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UNEVEN GROWTH: AUTOMATION'S IMPACT ON INCOME AND WEALTH INEQUALITY

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The benefits of new technologies accrue not only to high-skilled labor but also to owners of capital in the form of higher capital incomes. This increases inequality. To make this argument, we develop a tractable theory that links technology to the distribution of income and wealth—and not just that of wages—and use it to study the distributional effects of automation. We isolate a new theoretical mechanism: automation increases inequality by raising returns to wealth. The flip side of such return movements is that automation can lead to stagnant wages and, therefore, stagnant incomes at the bottom of the distribution. We use a multiasset model extension to confront differing empirical trends in returns to productive and safe assets and show that the relevant return measures have increased over time. Automation can account for part of the observed trends in income and wealth inequality.

KEYWORDS: Inequality, wealth, capital, returns, wages, labor share, technology, automation.

1. INTRODUCTION

OVER THE PAST 40 YEARS, economic growth in many advanced economies has been unevenly distributed. In the United States, while the aggregate economy has grown at roughly 2% per year, income percentiles corresponding to the lower half of the distribution have stagnated. At the same time, incomes at the 95th percentile have doubled and top 1% incomes tripled.[1]

One potential driver of these trends that is often cited by pundits and policy makers is technical change, and in particular automation. A large literature in macro and labor economics has studied how technology and automation affect the distribution of labor incomes.[2] But not all income is labor income and capital is an important income source, particularly at the top of the distribution where incomes have increased the most. Existing

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[1] See, for example, Census Bureau (2015) and Piketty, Saez, and Zucman (2018).

[2] See, for example, Katz and Murphy (1992), Krusell, Ohanian, Ríos-Rull, and Violante (2000), and Autor, Katz, and Kearney (2006).

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theories therefore paint an incomplete picture of technology's implications for overall income inequality. This shortcoming is particularly acute when it comes to automation, technical change that substitutes labor with capital and increases the importance of capital in the economy.

We argue that the benefits of new technologies—and in particular automation technologies—accrue not only to high-skilled labor but also to owners of capital in the form of higher capital incomes. This increases inequality. To make this argument, we develop a tractable framework that allows us to study the impact of technology on factor prices and the personal distribution of income and wealth—and not just that of wages—and use it to study the distributional effects of automation.

Our framework provides a complete characterization of how technology and, more generally, changes in the economy's production and market structure affect the personal distribution of income, wages, and capital ownership, as well as macroeconomic aggregates.[3] We achieve this by assuming that households differ in their skills and their wealth accumulation is subject to *dissipation shocks*, which leave them with zero assets and only their labor income, thereby capturing the hazards of accumulating and maintaining a fortune. Dissipation shocks allow us to deviate in a tractable fashion from models that admit a representative household (like variants of the neoclassical growth model) and introduce two key features relative to such models that are crucial for understanding how automation affects inequality and aggregates.[4] First, our theory generates well-defined and tractable steady-state distributions for wealth and income. Second, the long-run supply of capital is less than perfectly elastic and, therefore, determines the long-run effect of technology on returns to wealth (prices) and the amount of capital used in production (quantities).

The framework underscores two novel channels through which technology affects income inequality and contributes to the pattern of uneven growth described above. First, automation increases wealth and capital income inequality by raising returns to wealth. Second, relative to theories in which returns are unaffected, automation can lead to stagnant wages and, therefore, stagnant incomes at the bottom of the income distribution. The key to understanding both results is that the long-run capital supply in our model is upward-sloping. Automation increases the demand for capital relative to labor and, because supply is upward-sloping, this demand shift *permanently* increases returns to wealth. The first part, that automation directly increases wealth and income inequality, then follows because households receive a higher return to their assets and grow their fortunes more rapidly. The second part—that wages are more likely to stagnate—follows because some of the productivity gains from automation do not accrue to workers but rather to owners of capital in the form of a higher return to their wealth. Neither of these mechanisms would be operational in textbook representative household models, in which the long-run capital supply is perfectly elastic and, therefore, returns are unaffected by technology (see, e.g., Acemoglu and Restrepo (2018), Caselli and Manning (2019)).

We first illustrate these mechanisms using a simple baseline model and later extend this model in a number of directions. In the baseline model, capital is the only asset in the

[3]Importantly, ours is a theory of the *personal* income distribution and not just of the *factor* income distribution. The latter type of theory—for example, "two class models" with capitalists and workers—cannot speak to a number of empirical regularities in developed countries, for example, that individuals at the top of the labor income distribution typically also earn substantial capital incomes (and vice versa).

[4]Our notion of "models that admit a representative household" allows for skill heterogeneity and includes models that assume there are different skill types but these are all members of the same representative household, and models where Gorman aggregation holds (see Theorem 5.2 in Acemoglu (2009)).

economy and all households earn the same return to their wealth. We obtain two main analytical results that illustrate how automation affects inequality.

First, the steady-state return r exceeds the discount rate ρ by a *return gap* $r - \rho = p\sigma\alpha_{\text{net}}$, where p is the arrival rate of dissipation shocks, σ is the inverse of the intertemporal elasticity of substitution, and α_{net} is the net capital share—an object that rises with automation. In contrast to textbook representative household models in which this return equals the (fixed) discount rate, the return in our model increases with automation.

Second, the wealth accumulation process of individual households generates an exact Pareto distribution for both effective wealth (the sum of financial and human wealth) and income conditional on wages. Tail inequality—as measured by the inverse of the Pareto shape parameter—equals the net capital share α_{net}, meaning that both top income and wealth inequality increase with automation. Intuitively, the gap between the return to wealth and the discount rate determines the speed at which *individual households* accumulate wealth in the absence of a dissipation shock, which in turn determines the thickness of the wealth distribution. In equilibrium, this gap equals $p\sigma\alpha_{\text{net}}$ and, therefore, the net capital share α_{net} pins down the tail index of these distributions. This result illustrates how automation contributes to income inequality by permanently increasing returns to wealth and the concentration of capital ownership.[5]

The flip side of the finding that the return to wealth rises with automation is that wages of displaced workers are more likely to fall, not only *relative* to other skill types (as commonly emphasized in the literature), but also in *levels*. In our theory, the upward-sloping supply of capital limits the expansion of investment and output in response to technological improvements. As a result, automation can lead to stagnant or falling real wages even in the long run, especially for workers whose skills are more susceptible to automation.

Although our baseline model is intentionally stylized, the mechanisms underlying our results are more general. The two key ingredients behind our results are an upward-sloping long-run supply of capital—so that technology persistently affects asset returns—and a nexus between returns to wealth and inequality. The first ingredient, an upward-sloping capital supply, seems to us a more natural and less extreme starting point than the perfectly-elastic capital supply of the textbook growth model and its relatives. It is also a feature of overlapping generations (OLG) models, models with a life-cycle component, and models with labor income risk and precautionary savings (as in Aiyagari–Bewley–Huggett models). The second ingredient, a return-inequality nexus, emerges naturally in models in which stochastic wealth accumulation at the individual level generates a fat-tailed wealth distribution. This includes models with stochastic returns or discount rates (Krusell and Smith (1998), Benhabib, Bisin, and Zhu (2011, 2015)) and models with explosive growth coupled with a birth and death process (Wold and Whittle (1957), Steindl (1965), Benhabib and Bisin (2007), Jones (2015), Sargent, Wang, and Yang (2021)).[6] Also the argument of Piketty (2014, 2015) that top wealth inequality depends on "$r - g$" highlights precisely this nexus.

[5]This new channel differs from the common argument that a rise in the capital share leads to higher inequality because capital income is more unequally distributed than labor income (Meade (1964), Piketty (2014)). As we discuss in detail, such compositional effects are small relative to the data and to the changes in capital ownership generated by our model.

[6]See Benhabib and Bisin (2018) for a review of models capable of generating fat-tailed wealth distributions. Among the models surveyed, only models where the tail of the wealth distribution is induced by the tail of the distribution of labor earnings ("models with skewed earnings") lack a return inequality nexus. These include models with finite lives and no inheritances, and simple versions of Aiyagari–Bewley–Huggett models (Stachurski and Toda (2019)).

In our baseline model, automation has important distributional consequences because it raises *the* return to wealth. One of the main challenges in applying this theory to the data is that returns on different assets have displayed divergent empirical trends: treasury rates have declined over time (e.g., Rachel and Summers (2019)) while the return to other assets, including US business capital and equity have increased (e.g., Caballero, Farhi, and Gourinchas (2017), Gomme, Ravikumar, and Rupert (2011), Reis (2020)). What is then the relevant return to evaluate our theory? And how should we account for changes in the growth rate of the economy and other forces affecting returns?

To answer these questions, we extend our model to feature multiple assets with different returns, richer capital income risk, markups, taxation, and long-run growth. Despite the added complexity, the model remains tractable. In our extended model, the return to risky capital exceeds the return to safe bonds because of compensation for risk or financial frictions that prevent arbitrage.[7] We show that the key return measure determining wealth inequality is the overall return to investors' wealth, r_W, which depends on investors' portfolio allocation and leverage. The safe rate is not necessarily informative about this return. In fact, when investors are leveraged, lower safe rates *increase* this return, and hence wealth inequality. Moreover, automation leads to a higher return to investors' wealth and a widening spread between risky and safe returns, with ambiguous implications for the level of the safe rate. Finally, the model also shows that, in a growing economy, wealth inequality depends on the *return gap* $r_W - \rho - \sigma g$, and that this gap always rises with automation.

These results show that, when assessing the validity of our mechanism, one should focus on the return to investors' wealth and the return gap. Building on this insight, we provide a series of estimates for the return to investors' wealth r_W and the return gap. Contrary to the observed trend in safe rates and in line with our theory, the return to wealth rose by 0.5–2 percentage points and the return gap rose by 1.5–3.5 percentage points since 1980.

The addition of risky capital in the extended model generates higher levels of top-tail inequality. In our baseline model, top income inequality as measured by the inverse of the Pareto shape parameter equals $\alpha^*_{\text{net}} \approx 0.2$, which is much lower than the corresponding empirical values of around 0.7. In contrast, the extended model with capital income risk can generate the higher tail inequality that we see in the data.

After presenting our main analytical results, we turn to a numerical evaluation of the model. In our numerical exercise, we study how the automation of routine jobs contributed to overall income inequality. To do so, we infer changes in automation by percentile of the wage distribution using exposure to routine jobs, which the literature singles out as jobs that can be easily automated using computer software or other equipment (see Autor, Levy, and Murnane (2003), Autor, Katz, and Kearney (2006)). To benchmark the extent to which these jobs have been automated, we calibrate the automation of routine jobs since 1980 to match the declining share of labor in GDP during this period.

The automation of routine jobs generates a pattern of uneven growth reminiscent of that observed in the US over the last 40 years. Two features combine to produce this pattern. First, our model generates a decline in real wages at the bottom and middle of the income distribution, which accounts for part of the income stagnation observed at these percentiles. Second, it generates a rising concentration of capital income at the top of the distribution, which accounts for the sharp rise of incomes at the top. In particular,

[7]The observed divergence between real risk-free interest rates and the returns to capital has been previously emphasized by Caballero, Farhi, and Gourinchas (2017) and Farhi and Gourio (2018) who propose various candidate explanations ranging from rising market power to rising discounts on safe assets.

automation generates an increase in tail income inequality (measured by the inverse of the Pareto index) from 0.54 to 0.65, about 75% of the increase estimated by Piketty, Saez, and Zucman (2018).

The first contribution of our paper is to the literature on technological change and automation. Like most theory papers in this area, we use a task-based framework to model automation (Zeira (1998), Acemoglu and Autor (2011), Acemoglu and Restrepo (2018)). Papers in this literature focus on wage inequality (Autor, Levy, and Murnane (2003), Autor, Katz, and Kearney (2006), Acemoglu and Autor (2011), Hémous and Olsen (2022)), or study the effect of automation on aggregates and wages using a representative household framework (Acemoglu and Restrepo (2018), Caselli and Manning (2019)). One exception is Sachs and Kotlikoff (2012), who study the possibility of immiserizing growth in an OLG model. We contribute to this literature by moving beyond representative-household models and exploring the implications of automation for inequality of total incomes and wealth.

Our second contribution is to the literature exploring the determinants of wealth inequality. Several papers explore this question quantitatively in general equilibrium models, including Krusell and Smith (1998), Castañeda, Díaz-Giménez, and Ríos-Rull (2003), Kaymak and Poschke (2016), Straub (2019), and Hubmer, Krusell, and Smith (2021). Relative to this literature, we isolate a new theoretical mechanism: technical change affects income and wealth distribution via returns to wealth. In contrast, other papers studying how technology affects wealth distribution focus on its effects through wage dispersion. Our model is deliberately simple and abstracts from labor income risk and a realistic treatment of life cycle and bequests—important elements in quantitative theories of the wealth distribution. In exchange, we obtain analytical solutions for the steady-state distributions of wages, incomes, and wealth.[8] Furthermore, the economy aggregates, and solving for its transition dynamics is as easy as in the neoclassical growth model. Thus, our model is closer in spirit to models of the Gorman class (see Chatterjee (1994), Caselli and Ventura (2000)) in which policy functions are linear and aggregates do not depend on the wealth distribution.[9] However, in contrast to these models, dissipation shocks generate a determinate steady-state wealth distribution and imply that our model does not admit a representative household.

Section 2 lays out our baseline theory of uneven growth. Section 3 presents the extended model and a measurement exercise showing that the relevant return measures have increased in line with our theory. In Section 4, we take the extended model to the data with a calibration of changes in automation across the wage distribution, and in Section 5 we show that this generates a pattern of uneven growth like that observed in the data. Section 6 concludes. Appendices A and B contain proofs and derivations for the

[8]The mechanism generating a Pareto tail in our model is a common feature of random growth processes (see Gabaix (2009), for a review). In fact, the process with dissipation shocks in our model is a simple and tractable example of a random growth process, and the idea that it leads to a Pareto distribution has been used by many authors before us. Among those, the closest to our work are Cao and Luo (2017), who also derive formulas for the tail parameter of the wealth distribution in terms of aggregate statistics and provide comparative statics, but do not explore the role of automation as we do.

[9]Because policy functions are linear "macro matters for inequality, but inequality does not matter for macro." A large recent literature argues that models incorporating empirically realistic heterogeneity in household balance sheets and marginal propensities to consume often deliver different aggregate implications than do representative agent models, precisely because aggregates depend on distributions. We view linear policy functions and other abstractions with unrealistic implications as costs worth paying in return for our theory's analytical tractability.

baseline and extended models, respectively (Moll, Rachel, and Restrepo (2022)). Supplementary material available online contains analysis of dissipation shocks, equilibrium income composition, transitional dynamics as well as additional evidence on the main mechanisms and a detailed description of the calibration and data construction.

2. BENCHMARK MODEL OF UNEVEN GROWTH

The model is cast in continuous time. For expositional clarity, we outline the model in stationary form. Appendix A provides proofs and derivations. Appendix E describes the model along the transition path.

2.1. *Economic Environment*

Households. There is a unit continuum of households that differ in their skills z and supply their labor inelastically, with ℓ_z denoting the population share of each skill. Households maximize standard preferences over utility flows from consumption subject to a flow budget constraint and a natural debt limit:

$$\max_{\{c_{z,t},a_{z,t}\}_{t\geq 0}} \int_0^\infty e^{-\rho t}\frac{c_{z,t}^{1-\sigma}}{1-\sigma}\,dt$$
$$\text{subject to: } \dot{a}_{z,t} = w_z + ra_{z,t} - c_{z,t}, \text{ and } a_{z,t} \geq -w_z/r, \tag{1}$$

where $a_{z,t}$ denotes assets, $c_{z,t}$ consumption, r the return to wealth, and w_z wage income.

Without any modification, Gorman's aggregation theorem applies (see Theorem 5.2 in Acemoglu (2009)) and this model would admit a representative household. Independently of the production side, the steady state would involve a constant interest rate equal to the discount rate ρ (or $\rho + \sigma g$ in a growing economy). Moreover, the wealth distribution would be indeterminate (see Caselli and Ventura (2000)), and so the theory would be ill-suited to study how technology affects income and wealth inequality.

