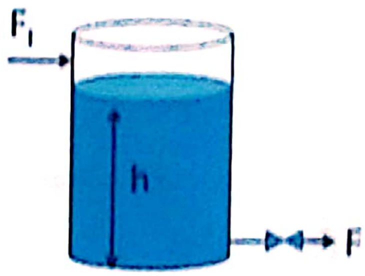
Applications of First Oder ODEs

Q1. For the cylindrical tank as shown below. It was assumed that the input and output flowrates could be independently varied. Consider a situation in which the outlet flow rate is a function of the height of liquid in the tank. Write the modeling equation for tank height assuming:

a) $F = \beta h$

b) $F = \beta \sqrt{h}$

c) Output flowrate is independent of level in the tank



where β is known as flow coefficient. List the state variables, parameters, as well as the input variables. And find the units of the coefficient for both cases.

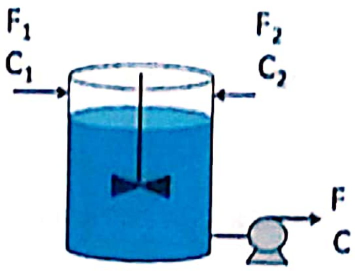
Q2. Derive a model for a mixing tank with two feed streams, as shown below. Assume that there are two components, solute A and water. C represents the mass concentration of A (kg/m³). (C_1 is the mass concentration of A in in stream 1 and C_2 is the mass concentration of A in stream 2). Consider the following cases:

a) Constant volume, constant density.

b) Constant volume, density varies linearly with concentration.

c) Variable volume, density varies linearly with concentration.

d) Replace the pump at the outlet with a valve such as in Q1/b.

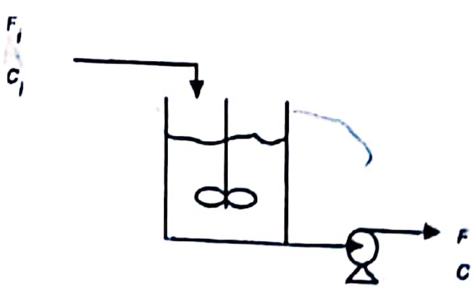


F_1 and F_2 and F are volumetric flowrates in m³/s.

Isothermal Chemical Reactor (Solved Problem)

Q3. Assume that two chemical species A and B, are in a solvent feedstream entering a liquid-phase chemical reactor that is maintained at a constant temperature. The two species react irreversibly to form a third species, P. Find the reactor concentration of each species as a function of time. Assume no P in the feed to the reactor.

Solution:



The reaction formula is

$$A + 2B \rightarrow P$$

And the reaction rate is $-r_A = kC_AC_B$

Solution

Writing a material balance for the three components

For component A $$\frac{dVC_A}{dt} = F_iC_{Ao} - FC_A - V(-r_A) \qquad (1)$$

For component B $$\frac{dVC_B}{dt} = F_iC_{Bo} - FC_B - V(-r_B) \qquad (2)$$

For component P $$\frac{dVC_P}{dt} = 0 - FC_P + Vr_P \qquad (3)$$

The accumulation term in equation (1) can be wrtitten as

$$\frac{dVC_A}{dt} = C_A\frac{dV}{dt} + V\frac{dC_A}{dt}$$

The reactor volume is constant with time (dV/dt=0)

Substituting into equation (1) to get

$$\frac{dC_A}{dt} = \frac{F_i}{V}(C_{Ao} - C_A) - kC_AC_B$$

For component B the rate of reaction $-r_B = 2kC_AC_B$. The material balance will be

$$\frac{dC_B}{dt} = \frac{F_i}{V}(C_{Bo} - C_B) - 2kC_AC_B$$

And finally for component P ($r_P = kC_AC_B$)

$$\frac{dC_P}{dt} = \frac{F_i}{V}(C_{Po} - C_P) + kC_AC_B$$

$$\frac{dC_P}{dt} = -C_P\frac{F_i}{V} + kC_AC_B$$

Q4. Consider a multiple reactions system (assume a constant volume reactor).

