BCSE304L
Theory of Computation

Lecture 3

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15-12-2022

Operations on Languages

- Complementation
- Union
- Intersection
- Concatenation
- Reversal
- Closure

Complementation

Let L be a language over an alphabet Σ.

The complementation of L, denoted by $\overline{L}$ (or L'), is $\Sigma^* - L$

Example:

Let $\Sigma = \{0, 1\}$ be the alphabet.

L = $\{\omega \in \Sigma^*$ | the number of 1's in ω is even$\}$

L' = $\{\omega \in \Sigma^*$ | the number of 1's in ω is not even$\}$

L' = $\{\omega \in \Sigma^*$ | the number of 1's in ω is odd$\}$

Union

Let L_1 and L_2 be languages over an alphabet Σ.

The union of L_1 and L_2 denoted by $L_1 \cup L_2$ is

$L_1 \cup L_2 = \{x \mid x$ is in L_1 or $L_2\}$.

Example:

$L_1 = \{ x \in \{0,1\}^* \mid x$ begins with 0 $\}$

$L_2 = \{x \in \{0,1\}^* \mid x$ ends with 0$\}$

$L_1 \cup L_2 = \{x \in \{0,1\}^* \mid x$ begins or ends with 0$\}$

Union

$$\{0,01,011,010,0110\} \cup \{0,10,110,0110\}$$

$$=\{0,01,011,010,10,110,0110\}$$

Intersection

Let L_1 and L_2 be languages over an alphabet Σ.

Intersection of L_1 and L_2, denoted by $L_1 \cap L_2$ is

$L_1 \cap L_2 = \{ x \mid x$ is in L_1 and $L_2\}$.

Example:

$L_1 = \{ x \in \{0,1\}^* \mid x$ begins with 0$\}$

$L_2 = \{ x \in \{0,1\}^* \mid x$ ends with 0$\}$

$L_1 \cap L_2 = \{ x \in \{0,1\}^* \mid x$ begins and ends with 0$\}$

Intersection

$$\{0,01,011,010,0110\} \cap \{0,10,110,0110\}$$

$$= \{0,0110\}$$

Concatenation

Let L_1 and L_2 be languages over an alphabet Σ.

The concatenation of L_1 and L_2, denoted by $L_1 \cdot L_2$ is

$L_1 \cdot L_2 = \{w_1 \cdot w_2 \mid w_1$ is in L_1 and w_2 is in $L_2\}$.

Example:

$L_1 = \{ x \in \{0,1\}^* \mid x$ begins with 0$\}$

$L_2 = \{ x \in \{0,1\}^* \mid x$ ends with 0$\}$

$L_1 \cdot L_2 = \{ x \in \{0,1\}^* \mid$ x begins and ends with 0 and length(x) $\geq 2\}$

$L_2 \cdot L_1 = \{ x \in \{0,1\}^* \mid x$ has 00 as a substring$\}$

Concatenation

{0,01,010,0110} .{0,0110,110}

=

{00,010,0100,01100,00110,010110,0100110,01100110,0110,01
110,010110,0110110}

{0,0110} . {0,01,010,0110}

={00, 001, 0010, 00110, 01100, 011001, 0110010, 01100110}

Reversal

Let L be a language over an alphabet Σ.

The reversal of L, denoted by L^R is $\{w^R \mid w$ is in $L\}$.

Example

$L =$ $\{x \in \{0,1\}^* \mid x$ begins with 0$\}$

$L^R = \{x \in \{0,1\}^* \mid x$ ends with 0$\}$

$L =$ $\{x \in \{0,1\}^* \mid x$ has 00 as a substring$\}$

$L^R = \{x \in \{0,1\}^* \mid x$ has 00 as a substring$\}$

Reversal

$\{0,01,011,0111,0110\}^R = \{0,10,110,1110,0110\}$

$\{00,001,01001,011001,00110\}^R =$

$\{00,100,10010,100110,01100\}$

Kleene's closure

Let L be a language over an alphabet Σ.

The Kleene's closure of L, denoted by L^* is
$\{x$ | for an integer $n \geq 0$, $x = x_1 x_2 \ldots x_n$ and $x_1, x_2, \ldots,$
x_n are in $L\}$.

$$L^* = \cup_{i=0}^{\infty} L^i$$

Kleene's closure

$L=\{a,ab\}$

$L^*=L^0 \cup L^1 \cup L^2 \cup \ldots$

$L^* = \{\varepsilon\} \cup \{a, ab\} \cup \{aa, aab, aba, abab\} \cup \quad \ldots$

Kleene's closure

Example:

Let Σ = {0,1} and

L_e = {$\omega \in \Sigma^*$ | the number of 1's in ω is even}

L_e* = {$\omega \in \Sigma^*$ | the number of 1's in ω is even}

($\overline{L}_e$)* = {$\omega \in \Sigma^*$| the number of 1's in ω is odd}*

= {$\omega \in \Sigma^*$| the number of 1's in ω > 0}

Kleene's closure

L_e = {110,1010110,101}

L_e* = {ε} $\cup$ {110,1010110,101} $\cup \{$ 110110, 1101010110, 110101, 1010110110, 10101101010110, 1010110101, 101110, 1011010110,101101 $\} \cup \ldots$