Statistics Learning Theorem

2019.02.02

Review the proof of finite hypothesis class

Solution for problem

Let x_i be the Bernoulli variable with parameter $p = Pr_{x \sim \mathcal{D}}\{x | h(x) \neq f(x)\}$. That is,

$$x_i = \begin{cases} 1 & p \\ 0 & 1 - p \end{cases}$$

Then $L_S(h) = \frac{\sum_{i=1}^{m} x_i}{m}$. The iid assumption implies that

$$E_{S \sim \mathcal{D}^m}[L_S(h)] = E_{S \sim \mathcal{D}^m}[\frac{\sum_{i=1}^{m} x_i}{m}] = \frac{\sum_{i=1}^{m} E_{x_i}[x_i]}{m} = E[x_i] = p = L_{\mathcal{D},f}(h)$$

The $E_{S \sim \mathcal{D}^m}[\frac{\sum_{i=1}^{m} x_i}{m}] = \frac{\sum_{i=1}^{m} E_{x_i}[x_i]}{m}$ follows from Fubini's theorem.

PAC Learnability

A hypothesis class $\mathcal{H}$ is PAC learnable if there exist a function $m_{\mathcal{H}} : (0, 1)^2 \to N$ and a learning algorithm with the following property: For every $, \delta \in (0, 1)$, for every distribution $\mathcal{D}$ over $\mathcal{X}$, and for every labeling function