~~Lec (4)~~

Mathematical

solution Methods of 1st order ODEs

$$\frac{dy}{dt} = f(y, t)$$

We have 2 Methods:-

1) Separable ODE

واتا ئەتوانین x, y
جیابکەینەوە یان هەر شتێکی تر

$$* \quad \frac{dy}{dx} = \frac{y}{x} \Rightarrow \frac{dy}{y} = \frac{dx}{x}$$

یان dependent جیابکەین
لە independent

$$* \quad \frac{dy}{dt} = y + 1 \Rightarrow \frac{dy}{y+1} = dt$$

2) Linear 1st order ODE

$$\frac{dy}{dt} + f(t)y = g(t)$$

$$e^{\int f(t)\,dt}\frac{dy}{dt} + f(t)\,e^{\int f(t)\,dt}\,y = g(t)\,e^{\int f(t)\,dt}$$

$$\frac{d}{dt}\left(e^{\int f(t)\,dt}\,y\right) = g(t)\,e^{\int f(t)\,dt}\,dt$$

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2

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Date / /

$I.F = e^{\int f(t)\,dt}$

Integrating . Factor

$\frac{dy}{dt} = y+1$ when $y(0) = 5$ at $t=0$, $y=5$

initial Condition

Method 1

$\int \frac{dy}{y+1} = \int dt$

$\ln(y+1) = t + C$

$e^{\ln} y+1 = e^{(t+C)}$ ~~$+C$~~

$y+1 = e^{t} \cdot e^{C}$

$y = e^{t} \cdot e^{C} - 1$

$5 = e^{0} e^{C} - 1 \Rightarrow e^{C} = 6 \Rightarrow \boxed{y = 6e^{t} - 1}$

3

Memo No.
Date

Method 2 Linear 1st order ODE

$$\frac{dy}{dt} = y+1$$

$$\frac{dy}{dt} - y = 1$$

$$I.F = e^{\int -dt} = e^{-t}$$

$$e^{-t}\frac{dy}{dt} - e^{-t} y = e^{-t}$$

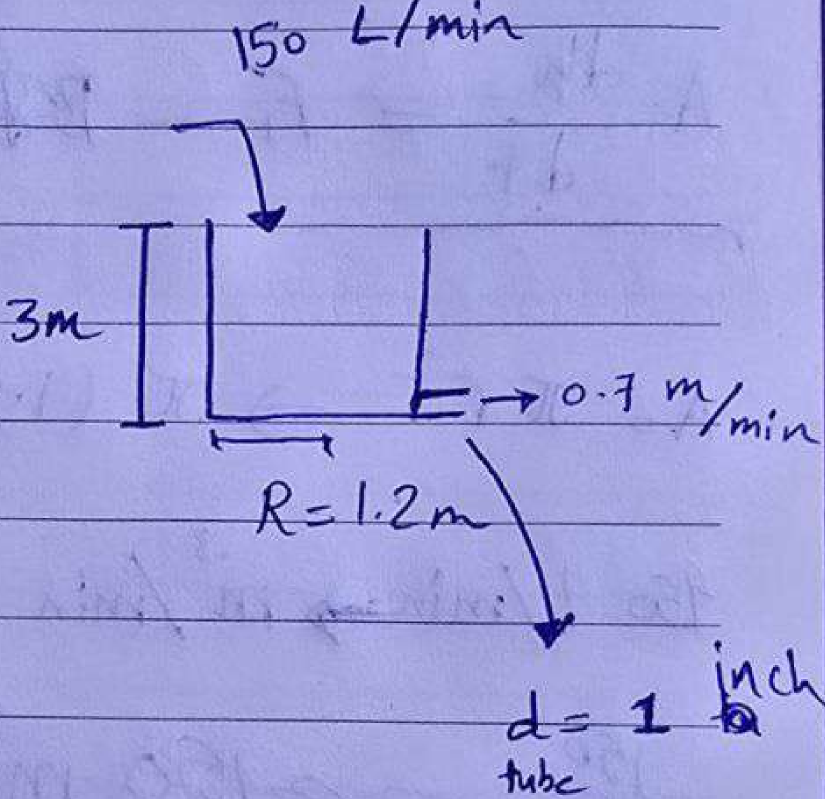
$$\int \frac{d}{dt}\left(e^{-t} y\right) = \int e^{-t} + C$$

