1. Perform the following divisions in binary and get the quotient and remainder. **Hint:** if you already have the answer for the unsigned division, you just need to consider the sign separately. If the Quotient or reminder turn out to be negative, please represent them as 2's complement.

a. 100/7 01110 R 010

b. (-100)/7 10010 R 10

c. 200/9 010110 R 010

d. (-200)/(-9) 010110 R 10

e. 200/(-9) 101010 R 010

2. Find the modulo of the following numbers:

a. 30 mod5 0

b. 33 mod 7 5

c. -21 mod4 3

d. -24 mod 6 0

3. Find x in the following equations (x represent the reciprocal of the number)

a. $4x\text{mod}5$ = 1 4

b. $5x\text{mod}7$ = 1 3

4. Perform the following multiplications using the simple two's complement 4-bit floating point representation that was covered in class. Make sure that your final answer is represented as a normalized fraction. If your final exponent is out of range you have to indicate an overflow. Please note that internally fractions are allowed to be un-normalized and exponents are allowed to overflow, but not the final answer. After the calculation is done, make sure to compare your results with the correct answer (calculated in base 10) and comment on precision:

- $(-1 \times 2^{-4}) \times (-1 \times 2^{-5})$ $0.5 * 2^{-8}$ Exactly precise!
- $(0.5 \times 2^{5}) \times (0.5 \times 2^{3})$ $0.5 * 2^{7}$ Exactly precise!
- $(0.875 \times 2^{3}) \times (0.875 \times 2^{3})$ $0.75 * 2^{6}$ Error of $0.5 * 2^{1}$
- $(0.875 \times 2^{3}) \times (-0.875 \times 2^{3})$ $-0.875 * 2^{6}$ Error of $0.875 * 2^{3}$

5. Perform the following additions using the simple two's complement 4-bit floating point representation that was covered in class. Make sure that your final answer is represented as a normalized fraction. If your final exponent is out of range you have to indicate an overflow. Please note that internally fractions are allowed to be un-normalized and exponents are allowed to overflow, but not the final answer. If the result was zero make sure to set your output to the right value. After the calculation is done, make sure to compare your results with the correct answer (calculated in base 10) and comment on precision:

- $(-1 \times 2^{-4}) + (-1 \times 2^{-5})$ $-0.75 * 2^{-3}$ Exactly precise!
- $(-1 \times 2^{-4}) - (-1 \times 2^{-5})$ $-0.5 * 2^{-4}$ Exactly precise!
- $(0.5 \times 2^{-3}) + (0.5 \times 2^{-4})$ $0.75 * 2^{-3}$ Exactly precise!
- $(0.5 \times 2^{3}) + (0.5 \times 2^{5})$ $0.625 * 2^{5}$ Exactly precise!

6. Using IEEE 754, find the floating-point representation of the numbers below in single and double precisions:

- 4
- -6

a) Single: 00000000000000000000000010000001
Double: 0000000000000000000000000000000000000000000000000000010000000001

b) Single: 11000000000000000000000010000001
Double: 1100000000000000000000000000000000000000000000000000010000000001