A + B → 2C (reaction 1)

2A + C → D (reaction 2)

Assume that no C is fed to the reactor. Assume that the reaction rate (generation) of A per unit volume for reaction 1 is characterized by expression

$$-r_{A1} = k_1C_AC_B$$

Assume that the reaction rate of A per unit volume for reaction 2 is characterized by the expression

$$-r_{A2} = k_2C_AC_C$$

Derive the model for concentrations of all involved compounds variation with time.

Q5. For the reaction system below. Find the concentrations of A, B, C, and D as a function of time, given that the reactions are elementary and these reactions are carried out in a constant volume vessel.

$$A + 2B \Leftrightarrow C \rightarrow D$$

$-r_A = k_1 C_A C_B^2$

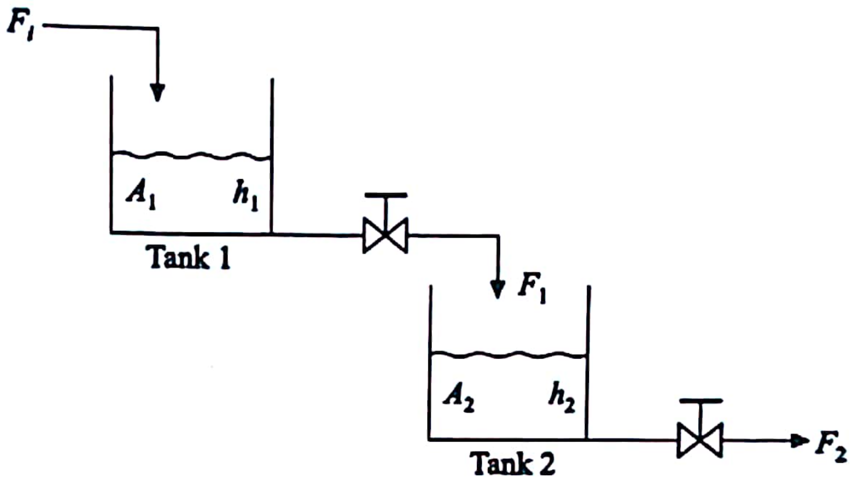
$-r_C = k_2 C_C$

$-r_C = k_3 C_C$

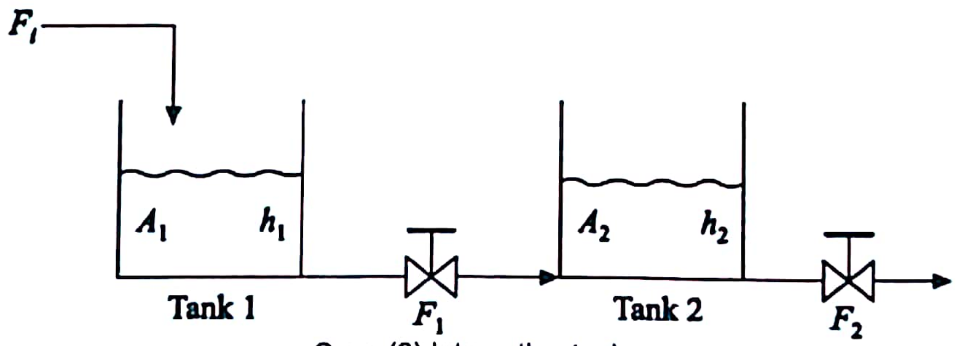
The reactions rate constants are k_1, k_2, and k_3.

Q6. Derive the dynamic mass balance equations for the following two simple cases (Figures below). Assume:

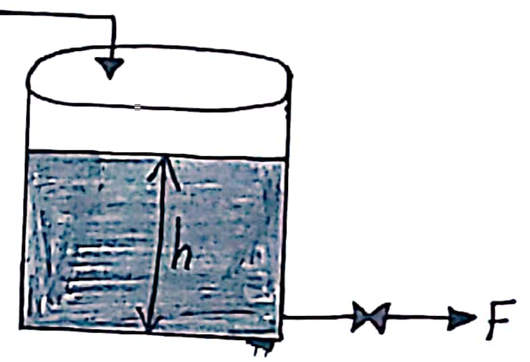
i. a linear relationship between liquid level and flow rate through the outlet valve, and
ii. constant liquid density.



