Supplementary Material for Revision 149

1 Proofs of Lemmas

Lemma 2: Given two transition matrices T and $\tilde{T}$ of the same size $n \times n$,

$$\|T^k - \tilde{T}^k\|_F \leq k \cdot \sqrt{n} \cdot \|T - \tilde{T}\|_F.$$

Proof. First, we need to prove the claim that if T is a transition matrix, $\forall k > 0$, T^k is also a transition matrix which satisfies that each element is between 0 and 1 and the row sum is 1. A simple proof using Mathematical Induction.

(i)The claim holds when $k' = 1$.

(ii)Supposed that when $k' = k - 1$ the claim holds, consider $T^k = T^{k-1} \cdot T$. Then, $\forall 0 \leq i, j \leq n, T^k_{i,j} = \sum_{a=1}^{n} T^{k-1}_{i,a} \cdot T_{a,j}$, so $T^k_{i,j} > 0$. For row i, $\sum_{j=1}^{n} T^{k-1}_{i,j} = 1$ and $\sum_{j=1}^{n} T_{i,j} = 1$. Next , calculate the row sum of T^k as follows:

$$\begin{aligned}
&\sum_{j=1}^{n} T^k_{i,j} = \sum_{j=1}^{n}[\sum_{a=1}^{n} T^{k-1}_{i,a} \cdot T_{a,j}] \\
&= \sum_{j=1}^{n}[T^{k-1}_{i,1} \cdot T_{1,j} + T^{k-1}_{i,2} \cdot T_{2,j} \cdots + T^{k-1}_{i,n} \cdot T_{n,j}] \\
&= [T^{k-1}_{i,1} \cdot T_{1,1} + T^{k-1}_{i,2} \cdot T_{2,1} \cdots + T^{k-1}_{i,n} \cdot T_{n,1}] + [T^{k-1}_{i,1} \cdot T_{1,2} + T^{k-1}_{i,2} \cdot T_{2,2} + \cdots + T^{k-1}_{i,n} \cdot T_{n,2}] \cdots \\
&+ [T^{k-1}_{i,1} \cdot T_{1,n-1} + T^{k-1}_{i,2} \cdot T_{2,n-1} + \cdots + T^{k-1}_{i,n} \cdot T_{n,n-1}] + [T^{k-1}_{i,1} \cdot T_{1,n} + T^{k-1}_{i,2} \cdot T_{2,n} + \cdots + T^{k-1}_{i,n} \cdot T_{n,n}] \\
&= \sum_{a=1}^{n} T^{k-1}_{i,a} \cdot (T_{1,1} + T_{1,2} \cdots + T_{1,n}) \\
&= \sum_{a=1}^{n} T^{k-1}_{i,a} \cdot 1 = 1.
\end{aligned}$$

Moreover, because each element of T is larger than or equal to 0 and row sum of T is 1, each element of T^k is less than or equal to 1.

Next, prove that Forbenius norm of T satisfies $\|T\|_F \leq \sqrt{n}$, so $\|T^k\|_F \leq \sqrt{n}$. Because $\sum_{j=1} |T_{i,j}| = 1$, when $0 \leq T_{i,j} \leq 1$, $\sum_{j=1} |T_{i,j}|^2 \leq (\sum_{j=1} |T_{i,j}|)^2 = 1$. Therefore,

$$\|T\|_F = \sqrt{\sum_{i=1} \sum_{j=1} |T_{i,j}|^2} \leq \sqrt{\sum_{i=1} 1} = \sqrt{n}, \Longrightarrow \|T^k\|_F \leq \sqrt{n}.$$

At last, prove the lemma. Because of $\|T^k\|_F \leq \sqrt{n}$, we have

$$\begin{aligned}
&\|T^k - \tilde{T}^k\|_F \\
&= \|(\tilde{T} - T)(\tilde{T}^{k-1} + \tilde{T}^{k-2} \cdot T + \cdots + \tilde{T} \cdot T^{k-2} + T^{k-1})\|_F \\
&\leq \|(\tilde{T} - T)\|_F \|(\tilde{T}^{k-1} + \tilde{T}^{k-2} \cdot T + \cdots + \tilde{T} \cdot T^{k-2} + T^{k-1})\|_F \\
&\leq \|\tilde{T} - T\|_F \cdot k \cdot \max\{\|T^k\|_F, \|\tilde{T}^k\|_F\} \\
&\leq \|\tilde{T} - T\|_F \cdot k \cdot \sqrt{n}.
\end{aligned}$$

□

Lemma 3: Given a graph G, let T be its transition matrix. Given $\rho(\leq rank(T))$, let T_ρ be the optimal ρ-rank approximation of T. If algorithm CUR-Trans sets the number of sampled columns and rows as $c = r = O(\rho \cdot \log(\rho))$, we have

$$\Pr\left(\|T - CUR\|_F \leq (1 + \phi)\|T - T_\rho\|_F\right) \geq 0.81, \tag{1}$$

where $\phi = \varphi^2 + 2\varphi$, $\varphi = \sqrt{\rho \cdot c} + \sqrt{c}$.

