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2019

22nd Annual High School Mathematical Contest in Modeling (HiMCM) Summary Sheet

(Please make this the first page of your electronic Solution Paper.)

Team Control Number: 10103

Problem Chosen: A

Summary

Do you have "low Battery anxiety"? Many of us worry about our electronic devices will run out of battery. Thus, many of us may get used to look for charging outlet at public places. Some of them served as free charging outlets. In our paper, we construct some mathematical models to determine the impacts and requirement of these "free" charging sites, compute their cost and identify who will pay for them, and come up with a practical economic initiative to save the cost.

In model 1, we first define the impacts and the requirements based on three divided categories: ***Government Sponsor***, ***Private Sponsor***, and ***Commercial***. Then, we use ***polynomial fitting*** to predict the future tendency of such public "free" charging outlets with energy consumption and charging demands.

In model 2, we determine the cost of charging outlets with the consideration of ***psychological factor***: different ***SOC (State of Charge)*** corresponds with user's charging reaction, under different public places as we have sorted in model 1. In fact, we sort the cost and energy consumption model into two major charging devices outlets: ***Electric Vehicles and Plug-in***. Then, we determine the lowest battery acceptances toward these two distinctive charging devices. After that, we determine initial value of SOC to calculate energy consumption. After we construct the model for ***"free" charging cost***, we apply our model for an ***Application*** of a ***Coffee Shop (Commercial)***. Moreover, we discuss one more Application for ***Universities (Government Sponsor)***.

In model 3, we find the ***Elasticity correlation*** between ***Quantity demanded*** and ***Price***. Through the regulation of price, we could indirectly vary the demand of sockets. Acknowledging the increase in the marginal revenue, we could keep a balance between revenue and cost. Implementing this to our Coffee Shop application, we have our result: when $\Delta P = 0.73$, in 224.06days, the shopkeepers could gain back the cost. In one year, they could actually gain 1014.712 more than before.

Keywords: ***Polynomial fitting***, ***Psychological factor, SOC (State of Charge), Elasticity correlation, Price, Quantity demanded.***

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Charge for Charging!

Everyone owns a cellphone. In this age of technology, which electronic devices play a more and more crucial role in the daily life, people send messages by cellphones, watch movies on their iPads, and travel by driving electric cars. Given this scenario, instead of charging at home, more people prefer to charge their devices and electric cars in public places, such as Starbucks Coffee, public library, and schools, and the reason for that is those charging places are "Free". However, as saying goes, after all, the wool still cons from the sheep's back. So, here comes the question: Does those free charging truly "FREE"?

According to the data and our statistic forecast, the amount of energy that people are going to use in 2023 in public charging stations (for electric cars) up to approximately 153,900,000,000 KWH, which is five and a half times of energy that those cars consumed in 2018. This huge amount of energy will cost up to about twenty-two billion USD electricity fees in 2023, which is paid by the free charging stations' providers. No one wants to pay for such a huge bill. As a result, part of that money should be paid by users invisibly.

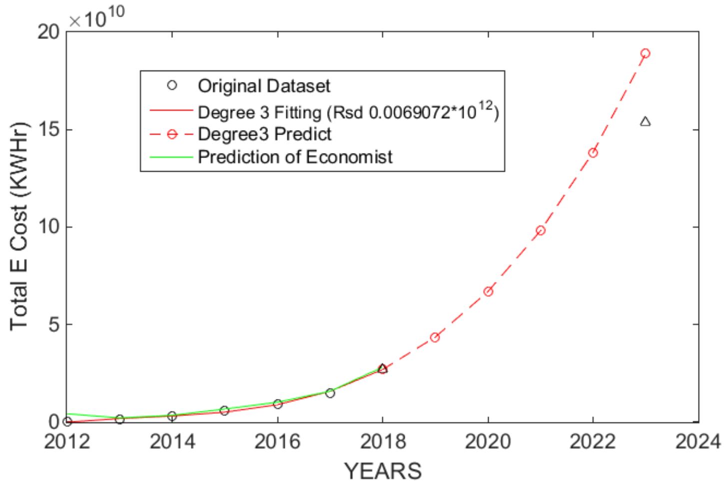

Figure. Future Predicted Tendency

For commercial places, such as Starbucks Stores, this bill should be paid by their customers. When those stores are providing free charging services, they will raise the price of their product appropriately in the meantime to cover the electricity bill. For instance, suppose a cup of coffee in a specific coffee bar sells \$3, and they have 363 customers each day. Now, the store manager decides to provide a free charging service, in which electric car charging stations are included. As a result, the price of each cup of coffee should increase \$0.73 dollars, or the store will be at a loss. It is true that this price raising makes the store loss some of their costumers, approximately 21.4%, but the free charging service would make profits for the store at least 1 thousand dollars after this service provided for costumers for one year.

No one willing to pay for all those bills, and all those bills are indirectly paid by costumers. To reduce the cost of such "free" charging cost, we develop a new model to calculate the how we can change price to determine the number of sockets. Moreover, we determine how many days later the shopkeepers could be repaid their cost for "free" charging outlets by their revenue.

1. Introduction

As the general awareness of energy and environment conservation across the world, the traditional gasoline-based automobiles are challenged to be the mainstream in the growing favor of battery-electric transportation. Along with those rising electricity demands, some public places "generously" provide "free" charging outlets from the ubiquitous cellphones and laptops to electric vehicles. The problem arises: how to manage the charging sites in various public places and who pays for them? In this paper we present a mathematical model for determining optimal numbers of charging outlets allows for broad-scope types of electric needs. The model includes a plan to depict and predict the energy consumption, a plan to calculate and distribute the resulting costs of those "free" charging outlets for distinctive public places, and a plan to figure out some initiatives in order to deduct those cost expenditures. In addition, we also wrap up a letter to the school newspaper to illustrate our findings and presents our recommendations. We believe our plans are able to bring reliable and scientific suggestion to "free"-charging-installed public places.

1.1 Analysis of the Question

As people "plug in" their electric items at the "free" charging sites in public places, what is the influence and what is the requirement for public places to settle the charging sites? Since various public places play their different functions, they construct the charging outlets with diverse purposes. Based on this fact, we define and analyze the impact and requirement with three divided categories:

① ***Government Sponsor:*** Charging sites served for the daily usage of energy consumption without required demand.
Representatives could be **(a)** ***Library***, **(b)** ***School,*** and **(c)** ***Airport***. In those listed public places, they are not targeted for profit. When installing such "free" charging sites, they target to either satisfy certain ratio of charging demands (Library) or raise the city figure (Airport).

② ***Private Sponsor:*** Settling charging sites in order to meet the necessary charging demands.
Representatives could be **(a)** ***Offices***. During working time, companies provide their employees with the "free" charging sites in order to let them charge computers to work on files or cellphones to keep contact with clients.

③ ***Commercial:*** Settling charging sites aimed for attracting potential consumers and increase revenue.
Representatives could be **(a)** ***Shopping malls*** **(b)** ***Coffee shops***. In those commercial and profit-orientated places, their aim is not to need to satisfy all the consumers demand which instead is the maximum revenue.

2. Assumptions and Variables

(1) ***Ratio of charging occurs in public places and at home is a constant.*** Although those free charging outlets are available in public places, charging at home is far more convenient than making trips to the gas station, because drivers can do most of their charging at home overnight. According to existed statistics, more than 90% of charging occurs at home. (Tom Saxton & filed under Features, Infrastructure Features.) As a result, we assume that this ratio could be constant.

(2) ***Energy consumption is most costed by people who obtain electric items.*** For instance, electric vehicles like Tesla Motors. The growth in sales of those electric products accompanied by more frequent the charging outlets are used by users. So, the growth rate of energy consumption can be deducted by the sales data of electric products. The correlation between the sales of electric items and their energy consumption is what we aimed to work out.

(3) ***The resulting cost we need to determine must base on our identified demand,*** which we previously defined as the amount of "free" public charging energy consumption. Moreover, as Question Part Two asked for the resulting costs of such increasing demand, we do not calculate the most efficient energy consumption, which brings the maximum benefit. Instead, we dedicate to the resulting cost of the public charging energy consumption in which such demand required.

