Single Value Decomposition Applications on MNIST DATASET

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***Abstract*- This paper explores the application of singular value decomposition (SVD) X=UΣV^T on the MNIST dataset, a collection of handwritten digits widely used for benchmarking for image processing systems. This project focuses on the SVD analysis of images of each digit, interpretation of the resultant matrices, singular value spectrum, and reconstruction of the images using ranked r approximation of X. The findings provide useful insights into the dimensional reduction capabilities of SVD and its applications in image classification.**

II. INTRODUCTION

THE MNIST dataset is composed of 70,000 20x20 black and white images of handwritten digits, which is split into 60,000 training images and 10,000 test images with their accompanying labels. Singular Value Decomposition (SVD) offers a promising approach to address these challenges by reducing dimensionality while preserving critical information. This project's goal is to go through a step-by-step process of SVD and show with experimentation the reduction and approximation of images, as well as calculating the Euclidean distance of a test image with the reconstructed image, which can be applied for classification purposes. This research contributes to the broader field of machine learning by demonstrating the practical utility of SVD in enhancing algorithmic performance on complex datasets.

III. IMAGE PROCCESSING WITH SINGULAR VALUE DECOMPOSITION (SVD)

Single value decomposition (SVD) is a unique matrix decomposition that exists for every complex-valued matrix.

$$X = U\Sigma V^{\mathrm{T}}$$

Let X be a *nxm* matrix the resulting matrices will have the dimensions of U_{nxn}, $\Sigma_{nxm,}$ and V_{mxm}. U, called the left singular values, and V, called the right singular values, are unitary matrices with orthonormal columns. The Σ is a diagonal matrix with real, non-negative entries, called singular values, organized hierarchically from the largest value being in the top left corner and the lowest being in the bottom right corner, depending on the rank of X, will sometimes of 0s on the bottom rows of Σ. The rank of X will determine how many non-zero values make up the diagonal of X.

For this project the matrices that are of interest are the X, U, and Σ.

I. X matrix holds the features of our dataset in the column vectors. Each column represents one sample of the dataset that has been flattened into a column vector.

$$\mathbf{X} = \begin{bmatrix} | & | & & | \\ \mathbf{x}_1 & \mathbf{x}_2 & \cdots & \mathbf{x}_m \\ | & | & & | \end{bmatrix}$$

Figure 1 representation of matrix X [1]

II. U matrix is square matrix and is the same size as the columns of X, and contains information on the column space of X, also called the eigen faces.

III. Σ determines how important the columns of U and V are in a hierarchical manner. Important for truncating and approximation of X.

IV. V contains information on the row space of X. For this project it isn't used for approximation or truncation of the images.

IV. RESULTS

A. *Normalizing the data*

a) *First, we get the mean or average of the training data set.*

$$\bar{X} = mean(X)$$

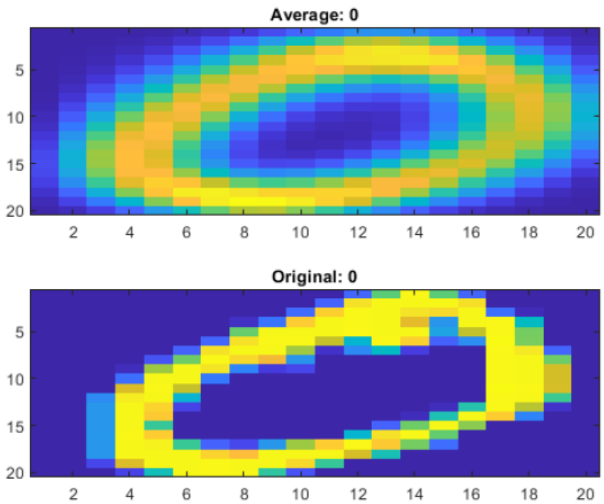

Figure 2 Average of digit(top) vs Original Image(bottom)

b) *Second, Mean subtracted.*

$$\tilde{X} = X - \bar{X}$$

This portion of the design process removes the biases that may be present in the images and improves the reconstruction of the image.

B. *Eigen Faces of matrix U of digit 0.*

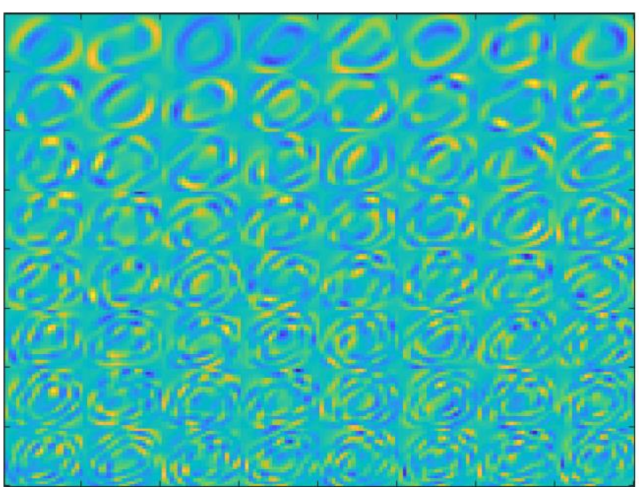

Figure 3 Eigen faces of digit 0.

This shows that most of the features are present in the first column of U (top left), compared with the lowest features present in the bottom right. This sets the foundation in which we can decrease the dimensions while getting an accurate reconstruction of the image.

