Petri Nets

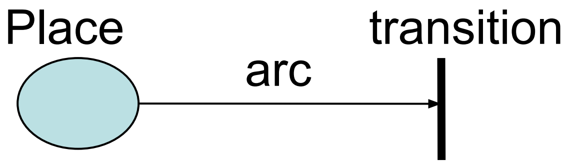
Lesson 4

Special thanks to J.P. Bahsoun

Petri nets: Model parallel activities

Definition:

- Oriented Bipartite Graph, consists of two sets of vertices (place, transitions) and one set of arcs.



Representation:

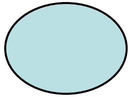
- An arc (arrow) links a place (resp. a transition) to a transition (resp. a place).
- A place expresses a state, an activity,
- a condition, a precondition or a postcondition
- A transition expresses an event

Places and transitions are numbered or labeled

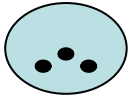
Place

A place has a name and expresses its semantics according to the presence of tokens

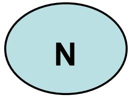
Empty canal



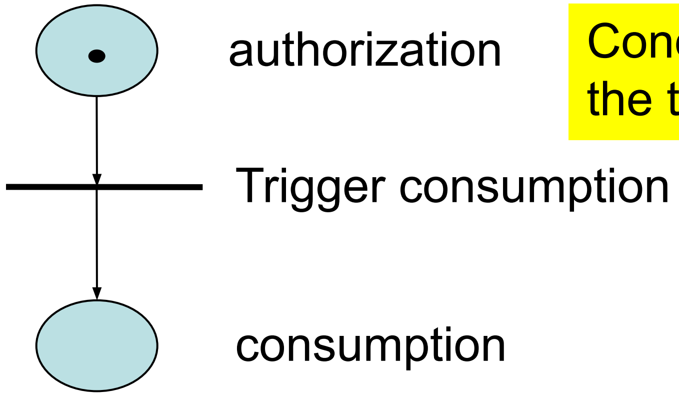
Canal of capacity 3



Canal of capacity N



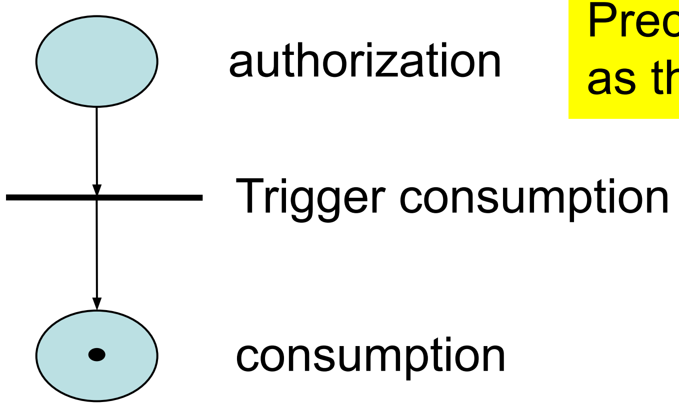
Example of Transition



Condition of
the transition

Transition armée, exprimée
Par la présence d’un jeton
dans la condition
Peut être déclenchée
À tout moment

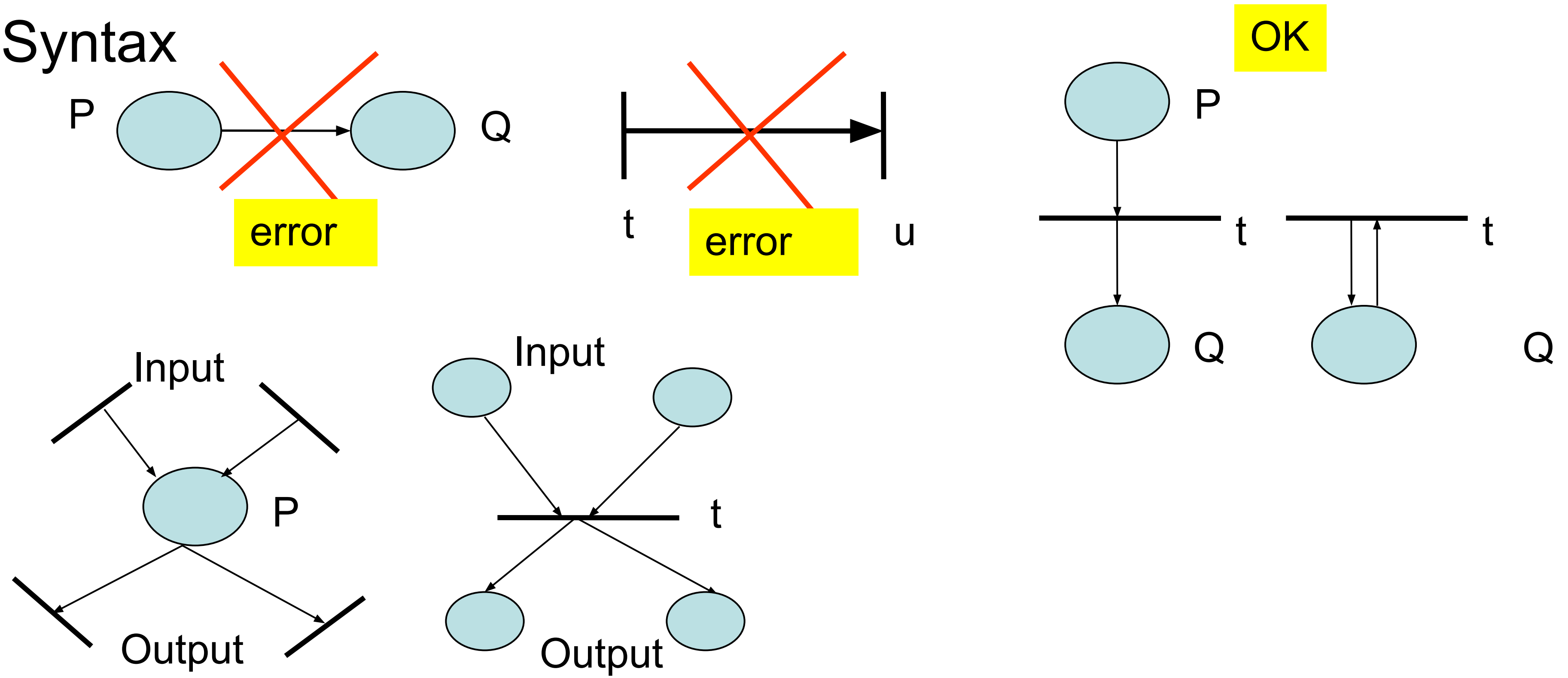
Example of Transition



Precondition not fulfilled as there is no token

Disarmed transition, cannot be activated

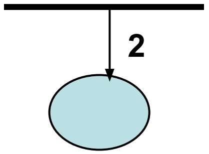
Syntax



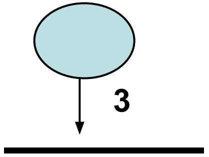
Syntax

An arc has a default weight of 1.