(4) ***Loss in the current transmission is not taken into account.*** The amount of loss in the current transmission is so small, and the changes in this loss depend on the types (wired or wireless charger) and brands of charger users are using. Those reasons force us to ignore the loss in the current transmission to simplify our models.

(5) ***The total electric quantity of cellphones and electric cars are the same for everybody.*** Everyone has a different brand of cellphone and electric cars. For example, Pixel Phone 3 is equipped with a 2915mAh battery while the iPhone's battery has only 2700mAh. Those variables can only make our models make complicated. As a result, we suppose that everyone has the same kind of appliance.

(6) ***No maintaining fee for the car charging stations and adapters.*** More specifically, according to our research, most of the charging stations have few or even no maintaining fees, (since most of the stations can be put into use for almost 8 years.). So, in our models, this kind of spending is ignored.

(7) ***All the people have a cellphone, but no more than one cellphone.*** Today, some businessman owns two cellphones, sometimes even three. For some people who are not so rich, they even have no cellphone. However, in our models, we assume that all people have exactly one cellphone, as most people do.

Table1. Variables

E_t	estimated energy consumption of EVs (in kWh)
S_{evo}	EV ownership
D_{amc}	average drove miles per EV car
E_{epc}	cost energy per EV car each mile (in kWh)
P_{cip}	represents the percentage EV cars charged in public sites
E	Energy consumption
CE	Cost for charging EV
d	Day
Q	Quantity
B	Unit price
x	SOC
R	Revenue
N/C	Number of people
T	Battery capacity

3. Predicted Public Charging Tendency

3.1 Introduction

In this model, we collect the data of the *past seven years* of *Tesla Motors*. According to those data, we developed a model to determine the fluctuation of the ***consumption in Public Electricity Chargers*** over time. To predict EV's future tendency in a precise result, we use ***polynomial fitting.*** Since we are discussing the increasing tendency of public charging consumption, the increasing in indoor housing outlets' energy consumption are excluded in this energy consumption calculation.

3.1.1 The Construction of Formula

From the Financial Report of Tesla Motors, we collect the data from the first sale year to most updated year 2018. We consider the *ownership* and the *energy consumption* by Tesla motors could be converted into the ***public charging energy consumption***. First, we calculate the whole energy consumption of Tesla Motors users. Then, after determining the *public energy charging portion* of Tesla Motors users, we estimated the public charging energy consumption. After obtaining the Using this dataset as base, we calculated the estimated energy consumption of those EVs by the following expression:

$$E_t = S_{evo} * D_{amc} * E_{epc} * P_{cip}$$

3.2 Degree of polynomial fitting

A suitable degree of polynomial fitting is the key to the precise result. In this process, we tried several values of degree to determine which value is the beast match for our dataset.

E_t(Y) = 200811963.499215 × Y^3 – 1212978819164.56 × Y^2+2442282071480420 × Y – 1639145078272600000

3.3 Results

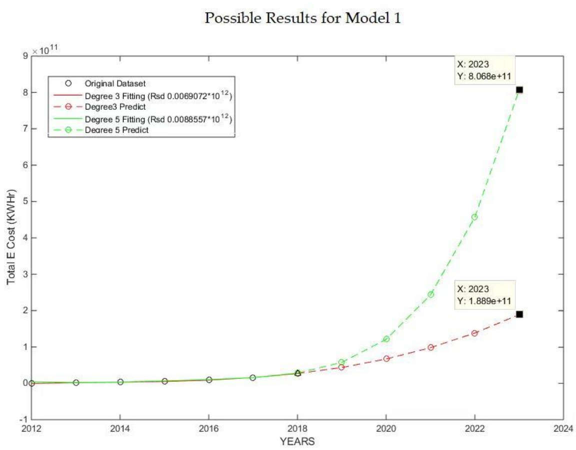

Fig.1.Predictions for Future Tendency

3.4 Verification and final result

For the fact that we just have seven-year dataset for Tesla motors, our group compare our models with other existing predicted result. If our predicted result has the approximate data result with other accurate predicted model, our model will be confirmed as a valid and reliable tool to predict the future tendency of charging electric products.

According to CNBC analysist and economist Adam Jonas from Morgan Stanley, till 2040, the ownership of Tesla will be 100 times than ownership today. Until the end of 2017, the number of Tesla would be 300,000. For 2018, the fluctuated rate would increase to 80% and the total number of Tesla cars would change to 531,000. Up till 2023, the ownership of Tesla will be 3*10^11. (Adam Jonas) For those past predicted result, we deal with them rationally since there are actual difference between the real number of Tesla motors in 2017 and the predicted number of Tesla motors by CNBC analysist. We express the difference by the graph below:

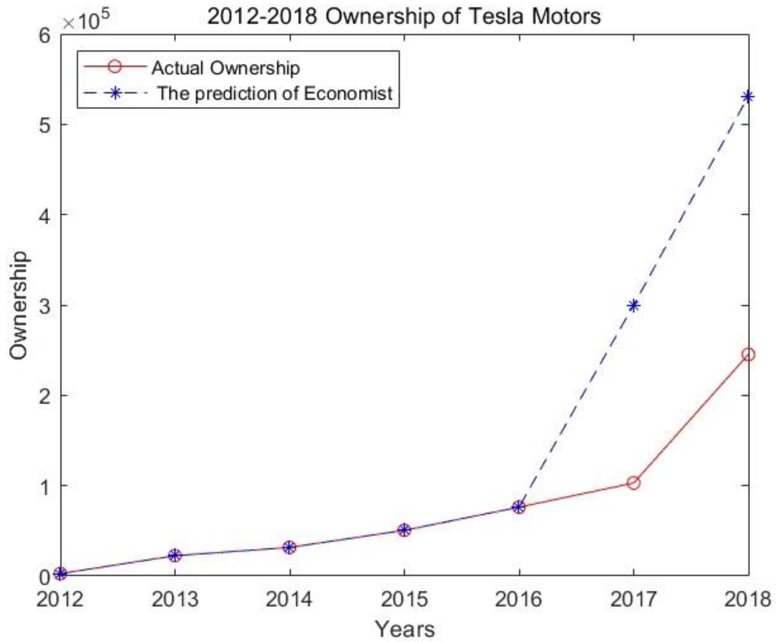

Fig.2. Deviation of Economist's Data

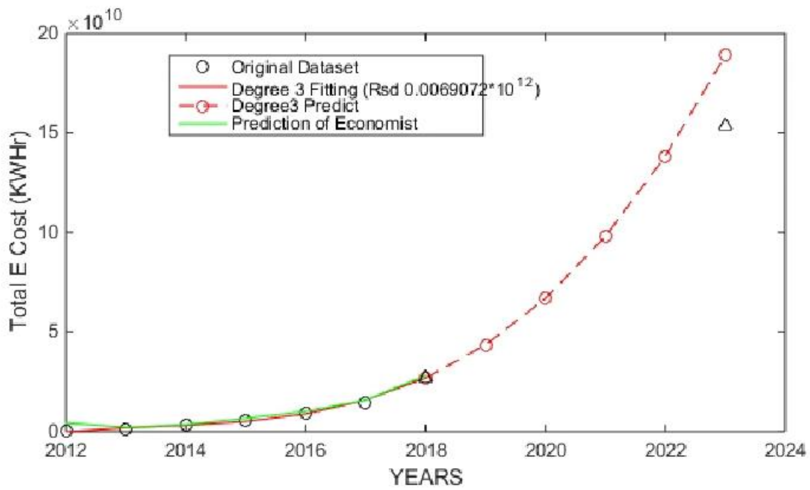

Fig.3.Result of Model 1

4. Costs and Profits of Free Charging

4.1 Introduction

With *basic economic application* and our *identified impact and requirement*, which is the demand and energy consumption of charging users, our group calculate the resulting cost according to the following expression:

$$CE_{total} = CE_{fixed} + CE_{variable}$$

Moreover, we consider *two types of electricity device* charging sites when we compute the resulting cost of "free" public charging:

① ***Electric Vehicles***
② ***Plug-in***

Based on our previous model, the total cost formula could be described as:

$$CE_{total} = CE_{EVs} + CE_{plug-in}$$

To calculate the CE_{total}, we should calculate and then add the result of CE_{EVs} and $CE_{plug-in}$. To this end, we should execute the following procedure step by step:

Step 1: Calculate E_{EVs} / $E_{plug-in}$

Step 2: Calculate $CE_{variable} = E_{EVs} \times F \times d$ / $CE_{variable} = E_{plug-in} \times F \times d$

Step 3: Calculate $Q_{EVs_{max}}$ / $Q_{plug-in}$

Step 4: Calculate $CE_{fixed} = Q_{EVs_{max}} \times B_{charger}$ / $CE_{fixed} = Q_{plug-in} \times B_{plug-in}$

Step 5: Get result of $CE_{total} = CE_{EVs} + CE_{plug-in}$

4.2 : EVs Costing Model

4.2.1 The construction of the Energy Consumption Formula

The total energy consumption is the accumulation of energy consumed by every single car. Thus, we first need to calculate the energy of single car $E_{for\ single\ car}$. To calculate $E_{for\ single\ car}$. Then, we need to identify the quantity demand $\boldsymbol{Q_d}$ of "free" public charging. Therefore, we need two possibilities: $P(x)$ & $f(x)$ and O_{EVS}.