We break Gorman aggregation by assuming that the accumulation of wealth is subject to *dissipation shocks*, which arrive at a rate $p > 0$. We operationalize this assumption in the simplest possible way by assuming that after receiving a dissipation shock, households consume all of their wealth and are left with zero assets, so that following a dissipation shock we have $a_{z,0} = 0$.[10] In our formulation and in all this section, t denotes the time elapsed since the last dissipation shock and not calendar time. Appendix C shows formally that in the presence of dissipation shocks, households solve (1) and discount the future at a rate $\rho := \varrho + p$, where ϱ is the pure rate of time preference. In what follows, we will refer to the case with $p = 0$ as the representative-household benchmark.

Dissipation shocks provide a reduced-form way of capturing the risks and hazards involved in raising and maintaining a fortune, both over time and across generations. These risks are a defining feature of wealth accumulation, as emphasized by recent work that documents substantial churn at the top of the wealth distribution (Gomez (forthcoming), Zheng (2019)). Appendix C.1 provides several microfoundations for dissipation shocks, many of which are key ingredients in existing theories of wealth inequality:

[10]For households with debt, we assume that dissipation shocks act as a cancellation of their debt. Alternatively, one could assume that households cannot borrow, or that dissipation shocks only hit households with a positive level of assets. These assumptions are not important for our results. As we will show, the steady state of our model involves all households holding positive assets, that is, no one holds debt.

- *perpetual youth*: individuals die with a constant probability p and are not altruistic toward their offspring. If there are no accidental bequests (either because of an annuity market, as in Blanchard (1985), or because people consume their wealth right before they die), newborns start life with zero assets and only their labor income, mimicking the effect of a dissipation shock.[11]
- *finite lives and stochastic altruism*: finitely-lived individuals are part of a dynasty and pass on their wealth over time. At rate p, the current member of the dynasty stops being altruistic and consumes all of her wealth so that the dynasty is interrupted.
- *population growth*: p captures net increases in population. Being born is isomorphic to receiving a dissipation shock because newborns start life with zero assets.
- *discount rate shocks*: at rate p, households become infinitely impatient and consume all of their wealth. This is a simple and tractable version of the more general process for discount rates assumed in Krusell and Smith (1998).
- *uninsured capital income risk*: there are multiple investment projects, households invest in a single project, and at rate p the capital produced in this project becomes obsolete and looses all its value, that is, a return of minus 100%.[12] This is a simple version of more general processes for idiosyncratic returns to wealth (e.g., Benhabib, Bisin, and Zhu (2011, 2015)).

Technology. Our description of the production process follows Acemoglu and Restrepo (2018) and emphasizes the role of tasks, which are either completed by workers or automated. Each skills z can be used to produce a different unit continuum of tasks, a fraction α_z of which are technologically automated and can be produced by capital. In addition, tasks are combined via a Cobb–Douglas production function. Appendix A.1 shows that, in this setup, the economy's production side reduces to an aggregate production function in terms of capital and labor

$$Y(K) := \mathcal{A}K^{\sum_z \alpha_z \eta_z} \prod_z (\psi_z \ell_z)^{(1-\alpha_z)\eta_z}, \tag{2}$$

where η_z denotes the importance in value added of the tasks that can be performed by skill z, ψ_z denotes the productivity of labor at these tasks, and K denotes the aggregate stock of capital in the economy. The productivity term $\mathcal{A}$ is a constant that depends on parameters $\{A, \alpha_z, \eta_z\}$, where A captures the role of factor-neutral technological improvements.

Labor of skill z is rented at a wage w_z. Capital is produced one-for-one from the final good and depreciates at a rate $\delta \geq 0$, which implies a rental rate of $R = r + \delta$. Therefore, the unit cost of producing a task with capital is R and that of producing it with labor is w_z/ψ_z where ψ_z is the productivity of skill z. The representation in equation (2) assumes

[11]When there is an annuity market, the return obtained by households is $r + p$, where the addition of p accounts for the income from annuities. See Appendix C.1.1 for a more detailed discussion.

This setup can be generalized to allow for some bequests (e.g., due to partial altruism or a "joy of giving" bequest motive) as in Benhabib and Bisin (2007), so long as individuals do not pass on their entire wealth to their offspring thereby forming a perfect infinitely-lived dynasty.

[12]This microfoundation requires two reasonable assumptions. First, households cannot be perfectly diversified across product lines. Second, risk-sharing is limited. Otherwise, an investment company could collect all investments and offer households a risk free return of $r - p$, which they would prefer to a return of r coupled with the risk of losing all of their capital with probability p.

that all tasks that can be produced by capital are allocated to capital in equilibrium, which imposes the following assumption:[13]

ASSUMPTION 1—Full adoption of automation technologies: *The cost of producing tasks with labor*, w_z/ψ_z, *exceeds the cost of producing them with capital*, R:

$$\frac{w_z}{\psi_z} > R \quad \text{for all } z.$$

Equation (2) represents aggregate output as a Cobb–Douglas production function with factor shares linked to the range of tasks performed by each factor. The share of capital is

$$\alpha := \sum_z \alpha_z \eta_z,$$

which gives the average degree of automation in the economy. Our main exercise studies the effects of improvements in automation technologies parameterized as exogenous increases in the α_z's. An increase in α_z captures the automation of tasks previously performed by workers of skill z in the sense that it expands the range of tasks in which capital can now substitute for these workers. For example, workers engaged in white-collar office work devote their time to tasks such as accounting, keeping and locating records, and customer support. Workers engaged in blue-collar work devote their time to tasks such as welding, painting, assembling, machining, and supervision. Over time, technological improvements have allowed the automation of some of these tasks, while others, like customer support or supervision, remain the domain of workers. As tasks that were the domain of skill z get automated, this skill loses importance in production ($1-\alpha_z$ declines) and capital gains importance (α rises). Assumption 1 ensures that an increase in α_z raises TFP and that these technologies are adopted. Changes in the α_z's thus provide a flexible way of capturing technological improvements that involve an expansion in the range of tasks that capital and machines can do at the expense of some workers.

Equilibrium. We assume competitive input and final good markets and define a steady- state equilibrium of the economy as follows.

DEFINITION 1: The steady state equilibrium is given by aggregate output and capital, factor prices, and consumption and saving policy functions such that

- factor prices equal marginal products, $w_z = (1-\alpha_z)\frac{\eta_z}{\ell_z}Y(K)$, $R = \alpha\frac{Y(K)}{K}$, and $r = R - \delta$;
- policy functions $a_{z,t}$, $c_{z,t}$ maximize utility given $a_{z,0} = 0$, w_z and r, where t denotes time since the last dissipation shock experienced by a household;
- capital markets clear: $K = \sum_z \ell_z \int_0^\infty a_{z,t} p e^{-pt}\, dt$, where pe^{-pt} is the share of households that have accumulated wealth without experiencing a dissipation shock for t periods.

We first study households' consumption and saving decisions, which determine the supply of capital in the economy.

[13]Assumption 1 involves endogenous factor prices, so it needs to be verified in equilibrium. Lemma 2 in Appendix A.1 shows that it holds when productivity A is high.

LEMMA 1—Policy functions: *Suppose that $r > (r-\rho)/\sigma$. Starting from $a_{z,0}=0$, the solution to* (1) *is given by policy functions that are linear in effective wealth*, $a_{z,t}+w_z/r$:

$$\dot{a}_{z,t}=\frac{r-\rho}{\sigma}\left(a_{z,t}+\frac{w_z}{r}\right), \qquad c_{z,t}=\left(r-\frac{r-\rho}{\sigma}\right)\left(a_{z,t}+\frac{w_z}{r}\right). \tag{3}$$

The lemma shows that households accumulate assets at a rate $(r-\rho)/\sigma$ and consume a constant share of their effective wealth. The linear policy functions imply that, in our model, the behavior of aggregates will be independent of the wealth and income distribution. In what follows, we will explore the behavior of aggregates in response to automation first, and then study how changes in factor prices and returns shape the income and wealth distribution.

2.2. *Behavior of Aggregates and Wages*

We characterize the equilibrium of the return to wealth and aggregates in steady state. We denote variables in the equilibrium steady state with an asterisk.

PROPOSITION 1—Return to wealth and aggregate effects of automation: *The steady state is unique. The return to wealth*, r^*, *is given by the solution to the equation*

$$\frac{1-\rho/r^*}{p\sigma+\rho-r^*}=\frac{\alpha}{1-\alpha}\frac{1}{r^*+\delta}. \tag{4}$$

Equivalently, denoting by $\alpha^*_{\text{net}}:=r^*K^*/(r^*K^*+\sum_z w^*_z\ell_z)$ *the net share of capital income, the return to wealth satisfies*

$$r^*=\rho+p\sigma\alpha^*_{\text{net}}. \tag{5}$$

The return to wealth, r^*, *the net capital share* α^*_{net}, *and the capital-output ratio*, $(K/Y)^*$, *are all increasing functions of* α. *Moreover, output increases in all* α_z.

We can think of the return to wealth as determined by the supply of and demand for capital. It is convenient to think in terms of capital relative to labor income

$$k:=\frac{K}{\bar{w}},$$

where $\bar{w}:=\sum_z w_z\ell_z$ is the wage bill in the economy. To derive the supply of capital, we use the characterization in Lemma 1. Integrating equation (3) yields $\dot{K}=\frac{r-\rho}{\sigma}(K+\frac{\bar{w}}{r})-pK$, where the term $-pK$ accounts for the wealth loss due to dissipation shocks. In steady-state $\dot{K}=0$, and hence

$$\frac{r-\rho}{\sigma}\left(K+\frac{\bar{w}}{r}\right)=pK.$$

Rearranging gives the (relative) supply of capital for $r\geq\rho$:

$$k^s=\frac{1-\rho/r}{p\sigma+\rho-r}. \tag{6}$$

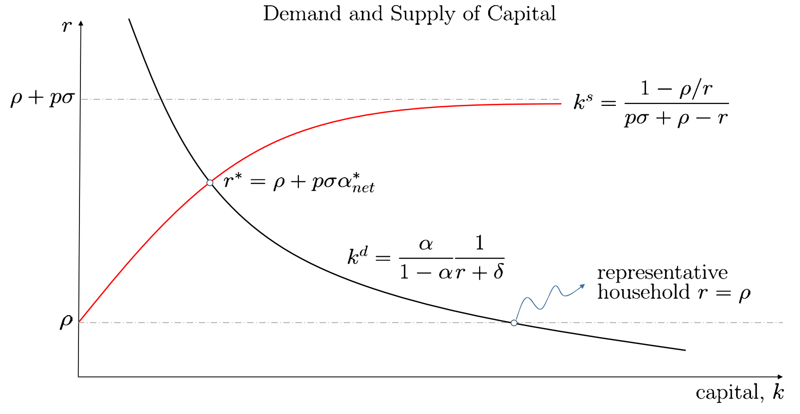

FIGURE 1.—The supply and demand for capital and the determination of the return to wealth.

This equation gives the upward-sloping long-run capital supply curve depicted in Figure 1. Intuitively, as the return to wealth rises, households accumulate and supply more capital (relative to their labor income).

Figure 1 also plots a capital demand curve, which we obtain from the economy's production side: combining $\bar{w} = (1-\alpha)Y(K)$ and $RK = \alpha Y(K)$ where $R = r + \delta$ yields

$$k^d = \frac{\alpha}{1-\alpha}\frac{1}{r+\delta}. \tag{7}$$

and so (normalized) capital demand is a decreasing function of the return to wealth.

Equation (4) gives the return r^* implicitly as the unique point at which demand equals supply. Figure 1 shows that r^* lies between ρ and $\rho + p\sigma$, which implies that the return to wealth exceeds ρ. Equation (5) shows that α^*_{net} determines exactly where in this range r^* lies, with the gap $r^* - \rho$ given by $\sigma p \alpha^*_{\text{net}}$.[14]

Figure 1 also illustrates the difference between our model and the representative-household benchmark. With a representative household, the supply of capital is perfectly elastic and the return to wealth is fixed at $r^* = \rho$. In such models, technology can have at most a short-lived impact on asset returns.[15]

The logic behind the upward-sloping supply of capital and the result that the return gap $r - \rho$ is linked to α^*_{net} in equilibrium is intuitive and general. The driving force behind this result is that all the financial wealth and capital in the economy derives from labor income that households save and invest over time. Because everyone at some point in time had nothing but their human wealth, the equilibrium return to wealth r must exceed the discount rate ρ so as to incentivize households to accumulate and supply capital. How

[14]The formula for r^* in equation (5) holds for *any* constant returns to scale production function. In particular, $0 = \dot{K} = \frac{r-\rho}{\sigma}(K + \frac{\bar{w}}{r^*}) - pK$ implies $r = \rho + p\sigma\frac{rK}{rK+\bar{w}} = \rho + p\sigma\alpha_{\text{net}}$. Thus, the exact modeling of automation and our assumed production function is not required for this formula. Likewise, any technology that increases the net capital share will raise the return to wealth and contribute to higher wealth inequality.

[15] Recall that our simple model does not feature sustained growth. The reason why the return to wealth exceeds ρ in our model is therefore different from the one embedded in Ramsey's formula $r = \rho + \sigma g$, which holds along a balanced-growth path of a *growing* economy. Equation (16) in Section 3 generalizes equation (5) to an environment with sustained growth and shows that the equilibrium return is given by $\rho + \sigma g + p\sigma\alpha^*_{\text{net}}$, where α^*_{net} is now the capital share in income net of depreciation and growth.

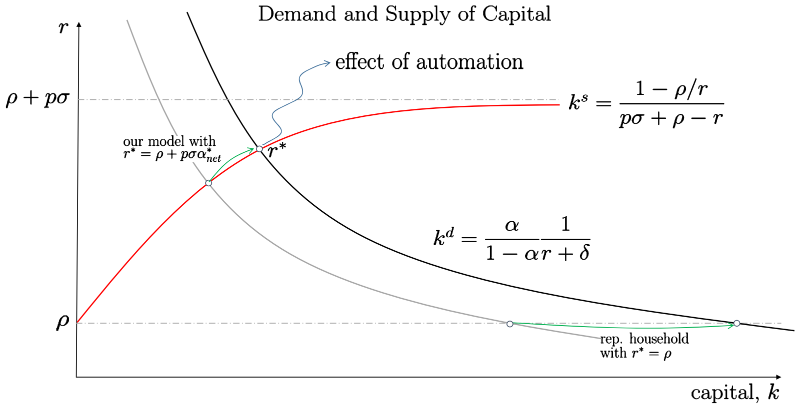

FIGURE 2.—Effect of automation—an increase in α—on the equilibrium return to wealth, r^*. The green arrows illustrate how the equilibrium changes in our model and in the representative-household benchmark.

high above ρ must the return to wealth be? This depends on the importance of capital relative to human labor. If the net capital share is high, and hence capital is important relative to human labor, households must accumulate capital rapidly and this requires that the return to wealth exceeds ρ by a large amount. This explanation also shows that all that is required for our theory is that households experience *some* wealth dissipation. The assumption that they lose all their wealth following a dissipation shock is made only for simplicity.

Proposition 1 shows that automation increases the net capital share and the return to wealth. This finding can be seen directly from Figure 2, where an increase in α raises the demand for capital relative to labor. This demand shift increases the return to wealth, the ratio of capital to human wealth, k^*, and the net share of capital.

Does the higher demand for capital following an increase in automation result primarily in a higher return to capital or in an expansion of the capital-to-output ratio? The answer depends on the capital-supply elasticity, which is inversely linked to p. To illustrate this, consider the special case $\delta = 0$ in which we obtain simple formulas for aggregates:

EXAMPLE: When $\delta = 0$, we have that $\alpha^*_{\text{net}} = \alpha$ and

$$r^* = \rho + p\sigma\alpha, \qquad (K/Y)^* = \frac{\alpha}{\rho + p\sigma\alpha}. \tag{8}$$

As these equations show, for high values of p, automation leads to a sizable increase in the return to capital and a mild expansion in the capital-to-output ratio. Instead, in the representative-household benchmark with $p = 0$, automation results in a larger expansion of capital and has no effect on the return to wealth.

