USS_193

October 12, 2021

###

Unsteady State Heat Transfer

0.0.1 Objective:

- To study unsteady state heat transfer using the lumped capacitance method for finite geometric shape.

0.0.2 Aim:

- To determine internal thermal resistance of the body by calculating Biot number for the solid cylinders.
- Draw a graph between time, t (s) verses $\ln\left[\frac{T-T_\infty}{T_i-T_\infty}\right]$ and from the slope calculate Bi and h and compare with average values.

[200]:
```
import numpy as np
from matplotlib import pyplot as plt
import pandas as pd
```

[89]:
```
#Given data
ro = 0.019
l = 0.14
#brass
rho_b = 8522            #kg/m3
cp_b = 385              #J/Kg K
k_b = 110.7             #W/m K
alpha_b = k_b/(rho_b*cp_b)
#stainless steel
rho_ss = 7817
cp_ss = 461
k_ss = 16.3
alpha_ss = k_ss/(rho_ss*cp_ss)

A = 2*np.pi*ro*l
V = np.pi*(ro**2)*l
```

[161]:
```
#For Brass @ 50 degrees with agitator
t_b1 = np.array([0,20,30,40,50,60,70,80,90,100,110,120])
```

```python
T_brass_1 = np.array([24.1,30.6,42.2,46.6,47.7,48.3,48.5,48.7,49,49.1,49.1,49.
↪1])
Ti_b1 = T_brass_1[0]
T_inf_b1 = 50
F_b1 = (alpha_b*t_b1)/(V/A)**2       #Fourier number
Te_b1 = (T_brass_1 - T_inf_b1) / (Ti_b1 - T_inf_b1)
Bi_b1 = np.log(Te_b1[1:])/(-1*F_b1[1:])
h_b1 = 2*k_b*Bi_b1/ro
Qi_b1 = h_b1*A*(T_inf_b1 - Ti_b1)*np.exp(-1*Bi_b1*F_b1[1:])
```

[162]:
```python
br_50_a = pd.DataFrame({'Time (t)': t_b1[1:],
                   'Te': Te_b1[1:],
                  'Fo': F_b1[1:],
                  'Bi':Bi_b1,
                  'h':h_b1,
                  'Qi':Qi_b1})
print(br_50_a)
```

```
    Time (t)        Te         Fo        Bi            h          Qi
0         20  0.749035   7.477013  0.038648   450.348251  146.019789
1         30  0.301158  11.215520  0.107005  1246.892482  162.549304
2         40  0.131274  14.954026  0.135781  1582.201980   89.908830
3         50  0.088803  18.692533  0.129535  1509.421505   58.022959
4         60  0.065637  22.431040  0.121422  1414.882212   40.200431
5         70  0.057915  26.169546  0.108859  1268.488137   31.800883
6         80  0.050193  29.908053  0.100036  1165.681382   25.327056
7         90  0.038610  33.646559  0.096718  1127.024431   18.836267
8        100  0.034749  37.385066  0.089865  1047.162033   15.751354
9        110  0.034749  41.123573  0.081695   951.965485   14.319413
10       120  0.034749  44.862079  0.074887   872.635028   13.126129
```

[167]:
```python
#For Brass @ 60 degrees with agitator
t_b2 = np.linspace(0,110,12)
T_brass_2 = np.array([31,41.5,50,54.5,56.8,57.7,58.4,58.7,58.8,59,59,59])
Ti_b2 = T_brass_2[0]
T_inf_b2 = 60
F_b2 = (alpha_b*t_b2)/(V/A)**2                         #Fourier number
Te_b2 = (T_brass_2 - T_inf_b2) / (Ti_b2 - T_inf_b2)
Bi_b2 = np.log(Te_b2[1:])/(-1*F_b2[1:])                 #Biot number
h_b2 = 2*k_b*Bi_b2/ro
Qi_b2 = h_b2*A*(T_inf_b2 - Ti_b2)*np.exp(-1*Bi_b2*F_b2[1:])
```