$$\left[e^{-t} y = -e^{-t} + C\right] \div e^{-t}$$

$$\frac{c}{e^{-t}} = c \cdot e^{+t}$$

$$y = -e^{-t} \cdot e^{t} + C \cdot e^{+t}$$

Case I



1) $\frac{dh}{dt} = ?$

Solution/

Make a M.B on the tank

Acc = in − out

$$\frac{dv}{dt} = F_1 - F_2 \qquad [V = A \cdot h]$$

$$\beta\sqrt{h}$$

5

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Date / /

$$\frac{d}{dt}(A \cdot h) = F_1 - \cancel{B\sqrt{h}}\ \beta h$$

∘ linear ~~diff~~

$$A\frac{dh}{dt} = F_1 - \cancel{B\sqrt{h}}\ Bh$$

$$A = \pi r^2 \Rightarrow \pi (1.2)^2 \Rightarrow 4.524\ m^2$$

$150\ L/min \rightarrow m^3/min$

$$\frac{150}{1000} = 0.150\ m^3/min$$

outlet flow = const

$0.7 * A_{Tube}$

$$A_{Tube} = \frac{\pi}{4} d^2$$

6

Memo No.
Date / /

$A_{tube} = 5.067 * 10^{-4} m^2$

$\frac{dh}{dt} = 0.033$ m/min

$A \frac{dh}{dt} = F_1 - Bh$

$\frac{dh}{dt} = \frac{F_1 - Bh}{A}$?

$\frac{dh}{dt} + \frac{B}{A} h = \frac{F_1}{A}$

$I.F = e^{\int \frac{B}{A} dt} = e^{\frac{B}{A} t}$

$e^{\frac{B}{A} t} \frac{dh}{dt} + e^{\frac{B}{A} t} \frac{B}{A} h = \frac{F_1}{A} e^{\frac{B}{A} t}$

$\int \frac{d}{dt} (e^{\frac{B}{A} t} h) = \int \frac{F_1}{A} e^{\frac{B}{A} t} + C$

7

$\frac{d}{dt} e^{5} = 5e^{5}$ But $\int e^{5t} = \frac{e^{5t}}{5}$

Memo No. Date

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To understand

$$e^{\frac{B}{A}t} h = \frac{F_1}{\cancel{A}} \cdot \frac{\cancel{A}}{B} e^{\frac{B}{A}t} + C$$

initial condition $h(0)=0$

$$h = \frac{F_1}{B} + C e^{-\frac{B}{A}t} \rightarrow 0 = \frac{F_1}{B} + C_1$$

8

$$C = -\frac{F_1}{B}$$

$$h = \frac{F_1}{B}\left(1 - e^{-\frac{B}{A}t}\right)$$

$$\frac{dh}{dt} = \left(\frac{F_1}{B} - \frac{F_1}{\cancel{B}} e^{-\frac{\cancel{B}}{A}t}\right)\frac{d}{dt}$$

$$\frac{dh}{dt} = 0 - \frac{F_1}{\cancel{B}} * \left(-\frac{\cancel{B}}{A}\right) e^{-\frac{B}{A}t}$$

$$\boxed{\frac{dh}{dt} = \frac{F_1}{A} e^{-\frac{B}{A}t}}$$

Memo No. Date

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Part ③ A

Case (2)

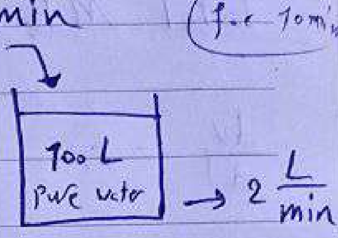
Memo No.
Date

Part ③ Assignment

Case (2)

Salt $\frac{0.5\,Kg}{\text{water } L}$

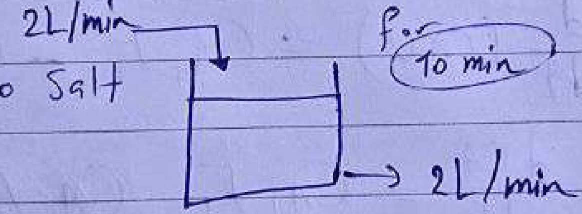
(for 10 min)



9

$\boxed{\text{After } \underline{10} \text{ min}}$

for (10 min)



