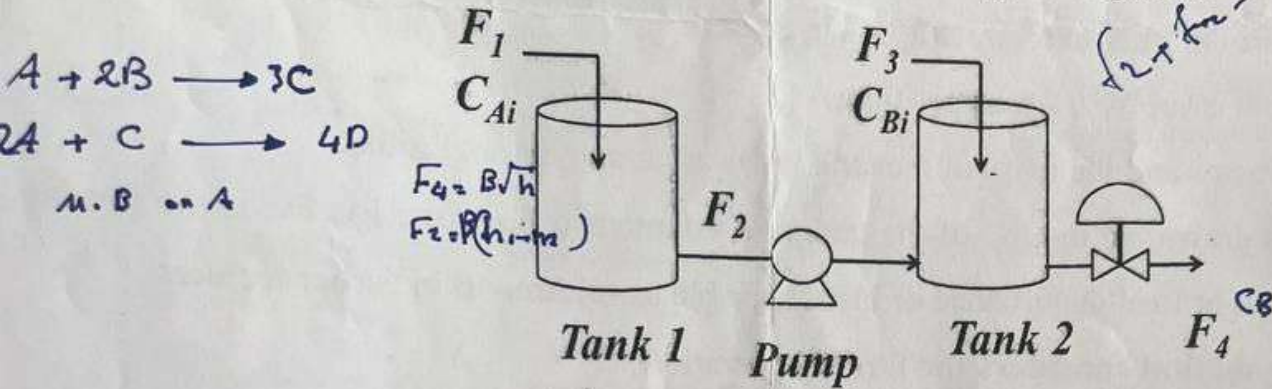


Department: Chemical Engineering
Stage/ Year: Third

Course Title: Mathematical Models
Course Code: KOU20451
Time Allowed: 2 hours

Answer All Questions

Q1. Consider the blending system shown with the following assumptions: [30 marks]



- Tanks 1 & 2 are on the same level and each has different cross-sectional area.
- F_1, F_2 are the volumetric flowrates of pure water.
- Two different solutes (A & B) are added in this process and their concentrations are expressed as C_A and C_B (kg solute/ m^3 water).
- Physical properties can be assumed to be constant everywhere.
- The valve is nonlinear such that $F \propto \sqrt{h}$ and the proportionality constants β .
- The reaction that can occur is $A + B \rightarrow C$ and is elementary.

Answer the followings:

- Is the system above interacting or none interacting and why?
- Write mathematical models to describe the liquid height in each tank.
- Write the models that describe the concentrations in each tank.

Department: Chemical Engineering
 Page/ Year: Third

Course Title: Mathematical Models
 Course Code:
 Time Allowed: 2 hours

Answer All Questions

Q1. Consider the system shown in Fig. 1 with the following assumptions:

[30 Marks]

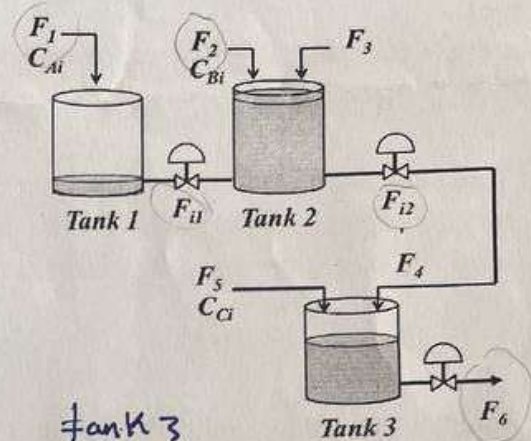


Fig. 1 Tanks System

The process above follows the Assumptions

- Tanks 1 & 2 are on the same level and tank 3 is beneath them and each has different cross-sectional area.
- F_1, F_2, \dots, F_6 are the volumetric flowrates of pure water.
- F_{11}, F_{12} are the volumetric flowrates of solution through the valves shown in Fig. 1.
- Three different solutes (A, B, and C) are added in this process and their concentrations are expressed as C_A, C_B , and C_C (kg solute/m³ water).
- F_3 stream is pure water.
- Physical properties can be assumed to be constant everywhere.
- The valves are nonlinear such that $F \propto \sqrt{h}$ and the proportionality constants are β_1, β_2 , and β_3 .
- No chemical reaction is taking place.