[168]:
```python
br_60_a = pd.DataFrame({'Time (t)': t_b2[1:],
                   'Te': Te_b2[1:],
                  'Fo': F_b2[1:],
                  'Bi':Bi_b2,
                  'h':h_b2,
```

```
                   'Qi':Qi_b2})
print(br_60_a)
```

```
    Time (t)        Te         Fo        Bi            h          Qi
0       10.0  0.637931   3.738507  0.120242  1401.134442  433.224533
1       20.0  0.344828   7.477013  0.142398  1659.309894  277.324991
2       30.0  0.189655  11.215520  0.148236  1727.343596  158.782607
3       40.0  0.110345  14.954026  0.147395  1717.536751   91.858113
4       50.0  0.079310  18.692533  0.135583  1579.896883   60.732070
5       60.0  0.055172  22.431040  0.129164  1505.105392   40.248379
6       70.0  0.044828  26.169546  0.118647  1382.546850   30.038948
7       80.0  0.041379  29.908053  0.106492  1240.914349   24.887688
8       90.0  0.034483  33.646559  0.100078  1166.177419   19.490641
9      100.0  0.034483  37.385066  0.090071  1049.559677   17.541577
10     110.0  0.034483  41.123573  0.081882   954.145161   15.946888
```

[170]:
```
#For Brass @ 70 degrees without agitator
t_b3 = np.linspace(0,220,23)
T_brass_3 = np.array([31.1,40.2,47,52.3,55.7,58.2,60.7,62.2,63.4,64.2,65.2,65.
 ↪7,66.2,66.6,66.8,67,67.2,67.3,67.5,67.7,
                     67.8,68,68])
Ti_b3 = T_brass_3[0]
T_inf_b3 = 70
F_b3 = (alpha_b*t_b3)/(V/A)**2                    #Fourier number
Te_b3 = (T_brass_3 - T_inf_b3) / (Ti_b3 - T_inf_b3)
Bi_b3 = np.log(Te_b3[1:])/(-1*F_b3[1:])            #Biot number
h_b3 = 2*k_b*Bi_b3/ro
Qi_b3 = h_b3*A*(T_inf_b3 - Ti_b3)*np.exp(-1*Bi_b3*F_b3[1:])
```

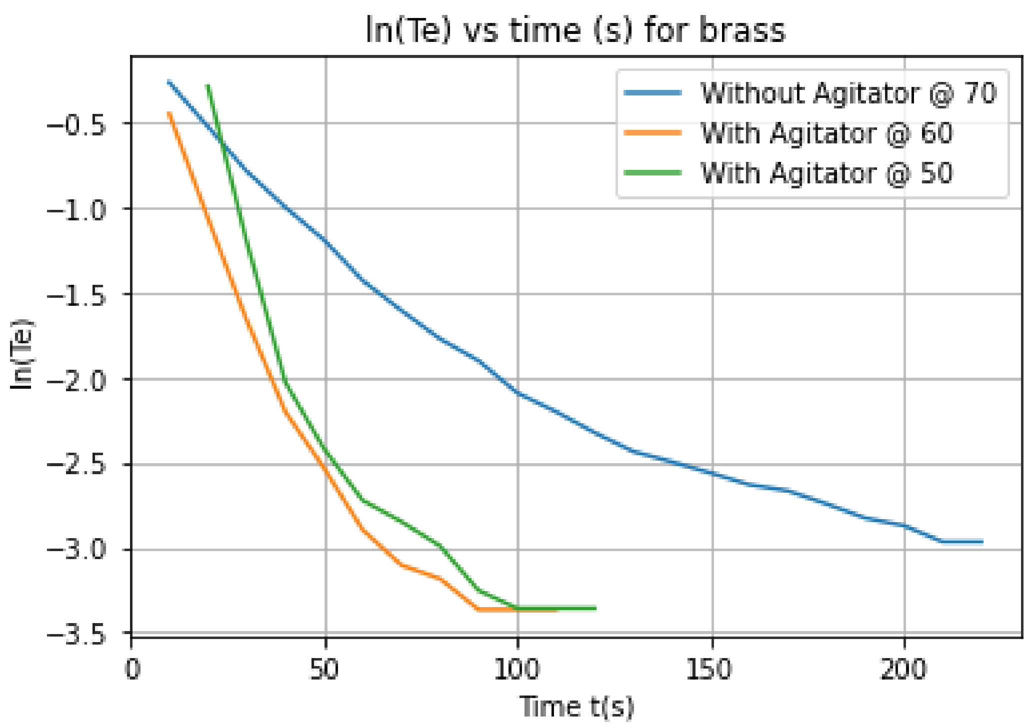
[171]:
```
br_70_wa = pd.DataFrame({'Time (t)': t_b3[1:],
                    'Te': Te_b3[1:],
                   'Fo': F_b3[1:],
                   'Bi':Bi_b3,
                   'h':h_b3,
                   'Qi':Qi_b3})
print(br_70_wa)
```

```
    Time (t)        Te         Fo        Bi           h          Qi
0       10.0  0.766067   3.738507  0.071281  830.615497  413.692644
1       20.0  0.591260   7.477013  0.070282  818.971178  314.816843
2       30.0  0.455013  11.215520  0.070209  818.118761  242.019926
3       40.0  0.367609  14.954026  0.066921  779.802886  186.372736
4       50.0  0.303342  18.692533  0.063817  743.631839  146.656558
5       60.0  0.239075  22.431040  0.063795  743.375310  115.545380
6       70.0  0.200514  26.169546  0.061402  715.498466   93.274905
7       80.0  0.169666  29.908053  0.059313  691.147966   76.238874
8       90.0  0.149100  33.646559  0.056563  659.102950   63.891451
```