Finally, the proposition shows that output rises with automation. Appendix A shows that, to a first-order approximation, the effect of automation on output is

$$d\ln Y^* = \underbrace{\frac{1}{1-\alpha}\sum_z \eta_z \ln\left(\frac{w^*_z}{\psi_z R^*}\right) d\alpha_z}_{:=d\ln \mathrm{TFP}_\alpha} + \frac{\alpha}{1-\alpha} d\ln(K/Y)^* > 0. \tag{9}$$

Output increases via two channels. First, by allowing the substitution of labor for cheaper capital, automation increases TFP (the term $d\ln \mathrm{TFP}_\alpha > 0$). The contribution of automation to TFP is a (weighted) sum of the cost-saving gains at the task-level $\ln(w_z^*/(\psi_z R^*))$ (which are positive due to Assumption 1). Second, output increases due to endogenous capital accumulation. For high values of p, this second force is modest, and automation generates a more limited output expansion.

The following proposition summarizes the effects of automation on real wages.

PROPOSITION 2—The long-run effects of automation on wages:
- *An increase in α_z reduces the wage w_z^* relative to other wages w_v^* for $v \neq z$.*
- *For a given increase in the α_z's, there exists a threshold $\bar{p} > 0$ such that, for $p > \bar{p}$, the average wage $\bar{w}^*$ falls; and for $p < \bar{p}$, $\bar{w}^*$ increases.*

The effect of automation on relative wages is unambiguous and follows from the fact that $w_z = (1-\alpha_z)\frac{\eta_z}{\ell_z}Y$ (see also Hémous and Olsen (2022), Acemoglu and Restrepo (2021)).

A more novel implication of the proposition is the possibility that automation may lead to stagnant wages for the average worker, which necessarily implies a more pronounced *real decline* in the wages of workers displaced by automation. Whether this is the case or not depends again on p, which determines how inelastic the supply of capital is in steady state.

To understand this, consider an improvement in automation technologies that raises TFP by $d\ln \mathrm{TFP}_\alpha > 0$. The dual version of the Solow residual implies that[16]

$$d\ln \mathrm{TFP}_\alpha = (1-\alpha)\, d\ln \bar{w} + \alpha\, d\ln R, \qquad R = r + \delta. \tag{10}$$

That is, productivity improvements accrue either to workers in the form of higher average wages or to capital owners in the form of a higher return to wealth. In the representative-household benchmark $p = 0$, supply is perfectly elastic, and hence $d\ln R = 0$ in the long run so that all productivity gains from automation eventually accrue to labor—the inelastic factor.[17] However, as p increases and capital supply becomes more inelastic, an increasingly larger share of these productivity gains accrues to capital in the form of a higher return. When $p > \bar{p}$ so that capital supply is sufficiently inelastic, most of these gains accrue to capital so that average wages fall permanently, $d\ln \bar{w} < 0$.[18]

[16]This result holds in general whenever aggregate output exhibits constant returns to scale and markets are competitive. Under these assumptions, we have $Y = \bar{w}L + RK$ where we now allow for movements in labor supply L to underline the argument's generality so that $\bar{w}$ denotes the average wage. Differentiating both sides of this identity, we get that, following a technological improvement, we have

$$d\ln Y = d\ln \mathrm{TFP}_\alpha + \alpha\, d\ln K + (1-\alpha)\, d\ln L = (1-\alpha)\, d\ln \bar{w} + \alpha\, d\ln R + \alpha\, d\ln K + (1-\alpha)\, d\ln L,$$

where $\alpha = RK/Y$. The expression in the main text follows by canceling the $d\ln K$ and $d\ln L$ terms. This derivation shows that the results in the proposition extend beyond our model: any type of technological change will raise wages provided that $d\ln R = 0$. See Jaffe, Minton, Mulligan, and Murphy (2019, Chapters 18/19) for a textbook treatment.

[17]This part of the proposition is in line with papers that studied the impact of automation in settings with a representative household or an infinitely elastic supply of capital, such as Simon (1965), Caselli and Manning (2019), Acemoglu and Restrepo (2021). In these papers, any improvement in technology must necessarily increase average wages, even if some groups see a reduction in real wages.

[18]A complementary intuition comes from the fact that $\bar{w} = (1-\alpha)Y$. From (9), we know that automation results in an output expansion, with the magnitude depending on the productivity increase $d\ln \mathrm{TFP}_\alpha$ and the

Transitional Dynamics and a Numerical Example. To illustrate the results from Propositions 1 and 2 and how the transition dynamics deviate from the representative-household benchmark, we present a numerical example that shows the response of aggregates to an improvement in automation technologies—a permanent increase in the α_z's.

Appendix E shows that the transition dynamics for the aggregate variables are governed by the stable arm of the system of differential equations

$$\begin{aligned}\dot{C} - p\dot{K} &= \frac{1}{\sigma}\big(Y'(K) - \delta - \rho\big)(C - pK) - \mu pK,\\ \dot{K} &= Y(K) - \delta K - C,\\ \frac{\dot{\mu}}{\mu} &= \mu - Y'(K) + \delta + \frac{1}{\sigma}\big(Y'(K) - \delta - \rho\big),\end{aligned}$$

where μ is the (common) marginal propensity to consume out of effective wealth. This is similar to the usual system of equations determining the equilibrium in the neoclassical growth model, but is now coupled with an additional forward-looking equation for μ. Moreover, $C - pK$—the consumption not triggered by dissipation shocks—follows a standard Euler equation, with an adjustment $-\mu pK$, which accounts for the share μ of dissipated wealth pK that would have been consumed.

For this example, we use a subset of the parameters from the calibration of our extended model in Section 4. In particular, we feed into the model a gradual increase in the α_z's that starts in 1980 and matches the observed increase in the capital share between 1980 and 2014 from 34.5% to 43%. We also assume that automated tasks see a cost reduction of 30% (so that aggregate TFP increases by 2.5% due to automation), which makes this a relatively small shock in terms of productivity gains. Finally, we set a value of $p = 4.5\%$, which implies a fairly high capital supply semi-elasticity, $d\ln K/dr$, of 50.

Figure 3 presents the transition dynamics for the labor share, output per worker, the net investment rate, the capital-output ratio, the return to wealth, and the average wage per hour. For comparison, we also plotted the transition dynamics for the representative-household benchmark.

In our model, automation leads to a modest expansion of output of 11%. Automation generates a modest expansion in the capital-output and investment-output ratios of 15% and permanently increases the return to wealth from 6.5% to 7.2%. Thus, our model with an upward-sloping long-run capital supply provides a partial answer to the question "if the decline in the US labor share was driven by automation, shouldn't investment and capital have increased?" The answer is "yes, but only slightly, precisely because capital supply is upward-sloping." Intuitively, while automation leads to an increase in the gross capital share $\alpha = RK/Y$, a substantial fraction of this increase may show up in returns, R, rather than quantities, K/Y.[19]

expansion in capital supply $d\ln(K/Y)^*$. The effect of automation on the average wage is therefore determined by the relative strength of this output expansion and the displacement effect captured by the term $1-\alpha$. With sufficiently inelastic capital supply (i.e., $p > \bar{p}$), the displacement effect dominates. With sufficiently elastic capital supply (i.e., $p < \bar{p}$), the output expansion dominates.

[19]To see this in more detail, recall from above that the capital share increased from 0.345 to 0.43% between 1980 and 2014, a 25% increase. How much would we expect the capital-output ratio to increase? The answer

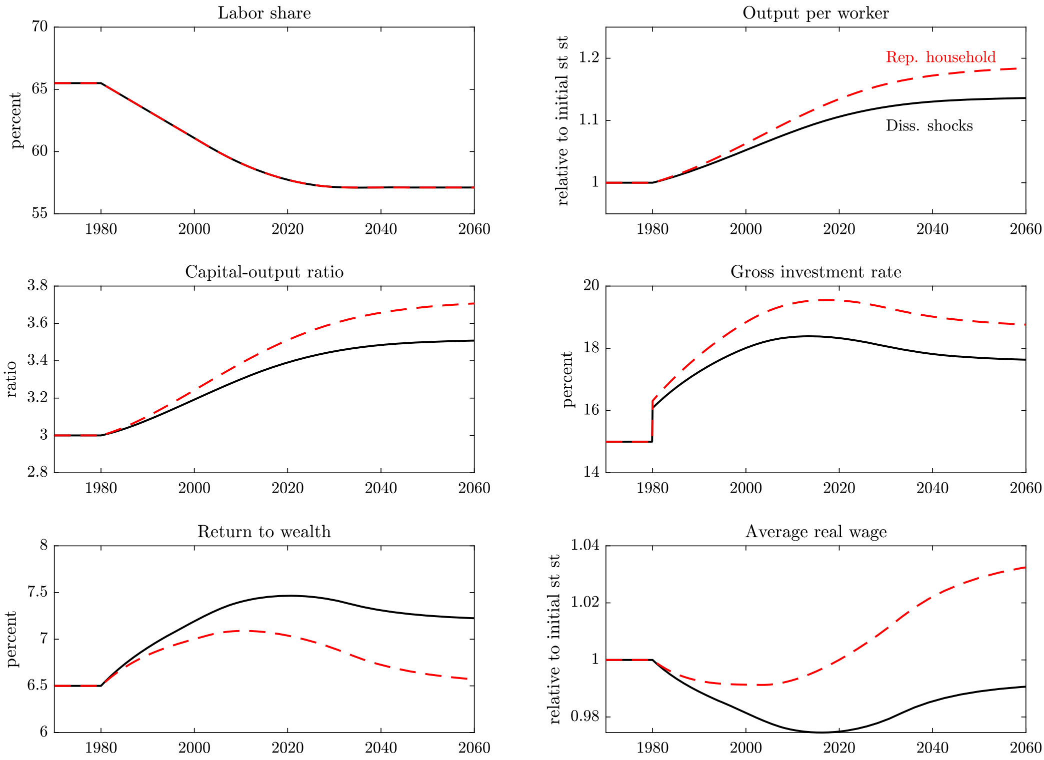

FIGURE 3.—Transitional dynamics following an improvement in automation technologies taking place from 1980 to 2014. The solid line presents the transition dynamics in our model. The dashed line presents the transition dynamics in the representative-household benchmark.

Despite the increase in the average output per worker, and in line with Proposition 2, mean wages go down by 3% in 2020 and by 1% in the long run.[20] This is in contrast to what would happen in the representative-household benchmark, where automation leads to a more pronounced economic expansion propelled by a boom in investment, a *temporary* increase in the return to wealth, and higher average wages in the long run.

The differences between these models underscore the importance of the capital-supply elasticity. Even though the supply of capital in our numerical example is fairly elastic (with

can be seen from manipulating the definition of the capital share $\alpha = RK/Y$:

$$\frac{(K/Y)_{2014}}{(K/Y)_{1980}} = \frac{\alpha_{2014}/\alpha_{1980}}{R_{2014}/R_{1980}}$$

Now consider two scenarios, and assume in both that $R_{1980} = 11.5\%$ as in our calibration. If the rental rate R is unchanged so that $R_{2014}/R_{1980} = 1$, as in the representative household model, the capital-output ratio must increase by 25%. But suppose instead that the rental rate R rises by one percentage as in our model. Then $\frac{(K/Y)_{2014}}{(K/Y)_{1980}} = \frac{0.43/0.345}{12.5\%/11.5\%} = 1.15$. Rather than increasing by 25%, the capital-output ratio increases by only 15%. As we discuss in Appendix J.1, this is well within the range observed in the data.

[20]Equation (10) shows why a small increase in the return to wealth of roughly 1 percentage point can have a large effect on wages. Using this equation with $\alpha = 0.345$—the base point of our calibration—shows that a small increase in R^* of 10% (or 1 percentage point) is enough to ensure that the 2.5% increase in TFP driven by automation results in a decline in real wages, $\bar{w}$, of roughly 1%.

a semielasticity $d\ln K/dr = 50$), the response of macroeconomic aggregates to automation differs significantly from what one would get in a representative household model.

2.3. *Wealth and Income Inequality*

We now study the wealth and income distributions. Recall that the steady-state return to wealth satisfies $r^* > \rho$. Therefore, while aggregate capital K is constant in steady state, individual households accumulate wealth at a positive rate, with some experiencing dissipation shocks and losing all their wealth at rate p. The combination of accumulation and dissipation gives rise to a unique, nondegenerate distribution of effective wealth, which in turn determines the income distribution.

PROPOSITION 3—Automation and the wealth and income distribution: *Denote households' effective wealth by $x_{z,t} := a_{z,t} + w_z^*/r^*$. The stationary distribution of effective wealth for skill type z is Pareto, with a PDF*

$$f_z(x) = \left(\frac{w_z^*}{r^*}\right)^{\zeta} \zeta x^{-\zeta-1}, \qquad \frac{1}{\zeta} := \frac{1}{p}\frac{r^* - \rho}{\sigma} = \alpha_{\text{net}}^*.$$

The conditional and unconditional wealth distributions satisfy

$$\Pr(\textit{wealth} \geq a|z) = \left(\frac{a + w_z^*/r^*}{w_z^*/r^*}\right)^{-\zeta}, \qquad \Pr(\textit{wealth} \geq a) = \sum_z \ell_z \left(\frac{a + w_z^*/r^*}{w_z^*/r^*}\right)^{-\zeta};$$

and the conditional and unconditional income distributions satisfy

$$\Pr(\textit{income} \geq y|z) = \left(\frac{\max\{y, w_z^*\}}{w_z^*}\right)^{-\zeta}, \qquad \Pr(\textit{income} \geq y) = \sum_z \ell_z \left(\frac{\max\{y, w_z^*\}}{w_z^*}\right)^{-\zeta}.$$

Moreover, the tail index $1/\zeta = \alpha_{\text{net}}^*$ increases with automation, α.

The proposition shows that the distribution of effective wealth for skill type z is Pareto with scale w_z^*/r^* and tail index $1/\zeta = \alpha_{\text{net}}^*$. The driving force behind this result is the random growth process governing the accumulation of wealth: starting from their human wealth $x_z(0) = w_z^*/r^*$, households accumulate capital over time at a rate $(r^* - \rho)/\sigma$. The distribution of wealth is stabilized by the dissipation shocks, which arrive at a rate p. Figure 4 depicts the process of accumulation. This random growth process gives rise to a Pareto distribution (see Wold and Whittle (1957), Steindl (1965), Jones (2015)) with tail index

$$\frac{1}{\zeta} = \frac{\text{individual household accumulation rate}}{\text{dissipation rate}} = \frac{(r^* - \rho)/\sigma}{p} = \alpha_{\text{net}}^*.$$

As the formula shows, inequality depends on the ratio of the rate at which households accumulate wealth and the probability with which their wealth dissipates, p, and in equilibrium this is exactly given by the net capital share α_{net}^*.

The reason why we get a Pareto tail is that, while the average household retains a constant level of wealth due to the dissipation shocks, some fortunate households manage to accumulate wealth exponentially at a rate $(r^* - \rho)/\sigma > 0$, populating the tail of the wealth

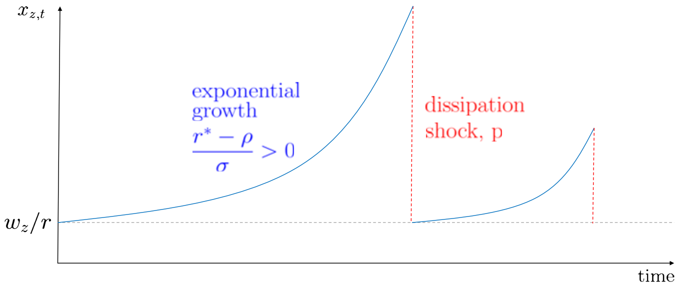

FIGURE 4.—Dynamics of effective wealth accumulation.

distribution over time. A higher return means more rapid accumulation and a fatter tail. Although our model delivers this insight in a stylized way, we see dissipation shocks as a tractable way to capture the nexus between returns to wealth and inequality that is present in a broader class of more realistic models where the stochastic accumulation of wealth results in fat-tailed distributions. This broader class includes models with stochastic bequest motives (see Benhabib and Bisin (2007)), models with stochastic returns or discount rates (see Krusell and Smith (1998), Benhabib, Bisin, and Zhu (2011, 2015)), or the extension of our model presented in Section 3. While the details differ, in all of these models wealth accumulation at the top of the distribution is governed by a random growth process and this creates a natural nexus between returns and inequality (see Gabaix, Lasry, Lions, and Moll (2016)). In these models, technological changes that lead to a higher return to wealth increase the rate at which successful households accumulate wealth, and hence lead to a more fat-tailed wealth distribution.