Bases on the case above answer the followings:

- Without any math. Describe what would happen once the process is started and the valves are opened and discuss whether this behavior changes or not after a while from the start of operation. (10 Marks)
- Write models that describe the change in the liquid height in each tank. (10 Marks)
- Write models that describe the change in the concentrations of A, B, and C. (10 Marks)

Q2. Consider the vaporizer sketched in Fig. Liquefied petroleum gas (LPG) is fed into a pressurized tank to hold the liquid level in the tank. We will assume that LPG is a pure component: propane.

Assumptions:

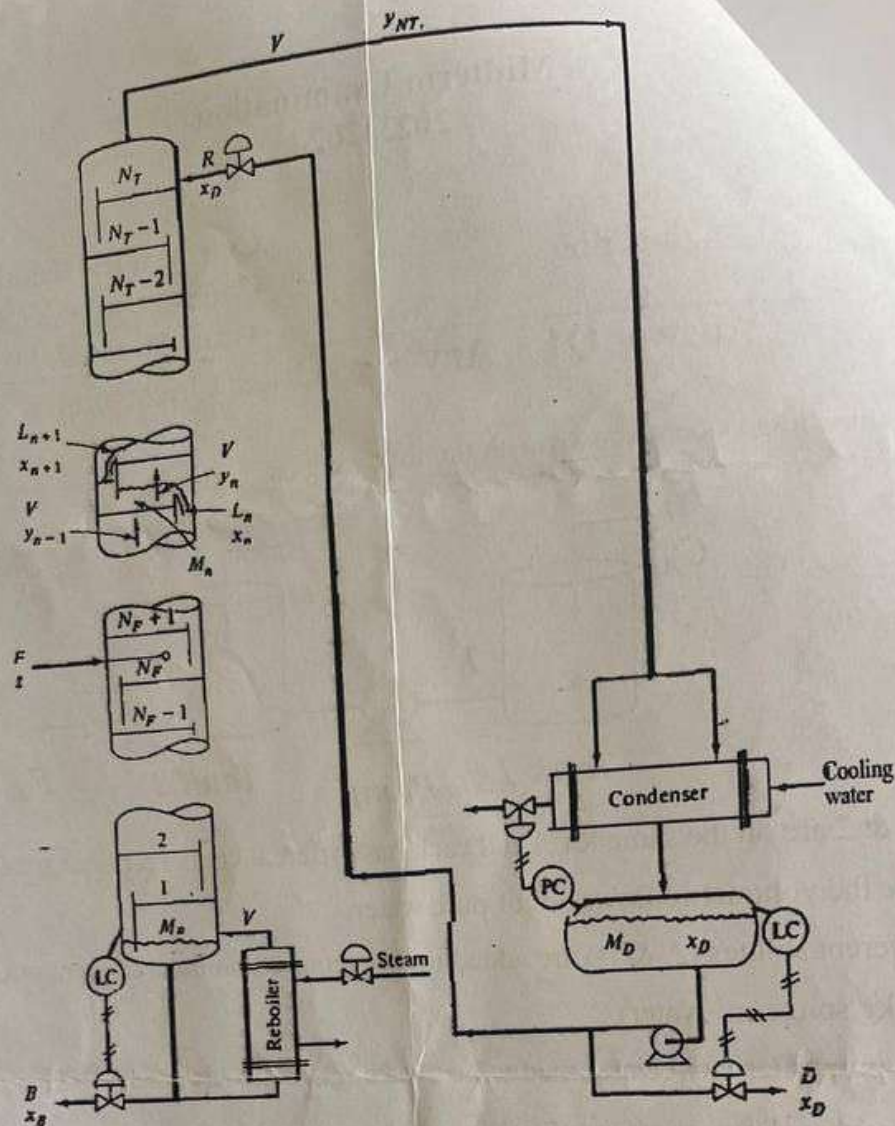
1- tank 1, 2 began will begin to fill with water from stre F_1, F_2 mean

2- Solute A, B, C Page 1-2 enter with flow 4, 5, 6.

