PHYS 2311 Ch. 10 HW

Corbin Hibler (c-hibler@onu.edu)

November 14, 2024

Problem 1.

(a) 0.785 rad, $\frac{\pi}{4}$ rad

(b) 1.0471 rad, $\frac{\pi}{3}$ rad

(c) 1.571 rad, $\frac{\pi}{2}$ rad

(d) 6.283 rad, 2π rad

(e) 7.7667 rad, $\frac{89\pi}{36}$ rad

Problem 4.

$$8500\text{rpm} \cdot \frac{1\,\text{min}}{60\,\text{s}} \cdot \frac{2\,\pi\text{rad}}{1\,\text{rev}} = 890.12\,\text{rad/s} = \omega$$

$$t = 4.0$$

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{0 - 890.12}{4.0} = \boxed{-223\,\text{rad/s}^2}$$

Problem 5.

(a)

$$7200\text{rpm} \cdot \frac{1\,\text{min}}{60\,\text{s}} \cdot \frac{2\,\pi\text{rad}}{1\,\text{rev}} = \boxed{240\pi\,\text{rad/s}}$$

(b)

$$r = 3.00\,\text{cm} = 0.03\,\text{m}$$

$$v = r\omega = (0.03)(240\pi) = \boxed{22.62\,\text{m/s}}$$

(c)

$$22.62 \cdot \frac{1\,\text{b}}{0.5\,\mu\text{m}} = \boxed{45.24 \times 10^6\,\text{bps}}$$

Problem 6.

$$C = \frac{\text{distance}}{\text{revolutions}} = \frac{3.1}{12.0} = 0.258\,\text{m}$$
$$C = \pi \cdot d = \frac{0.258\,\text{m}}{\pi} = d = 0.822\,\text{m} = \boxed{8.2\,\text{cm}}$$

Problem 19.

$$15000\text{rpm} \cdot \frac{1\,\text{min}}{60\,\text{s}} \cdot \frac{2\,\pi\text{rad}}{1\,\text{rev}} = 1.57\,\text{rad/s}$$
$$\alpha = \frac{\omega_f - \omega_0}{t}$$
$$\alpha = \frac{1570.8\,\text{rad/s} - 0}{220\,\text{s}} = 7.14\,\text{rad/s}^2$$
$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$
$$\theta = \frac{1}{2} \times 7.14\,\text{rad/s}^2 \times (220\,\text{s})^2 = 172,960\,\text{rad}$$
$$\text{rev} = \frac{172,960\,\text{rad}}{2\pi\,\text{rad/rev}} = \boxed{27\,855\,\text{rev}}$$

Problem 20.

(a)

$$\theta(t) = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$
$$\text{rev} = 26 \cdot 2\pi = 52\pi\,\text{rev}$$

$$52\pi = \frac{1}{2}\alpha(60)^2$$
$$\alpha = \frac{(52\pi)(2)}{60^2} = \boxed{0.0909\,\text{rad/s}^2}$$

(b)

$$\omega_f = \omega_0 + \alpha t$$
$$\omega_f = 0 + (0.0909\,\text{rad/s}^2)(60\,\text{s})$$
$$\omega_f = 5.454\,\text{rad/s}$$
$$\omega_f = 5.454\,\text{rad/s} \times \frac{60\,\text{s}}{2\pi\,\text{rad}}$$
$$\omega_f \approx \boxed{52.0\,\text{rpm}}$$

]

Problem 21.

(a)

$$\omega_0 = 780\,\text{rev/min} \times \frac{2\pi\,\text{rad}}{60\,\text{s}} = 81.68\,\text{rad/s}$$

$$\theta = 1250\,\text{rev} \times 2\pi\,\text{rad/rev} = 2500\pi\,\text{rad}$$

$$\omega_f^2 = \omega_0^2 + 2\alpha\theta$$

$$0 = (81.68\,\text{rad/s})^2 + 2\alpha(2500\pi\,\text{rad})$$

$$2\alpha(2500\pi) = -(81.68)^2$$

$$\alpha = \frac{-(81.68)^2}{2 \times 2500\pi}$$

$$\alpha \approx \boxed{-0.42\,\text{rad/s}^2}$$

(b)

$$\omega_f = \omega_0 + \alpha t$$

$$0 = 81.68\,\text{rad/s} + (-0.42\,\text{rad/s}^2)t$$

$$t = \frac{81.68\,\text{rad/s}}{0.42\,\text{rad/s}^2}$$

$$t \approx \boxed{192.3\,\text{s}}$$

Problem 25.