9	100.0	0.123393	37.385066	0.055968	652.177901	52.320131
10	110.0	0.110540	41.123573	0.053555	624.058470	44.849256
11	120.0	0.097686	44.862079	0.051848	604.161514	38.370562
12	130.0	0.087404	48.600586	0.050148	584.355364	33.206068
13	140.0	0.082262	52.339092	0.047724	556.112995	29.742298
14	150.0	0.077121	56.077599	0.045694	532.449562	26.696925
15	160.0	0.071979	59.816105	0.043991	512.611800	23.988779
16	170.0	0.069409	63.554612	0.041976	489.126111	22.072225
17	180.0	0.064267	67.293119	0.040787	475.279189	19.858677
18	190.0	0.059126	71.031625	0.039814	463.943123	17.834218
19	200.0	0.056555	74.770132	0.038418	447.673600	16.460600
20	210.0	0.051414	78.508638	0.037803	440.502207	14.724467
21	220.0	0.051414	82.247145	0.036084	420.479379	14.055173

[172]:

```
#graph for brass
plt.plot(t_b3[1:],np.log(Te_b3[1:]),label="Without Agitator @ 70")
plt.plot(t_b2[1:],np.log(Te_b2[1:]),label="With Agitator @ 60")
plt.plot(t_b1[1:],np.log(Te_b1[1:]),label="With Agitator @ 50")
plt.title("ln(Te) vs time (s) for brass")
plt.xlabel("Time t(s)")
plt.ylabel("ln(Te)")
plt.grid()
plt.legend()
plt.show()
```



[173]:
```
#For Stainless steel @ 50 degrees with agitator
t_1 = np.linspace(0,100,11)
T_ss_1 = np.array([24.1,25,35.6,41.7,44.8,48.2,48.8,49.1,49.2,49.3,49.3])

Ti_1 = T_ss_1[0]
T_inf_1 = 50
F_1 = (alpha_ss*t_1)/(V/A)**2                    #Fourier number
Te_1 = (T_ss_1 - T_inf_1) / (Ti_1 - T_inf_1)
Bi_1 = np.log(Te_1[1:])/(-1*F_1[1:])           #Biot number
h_1 = 2*k_ss*Bi_1/ro
Qi_1 = h_1*A*(T_inf_1 - Ti_1)*np.exp(-1*Bi_1*F_1[1:])
```