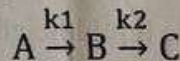
3. the out flows F_{11}, F_{12}, F_6 will higie reach certain highe

4- at beging liquid lvel increse in highe than ou

4- at time pass out increse \sqrt{h} .

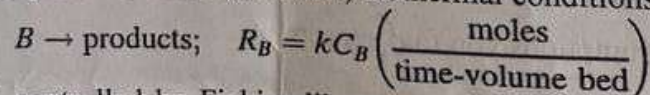


- Q3. A constant-volume batch reactor undergoes the series reaction sequence [30 marks]



The initial concentrations of A is denoted by C_{A0} , whereas B and C are initially nil. The reaction rates per unit reactor volume are elementary. Find the solution of the differential equations describing $C_B(t)$ and $C_C(t)$.

- Q4. Find the relation to predict the composition profile in a packed tube reactor undergoing isothermal linear kinetics with axial diffusion. The packed tube, heterogeneous catalytic reactor is used to convert species B by way of the reaction into products under (assumed) isothermal conditions. [30 marks]



Diffusion along the axis is controlled by Fickian-like expression so that, in parallel with transport by convection due to superficial velocity v_0 , there is also a diffusion-like flux represented by a Fickian relation

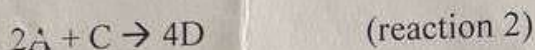
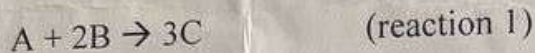
$$J_E = -D_E \frac{\partial C_B}{\partial z} \left(\frac{\text{mole}}{\text{area-time}} \right)$$

Department: Chemical Engineering
Stage/ Year: Third

Course Title: Mathematical Models
Course Code: KOU20451
Time Allowed: 90 minutes

Answer All Questions

Q1. Consider a multiple reactions system (assume a constant volume reactor).



Assume that no C is fed to the reactor. Assume that the reactions are elementary.
Derive the model for concentrations of all involved compounds variation with time.

[30 marks]

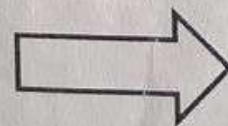
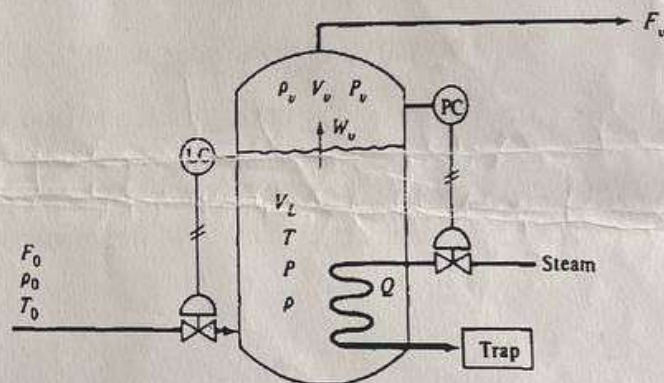
Q2. Consider the vaporizer sketched in Fig. Liquefied petroleum gas (LPG) is fed into a pressurized tank to hold the liquid level in the tank. We will assume that LPG is a pure component: propane.

Assumptions:

[30 marks]

1. The liquid in the tank is assumed perfectly mixed.
2. Heat is added at a rate Q to hold the desired pressure in the tank by vaporizing the liquid at a rate W_v , (mass per time).
3. Heat losses and the mass of the tank walls are assumed negligible.
4. Gas is drawn off the top of the tank at a volumetric flow rate F_v . F_v , is the forcing function or load disturbance or input variable as referred to in earlier lectures.

Write a model that considers both the vapor and liquid dynamics of the vaporizer.



