

The problem of designing revenue-maximizing combinatorial auctions is one of the most fundamental problems in computational economics, yet it remains unsolved even for two bidders and two items for sale. In traditional economic models, it is assumed that the bidders' valuations are drawn from an underlying distribution and that the auction designer has perfect knowledge of this distribution. However, this assumption is often unrealistic, and despite this, the revenue-maximizing combinatorial auction remains unknown.

# Automated Mechanism Design

In recent years, automated mechanism design has emerged as a promising approach to designing high-revenue combinatorial auctions. The most scalable automated mechanism design algorithms take as input samples from the bidders' valuation distribution and then search for a high-revenue auction in a "sample complexity" framework. This framework measures the number of samples needed to achieve a certain level of revenue guarantee.

# Sample Complexity

The article discusses the sample complexity of automated mechanism design algorithms and provides upper and lower bounds on the sample complexity for various auction settings. The authors also discuss the trade-off between sample complexity and revenue guarantee, showing that as the revenue guarantee increases, the sample complexity also increases. Overall, the article highlights the importance of automated mechanism design in solving the problem of revenue-maximizing combinatorial auctions and provides insights into the sample complexity of these algorithms.

# Sample Complexity Analysis for Combinatorial Auction Classes

## The Problem:

- Designing automated mechanisms for combinatorial auctions
- Need to ensure empirical revenue is close to expected revenue
- Sample complexity analysis for standard hierarchy of deterministic combinatorial auction classes

# Tight Sample Complexity Bounds

## The Results:

- Tight sample complexity bounds provided for each auction class in hierarchy
- Hypothesis functions defined through multi-stage combinatorial optimization procedures
- Different from simple decision boundaries in machine learning
- Pushes boundaries of learning theory

# Extensive Study of Combinatorial Auctions

## The Introduction:

- Multi-item, multi-bidder auctions studied in economics, operations research, and computer science
- Automated mechanism design important for setting firm foundations
- Reference to source on combinatorial auctions by Cramton

# Combinatorial Auctions

- Bidders can submit bids on bundles of goods
- Allows for expression of complex valuation functions
- Practical applications in various industries

# Optimal Auction Design

- Design auctions that maximize seller's expected revenue
- Bidders' valuations assumed to be drawn from underlying distribution
- Mechanism designer has perfect information about distribution
- Open question in computational economics



# Conclusion

- Combinatorial auctions and optimal auction design important in various industries
- Ongoing research in computational economics to improve mechanisms

The text describes a design problem that the authors encountered.

- The problem is not explicitly stated in the text.
- However, the authors provide details about their approach to solving the problem.

# Authors' Addresses

The authors of the text are affiliated with Carnegie Mellon University's School of Computer Science.

- The authors' addresses are provided in the text.
- The authors' email addresses are also included.

The text ends with a reference to an arXiv article with the identifier 1606.04145v1 in the cs.LG category, dated June 13, 2016.

- The authors may have published their work in this article.
- The article may provide more details about the design problem and the authors' approach to solving it.

# Optimal Auction Design

The optimal auction design problem has been a challenge even for auctions with just two distinct items for sale and two bidders. However, a significant breakthrough was made in the study of optimal auction design with the characterization of the optimal 1-item auction by Myerson in 1981. In this auction, the winner and payment are determined based on virtual valuations, which are transformations of the bids that make weak bidders more competitive. This auction was later extended to the case of selling multiple copies of the same item by Maskin and Riley in 1989. However, the characterization of revenue-maximizing multi-item auctions has only been obtained for special cases of the two-item two-bidder setting by Avery and Hendershott in 2000 and Armstrong in 2000.

# Optimal Auction Design

Although it may be surprising that the revenue-maximizing common auction is unknown, it is expected when viewed through a computational lens. Conitzer and Sandholm proved that the problem of finding the optimal auction is computationally intractable, even for simple cases. Therefore, finding the optimal auction design requires a combination of economic theory and computational complexity analysis.

# Optimal Auction Design

- Optimal auction design is a challenging problem even for simple cases.
- Myerson's characterization of the optimal 1-item auction was a significant breakthrough.
- The revenue-maximizing multi-item auction is unknown, except for special cases.
- The problem of finding the optimal auction is computationally intractable.
- Economic theory and computational complexity analysis are required to find the optimal auction design.

The problem of finding a revenue-maximizing combinatorial auction (CA) among all deterministic CAs with discrete types is NP-complete, according to Conitzer and Sandholm (2004). This suggests that a concise characterization of revenue-maximizing CAs among deterministic CAs may not even exist.



# Automated Mechanism Design

However, in recent years, a new approach called automated mechanism design (AMD) has been used to tackle the revenue-maximizing auction design problem. In one strand of AMD research, the distribution of bidders' valuations is discretized, and the input to the design algorithm is a probability for each support point. This approach faces the challenge that the input is doubly exponential in the number of items.

# Independent-Private-Values Setting

In an independent-private-values setting, the number of support points is  $nk^2m$ , where  $n$  is the number of bidders,  $k$  is the number of discrete value levels a bidder can assign to a bundle, and  $m$  is the number of items.

# Challenges of Designing Auctions in AMD

- Bidders may have correlated valuations
- Mechanism designer may not have perfect information about bidders' valuation distribution

# Two Strands of AMD Research

- First strand: using prior information about bidders' valuations to design mechanisms (not scalable)
- Second strand: using algorithms that take samples from bidders' valuation distributions as input and optimize over a rich class of auctions (yielded deterministic mechanisms with highest known revenues in empirical evaluations)

# Future Research in AMD

- No formal characterization of the number of samples required for AMD algorithms to be effective
- Important area for future research
- AMD has potential to improve auction design and increase revenue for sellers

- The authors aim to provide a missing link in the design of auction mechanisms.
- They guarantee that the empirical revenue of the designed mechanism on the samples is close to its expected revenue on the underlying, unknown distribution over bidder valuations.
- The auctions considered in the paper achieve significantly higher revenue than the VCG baseline by weighting bidders and boosting outcomes.

# Sample Complexity Guarantees

- The authors present tight sample complexity guarantees over an extensive hierarchy of expressive CA families.
- These are the most commonly used auction families in AMD.
- The classes in the hierarchy are based on the classic VCG mechanism, which is a generalization of the well-known second-price, or Vickrey, single-item auction.

# Conclusion

- The authors have presented a mechanism that guarantees close empirical revenue to expected revenue on the underlying distribution.
- The mechanism achieves significantly higher revenue than the VCG baseline by weighting bidders and boosting outcomes.
- The authors have provided tight sample complexity guarantees over an extensive hierarchy of expressive CA families.