Generates 2 tokens

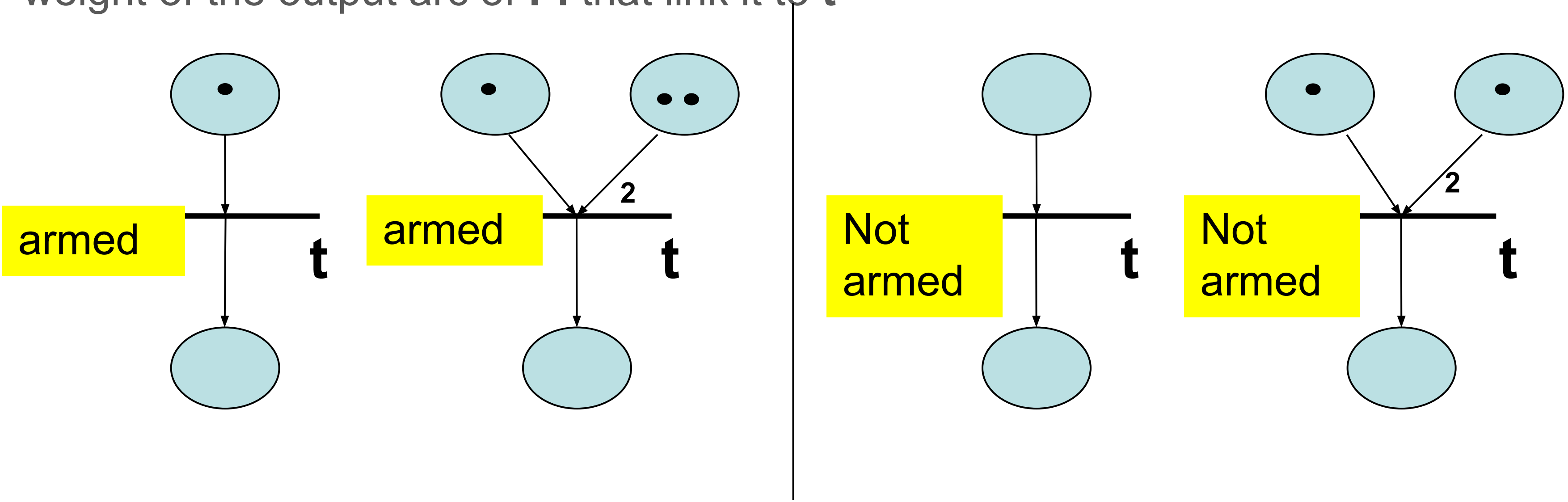


At least 3 tokens needed to arm the transition



Semantic

A transition **t** is armed if each input place **Pi** contains tokens of number >= the weight of the output arc of **Pi** that link it to **t**

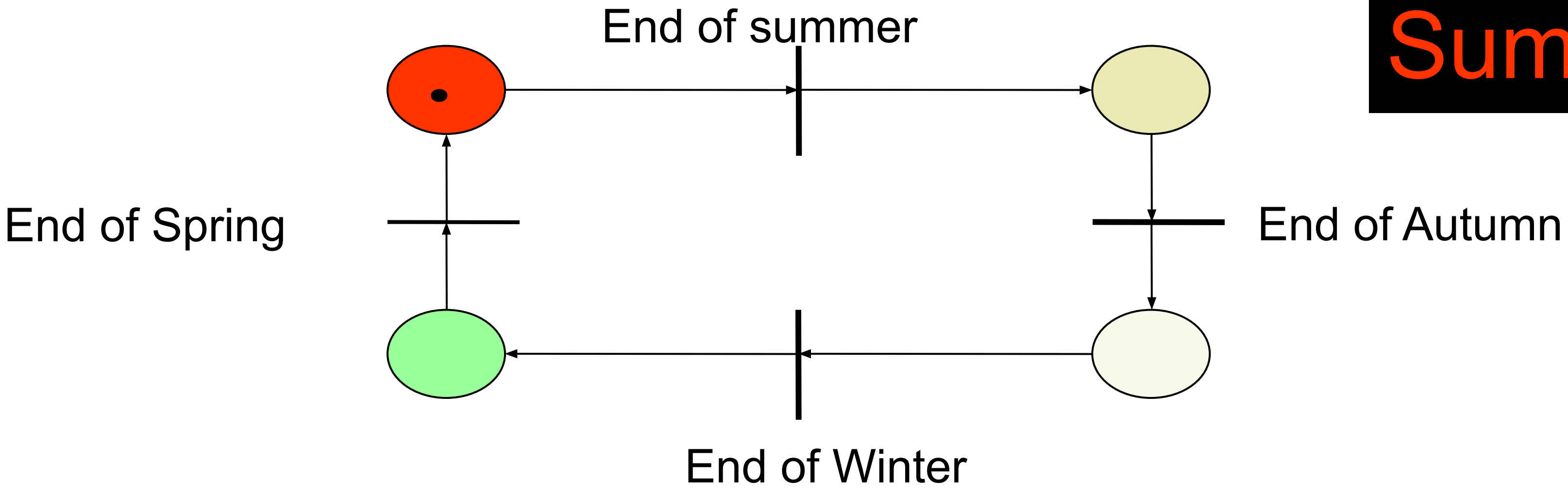


Dynamic modeling

One transition is triggered at a time

The triggering of a transition:

- eventually leads to the circulation of tokens
- eventually leads the RoP from one state to another