Midterm Examination
2024-2025

Department: Chemical Engineering
Year: Third

Course Title: Mathematical Models

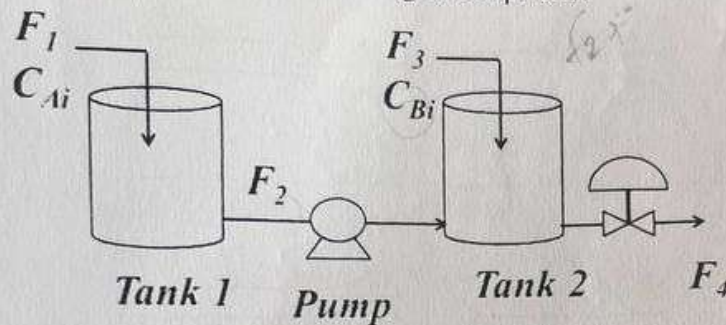
Course Code: KOU20451

Time Allowed: 90 minutes

Answer Q1 + Any Two Other Questions

Consider the blending system shown with the following assumptions:

[40 marks]



1. Tanks 1 & 2 are on the same level and each has different cross-sectional area.
2. F_1, F_2 are the volumetric flowrates of pure water.
3. Two different solutes (A & B) are added in this process and their concentrations are expressed as C_A and C_B (kg solute/ m^3 water).
4. Physical properties can be assumed to be constant everywhere.
5. The valve is nonlinear such that $F \propto \sqrt{h}$ and the proportionality constants β .
6. No reaction is taking place.

Write mathematical models to describe the liquid height and the concentrations in each tank.

2. For the case of plug flow with heat transfer, derive a mathematical model for the temperature distribution in the pipe assuming:

[30 marks]

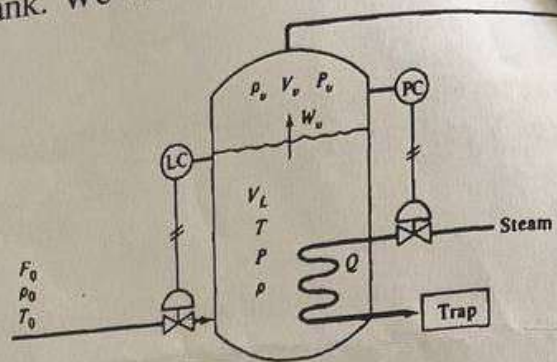
- a. A steady-state conditions.
- b. The physical properties (density, specific heat, thermal conductivity, etc.) of the fluid remain constant.
- c. The wall temperature is constant and uniform (i.e., does not change in the z or r direction) at a value T_w .
- d. The inlet temperature is constant and uniform (does not vary in r direction) at a value T_0 , where $T_0 > T_w$.
- e. The velocity profile is plug shaped or flat, hence it is uniform with respect to z or r .
- f. The fluid is well mixed (highly turbulent), so the temperature is uniform in the radial direction.
- g. Thermal conduction of heat along the axis is small relative to convection.

Q2. Consider the vaporizer sketched in Fig. Liquefied petroleum gas (LPG) is fed into a pressurized tank to hold the liquid level in the tank. We will assume that LPG is a single component: propane.

Assumptions:

[30 marks]

1. The liquid in the tank is assumed perfectly mixed.
2. Heat is added at a rate Q to hold the desired pressure in the tank by vaporizing the liquid at a rate W_v , (mass per time).
3. Heat losses and the mass of the tank walls are assumed negligible.
4. Gas is drawn off the top of the tank at a volumetric flow rate F_v . F_v is the forcing function or load disturbance or input variable as referred to in earlier lectures.



Write a mode that considers the liquid dynamics.

Q3. Water containing 0.5 kg of salt per liter (L) is poured into a tank at a rate of 2 L/min, and the well-stirred mixture leaves at the same rate. After 10 min, the process is stopped and fresh water is poured into the tank at a rate of 2 L/min, with the new mixture leaving at 2 L/min. Determine the amount (kg) of salt in the tank at the end of 20 min if there were 100 L of pure water initially in the tank.