Case (1) Noninteracting tanks



Case (2) Interacting tanks



a) $F = \beta h$

M: total mass in the tank at any time *
m_1 = input mass flowrate *
m = output mass flowrate *
Area is constant *
$F = \frac{m^3}{s}$ *
β = constant *

Mass Balance :-

Acc. = Input - output

$$\frac{dM}{dt} = \dot{m}_1 - m \text{ ---- (1)}$$

$$\rho = \frac{M}{V} \rightarrow M = \rho V \rightarrow M = \rho A h \text{ --- (2)}$$

$m_1 = \rho_1 F_1$ --- (3)

$m = \rho F$ --- (4)

sub eq. 2, 3, 4 in eq. 1

$$\frac{d(\rho A h)}{dt} = \rho_1 F_1 - \rho F \text{ --- (5)}$$

$F = \beta h$ --- (6)

sub eq. 6 in eq. 5

$$\boxed{\frac{d(\rho A h)}{dt} = \rho_1 F_1 - \rho \beta h}$$

ex. 1
A

Ex. 2

b) $F = \beta\sqrt{h}$

Mass Balance :-

Acc. = Input − output

$$\frac{dM}{dt} = m_1 - m \quad \text{---} (1)$$

$$\rho = \frac{M}{V} \rightarrow M = \rho V \rightarrow M = \rho A h \quad \text{---} (2)$$

$$m_1 = \rho_1 F_1 \quad \text{---} (3)$$

$$m = \rho F \quad \text{---} (4)$$

sub eq. 2, 3, 4 in eq. 1

$$\frac{d(\rho A h)}{dt} = \rho_1 F_1 - \rho F \quad \text{---} (5)$$

$$F = \beta h^{0.5} \quad \text{---} 6$$

sub equation 6 in eq. 5

$$\boxed{\frac{d(\rho A h)}{dt} = \rho_1 F_1 - \rho \beta h^{0.5}}$$

c) output flowrate is independent of level in the tank ($F \neq \beta h$)

Mass Balance :-

Acc. = Input − output

$$\frac{dM}{dt} = m_1 - m \quad \text{---} (1)$$

$$\rho = \frac{M}{V} \rightarrow M = \rho V \rightarrow M = \rho A h \quad \text{---} (2)$$

$$m_1 = \rho_1 F_1 \quad \text{---} (3)$$

$$m = \rho F \quad \text{---} (4)$$

sub eq. 2, 3, 4 in eq. 1

$$\boxed{\frac{d(\rho A h)}{dt} = \rho_1 F_1 - \rho F}$$

B

Ex:

Case 1 $F=\beta h$ $\qquad F=\frac{m^3}{s}$ ✱

$$\frac{\text{Area}\cdot\cancel{\text{height}}}{\text{time}}=\beta\,\cancel{\text{height}}$$

$$\beta=\frac{\text{Area}}{\text{time}}$$

$$\boxed{\beta=\frac{m^2}{s}}$$

Case 2

$$F=\beta h^{0.5}$$

$$\frac{\text{Vol.}}{\text{time}}=\beta\,(\text{length})^{0.5}$$

$$\boxed{\beta=\frac{\text{Vol.}}{\text{time}\cdot\text{length}^{0.5}}}$$

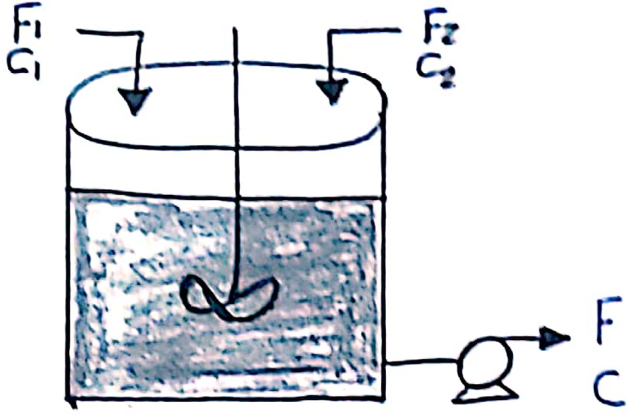
State variable / the variable that represent either mass, energy, momentum in a system. $\boxed{V, h, T, P, \text{concentrantion}}$