Dynamic modeling

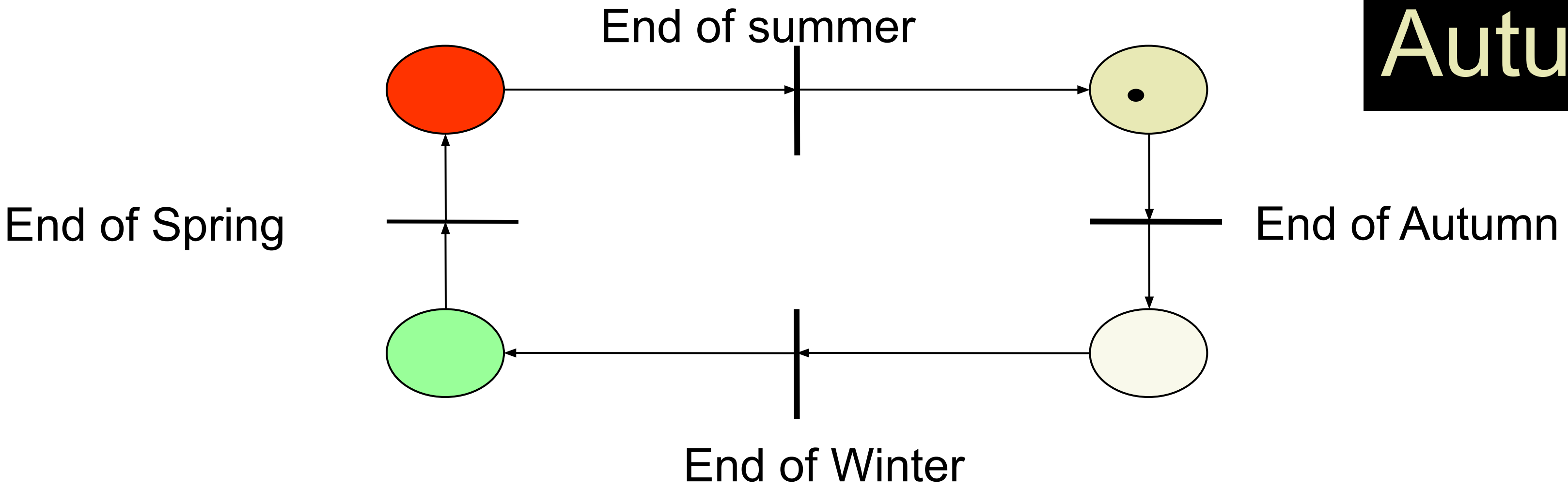
Summer Autumn Winter Spring

One transition is triggered at a time

The triggering of a transition:

- eventually leads to the circulation of tokens
- eventually leads the RoP from one state to another

Autumn



Dynamic modeling

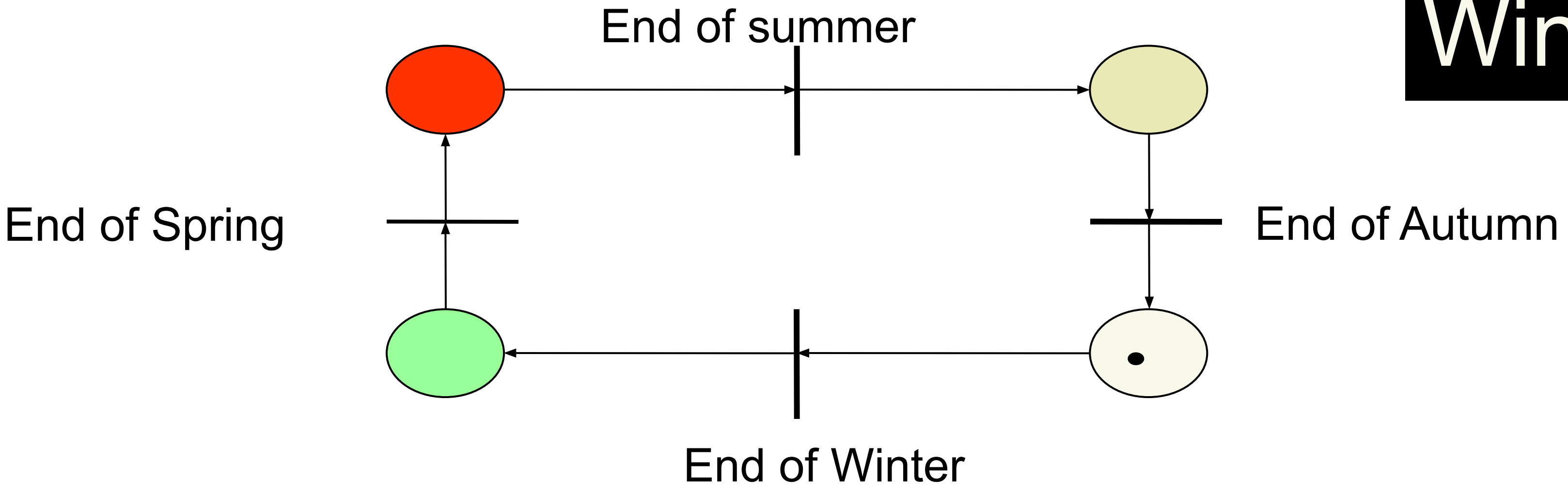
Summer Autumn Winter Spring

One transition is triggered at a time

The triggering of a transition:

- eventually leads to the circulation of tokens
- eventually leads the RoP from one state to another

Winter



Dynamic modeling

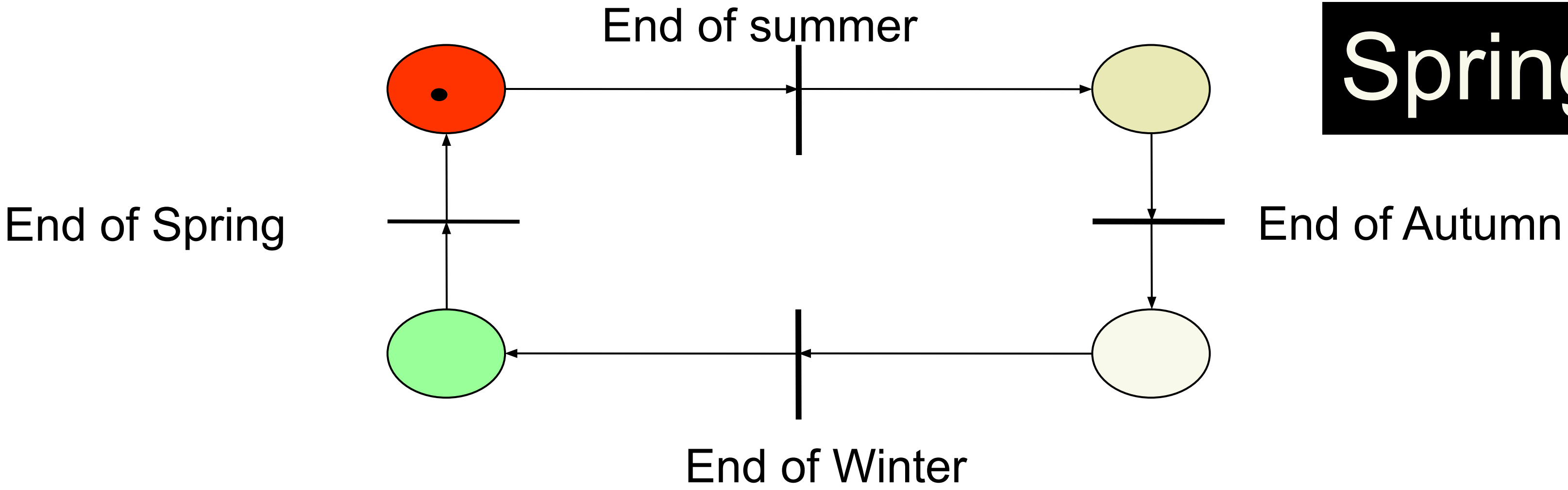
Summer Autumn Winter Spring

One transition is triggered at a time

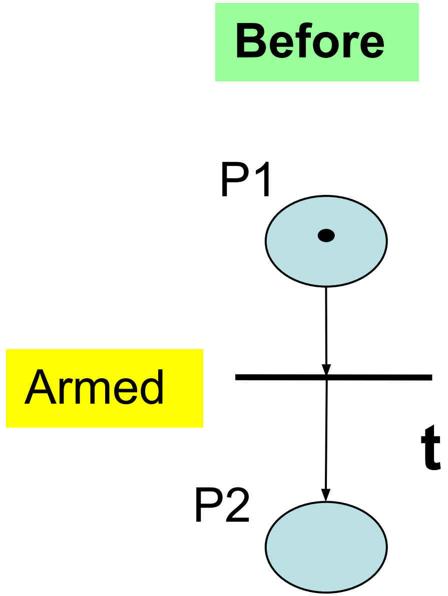
The triggering of a transition:

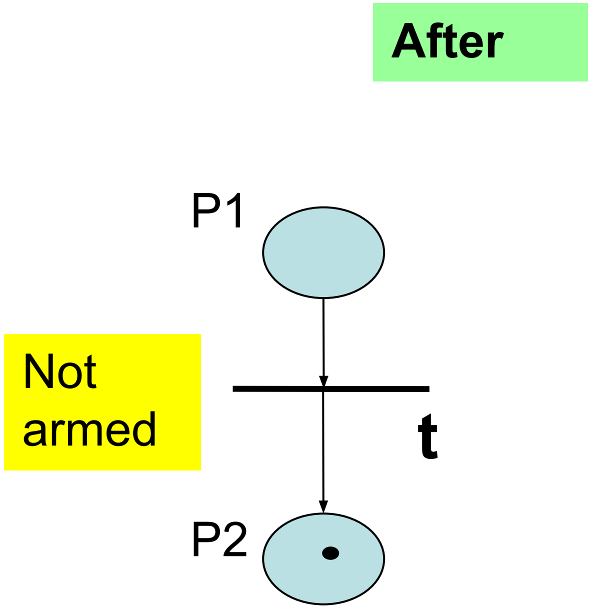
- eventually leads to the circulation of tokens
- eventually leads the RoP from one state to another