In short, Total energy of EVs could be expressed as following:

$$E_{EVs}(x) = \boldsymbol{Q_d} \times E_{single\ car}$$

(I) Probability of Charging Action $\boldsymbol{P(x)}$with Utilization of SOC

Firstly, no matter for EVs or Plug-in devices, the charging demand, which is the ***possibility of charging action,*** is the factor that need to calculate. It is hard to compute such *varied psychological factor*, but we consider utilizing ***SOC (State of Charge)***. SOC, means the percentage of energy consumption of a car. *For instance*, if SOC of a car equals to 10%, it represents 10% energy of this car battery has been used up (the remaining battery level is 90%).

When the SOC decreases to certain value, which we called the ***Minimum Acceptance SOC***, user's demand begin to raise: they begin to find charging sites to charge for their electric vehicles. Due to the fact that Minimum Acceptance SOC Value varies for each user, users' demand for charging sites changes as well. Accordingly, not all the users will charge their electric vehicles in the "free" public charging sites and the ratio of their charging demands depends on the calculation of varied minimum SOC acceptance value of each consumer

.

Utilizing SOC to predict the possibility of user's charging demand, we firstly *fitting* the data a collected by the NHTS2009 (National Household Travel Survey 2009) to determine the possibility of charging demand in diverse public places $P(x)$ [1]

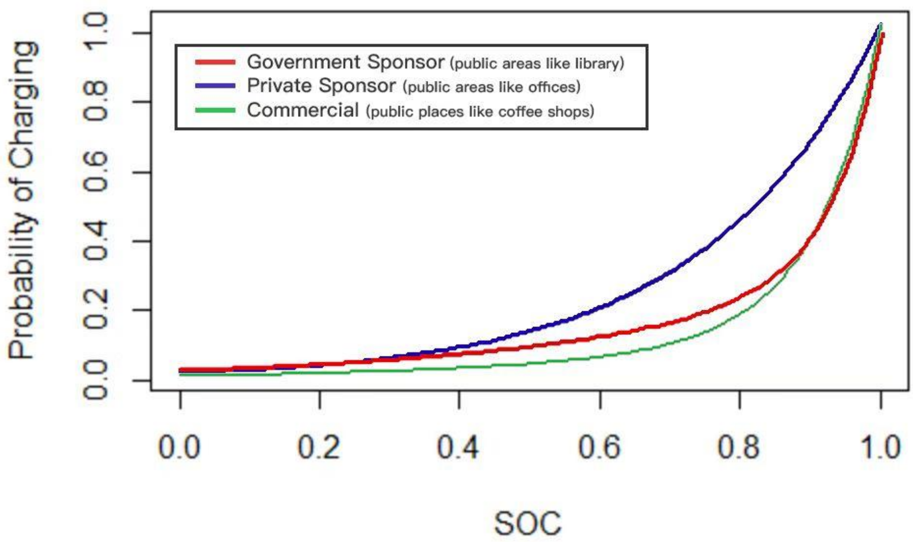

Fig4. P(x) in different public places

The resulting graph illustrates the relationship between the Minimum Acceptance SOC and the possibility of charging demand in three "free" public charging categories we have defined in part 1.1:

① ***Government Sponsor*** (public areas like library):

$$p_g(x) = 0.02576e^{2.566x} + 2.244 \times 10^{-7}e^{14.86x}$$

② ***Private Sponsor*** (public areas like offices):

$$p_p(x) = 0.007483e^{-1.289x} + 0.01961e^{3.95x}$$

③ ***Commercial*** (public places like coffee shops):

$$p_c(x) = 0.01272e^{2.474x} + 1.528 \times 10^{-5}e^{10.95x}$$

(II) Probability of Initial SOC Value $f(x)$.

Then we initiate our ***second step***: compute the ***energy consumption per car.*** According to the EV Project of US of Energy Department, we make the statistics relate to the correlation between the ***frequency*** and ***probability density of Initial SOC***, which is likely to be a normal distribution and defined as $\boldsymbol{f(x)}$.

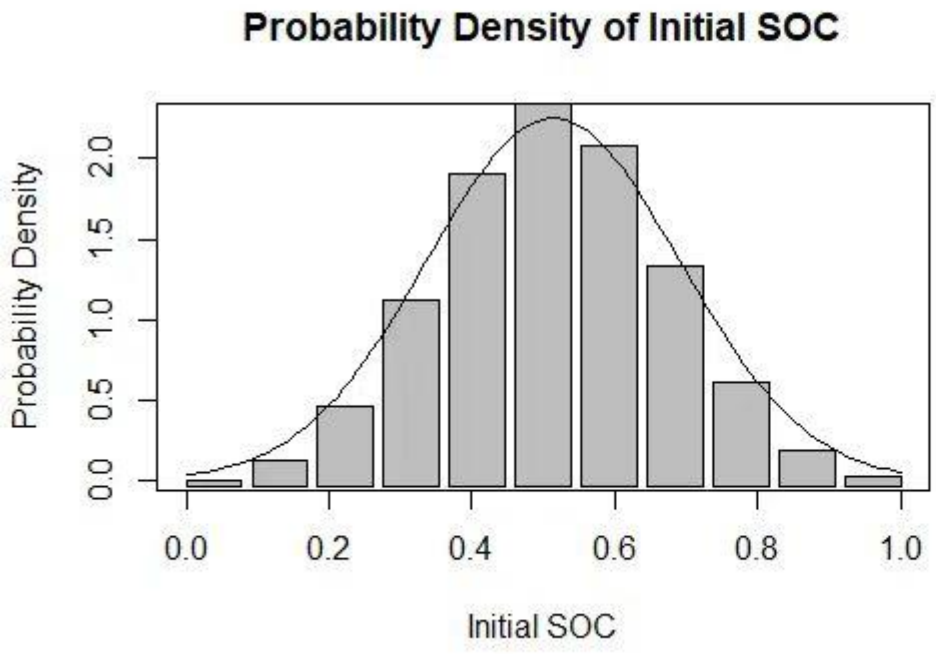

Fig.5 Frequency and probability density of initial SOC

The initial SOC could assist in determining the ***initial battery level*** of a car come for charging in public places. As our assumption for the charging demand to 100% battery level, we can know how much electricity needed for charging to 100% battery level based on the initial battery level is initial SOC.