In order to proof Lemma 3, we need to proof some claims at first.

For $T \in \mathbf{R}^{n\times n}$, $C \in \mathbf{R}^{n\times c}$ is constructed by sampling c columns from T without replacement using the probability of equation (2).

$$p_j^c = \frac{\sum_{i=1}^n |T_{i,j}|^2}{\sum_{j=1}^n \sum_{i=1}^n |T_{i,j}|^2}. \tag{2}$$

Then, sample rows from C without replacement using the probability of equation (3).

$$p_i^r = \frac{\sum_{j=1}^n |T_{i,j}|^2}{\sum_{i=1}^n \sum_{j=1}^n |T_{i,j}|^2}. \tag{3}$$

Let S be a diagonal matrix denoted the sampled rows, if row i is sampled, the $S_{i,i} = 1$, else $S_{i,i} = 0$. Therefore, $R = SC$. If rank of C is μ , the SVD decomposition of C is $C_{n\times c} = U_C \Sigma_C V_C^T = C_{n\times c} = U_{C,\mu}\Sigma_{C,\mu}V_{C,\mu}^T$. If ρ is a parameter not less than μ, $C = C_\rho = U_{C,\rho}\Sigma_{C,\rho}V_{C,\rho}^T, \mu \le \rho$.

Claim 1: For $c = r = O(100\rho \cdot \log(\rho))$, suppose $\tilde{\mu} = rank\left(SU_{C,\mu}\right)$, then with a probability at least 0.9, we have $\mu = \tilde{\mu}$ which means $rank\left(SU_{C,\rho}\right) = rank(C_\rho)$. Moreover, we have

$$(SC_\rho)^+ = V_{C,\rho}\Sigma_{C,\rho}^{-1}(SU_{C,\rho})^+,$$

where $(SU_{C,\rho})^+$ is Moore–Penrose pseudoinverse of matrix $SU_{C,\rho}$.

Proof. D is a diagonal matrix denoted the scaling of sampled rows, if row i is sampled, $D_{i,i} = \frac{1}{\sqrt{rp_i}}$, else $D_{i,i} = 0$. Let $\mathbb{S} = D\cdot S$. We have

$$\begin{aligned}
|1-\sigma_i^2(\mathbb{S}U_{C,\rho})| &= |\sigma_i(U_{C,\rho}^T U_{C,\rho}) - \sigma_i(U_{C,\rho}^T \mathbb{S}^T\mathbb{S}U_{C,\rho})| && (4)\\
&\le \|U_{C,\rho}^T U_{C,\rho} - U_{C,\rho}^T\mathbb{S}^T\mathbb{S}U_{C,\rho}\|_2 && (5)\\
&\le 10\mathbf{E}[\|U_{C,\rho}^T U_{C,\rho} - U_{C,\rho}^T\mathbb{S}^T\mathbb{S}U_{C,\rho}\|_2] && (6)\\
&\le 10O(1)\sqrt{\frac{\log(r)}{r}}\|U_{C,\rho}^T U_{C,\rho}\|_F\|U_{C,\rho}^T U_{C,\rho}\|_2 && (7)\\
&= O(1)10\sqrt{\frac{\log(r)}{r}}\sqrt{\rho}. && (8)
\end{aligned}$$

Equation (4) follows from properties of SVD that $U_{C,\rho}^T U_{C,\rho} = I$ and $\sigma_i^2(\mathbb{S}U_{C,\rho}) = \sigma_i(U_{C,\rho}^T\mathbb{S}^T\mathbb{S}U_{C,\rho})$. Equation (5) follows from $|\sigma(C+E)-\sigma(C)| \le \|E\|_2$ [2]. Equation (6) follows from the Markov's inequity [3], which is that given $a > 0$, $P(X \ge a) \le \frac{E(X)}{a}$. Then, we use it and give a probability $\frac{E(X)}{a} = 0.9$, so $a = 10\mathbf{E}[\|U_{C,\rho}^T U_{C,\rho} - U_{C,\rho}^T\mathbb{S}^T\mathbb{S}U_{C,\rho}\|_F]$. Equation (7) follows because of Theorem 7 in work [1]. Equation (8) follows because $\|U_{C,\rho}^T U_{C,\rho}\|_F^2 = \rho$ and $\|U_{C,\rho}^T U_{C,\rho}\|_2 = 1$. When use the $r = O(\rho\cdot\log(\rho))$ combined with section6.3.5 and Lemma 1 of previous work [1], we have $|1-\sigma_i^2(\mathbb{S}U_{C,\rho})| < 1$, which means all singular values are strictly positive. Therefore, $rank(\mathbb{S}U_{C,\rho}) = rank\left(U_{C,\rho}\right)$. D is a linear transformation matrix for $SU_{C,\rho}$, which will not change the rank, so with a probability at least 0.9, we have $rank\left(SU_{C,\rho}\right) = rank\left(U_{C,\rho}\right) = rank(C_\rho) = \mu = \tilde{\mu}$.