The distribution of effective wealth is important in and of itself because it tells us about inequality in consumption and welfare (a corollary of Lemma 1). But our model also allows us to characterize the conditional and unconditional distributions of wealth and income, which is what we typically measure in the data. The remaining formulas in Proposition 3 characterize the wealth and income distributions. The formulas show that technology affects both distributions through two channels: first, via wages which determine the distributions' scale parameters; second—and this is more novel—via the net capital share, which determines the return to wealth and the distributions' tail index, $1/\zeta$.

The formulas in Proposition 3 illustrate why automation might generate uneven growth. By depressing the real wages of displaced workers, automation reduces incomes at the bottom of the distribution, especially for displaced workers with no assets. By raising the return to wealth, automation generates rising income inequality at the top driven by the rapid and uninterrupted capital accumulation of some households. Both effects depend crucially on the less than perfectly elastic supply of capital, which implies that a smaller share of the productivity gains from automation accrues to workers and more accrues to wealthy capital owners in the form of higher returns to their wealth.

Composition of Top Income Earners. Proposition 3 can be used to study the composition of top income earners and quantify their share of income. As we show in Appendix D, for small q, the share of national income held by the top q (e.g., the top 0.1%) is

$$S(q) = \Lambda q^{1-\alpha^*_{\text{net}}}, \tag{11}$$

where Λ is a constant that depends on the wage distribution. When there is no heterogeneity in wages, $\Lambda = 1$ and we obtain the usual formula for the top q percent share in a

Pareto distribution (see Jones (2015)). One implication of this formula is that technology might affect $S(q)$ through wages (via the Λ term) but this would cause a proportional increase in $S(q)$ for all $q < \bar{q}$. Instead, by raising α^*_{net}, technology will increase the share of income held by higher percentiles disproportionately.

Moreover, the probability that someone with a wage w_z reaches the top q percent is

$$\Pr(\text{skill} = z | \text{top } q) = \frac{\ell_z w_z^{1/\alpha^*_{\text{net}}}}{\sum_v \ell_v w_v^{1/\alpha^*_{\text{net}}}},$$

which shows that high wage households are overrepresented at the top of the income distribution as is the case in the data. Interestingly, wages become less relevant in determining who reaches the top in a more automated world with higher returns. The Appendix also shows that, as we move up the income distribution, households derive more of their income from capital ownership, since the tail is increasingly made of successful investors for whom labor income represents a small part of their earnings.

Other Mechanisms. Starting with Meade (1964) and more recently with Piketty (2014), several authors have emphasized that a rise in the net capital share will generate income inequality via a compositional effect. The argument is that because capital is more unequally distributed than labor income, a rise in the relative importance of capital mechanically increases income inequality. This argument differs from ours, since we emphasize how technology might lead to a more concentrated ownership of wealth.

To see the difference, denote by $S(q)$ the top q percent income share. Also denote by $\tilde{S}_k(q)$ and $\tilde{S}_\ell(q)$ the share of aggregate capital income and labor income earned by the top q percent of the distribution of *total* income. It then follows that

$$S(q) = \alpha_{\text{net}} \tilde{S}_k(q) + (1 - \alpha_{\text{net}}) \tilde{S}_\ell(q). \tag{12}$$

This formula was already used by Meade (1964, p. 34) and we rederive it in Appendix D. Next, we decompose changes over time in $S(q)$ as

$$dS(q) = \underbrace{\big(\tilde{S}_k(q) - \tilde{S}_\ell(q)\big)\, d\alpha_{\text{net}}}_{\text{Compositional effect at } q} + \underbrace{\alpha_{\text{net}}\, d\tilde{S}_k(q) + (1 - \alpha_{\text{net}})\, d\tilde{S}_\ell(q)}_{\text{Changes in within factor distribution at } q}.$$

This decomposition highlights the difference between compositional effects and our mechanism. In our mechanism, automation increases the concentration of capital ownership, leading to a rise in income inequality via changes in the within factor term. The decomposition also shows that composition effects are small in practice. For example, for the top 1% in the US in 1980 we had $\tilde{S}_\ell(q) = 8.3\%$ and $\tilde{S}_k(q) = 15.3\%$, so that an increase in the net capital share of one percentage point would yield an increase in the top 1% income share of only $(\tilde{S}_k(q) - \tilde{S}_\ell(q)) d\alpha_{\text{net}} = 0.07 \times 0.01 = 0.07$ percentage points.[21] This effect is small when compared to the effects in our model and the observed increase of top tail inequality in the data, where the top 1% income share increased from 10% to 20% in 1980–2012.

[21] In 1980, the top 1% earned 10% of US income. 35% of the income of that group was capital income and 65% was labor income (as opposed to a 20% and 80% breakdown in the aggregate) (see Piketty and Saez (2003)). Using these numbers, we have $\tilde{S}_k(q) = \frac{\alpha_{\text{net}}(q)}{\alpha_{\text{net}}} S(q) = 15.3\%$ and $\tilde{S}_\ell(q) = \frac{1 - \alpha_{\text{net}}(q)}{1 - \alpha_{\text{net}}} S(q) = 8.3\%$.

3. EXTENDED MODEL: MULTIPLE ASSETS AND GROWTH

We now present an extended version of our model that clarifies how automation and concurrent changes in capital markets affect the returns of different assets. This extension identifies the key returns that are linked to rising wealth inequality and explains how these returns can be measured. All proofs and derivations are in Appendix B.

3.1. *An Extended Version of the Model*

The extension modifies the baseline model in four ways:

1. We assume that there are two assets, risky capital (or equity) and safe bonds, and that only a fraction of the population can invest in capital, but are constrained to do so because of risk or financial frictions.
2. We include markups in the production sector, which allows for the possibility that part of the decline in the labor share might be due to markups, and that the return to capital may include some of the profits generated by markups.
3. We model the taxation of capital, which will also affect capital markets.
4. We allow for factor-neutral technological change causing sustained growth at a rate g.

Households and Investors. Households maximize their expected lifetime utility. We work with Duffie–Epstein–Zin preferences

$$v_0 = \mathbb{E}_0 \int_0^\infty f(c_t, v_t)\, dt \quad \text{with } f(c, v) = \frac{\rho(1-\gamma)v}{1-\sigma}\left(\left(\frac{c}{\big((1-\gamma)v\big)^{1/(1-\gamma)}}\right)^{1-\sigma} - 1\right),$$

where $\gamma > 0$ measures risk aversion and $\sigma > 0$ is the inverse of the IES. As in the baseline model, households face dissipation shocks at rate p so that the discount rate ρ equals $\varrho + p$.

A fraction $\chi \in (0, 1)$ of households can invest in both capital and safe bonds. We refer to these households as investors, and assume that investors come with equal probability from all skill groups. The remaining $1 - \chi$ households can only invest in bonds.

The bond pays a safe return r_B. The return to capital instead features idiosyncratic risk: the after-tax return over a time interval of length dt equals $dR_t = r_K\, dt + \nu\, dW_t$, where r_K is the average after-tax return per dollar of capital, ν is the standard deviation of returns, and W_t is an idiosyncratic standard Brownian motion. This return risk might arise from shocks to the productivity of businesses or might also capture idiosyncratic capital gains.

The budget constraint of an investor with wage w_z is therefore given by

$$da_{z,t} + db_{z,t} = (r_K a_{z,t} + r_B b_{z,t} + w_z + T - c_{z,t})\, dt + a_{z,t}\nu\, dW_t.$$

Taxation. Capital income from bond holdings and equity is taxed at a rate τ and the revenue is returned to households via a common lump-sum transfer T.[22]

[22]To simplify the exposition, we assume that idiosyncratic capital returns—the term $a_{z,t}\nu\, dW_t$ in investors' budget constraint—are not taxed. If these were taxed, the after-tax income risk would be $(1-\tau)a_{z,t}\nu\, dW_t$, and taxes would reduce capital income risk. Instead, here we are only interested in the effect of taxes on the expected after-tax returns r_K and r_B.

Technology and Market Structure. Firms operate the same technology as in our baseline model but now charge a constant markup $\varphi \geq 1$ with all profits accruing to equity owners. In addition, to generate balanced growth, we assume that ψ_z grows at a constant rate of $g > 0$ for all skill groups z.

Balanced-Growth Equilibrium (BGE). In a balanced-growth equilibrium, output, capital, bond holdings, and wages grow at a rate $g > 0$, and the returns r_K and r_B are constant, with $r_B > g$ so that human wealth $(w_z + T)/(r_B - g)$ is finite. Moreover, in the main text, we will focus on characterizing the BGE of a closed economy in which bonds are in zero net supply. We assume that $\rho + (\sigma - 1)g > 0$, which is a sufficient condition to ensure that the equilibrium exists and features finite wealth.

Define the return to investors' wealth as

$$r_W := \underbrace{\kappa r_K + (1-\kappa) r_B}_{\text{portfolio return}} + \underbrace{\frac{1}{2}(\sigma - 1)\gamma \nu^2 \kappa^2}_{\text{risk adjustment}}, \tag{13}$$

where $\kappa := \frac{r_K - r_B}{\gamma \nu^2}$ denotes the share of investors' effective wealth invested in risky capital. The formula for r_W takes the return to investors' portfolio, $\kappa r_K + (1-\kappa) r_B$, and adjusts it by the extra term $\frac{1}{2}(\sigma - 1)\gamma \nu^2 \kappa^2$, which accounts for the effect of capital income risk on investors' saving decisions.[23] The return r_W determines investors' optimal wealth accumulation. In particular, their optimally chosen effective wealth (defined now as $x_{z,t} = a_{z,t} + b_{z,t} + \frac{w_z + T}{r_B - g}$) follows a random growth process of the form

$$dx_{z,t} = \frac{r_W - \rho}{\sigma} x_{z,t}\, dt + \kappa \nu x_{z,t}\, dW_t,$$

where, in addition, $x_{z,t}$ is reset to $x_{z,0} = \frac{w_z + T}{r_B - g}$ with probability p. Noninvestors solve the same problem as in the baseline model so that Lemma 1 applies and their effective wealth (given by $x_{z,t} = b_{z,t} + \frac{w_z + T}{r_B - g}$) also resets with probability p and otherwise evolves as

$$\dot{x}_{z,t} = \frac{r_B - \rho}{\sigma} x_{z,t}.$$

In what follows, define the *after-tax share of capital inclusive of profits* in GDP

$$\widetilde{\alpha} := (1 - \tau)\left(\frac{\varphi - 1}{\varphi} + \frac{\alpha}{\varphi}\right), \tag{14}$$

which increases in α, φ and the keep rate $1 - \tau$. As we will see, all these primitives have the same effects on returns and wealth inequality operating through $\widetilde{\alpha}$.

PROPOSITION 4: *The model admits a BGE where aggregates grow at a constant rate g.*

[23] In the presence of capital income risk, investors recognize that a given investment will bring them a lower future consumption equivalent. This generates an income effect (the lower value of future consumption pushes current consumption down and savings up) and a substitution effect (the lower consumption equivalent pushes current consumption up and savings down). For $\sigma = 1$, these effects cancel, but for $\sigma > 1$, as in most calibrations of the (inverse of the) intertemporal elasticity of substitution, the income effect dominates, and capital income risk leads to higher savings, which shows up in our formula as if investors faced a higher return. See also the discussion in Obstfeld (1994) and Angeletos (2007).

- *If investors are risk neutral,* $r_W^* = r_K^* = r_B^* = r^* > \rho + \sigma g$*, and* r^* *is increasing in* $\widetilde{\alpha}$*.*
- *If investors are risk averse,*

$$r_W^*, r_K^* > r_B^*, \quad \rho + \sigma g.$$

- *In both cases, the distributions of wealth and income have Pareto tails with tail index*

$$\frac{1}{\zeta} := \frac{r_W^* - \rho - \sigma g - \frac{\sigma \kappa^{*2} \nu^2}{2} + \sqrt{\left(r_W^* - \rho - \sigma g - \frac{\sigma \kappa^{*2} \nu^2}{2}\right)^2 + 2\sigma^2 \kappa^{*2} \nu^2 p}}{2p\sigma}. \tag{15}$$

The first part of the Proposition shows that, when investors are risk neutral, the return to capital equals that on bonds. This common return satisfies a generalization of (5):

$$r^* = \rho + \sigma g + p\sigma \alpha_{\text{net}}^*, \quad \alpha_{\text{net}}^* := \frac{(r^* - g)K^*}{(r^* - g)K^* + \sum_z w_z^* \ell_z}, \tag{16}$$

with the difference that now, α_{net}^* is the capital share net of depreciation, capital taxes, and growth (but including all profits). Appendix B derives demand and supply curves that uniquely define r^* and shows that both r^* and α_{net}^* increase in $\widetilde{\alpha}$.

The second part of the proposition shows that, if investors are risk averse, the return to capital and investors' wealth exceeds the return on bonds, and also $\rho + \sigma g$ – the level of returns in a BGE of the representative-household benchmark.[24]

The third part of the proposition shows that top-tail inequality depends on the return to investors' wealth r_W^*. More precisely, top tail inequality depend on the *return gap*:

$$\text{return gap} := r_W^* - \rho - \sigma g. \tag{17}$$

To see why, note that the wealth and income of an average household grow at a rate of g. Thus, the tail of these distributions will be populated by successful investors who manage to accumulate wealth uninterruptedly at an average rate of $(r_W^* - \rho)/\sigma$ that exceeds g. The return gap measures precisely how fast successful investors accumulate wealth relative to the rest of the economy, determining the thickness of the income and wealth distribution. In a way, our formula for the return gap and its link to top-tail inequality generalizes Piketty's "$r - g$" insight. In fact, the return gap is equal to $r - g$ in the special case when preferences are logarithmic and there is no discounting (so that investors reinvest all their wealth).

The expression for the return gap (17) and its link to top inequality in equation (15) clarify how wealth inequality responds to changes in equilibrium safe returns r_B^* and returns to capital r_K^*. The answer depends on investors' portfolio shares κ^*. If investors are primarily invested in business capital (κ^* close to one), inequality will be largely unaffected by movements in the safe return r_B^*. In fact, (17) shows that if investors are leveraged, $\kappa^* > 1$, a *reduction* in safe rates increases inequality. Instead, the return to business capital r_K^* is tightly connected to inequality.

[24]Appendix B provides a more general version of the model where investors face borrowing constraints. This is another force that could drive a wedge between bond rates and the return to capital.

Equation (15) shows that tail inequality will also depend on investors' exposure to risky capital, κ. This is because returns are stochastic, and so some investors will obtain even larger returns in a sustained way, generating a thicker tail for income. This second effect implies that, relative to the formula for the tail index in Proposition 3, the extended model with additional capital income risk generates higher income inequality at par with what we see in the data.[25]

The implications of our extended model for returns and inequality and their connection to technology and other factors affecting the demand for capital can be illustrated by approximating the equilibrium around $\widetilde{\alpha} = 0$.[26]

PROPOSITION 5: *For small values of $\widetilde{\alpha}$, the economy admits a unique BGE. In this equilibrium, aggregates grow at a constant rate g and, to a first-order approximation, we have*

$$r_K^* = \rho + \sigma g + \left(p\sigma + \frac{\gamma \nu^2}{\chi}\right)\alpha_{\text{net}}^*, \qquad r_B^* = \rho + \sigma g + p\sigma\alpha_{\text{net}}^*, \qquad \kappa^* = \frac{1}{\chi}\alpha_{\text{net}}^*,$$

where $\alpha_{\text{net}}^* = \frac{\rho + (\sigma - 1)g}{\rho + \sigma g + \delta}\widetilde{\alpha}$ gives the capital share net of depreciation and growth. Along this BGE, inequality satisfies (to a first-order approximation)

$$\frac{1}{\zeta^*} = \alpha_{\text{net}}^* + \alpha_{\text{net}}^* \frac{\dfrac{\nu^2}{p\chi^2}}{1 + \sqrt{1 + 2\dfrac{\nu^2}{p\chi^2}}},$$

and, to a second-order approximation, the return gap is given by

$$r_W^* - \rho - \sigma g = p\sigma\alpha_{\text{net}}^* + \frac{1}{2}(\sigma + 1)\frac{\gamma\nu^2}{\chi^2}\left(\alpha_{\text{net}}^*\right)^2.$$

The proposition shows that the return gap and top-tail inequality depend on $\widetilde{\alpha}$ and its effect via the net capital share α_{net}^*. Qualitatively, this is the same insight as in our baseline model. But now, our model explicitly distinguishes between the return to capital and the return to bonds, and provides us with an explicit formula for the return gap in this extended model. Importantly, our model now shows that automation widens the spread between $r_K - r_B = \frac{\gamma\nu^2}{\chi}\alpha_{\text{net}}^*$. The extended model can therefore make sense of a situation where a decline in ρ or g reduce r_B (with no effect on inequality) while at the same time automation raises r_K, the return gap, and top-tail inequality.