A = $m_1 = m_2$ [40 marks]

$$I- \frac{dM_{x0}}{dt} = V_{NT} x_0 (R + D) x_0$$

$$\frac{dM_{xNT}}{dx} = R - I_{NT} -$$

$$P_v F_v () = Q_v$$

$$\rho \frac{dM_{xL}}{dt} = P_0 F_0 - P_v F_v$$

$$c_p \rho (M \frac{d(T_0 - T_{ref})}{dt}) = P_0 c_p F_0 (T_0 - T_{ref}) -$$

$$P_v F_v (c_p (T_0 - T_{ref}) + \lambda_{ref})$$

Midterm Examination
2023-2024

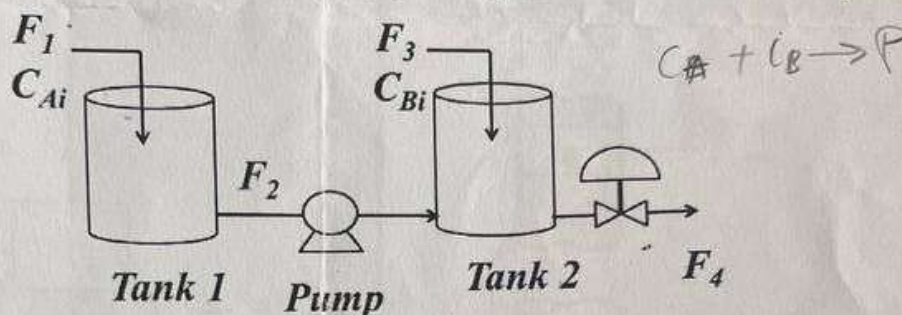
Department: Chemical Engineering
Stage/ Year: Third

Course Title: Mathematical Models
Course Code: KOU20451
Time Allowed: 90 minutes

Answer Q1 + Any Two Other Questions

Q1. Consider the blending system shown with the following assumptions:

[40 marks]



1. Tanks 1 & 2 are on the same level and each has different cross-sectional area.
2. F_1, F_2 are the volumetric flowrates of pure water.
3. Two different solutes (A & B) are added in this process and their concentrations are expressed as C_A and C_B (kg solute/ m^3 water).
4. Physical properties can be assumed to be constant everywhere.
5. The valve is nonlinear such that $F \propto \sqrt{h}$ and the proportionality constants β .
6. ~~The reaction taking place is (reaction 1) described in Question 1.~~

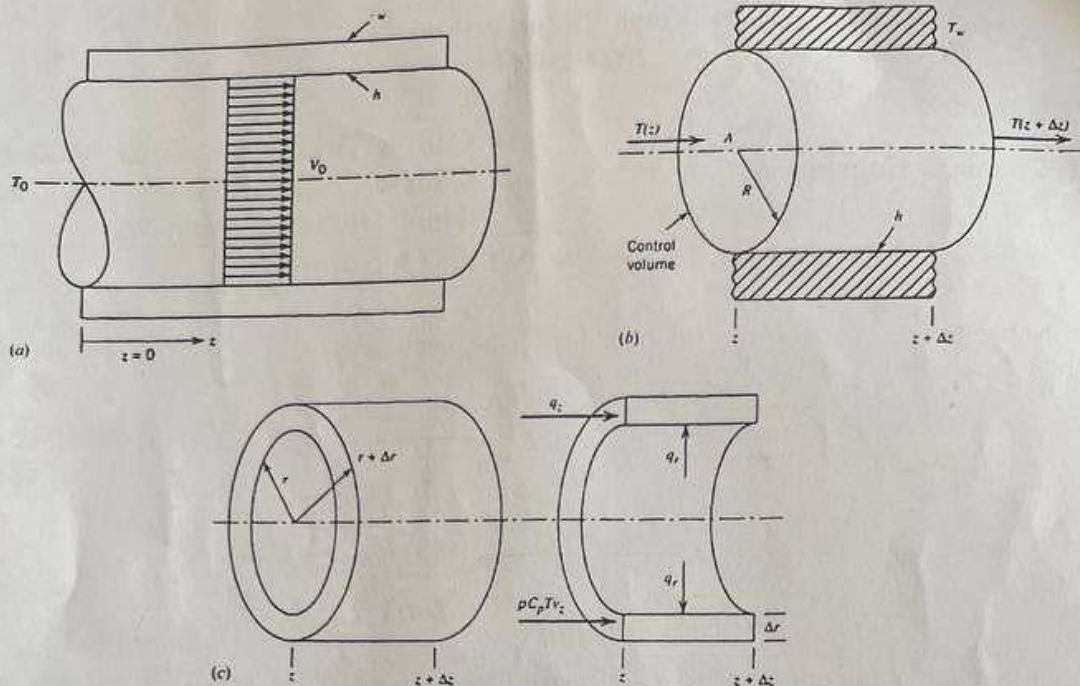
Write mathematical models to describe the liquid height and the concentrations in each tank.

