Universität Zürich[UZH]

Introduction to Artificial Intelligence
Exercise Sheet 8

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Exercise 8.1

(a)

$\boxed{8.1}$

random variables: C : coin, $\text{dom}(C) = \{\overset{\text{fake}}{f}, \overset{\text{real}}{r}\}$

T_c : throw of coin c, $\text{dom}(T_c) = \{\overset{\text{heads}}{h}, \overset{\text{tails}}{t}\}$

we have: $P(C=f) = \frac{1}{n}$ $\qquad P(T_f = h) = 1$

$P(C=r) = \frac{n-1}{n} = 1 - \frac{1}{n}$ $\qquad P(T_r = h) = \frac{1}{2}$

a) $$P(C=f \mid T_c = h) = \frac{P(T_c = h \mid C = f)\, P(C=f)}{P(T_c = h)}$$

Law of total probability:

$$\boxed{\begin{aligned} P(T_c = h) &= P(C=f)\,P(T_f = h) + P(C=r)\,P(T_r = h) \\ &= \frac{1}{n} \cdot 1 + \frac{n-1}{n} \cdot \frac{1}{2} = \frac{1}{2n}(n+1) \end{aligned}}$$

$$= \frac{1 \cdot \frac{1}{n}}{\frac{1}{2n}(n+1)} = \frac{2}{n+1}$$

(b) We assume that the event is dependent of getting k heads and the fake coin.
$\mathbb{P}(\text{c = fake} \mid \text{k heads}) = \frac{\mathbb{P}(kheads|c=fake)*\mathbb{P}(c=fake)}{\mathbb{P}(kheads)} = \frac{\frac{1}{n}}{\frac{1}{n}+\frac{n-1}{n}*(\frac{1}{2})^k}$

(c) $\mathbb{P}(c = normal \mid kheads) = \frac{\mathbb{P}(kheads|c=normal)*\mathbb{P}(c=normal)}{\mathbb{P}(kheads)} = \frac{(\frac{1}{2})^k*\frac{n-1}{n}}{\frac{1}{n}*1+\frac{n-1}{n}*(\frac{1}{2})^k} = \frac{(\frac{1}{2})^k*(n-1)}{1+(\frac{1}{2})^k*(n-1)}$
$= \frac{1}{1+\frac{1}{(\frac{1}{2})^k*(n-1)}}$

Exercise 8.2

(a) $\mathbb{P}(\text{positive result} \mid \text{Test A} = \text{positive}) = \frac{\mathbb{P}(TestA=positive|positiveresult)*\mathbb{P}(positiveresult)}{\mathbb{P}(TestA=positive)} = \frac{0.95*0.01}{0.01*0.95+0.99*0.01} = \frac{0.095}{0.095+0.099} = 0.0876$
$\mathbb{P}(\text{positive result} \mid \text{Test B} = \text{positive}) = \frac{\mathbb{P}(TestB=positive|positive)*\mathbb{P}(positive)}{\mathbb{P}(TestB=positive)} = \frac{0.9*0.01}{0.9*0.01+0.99*0.05} = \frac{0.009}{0.009+0.0495} = 0.155$
(b) $\mathbb{P}(\text{positive result} \mid \text{Test A} = \text{positive}) = \frac{\mathbb{P}(TestA=positive|positiveresult)*\mathbb{P}(positiveresult)}{\mathbb{P}(TestA=positive)} = \frac{0.95*0.3}{0.3*0.95+0.7*0.1} = 0.802$
$\mathbb{P}(\text{positive result} \mid \text{Test B} = \text{positive}) = \frac{\mathbb{P}(TestB=positive|positive)*\mathbb{P}(positive)}{\mathbb{P}(TestB=positive)} = \frac{0.9*0.3}{0.9*0.3+0.7*0.05} = 0.885$

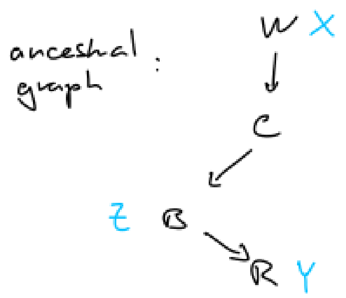
Exercise 8.3

(a) and (b)

a) NegligentDriver
BatteryProblem
Cold

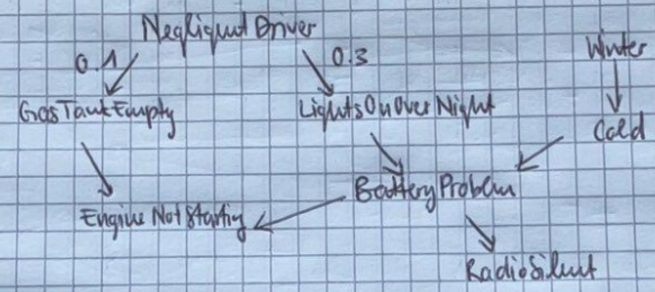
b) 1) false
2) true
3) true
4) true analogous to 3)

ancestral graph:



(c)

c) P(EngineNotStarting | NegligentDriver, ¬Cold)



→ 0.9 x 0.8 x 0.7 x 0.8 x 0.01 x 0.3 x 0.1 = 0.00012096

P(EngineNotStarting | GasTankEmpty, BatteryProblem) = 0.9
x P(" | , ¬) = 0.8
x P(" | ¬ ,) = 0.7
x P(BatteryProblem | ¬ Cold, LightsOnOverNight) = 0.8
x P(GasTankEmpty | NegligentDriver) = ~~ohen~~ 0.1
x P(BatteryProblem | ¬ Cold, ¬ LightsOnOverNight) = 0.01
x P(LightsOnOverNight | NegligentDriver) = 0.3