C. *Approximation of the image of the Image*

a) X_{test} is represents an image part of the test data, a new image separate from the SVD. A series of truncation values(r) will be used to show the different approximations.

$$\tilde{X}_{test} = \tilde{X} + \tilde{U}\tilde{U}^T X_{test}$$

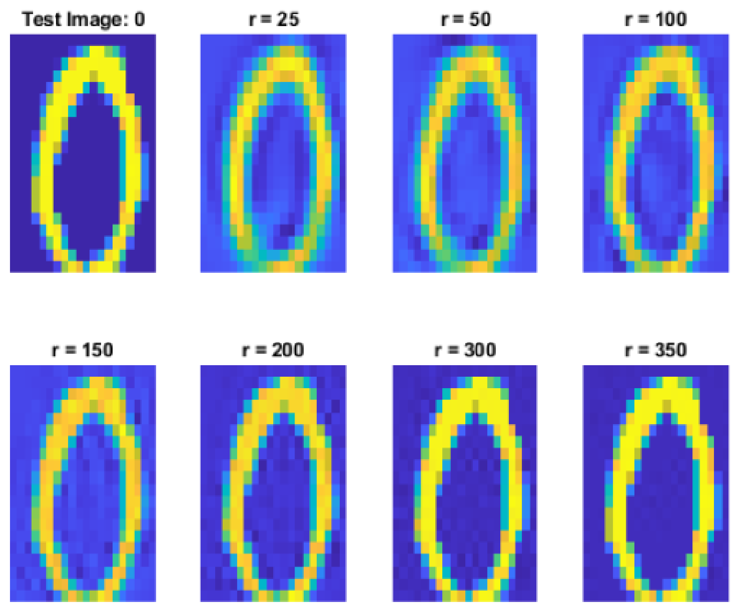

*Figure 4 Different Approximations of Digit 0 with different **r** values.*

D. *Singular Value Spectrum and Cumulative Sum*

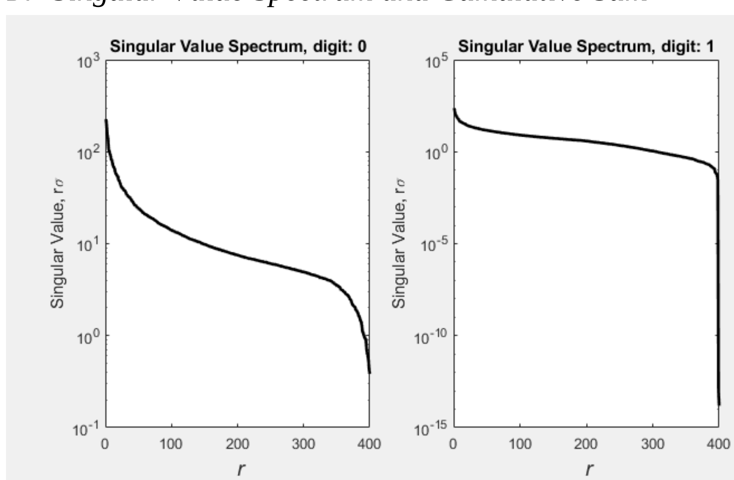

Figure 5 Singular Value Spectrum Digit 0 and 1

01

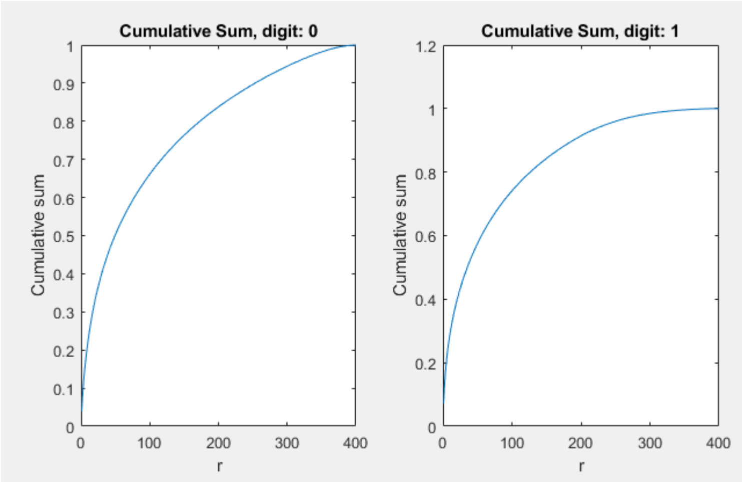

Figure 6 Cumulative Sum digit 0 and digit 1

This is important if you want to estimate what the best truncation value(r) to use for decreasing the dimensionality and getting the best approximation of the image.

E. *Euclidean Distance*

$$Distance = \left| X_{test} - \tilde{X} \right|$$

The Euclidean Distance between a test image and our approximate image is useful for classification purposes. The smaller the distances the more similar images and a prediction algorithm can determine what image be shown is. In contrast the larger the distances the more dissimilar the images are.

V. CONCLUSION

The value of SVD was shown through this project by reducing a high dimensional object into a lower dimensional object and approximating with significant accuracy the data. Outside of using SVD for images, anything with large amounts of data can be reduced and approximated, which could then reduce time and resources.

VI. REFERENCES

[1] S. L. Brunton and J. N. Kutz, *Data-Driven Science and Engineering: Machine Learning, Dynamical Systems, and Control*. Cambridge: Cambridge University Press, 2022.