According to the fitting result, the function for ***Frequency possibility of initial SOC*** is shown below:

$$f(\boldsymbol{x}) = \frac{1}{\sqrt{2\pi} \times 0.1772} e^{-\frac{(x-0.5173)^2}{2\times 0.1772^2}}$$

(III) Result of Total energy of EVs

Now the following steps for us is to calculate the $E_{for\ single\ car}$ which express as:

$$E_{single\ car} = T \times x$$

The demand of users using public places charging sites could be consisted of variables below and form the expression:

$$\begin{aligned} Q_d &= P_{the\ SOC\ for\ car\ owner\ is\ x} \times P_{the\ E-car\ owner\ wamt\ to\ charge} \times C_{EVS} \\ &= f(x) \times p(x) \times O_{EVS} \end{aligned}$$

Then, we use the Q_d times the $E_{for\ single\ car}$ in order to get the $E_{EVs}(x)$:

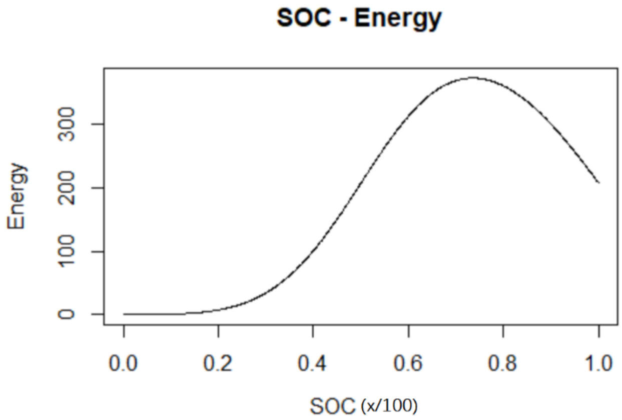

Fig.5 Energy of EVs

$$E_{EVs}(x) = \sum\nolimits_{x=1}^{10} f\left(\frac{x}{10}\right) \times p\left(\frac{x}{10}\right) \times O_{EVs} \times T \times \frac{x}{10}$$

When we compute the formula, we can expand and rewrite it as:

$$E_{EVs}(x) = \sum\nolimits_{x=1}^{10} \frac{1}{\sqrt{2\pi} \times 0.1772} e^{-\frac{\left(\frac{x}{10}-0.5173\right)^2}{2\times 0.1772^2}} \times p\left(\frac{x}{10}\right) \times O_{EVs} \times T \times \frac{x}{10}$$

4.2.2 The construction of the Resulting Cost Formula

As we mentioned before, the Total Cost is composed of two parts:

$$CE_{total} = CE_{fixed} + CE_{variable}$$

(I) CE variable

Since the energy of single EV varies, we utilize the total energy E_{EVs} to represent the sum of various $E_{for\ single\ car}$ and then calculate the ***Variable Cost (*** $\boldsymbol{CE_{variable}}$***)***, which could be expressed as below:

$$CE_{variable} = E_{EVs} \times F$$

(II) CE fixed

Q_d of *EVs chargers* and the *installation fee* $B_{charger}$ for each charger consist of the ***Fixed Cost*** **(**$\boldsymbol{CE_{fixed}}$**)**, which could be expressed as below:

$$CE_{fixed} = Q_{EVs_{max}} \times B_{charger}$$

As we aim to calculate the quantity which satisfy the maximum demand of consumers in "free" public charging places, we adopt the $N(t)$, the maximum Customer Flow to calculate $Q_{EVs_{max}}$, shown as below:

$$Q_{EVs_{max}} = G(x) \times N(t) \times P_{EVs}|_{\frac{\partial C}{\partial t}=0;\frac{\partial G}{\partial x}=0}$$

Ultimately, CE_{total} could be described as:

$$CE_{total} = CE_{fixed} + CE_{variable}$$
$$= Q_{EVs_{max}} \times B_{charger} + F \times E_{EVs}(x) \times d$$

According to our model, we find how to discover the total cost of EVs.

4.3 Plug-In Costing Model

4.3.1 The construction of the Energy Consumption Formula

In plug-in model, we first calculate the total energy of plug-in charging sites. As same as EVs, we also need to figure out $\boldsymbol{Q_d}$ and $\boldsymbol{E_{single}}$. For $\boldsymbol{Q_d}$, we need $P_{plug-in}$which could be expressed as below:

$$E_{plug-in}(x) = Q_d \times E_{single}$$

(I) Possibility of plug-in charging

According to the research by LG, when the phone' s power is less than 20%, people begin to show sign of charging demand. Therefore, the probability that a consumer in a shop need to charge his/her phone is:

$$Q_d = P_{p<20\%} \times P_{charge}$$

In order to get $P_{p<20\%}$, we collect the data from the graph below(), which shows the daily charging behavior of a person.

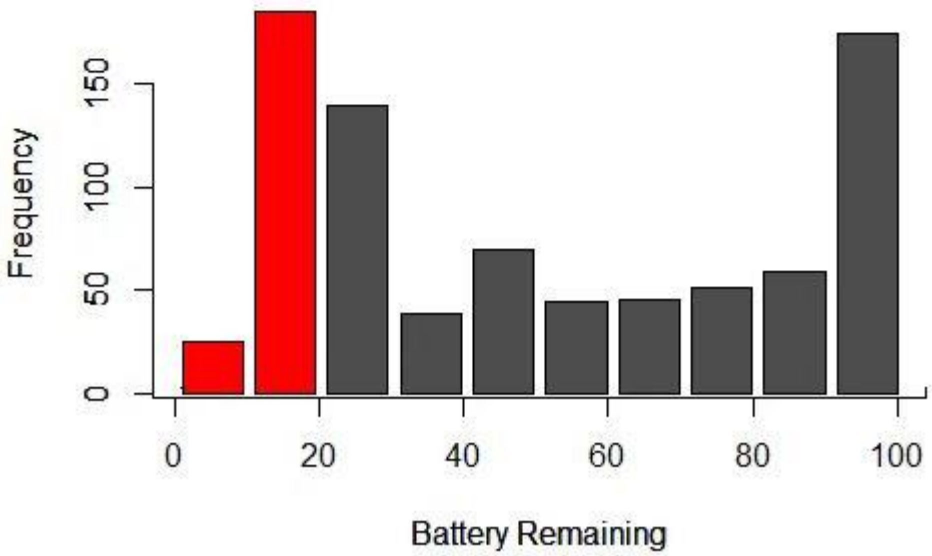

Fig.6 Daily charging behavior

As we can see in the graph above, red part represents the situation of $P_{p<20\%}$. Thus, we take the percentage of the red part, which occupies 25.21008% of the whole data, as the value we given for $P_{p<20\%}$.

$$P_{p<20\%} = 25.21008\%$$

Then, we draw our attention on energy consumption E_{single}, Considering the efficiency of charging is not 100%, E_{single} could be expressed below:

$$E_{single} = T_{phone} \times (1+\sigma) \times (1-20\%)$$

Now we multiply E_{single} and Q_d together to get $E_{plug-in}$:

$$\begin{aligned} E_{plug-in} &= Q_d \times E_{single} \\ &= P_{p<20\%} \times P_{charge} \times N \times T_{phone} \times (1+\sigma) \times 0.8 \\ &= P_{p<20\%} \times P_{charge} \times N \times 0.015552kWh \\ &\approx 0.004 \times N \end{aligned}$$

4.3.2 The construction of the Resulting Cost Formula

As same as the EVs costing model, Plug-in cost also consisted of ***Fixed Cost*** **(CE_{fixed})** and ***Variable Cost ($CE_{variable}$)***.

$$CE_{total} = CE_{fixed} + CE_{variable}$$

(I) ***Variable Cost ($CE_{variable}$)***

In order to calculate ***Variable Cost ($CE_{variable}$),*** we need to find out$E_{plug-in}$. When we get the amount of energy would be consumed in Plug-in devices, we time the amount with F to obtain the total electricity fee. Then, we multiply by days to attain the total cost for Energy consumption. We have our ***Variable Cost*** below:

$$CE_{variable} = E_{plug-in} \times F \times d$$

(II) ***Fixed Cost*** **(CE_{fixed})**

What we consider for $Q_{plug-in}$ is to satisfy the maximum quantity demanded. Thus, we choose the apex of the Customer Flow. Once we satisfy the demands when most consumers coming, we satisfy the highest charging demands.

$$Q_{plug-in} = N(t_{P<0.2})$$

Also, due to fixed cost includes first time installation fee, we also multiply $Q_{plug-in}$ with $B_{plug-in}$ to get the total ***Fixed Cost.***

$$CE_{fixed} = Q_{plug-in} \times B_{plug-in}$$

As a result, we have our Plug-in CE_{total}

$$CE_{total} = N(t_{P<0.2}) \times B_{plug-in} \times E_{plug-in} \times F \times d$$

4.4 Application of Coffee Shop

How could our model be used in the reality to actually help computing the cost? To this end, we apply our model to a coffee shop simulation to verify the feasibility and reliability of our model.