The formula for $1/\zeta^*$ highlights the fact that top-tail inequality is higher than in our baseline model. The first term in the formula captures the fact that a high value of $\widetilde{\alpha}$ requires a higher drift in the process of capital accumulation among investors—the same

[25]Our extended model provides a more realistic description of the process by which investors reach and leave the top of the distribution. Because of capital income risk, some investors will quickly reach the top after obtaining a series of high returns. There is also much more churn among this group, with investors dropping out of the top after a dissipation shock or obtaining a series of low returns. Recent work documents the importance of churn at the top of the wealth distribution (Gomez (forthcoming), Zheng (2019)).

[26]Proposition 7 in Appendix B provides a complementary result and shows that, for $\widetilde{\alpha} < \bar{\alpha}$, r_W^*, $r_W^* - \rho - \sigma g$, $r_K^* - r_B^*$, and $1/\zeta^*$ increase in $\widetilde{\alpha}$ in the full nonlinear model.

force as in our baseline model. In particular, when $\nu = 0$, the formula in (15) boils down to $(r_W^* - \rho - \sigma g)/(p\sigma) = \alpha_{\text{net}}^*$, which is the natural extension of the formula in our baseline model to a growing economy. The second term captures the fact that returns are stochastic, and investors' exposure to this extra risk is also determined by $\widetilde{\alpha}$ and given by $\kappa^* = \alpha_{\text{net}}^*/\chi$. This extra source of capital income risk generates a fatter tail, as some investors are lucky to obtain high returns consistently.[27] For example, taking $\nu = 7.7\%$, $p = 4.5\%$, and $\chi = 6.6\%$ (our calibrated parameters below), and for $\alpha_{\text{net}}^* = 15\%$, we get $1/\zeta^* = 0.66$. The extended model is thus capable of producing levels of wealth inequality that match the data.

3.2. *Other Drivers of an Increasing Net Capital Share*

Our extended model implies that automation is not the only driver of rising wealth inequality. Higher markups (φ) or lower capital taxation (τ) have similar effects. Higher markups and lower capital taxes also increase the *after-tax share of capital inclusive of profits* $\widetilde{\alpha}$ defined in equation (14) and Proposition 5 showed that it is this object that determines asset returns and top wealth inequality. Intuitively, all of automation, higher markups, and lower capital taxes increase the demand for capital, and hence the return to wealth, the return gap, and the spread between the return to capital and safe assets.

While our paper's main focus is on understanding the consequences of automation on income and wealth inequality, we view our framework's ability to speak to these alternative trends as one of its key advantages.[28] In our quantitative exercise below and the discussion that follows, we study the effects of automation on inequality. But readers should keep in mind that the model-implied effects of automation on capital markets, returns, and wealth inequality at the top of the distribution are identical to those generated by an increase in markups or a reduction in capital taxes causing the same reduction in the after-tax share of capital inclusive of profits. That being said, these other forces would have different implications for wage inequality and aggregates such as investment.

3.3. *Has the Return Gap Risen?*

The extended model shows that top-tail inequality depends on the return gap obtained by investors, which is in turn determined in equilibrium by the extent of automation, markups, and capital taxation in the economy. We turn to a measurement exercise that shows that the return gap has risen since 1980 despite a secular decline in safe rates. We highlight the main takeaways and relegate a detailed discussion of data and measurement to Appendix H.

Figure 5 plots estimates of returns on bonds—our proxy for r_B—and three measures of returns to business capital—our proxy for r_K—in the US. The solid line depicts the

[27]This mechanism continues to operate and becomes stronger when labor income is risky. In this case, automation reduces the importance of labor income risk (since wages became a smaller share of income) and increases the importance of capital income risk for investors. This shift in the nature of risk still contributes to higher inequality at the top because tail inequality depends on the multiplicative capital income risk and not on additive labor income risk. Moreover, risk averse investors will respond to the decline in background labor income risk by increasing their exposure to risky capital (see, e.g., Section 6.1.1 in Campbell and Viceira (2002)), which leads to a further expansion of κ and a more pronounced increase in wealth inequality in response to improvements in automation technologies.

[28]For example, Cao and Luo (2017) use a similar framework to study the role of financial deregulation and lower capital taxes on wealth inequality.

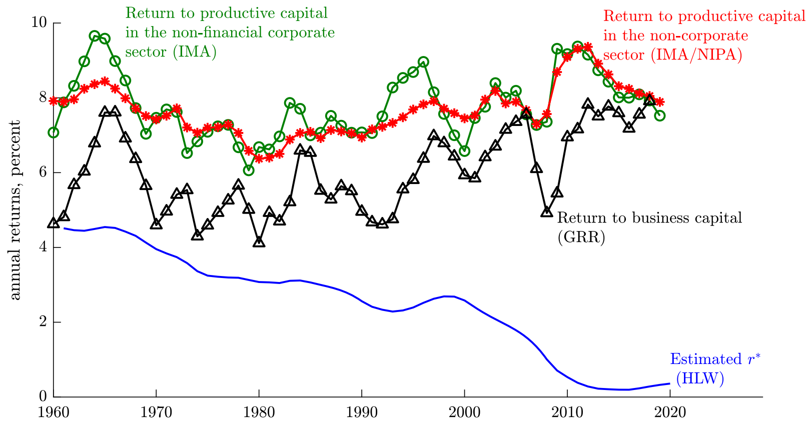

FIGURE 5.—Estimates of the return to US capital, r_K, and the safe rate, r_B, *Notes*: see Appendix H for data sources and measurement details.

long-run return to safe assets, measured using Holston, Laubach, and Williams (2017) estimate of the real natural interest rate for the United States. The safe rate declined by 3 percentage points in 1960–2020. Most of the decline can be explained by demographic factors or developments in international markets, like the "savings glut" (captured by a lower ρ in our model) and slower trend productivity growth (see Rachel and Summers (2019), Auclert, Malmberg, Martenet, and Rognlie (2021)).

In contrast with what we found for the safe rate, all three measures of returns to capital in Figure 5 exhibit an increasing trend, rising by about 1–2 percentage points from 1970–2020. The first series is from Gomme, Ravikumar, and Rupert (2011), who measure the return to US business capital as $r_K := Y_k/K + \Pi_k$. Here, Y_k is net capital income (net output generated by business capital minus labor compensation and taxes), K is the stock of capital measured at replacement cost, and Π_K captures expected capital gains arising from changes in the price of capital over time.[29] We plot a version of their measure that sets expected capital gains to their average realization over the 1960–2020 period. We complement this series with two additional measures of the return to capital: the return to productive (i.e., nonfinancial) assets in the nonfinancial corporate sector, and the return to business capital in the non-corporate sector. These returns are also measured as $r_K := Y_k/K + \Pi_k$, with productive capital valued at replacement cost using data from NIPA and the Integrated Macroeconomic Accounts—IMAs, as described in Appendix H.[30] All three series paint a consistent picture of a gradual increase in the return to capital since 1980.

[29]Appendix H Figure H.1 shows that, relative to the income component, capital gains are more volatile but exhibit no visible trend. Thus, the upward trend in returns to business capital is driven by the income component of returns. Also, the series from Gomme, Ravikumar, and Rupert (2011) excludes housing and focuses only on productive assets.

[30]Appendix H provides different proxies for returns to equity, including the dividend yield, realized 10-year returns, and realized yearly returns. Depending on the measure, returns to equity have either remained stable or have increased over time. The exception is a decline in the dividend yield in the recent period since the 1980s; however, the dividend yield does not display a clear trend when viewed over a longer horizon since the 1960s. These findings align with Greenwald, Lettau, and Ludvigson (2019) who document an increase in realized stock market returns over the postwar period and estimate that 2.1 pp of this increase are accounted for by persistent factor-share shocks, which in our model correspond to increases in $\tilde{\alpha}$.

We now turn to estimates of the return to investors' wealth, r_W, and the return gap $r_W - \rho - \sigma g$. We compute the return to investors' wealth as $r_W = \kappa r_K + (1-\kappa) r_B$, where r_K is the return to productive capital from Gomme, Ravikumar, and Rupert (2011) and r_B is given by Holston, Laubach, and Williams (2017) estimate of the US natural interest rate.[31]

We provide several estimates for the return to investors' wealth and the return gap based on different measures of κ. In our model, κ equals the productive capital held by investors divided by their net worth with values exceeding 1 meaning that investors are leveraged. However, when computing κ, we have to adjust for the fact that: (i) not just investors but also the businesses they invest in may be leveraged (so that these businesses' assets exceed their equity); (ii) not all equity is held by investors; and (iii) investors also hold government and corporate bonds. This implies

$$\kappa = \frac{\text{business capital}}{\text{business equity} + \text{bonds} - \text{personal debt}},$$

where the business capital held by investors is given by

$$\text{business capital} = \text{equity share held by investors} \times \text{firms' total assets}.$$

This approach accounts for (i) because part of firms' assets are financed through liabilities, and so firms' productive capital will typically exceed their equity. Thus, our measure for κ will exceed 1 when investors are leveraged themselves or invest in highly leveraged businesses.

We compute κ for households at the top of the income and wealth distribution, which we identify with investors in our model. We measure all the components using data from Distributional National Accounts (DINAs) of Piketty, Saez, and Zucman (2018) and the Federal Reserve's Distributional Financial Accounts (DFAs). We find that κ lies between 1 and 1.5 depending on the data source, and on whether we include government bonds in our calculations. The measures that exclude government bonds, and can therefore be more closely mapped to our model, are all close to 1.3 in 1980 and exhibit a mild upward trend since then. These values suggest that investors at the top of the distribution top up every dollar of their own resources with 30 cents of borrowing from households. These findings are consistent with the evidence of substantial leverage at the top, including on the balance sheets of businesses held by those at the top (see, e.g., Gomez and Gouin-Bonenfant (2020), who estimate values of κ of 1.5 for entrepreneurs).

Figure 6 provides our estimates for the return to investors' wealth based on our estimates for r_K, r_B, and κ. We provide estimates for the top 1% of earners (from the DINAs) and the wealthiest 1% (from the DFAs). Since our model does not feature government bonds, we also report estimates that exclude government bond holdings from investors' portfolios (the dashed lines). Finally, we also show the return that accrues to a fictional investor who holds no corporate bonds or personal debt (the solid black line). All of the resulting measures of returns to wealth have increased over time, by 0.5–2 percentage points since the 1980s. This increase is driven both by the rise in the return to productive capital r_K and by the decline in the return to bonds r_B, which raises the return to investors' wealth when they take leverage positions with $\kappa > 1$, as in the data.

[31]Our calculations exclude housing and mortgages from investors' portfolios. We also focus on investors' financial portfolios. Thus our calculations are particularly relevant for the tail of the wealth distribution, where the importance of human wealth diminishes. We also ignore the risk-adjustment term in equation (13), which generates conservative estimates of the increase in r_W (if $\sigma \geq 1$ as in our calibration).

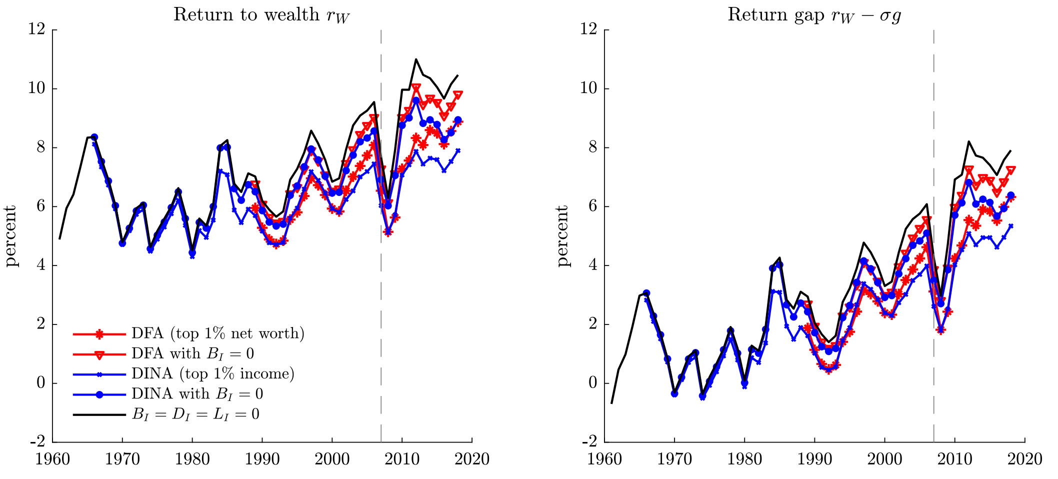

FIGURE 6.—Estimated returns to wealth (left panel) and the return gap for $\sigma = 2$ (right panel). *Notes*: see Appendix H for details.

Our theory's precise implication is that it is the return gap $r_W - \rho - \sigma g$, rather than the return to wealth itself, that drives wealth and income inequality. The right panel in Figure 6 combines the data on returns shown in the left panel with the estimates of the trend growth rate of the economy from the Congressional Budget Office, to compute the return gap over time as $r_W - \sigma g$.[32] The decline in trend growth rate means that the return gap has increased by more than the return to investor's wealth since the 1980s.

These estimates support the idea that, despite the large decline in the safe rate of return, returns to wealth, and in particular the return gap rose by 1.5–3.5 percentage points since the 1980s. This divergence is driven by (i) the increase in the returns to US productive capital, r_K, (ii) the fact that households at the top of the income and wealth distribution are leveraged, and so the decline in safe rates actually increased the return to their portfolios, and (iii) the lower trend growth during this period. Our theory highlights the fact that this growing return gap is a potential driver of the observed rise in wealth and income inequality.

4. CALIBRATION AND IMPLICATIONS FOR AGGREGATES

To study the effects of automation, we feed exogenous changes in α_z into the extended model and explore the consequences for aggregates and inequality. We will focus on changes in automation between 1980 and 2014, a period with a marked shift in technology toward automation (Acemoglu and Restrepo (2019)), especially of routine tasks (Autor, Levy, and Murnane (2003), Acemoglu and Autor (2011)).[33] This exercise isolates

[32]In our calculations, we use a 20-year moving average of the growth rate of per-capita real potential GDP, together with the CBO latest (2021) forecast, as the proxy for the expected trend growth rate of the economy. Our measure of g declines by around a 1.2 percentage point relative to 1980. We also ignore the discounting term ρ. This simplification is conservative because the effective discount rate has arguably declined over time, for example, due to demographic trends, so that it would have pushed up the return gap.

[33]Our focus on this period does not imply that there was no automation before. As discussed in Acemoglu and Restrepo (2019), before 1980 jobs were automated in some specific industries and tasks, but this was counteracted by other technological improvements that raised labor shares in other industries or introduced new

the effects of automation while holding other drivers of distributional and macroeconomic trends at their 1980 levels.