[174]:
```
ss_50_a = pd.DataFrame({'Time (t)': t_1[1:],
                   'Te': Te_1[1:],
                  'Fo': F_1[1:],
                  'Bi':Bi_1,
                  'h':h_1,
                  'Qi':Qi_1})
print(ss_50_a)
```

```
   Time (t)        Te        Fo        Bi            h          Qi
0      10.0  0.965251  0.501186  0.070567   121.077831   50.590171
1      20.0  0.555985  1.002373  0.585625  1004.809355  241.828603
2      30.0  0.320463  1.503559  0.756862  1298.616337  180.144273
3      40.0  0.200772  2.004746  0.800892  1374.161497  119.427028
4      50.0  0.069498  2.505932  1.064058  1825.698713   54.924122
5      60.0  0.046332  3.007119  1.021550  1752.764196   35.153312
6      70.0  0.034749  3.508305  0.957614  1643.064550   24.714888
7      80.0  0.030888  4.009492  0.867289  1488.084599   19.896611
8      90.0  0.027027  4.510678  0.800527  1373.535058   16.069386
9     100.0  0.027027  5.011865  0.720474  1236.181552   14.462448
```

[175]:
```
#For Stainless steel @ 60 degrees with agitator
t_2 = np.linspace(0,160,17)
T_ss_2 = np.array([30.2,30.8,36.8,42.6,47.7,50.5,53.1,54.7,56.1,56.9,57.5,58,58.
↪2,58.4,58.5,58.5,58.5])

Ti_2 = T_ss_2[0]
T_inf_2 = 60
F_2 = (alpha_ss*t_2)/(V/A)**2                    #Fourier number
Te_2 = (T_ss_2 - T_inf_2) / (Ti_2 - T_inf_2)
Bi_2 = np.log(Te_2[1:])/(-1*F_2[1:])           #Biot number
h_2 = 2*k_ss*Bi_2/ro
Qi_2 = h_2*A*(T_inf_2 - Ti_2)*np.exp(-1*Bi_2*F_2[1:])
```

[176]:
```
ss_60_a = pd.DataFrame({'Time (t)': t_2[1:],
                   'Te': Te_2[1:],
```

```
                    'Fo': F_2[1:],
                    'Bi':Bi_2,
                    'h':h_2,
                    'Qi':Qi_2})
print(ss_60_a)
```

```
    Time (t)        Te        Fo        Bi           h          Qi
0       10.0  0.979866  0.501186  0.040583   69.631997   33.982334
1       20.0  0.778523  1.002373  0.249763  428.541465  166.166067
2       30.0  0.583893  1.503559  0.357843  613.983201  178.553037
3       40.0  0.412752  2.004746  0.441407  757.361681  155.693307
4       50.0  0.318792  2.505932  0.456204  782.750148  124.282010
5       60.0  0.231544  3.007119  0.486508  834.745053   96.264121
6       70.0  0.177852  3.508305  0.492204  844.518248   74.807719
7       80.0  0.130872  4.009492  0.507179  870.213131   56.722027
8       90.0  0.104027  4.510678  0.501722  860.849206   44.601584
9      100.0  0.083893  5.011865  0.494470  848.406702   35.449132
10     110.0  0.067114  5.513051  0.489994  840.726269   28.102575
11     120.0  0.060403  6.014238  0.466680  800.723830   24.088889
12     130.0  0.053691  6.515424  0.448859  770.146992   20.594683
13     140.0  0.050336  7.016611  0.425995  730.918259   18.324055
14     150.0  0.050336  7.517797  0.397596  682.190375   17.102451
15     160.0  0.050336  8.018984  0.372746  639.553477   16.033548
```