4.4.1 Energy consumption for EVs

$$E_{EVs}(x) = \sum_{x=1}^{10} f\left(\frac{x}{10}\right) \times p\left(\frac{x}{10}\right) \times C_{EVs} \times T \times \frac{x}{10}$$

(I) Basic information of the coffee shop

1. From the graph below, we can collect the data for the total $N(t) = 363$. As we know the portion of people who own EVs in America, we apply this ratio in our coffee shop simulation, assuming 21.6% of all consumers own EVs.

$$O_{EVs} = \sum_{x=0}^{24} N(t) \times P(x) = 363 \times 21.6\% \approx 78$$

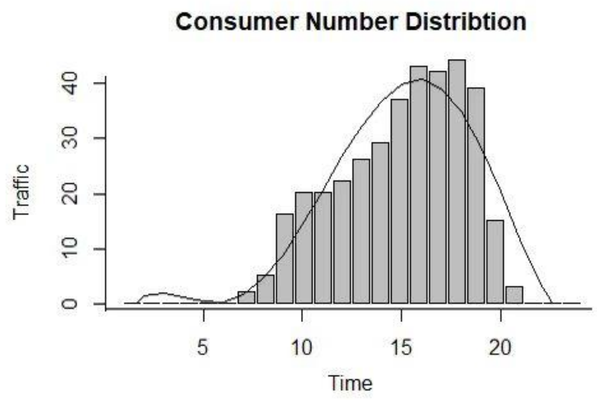

Fig.7 Customer Flow of a Starbuck in Switzerland

2. We calculate the P_{EVs}, approximately ratio of 21.6% for population who own EVs. [3]

3. Our application demo Coffee shops should be attributed into the Commercial category, which is $p_c(x)$.

$$p_c(x) = 0.01272e^{2.474x} + 1.528 \times 10^{-5}e^{10.95x}$$

Now we put our determined variables into our $E_{EVs}(x)$ formula:

$$E_{EVs}(x) = \sum_{x=1}^{10} \frac{1}{\sqrt{2\pi} \times 0.1772} e^{-\frac{\left(\frac{x}{10}-0.5173\right)^2}{2\times 0.1772^2}} \times p\left(\frac{x}{10}\right) \times C_{EVs} \times T \times \frac{x}{10}$$

Whereas:

$$\begin{cases} p\left(\frac{x}{10}\right) = 0.1272e^{\frac{2.474x}{10}} + 1.528 \times 10^{-5}e^{\frac{10.95x}{10}} \\ C_{EVs} = 78 \\ T = 50 \end{cases}$$

Therefore, $E_{e-car} = 1903.946\ kWh$, which represent the energy cost for charging station. Then, we calculate the cost of the consumption of energy. As shown before,

$$Q_{EVs_{max}} = G(x) \times C(t) \times P_{EVs}|_{\frac{\partial C}{\partial t}=0;\frac{\partial G}{\partial x}=0}$$

Considered the situation of coffee bar, we can solve out the number of charging stations:

$$Q_{EVs_{max}} = 0.1379595 \times 44 \times 0.216 \approx 1.3$$

And for the F, we use the American electric charge: F=0.12 dollar/kWh, and for the $B_{charger}$ the most common charging devices cost: \$1,000. Then, solve it out:

$$\begin{aligned} CE_{total} &= Q_{e-car} \times B_{charger} + F \times E_{e-car} \times d \\ &= 1.3 \times 1000 + 0.12 \times 1903.946 \times d \\ &= 1300 + 228.47352 \times d \end{aligned}$$

4.4.2 Energy consumption for Plug-in

When the demand of the plug-in at the highest point, $C(t) = 44$. Therefore, the quantity of plug-in should be at least 44 to satisfy all the consumer's demand. According to our *cost model for plug in*, the cost of plug-in should be:

$$\begin{aligned} &CP_{total} = E_{plug-in} \times F \times d + Q_{plug-in} \times B_{plug-in} \\ &= 1.372 \times 0.12 \times d + 44 \times 10 \\ &= 0.16464 \times d + 440 \end{aligned}$$

4.4.3: Total cost:

According to the cost model for plug-in and EV, we gain the *total cost model:*

$$\begin{aligned} \boldsymbol{C_{total}} &= \boldsymbol{CE_{total}} + \boldsymbol{CP_{total}} \\ &= 228.63816 \times \boldsymbol{d} + \mathbf{1740} \end{aligned}$$

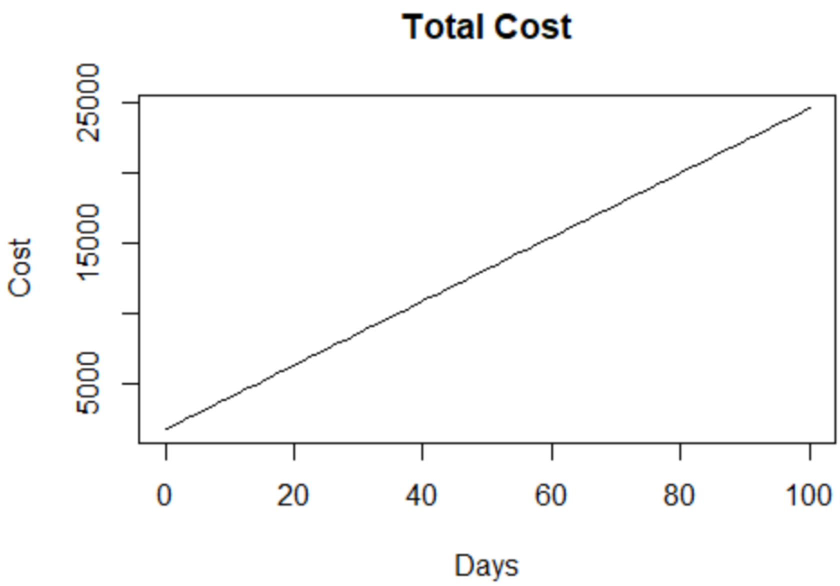

Fig.7 Total Cost for Plug-in

Whereas 1740 is the fixed cost and 228.63816 is the energy cost per day.

4.5 Extended Application of University

It will be thoroughly beneficial for all sorts of public places to be sufficiently acknowledged in this aspect: school and offices are able to pay costs by adding to tuition or wages; airports could contain the charge fee in the tickets; sharp-sighted coffee shops and shopping malls can increase their revenue by attracting more consumers with "free" charging initiatives.

For different places, like universities and coffee shops, we should choose distinct models. In this part, we use an example of a public place to describe how our model changes. We use university of MIT as an instance, to analyze the demand of the E-car charging station to satisfy 11376 students' demands in charging. For its $P(x)$function, since university could be attributed to the Government Sponsor category, it is different to coffee shop.

$$p\,(x) = 0.02576\mathrm{e}^{2.566\mathrm{x}} + 2.244 \times 10^{-7}\mathrm{e}^{14.86\mathrm{x}}$$

For government places, like a university, the propose they provide free charging service is to meet the demand of students and professors, but nor for making money by attracting consumers through free charging service. Plus, the current sockets have already satisfied students' demand and the energy-consuming for plug-in is relatively small compared to that of EV charger. As a result, we neglect the factor of plug-in in this model.

Consider that approximately 80% of students own their cars. Therefore, in the foundation that 21.6% people own EV, we time it by 80% to solve the number of college students who own EV

$$\begin{aligned} C_{EVs} &= 11376 \times 0.216 \times 0.7 \\ &\approx 1721 \end{aligned}$$

Then, we can apply the energy function:

$$\begin{cases} p(x) = 0.02576\mathrm{e}^{2.566\frac{\mathrm{x}}{10}} + 2.244 \times 10^{-7}\mathrm{e}^{14.86\frac{\mathrm{x}}{10}} \\ C(t) = 1721 \\ T = 50 \end{cases}$$
$$E_{e-car}(x) = 61473.54kWh$$

Therefore, MIT will consume 61473.54kWh for charging EV if they satisfy all the student's demands.

For solving out the quantity of charging station, we firstly find the maximum value of $G(x)$.