Then, if $\mu = \tilde{\mu}$, we can prove $(SC_\rho)^+ = V_{C,\rho}\Sigma_{C,\rho}^{-1}(SU_{C,\rho})^+$. We have

$$\begin{aligned}
(SC_\rho)^+ &= (SU_{C,\rho}\Sigma_{C,\rho}V_{C,\rho}^T)^+\\
&= (U_{SU_{C,\rho}}\Sigma_{SU_{C,\rho}}V_{SU_{C,\rho}}^T\Sigma_{C,\rho}V_{C,\rho}^T)^+\\
&= V_{C,\rho}(\Sigma_{SU_{C,\rho}}V_{SU_{C,\rho}}^T\Sigma_{C,\rho})^+U_{SU_{C,\rho}}^T\\
&= V_{C,\rho}(\Sigma_{SU_{C,\rho}}V_{SU_{C,\rho}}^T\Sigma_{C,\rho})^{-1}U_{SU_{C,\rho}}^T && (9)\\
&= V_{C,\rho}\Sigma_{C,\rho}^{-1}V_{SU_{C,\rho}}\Sigma_{SU_{C,\rho}}^{-1}U_{SU_{C,\rho}}^T && (10)\\
&= V_{C,\rho}\Sigma_{C,\rho}^{-1}(SU_{C,\rho})^+.
\end{aligned}$$

Equation (9) follows because when $\mu = \tilde{\mu}$, matrices $\Sigma_{SU_{C,\rho}}$, $V_{SU_{C,\rho}}$, and $\Sigma_{C,\rho}$ are full rank $\mu\times\mu$ square matrix matrices, which means that they are invertible. When the matrix is invertible, the pseudo-inverse is equal to the inverse. If A,B,C are invertible matrices, then $(ABC)^{-1} = C^{-1}B^{-1}A{-}1$, so equation (10) follows. □

Claim 2: Define $\Omega = (SU_{C,\rho})^{+} - (SU_{C,\rho})^{T}$, then

$$\|\Omega\|_2 = \|\Sigma_{SU_{C,\rho}}^{-1} - \Sigma_{SU_{C,\rho}}\|_2 \leq 1.$$

Proof.

$$\begin{aligned}
\|\Omega\|_2 &= \|(SU_{C,\rho})^{+} - (SU_{C,\rho})^{T}\|_2 \\
&= \|(U_{SU_{C,\rho}}\Sigma_{SU_{C,\rho}}V_{SU_{C,\rho}}^{T})^{+} - (U_{SU_{C,\rho}}\Sigma_{SU_{C,\rho}}V_{SU_{C,\rho}}^{T})^{T}\|_2 \\
&= \|V_{SU_{C,\rho}}(\Sigma_{SU_{C,\rho}}^{-1} - \Sigma_{SU_{C,\rho}})U_{SU_{C,\rho}}^{T}\|_2 \\
&= \|\Sigma_{SU_{C,\rho}}^{-1} - \Sigma_{SU_{C,\rho}}\|_2 \\
&= \max_{i,j\in[1,\rho]} |\sigma_i(SU_{C,\rho}) - \frac{1}{\sigma_j(SU_{C,\rho})}|.
\end{aligned}$$

With $\sigma_i(SU_{C,\rho}) \geq 0$, therefore, $\sigma_i(SU_{C,\rho}) \in [0,1]$. Use the SVD decomposition of $SU_{C,\rho}$, and $V_{SU_{C,\rho}}$, $U_{SU_{C,\rho}}$ are matrices with orthonormal columns. We have

$$\|\Omega\|_2 = \max_{i,j\in[1,\rho]} |\sigma_i(SU_{C,\rho}) - \frac{1}{\sigma_j(SU_{C,\rho})} \leq |0-1| = 1.$$