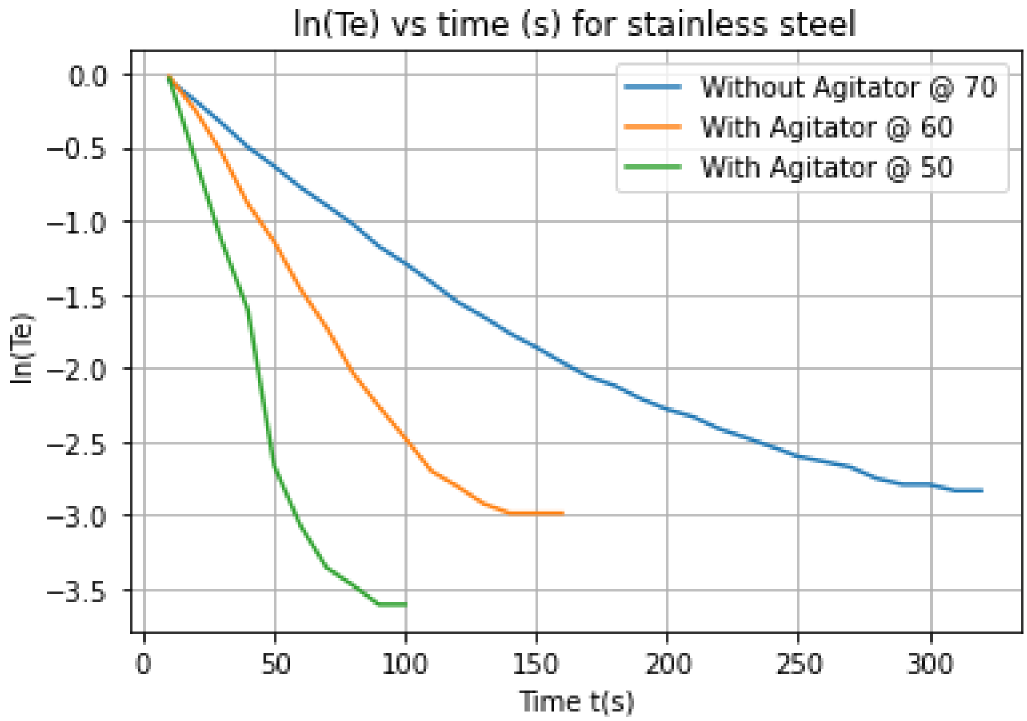
[177]:
```
#For Stainless steel @ 60 degrees without agitator
t_3 = np.linspace(0,320,33)
T_ss_3 = np.array([30.8,32.5,37.4,42,46.2,49.1,51.9,54,55.9,57.9,59.2,60.5,61.
 ↪7,62.5,63.3,63.9,64.5,65,65.3,65.7,66,
                    66.2,66.5,66.7,66.9,67.1,67.2,67.3,67.5,67.6,67.6,67.7,67.7])
Ti_3 = T_ss_3[0]
T_inf_3 = 70
F_3 = (alpha_ss*t_3)/(V/A)**2                    #Fourier number
Te_3 = (T_ss_3 - T_inf_3) / (Ti_3 - T_inf_3)
Bi_3 = np.log(Te_3[1:])/(-1*F_3[1:])            #Biot number
h_3 = 2*k_ss*Bi_3/ro
Qi_3 = h_3*A*(T_inf_3 - Ti_3)*np.exp(-1*Bi_3*F_3[1:])
```

[178]:
```
ss_60_wa = pd.DataFrame({'Time (t)': t_3[1:],
                     'Te': Te_3[1:],
                    'Fo': F_3[1:],
                    'Bi':Bi_3,
                    'h':h_3,
                    'Qi':Qi_3})
print(ss_60_wa)
```

```
    Time (t)        Te         Fo        Bi           h          Qi
0       10.0  0.956633   0.501186  0.088462  151.781670   95.128818
```