Spring



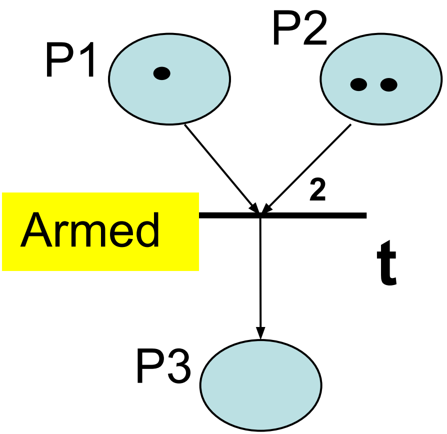
Examples



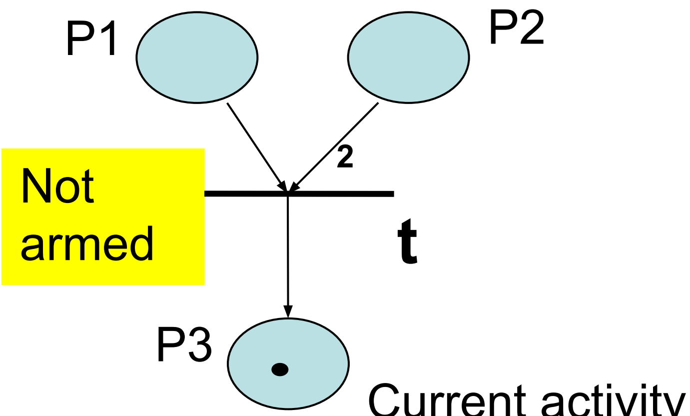


Examples: multiple pre-conditions

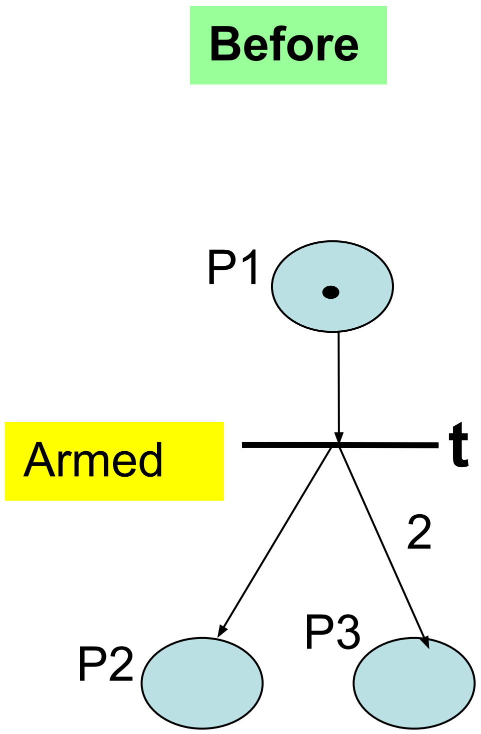
Before

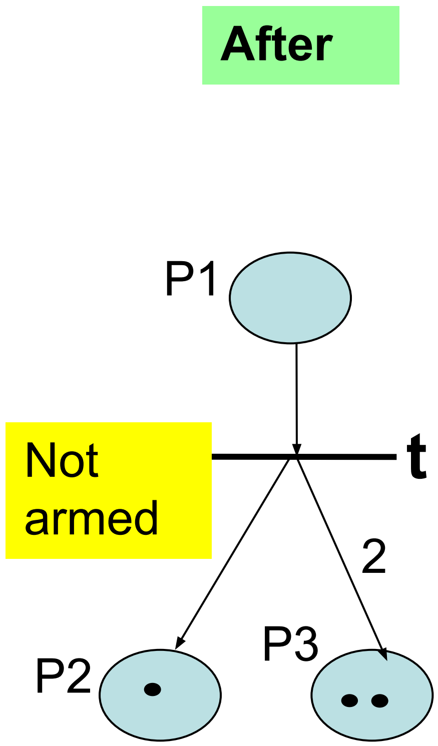


After

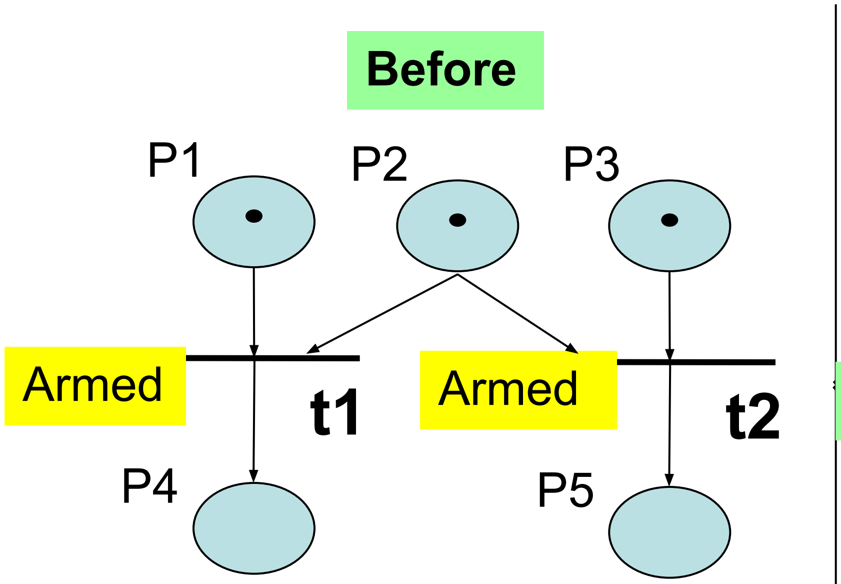


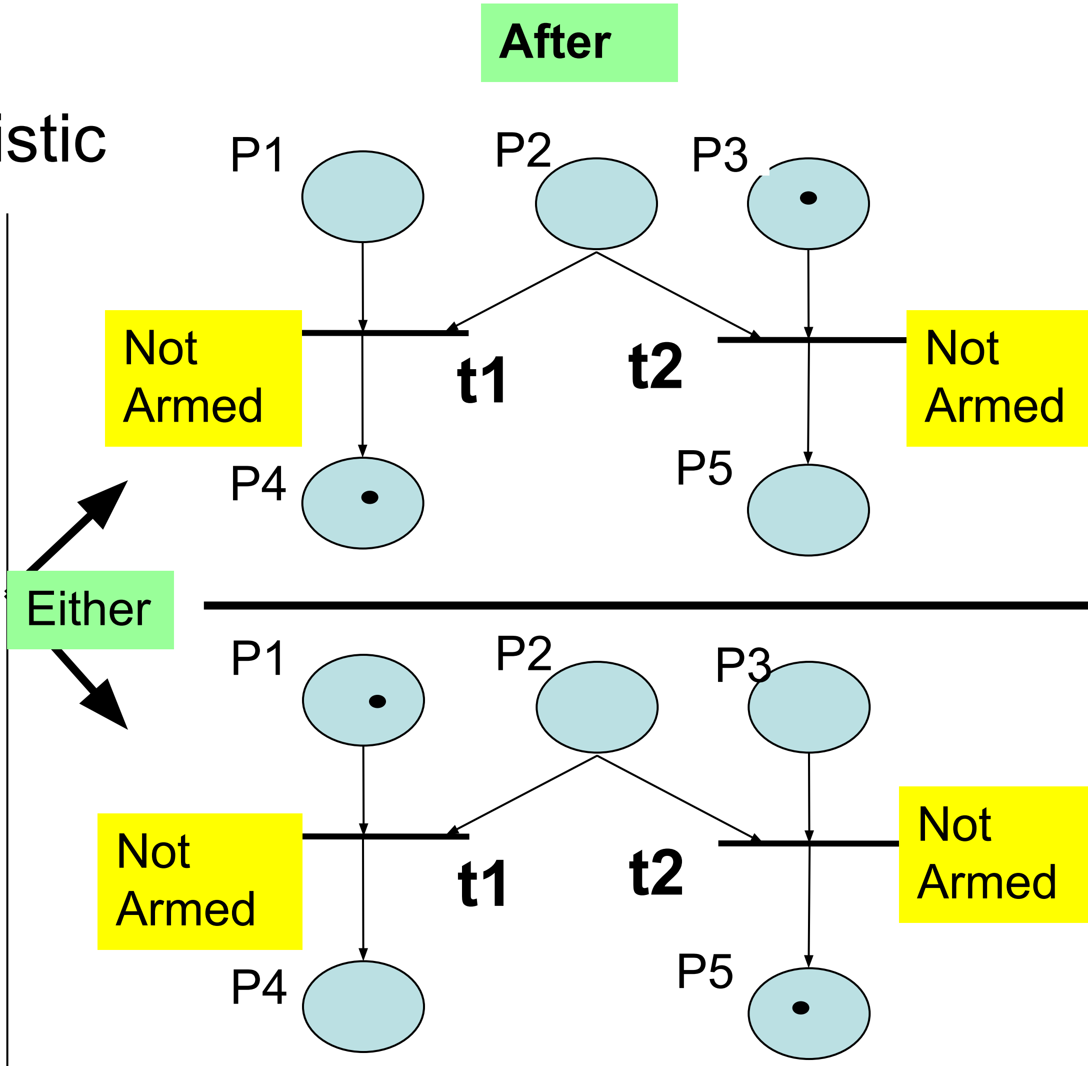
Examples: Diffusion



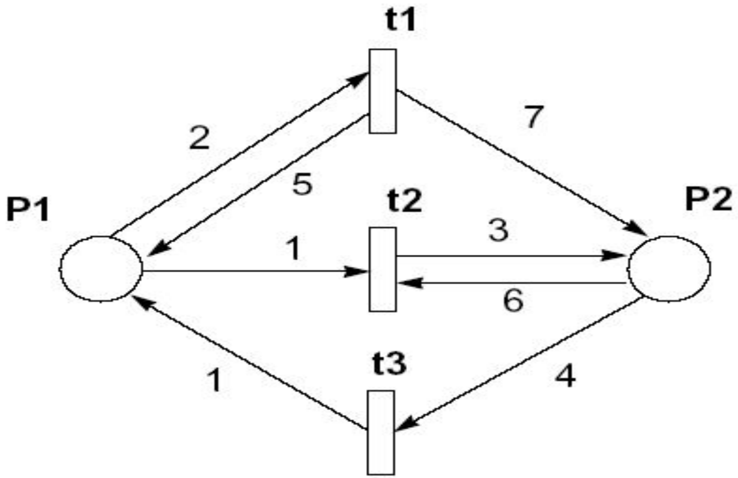


Examples: Non-deterministic





Matrix representation



Input

$$\begin{array}{c|c} & \begin{matrix} t1 & t2 & t3 \end{matrix} \\ \begin{matrix} P1 \\ P2 \end{matrix} & \begin{bmatrix} 2 & 1 & 0 \\ 0 & 6 & 4 \end{bmatrix} \end{array}$$

