Twinkle kumari

**Indian Institute of Information Technology, Bhagalpur**

**Class Test - MA101: Engineering Mathematics I**

Full Marks - 15 Time - 30 Min

1. Find the inverse of the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}$ by Gauss Jordan method. [4]

2. Solve the following system of linear equations by using the Gauss elimination method: [5]

$$\begin{aligned} x_1 + x_2 - 2x_3 + 4x_4 &= 5 \\ 2x_1 + 2x_2 - 3x_3 + x_4 &= 3 \\ 3x_1 + 3x_2 - 4x_3 - 2x_4 &= 1 \end{aligned}$$

3. Check whether the following transformations are linear or not. [6]

(i) $T : \mathbf{R}^3 \to \mathbf{R}^2$, $T(x, y, z) = (x + 1, y + z)$

(ii) $T : \mathbf{R}^2 \to \mathbf{R}$, $T(x, y) = xy$

(iii) $T : \mathbf{R}^3 \to \mathbf{R}^2$, $T(x, y, z) = (|x|, 0)$

... with a diagram.

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# Indian Institute of Information Technology, Bhagalpur

## End Semester Examination – 2024

MA101: Engineering Mathematics – I

Full Marks – 30 Time – 2 Hours

1. Solve the following system of equations by Cramer's rule [3]

$$2x - 5y + 2z = 2;\ x + 2y - 4z = 5;\ 3x - 4y - 6z = 1$$

2. Solve the following system of linear equations by using the Gauss elimination method: [3]

$$\begin{aligned} x_1 - x_2 + x_3 &= 0, \\ -x_1 + x_2 - x_3 &= 0, \\ 10x_2 + 25x_3 &= 90, \\ 20x_1 + 10x_2 &= 80. \end{aligned}$$

3. Reduce the following matrix to row-echelon form: $\begin{bmatrix} 3.0 & 2.0 & 2.0 & -5.0 & 8.0 \\ 0.6 & 1.5 & 1.5 & -5.4 & 2.7 \\ 1.2 & -0.3 & -0.3 & 2.4 & 2.1 \end{bmatrix}$. [3]

4. Using the Gauss-Jordan method, find the inverse of the matrix: $\begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$. [3]

5. Prove that a linear transformation $T : \mathbf{V} \to \mathbf{W}$ is one-to-one if and only if $\mathrm{Ker(T)} = \{0\}$. [2]

6. Check whether the following transformations are linear or not. [6]
   (a) $T : \mathbf{M}_{22} \to \mathbf{R}$ defined by $T(A) = det(A)$.
   (b) $T : \mathbf{R} \to \mathbf{R}$ defined by $T(x) = 2^x$.
   (c) $T : \mathbf{R} \to \mathbf{R}$ defined by $T(x) = x + 1$.

7. Let $T : \mathbf{V} \to \mathbf{W}$ be a linear transformation, then [4]
   (a) the kernal of $T$ is a subspace of $\mathbf{V}$.
   (b) the range of $T$ is a subspace of $\mathbf{W}$

8. Suppose $T$ is a linear transformation from $R^2$ to $P_2$ such that $T\begin{bmatrix}1\\1\end{bmatrix} = 2 - 3x + x^2$ and $T\begin{bmatrix}2\\3\end{bmatrix} = 1 - x^2$. Find $T\begin{bmatrix}-1\\2\end{bmatrix}$ and $T\begin{bmatrix}a\\b\end{bmatrix}$ [3]

9. Find the kernel and range of the differential opperator $D : \mathbf{P}_3 \to \mathbf{P}_2$ defined by $D(p(x)) = p'(x)$. [3]

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**Indian Institute of Information Technology, Bhagalpur**

**End Semester Examination**

MA101: Engineering Mathematics I

Full Marks – 50 Time – 3 Hours

1. What is the coordinate vector of $X = (-15, 35, 2)$ related to the basis $B = \{(1, 3, 2), (-2, 4, -1), (2, -2, 0)\}$. [3]

2. Solve the following system of equations by Cramer's rule [4]

$$\begin{aligned} 3x - 2y + z &= 13 \\ -2x + y + 4z &= 11 \\ x + 4y - 5z &= -31 \end{aligned}$$

3. Find the rank of the matrix $A = \begin{bmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{bmatrix}$ by row reduction. [2]

4. Find the determinants of the matrix $B = \begin{bmatrix} 1 & -1 & 2 & 4 \\ 0 & 2 & 0 & 4 \\ 4 & -8 & 8 & 8 \\ 3 & 4 & 1 & 5 \end{bmatrix}$. [3]