1	20.0	0.831633	1.002373	0.183928	315.581727	171.945555
2	30.0	0.714286	1.503559	0.223784	383.965870	179.685139
3	40.0	0.607143	2.004746	0.248905	427.068469	169.877543
4	50.0	0.533163	2.505932	0.250975	430.621078	150.419131
5	60.0	0.461735	3.007119	0.256978	440.920944	133.383100
6	70.0	0.408163	3.508305	0.255419	438.245309	117.192215
7	80.0	0.359694	4.009492	0.255020	437.561196	103.114423
8	90.0	0.308673	4.510678	0.260597	447.130362	90.423442
9	100.0	0.275510	5.011865	0.257216	441.328083	79.661196
10	110.0	0.242347	5.513051	0.257096	441.123073	70.039798
11	120.0	0.211735	6.014238	0.258124	442.887038	61.437363
12	130.0	0.191327	6.515424	0.253824	435.509245	54.590886
13	140.0	0.170918	7.016611	0.251770	431.983607	48.373061
14	150.0	0.155612	7.517797	0.247465	424.596986	43.288072
15	160.0	0.140306	8.018984	0.244910	420.213854	38.627319
16	170.0	0.127551	8.520170	0.241690	414.688929	34.654046
17	180.0	0.119898	9.021357	0.235121	403.418859	31.689513
18	190.0	0.109694	9.522543	0.232087	398.213010	28.618404
19	200.0	0.102041	10.023730	0.227698	390.681687	26.118279
20	210.0	0.096939	10.524916	0.221729	380.439715	24.161893
21	220.0	0.089286	11.026103	0.219109	375.944203	21.991403
22	230.0	0.084184	11.527289	0.214687	368.356969	20.316287
23	240.0	0.079082	12.028476	0.210939	361.926913	18.751848
24	250.0	0.073980	12.529662	0.207824	356.582434	17.283013
25	260.0	0.071429	13.030849	0.202524	347.488246	16.261465
26	270.0	0.068878	13.532035	0.197710	339.229534	15.308017
27	280.0	0.063776	14.033222	0.196134	336.523931	14.061041
28	290.0	0.061224	14.534408	0.192179	329.738702	13.226431
29	300.0	0.061224	15.035595	0.185773	318.747411	12.785550
30	310.0	0.058673	15.536781	0.182520	313.165267	12.038238
31	320.0	0.058673	16.037968	0.176816	303.378852	11.662043

[179]:
```
#Graph for stainless steel
plt.plot(t_3[1:],np.log(Te_3[1:]),label="Without Agitator @ 70")
plt.plot(t_2[1:],np.log(Te_2[1:]),label="With Agitator @ 60")
plt.plot(t_1[1:],np.log(Te_1[1:]),label="With Agitator @ 50")
plt.title("ln(Te) vs time (s) for stainless steel")
plt.xlabel("Time t(s)")
plt.ylabel("ln(Te)")
plt.grid()
plt.legend()
plt.show()
```



0.0.3 Graph:

[221]:
```
#Graph for stainless steel
plt.plot(t_3[1:],np.log(Te_3[1:]),label="Brass without Agitator @ 70")
plt.plot(t_2[1:],np.log(Te_2[1:]),label="Brass with Agitator @ 60")
plt.plot(t_1[1:],np.log(Te_1[1:]),label="Brass with Agitator @ 50")
plt.plot(t_b3[1:],np.log(Te_b3[1:]),label="SS without Agitator @ 70")
plt.plot(t_b2[1:],np.log(Te_b2[1:]),label="SS with Agitator @ 60")
plt.plot(t_b1[1:],np.log(Te_b1[1:]),label="SS with Agitator @ 50")
plt.title("ln(Te) vs time (s) for brass & SS")
plt.xlabel("Time t(s)")
plt.ylabel("ln(Te)")
plt.grid()
plt.legend()
plt.rcParams['figure.figsize'] = [15, 8]
#trendlines
slope1, intercept1 = np.polyfit(t_3[1:],np.log(Te_3[1:]), 1)
plt.plot(t_3[1:], t_3[1:]*slope1 + intercept1, '--',color="blue")

slope2, intercept2 = np.polyfit(t_2[1:],np.log(Te_2[1:]), 1)
plt.plot(t_2[1:], t_2[1:]*slope2 + intercept2, '--',color="orange")

slope3, intercept3 = np.polyfit(t_1[1:],np.log(Te_1[1:]), 1)
```

