



**Universität
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Introduction to Artificial Intelligence Exercise Sheet 8

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Exercise 8.1

(a) $\mathbb{P}(c = \text{fake}) = 1 \div n$

(b) We assume that the event is dependent of getting k heads and the fake coin.

$$\mathbb{P}(c = \text{fake} \mid k \text{ heads}) = \frac{\mathbb{P}(k \text{ heads} \mid c = \text{fake}) * \mathbb{P}(c = \text{fake})}{\mathbb{P}(k \text{ heads})} = \frac{\frac{1}{n}}{\frac{1}{n} + \frac{n-1}{n} * (\frac{1}{2})^k}$$

$$\begin{aligned} \text{(c) } \mathbb{P}(c = \text{normal} \mid k \text{ heads}) &= \frac{\mathbb{P}(k \text{ heads} \mid c = \text{normal}) * \mathbb{P}(c = \text{normal})}{\mathbb{P}(k \text{ heads})} = \frac{(\frac{1}{2})^k * \frac{n-1}{n}}{\frac{1}{n} * 1 + \frac{n-1}{n} * (\frac{1}{2})^k} = \frac{(\frac{1}{2})^k * (n-1)}{1 + (\frac{1}{2})^k * (n-1)} \\ &= \frac{1}{1 + \frac{1}{(\frac{1}{2})^k * (n-1)}} \end{aligned}$$

Exercise 8.2

$$\begin{aligned} \text{(a) } \mathbb{P}(\text{positive result} \mid \text{Test A} = \text{positive}) &= \frac{\mathbb{P}(\text{Test A} = \text{positive} \mid \text{positive result}) * \mathbb{P}(\text{positive result})}{\mathbb{P}(\text{Test A} = \text{positive})} = \\ &= \frac{0.95 * 0.01}{0.01 * 0.95 + 0.99 * 0.01} = \frac{0.095}{0.095 + 0.099} = 0.0876 \end{aligned}$$

$$\begin{aligned} \mathbb{P}(\text{positive result} \mid \text{Test B} = \text{positive}) &= \frac{\mathbb{P}(\text{Test B} = \text{positive} \mid \text{positive}) * \mathbb{P}(\text{positive})}{\mathbb{P}(\text{Test B} = \text{positive})} = \frac{0.9 * 0.01}{0.9 * 0.01 + 0.99 * 0.05} \\ &= \frac{0.009}{0.009 + 0.0495} = 0.155 \end{aligned}$$

$$\begin{aligned} \text{(b) } \mathbb{P}(\text{positive result} \mid \text{Test A} = \text{positive}) &= \frac{\mathbb{P}(\text{Test A} = \text{positive} \mid \text{positive result}) * \mathbb{P}(\text{positive result})}{\mathbb{P}(\text{Test A} = \text{positive})} = \\ &= \frac{0.95 * 0.3}{0.3 * 0.95 + 0.7 * 0.1} = 0.802 \end{aligned}$$

$$\begin{aligned} \mathbb{P}(\text{positive result} \mid \text{Test B} = \text{positive}) &= \frac{\mathbb{P}(\text{Test B} = \text{positive} \mid \text{positive}) * \mathbb{P}(\text{positive})}{\mathbb{P}(\text{Test B} = \text{positive})} = \frac{0.9 * 0.3}{0.9 * 0.3 + 0.7 * 0.05} = \\ &= 0.885 \end{aligned}$$

Exercise 8.3

(a)

(b)

(c)