□

Claim 3: $U_{C,\rho}$ is an $n\times\rho$ matrix contained the top ρ left singular vectors of C. $U_{C,\rho}^{\perp}$ is an $n\times(n-\rho)$ contained the bottom $\mu-\rho$ non-zero left singular vectors of C. $V_{C,\rho}^{\perp}$ and $\Sigma_{C,\rho}^{\perp}$ are defined by the same way. We have that

$$\|U_{C,\rho}^{T}S^{T}SU_{C,\rho}^{\perp}(U_{C,\rho}^{\perp})^{T}T\|_F \leq \sqrt{r\rho}\|U_{C,\rho}^{\perp}(U_{C,\rho}^{\perp})^{T}T\|_F,$$

$$\text{and } \|SU_{C,\rho}^{\perp}(U_{C,\rho}^{\perp})^{T}T\|_F = \sqrt{r}\|U_{C,\rho}^{\perp}(U_{C,\rho}^{\perp})^{T}T\|_F.$$

Proof.

$$\|U_{C,\rho}^{T}S^{T}SU_{C,\rho}^{\perp}(U_{C,\rho}^{\perp})^{T}T\|_F \leq \|U_{C,\rho}^{T}\|_F\|S^{T}S\|_F\|U_{C,\rho}^{\perp}(U_{C,\rho}^{\perp})^{T}T\|_F = \sqrt{\rho}\sqrt{r}\|U_{C,\rho}^{\perp}(U_{C,\rho}^{\perp})^{T}T\|_F.$$

Similarly, we have

$$\|SU_{C,\rho}^{\perp}(U_{C,\rho}^{\perp})^{T}T\|_F \leq \|S\|_F\|U_{C,\rho}^{\perp}(U_{C,\rho}^{\perp})^{T}T\|_F = \sqrt{r}\|U_{C,\rho}^{\perp}(U_{C,\rho}^{\perp})^{T}T\|_F.$$

□

Claim 4: For $r = O(\rho\cdot\log(\rho))$, with a probability at least 0.9, we have

$$\Pr\left(\|T - C(SC)^{+}ST\| \leq (1+\varphi)\|T - CC^{+}T\|\right) \geq 0.9,$$

where $\varphi = \sqrt{\rho r} + \sqrt{r}$. Similarly, we have

$$\Pr\left(\|T - ST(SC)^{+}C\| \leq (1+\varphi)\|T - TC^{+}C\|\right) \geq 0.9.$$

Proof. Suppose $\tilde{X} = (SC)^{+}(ST)$ and $X = C^{+}T$. Next, we find the distance between $\|T - CX\|$ and $\|T - C\tilde{X}\|$. Given $\rho \geq \mu$, $C = C_\rho = U_{C,\rho}\Sigma_{C,\rho}V_{C,\rho}^{T}$.

$$\begin{aligned}
T - C_\rho\tilde{X} &= T - C_\rho(SC_\rho)^{+}(ST) \\
&= T - U_{C,\rho}(SU_{C,\rho})^{+}(ST) && (11)\\
&= T - U_{C,\rho}(SU_{C,\rho})^{+}SU_{C,\rho}U_{C,\rho}^{T}T - U_{C,\rho}(SU_{C,\rho})^{+}SU_{C,\rho}^{\perp}(U_{C,\rho}^{\perp})^{T}T && (12)\\
&= U_{C,\rho}^{\perp}(U_{C,\rho}^{\perp})^{T}T - U_{C,\rho}(SU_{C,\rho})^{+}SU_{C,\rho}^{\perp}(U_{C,\rho}^{\perp})^{T}T && (13)\\
&= U_{C,\rho}^{\perp}(U_{C,\rho}^{\perp})^{T}T + U_{C,\rho}(SU_{C,\rho})^{T}SU_{C,\rho}^{\perp}(U_{C,\rho}^{\perp})^{T}T + U_{C,\rho}\Omega SU_{C,\rho}^{\perp}U_{C,\rho}^{\perp}T.
\end{aligned}$$