(a)

$$\theta = 9.5t - 13.0t^2 + 1.6t^4$$

$$\omega = 9.5 - 26.0t + 6.4t^3$$

(b)

$$\alpha = -26.0 + 19.2t^2$$

(c)

$$\omega(3.0) = 9.5 - 26.0(3.0) + 6.4(3.0)^3 = \boxed{104.3\,\text{rad/s}}$$

$$\alpha(3.0) = -26.0 + 19.2(3.0)^2 = \boxed{146.8\,\text{rad/s}^2}$$

(d)

$$\omega_{avg} = \frac{\Delta\theta}{\Delta t} = \frac{\theta(3.0) - \theta(2.0)}{3.0 - 2.0}$$

$$= \frac{(9.5(3.0) - 13.0(3.0)^2 + 1.6(3.0)^4) - (9.5(2.0) - 13.0(2.0)^2 + 1.6(2.0)^4)}{1.0} = \boxed{48.5\,\text{rad/s}}$$

(e)

$$\alpha_{avg} = \frac{\Delta\omega}{\Delta t} = \frac{\omega(3.0) - \omega(2.0)}{3.0 - 2.0}$$

$$= \frac{(9.5 - 26.0(3.0) + 6.4(3.0)^3) - (9.5 - 26.0(2.0) + 6.4(2.0)^3)}{1.0} = \boxed{95.6\,\text{rad/s}^2}$$

Problem 28.

$$\tau_1 = mg\ell_1, \quad \tau_2 = -mg\ell_2$$

$$\tau_{net} = \tau_1 - \tau_2 = mg\ell_1 - mg\ell_2 = mg(\ell_1 - \ell_2)$$

$$\ell_1 > \ell_2$$

The direction of the net torque is clockwise.

Problem 29.

(a)

$$\tau = Fd = (37)(0.96) = \boxed{35.52\,\text{Nm}}$$

(b)

$$\tau = Fd\sin\theta = (37)(0.96)\sin(60) = \boxed{30.66\,\text{Nm}}$$

Problem 30.

$$\tau_1 = r_1F_1\sin\theta$$

$$\tau_1 = (0.12)(35)\sin(135^\circ)$$

$$\sin(135^\circ) = \sin(45^\circ) = \frac{\sqrt{2}}{2}$$

$$\tau_1 = (0.12)(35)(\frac{\sqrt{2}}{2})$$

$$\tau_1 = 2.97\,\text{Nm (counterclockwise)}$$

$$\tau_2 = r_2F_2\sin 90^\circ$$
$$\tau_2 = (0.24)(28)$$
$$\tau_2 = 6.72\,\text{Nm (counterclockwise)}$$

$$\tau_3 = r_2F_3\sin 90^\circ$$
$$\tau_3 = (0.24)(18)$$
$$\tau_3 = 4.32\,\text{Nm (clockwise)}$$

$$\tau_{\text{net}} = \tau_1 + \tau_2 - \tau_3 - \tau_{\text{friction}}$$
$$\tau_{\text{net}} = 2.97 + 6.72 - 4.32 - 0.60$$
$$\tau_{\text{net}} = \boxed{4.77\,\text{Nm (counterclockwise)}}$$

Problem 34.

The radius near the hub of the bicycle is effectively zero, as it is so small that it's considered negligible.

$$R = \frac{D}{2} = \frac{0.67}{2} = 0.335\,\text{m}$$

$$I = mR^2 = (1.1)(0.335)^2 = \boxed{0.123\,\text{kg}\cdot\text{m}^2}$$

Problem 35.

$$\alpha = \frac{0.72}{34} = 0.0212\,\text{rad/s}^2$$

$$\tau = m\frac{1}{2}R^2\alpha = (31000)\frac{1}{2}(7.0)^2(0.0212) = \boxed{16\,100\,\text{Nm}}$$

Problem 37.

(a)

$$I = \frac{1}{2}mR^2 = \frac{1}{2}(0.380)(0.0850)^2 = \boxed{1.37\times10^{-3}\,\text{kg}\cdot\text{m}^2}$$

(b)

$$1950\,\text{rpm}\times\frac{2\pi\,\text{rad/rev}}{60\text{s/min}} = 204.2\,\text{rad/s}$$

$$1500\,\text{rpm}\times\frac{2\pi\,\text{rad/rev}}{60\text{s/min}} = 157.1\,\text{rad/s}$$

$$\alpha = \frac{\omega_f - \omega_i}{t} = \frac{204.2-0}{5.00} = 40.84\,\text{rad/s}^2$$

$$\alpha_{\text{friction}} = \frac{0-157.1}{55.0} = -2.856\,\text{rad/s}$$

$$\tau = I\alpha = (1.37\times10^{-3})(40.84) = 5.596\times10^{-2}\,\text{Nm}$$

$$\tau_{friction} = I\alpha_{\text{friction}} = (1.37\times10^{-3})(-2.856) = 3.9\times10^{-3}\,\text{Nm}$$

$$\tau_{\text{net}} = 5.596\times10^{-2} + 1.37\times10^{-3} = \boxed{6.0\times10^{-2}\,\text{Nm}}$$

Problem 53.

$$I = I_{\text{cm}} + Md^2$$

$$I_{\text{cm}} = \frac{1}{12}ML^2$$

$$d = \frac{L}{2}$$

$$I = \frac{1}{12}ML^2 + M\left(\frac{L}{2}\right)^2$$

$$I = \frac{1}{12}ML^2 + M\left(\frac{L^2}{4}\right)$$

$$I = \frac{1}{12}ML^2 + \frac{1}{4}ML^2$$

$$I = \frac{1}{12}ML^2 + \frac{3}{12}ML^2$$

$$I = \frac{4}{12}ML^2 = \boxed{\frac{1}{3}ML^2}$$

Problem 54.