By calculating, we find the max point is $G(0.6)$, so,

$$\begin{aligned} Q_{e-car} &= p(x) \times f(x) \times C(t) \\ &= G(0.6) \times 1721 \\ &\approx 420 \end{aligned}$$

Therefore, $\mathrm{CE}_{\mathrm{total}}$ will be:

$$C_{total} = 420000 + 885.218976 \times d$$

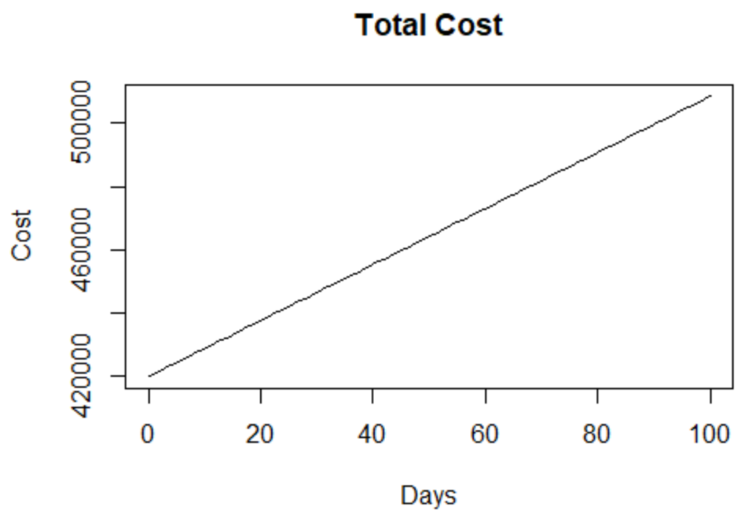

Fig.8. CE total cost

Conclusion

For MIT, to satisfy all the student demand in charging EV. Is should at least build 420 charging stations, the fixed cost is $420000, and the energy consumption for charging the EV will be $885.218976 per day.

5. Initiatives to reduce the cost

For commercial places like coffee bars or supermarkets, some of them provide free charging service. However, the dramatical increase of energy consumption will cost a lot. Therefore, it is impossible for them to provide this service for free forever. One way to solve this problem is to raise the price of goods. In this case, consumers are actually paying for this service without noticing. However, according to the basic principle of demand, when the price of a commodity increases, the demand for it will decrease because the consumer becomes less willing to buy it. Therefore, we build a model to find the adaptive price increment for shop keepers.

First of all, we can describe the revenue as:

$$R = P \times C \times d$$

Whereas the P represents the revenue per commodity, C means the number of consumers per day and d means the number of the day. For the revenue that before the shop builds the charging devices, the revenue function is:

$$R_{before} = P_{origin} \times C_{origin} \times$$

However, after the shop starts to build free charging devices and began to increase the price of their commodity, the revenue model will become like:

$$R_{after} = P_{after} \times C_{after} \times d - Cost$$
$$= P_{after} \times C_{after} \times d - Cost_{fixed} - Cost_{variable}$$

Whereas the Cost represents the cost for building the charging devices (include E-car charging station and plug-in). $Cost_{fixed}$ means the fixed cost and $Cost_{variable}$ means variable cost.

In economics, it is known to us that when the price of a good increased, the demand for this goodwill decrease by a linear relationship. This kind of correlation is called price elastic, which is shown below:

$$\epsilon = \frac{\Delta Q}{\Delta P}$$

Whereas the Q above means the quantity of demand, and P_{origin} means the price. We also define a variable σ to represent the change ratio of demand as the price changes.

$$\sigma = 1 - \frac{\Delta P}{P_{origin}} \times PE$$

For the P_{after} *and* C_{after} in R_{after} function, each of them also equal to:

$$P_{after} = P_{origin} + \Delta P$$
$$C_{after} = C_{origin} \times \sigma$$

For the $Cost_{fixed}$, and $Cost_{variable}$, according to the Energy model in question 2. Both of them $\propto$ C. Therefore, for $Cost$, which equal to $Cost_{fixed} + Cost_{variable}$, also $\propto C$

Finally, we can build the R_{after} model as below:

$$R_{after} = (P_{origin} + \Delta P) \times (C_{origin} \times \sigma) \times d - Cost_{fix-origin} \times \sigma - Cost_{variable-origin} \times \sigma \times d$$

$$\Delta R = R_{after} - R_{before}$$

For the ΔR function, it means the revenue after shop builds charging devices. To achieve their goal that making money by providing free charging service, we should find the ΔP when the ΔR function reaches the max value.

Application

We will still use the example of the coffee bar we mentioned in question 2 to calculate the price that the shop should increase to make up their price for free charging,

Assume all the commodity in this shop cost $P_{origin} = 3$ and the revenue for every single commodity are $P = 1$. For the coffee shop, they hope that they can begin to make money from their charging service after one year. The data below shows the information about this shop:

Table1: Basic information before offering 'free' charging service

P_{origin}	P	C	Cost	
			Fixed	Variable
3	1	363	1740	228

Table2: Basic information after offering 'free' charging service

P_{origin}	P	C	Cost	
			Fixed	Variable
3+ΔP	1+ΔP	363× σ	1740× σ	228 × σ

By applying the model, we can get two functions:

$$\Delta R = (1 + \Delta P) \times (363\sigma) \times 100 - 1740\sigma - 22863.816\sigma - 36300$$

$$\sigma = 1 - \frac{\Delta P \times 0.03}{3 \times 0.1}$$

Assuming that the shop keeper wants to recover their investment in one year, the ΔR vs ΔP is shown below:

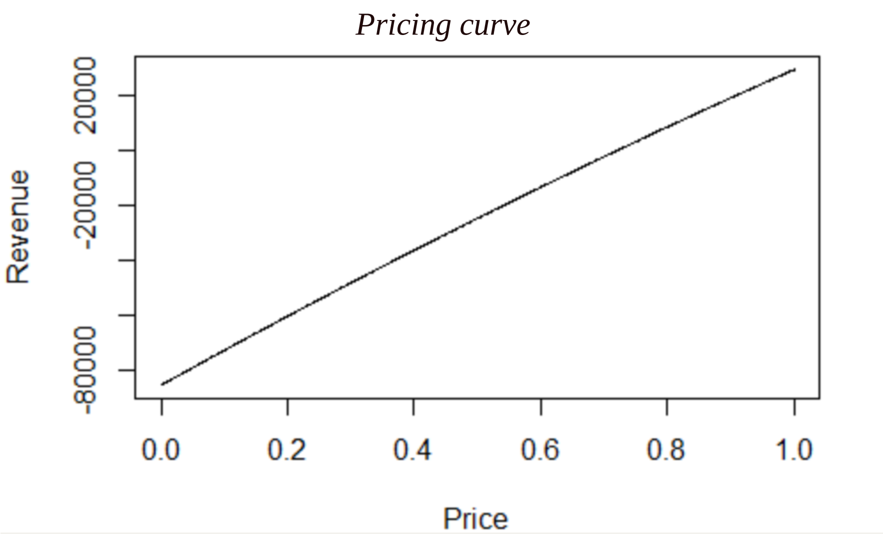

Fig.8. Price Curve

Considered it is a Quadratic function when ΔP lays between 0.72 and 0.73,. Therefore, to ensure the coffee shop start making money after one year, the price should be added by 0.73. In this situation, this coffee shop can get back revenue after 224.06 days.

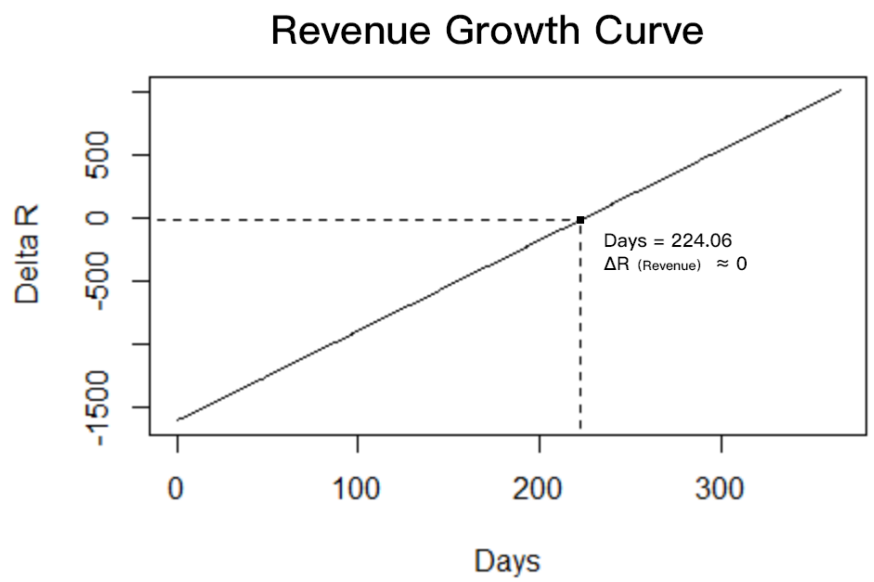

Fig.9. Revenue Growth Curve

From the graph above, the shopkeepers can be repaid back 224.06 days. When times go to 365^{th} day, the shopkeepers can earn 1014.712 than before.