Q2. For the binary distillation column shown in Fig below, derive the component and total balances for:

1. Feed stage.
2. Top stage.
3. Bottom stage.

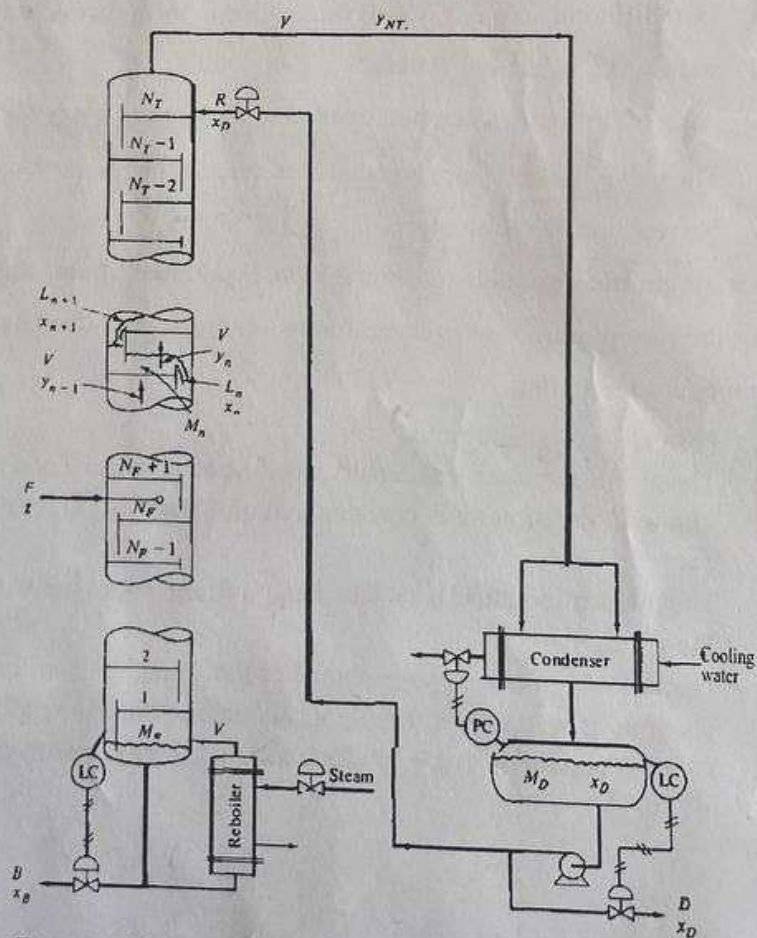
[30 marks]

Note: Assume any assumptions you find appropriate.



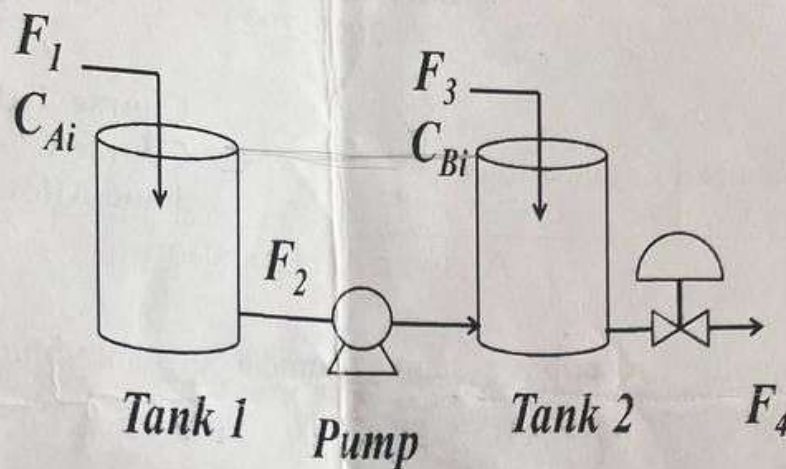
Q3. For the binary distillation column shown in Fig below, derive the component and total balances for: Feed stage, Top stage and Bottom stage. Assume any assumptions you find appropriate. [30 marks]