HW 3

1a) 0001110 R010

111 | 1100100

100/7

−0111

1011

−0111

1000

−111

10

1b) (−100)/7 10010 R 10

1c) 200/9 00010110 R 010

1001 | 11001000

−1001

1110

−1001

1010

−1001

10

1d) (−200)/(−9) 010110 R 10

1e) 200/(−9) 101010 R 010

2a) 30 mod 5 0

2b) 33 mod 7

33−7=26−7=19−7=12−7=5

2c) −21 mod 4

−21+4=−17+4=−13+4=−9+4=−5

−5+4=−1+4=3

2d) $-24 \bmod 6$

$-24+6=-18+6=-12+6=-6+6=0$

3a) $4x \bmod 5 = 1$

$4(4)=16$ $\quad$ 4

$16 \bmod 5 = 1$

$16-5=11-5=6-5=1$

3b) $5x \bmod 7$

$5(3)=15$ $\quad$ 3

$15-7=8-7=1$

4a) $(-1 \cdot 2^{-4}) \cdot (-1 \cdot 2^{-5})$

$(1000 \cdot 1000) \cdot (-4 + -5 = -9)$

$01.000000 \cdot 2^{-9}$

$0.100000 \cdot 2^{-8}$

$0.5 \cdot 2^{-8}$

4b) $(0.5 \cdot 2^{5}) \cdot (0.5 \cdot 2^{3})$

$(0100 \cdot 0100) \quad (5+3=8)$

$\cancel{0}0.010\cancel{0}00 \cdot 2^{8}$

$0.100 \quad \cdot 2^{7}$

$0.5 \quad \cdot 2^{7}$

4c) $(0.875 \cdot 2^3) \cdot (0.875 \cdot 2^3)$

$(0111 \cdot 0111)$ $(3+3=6)$

```
      0111
    x 0111
  00000111 PPO
  00001110 PP1
+ 00011100 PP2
  00110001
```

$\cancel{00}.110\cancel{0}01 \cdot 2^6$ source of error $\rightarrow .\cancel{110001} \cdot 2^6$

01100110

$\boxed{0.75 \cdot 2^6}$

$.1 \cdot 2^1$

$\boxed{0.5 \cdot 2}$

1

4d) $(0.875 \cdot 2^3) \cdot (-0.875 \cdot 2^3)$

$(0111 \cdot 1001)$ $(3+3=6)$

```
      0111
    x 1001
  00000111 PPO
+ 00111000 PP3
  00111111 → 2's comp!
```

```
  00000111
+ 11001000
  11.001111
```

$\cancel{11}.00\cancel{1111} \cdot 2^6$ source of error $\rightarrow .000111 \cdot 2^6$

10010110

$.111 \cdot 2^3$

$-0.875 \cdot 2^6$

$0.875 \cdot 2^3$

5a) $(-1 \cdot 2^{-4}) + (-1 \cdot 2^{-5}) = -0.09375$

$1.000 \quad E_1 = -4$

$1.1000 \quad E_2 = -5+1 = -4$

$$\begin{array}{r} 1.000 \\ + \ 1.100 \\ \hline 1 \ 0.100 \end{array}$$

~~overflow~~

$\boxed{1010 \mid 1101}$

$$\begin{array}{r} 1.100 \quad E=-3 \\ + \ 1.110 \\ \hline 1 \ 1.010 \end{array}$$

$\boxed{-0.75 \cdot 2^{-3}}$

~~overflow~~

5b) $(-1 \cdot 2^{-4}) - (-1 \cdot 2^{-5}) = -0.03125$

$1.000 \quad E_1 = -4$

2's compl. $1.000 \quad E_2 = -5+1 = -4$

$$\begin{array}{r} 1.000 \\ + \ 0.100 \\ \hline 1.100 \end{array}$$

$\boxed{1100 \mid 1100}$

$\boxed{-0.5 \cdot 2^{-4}}$

5c) $(0.5 \cdot 2^{-3}) + (0.5 \cdot 2^{-4})$

$0.100 \quad E_1 = -3$

$0.0100 \quad E_2 = -4 + 1 = -3$

$$\begin{array}{r} 0.100 \\ +\ 0.010 \\ \hline 0.110 \end{array}$$

$\boxed{0110}\boxed{1101}$

$\boxed{0.75 \cdot 2^{-3}}$

5d) $(0.5 \cdot 2^{3}) + (0.5 \cdot 2^{5})$

$0.00100 \quad E_1 = 3 + 2 = 5$

$0.100 \quad E_2 = 5$

$$\begin{array}{r} 0.001 \\ +\ 0.100 \\ \hline 0.101 \end{array}$$

$\boxed{0101}\boxed{0101}$

$\boxed{0.625 \cdot 2^{5}}$

6a) $N = (-1)^{S} \cdot 1.F \cdot 2^{E}$

$4 = (-1)^{0} \cdot 1.0 \cdot 2^{2}$

$E + 127 = 129$

$E + 1023 = 1025$

6b) $-6 = (-1)^{1} \cdot 1.5 \cdot 2^{2}$

$E + 127 = 129$

$E + 1023 = 1025$