Resonance

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※ Lecture 1 (1/21)

1.1 Basic Definitions

Resonance: The property of cancellation of reactance when inductive and capacitive reactance are in series and cancellation of susceptance when they are in parallel is called Resonance.

Figure of merit or Q factor: Q factor is a measure of efficiency of the circuit. It is respresented by the following expression:

$$Q = 2\pi \text{ X } \frac{\text{Maximum energy stored per cycle}}{\text{Energy dissipated per cycle}}$$

[Dimensionless]

1.2 Q Factor, Figure of merit

As mentioned above, Q factor is used as a measure of efficiency of a circuit. In terms of maximum current, the maximum energy stored in an inductor is $LI_m^2/2$ and energy dissipation $I_m^2 R_s/2f$ when R_s is in series with the inductor. So we can write the Q factor as:

$$Q = \frac{2\pi L I_m^2}{I_m^2 R_s/2f} = \frac{2\pi f L}{R_s} = \frac{\omega L}{R_s}$$

Similarly with capacitance in parallel with resistance R_s

$$Q = \frac{2\pi C E_m^2}{E_m^2/R_p f} = 2\pi f C R_p = \omega C R_p$$

Where E_m is the voltage across C and R_p

1.3 Series Resonance

Total energy in the circuit would be

$$E = E_R + E_L + E_C$$

Where E_R, E_L, E_C are energy across resistor, inductor and capacitor respectively. Rms value of this energy is:

$$E_{rms} = I_{rms} R + I_{rms} Z_L + I_{rms} Z_C$$

$$E_{rms} = I_{rms} R + I_{rms}(j\omega L) + I_{rms}(\frac{-j}{\omega C})$$

$$E_{rms} = I_{rms} Z$$

Where,

$$Z = R + j(\omega L - \frac{1}{\omega C})^2$$

We may draw a vector diagram of Impedance. Obtaining the magnitude:

$$|Z| = \sqrt{R^2 + j(\omega L - \frac{1}{\omega C})^2}$$

And Phase angle(Angle between x axis and Impedance vector):

$$tan\delta = \frac{\omega L - \frac{1}{\omega C}}{R}$$

As ω changes, ωL and $\frac{1}{\omega C}$ will change as well and at a certain frequency, say $\omega = \omega_0$, they will become equal.

$$\omega L = \frac{1}{\omega C}$$

At this frequency, Impedance of the circuit becomes minimum and current in the circuit is maximum. At resonance,

$$(I_{rms})_R = \frac{E_{rms}}{R}$$

$$\omega_r L = \frac{1}{\omega_r C}$$

Where ω_{ar} is the angular frequency at resonance.

$$\omega_r^2 = \frac{1}{LC}$$

$$2\pi f_r = \frac{1}{\sqrt{LC}}$$

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

References

[1] John D Ryder *Network Lines and Fields*.