Find Kg Salt
after 20 min

Memo No.
Date / /

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10

For 10 min

Total M.B

$$\frac{dv}{dt} = F_1 - F_2 = 2 - 2 = 0$$

$V =$ Constant $= 100\ L$

M.B on Salt

$$\frac{d}{dt}(C_A V) = C_{A1} F_1 - C_A F_2$$

$$V \frac{dC_A}{dt} = C_{A1} F_1 - C_A F_2$$

$$100 \frac{dC_A}{dt} = 2(0.5) - 2\,C_A$$

$$100 \frac{dC_A}{dt} = 1 - 2\,C_A$$

$$\int \frac{dC_A}{1-2C_A} =$$

$$-\frac{1}{2}\ln(1-2C_A$$

$$\ln(1-2C_A$$

$$e^{\ln(1-2C_A)}$$

$$1-2C_A =$$

$$C_A = \frac{1-}{}$$

Memo No.
Date / /

$$\int \frac{dCA}{1-2CA} = \int \frac{dt}{100} + K$$

$$\left[-\frac{1}{2}\ln(1-2CA) = \frac{t}{100} + K\right] \cancel{x} -2$$

$$\ln(1-2CA) = \frac{-\cancel{2}t}{\cancel{100}_{50}} - K$$

$$e^{\ln(1-2CA)} = e^{\left(-\frac{t}{50} - K\right)}$$

$$1-2CA = e^{-\frac{t}{50}} - K$$

$$CA = \frac{1 - e^{-\frac{t}{50}} + K}{2}$$

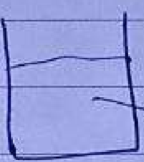
11

لە (initial condition) ئەوەندەی دەرەوەی تانکەکە

حساب ناکەین بەولای ناو تانکەکە

حسابی بۆ

هەبێ $(0.5\ kg/L)$



$CA(0)=0$

$$CA = 0.5\left(1-e^{-t/50}\right)$$

For 2nd (10) min, $CA_1=0$ (pure water) چونکە

$$V\frac{dCA}{dt} = \cancel{CA_1}F_1 - CA\,F_2$$

$$V\frac{dCA}{dt} = -CA\,F_2$$

Final Value will be the

$$\frac{dCA}{CA} = -\frac{F_2}{V} d$$

The initial

phase is

1st phase

$CA = e^{-\frac{F_2}{V}}$

$CA(10) = $

$= 0.$

$CA = $

Final Value ~~for CA~~ in 1st 10min
will Be the initial Value for 2nd 10 min

$$\frac{dC_A}{C_A} = -\frac{F_2\,dt}{V} \Rightarrow C_A = e^{-\frac{F_2}{V}t} K_2$$

The initial condition of the 2nd phase is the final Value of the 1st phase

13

$$C_A = e^{-\frac{F_2}{V}t} K_2$$

$$C_A(10) = 0.5\left(1 - e^{-\frac{10}{50}}\right)$$

$$= 0.0906 \text{ Kg/L}$$

$$C_A = 0.1106\, e^{-\frac{t}{50}} \text{ Kg/L}$$

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Memo No.
Date / /

$$\cancel{CA}\ CA = \left(0.1106\, e^{-20/50}\right) \frac{Kg}{\cancel{L}} \cdot 100\, \cancel{L}$$

$CA = 7.413$ Kg Salt

14

Case 3) $A \xrightarrow{k_1} B \xrightarrow{k_2} C$

C_{A0} : initial conc. of A

$C_{B0} = C_{C0} = 0$

Find $C_B(t)$

M.B on B)

$$\frac{d}{dt}(\cancel{V} C_B) = \cancel{V} k_1 C_A - \cancel{V} k_2 C_B$$

$$\frac{dC_B}{dt} = k_1 C_A - k_2 C_B$$

$$\frac{dC_B}{dt} + k_2 C_B = k_1 C_A$$

M.B. on A)