Input variables / are the variable chosen to specify data in the problem. $\boxed{(F_i, F)}$

Parameter / Are properties for (mechanical) the system such as $\boxed{\text{Area}, \beta}$

C

Ex.01



Total Mass Balance :-

$$Acc. = Input_1 + Input_2 - output$$

$$\frac{dM}{dt} = m_1 + m_2 - m \text{ ---- (1)}$$

$$\rho = \frac{M}{V} \rightarrow M = \rho V \text{ --- (2)}$$

$$m_1 = \rho_1 F_1 \text{ --- (3)}$$

$$m_2 = \rho_2 F_2 \text{ --- (4)}$$

$$m = \rho F \text{ --- (5)}$$

sub eq. 2,3,4,5 in eq.1

$$\frac{d(\rho V)}{dt} = F_1\rho_1 + F_2\rho_2 - F\rho \text{ ---- (6)}$$

$$C = \frac{kg \text{ of } A}{m^3 \text{ total}} \quad *$$

$$\rho = \frac{kg \text{ total}}{m^3 \text{ total}} \quad *$$

$$\dot{m} = F * \rho \quad *$$

$$\frac{m^3}{s} * \frac{kg}{m^3} = \frac{kg}{s}$$

$$\dot{m}_A = F * C \quad *$$

$$= \frac{m^3}{s} * \frac{kgA}{m^3} = \frac{kgA}{s}$$

* بوّ Input و output همووكات (F).

Mass Balance on A :-

$$\frac{d(M_A)}{dt} = m_{A_1} + m_{A_2} - m_A \text{ --- (7)}$$

$$M_A = CV \text{ -- (8)}$$

$$m_{A_1} = F_1 C_1 \text{ --- (9)}$$

$$m_{A_2} = F_2 C_2 \text{ --- (10)}$$

$$m_A = FC \text{ --- (11)}$$

sub eq. 8,9,10,11 in eq.6

$$\frac{d(CV)}{dt} = F_1 C_1 + F_2 C_2 - Fc \text{ ---- (12)}$$

ex2

A

a) Constant volume, Constant density

$$\frac{d(\rho V)}{dt} = F_1\rho_1+F_2\rho_2-F\rho \text{ ---- } (6)$$

$$\frac{d(\rho V)}{dt} = \rho\,\cancel{\frac{dV}{dt}}^{\,0} + V\,\cancel{\frac{d\rho}{dt}}^{\,0} = 0 \qquad \rho_1=\rho_2=\rho=\text{constant}$$

$$0 = F_1\cancel{\rho_1}+F_2\cancel{\rho_2}-F\cancel{\rho}$$

$$\boxed{F_1+F_2-F=0}$$

$$\frac{d(CV)}{dt} = F_1C_1+F_2C_2-FC \text{ ---- } (12)$$

$$\frac{d(CV)}{dt} = C\,\cancel{\frac{dV}{dt}}^{\,0} + V\frac{dc}{dt}$$

$$\boxed{V\cdot\frac{dc}{dt} = F_1C_1+F_2C_2-FC}$$

B

b) Constant volume, variable density

$$\frac{d(\rho V)}{dt} = F_1\rho_1 + F_2\rho_2 - F\rho \quad ---(6)$$

$$\frac{d(\rho V)}{dt} = \rho \overset{0}{\cancel{\frac{dV}{dt}}} + V\frac{d\rho}{dt}$$