5. Solve the following system of linear equations by using the Gauss elimination method: [4]

$$\begin{aligned} x_1 + x_2 - 2x_3 + 4x_4 &= 5 \\ 2x_1 + 2x_2 - 3x_3 + x_4 &= 3 \\ 3x_1 + 3x_2 - 4x_3 - 2x_4 &= 1 \end{aligned}$$

6. Let $A = \begin{bmatrix} 1 & \sqrt{2} \\ -\sqrt{2} & -1 \end{bmatrix}$. Compute the matrix $B = 3A - 2A^2 - A^3 - 5A^4 + A^6$. [2]

7. Let $A \in M_2(\mathbb{R})$, $tr(A) = 2$ and $det(A) = 3$. Find the characteristic polynomial of $A^{-1}$. [2]

8. Find the inverse of the matrix $C = \begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}$ by Gauss Jordan method. [3]

9. Find the reduce row echelon form for the matrix $D = \begin{bmatrix} 1 & 2 & -3 & 1 & 2 \\ 2 & 4 & -4 & 6 & 10 \\ 3 & 6 & -6 & 9 & 13 \end{bmatrix}$, which are assumed to be over $\mathbb{R}$. [4]

10. Let $V(\mathbb{R}) = P_n$ be vector space of polynomials in $t$ over the field of real numbers of degree $\leq n$. Show that the set $S = \{1, t, t^2, \cdots t^n\}$ is basis of $V$. [3]

11. Show that the solution set $W$ of a homogeneous system $AX = 0$ in $n$ unknowns is a subspace of $\mathbb{R}^n$. Is solution set of a nonhomogeneous system $AX = B$ a subspace of $\mathbb{R}^n$ ? Why? [4]

12. If the vectors $(0, 1, a), (1, a, 1)$ and $(a, 1, 0)$ of the vector space $\mathbb{R}^3(\mathbb{R})$ be linearly dependent, then find the value of $a$. [2]

13. Show that in a finite dimensional vector space $V$ over the field $F$, basis set $B = \{\alpha_1, \alpha_2, \alpha_3, \cdots, \alpha_n\}$, every vector $\alpha \in V$ is uniquely expressible as a linear combination of the vectors in $B$. [2]

14. Let $A$ be a matrix whose entries are real numbers. Prove that any system of linear equations $AX = b$ has either no solution or unique solution or infinitely many solutions. [3]

15. Show that the vectors $(1, 0, 0), (0, 1, 0), (0, 0, 1)$ and $(1, 2, 3)$ generate the same space as generated by the vectors $(1, 0, 0), (0, 1, 0)$ and $(0, 0, 1)$. [3]

16. Find a basis of $\mathbb{R}^3$ containing the vectors $(1, 1, 0), (1, 1, 1)$. [3]

17. Let $S = \{(x, y, z) \in \mathbb{R}^3 : 3x - y + z = 0\}$. Show that $S$ is a subspace of $\mathbb{R}^3$. Find a basis for $S$. [3]

**Indian Institute of Information Technology, Bhagalpur**

**End Semester Examination – 2024**

MA101: Engineering Mathematics – I

Full Marks – 50 Time - 3 Hours

1. Solve the following system of equations by Cramer's rule [3]

$$x + 2y + z = 9;\ 2x + 4y + z = 18;\ 3x + 5y + z = 24.$$

2. Find the rank and the nullity of the linear transformation $S : P_1 \to \mathbb{R}$ defined by

$$S(p(x)) = \int_0^1 p(x)dx. \qquad [3]$$

3. Reduce the following matrix to row-echelon form: $\begin{bmatrix} 1 & -3 & 1 & 0 & 4 \\ 2 & 1 & 0 & -3 & -2 \\ -4 & 1 & -2 & 3 & 1 \\ 3 & -1 & 0 & 2 & -2 \end{bmatrix}$. [3]

4. Let $V$ and $W$ be two finite dimensional vector spaces (over the same field of scalars). Then prove that $V$ is isomorphic to $W$ iff $\dim V = \dim W$. [3]

5. Using the Gauss-Jordan method, find the inverse of the matrix: $\begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$. [3]

6. Prove that a linear transformation $T : \mathbf{V} \to \mathbf{W}$ is one-to-one if and only if $\mathrm{Ker}(T) = \{0\}$. [2]

7. Check whether the following transformations are linear or not. [4]

   (a) $T : \mathbb{R}^3 \to \mathbb{R}^2$ defined by $T(x, y, z) = (|x|, y + z)$.

   (b) $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x, y) = (x + y, x)$.

8. Let $T : \mathbf{V} \to \mathbf{W}$ be a linear transformation, then prove that [4]

   (a) the kernal of $T$ is a subspace of $\mathbf{V}$.

   (b) the range of $T$ is a subspace of $\mathbf{W}$.