Output

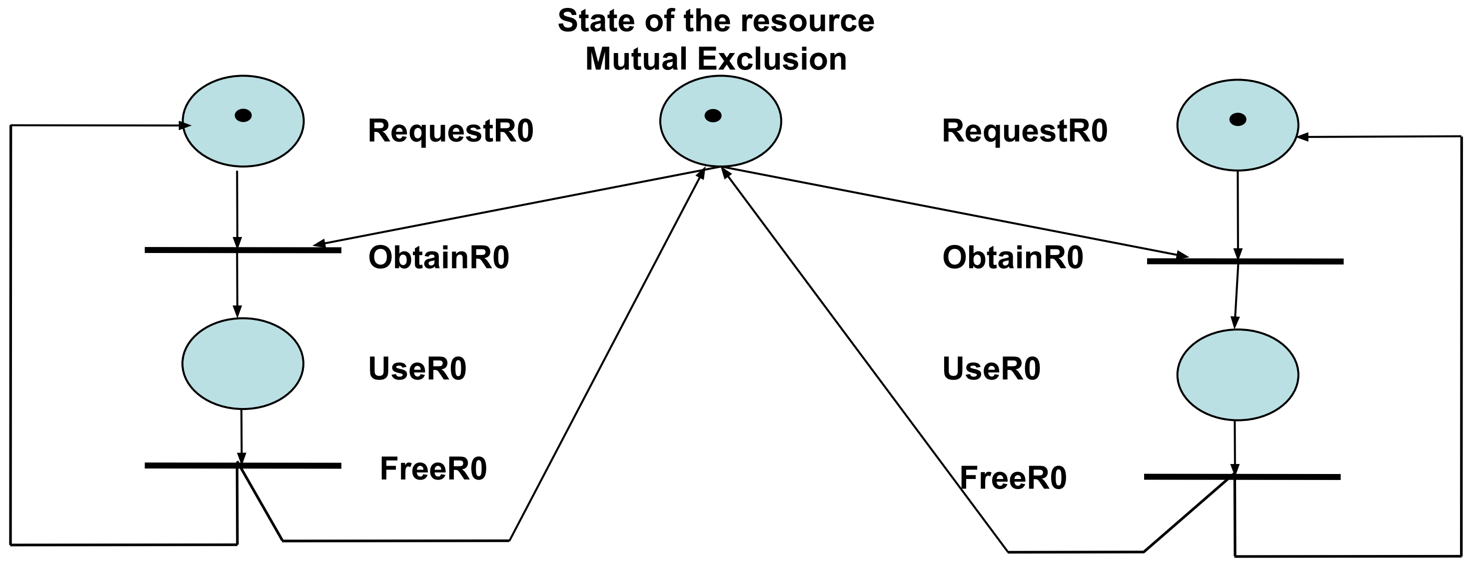
$$\begin{array}{c|c} & \begin{matrix} t1 & t2 & t3 \end{matrix} \\ \begin{matrix} P1 \\ P2 \end{matrix} & \begin{bmatrix} 5 & 0 & 1 \\ 7 & 3 & 0 \end{bmatrix} \end{array}$$

Example: Model two cyclic processes that share a resource in mutual exclusion

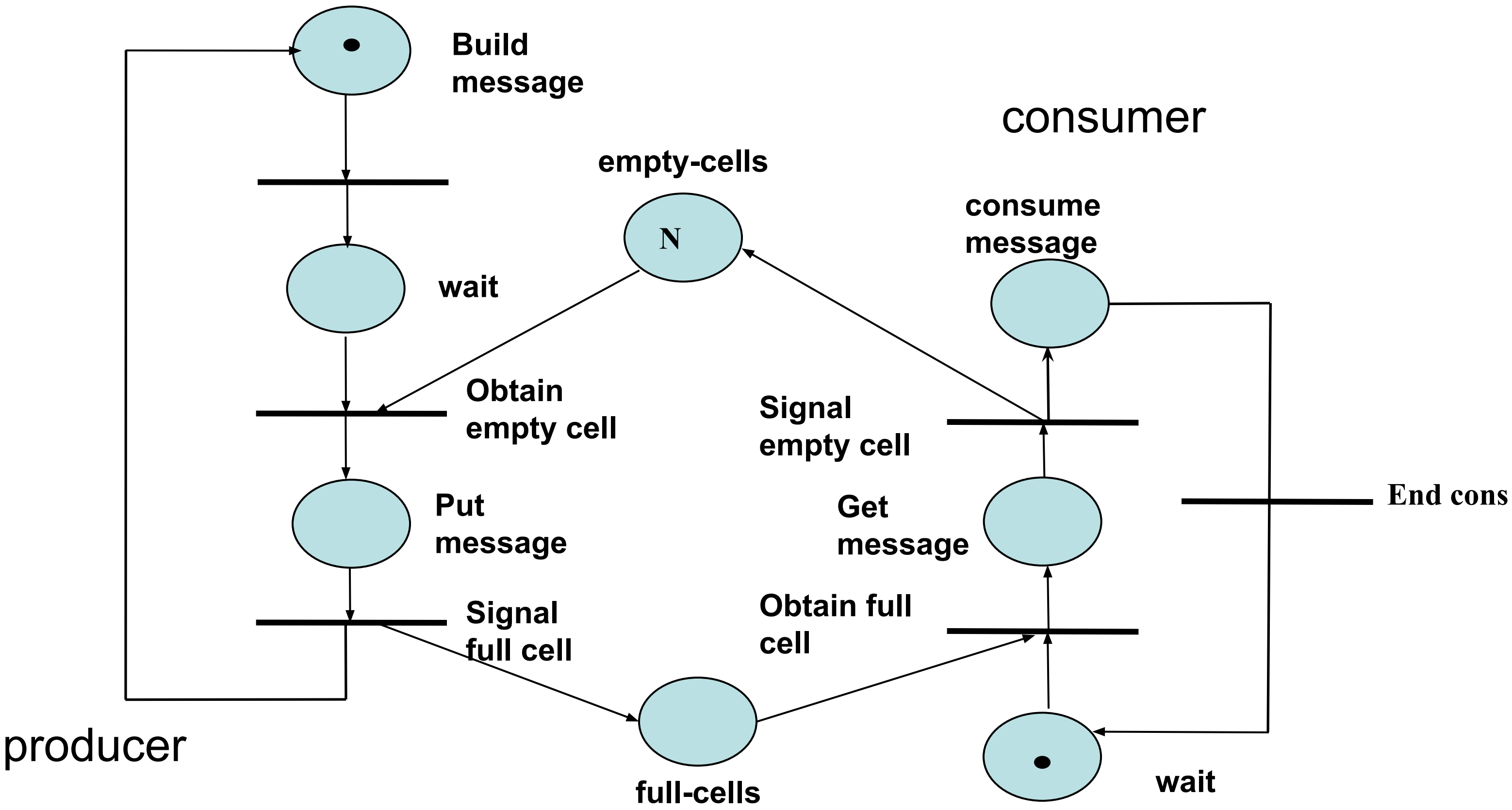
User process modeling. What about R0?

Example: Model two cyclic processes that share a resource in mutual exclusion

Ensure mutual exclusion



Example: Producer-consumer



Graph of markings

Detection of absence of global or partial blocking.

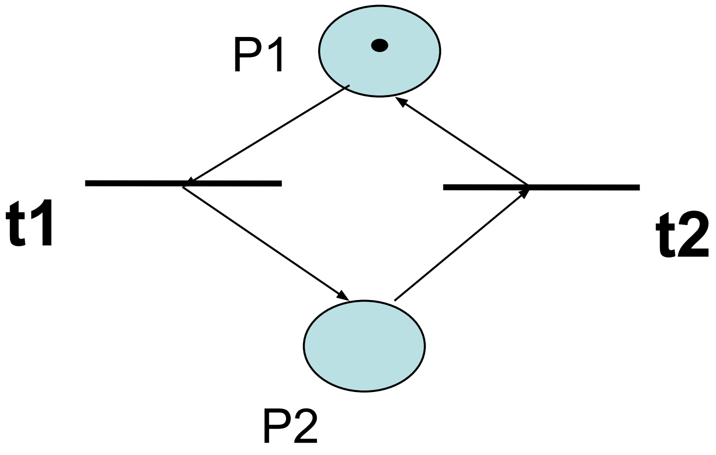
The construction of the graph is done from the tree of markings.