$$V\frac{d\rho}{dt} = F_1\rho_1 + F_2\rho_2 - F\rho$$

$$\rho = f(T, P, C)$$

$$\rho \propto C$$

$$\boxed{\rho = XC}$$

$$\boxed{V\frac{d(XC)}{dt} = F_1XC_1 + F_2XC_2 - FXC}$$

$$\frac{d(CV)}{dt} = F_1C_1 + F_2C_2 - FC \quad ---(12)$$

$$\frac{d(CV)}{dt} = C\overset{0}{\cancel{\frac{dV}{dt}}} + V\frac{dc}{dt}$$

$$\boxed{V\frac{dc}{dt} = F_1C_1 + F_2C_2 - FC}$$

C

EqXoil

c) variable volume, variable density

$$\frac{d(\rho V)}{dt} = F_1\rho_1 + F_2\rho_2 - F\rho$$

$$\frac{d(\rho V)}{dt} = \rho\frac{dV}{dt} + V\frac{d\rho}{dt}$$

$$\rho\frac{dV}{dt} + V\frac{d\rho}{dt} = F_1\rho_1 + F_2\rho_2 - F\rho$$

$$\rho = f(\cancel{T,P},C)$$

$$\rho \propto C$$

$$\boxed{\rho = XC}$$

$$\boxed{XC\frac{dV}{dt} + V\frac{dXC}{dt} = F_1XC_1 + F_2XC_2 - FXC}$$

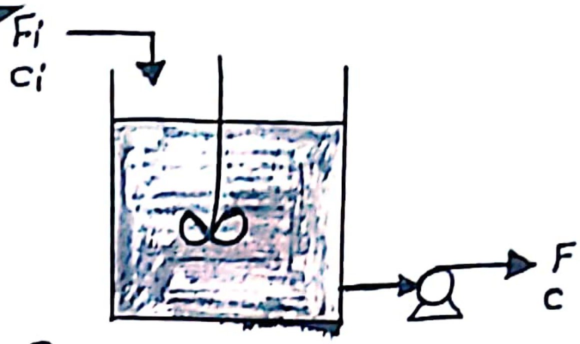
$$\frac{d(CV)}{dt} = F_1C_1 + F_2C_2 - FC \text{ --- } (12)$$

$$\frac{d(CV)}{dt} = C\frac{dV}{dt} + V\frac{dC}{dt}$$

$$\boxed{C\frac{dV}{dt} + V\frac{dC}{dt} = F_1C_1 + F_2C_2 - FC}$$

D

Ex.



$A + 2B \longrightarrow P$

$-r_A = k C_A C_B$

Assumption :-

1) Isothermal T constant
2) $V \neq$ constant
3) we have chemical reaction

$N_A = C * V$ — $\frac{mol}{m^3} * m^3$

$V * r$ — $m^3 * \frac{mol}{m^3 * s}$

consumption و gen به V(r) دەپێوسێن.

If $V = const.$ set $\frac{dV}{dt} = 0$

(Ele.) وەکو ژمارەی mol ی پێش خۆیەتی. (Non Ele.) وەکو ژمارەی mol ی پێش خۆی نیە.

M.B. on A

$$Acc. = Input - output + \overset{0}{\cancel{gen.}} - consumption$$

$$\frac{d(C_A V)}{dt} = C_{Ai} F_i - C_A F - V(-r_A)$$

$$C_A \frac{dV}{dt} + V \frac{dC_A}{dt} = C_{Ai} F_i - C_A F - V k C_A C_B$$

M.B. on B

$$Acc. = Input - output + \overset{0}{\cancel{gen.}} - consumption$$

$$\frac{d(C_B V)}{dt} = C_{Bi} F_i - C_B F - V(-r_B)$$

$$\frac{-r_B}{2} = \frac{-r_A}{1} \longrightarrow -r_B = 2(-r_A)$$

constant

$$C_B \cancel{\frac{dV}{dt}} + V \frac{dC_B}{dt} = C_{Bi} F_i - C_B F - 2V k C_A C_B$$

M.B. on P

$$Acc. = \overset{0}{\cancel{Input}} - output + gen. - \overset{0}{\cancel{consumption}}$$

$$\frac{d(C_P V)}{dt} = -C_P F + V(r_P)$$

$$\frac{r_P}{1} = \frac{-r_A}{1} \longrightarrow r_P = -r_A$$

$$C_P \frac{dV}{dt} + V \frac{dC_P}{dt} = -C_P F + V k C_A C_B$$

ex.3

A

Ex.4

$$A + B \longrightarrow 2C \quad ---①\qquad -r_{A1} = k_1 C_A C_B$$

$$2A + C \longrightarrow D \quad ---②\qquad -r_{A2} = k_2 C_A C_C$$

* (Concentration of C in feed = 0), (V = constant)