6. Strengths and Weaknesses

Strength

1) The SOC model has been used by us, which excellently measured the probability of several situations.
2) We introduce several concepts in the field of economy, building up an economic model, which successfully makes the number of plugs into a dynamic variable, instead of a constant.

Weakness

1) We supposed that every person should charge their devices or electric cars until the battery is full, which is a little unreasonable.

7. Conclusion

For question 1. We use the polynomial fitting to find the pattern of "usage of electricity" and predict that it will become 1.889*10^11 in 2022. For our model 2, we build a model to find the relationship between the energy consumption and the consumer number in order for shops to know the number of EV charge stations and plugs they need to build to satisfied consumer's demand. Plus, we use a real place—A Starbucks in Swiss, as an example, to use our model to give them a plan about free charging devices—build 2 charging stations, 44 plugs, consume them 228kWh per day and 1740 for fixed cost. In model 3, we apply the model in question2 to a different situation—a university, and find MIT need to build 420 EV charging station and they will pay 885.218976 per day for free charging devices with 420000 fixed costs. In model 4, we build a model to find a way for the place where provide free charging services to gain benefits. As a result, for the coffee bar in question 2, they need to increase their commodity by 0.73 in order to recover their investment in one year.

References

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[3] "Figure 2f from: Irimia R, Gottschling M (2016) Taxonomic Revision of Rochefortia Sw. (Ehretiaceae, Boraginales). Biodiversity Data Journal 4: e7720. Https://Doi.org/10.3897/BDJ.4.e7720." doi:10.3897/bdj.4.e7720.figure2f.
[4]"Figure 2f from: Irimia R, Gottschling M (2016) Taxonomic Revision of Rochefortia Sw. (Ehretiaceae, Boraginales). Biodiversity Data Journal 4: e7720. Https://Doi.org/10.3897/BDJ.4.e7720." doi:10.3897/bdj.4.e7720.figure2f.LG Electronics USA. "'Low Battery Anxiety' Grips 9 Out Of Ten People." PR Newswire: Press Release Distribution, Targeting, Monitoring and Marketing, 29 June 2018, www.prnewswire.com/news-releases/low-battery-anxiety-grips-9-out-of-ten-people-300271604.html.
[5] "Figure 2f from: Irimia R, Gottschling M (2016) Taxonomic Revision of Rochefortia Sw. (Ehretiaceae, Boraginales). Biodiversity Data Journal 4: e7720. Https://Doi.org/10.3897/BDJ.4.e7720." doi:10.3897/bdj.4.e7720.figure2f.

Appendix

- Question 1 Prediction.m

```
close all
predictYears = 5;

EData=[
135958250
1289140735
2913200510
5508207410
9419187560
14703602560
27285640760
];

EconPdData = [
    27242955000.00,
    153915000000.00

]

EconPdYears = [
    2018,
    2023
]

n = length(EData);
x = [1:n];
x = x+2011;
pdx = [n+2011: n+2011+predictYears]
plot(x,EData, 'ko'); hold on;

%%% Degree 3 %%%
% Fit
p3 = polyfit(x,EData,3);
fx3 = p3(1)*x.^3+p3(2)*x.^2+p3(3)*x.^1+p3(4)
plot(x,fx3, '-r'); hold on;
% Resudual
sum = 0;
for i = 1:n
    sum = sum + abs(EData(i) - fx3(1));
```

```
end;
resd3 = sum/n;
% Predict
pdfx3 = p3(1)*pdx.^3+p3(2)*pdx.^2+p3(3)*pdx.^1+p3(4)
plot(pdx,pdfx3, '--or'); hold on;

%%% Degree 5 %%%
% Fit
p5 = polyfit(x,EData,5);
fx5 = p5(1)*x.^5+p5(2)*x.^4+p5(3)*x.^3+p5(4)*x.^2+p5(5)*x.^1+p5(6)
plot(x,fx5, '-g'); hold on;
% Resudual
sum = 0;
for i = 1:n
   sum = sum + abs(EData(i) - fx5(1));
end;
resd5 = sum/n;
% Predict
pdfx5 = p5(1)*pdx.^5+p5(2)*pdx.^4+p5(3)*pdx.^3+p5(4)*pdx.^2+p5(5)*pdx.^1+p5(6)
plot(pdx,pdfx5, '--og'); hold on;

%%% Labeling %%%
plot(EconPdYears,EconPdData, '^k'); hold on;
legend('Original Dataset', ['Degree 3 Fitting (Rsd ', num2str(resd5/(10^12)), '*10^1^2)'], 'Degree3 Predict',
 'Prediction of Economist');
xlabel('YEARS'); hold on;
ylabel('Total E Cost (KWHr)');
set(gcf,'color','w')
set(gca,'fontsize',14)
```

- **BarPlotting**

```
data <- read.csv("~/HiMCM-20191108/Tables/2019-11-09.csv", dec=",");
thisData <- data[[4]];

thisData <- sort(thisData);
plotData <- matrix(0, 1, 100);

for (i in 1:length(thisData)){
  thisVal <- thisData[[i]]
  plotData[[thisVal]] <- plotData[[thisVal]]+1;
}

barData <- matrix(0, 1, 10);
#plot(1:100, plotData, type='l')

for (i in 1:100){
```

```
  thisVarVal <- ceiling(i/10)
  thisVal <- plotData[[i]]
  barData[[thisVarVal]] <- barData[[thisVarVal]]+thisVal;
}

barData20 <- matrix(0, 1, 10);
for (i in 1:100){
  thisVarVal <- ceiling(i/10)
  thisVal <- plotData[[i]]
  barData20[[thisVarVal]] <- barData20[[thisVarVal]]+thisVal;
  if(i>20){
    barData20[[thisVarVal]] <- 0
  }
}

barplot(barData, names=1:10, xlab="Battery Remaining", ylab="Frequency")
par(new=TRUE)
barplot(barData20, xlab="Battery Remaining", ylab="Frequency", col='red')
```

- **Costumer_Number_Distribution.r**

```
oriData <- c(0, 0, 0, 0, 0, 0, 2, 5, 16, 20, 20, 22, 26, 29, 37, 43, 42, 44, 39, 15, 3, 0, 0, 0)
x <- 1:24
# barplot(oriData)
par(new=TRUE)

###################
#    FITTING    #
###################
model <- lm(oriData ~ poly(x, degree=5))
confint(model, level = 0.95)

###################
#    PLOTING    #
###################
pdPlot <- matrix(0, 1, 24);
for(i in 1:24){
  thisPd <- predict(model,data.frame(x = i))
  pdPlot[[i]] <- thisPd
}
plot(1:24, pdPlot, type='l', xlab='Time', ylab='Traffic') + title('Consumer Number Distribtion')

###################
#   OUTPUTING   #
###################
```