4.1. *Calibration*

We interpret z as indexing workers in a given percentile of the wage distribution, so that we have 100 skill groups. We think of increases in α_z from 1980 to 2014 as driven by the automation of routine tasks in which group z specializes. To capture this, we adopt the shift-share specification

$$\frac{1}{1-\alpha_{z,2014}} - \frac{1}{1-\alpha_{z,1980}} = \omega_z^R\left(\frac{1}{1-\alpha_{2014}} - \frac{1}{1-\alpha_{1980}}\right), \tag{18}$$

which assumes that the observed decline in the labor share from 1980 to 2014 was entirely driven by automation, and then apportions the increase in automation across skill groups based on their relative exposure to routine jobs. The change in α_z depends on two factors: First, we have the revealed comparative advantage of workers in each percentile in routine jobs, ω_z^R, which adjusts for the incidence of automation.[34] Second, we have a common shift term that captures the overall extent of automation taking place in this period. We benchmark this shift by assuming that the observed decline in the labor share during 1980–2014 is due to automation, which implies that the shift term matches the increase in the capital share during this period (from $\alpha_{1980} = 34.5\%$ to $\alpha_{2014} = 42.8\%$ in the BLS series for the nonfinancial corporate sector).[35] Finally, we normalize $\alpha_{z,1980}$ in 1980 to be equal across all z, which requires $\alpha_{z,1980} = \alpha_{1980} = 34.5\%$—the gross capital share in 1980.

This procedure results in the 1980 and 2014 values for α_z plotted in Figure 7, which shows greater exposure to the automation of routine jobs among workers at the middle and bottom of the wage distribution. The average change in α_z is of 8.4 percentage points, which roughly matches the observed decline in the labor share during this period. Finally, in terms of timing, we feed a smooth increase in α_z from its value in 1980 to its final value in 2014, though this only affects transition dynamics.

For the remaining labor parameters, we calibrate η_z to match the wage distribution in 1980 (obtained from the 1980 Census). The η_z's might have changed over time as a result of other forms of skill biased technical change not modeled here, but we do not explore this possibility. We pick ψ_z to ensure that human labor is 30% more costly than capital in automated tasks. This number is in line with studies exploring the cost-saving gains from industrial robots (see Acemoglu and Restrepo (2020)). As already discussed, this implies that automating a task reduces its cost by 30%, and that, to a first-order approximation,

labor-intensive roles for labor in production. As a result, the aggregate labor share—the key object determining how technology affects wealth inequality—remained stable before 1980.

[34] This is given by the share of labor income derived by workers in percentile z from routine jobs relative to the labor income derived by all workers from routine jobs. We measure this term using data from the 2000 Census—a point in the middle of our study period—and based on the ONET definition of routine jobs in Acemoglu and Autor (2011). Appendix G provides the details and a derivation of (18).

[35] In particular, we view this calibration as asking: if the observed decline in the labor share were explained by the automation of routine jobs, what would the distributional and aggregate consequences be? We recognize that there are other forces behind the decline in the labor share, including rising markups (Autor, Dorn, Katz, Patterson, and Van Reenen (2020), Loecker, Eeckhout Jan, and Unger (2020)). Although our calibration assumes no markups, one could also use our model and feed a combination of rising markups and automation that matches the observed decline in the labor share. The extended model in Section 3 shows that this would create similar implications for wealth inequality at the top, since both shocks affect top inequality via $\tilde{\alpha}$.

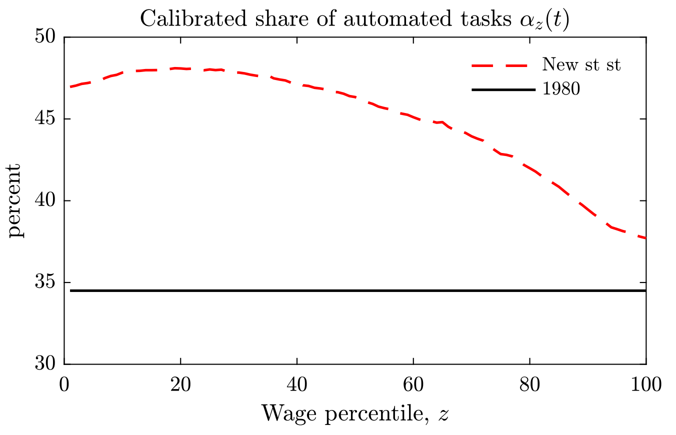

FIGURE 7.—Calibrated α_z by wage percentile in 1980 and the new steady state.

the productivity gains from the calibrated increase in $\alpha_{z,t}$ between 1980 and 2014 are only 2.4%—close to 10% of the total increase in TFP during this period (Fernald (2014)).

Turning to investors, we focus on a calibration with risk averse investors with $\gamma = 2$. We jointly calibrate the share of investors in the economy χ and the extent of capital income risk ν to match the observed tail index of the income distribution in 1980 ($1/\zeta_{1980} = 0.54$) and our estimates of investors' exposure to business capital $\kappa = 1.3$.[36] This procedure yields $\nu = 7.7\%$ and $\chi = 6.6\%$.[37]

The remaining parameters are chosen to match aggregates in 1980 and estimates of the elasticity of capital supply. We take a capital-output ratio of $K/Y = 3$, which implies a rental rate of capital $R = 11.5\%$. We take a depreciation rate of 5% so that the return to capital equals $r_K = 6.5\%$. We pick ρ to ensure a 6.5% return to capital and p to target a long-run elasticity of capital supply $d \ln K/dr$ of about 50. This choice of p is conservative, in the sense that much of the evidence suggests a more inelastic supply of capital, as discussed in Appendix G. Table I summarizes the parameters used in our exercise.

4.2. *Implications for Aggregates and Returns*

We now explore the effects of changes in the α_z's on macroeconomic aggregates. Figure 3 already plotted the transition dynamics of our baseline model following a gradual increase in automation. In what follows, we focus on steady-state comparisons.

Figure 8 summarizes the steady-state effects of automation, which we plot against $\alpha = \sum_z \alpha_z \eta_z$—the average extent of automation in the economy, which also equals the

[36]The share of income held by the top 0.1% rose from 4% in 1980 to a peak of 12% in 2007 (Jones (2015), Piketty and Saez (2003)), which implies a tail index of 0.54 in 1980 and 0.7 in 2007. In this model, the tail index of the wealth distribution and the income distribution coincides, and so by matching the tail of the income distribution we cannot match the thicker tail of wealth inequality observed in the data.

[37]The calibrated capital income risk ν is reasonable and in line with estimates in the literature. For example, Fagereng, Guiso, Malacrino, and Pistaferri (2020) estimate a standard deviation of value-weighted before-tax real returns to net worth of 8.6% and an unweighted standard deviation of 22.1% (see their Table 3); They also estimate that 74% of return differences are due to idiosyncratic risk as opposed to individual fixed effects (see their Table 8), which would imply a value-weighted risk-only standard deviation of $74\% \times 8.6\% = 6.4\%$. We show in Appendix G that our results are quantitatively robust to targeting different values of κ, including $\kappa = 1$ and $\kappa = 1.5$. We also present results for a low risk aversion calibration in this Appendix, with $\gamma = 0.1$, and results where investors are borrowing constrained.

TABLE I

LIST OF CALIBRATED PARAMETER VALUES.

Description		*Value*	Target/Source
Preferences			
σ	Inverse IES	2	Standard calibration
γ	Risk aversion	2	Standard calibration
p	Dissipation rate (yearly)	4.5%	Target capital-supply elasticity $d\ln K/dr \approx 50$
ρ	Discount rate (yearly)	1%	Target $r = 6.5\%$
χ	Share investors	6.6%	Target $\kappa_{1980} = 1.3$ and $\frac{1}{\zeta_{1980}} = 0.54$
ν	Capital risk (yearly)	7.7%	Target $\kappa_{1980} = 1.3$ and $\frac{1}{\zeta_{1980}} = 0.54$
Technology			
g	Growth rate of ψ_z (yearly)	1.5%	Standard calibration
δ	Depreciation rate (yearly)	5%	Standard calibration
A	Hicks-neutral productivity term	0.14	Y/L in 1980
η_z	Skill demand shifters in 1980	vector	Wage levels in 1980 Census/ACS
$\psi_{z,1980}$	Productivity of labor relative to capital	vector	Automation reduces costs by 30% $(= \frac{w_z}{\psi_z R})$
Automation Shock			
α_{1980}	Capital share in 1980	0.345	BLS labor share in 1980
α_{2014}	Capital share in 2014	0.428	BLS labor share in 2014
ω_z^R	Exposure to routine jobs by pctile	vector	2000 Census/ACS

Note: The table provides the parameters used in our baseline calibration of the model. Our calibration assumes no markups ($\varphi = 0$) and no capital taxes ($\tau = 0$). For details, see the main text and Appendix G.

capital share. The figure thus shows how different levels of automation, moving the capital share from 34.5% (its 1980 level) and up to 42.8% (its 2014 level) would affect the steady-state behavior of aggregates. In line with the discussion in Section 3, automation leads to a sharp rise in the return to capital; while the return on bonds and safe assets

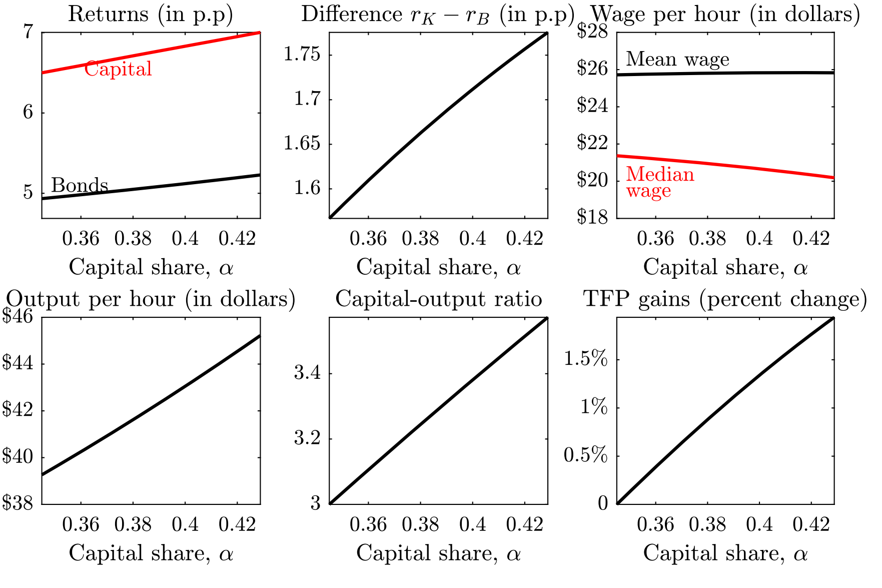

FIGURE 8.—Steady-state effects of changes in $\alpha = \sum_z \alpha_z \eta_z$.

increases slightly. As a result, we see a widening gap between r_K and r_B. Because some of the productivity gains from automation accrue to investors, mean wages are stagnant and median wages decline, and we see a modest expansion in output and the capital-output ratio. Finally, despite the large distributional implications that we will document, automation generates an increase in TFP of 2%—smaller but of a similar magnitude to our first-order approximation. Appendix J.1 shows that the modest increase in the capital-output ratio of 15% implied by our model is in line with that observed in the data.

5. UNEVEN GROWTH: INEQUALITY IN MODEL AND DATA

We now turn to the implications of our model for wage and income inequality. Section 5.1 briefly describes our model's implications for changes in wages and wage inequality. Section 5.2 then presents our model's implications for inequality of overall income, including not only wages but also capital income, and Section 5.3 confronts these model implications with the data. Section 5.4 briefly discusses a discrepancy between model and data regarding the speed at which these changes occur at the top of the income distribution.

5.1. *Wage Inequality*

As we already saw in Figure 8, average wages in our model are roughly stagnant when the capital share increases. However, the constant average wage masks substantial heterogeneity. This can be seen in Figure 9, which plots the change in steady-state wages by wage percentile. The real wage of workers below the 80th percentile of the wage distribution declines over time, but the most pronounced effects are for workers at the 25th percentile of the wage distribution, whose real wages fall by 10%. In contrast, the real wages of workers at the 95th percentile of the wage distribution rise by 7%. For comparison, the figure also plots on a different vertical axis the observed change in wages by percentile between 1980 and 2012–2016 (using data from the US Census and the American Community Survey (ACS)).

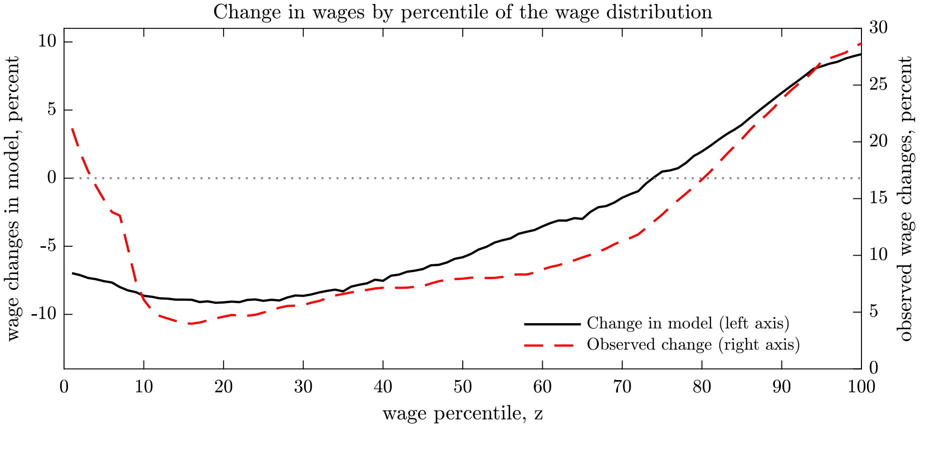

FIGURE 9.—Predicted change in wages by wage percentile (left axis) and observed change in wages by wage percentile (right axis). *Notes*: Observed wage changes computed using the 1980 Census and 2012–2016 ACS. See Appendix G for details.

5.2. *Uneven Growth in the Model*

We now turn to our model's implications for *income* inequality, including also capital income. Figure 10 presents the change in total income for different percentiles of the income distribution across steady states. The figure reveals a pattern of uneven growth. Below the 50th percentile, households experienced declines in total income of 5–10%. Between the 50th and 80th percentile, households experienced stagnant incomes. This is in contrast to the top income quantiles, which experienced an increase in income ranging from 20% (for households in the top 1%–0.5%) to 55% (for the top 0.1%). The figure shows that automation is capable of generating substantial income gains at the very top of the distribution and stagnant or declining incomes at the bottom.

Both wage and wealth inequality combine to produce the pattern observed in Figure 10. The shaded areas plot the contribution of changes in labor and capital income. The fall in real wages for households at the bottom of the wage distribution contributed to declining incomes for households at the middle and bottom of the income distribution. The contribution of changes in capital income is uniformly positive, as everyone benefits from a higher return to wealth. But the benefits from a higher return to wealth are highly dispersed. People at the bottom of the distribution have few assets and only invest in bonds and safe assets, and so do not benefit as much from an increase in the return to capital. In contrast, a higher return to capital allows investors at the top of the income distribution to accumulate large swaths of wealth and earn a high capital income.