Equation (11) follows from **Claim 1** with a probability at least 0.9. Equation (12) follows by inserting $U_{C,\rho}U_{C,\rho}^T + U_{C,\rho}^{\perp}(U_{C,\rho}^{\perp})^T = I_n$. Equation (13) follows since $(SU_{C,\rho})^+SU_{C,\rho} = I_\rho$ and $\Omega = (SU_{C,\rho})^+ - (SU_{C,\rho})^T$ in **Claim 2**. Then, with a probability at least 0.9, combining **Claim 2** and **Claim3**,we have

$$\begin{aligned}
\|T - C\tilde{X}\|_F &= \|T - C_\rho \tilde{X}\|_F \\
&\leq \|U_{C,\rho}^{\perp}(U_{C,\rho}^{\perp})^T T\|_F + \|U_{C,\rho}(SU_{C,\rho})^T SU_{C,\rho}^{\perp}(U_{C,\rho}^{\perp})^T T\|_F + \|U_{C,\rho}\Omega SU_{C,\rho}^{\perp}U_{C,\rho}^{\perp}T\|_F \\
&\leq \|U_{C,\rho}^{\perp}(U_{C,\rho}^{\perp})^T T\|_F + \|U_{C,\rho}^T S^T SU_{C,\rho}^{\perp}(U_{C,\rho}^{\perp})^T T\|_F + \|\Omega\|_2\|SU_{C,\rho}^{\perp}(U_{C,\rho}^{\perp})^T T\|_F, \\
&\leq (1 + \sqrt{\rho r} + \sqrt{r})\|U_{C,\rho}^{\perp}(U_{C,\rho}^{\perp})^T T\|_F \\
&= (1 + \sqrt{\rho r} + \sqrt{r})\|T - C_\rho C_\rho^+ T\|_F \\
&\leq (1 + \sqrt{\rho r} + \sqrt{r})\|T - CC^+T\|_F.
\end{aligned}$$

Then, if we let $\varphi = \sqrt{\rho r} + \sqrt{r}$, the claim is proved. □

At last, prove the lemma 3.

Proof. Let S_C be the diagonal matrix that records the sampled columns, where if column i is sampled, $(S_C)_{i,i} = 1$, otherwise, $(S_C)_{i,i} = 0$. Similarly, S_R is the diagonal matrix that records the sampled rows. According to **Claim 4**, given $c = O(\rho \cdot \log(\rho))$, with a probability at least 0.9, we have $\|T - CUR\|_F = \|T - C(S_R C)^+ S_R T\|_F \leq (1 + \varphi_1)\|T - CC^+T\|_F$. Therefore,

$$\Pr(\|T - CUR\|_F \leq (1 + \varphi_1)\|T - CC^+T\|_F) \geq 0.9 \tag{14}$$

where $\varphi_1 = \sqrt{\rho r} + \sqrt{r}$,and C^+ is the Moore–Penrose pseudo-inverse of C and $\|T - CC^+T\|$ is the projection of T on the subspace spanned by the columns of C.

Similarly, according to **Claim 4**, given $r = O(\rho \cdot \log(\rho))$ and T_ρ is the optimal ρ-rank approximation of T, $\varphi = \sqrt{\rho c} + \sqrt{c}$, with a probability at least 0.9, we have $\|T - CC^+T\|_F = \|T - TS_C(TS_C)^+T\|_F \leq \|T - TS_C(T_\rho S_C)^+ T_\rho\|_F \leq (1 + \varphi_2)\|T - TT_\rho^+ T_\rho\|_F = (1 + \varphi_2)\|T - T_\rho\|_F$. Therefore,

$$\Pr(\|T - CC^+T\|_F \leq (1 + \varphi_2)\|T - T_\rho\|_F) \geq 0.9 \tag{15}$$

Combining Equations(14) and (15), we have

$$\Pr(\|T - CUR\|_F \leq (1 + \varphi_1)(1 + \varphi_2)\|T - T_\rho\|_F) \geq 0.81.$$

Because $c = O(\rho \cdot \log(\rho)), r = O(\rho \cdot \log(\rho))$, we can set $c = r$. Therefore, we can set $\phi = 2\varphi + \varphi^2$ and $\varphi = \sqrt{\rho r} + \sqrt{r}$. Equation (1) can be derived, and thus the proof is completed. □

Lemma 5: According to algorithm T^2-Approx, we have

$$\|T^2 - X \cdot Y\|_F \leq \sqrt{(n - c)}\|T\|_F^2.$$

Proof. Consider $T^2 = T^{(L)} \cdot T^{(R)}$. Let S be a diagonal matrix, where if column i of $T^{(L)}$ and row i of $T^{(R)}$ are sampled, $S_{i,i} = 1$, otherwise, $S_{i,i} = 0$. Then, we have $X = T \cdot S$ and $Y = S^T \cdot T$. Therefore,

$$\begin{aligned}
\|T^{(L)}T^{(R)} - X \cdot Y\|_F &= \|T^{(L)}T^{(R)} - T^{(L)}SS^T T^{(R)}\|_F \\
&\leq \|T^{(L)}\|_F\|T^{(R)}\|_F\|I - SS^T\|_F \\
&= \sqrt{n - c}\|T^{(L)}\|_F\|T^{(R)}\|_F \\
&= \sqrt{(n - c)}\|T\|_F^2
\end{aligned}$$

□

2 Supplementary Experiments

2.1 Effect of the number of iterations

Figure 1 shows that how the errors of the two proposed algorithms change with the number of iterations on dataset Orkut. As the number of iterations increases from 1 to 3, the errors decrease rapidly. When the number of iterations exceeds 4, the errors do not change much and the algorithms converge.