- FunctionE_integrate.r

```
E_Fun <- function(X, T) {
  E <- function(x)(1/sqrt(2*pi)*exp(((x-0.5137)^2/(-
2*0.1772^2))) * ((0.01272*exp(2.474*x)) + 1.528 * 10^(-5)*exp(10.95*x)) * x * T)
  integrate(E, 0, 1)
}

E_Fun2 <- function(X, T, footstep = 0.001, upper = 1, lower = 0) {
  E <- function(x)(1/sqrt(2*pi)*exp(((x-0.5137)^2/(-
2*0.1772^2))) * ((0.01272*exp(2.474*x)) + 1.528 * 10^(-5)*exp(10.95*x)) * x * T)
  cuts = (upper - lower)/footstep
  area = 0
  for (thisCut in 0: cuts) {
    wid = thisCut * footstep
    hei = E(wid)
    area = area + (footstep*hei)
  }
  return(area)
}

E_Plot <- function(X, T, leftscale = 0, rightscale = 1, footstep = 0.01) {
  E <- function(x)(1/sqrt(2*pi)*exp(((x-0.5137)^2/(-
2*0.1772^2))) * ((0.01272*exp(2.474*x)) + 1.528 * 10^(-5)*exp(10.95*x)) * x * T)
  cuts = (rightscale - leftscale)/footstep
  datasx = list()
  datasy = list()
  for (thisCut in 0: cuts) {
    wid = thisCut * footstep
    hei = E(wid)
    datasx = append(datasx,wid)
    datasy = append(datasy,hei)
  }
  plot(datasx, datasy, xlab='SOC', ylab='Energy consumption', type='l')
  return()
}

E <- function(X, T){
  print(E_Fun(X, T))
  print(E_Fun2(X, T))
  E_Plot(X, T)
}
```

- P(f(x)).r

```
accu <- 0.0001

funF <- function(x)((1/(sqrt(2*pi)*0.1772))*exp(0-((x-0.5137)^2)/(2*0.1772^2)))
funP <- function(x)(0.01272*exp(2.474*x)+1.528*10^(0-5)*exp(10.95*x))

# plot(seq(from=-0, to=1, by=0.01), funF(seq(from=-
0, to=1, by=0.01)) ， type='l', xlab='Initial SOC', ylab='Probability Density', ) + title('Probability Density o
f Initial SOC') + barplot(funF(seq(from=-0, to=1, by=0.1)))

###################
# MAXIMUM FINDING #
###################
funFMaxy <- max(funF(seq(from=0, to=1, by=accu)))
funFMaxx <- which(funF(seq(from=0, to=1, by=accu)) == funFMaxy)*accu

###################
# FIND  FUN  P(X) #
###################
funPxVal <- funP(funFMaxx)
```

- university_MIT_cal.r

```
funE_Car <- function(x)((1/(sqrt(2*pi)*0.1772))*exp((x/10-0.5137)^2/(0-
2*0.1772^2))*(0.02576*exp(2.566*(x/10))+2.244*(10^(0-7))*exp(14.86*(x/10)))*1721*50*(x/10))

sum <- 0
for(i in 0:10){
  sum <- sum + funE_Car(i)
}

plot(seq(from = 0, to = 10, by = 0.01), funE_Car(seq(from = 0, to = 10, by = 0.01)), type='l', xlab='SOC', y
lab='Energy') + title('SOC - Energy (in MIT)')
print(sum)
```

- PcShop,PcWork,andPcRecreation.r

```
PcWork <- function(x)(0.007483*exp(0-1.289*x) + 0.01961*exp(3.95*x))
PcShop <- function(x)(0.01272*exp(2.474*x)+1.528*(10^-5)*exp(10.95*x))
PxRecreation <- function(x)(0.02576*exp(2.566*x)+2.244*(10^(-7))*exp(14.86*x))

accu <- 0.01
upper <- 1
lower <- 0

plot(seq(from=lower, to=upper, by=accu), PcWork(seq(from=lower, to=upper, by=accu)), type='l')
```

```
par(new=TRUE)
plot(seq(from=lower, to=upper, by=accu), PcShop(seq(from=lower, to=upper, by=accu)), type='l')
par(new=TRUE)
plot(seq(from=lower, to=upper, by=accu), PxRecreation(seq(from=lower, to=upper, by=accu)), type='l', xlab='SOC', ylab='Probability of Charging') + title()
```

- price&days.r

```
####################
#     PRICE     #
####################
sigma <- function(x)1-(x*0.03)/(3*0.1)
days <- 365
Rx <- function(x)(1+x)*363*sigma(x)*days - 1740*sigma(x) - 228.63816*sigma(x)*days - 1*363*days

plot(seq(from=0, to=1, by=0.001), Rx(seq(from=0, to=1, by=0.001)), type='l', xlab='Price', ylab='Revenue')

for (i in seq(from=0, to=1, by=0.0000001)){
  if(Rx(i) > 0){
    print(i)
    print(Rx(i))
    break
  }
}

####################
#     DAYS     #
####################
x <- 0.73    # Value of 0.73 comes from results of previous section.

plot(0:5, Rx(0:5), type='l')

for (i in seq(from=0, to=300, by=0.01)){
  sigma <- function(x)1-(x*0.03)/(3*0.1)
  days  <- i
  Rx    <- function(x)(1+x)*363*sigma(x)*days - 1740*sigma(x) - 228.63816*sigma(x)*days - 1*363*days

  if(Rx(x) > 0){
    print(i)
    print(sigma(i))
    break
  }
}
Rx    <- function(days)(1+x)*363*sigma(x)*days - 1740*sigma(x) - 228.63816*sigma(x)*days - 1*363*days
```

```
plot(0:365, Rx(0:365), type='l', xlab='Days', ylab='Delta R')
```

- **SOC-Energy.r**

```
accu <- 0.001

funE_Car <- function(x)((1/(sqrt(2*pi)*0.1772))*exp((x-0.5137)^2/(0-
2*0.1772^2))*(0.01272*exp(2.474*x)+1.528*(10^(0-5))*exp(10.95*x))*78*50*x)
plot(seq(from=0, to=1, by=accu), funE_Car(seq(from=0, to=1, by=accu)), type='l', xlab='SOC', ylab='Ener
gy') + title('SOC - Energy')

area <- function(footstep = 0.001, upper = 1, lower = 0) {
  funE_Car <- function(x)((1/(sqrt(2*pi)*0.1772))*exp((x-0.5137)^2/(0-
2*0.1772^2))*(0.01272*exp(2.474*x)+1.528*(10^(0-5))*exp(10.95*x))*78*50*x)
  cuts = (upper - lower)/footstep
  area = 0
  for (thisCut in 0: cuts) {
    wid = thisCut * footstep
    hei = funE_Car(wid)
    area = area + (footstep*hei)
  }
  return(area)
}
```

- **SOC-Energy_of_f(x)-g(x)**

```
PxRecreation <- function(x)(0.02576*exp(2.566*x)+2.244*(10^(-7))*exp(14.86*x))
FxC          <- function(c)(1/(sqrt(2*pi)*0.1772))*exp(0-(((c-0.5137)^2)/(2*0.1772^2)))

from <- 0
to   <- 1
by   <- 0.001

plotData <- seq(from=from, to=to, by=by)

for(i in 1:((to-from)/by+1)){
  plotData[i] <- PxRecreation(i*by + from) * FxC(i*by + from)
}

plot(seq(from=from, to=to, by=by), plotData, type='l', xlab='SOC', ylab='Probability') + title(('F(x) * G(x)'
))

## FIND MAXIMUM ##
yMax <- max(plotData)

for(i in 1:((to-from)/by+1)){
  thisData <- plotData[i]
```

```
  if(thisData == yMax){
    xMax <- i*by
    break;
  }
}
```

- SOC-Probability.r

```
accu <- 0.0001

funF <- function(x)((1/(sqrt(2*pi)*0.1772))*exp(0-((x-0.5137)^2)/(2*0.1772^2)))
funP <- function(x)(0.01272*exp(2.474*x)+1.528*10^(0-5)*exp(10.95*x))

lower <- 0
upper <- 1
cuts  <- (upper-lower)/accu
plotData <- matrix(0, 1, cuts)
for(i in 1:cuts){
  thisX <- lower+accu*i
  thisY <- funF(thisX)*funP(thisX)
  plotData[[i]] <- thisY
}
# plot(1:length(plotData), plotData)
plot(seq(from=0, to=1, by=accu), funF(seq(from=0, to=1, by=accu))*funP(seq(from=0, to=1, by=accu)), t
ype='l', xlab='SOC', ylab='Probability') + title('SOC - Probability')

## Find Maximum ##
maxX <- max(plotData)
maxY <- max.col(plotData)*accu
```

- Get_starbucks_popu_info.py

```
import populartimes as pt
import numpy as np

sum_data = np.zeros(24)

api_key = ''
place_id = ''


data = pt.get_id(api_key,place_id)

times = data['data']

for i in times:
```

```
    sum_data += times['time']

print(sum_data)
print([int(i/7) for i in sum_data])
```