Interestingly, although automation is skill-biased and raises wages at the top of the wage distribution (see Figure 9), the contribution of wages at the top of the overall income distribution is small or even negative. As discussed in Section 2.3, there are two effects at play here. On the one hand, the skill-biased nature of automation raises the wages

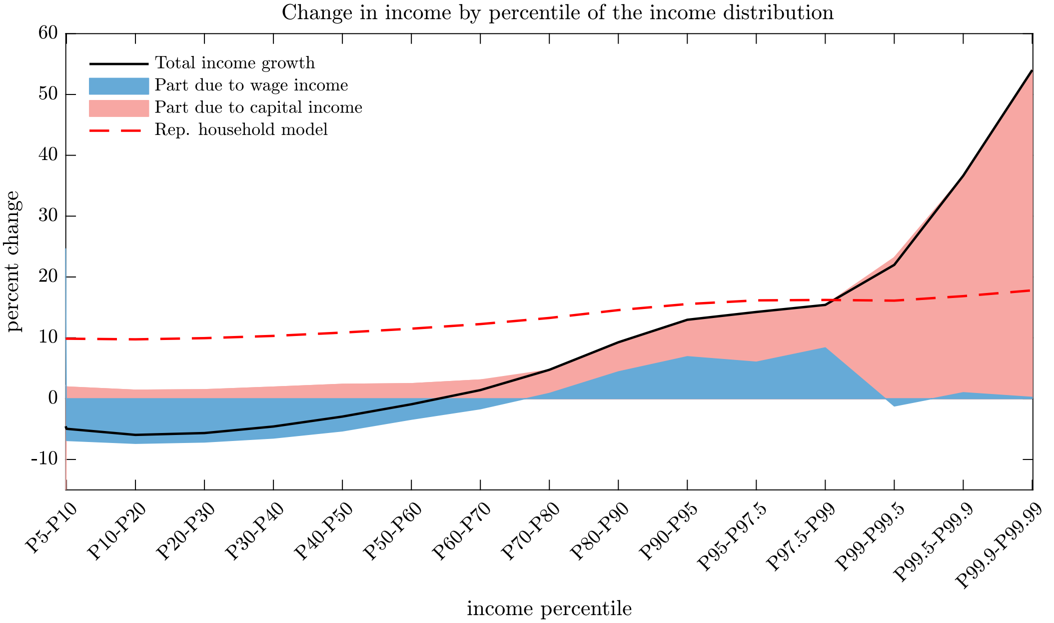

FIGURE 10.—Predicted change in income by income percentile decomposed into the contribution of capital and labor income. The dashed line plots the results from the representative-household benchmark. *Notes*: see Appendix F.3 for a detailed explanation of the figure's construction.

of households at the top of the distribution more than at other percentiles. However, the permanent increase in returns also means that the top of the income distribution becomes increasingly populated by low-wage investors with very high capital incomes. This shift in the composition of top earners dominates at the top of the income distribution and generates the observed low or negative contribution of labor income.[38]

This finding can be contrasted with the representative-household benchmark. In this model, the steady-state wealth distribution is indeterminate, but one can still trace its evolution starting from a given initial distribution, and so we assume the same initial distribution of wages and wealth as in the extended model with dissipation shocks (see Appendix E for details). The dashed line in Figure 10 shows the change in income for different percentiles of the income distribution in this model. The representative household model does not generate uneven growth: there is a fairly uniform increase in income between 10% and 18% for all quantiles. This reflects two differences. First, there is no wage stagnation in the representative household model, since the infinitely elastic supply of capital implies that all productivity gains from automation accrue to workers. Second, the temporary increase in returns to wealth benefits all households equally, as they are all able to scale their effective wealth by the same amount. This is in contrast to our model with dissipation shocks, where only a few households benefit from the higher return to capital.

How strong is the new mechanism in our model linking changes in the capital share due to automation, returns, and the resulting increase in income inequality? To address this question, Figure 11 plots the behavior of the return gap $r_W^* - \rho - \sigma g$ and the resulting tail index of the income distribution as functions of the capital share in our calibrated model.

The return gap in our calibrated model rises by 1.2 percentage points from 4 to 5.2 percentage points per year. Despite this relatively modest rise, the tail index of the income distribution rises from 0.54 (the targeted level in 1980) to 0.65, which is roughly 70% of the observed increase in the data. A large part of this increase in tail inequality is due

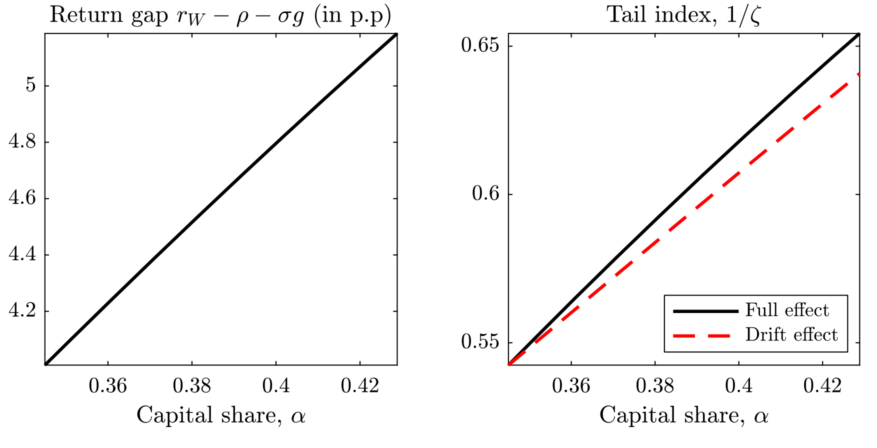

FIGURE 11.—Steady-state inequality effects of changes in $\alpha = \sum_z \alpha_z \eta_z$.

[38]Formally, the expected wage of households at the top of the income distribution is $\sum_z \ell_z w_z^{1+1/\alpha^*_{\text{net}}} / \sum_z \ell_z w_z^{1/\alpha^*_{\text{net}}}$. Skill-biased changes in wages raise this expected wage, but increases in α^*_{net} reduce it. Note that as we keep moving up the tail, the contribution of labor income converges to zero since households' own mostly capital income.

to the effect described in our baseline model, namely that investors' return to wealth r_W increases and that they therefore accumulate wealth at a faster rate. However, in our extended model with idiosyncratic capital income risk, there is also an additional effect: when the return spread $r_K - r_B$ increases, investors increase the share of equity in their portfolios and become more exposed to this risky capital. The right panel of the figure shows that the higher return to wealth (the "drift effect") generates an increase in the tail index from 0.54 to 0.63; while greater exposure to risk explains the increase from 0.63 to 0.65.

5.3. *Uneven Growth in the Data*

The main implication of our model is that technological change involving the automation of tasks performed by low and middle-wage workers generates the pattern of uneven growth documented in Figure 10: stagnant or decreasing labor incomes at the bottom and rising incomes at the top, with an increasing role for capital income at the top of the distribution.

We explore whether the evolution of capital and labor income in the US economy has followed a similar pattern of uneven growth using two data sources: the NBER IRS public use sample from 1980 to 2012 and the synthetic microfiles from the Distributional National Accounts (DINAs) of Piketty, Saez, and Zucman (2018) for the same period.[39] The IRS data set is based on administrative tax records and yields reliable information on incomes for the very top of the distribution. However, it only records *fiscal income* and, therefore, omits pensions and other sources of income that are tax-exempt. Likewise, fiscal income misses income flows such as corporate taxes, retained earnings, or housing services for homeowners. The DINAs account for this "missing income" by imputing tax-exempt sources of income and measuring corporate income and other sources of capital income using a "capitalization approach." This approach allocates aggregate measures of equity from the IMAs to households according to positive corporate income reported to the IRS, and then imputes corporate income based on these stock measures. The main advantage of the DINAs are the more comprehensive income concept being used that adds up to national income but this comes up at the cost of numerous imputations. Conversely, the main advantage of the IRS data is the transparency and simplicity of the data construction but this comes up at the cost of missing income and, in particular, missing capital income.

Figure 12 provides our analysis of uneven growth in the United States. Panel (a) uses the IRS data and panel (b) the DINAs. The dash-dotted black lines labeled "total income" plot the average annual income growth between 1980 and 2012 for different percentiles of the income distribution. The income percentiles corresponding to the lower half of the distribution have stagnated or declined between 1980 and 2012. In contrast, top income percentiles have grown rapidly. For example, in both data sets, the top 0.1 percentile increased at a yearly rate of 5%. The hockey stick shape of income growth at

[39]The NBER IRS public use sample is available through the NBER. See https://users.nber.org/∼taxsim/gdb/ and https://www.nber.org/taxsim-notes.html. The DINAs merge semipublic and private IRS data with national accounts and the US Census/ACS to construct consistent measures of wealth and income across the income and wealth distribution. The synthetic microfiles for the Distributional National Accounts can be accessed via Gabriel Zucman's website http://gabriel-zucman.eu/usdina/. We present additional decompositions for 1980–2007 in Appendix I.

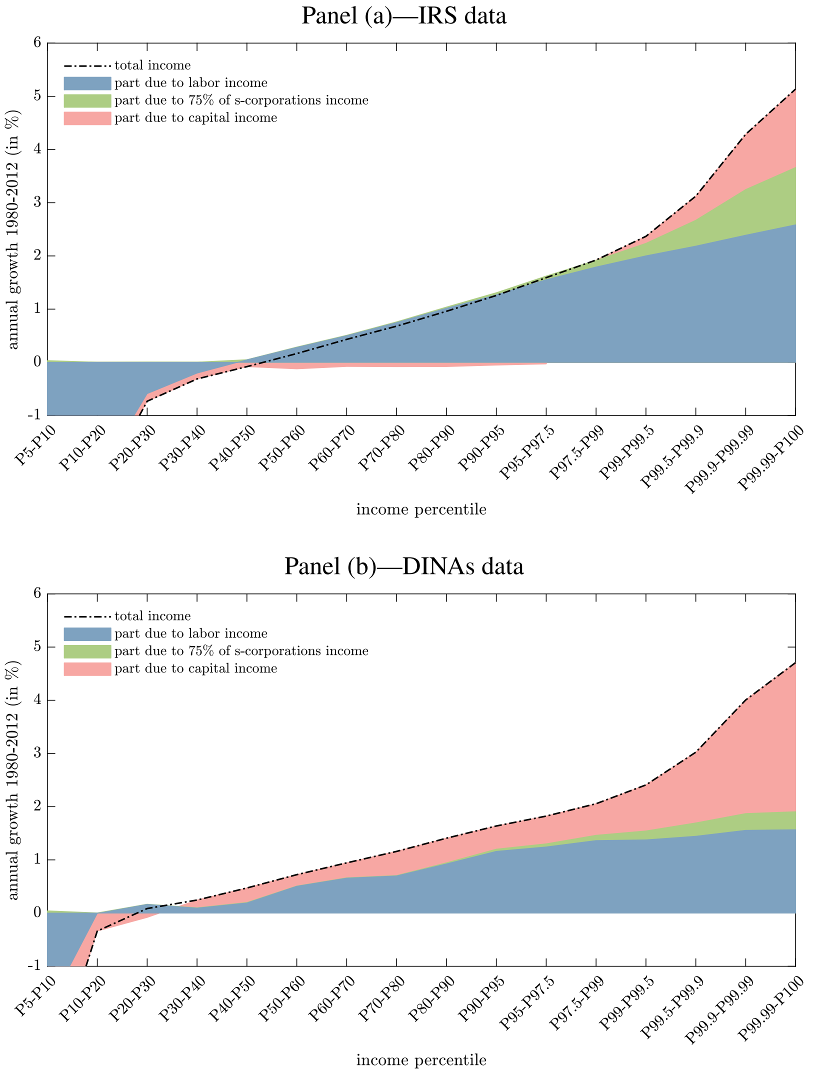

FIGURE 12.—Income growth by percentile between 1980–2012 and decomposition into capital and labor income. *Notes*: see Appendix I for details.

different percentiles in the figure is qualitatively similar to that predicted by our model (see Figure 10).[40]

What is the contribution of capital and labor income to the pattern of uneven growth in Figure 12? Decomposing income growth into these components is difficult because a substantial share of income is derived from self-employment or ownership of pass-through

[40]The change in total incomes in panels (a) and (b) is qualitatively similar but there are some quantitative differences in terms of both the average growth and the relative growth across different income percentiles. The dashed line in panel (b) labeled "total income" is the same as that in Figure II(a) of Piketty, Saez, and Zucman (2018) and they discuss this discrepancy. In particular, they note that their series yields "more growth for the bottom 90% since 1980 than suggested by the fiscal data studied by Piketty and Saez (2003)," which is essentially our IRS series.

entities, and these cannot be easily classified as labor or capital income. As a starting point, we follow Piketty, Saez, and Zucman (2018) who propose measuring capital income as the sum of corporate income (including the income of both C- and S-corporations) plus 30% of the mixed income from noncorporate pass-through businesses (including self-proprietorships and partnerships). However, Smith, Yagan, Zidar, and Zwick (2019) criticize this approach and argue that 75% of S-corporations' income should be counted as labor income rather than capital income (in addition to the 70% of noncorporate pass-through income already classified as wage income by Piketty, Saez and Zucman) . The argument is that these 75% of S-corporation income are, in fact, returns to the human capital of owners and are only reported as corporate income to minimize tax obligations. To address this criticism, we also present an alternative measure of capital income that relabels 75% of S-corporation income as labor income.

The shaded areas in both panels of Figure 12 decompose the growth in total income into parts due labor income, capital income, and the ambiguous S-corporation income.[41] The first area is the contribution of what both Piketty, Saez, and Zucman (2018) and Smith et al. (2019) classify as labor income and the area at the top is the contribution of what they agree is capital income. The area in the middle corresponds to the contribution of the 75% of S-corporations income that Piketty, Saez, and Zucman (2018) attribute to capital and Smith et al. (2019) attribute to the human capital of owners. The sum of the last two areas provides the contribution of capital to income growth under the classification favored by Piketty, Saez, and Zucman (2018), while the last area provides the contribution of capital to income growth under the classification favored by Smith et al. (2019).[42]

In both data sets, capital income is unimportant for income growth at the bottom of the distribution. As in our model, the stagnant incomes at the bottom of the distribution are driven by declining or stagnant real wages. However, further up in the distribution and especially within the top 1%, capital income becomes more important. The extent to which capital income accounts for the observed uneven growth and at which percentiles it starts to be important differs between data sets. The IRS data in panel (a) suggest that up to the 90th percentile, essentially all growth in total income is accounted for by labor income. Capital income becomes important within the top 1% and especially at the top 0.1% where it accounts for around 60% of the cumulative growth from 1980–2012, or 30% if one excludes income from S-corporations. In contrast, the DINA data in panel (b) suggest that capital income plays an important role already around the median of the income distribution, accounts for around half of cumulative growth of the top 1% and is twice as important as labor income for the rise in top 0.01% incomes. When using the DINA data, the relabeling of S-corporations income does not alter these conclusions.[43]

As a whole, the empirical patterns in both data sets are consistent with those generated by our model. Our analysis suggests that the exact contribution of capital to income

[41]Denoting by $y_t(q)$ the qth income percentile at time t and by $y_{\ell,t}(q)$ and $y_{k,t}(q)$ that percentile's labor and capital income, we decompose the annualized growth rate from a base year $t = 0$, $\frac{1}{T}\frac{y_T(q)-y_0(q)}{y_0(q)}$, into a part due to labor income $\frac{1}{T}\frac{y_{\ell,T}(q)-y_{\ell,0}(q)}{y_0(q)}$ and a part due to capital income $\frac{1}{T}\frac{y_{k,T}(q)-y_{k,0}(q)}{y_0(q)}$.

[42]In these figures, we measure S-corporation income using reported *fiscal income*. Appendix I shows an alternative strategy using DINAs where we compute corporate income from S-corporations using the capitalization method. Both approaches yield similar quantitative results.

[43]These discrepancies across data sets are to be expected, given the differences in coverage between the IRS and DINAs. In particular, the share of capital in fiscal income using the IRS data is of less than 10% whereas this share rises to 25–30% in the DINAs (a level that matches the net capital share in national income). Thus, the IRS data is likely to understate the contribution of capital to income growth, and relabeling part of S-corporations income as labor income is more consequential when using the IRS data.

growth at the very top of the distribution depends on the data set and assumptions used, but ranges from 30% (in the IRS data) to 70% (in the DINAs). Moreover, both sources of data suggest that incomes at the middle and bottom have stagnated due to slow labor-income growth.

The main difference between the patterns in these figures and that in our model is that, in the data, wage income plays a more prominent and positive role at the top of the income distribution. This suggests that there are forms of skill-biased technical change other than automation affecting relative wages at the top which are not included in our model (see the discussion of Figure 10).

Our discussion focused on US trends since the 1980s. But it is natural to ask whether our model's prediction that increases in the net capital share are accompanied by large increases in top income inequality receives support from other countries or time periods. Appendix J.2 discuss the available evidence, which shows that this link is visible in a panel of countries (see Bengtsson and Waldenström (2018)) and during historical periods of rapid mechanization, such as the industrial revolution in Britain (see Allen (2009), Lindert (2000)).

5.4. *Speed of Top Inequality Dynamics*

Figure 12 showed that the data features not only very large but also very fast changes in top inequality from 1980 to 2012. While our theory can account for large changes in inequality between steady states, it cannot generate rapid transition dynamics of top income inequality. That theories like ours cannot generate fast transition dynamics is a known result: Gabaix et al. (2016) have shown that standard theories of the Pareto tails of the income and wealth distributions, which build on a random growth mechanism, generate transition dynamics that are too slow relative to those observed in the data. Our theory is exactly a special case of such a theory (see Proposition 3) and is therefore subject to the same criticism.