9. Find the kernel and range of the differential opperator $D : \mathbf{P}_3 \to \mathbf{P}_2$ defined by $D(p(x)) = p'(x)$. [3]

10. Find the determinants of the matrix $B = \begin{bmatrix} 5 & 4 & 2 & 1 \\ 2 & 3 & 1 & -2 \\ -5 & -7 & -3 & 9 \\ 1 & -2 & -1 & 4 \end{bmatrix}$. [3]

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11. Show that the solution set $W$ of a homogeneous system $AX = 0$ in n unknowns is a subspace of $\mathbb{R}^n$. Will the solution set of a non-homogeneous system form a subspace? [4]

12. For the linear transformation $T : \mathbb{R}^2 \to \mathbf{P}_1$ defined by $T\begin{bmatrix} a \\ b \end{bmatrix} = a + (a+b)x$. Find $T^{-1}$. [3]

13. For the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x+3y \\ 2x+2y \end{bmatrix}$. If possible, find a basis $C$ for $\mathbb{R}^2$ such that the matrix of $T$ with respect to $C$ is diagonal. [3]

14. Let $T : \mathbb{R}^2 \to P_1$ and $S : P_1 \to P_2$ be the linear transformations defined by $T\begin{bmatrix} a \\ b \end{bmatrix} = a + (a+b)x$ and $S(p(x)) = xp(x)$. Find $(SoT)\begin{bmatrix} 3 \\ -2 \end{bmatrix}$ and $(SoT)\begin{bmatrix} a \\ b \end{bmatrix}$. [3]

15. Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be the linear transformation defined by $T\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x-2y \\ x+y-3z \end{bmatrix}$ and let $B = \{e_1, e_2, e_3\}$ and $C = \{e_2', e_1'\}$ be bases for $\mathbb{R}^3$ and $\mathbb{R}^2$, respectively. Find the matrix of $T$ with respect to $B$ and $C$. [3]

16. Let $T : V \to W$ be a one-to-one linear transformation. If $S = \{v_1, v_2, ..., v_k\}$ is a linearly independent set in $V$, then show that $T(s) = \{T(v_1), T(v_2), ..., T(v_k)\}$ is a linearly independent set in $W$. [3]

## Indian Institute of Information Technology, Bhagalpur

**Mid-semester -2023**

MA101: Engineering Mathematics I

Full Marks - 30 Time 2 Hours

1. Are the vectors $(0, 2, 1, 0)^T$, $(1, 0, 0, 1)^T$ and $(1, 0, 1, 1)^T$ in $\mathbb{R}^4$ are independent? [2]

2. Solve the following system of equations by Cramer's rule [3]

$$\begin{aligned} 3x - 2y + z &= 13 \\ -2x + y + 4z &= 11 \\ x + 4y - 5z &= -31 \end{aligned}$$

3. Find rank of the following matrices by row reduction: [3+2]

$$A = \begin{bmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & -1 & 2 & 4 \\ 0 & 2 & 0 & 4 \\ 4 & -8 & 8 & 8 \end{bmatrix}.$$

4. Find the determinants of the matrix $B = \begin{bmatrix} 1 & -1 & 2 & 4 \\ 0 & 2 & 0 & 4 \\ 4 & -8 & 8 & 8 \\ 3 & 4 & 1 & 5 \end{bmatrix}$. [4]

5. Solve the following system of linear equations by using the Gauss elimination method: [3]

$$\begin{aligned} x_1 + x_2 - 2x_3 + 4x_4 &= 5 \\ 2x_1 + 2x_2 - 3x_3 + x_4 &= 3 \\ 3x_1 + 3x_2 - 4x_3 - 2x_4 &= 1 \end{aligned}$$

6. Find the reduced row echelon form for the matrix $D = \begin{bmatrix} 1 & 2 & -3 & 1 & 2 \\ 2 & 4 & -4 & 6 & 10 \\ 3 & 6 & -6 & 9 & 13 \end{bmatrix}$, which are assumed to be over $\mathbb{R}$. [3]

7. Consider the following system of linear equations [2+3]

$$\begin{aligned} x_1 + 2x_2 + 4x_3 + 4x_4 &= 7 \\ x_2 + x_3 + 2x_4 &= 3 \\ x_1 + 2x_3 &= 1 \end{aligned}$$

(a) Let $A$ be the coefficient matrix of the associated homogeneous system. Find the reduced form of $A$.

(b) Determine whether the system is consistent and if so, find the general solution.