Q4. For the reaction system below. Find the concentrations of A, B, C, and D as a function of time, given that the reactions are elementary and these reactions are carried out in a constant volume vessel. [30 marks]



the blending system shown with the following assumptions:

[40 marks]

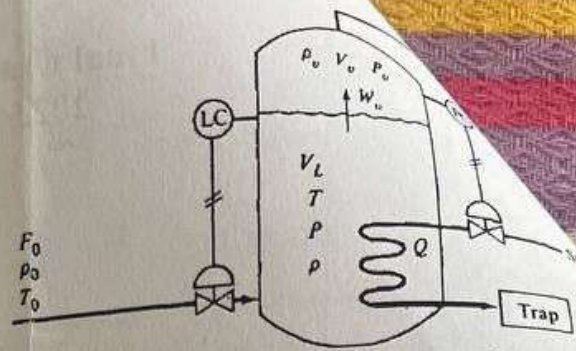


1. Tanks 1 & 2 are on the same level and each has different cross-sectional area.
2. F_1 , F_2 are the volumetric flowrates of pure water.
3. Two different solutes (A & B) are added in this process and their concentrations are expressed as C_A and C_B (kg solute/m³ water).
4. Physical properties can be assumed to be constant everywhere.
5. The valve is nonlinear such that $F \propto \sqrt{h}$ and the proportionality constants β . *at port* $F = \beta \sqrt{h}$
6. The reaction taking place is (reaction 1) described in Question 1.

Write mathematical models to describe the liquid height and the concentrations in each tank.

1. The liquid in the tank is assumed perfectly mixed.
2. Heat is added at a rate Q to hold the desired pressure in the tank by vaporizing the liquid at a rate W_v , (mass per time).
3. Heat losses and the mass of the tank walls are assumed negligible.
4. Gas is drawn off the top of the tank at a volumetric flow rate F_v . F_v is the forcing function or load disturbance or input variable as referred to in earlier lectures.

Write a mode that considers the vapor and liquid dynamics.



$$W_v = K_{MT} \frac{(P - P_v)}{(P - P_0)}$$

[25 Marks]

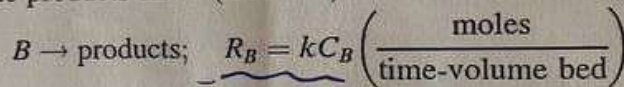
- Q3. Water containing 0.1 kg of salt per liter (L) is poured into a tank at a rate of 2 L/min, and the well-stirred mixture leaves at 1.5 L/min. Determine the amount (kg) of salt in the tank after 30 min if there were 100 L of water and 5 kg of salt initially in the tank.

[20 Marks]

Vo

- Q4. Find the relation to predict the composition profile in a packed tube reactor undergoing isothermal linear kinetics with axial diffusion. The packed tube, heterogeneous catalytic reactor is used to convert species B by way of the reaction into products under (assumed) isothermal conditions.

[25 marks]



Diffusion along the axis is controlled by Fickian-like expression so that, in parallel with transport by convection due to superficial velocity v_0 , there is also a diffusion-like flux represented by a Fickian relation

$$J_E = -D_E \frac{\partial C_B}{\partial z} \left(\frac{\text{mole}}{\text{area-time}} \right)$$

over all M.B

$$\text{In-out} + \text{generation} = \text{acc}$$

because Steady State $\text{acc} = 0$

$$\text{Convection} = v_0 C_B$$

$$\text{diffusion flux} = J_E = D_E \frac{dC_B}{dz}$$

$$\text{generation} = r = -K C_B$$

- Because

B consumed

$$\textcircled{1} \quad \frac{d}{dz} \left(-D_E \frac{dC_B}{dz} + v_0 C_B \right) = -K C_B$$

arrange

$$\left[+ D_E \frac{d^2 C_B}{dz^2} + v_0 \frac{dC_B}{dz} + K C_B \right] = 0$$

$$D_E \frac{d^2 C_B}{dz^2} + v_0 \frac{dC_B}{dz} + K C_B = 0$$

Best wishes

Assistant Prof. Dr. Saif T. Manji