$C_A * V$ * $\frac{kgmol}{m^3} * m^3$ $\boxed{kgmol}$

$C_A * F$ * $\frac{kgmol}{m^3} * \frac{m^3}{s}$ $\boxed{kgmol/s}$

Mole Balance on A :-

Acc. = Input − output − consumption$_1$ − consumption$_2$

$$\frac{d(VC_A)}{dt} = C_{Ai}F_i - C_AF - (-r_{A1}V) - (-r_{A2}V)$$

$$C_A \cancel{\frac{dV}{dt}}^{0} + V\frac{dC_A}{dt} = C_{Ai}F_i - C_AF - Vk_1C_AC_B - Vk_2C_AC_C$$

$$\boxed{V\frac{dC_A}{dt} = C_{Ai}F_i - C_AF - Vk_1C_AC_B - Vk_2C_AC_C}$$

Mole Balance on B :-

Acc. = Input − output − consumption

$$\frac{d(VC_B)}{dt} = C_{Bi}F_i - C_BF - (-r_BV)$$

$$\frac{-r_B}{1} = \frac{-r_A}{1}$$

$$\boxed{-r_B = -r_A}$$

$$C_B \cancel{\frac{dV}{dt}}^{0} + V\frac{dC_B}{dt} = C_{Bi}F_i - C_BF - Vk_1C_AC_B$$

$$\boxed{V\frac{dC_B}{dt} = C_{Bi}F_i - C_BF - Vk_1C_AC_B}$$

ex.4

A

Extra

Mole Balance on C :-

Acc. = Input − output + gen. − cons.

$$\frac{d(VC_C)}{dt} = \cancel{C_{C0}F_0}^{\,0} - C_C F + r_{C1}V - r_{C2}V$$

$$\frac{r_{C1}}{2} = \frac{-r_{A1}}{1}$$

$$\boxed{r_{C1} = 2(-r_{A1})}$$

$$\cancel{C_C\frac{dV}{dt}}^{\,0} + V\frac{dC_C}{dt} = -C_C F + 2V(-r_{A1}) - \frac{1}{2}V(-r_{A2})$$

$$\frac{r_{C2}}{1} = \frac{-r_{A2}}{2}$$

$$\boxed{r_{C2} = \frac{1}{2}(-r_{A2})}$$

$$\boxed{V\frac{dC_C}{dt} = -C_C F + 2Vk_1C_AC_B - \frac{1}{2}Vk_2C_AC_C}$$

ئەگەر نە یەکۆتبا assume بکە (Feed=0) ئەوا سفرت نەدەکرد،

Mole Balance on D :-

flow rate

Acc. = Input − output + gen.

flow rate

$$\frac{d(VC_D)}{dt} = C_{Di}F_i - C_D F + Vr_D$$

$$\frac{r_D}{1} = \frac{-r_{A2}}{2}$$

$$\boxed{r_D = \frac{1}{2}(-r_{A2})}$$

$$\cancel{C_D\frac{dV}{dt}}^{\,0} + V\frac{dC_D}{dt} = 0 - C_D F + \frac{1}{2}V(-r_{A2})$$

$$\boxed{V\frac{dC_D}{dt} = -C_D F + \frac{1}{2}Vk_2C_AC_C}$$

B

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Ex.7

$$A + 2B \rightleftharpoons C \longrightarrow D$$

$A + 2B \longrightarrow C$ $\quad -r_A = k_1 C_A C_B^2$

$C \longrightarrow A + 2B$ $\quad -r_{C_1} = k_2 C_C$

$C \longrightarrow D$ $\quad -r_{C_2} = k_3 C_C$ $\quad$ (V = constant)

Mole Balance on A :-

Acc. = Input − output + gen. − comp.