$$I_{\text{cm}} = \frac{1}{12}ML^2$$

$$d = \frac{L}{2}$$

$$I = I_{\text{cm}} + Md^2$$

$$I = \frac{1}{12}ML^2 + M\left(\frac{L}{2}\right)^2$$

$$I = \frac{1}{12}(19\,\text{kg})(1.0\,\text{m})^2 + 19\,\text{kg}\left(\frac{1.0\,\text{m}}{2}\right)^2$$

$$I = \frac{1}{12}(19)(1.0)^2 + 19\left(\frac{1.0^2}{4}\right) = \boxed{\frac{19}{3}\,\text{kg}\cdot\text{m}^2}$$

Problem 55.

(a)

$$I_{\text{sphere}} = \frac{2}{5}Mr_0^2$$

$$d = \frac{3r_0}{2}$$

$$I = 2\left(I_{\text{sphere}} + Md^2\right) = 2\left(\frac{2}{5}Mr_0^2 + M\left(\frac{3r_0}{2}\right)^2\right) = 2\left(\frac{2}{5}Mr_0^2 + M\frac{9r_0^2}{4}\right)$$

$$= 2\left(\frac{2}{5}Mr_0^2 + \frac{9}{4}Mr_0^2\right) = \frac{4}{5}Mr_0^2 + \frac{18}{4}Mr_0^2$$

$$= \frac{4}{5}Mr_0^2 + \frac{45}{10}Mr_0^2 = \boxed{5.3Mr_0^2}$$

(b)

$$I_{\text{simple}} = 2M\left(\frac{3r_0}{2}\right)^2 = 2M\frac{9r_0^2}{4}$$

$$= 4.5Mr_0^2$$

$$\text{Error} = \left|\frac{I - I_{\text{simple}}}{I}\right| \times 100\%$$

$$= \left|\frac{5.3Mr_0^2 - 4.5Mr_0^2}{5.3Mr_0^2}\right| \times 100\%$$

$$= \left|\frac{0.8}{5.3}\right| \times 100\%$$

$$\approx \boxed{15.1\%}$$

Problem 64.

$$M = 220\,\text{kg}, \quad R = 5.5\,\text{m}, \quad f = 3.8\,\text{rev/s}, \quad t = 18\,\text{s}$$

$$\omega = 3.8 \cdot 2\pi = 23.88\,\text{rad/s}$$

$$\tau = I\alpha = (\frac{1}{2}MR^2)(\frac{\omega}{t}) = (\frac{1}{2}(220)(5.5)^2)(\frac{23.88}{18}) = 4413.76\,\text{Nm}$$

$$P = \tau \cdot \omega = (4413.76)(23.88) = 105\,383\,\text{W}$$

$$hp = \frac{W}{746} = \frac{105383}{746} = \boxed{141.3\,\text{hp}}$$

Problem 67.

$$U_{\text{initial}} = m_B g h$$

$$U_{\text{final}} = m_A g h$$

$$K_{\text{trans}} = \frac{1}{2}m_A v^2 + \frac{1}{2}m_B v^2 = \frac{1}{2}(m_A + m_B)v^2$$

$$K_{\text{rot}} = \frac{1}{2}I\omega^2 = \frac{1}{2}\cdot\frac{1}{2}MR^2\left(\frac{v}{R}\right)^2 = \frac{1}{4}Mv^2$$

$$m_B gh = m_A gh + \frac{1}{2}(m_A + m_B)v^2 + \frac{1}{4}Mv^2$$

$$m_B gh - m_A gh = \left(\frac{1}{2}(m_A + m_B) + \frac{1}{4}M\right)v^2$$

$$v = \sqrt{\frac{m_B gh - m_A gh}{\frac{1}{2}(m_A + m_B) + \frac{1}{4}M}}$$

$$v = \sqrt{\frac{(41.0)(9.81)(2.5) - (35.0)(9.81)(2.5)}{\frac{1}{2}(35.0 + 41.0) + \frac{1}{4}(3.1)}}$$

$$v \approx \boxed{1.95\,\text{m/s}}$$

Problem 69.

$$U = mgh, \quad K_{\text{trans}} = \frac{1}{2}mv^2, \quad K_{\text{rot}} = \frac{1}{2}I\omega^2$$

$$U = K_{\text{trans}} + K_{\text{rot}}$$

$$mgh = \frac{1}{2}mv^2 + (\frac{1}{2})(\frac{1}{2}mR^2)(\frac{v}{R})^2$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{4}mv^2$$

$$gh = \frac{3}{4}v^2$$

$$v = \sqrt{\frac{4gh}{3}} = \sqrt{\frac{4(9.8)(6.5)}{3}} = \boxed{9.22\,\text{m/s}}$$