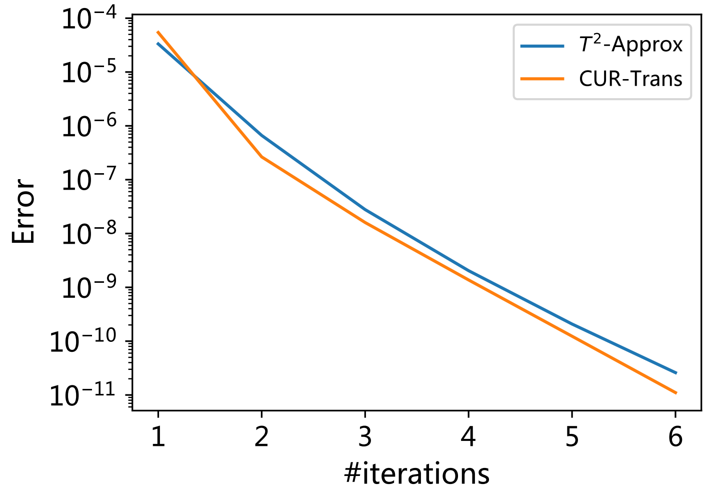
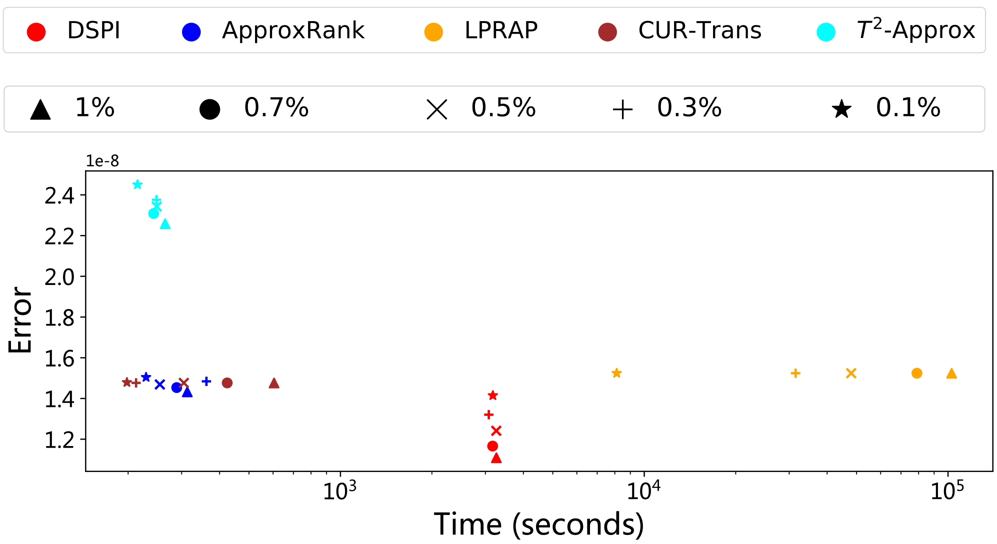
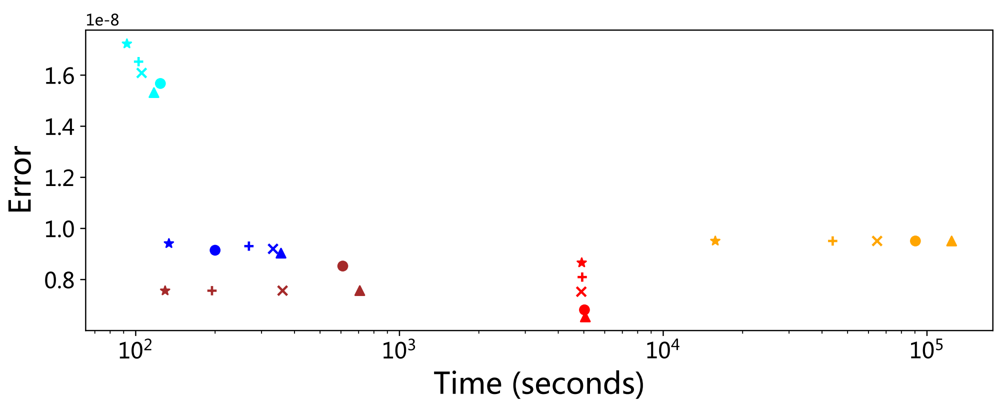


Figure 1: Errors affected by the number of iterations (Orkut).