$\frac{r_{A_1}}{1} = \frac{-r_{C_1}}{1}$

$\boxed{-r_{C_1} = r_{A_1}}$

$$\frac{d(C_A V)}{dt} = C_{A1}F_1 - C_A F + (r_{A_1} V) - (-r_A V)$$

$$V\frac{dC_A}{dt} + \cancel{C_A \frac{dV}{dt}}^{0} = C_{A1}F_1 - C_A F + (-r_{C_1} V) - (-r_A V)$$

$$\boxed{V\frac{dC_A}{dt} = C_{A1}F_1 - C_A F + V k_2 C_C - V k_1 C_A C_B^2}$$

Mole Balance on B :-

Acc. = Input − output − consup + gen.

$\frac{-r_{B_1}}{2} = \frac{-r_A}{1}$

$\boxed{-r_{B_1} = 2(-r_A)}$

$\frac{r_{B_2}}{2} = \frac{-r_{C_1}}{1}$

$\boxed{r_{B_2} = 2(-r_{C_1})}$

$$\frac{d(C_B V)}{dt} = C_{B1}F_1 - C_B F - (-r_{B_1} V) + (V r_{B_2})$$

$$V\frac{dC_B}{dt} + \cancel{C_B \frac{dV}{dt}}^{0} = C_{B1}F_1 - C_B F - 2V(-r_A) + 2V(-r_{C_1})$$

$$\boxed{V\frac{dC_B}{dt} = C_{B1}F_1 - C_B F - 2V k_1 C_A C_B^2 + 2V k_2 C_C}$$

ex.5

A

Ex

Mole Balance on C :-

Acc. = Input − output − consp.1 − consp.2 + gen.

$$\frac{d(VC_c)}{dt} = C_{ci}F_i - C_cF - (-r_{c_1}V) - (-r_{c_2}V) + (r_{c_3}V)$$

$$\frac{r_{c_3}}{1} = \frac{-r_A}{1}$$

$$\boxed{r_{c_3} = -r_A}$$

$$V\frac{dC_c}{dt} + C_c\cancel{\frac{dV}{dt}}^{0} = C_{ci}F_i - C_cF - (-r_{c_1}V) - (-r_{c_2}V) + (-r_AV)$$

$$\boxed{V\frac{dC_c}{dt} = C_{ci}F_i - C_cF - Vk_2C_c - Vk_3C_c + Vk_1C_AC_B^2}$$

Mole Balance on D :-

Acc. = Input − output + gen.

$$\frac{d(VC_D)}{dt} = C_{Di}F_i - C_DF + (r_DV)$$

$$\frac{r_D}{1} = \frac{-r_{c_2}}{1}$$

$$\boxed{r_D = -r_{c_2}}$$

$$V\frac{dC_D}{dt} + C_D\cancel{\frac{dV}{dt}}^{0} = 0 - C_DF + (-r_{c_2}V)$$

$$\boxed{V\frac{dC_D}{dt} = -C_DF + Vk_3C_c}$$

B

Case I: Non Interacting Tanks

Mass Balance on Tank 1 (ρ = constant)

Acc. = Input − output

$$\frac{d(M_1)}{dt} = m_i - m_1 \quad \cdots (1)$$

$$\rho = \frac{M_1}{V_1} \rightarrow M_1 = \rho_1 V_1 \quad \cdots (2)$$

$$m_i = \rho_i F_i \quad \cdots (3)$$

$$m_1 = \rho F \quad \cdots (4)$$

sub eq 2, 3, 4 in eq. 1

$$\frac{d(\rho_1 V_1)}{dt} = \rho_i F_i - \rho_1 F_1$$

$$\cancel{\rho_1}\frac{dV_1}{dt} = \cancel{\rho_i} F_i - \cancel{\rho_1} F_1$$

$$\frac{dV_1}{dt} = F_i - F_1 \quad \cdots (5)$$

$$V_1 = A_1 h_1 \quad \cdots (6)$$

$$F_1 = \beta_1 h_1 \quad \cdots (7)$$

sub eq 6, 7 in eq. 5

$$\frac{d(A_1 h_1)}{dt} = F_i - \beta_1 h_1$$

$$\boxed{A_1 \frac{dh_1}{dt} = F_i - \beta_1 h_1}$$

* (holdup) واتە تەواوی بڕی liquid ەکەی لە
تانکەکەدا هەیە بەشێکی بچێ دەرچووبێتی،

* کە گوتر بە volume ـەکە پێک بوو لەبەر (ρ=const)،

* dynamic = transient = unsteady state

* ($F_1 = \beta_1 h_1$) تا سادەتر بێت هاوکێشەکە
بەو پەیوەندی نێوان flow rate و height.