There are two cases:

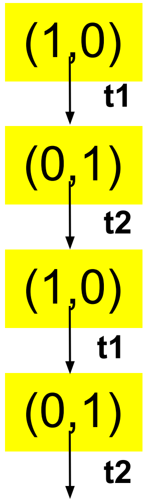
- Finite number of markings: markings graph
- Infinite number of markings: cover graph

Graph of markings

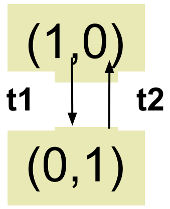
Petri Net



Tree of markings

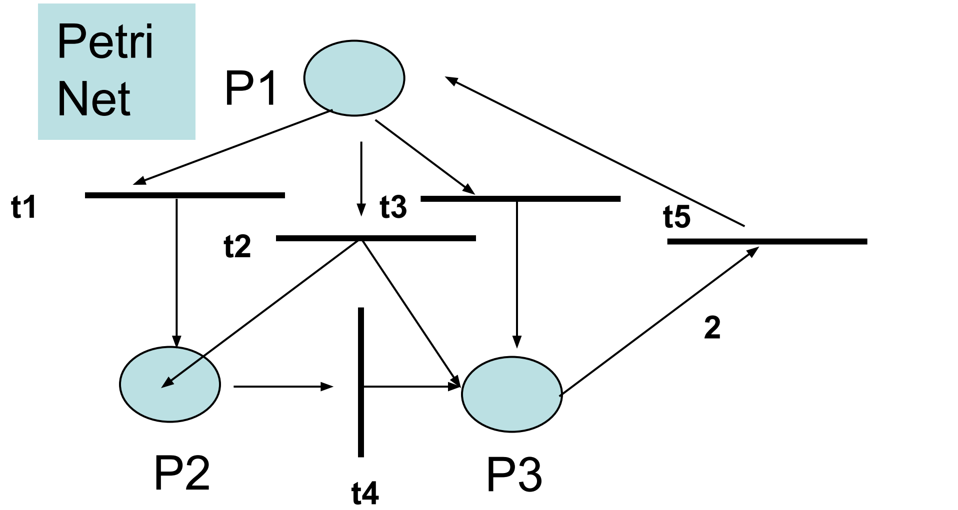


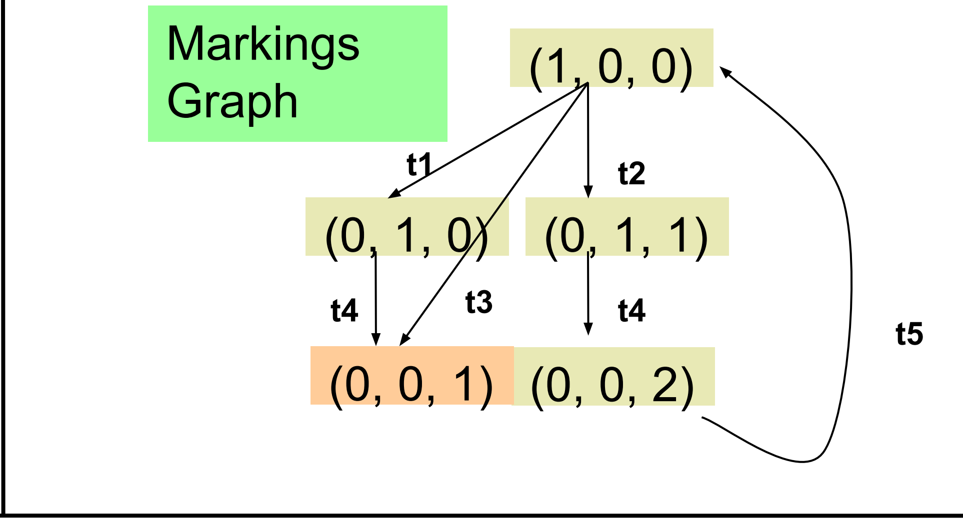
Graph of markings



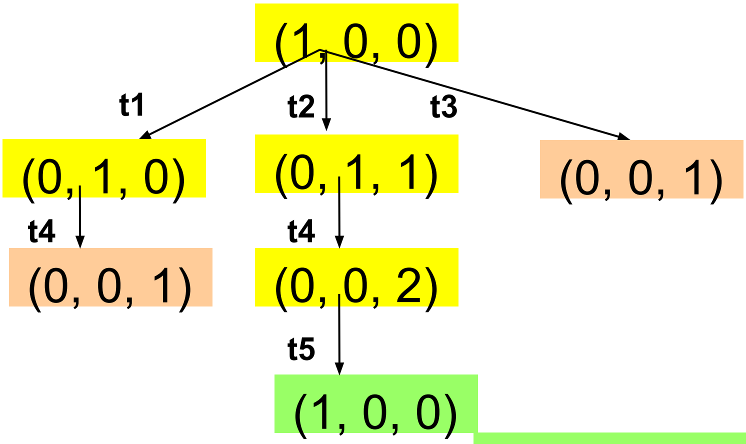
A marking is a vector of which each component represent the number of tokens in a place
It represents the state of a Petri Net at a given time

P1 P2
M0(1 , 0)