The good news is that we know how to "fix" random growth theories to deliver fast transition dynamics like those observed in the data. Gabaix et al. (2016) show that what is needed are particular deviations from Gibrat's law, what they call "type- and scale-dependence." For the case of wealth dynamics, heterogeneous and persistent rates of return to wealth are one candidate for generating such type- and scale-dependence and seem to be a prevalent feature of the data (Fagereng et al. (2020), Bach, Calvet, and Sodini (2020)). Future work should build more quantitatively serious theories of the general-equilibrium interaction between technology and income and wealth distribution that feature these model elements.

6. CONCLUSION

In this paper, we developed a tractable framework to study the effects of technology on income inequality. Our theory allowed us to go beyond wages and to explore how technology affects wealth inequality and overall income inequality. We used our framework to study the effects of automation and identified a new channel through which technology affects inequality. The benefits of new technologies accrue not only to high-skilled labor but also to owners of capital in the form of higher capital incomes and returns.

There are several fruitful avenues for future work. First, one could use our framework to study the distributional consequences of other types of technical change, changes in market structure and markups, and government policies, like the taxation of capital or estates, or redistributive policies. For example, our analytically tractable theory featuring a less-than-perfectly-elastic capital supply and nondegenerate wealth distribution may serve as a useful laboratory for exploring the optimal taxation of capital income and wealth.

Second, one could devise more elaborate quantitative models to study the effect of technologies on inequality. As explained in the Introduction, these more elaborate models should retain the two key features underscored by our analysis: an upward-sloping supply of capital and a return inequality nexus. Our model in Section 3 is a first step in that direction. More elaborate versions of our model could include realistic life-cycle structures, a careful treatment of intergenerational transfers, and additional sources of heterogeneity in portfolio and return rates. Successful quantitative extensions should also include some form of scale-dependence and type-dependence to account for the rapid rise of inequality in the data. Alternatively, theories with changing asset prices—another salient feature of the data that we do not model—are promising for generating fast wealth inequality dynamics.

Finally, ours is a model where "macro matters for inequality, but inequality does not matter for macro." An implication is that the supply curve of capital does not depend on the wage distribution. Therefore, in our Figure 2, automation shifts only capital demand but not capital supply (even though it shifts the wage distribution). In more general models, however, additional mechanisms shifting the capital supply curve can be at play. For example, Straub (2019) proposes a model in which high permanent-income types have a higher saving rate, a form of scale dependence. Skill-biased technical change like automation then reallocates permanent income from households with low saving rates to those with high saving rates, and hence expands the supply of saving. Therefore, to the extent that high permanent-income types empirically have higher saving rates, automation would shift not only capital demand but also capital supply to the right and this would provide an offsetting effect on returns to wealth. Our theory clarifies that, even in the case where such additional mechanisms are operational, the net effect of technology on top-tail income and wealth inequality depends on whether the higher demand for capital brought by automation translates into a higher rate of wealth accumulation for rich households in equilibrium. Exploring this question in models where "inequality matters for macro" is another interesting area for future research.

REFERENCES

ACEMOGLU, DARON (2009): *Modern Economic Growth*. New York: Princeton University Press. [2646,2650]

ACEMOGLU, DARON, AND DAVID AUTOR (2011): "Skills, Tasks and Technologies: Implications for Employment and Earnings," Handbook of Labor Economics, Vol. 4, 1043–1171. [2649,2669,2670]

ACEMOGLU, DARON, AND PASCUAL RESTREPO (2019): "Automation and New Tasks: How Technology Displaces and Reinstates Labor," *Journal of Economic Perspectives*, 33, 3–30. [2669]

——— (2018): "The Race Between Man and Machine: Implications of Technology for Growth, Factor Shares, and Employment," *American Economic Review*, 108, 1488–1542. [2646,2649,2651]

——— (2020): "Robots and Jobs: Evidence From US Labor Markets," *Journal of Political Economy*, 128, 2188–2244. [2670]

——— (2021): "Tasks, Automation, and the Rise in US Wage Inequality," Working Paper 28920, National Bureau of Economic Research. [2656]

ALLEN, ROBERT C. (2009): "Engels' Pause: Technical Change, Capital Accumulation, and Inequality in the British Industrial Revolution," *Explorations in Economic History*, 46, 418–435. [2679]

ANGELETOS, GEORGE-MARIOS (2007): "Uninsured Idiosyncratic Investment Risk and Aggregate Saving," *Review of Economic Dynamics*, 10, 1–30. [2663]

AUCLERT, ADRIEN, HANNES MALMBERG, FREDERIC MARTENET, AND MATTHEW ROGNLIE (2021): "Demographics, Wealth, and Global Imbalances in the Twenty-First Century," Working Paper 29161, National Bureau of Economic Research. [2667]

AUTOR, DAVID, DAVID DORN, LAWRENCE F. KATZ, CHRISTINA PATTERSON, AND JOHN VAN REENEN (2020): "The Fall of the Labor Share and the Rise of Superstar Firms," *The Quarterly Journal of Economics*, 135, 645–709. [2670]

AUTOR, DAVID H., LAWRENCE F. KATZ, AND MELISSA S. KEARNEY (2006): "The Polarization of the U.S. Labor Market," *American Economic Review*, 96, 189–194. [2645,2648,2649]

AUTOR, DAVID H., FRANK LEVY, AND RICHARD J. MURNANE (2003): "The Skill Content of Recent Technological Change: An Empirical Exploration," *The Quarterly Journal of Economics*, 118, 1279–1333. [2648, 2649,2669]

BACH, LAURENT, LAURENT E. CALVET, AND PAOLO SODINI (2020): "Rich Pickings? Risk, Return, and Skill in Household Wealth," *American Economic Review*, 110, 2703–2747. [2679]

BENGTSSON, ERIK, AND DANIEL WALDENSTRÖM (2018): "Capital Shares and Income Inequality: Evidence From the Long Run," *The Journal of Economic History*, 78, 712–743. [2679]

BENHABIB, JESS, AND ALBERTO BISIN (2007): "The Distribution of Wealth: Intergenerational Transmission and Redistributive Policies," Tech. rep, New York University. [2647,2651,2660]

——— (2018): "Skewed Wealth Distributions: Theory and Empirics," *Journal of Economic Literature*, 56, 1261–1291. [2647]

BENHABIB, JESS, ALBERTO BISIN, AND SHENGHAO ZHU (2011): "The Distribution of Wealth and Fiscal Policy in Economies With Finitely Lived Agents," *Econometrica*, 79, 123–157. [2647,2651,2660]

——— (2015): "The Wealth Distribution in Bewley Economies With Capital Income Risk," *Journal of Economic Theory*, 159, 489–515. [2647,2651,2660]

BLANCHARD, OLIVIER J. (1985): "Debt, Deficits, and Finite Horizons," *Journal of Political Economy*, 93, 223–247. [2651]

CABALLERO, RICARDO J., EMMANUEL FARHI, AND PIERRE-OLIVIER GOURINCHAS (2017): "Rents, Technical Change, and Risk Premia Accounting for Secular Trends in Interest Rates, Returns on Capital, Earning Yields, and Factor Shares," *American Economic Review*, 107, 614–620. [2648]

CAMPBELL, JOHN Y., AND LUIS M. VICEIRA (2002): *Strategic Asset Allocation: Portfolio Choice for Long-Term Investors,* Oxford University Press. [2666]

CAO, DAN, AND WENLAN LUO (2017): "Persistent Heterogeneous Returns and Top End Wealth Inequality," *Review of Economic Dynamics*, 26, 301–326. [2649,2666]

CASELLI, FRANCESCO, AND ALAN MANNING (2019): "Robot Arithmetic: New Technology and Wages," *American Economic Review: Insights*, 1, 1–12. [2646,2649,2656]

CASELLI, FRANCESCO, AND JAUME VENTURA (2000): "A Representative Consumer Theory of Distribution," *American Economic Review*, 90, 909–926. [2649,2650]

CASTAÑEDA, ANA, JAVIER DÍAZ-GIMÉNEZ, AND JOSÉ-VÍCTOR RÍOS-RULL (2003): "Accounting for the U.S. Earnings and Wealth Inequality," *Journal of Political Economy*, 111, 818–857. [2649]

CENSUS BUREAU, US (2015): "Real Household Income at Selected Percentiles: 1967 to 2014," Tech. rep., U.S. Census Bureau, Current Population Survey, 1968 to 2015 Annual Social and Economic Supplements. [2645]

CHATTERJEE, SATYAJIT (1994): "Transitional Dynamics and the Distribution of Wealth in a Neoclassical Growth Model," *Journal of Public Economics*, 54, 97–119. [2649]

FAGERENG, ANDREAS, LUIGI GUISO, DAVIDE MALACRINO, AND LUIGI PISTAFERRI (2020): "Heterogeneity and Persistence in Returns to Wealth," *Econometrica*, 88, 115–170. [2671,2679]

FARHI, EMMANUEL, AND FRANÇOIS GOURIO (2018): "Accounting for Macro-Finance Trends: Market Power, Intangibles, and Risk Premia," *Brookings Papers on Economic Activity*, Fall, 147–250. [2648]

FERNALD, JOHN (2014): "A Quarterly, Utilization-Adjusted Series on Total Factor Productivity," Tech. rep, Federal Reserve Bank of San Francisco. [2671]

GABAIX, XAVIER (2009): "Power Laws in Economics and Finance," *Annual Review of Economics*, 1, 255–294. [2649]

GABAIX, XAVIER, JEAN-MICHEL LASRY, PIERRE-LOUIS LIONS, AND BENJAMIN MOLL (2016): "The Dynamics of Inequality," *Econometrica*, 84, 2071–2111. [2660,2679]

GOMEZ, MATTHIEU (forthcoming): "Decomposing the Growth of Top Wealth Shares," *Econometrica*. [2650, 2665]

GOMEZ, MATTHIEU, AND EMILIEN GOUIN-BONENFANT (2020): "A Q-Theory of Inequality," Working Paper. [2668]

GOMME, PAUL, B. RAVIKUMAR, AND PETER RUPERT (2011): "The Return to Capital and the Business Cycle," *Review of Economic Dynamics*, 14, 262–278. [2648,2667,2668]

GREENWALD, DANIEL L., MARTIN LETTAU, AND SYDNEY C. LUDVIGSON (2019): "How the Wealth Was Won: Factors Shares as Market Fundamentals," Working Paper 25769, National Bureau of Economic Research. [2667]

HÉMOUS, DAVID, AND MORTEN OLSEN (2022): "The Rise of the Machines: Automation, Horizontal Innovation, and Income Inequality," *American Economic Journal: Macroeconomics*, 14, 179–223. [2649, 2656]

HOLSTON, KATHRYN, THOMAS LAUBACH, AND JOHN C. WILLIAMS (2017): "Measuring the Natural Rate of Interest: International Trends and Determinants," *Journal of International Economics*, 108, S59–S75. [2667, 2668]

HUBMER, JOACHIM, PER KRUSELL, AND ANTHONY A. SMITH (2021): "Sources of US Wealth Inequality: Past, Present, and Future", *NBER Macroeconomics Annual*, 35, 391–455. [2649]

JAFFE, SONIA, ROBERT MINTON, CASEY B. MULLIGAN, AND KEVIN M. MURPHY (2019): *Chicago Price Theory*. Princeton University Press. [2656]

JONES, CHARLES I. (2015): "Pareto and Piketty: The Macroeconomics of Top Income and Wealth Inequality," *Journal of Economic Perspectives*, 29, 29–46. [2647,2659,2661,2671]

KATZ, LAWRENCE, AND KEVIN M. MURPHY (1992): "Changes in Relative Wages, 1963–1987: Supply and Demand Factors," *The Quarterly Journal of Economics*, 107, 35–78. [2645]

KAYMAK, BARIS, AND MARKUS POSCHKE (2016): "The Evolution of Wealth Inequality Over Half a Century: The Role of Taxes, Transfers and Technology," *Journal of Monetary Economics*, 77, 1–25. [2649]

KRUSELL, PER, AND ANTHONY A. JR. SMITH (1998): "Income and Wealth Heterogeneity in the Macroeconomy," *Journal of Political Economy*, 106, 867–896. [2647,2649,2651,2660]

KRUSELL, PER, LEE E. OHANIAN, JOSÉ-VÍCTOR RÍOS-RULL, AND GIOVANNI L. VIOLANTE (2000): "Capital-Skill Complementarity and Inequality: A Macroeconomic Analysis," *Econometrica*, 68, 1029–1053. [2645]

LINDERT, PETER (2000): "Three Centuries of Inequality in Britain and America," in *Handbook of Income Distribution* (First Ed.), Vol. 1, Chapter 03, ed. by A. B. Atkinson and F. Bourguignon. Elsevier, 167–216. [2679]

LOECKER, DE, JAN EECKHOUT JAN, AND GABRIEL UNGER (2020): "The Rise of Market Power and the Macroeconomic Implications," *The Quarterly Journal of Economics*, 135, 561–644. [2670]

MEADE, JAMES E. (1964): "Efficiency, Equality and the Ownership of Property", George Allen & Unwin, Available at http://benjaminmoll.com/meade/. [2647,2661]

MOLL, BENJAMIN, LUKASZ RACHEL, AND PASCUAL RESTREPO (2022): "Supplement to 'Uneven Growth: Automation's Impact on Income and Wealth Inequality'," *Econometrica Supplemental Material*, 90, https://doi.org/10.3982/ECTA19417. [2650]

OBSTFELD, MAURICE (1994): "Risk-Taking, Global Diversification, and Growth", *American Economic Review*, 84, 1310–1329. [2663]

PIKETTY, THOMAS (2014): *Capital in the Twenty-First Century*. Cambridge: Harvard University Press. [2647, 2661]

——— (2015): "About Capital in the Twenty-First Century," *American Economic Review*, 105, 48–53. [2647]

PIKETTY, THOMAS, AND EMMANUEL SAEZ (2003): "Income Inequality in the United States, 1913-1998," *The Quarterly Journal of Economics*, 118, 1–41. [2661,2671,2677]

PIKETTY, THOMAS, EMMANUEL SAEZ, AND GABRIEL ZUCMAN (2018): "Distributional National Accounts: Methods and Estimates for the United States," *The Quarterly Journal of Economics*, 133, 553–609. [2645, 2649,2668,2676-2678]

RACHEL, LUKASZ, AND LAWRENCE H. SUMMERS (2019): "On Secular Stagnation in the Industrialized World," Brookings Papers on Economic Activity, Spring, 1–76. [2648,2667]

REIS, RICARDO (2020): "The Constraints on Public Debt When $r < g$ but $g < m$," Working paper, London School of Economics. [2648]

SACHS, JEFFREY D., AND LAURENCE J. KOTLIKOFF (2012): "Smart Machines and Long-Term Misery," Working Paper 18629, National Bureau of Economic Research. [2649]

SARGENT, THOMAS J., NENG WANG, AND JINQIANG YANG (2021): "Earnings Growth and the Wealth Distribution", *Proceedings of the National Academy of Sciences*, 118. [2647]

SIMON, HERBERT ALEXANDER (1965): *The Shape of Automation for Men and Management*, Vol. 13. New York: Harper & Row. [2656]

SMITH, MATTHEW, DANNY YAGAN, OWEN ZIDAR, AND ERIC ZWICK (2019): "Capitalists in the Twenty-First Century," *The Quarterly Journal of Economics*, 134, 1675–1745. [2678]

STACHURSKI, JOHN, AND ALEXIS AKIRA TODA (2019): "An Impossibility Theorem for Wealth in Heterogeneous-Agent Models With Limited Heterogeneity," *Journal of Economic Theory*, 182, 1–24. [2647]

STEINDL, JOSEF (1965): *Random Processes and the Growth of Firms: A Study of the Pareto Law*. London: Griffin. [2647,2659]

STRAUB, LUDWIG (2019): "Consumption, Savings, and the Distribution of Permanent Income," Tech. rep, Harvard University. [2649,2680]

WOLD, HERMAN O. A., AND PETER WHITTLE (1957): "A Model Explaining the Pareto Distribution of Wealth," *Econometrica*, 25, 591–595. [2647,2659]

ZEIRA, JOSEPH (1998): "Workers, Machines, and Economic Growth," *The Quarterly Journal of Economics*, 113, 1091–1117. [2649]

ZHENG, GEOFFERY (2019): "Wealth Shares in the Long Run," Tech. rep, UCLA Anderson School of Management. [2650,2665]

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