Take home midterm - 213

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Abstract

This midterm is a take home - use whatever it takes to solve it. Its not going to be easy! When you are done, scan the test and send it directly to Brian. typed answers are better.

1 Numerical ODE stability

The Forward Euler method for the solution of an ODE of the form

$$\dot{y} = f(y)$$

reads

$$y_{j+1} = y_j + hf(y_j).$$

To analyze the stability of the forward Euler method we used the test equation

$$\dot{y} = \lambda y$$

and wrote the method for the problem as

$$y_{j+1} = (1 + \lambda h)y_j$$

We then explored for what values of λh the method is stable.

We also explored the backward Euler

$$y_{j+1} = y_j + hf(y_{j+1}).$$

and discussed stability for it, which lead to (when applied to the test equation).

$$y_{j+1} = (1 - h\lambda)^{-1}y_j$$

The fractional θ method reads

$$y_{j+1} = y_j + h\left(\theta f(y_{j+1}) + (1-\theta)f(y_j)\right).$$

For $\theta = 0$ we obtain the forward Euler method and for $\theta = 1$ we obtain the backward Euler. Analyze the stability of the method.

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2 Match the method to the numerical solution of the ODE

Assume that you can use three ODE solvers, the forward Euler, the backward Euler and the midpoint method. Which one should you use for the system of ODE's (and why) $\dot{y} = Ay$ where A is

1. $\begin{pmatrix} -2 & 1 \\ 1 & -2 \end{pmatrix}$
2. $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$
3. $\begin{pmatrix} -10^3 & 1 \\ 1 & -10^-3 \end{pmatrix}$

3 Fluxes

The curl of a 3D field is defined by

$$\nabla \times \vec{J} = \begin{pmatrix} 0 & \partial_z & -\partial_y \\ -\partial_z & 0 & \partial_x \\ \partial_y & -\partial_x & 0 \end{pmatrix} \begin{pmatrix} j_x \\ j_y \\ j_z \end{pmatrix}.$$

- Show that $\nabla \times (\nabla \rho) = 0$
- Show that $\nabla \cdot (\nabla \times \vec{J}) = 0$
- Show that $\nabla \times \nabla \times \vec{J} - \nabla\nabla \cdot \vec{J} = -\nabla^2 \vec{J}$ where $\nabla^2 \vec{J} = [\nabla^2 j_x, \nabla^2 j_y, \nabla^2 j_z]$.