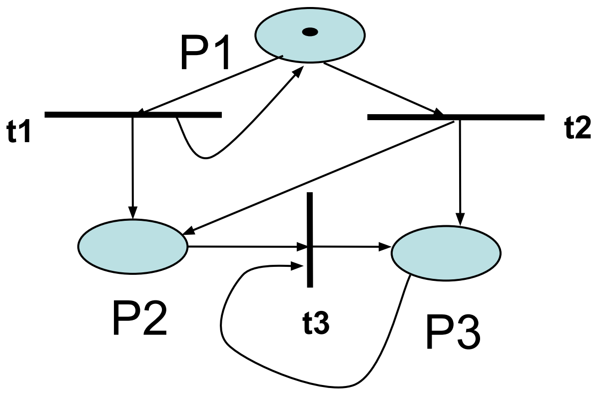
Markings Tree



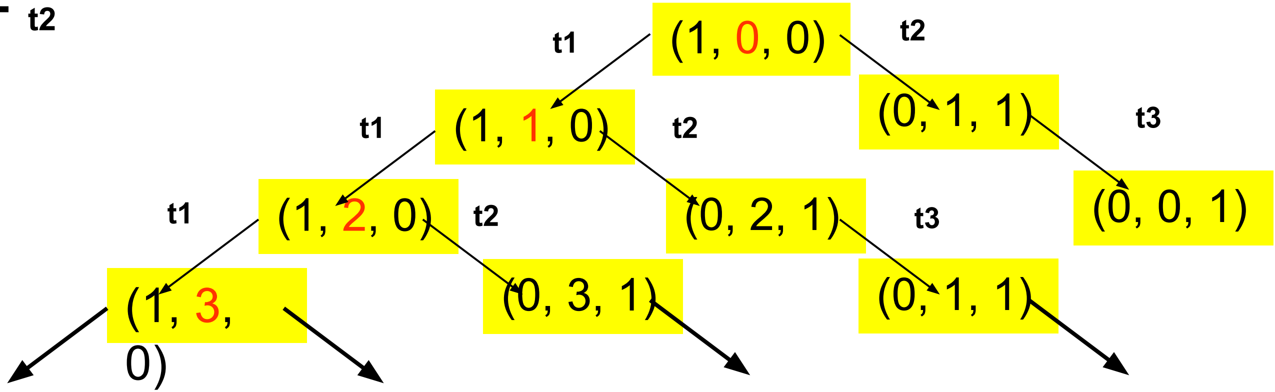
Terminal blocking marking.
All transitions from this marking onwards are disarmed.

Terminal blocking marking.
All transitions from this marking onwards are disarmed.

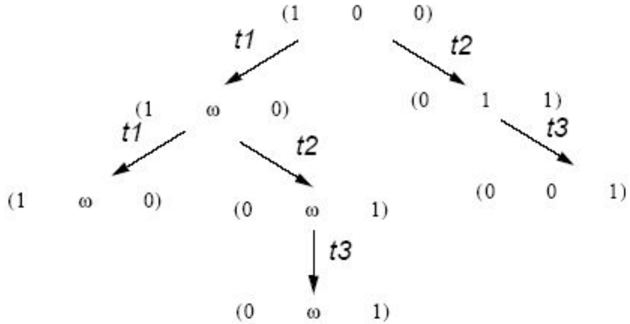
Terminal blocking marking. We loop



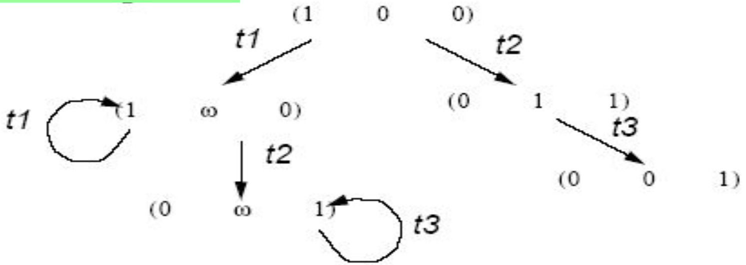
Infinite tree



Finite tree
Usage of ω



Graph of marking



ω symbol

ω symbol:

- Arbitrarily large quantity of tokens.
- Can be approximated as infinite: $\omega \notin \mathbb{N}$
- Properties (valid for all n in $\mathbb{N}$) :
  - $\omega + n = \omega$
  - $\omega - n = \omega$
  - $n < \omega$
  - $\omega \leq \omega$
- Used to construct the spanning tree in the case of an infinite markings graph

Build the graph

We start from the initial marking
Repeat
For each non-terminal leaf marking, we draw all the armed transitions
    For each new marking obtained:
    we label it as terminal:
        { if there is no armed transition
            Or
        if it is already encountered on the path from the root (parent)
        }
    if it is not labeled then if it is > than a parent marking then we replace the > component by ω
Until (any leaf is terminal)

Mi > Mj if all components of Mi are >= than the one of Mj except at least one which is >.

Examples:

- Mi(0,0,0,1) > Mj(0,0,0,0)
- Mi(0,2,0,1) > Mj(0,1,0,0)

Problem 1

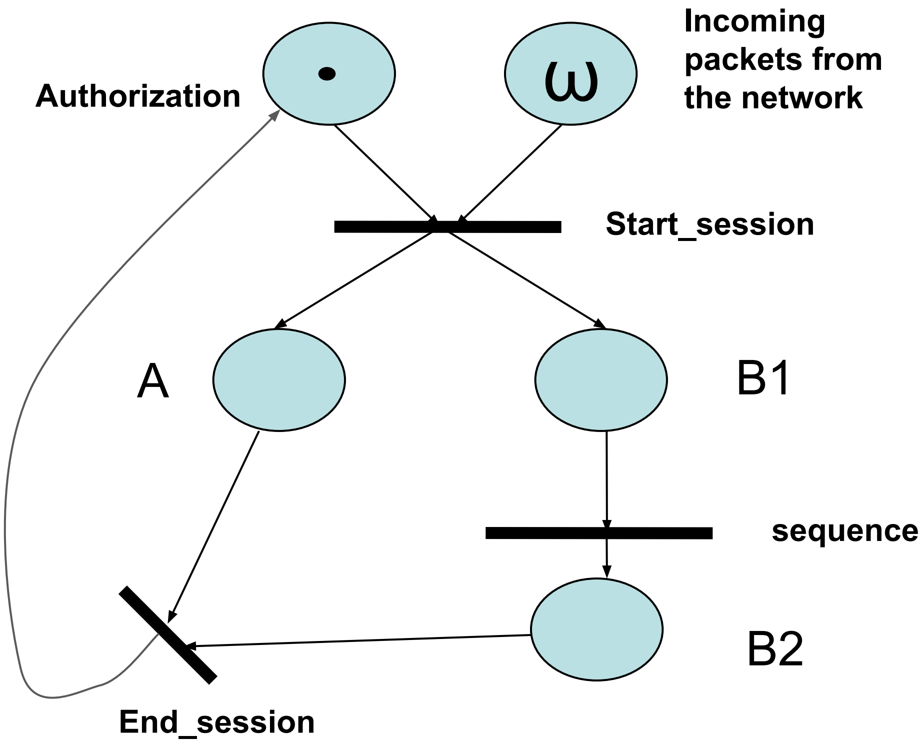
Model a system that receives a message from the network, launches two tasks in parallel:

- The first one is composed by an action A
- The second is composed by a sequence of actions B1; B2.

A new session can only be started if the current processing is finished.

Model this system by a Petri Net

Construct the markings graph of the PN



$$\begin{tikzcd}
(1, \omega, 0, 0, 0) \arrow[d] \\
(0, \omega, 1, 1, 0) \arrow[d] \\
(0, \omega, 1, 0, 1) \arrow[d] \\
(1, \omega, 0, 0, 0)
\end{tikzcd}$$

Tree of markings

$$\begin{tikzcd}
& (1, \omega, 0, 0, 0) \arrow[dl] \\
(0, \omega, 1, 1, 0) \arrow[d] & \\
(0, \omega, 1, 0, 1) \arrow[uur] &
\end{tikzcd}$$

Graph of markings

Problem 2

Model a system that receives a message from the network, launches two tasks in parallel:

- The first one is composed by an action A
- The second is composed by N actions B

A new session can only be started if the current processing is finished.

Model this system by a Petri Net

Construct the markings graph of the PN