements

## 4.6 Orbit evolution representation

## 4.7 HF filtering

For the filtering we have used the simple *movmean* function integrated in MATLAB which computes the average at each time instant between the current point and the neighboring ones with a cutoff period of $3T_{orbit}$.

We can see in the *secular evolution* that there is a periodic increase and decrease in eccentricty due to the acceleration from the SRP being always in the same direction and the secular evolution has an oscillatory behaviour caused by Earth's revolution around the Sun.

## 4.8 Comparison with real data

### 4.8.1 Satellite selection

### 4.8.2 Comparison with our model

# Bibliography


[1] Curtis,H.D. *Orbital mechanics for engineering students*. Butterworth-Heinemann , 2014. Chapter 12

[2] Vallado, D.A. *Fundamental of Astrodynamics and Applications, 4th Ed*, Microcosm Press, 2013. Chapters 8 and 9

[3] Battin, R, *An Introduction to the Mathematics and Methods of Astrodynamics*, AIAA Education Series, 1999. Chapter 10

[4] Curtis,H.D. *Orbital mechanics for engineering students*. Butterworth-Heinemann , 2019. Chapter 10.9