(a) Friendster.



(b) UKDomain.

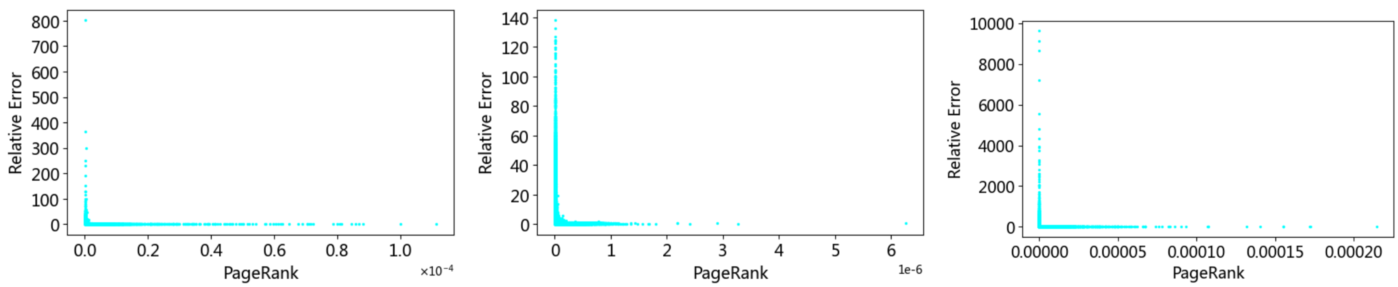
Figure 2: Scatter plots of computation time and errors.

2.2 Computation time vs. average errors

Figure 2 shows the scatter plots of the computation time and the average errors of all the algorithms when using different edge sampling ratios on two large datasets Friendster and UKDomain. It is observed that LPRAP is the slowest and its average error is comparable with CUR-Trans on Friendster but worse than CUR-Trans on UKDomain. The average error of DSPI is sometimes better than CUR-Trans, but it is significantly slower than CUR-Trans. ApproxRank is comparable with CUR-Trans on Friendster, but is worse than CUR-Trans in terms of the average error on UKDomain. T^2-Approx is the most efficient algorithm.

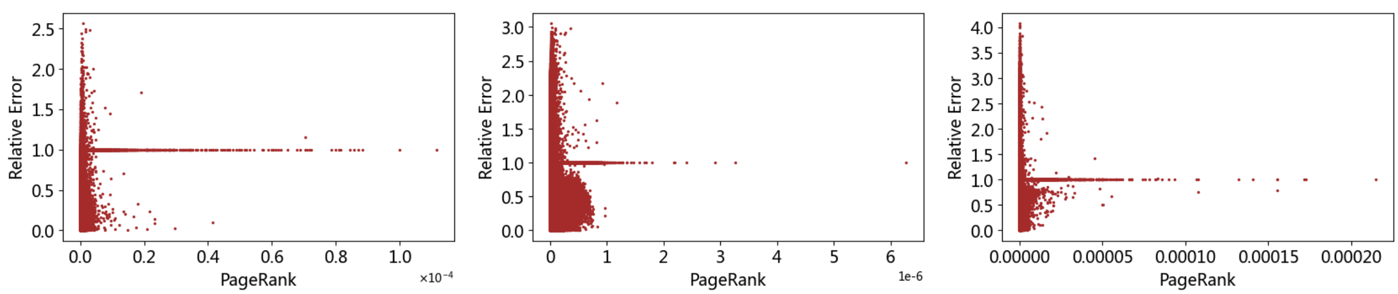
2.3 Relative errors of different algorithms

Figures 3, 4, 5, 6, and 7 show the scatter plots of the relative error and the estimated PageRank value of each vertex. The relative errors of the vertices with high PageRank values are high in algorithms LPRAP and ApproxRank. The relative errors of the vertices with low PageRank values are high in algorithms DSPI, CUR-Trans and T^2-Approx. If two vertices have the same absolute error, the relative error of the vertex with low PageRank value is higher than that of the vertex with high PageRank value. Thus, the average relative value may not be a good metric for evaluating the algorithms. In practice, making estimation for the vertices with high PageRank values is more important than that for the vertices with low PageRank values.