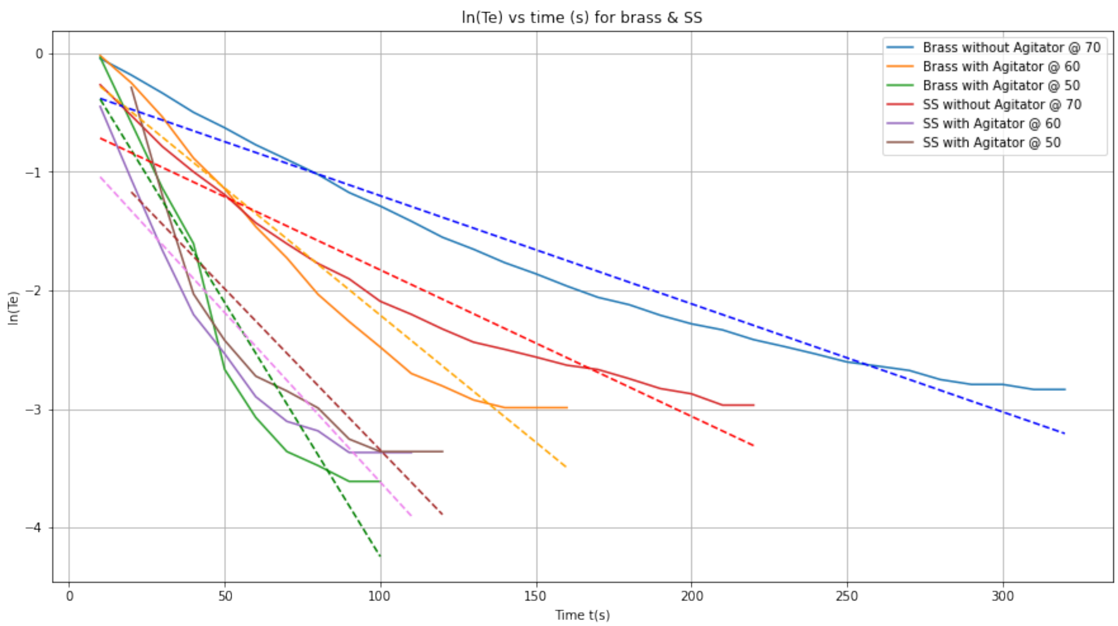
```python
plt.plot(t_1[1:], t_1[1:]*slope3 + intercept3, '--',color="green")

slope4, intercept4 = np.polyfit(t_b3[1:],np.log(Te_b3[1:]), 1)
plt.plot(t_b3[1:], t_b3[1:]*slope4 + intercept4, '--',color="red")

slope5, intercept5 = np.polyfit(t_b2[1:],np.log(Te_b2[1:]), 1)
plt.plot(t_b2[1:], t_b2[1:]*slope5 + intercept5, '--',color="violet")

slope6, intercept6 = np.polyfit(t_b1[1:],np.log(Te_b1[1:]), 1)
plt.plot(t_b1[1:], t_b1[1:]*slope6 + intercept6, '--',color="brown")
```

[221]: [<matplotlib.lines.Line2D at 0x7fb587e096a0>]



$$Slope = -\frac{hA}{\rho C_P V}$$

[228]:
```python
#BRASS
h1_b = -1*slope1*rho_b*cp_b*V/A
h2_b = -1*slope2*rho_b*cp_b*V/A
h3_b = -1*slope3*rho_b*cp_b*V/A
Bi_1 = h1_b*V/(A*k_b)
Bi_2 = h2_b*V/(A*k_b)
Bi_3 = h3_b*V/(A*k_b)
hb_avg = (h1_b+h2_b+h3_b)/3
print("The avg heat transfer coefficient for brass is {} ".format(hb_avg))
```

```
print("The biot numbers for 50, 60 and 70 degrees are {}, {},{} resp.".
↪format(Bi_1,Bi_2,Bi_3))
```

```
The avg heat transfer coefficient for brass is 762.6749572598286
The biot numbers for 50, 60 and 70 degrees are 0.02437870456777043,
0.05734089510633207,0.11463303159875307 resp.
```

[229]:
```
#STAINLESS STEEL
h1_s = -1*slope1*rho_b*cp_ss*V/A
h2_s = -1*slope2*rho_b*cp_ss*V/A
h3_s = -1*slope3*rho_b*cp_ss*V/A
Bi_1s = h1_s*V/(A*k_ss)
Bi_2s = h2_s*V/(A*k_ss)
Bi_3s = h3_s*V/(A*k_ss)
hs_avg = (h1_s+h2_s+h3_s)/3
print("The avg heat transfer coefficient for brass is {} ".format(hs_avg))
print("The biot numbers for 50, 60 and 70 degrees are {}, {},{} resp.".
↪format(Bi_1s,Bi_2s,Bi_3s))
```

```
The avg heat transfer coefficient for brass is 913.2289747968338
The biot numbers for 50, 60 and 70 degrees are 0.19824892304926428,
0.4662992108505957,0.9322019140577938 resp.
```

[ ]:
```
```