Introduction to Artificial Intelligence

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Exercise Sheet 8

Due: May 4, 2022

Points total: 20 marks

Exercise 8.1 – Conditional Independence — 8 marks

Suppose you are given a bag containing n unbiased coins (i.e., both siedes come up with equal probability 0.5). You are told that $n-1$ of these coins are normal, with heads on one side and tails on the other, whereas one coin is a fake, with heads on both sides.

(a) Suppose you reach into the bag, pick out a coin c at random, flip it, and get a head. What is the (conditional) probability that c is the fake coin? (2 marks)

(b) Suppose you continue flipping c for a total of k times after picking it and see k heads. Now what is the conditional probability that you picked the fake coin? (4 marks)

(c) Suppose you wanted to decide whether the chosen coin was fake by flipping it k times. The decision procedure returns FAKE if all k flips come up heads; otherwise it returns NORMAL. What is the (unconditional) probability that this procedure makes an error? (2 marks)

Exercise 8.2 – Bayes Rule — 6 marks

Consider two medical tests, A and B, for a virus. Test A is 95% effective at recognizing the virus when it is present, but has a 10% false positive rate (indicating that the virus is present, when it is not). Test B is 90% effective at recognizing the virus, but has a 5% false positive rate. The two tests use independent methods of identifying the virus. The virus is carried by 1% of all people.

(a) You want to test yourself for the virus using only one of the tests, and assume that the test comes back positive for carrying the virus.

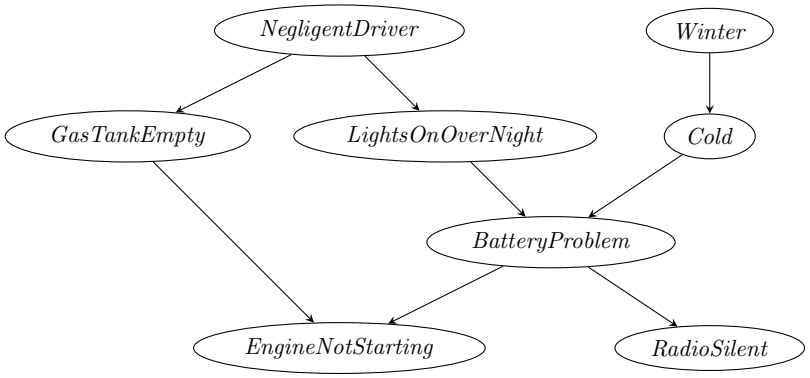
Which test returning positive is more indicative of you really carrying the virus? Apply Bayes rule and give the results for both tests.

(4 marks)

(b) A massive outbreak of the virus occured and now 30% of all people carry the virus. Do the results of part (a) change because of this? If yes give the new results, if not explain why not. (2 marks)

Exercise 8.3 – Bayes Nets **6 marks**

Consider the following Bayes net:



(a) List all nodes in the Markov-Blanket for node *LightsOnOverNight*. (1 mark)

(b) Which of the following conditional independencies follow from the structure of the Bayes net. We use $Ind(U, V \mid W)$ to denote that U is conditionally independent from V given W and $Ind(U, V)$ to denote the unconditional independence of U from V.

1. $Ind(Cold, Winter)$
2. $Ind(Winter, NegligentDriver)$
3. $Ind(Winter, RadioSilent \mid BatteryProblem)$
4. $Ind(Winter, EngineNotStarting \mid BatteryProblem)$

(2 marks)

(c) Compute $P(EngineNotStarting \mid NegligentDriver, \neg Cold)$ based on the following entries of a probability table:

$$P(LightsOnOverNight \mid NegligentDriver = 0.3$$
$$P(LightsOnOverNight \mid \neg NegligentDriver = 0.02$$
$$P(GasTankEmpty \mid NegligentDriver) = 0.1$$
$$P(GasTankEmpty \mid \neg NegligentDriver) = 0.01$$
$$P(BatteryProblem \mid Cold, LightsOnOverNight) = 0.9$$
$$P(BatteryProblem \mid Cold, \neg LightsOnOverNight) = 0.2$$
$$P(BatteryProblem \mid \neg Cold, LightsOnOverNight) = 0.8$$
$$P(BatteryProblem \mid \neg Cold, \neg LightsOnOverNight) = 0.01$$
$$P(EngineNotStarting \mid GasTankEmpty, BatteryProblem) = 0.9$$
$$P(EngineNotStarting \mid GasTankEmpty, \neg BatteryProblem) = 0.8$$
$$P(EngineNotStarting \mid \neg GasTankEmpty, BatteryProblem) = 0.7$$
$$P(EngineNotStarting \mid \neg GasTankEmpty, \neg BatteryProblem) = 0.05$$

(3 marks)

Submission rules:

- Exercise sheets must be submitted in groups of three students. Please submit a single copy of the exercises per group (only one member of the group does the submission).
- Create a single PDF file (ending `.pdf`) for all non-programming exercises. Use a file name that does not contain any spaces or special characters other than the underscore "_". If you want to submit handwritten solutions, include their scans in the single PDF. Make sure it is in a reasonable resolution so that it is readable, but ensure at the same time that the PDF size is not astronomically large. Put the names of all group members on top of the first page. Make sure your PDF has size A4 (fits the page size if printed on A4). Submit your single PDF file to the corresponding exercise assignment in MOODLE.
- For programming exercises, only create those code text files required by the exercise. Put your names in a comment on top of each file. Make sure your code compiles and test it. Code that does not compile or which we cannot successfully execute will not be graded. Create a ZIP file (ending `.zip`, `.tar.gz`, or `.tgz`; *not* `.rar` or anything else) containing the code text file(s) (ending `.py`) and nothing else. Do not use directories within the ZIP, i.e., zip the files directly.
- Do not upload several versions to MOODLE, i.e., if you need to resubmit, use the same file name again so that the previous submission is overwritten.