8. If the vectors $(0, 1, a), (1, a, 1)$ and $(a, 1, 0)$ of the vector space $\mathbb{R}^3(\mathbb{R})$ be linearly dependent, then find the value of $a$. [2]

9. Let $A$ be a matrix whose entries are real numbers. Prove that any system of linear equations $AX = b$ has either no solution or a unique solution or infinitely many solutions. [3]

# Indian Institute of Information Technology, Bhagalpur

## Class Test -2023

MA101: Engineering Mathematics I

Full Marks - 15 Time - 30 Min

1. Find the inverse of the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}$ by Gauss Jordan method. [5]

2. Find the eigenspace of the matrix $B = \begin{bmatrix} 3 & -2 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$. [5]

3. Let $A$ be a matrix whose entries are real numbers. Prove that any system of linear equations $AX = b$ has either no solution or a unique solution or infinitely many solutions. [5]

Quiz-MA101 date 4-11-23 Time: 30 minutes

1) Diagonalize the Matrix $A = \begin{bmatrix} 6 & -1 \\ 2 & 3 \end{bmatrix}$ (find eigen values, eigen vectors, argue why diagonalizable, write in the diagonalizable form $A = T^{-1}DT$ ) [2+2+1+1=6 marks]

2) Find an orthonormal basis of the subspace: $span\left(\left\{\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}\right\}\right)$ [ 6 marks]

3) Find the matrix $M_{CB}$ of the Linear transform $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x - y \\ x + y \end{bmatrix}$ where $C = \left\{\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \end{bmatrix}\right\}$ and $B = \left\{\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}\right\}$.

[3 marks]

VIJAY KUMAR
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# Indian Institute of Information Technology, Bhagalpur

## End Semester -2023

MA101: Engineering Mathematics I

Full Marks - 50 Time - 3 Hours

1. Are the vectors $(0,2,1,0)^T$, $(1,0,0,1)^T$ and $(1,0,1,1)^T$ in $\mathbb{R}^4$ linearly independent? [2]

2. What is the coordinate vector of $X = (-15, 35, 2)$ related to the basis $B = \{(1,3,2), (-2,4,-1), (2,-2,0)\}$. [3]

3. Is the vector $w = (42, 6, 76)$ in the span of this set of three vectors $(1,2,11)$, $(3,1,4)$ and $(7,-4,3)$ ? [3]

4. Solve the following system of equations by Cramer's rule [3]

$$2x - 5y + 2z = 2;\ x + 2y - 4z = 5;\ 3x - 4y - 6z = 1.$$

5. Find the determinants of the matrix $B = \begin{bmatrix} 5 & 4 & 2 & 1 \\ 2 & 3 & 1 & -2 \\ -5 & -7 & -3 & 9 \\ 1 & -2 & -1 & 4 \end{bmatrix}$. [3]

6. Solve the following system of linear equations by using the Gauss elimination method: [3]

$$x + 2y + 3z = 3;\ 2x + 3y + 8z = 4;\ 5x + 8y + 19z = 11.$$

7. Find the reduced row echelon form for the matrix $D = \begin{bmatrix} 1 & 2 & -3 & 1 & 2 \\ 2 & 4 & -4 & 6 & 10 \\ 3 & 6 & -6 & 9 & 13 \end{bmatrix}$, which are assumed to be over $\mathbb{R}$. [3]

8. Consider the following system of linear equations [2+3]

$$x_1 + 2x_2 + 4x_3 + 4x_4 = 7,\ x_2 + x_3 + 2x_4 = 3,\ x_1 + 2x_3 = 1.$$

(a) Let $A$ be the coefficient matrix of the associated homogeneous system. Find the reduced form of $A$.

(b) Determine whether the system is consistent and if so, find the general solution.

-24
-15
-39

9. Find the inverse of the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}$ by Gauss Jordan method. [4]

87
-54
-33

10. Find the eigenspace of the matrix $B = \begin{bmatrix} 3 & -2 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$. [4]

11. Let $A$ be a matrix whose entries are real numbers. Prove that any system of linear equations $AX = b$ has either no solution or a unique solution or infinitely many solutions. [3]

12. Show that $\langle A, B\rangle = tr(AB^T)$, where '$tr$' is the trace function, is an inner product in $M_{mn}$. [3]

13. Check whether the following transformations are linear or not. [4]

(a) $T: M_{22} \to R$ defined by $T(A) = det(A)$.

(b) $T: R \to R$ defined by $T(x) = x + 1$.

14. Suppose $T$ is a linear transformation from $R^2$ to $P_2$ such that $T\begin{bmatrix} 1 \\ 1 \end{bmatrix} = 2 - 3x + x^2$ and $T\begin{bmatrix} 2 \\ 3 \end{bmatrix} = 1 - x^2$. Find $T\begin{bmatrix} -1 \\ 2 \end{bmatrix}$ and $T\begin{bmatrix} a \\ b \end{bmatrix}$. [3]

15. Find the kernel and range of the differential operator $D: P_3 \to P_2$ defined by $D(P(x)) = p'(x)$. Also, find the rank and nullity of $D$. [4]