ex. 6

A

Mass Balance on Tank$_2$ (Case 1)

$$Acc. = Input - output$$

$$\frac{d(M_2)}{dt} = m_1 - m_2 \quad \cdots (1)$$

$$M_2 = \rho_2 V_2 \quad \cdots (2)$$

$$m_1 = \rho_1 F_1 \quad \cdots (3)$$

$$m_2 = \rho_2 F_2 \quad \cdots (4)$$

sub eq. 2, 3, 4 in eq. 1

$$\frac{d(\rho_2 V_2)}{dt} = \rho_1 F_1 - \rho_2 F_2$$

$$\cancel{\rho_2} \frac{dV_2}{dt} = \cancel{\rho_1} F_1 - \cancel{\rho_2} F_2$$

$$\frac{dV_2}{dt} = F_1 - F_2 \quad \cdots (5)$$

$$V_2 = A_2 h_2 \quad \cdots (6)$$

$$F_1 = \beta_1 h_1 \quad \cdots (7)$$

$$F_2 = \beta_2 h_2 \quad \cdots (8)$$

sub eq. 6, 7, 8 in eq. 5

$$\frac{d(A_2 h_2)}{dt} = \beta_1 h_1 - \beta_2 h_2$$

$$\boxed{A_2 \frac{dh_2}{dt} = \beta_1 h_1 - \beta_2 h_2}$$

B

(Case 2) Interacting Tanks

(ρ = constant)

Mass Balance on Tank 1

Acc. = Input − output

$$\frac{d(M_1)}{dt} = m_i - m_1 \quad \cdots (1)$$

(F_1) [illegible] (Inter) [illegible]
(M.B.1) [illegible]

$M_1 = \rho_1 V_1 \quad \cdots (2)$

$m_i = \rho_i F_i \quad \cdots (3)$

$m_1 = \rho_1 F_1 \quad \cdots (4)$

sub eq. 2, 3, 4 in eq. 1

$$\frac{d(\rho_1 V_1)}{dt} = \rho_i F_i - \rho_1 F_1$$

$$\rho_1 \frac{dV_1}{dt} = \rho_i F_i - \rho_1 F_1$$

$$\frac{dV_1}{dt} = F_i - F_1 \quad \cdots (5)$$

$V_1 = A_1 h_1 \quad \cdots (6)$

sub eq. 6 in eq. 5

$$\frac{d(A_1 h_1)}{dt} = F_i - F_1$$

$$\boxed{A_1 \frac{dh_1}{dt} = F_i - F_1}$$

Mass Balance on Tanks — Case 2

Acc. = Input − output

$$\frac{d(M_2)}{dt} = m_1 - m_2 \quad \text{---} (1)$$

$$M_2 = \rho_2 V_2 \quad \text{---} (2)$$

$$m_1 = \rho_1 F_1 \quad \text{---} (3)$$

$$m_2 = \rho_2 F_2 \quad \text{---} (4)$$

sub eq 2, 3, 4 in eq (1)

$$\frac{d(\rho_2 V_2)}{dt} = \rho_1 F_1 - \rho_2 F_2$$

$$\cancel{\rho_2} \frac{dV_2}{dt} = \cancel{\rho_1} F_1 - \cancel{\rho_2} F_2$$

$$\frac{dV_2}{dt} = F_1 - F_2 \quad \text{---} (5)$$

$$V_2 = A_2 h_2 \quad \text{---} (6)$$

$$F_1 = \beta_1 (h_1 - h_2) \quad \text{---} (7)$$

$$F_2 = \beta_2 h_2 \quad \text{---} (8)$$

sub eq. 6, 7, 8 in eq. 5

$$\frac{d(A_2 h_2)}{dt} = \beta_1 (h_1 - h_2) - \beta_2 h_2$$

$$\boxed{A_2 \frac{dh_2}{dt} = \beta_1 (h_1 - h_2) - \beta_2 h_2}$$

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