$$\frac{dC_A}{dt} = -k_1 C_A \Rightarrow \frac{dC_A}{C_A} = -k_1 dt$$

$$\ln C_A = -k_1 t + K$$

$$C_A = K e^{-k_1 t} \qquad C_A(0) = C_{A0}$$

$$C_{A0} = K$$

$$C_A = C_{A0} e^{-k_1 t}$$

$$\frac{dC_B}{dt} + k_2 C_B = k_1 C_{A0} e^{-k_1 t}$$

$$IF = e^{\int k_2 dt} = e^{k_2 t}$$

$$e^{k_2 t}\frac{dC_B}{dt} + k_2 e^{k_2 t} C_B = k_1 C_{A0} e^{(k_2-k_1)t}$$

$$\int \frac{d}{dt}\left(e^{k_2 t} C_B\right) = \int k_1 C_{A0} e^{(k_1-k_2)t} + \lambda$$

$$e^{k_2 t} C_B = \frac{k_1}{k_2-k_1} C_{A0} e^{(k_2-k_1)t} + \lambda$$

$$C_B = \frac{k_1}{k_2-k_1} C_{A0} e^{(\cancel{k_2}-k_1-\cancel{k_2})t} + \lambda e^{-k_2 t}$$

$$C_B = \frac{k_1}{k_2-k_1} C_{A0} e^{-k_1 t} + \lambda e^{-k_2 t}$$

$$C_{B(0)} = 0$$

$$0 = \frac{k_1}{k_2-k_1} C_{A0} + \lambda \Rightarrow C_B = \frac{k_1}{k_2-k_1}\left(e^{-k_1 t} - e^{-k_2 t}\right)$$

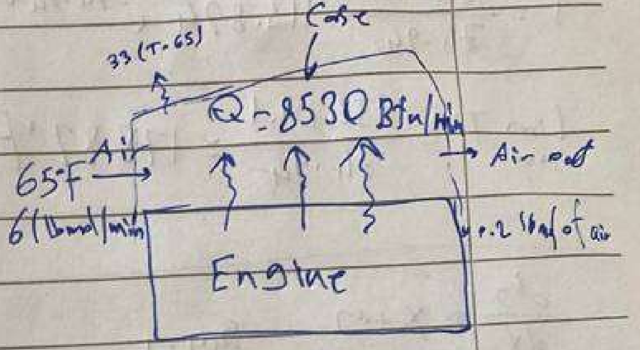
M.B on C

$$\frac{dC_C}{dt} = k_2 C_B = \frac{k_1 k_2}{k_2-k_1} C_{A0}\left(e^{-k_1 t} - e^{-k_2 t}\right)$$

Case 4

$M = 0.2\ \text{lbmol}$ $\quad \frac{dM}{dt} = 0$

$\frac{dM}{dt} = 0 = \dot m_1 - \dot m \Rightarrow \dot m_1 = \dot m$



1)

Energy Balance

~~$E = U + KE + PE$~~

~~Acc. = heat in - heat out + gen. - loss~~

$T_{(0)} = 65°F$

$$\frac{d}{dt}\left(M C_v (T - T_R)\right) = \dot m_1 C_p \overset{(T_1 - T_R)}{\Delta T} - \dot m C_p \overset{(T - T_R)}{\Delta T} + 8530 - 33(T-65)$$

$C_p = C_v + R = 5 + 1.99 = 6.99 \ \frac{\text{Btu}}{\text{lbmol.F}}$

$$M C_v \frac{dT}{dt} = \dot m_1 C_p (T_1 - T) + 8530 - 33(T-65)$$

$$0.2(5)\frac{dT}{dt} = 6(6.99)(65 - T) + 8530 - 33(T-65)$$

$$\boxed{\frac{dT}{dt} + 74.94T = 13401.1}$$

$$I.F. = e^{\int a\,dt} = e^{at}$$ where $a = 74.94$

$$e^{at}\frac{dT}{dt} + a e^{at} T = b e^{at}$$ where $b = 13401.1$

$$\int \frac{d}{dt}(e^{at}T) = \int b e^{at} + C$$

$$\left(e^{at} T = \frac{b}{a} e^{at} + C\right) \div e^{at}$$

$$T = \frac{b}{a} + C e^{-at}$$

$T_{(0)} = 65°F$

$65 = \frac{b}{a} + C \Rightarrow C = -113.824$

4-1

4-2

التاريخ / /

$$T = \frac{13401.1}{74.94} - 113.824\, e^{-74.94t}$$

$$\lim_{t \to \infty} T = \frac{13401.1}{74.94} = 178.82\,^{\circ}F = 81.11\,^{\circ}C$$