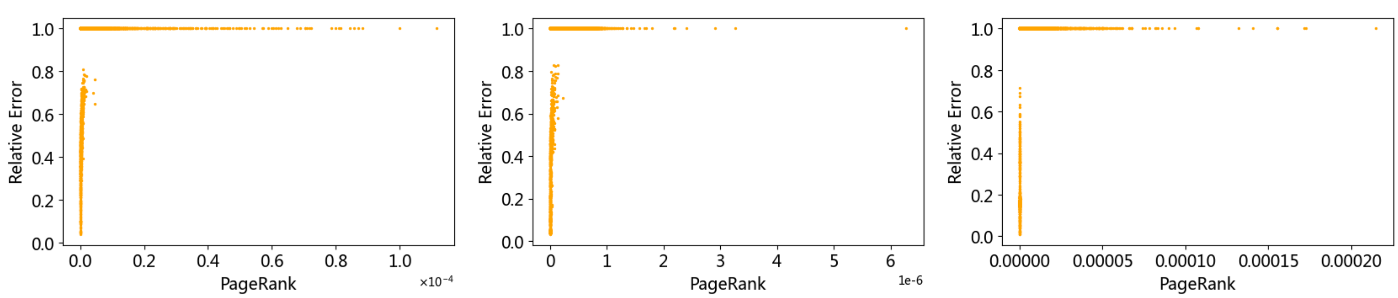
(a) Orkut. (b) Friendster. (c) UKDomain.

Figure 3: Relative error results for different PageRank values of DSPI on different datasets .



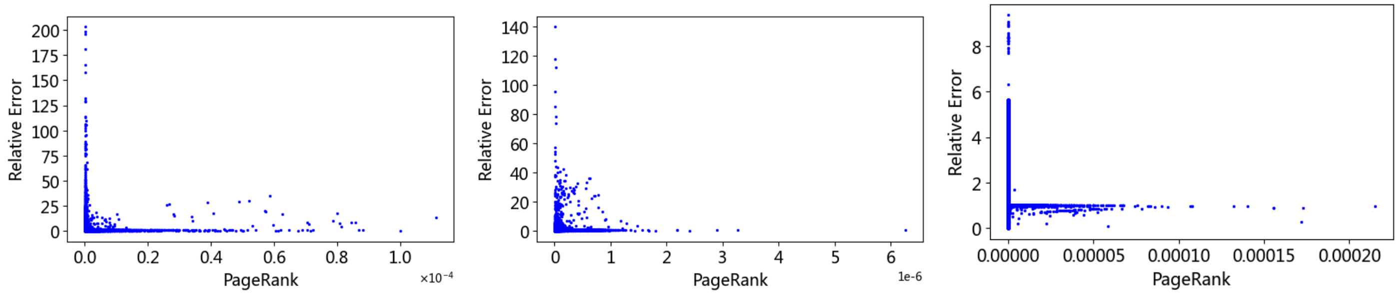
(a) Orkut. (b) Friendster. (c) UKDomain.

Figure 4: Relative error results for different PageRank values of ApproxRank on different datasets.



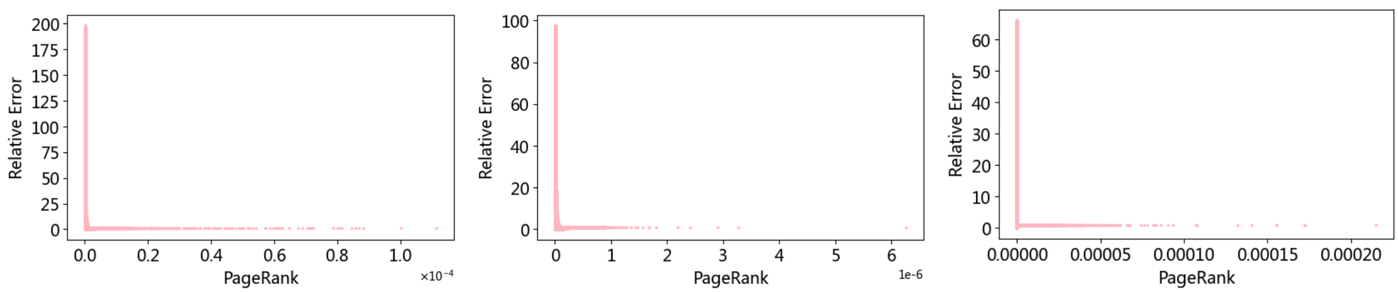
(a) Orkut. (b) Friendster. (c) UKDomain.

Figure 5: Relative error results for different PageRank values of LPRAP on different datasets.



(a) Orkut. (b) Friendster. (c) UKDomain.

Figure 6: Relative error results for different PageRank values of CUR-Trans on different datasets.



(a) Orkut. (b) Friendster. (c) UKDomain.

Figure 7: Relative error results for different PageRank values of T^2-Approx on different datasets.

References

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