CHAPTER 9

# Frequency Response

## IN THIS CHAPTER YOU WILL LEARN

1. How coupling and bypass capacitors cause the gain of discrete-circuit amplifiers to fall off at low frequencies, and how to obtain an estimate of the frequency $f_L$ at which the gain decreases by 3 dB below its value at midband.
2. The internal capacitive effects present in the MOSFET and the BJT and how to model these effects by adding capacitances to the hybrid-$\pi$ model of each of the two transistor types.
3. The high-frequency limitation on the gain of the CS and CE amplifiers and how the gain falloff and the upper 3-dB frequency $f_H$ are mostly determined by the small capacitance between the drain and gate (collector and base).
4. Powerful methods for the analysis of the high-frequency response of amplifier circuits of varying complexity.
5. How the cascode amplifier studied in Chapter 7 can be designed to obtain wider bandwidth than is possible with the CS and CE amplifiers.
6. The high-frequency performance of the source and emitter followers.
7. The high-frequency performance of differential amplifiers.
8. Circuit configurations for obtaining wideband amplification.

# Introduction

Except for brief comments in Sections 5.6.8 and 6.6.8, our study of transistor amplifiers in Chapters 5 through 8 has assumed that their gain is constant independent of the frequency of the input signal. This would imply that their bandwidth is infinite, which of course is not true! To illustrate, we show in Fig. 9.1 a sketch of the magnitude of the gain versus the frequency of the input signal of a discrete-circuit BJT or MOS amplifier. Observe that there is indeed a wide frequency range over which the gain remains almost constant. This is the useful frequency range of operation for the particular amplifier. Thus far, we have been assuming that our amplifiers are operating in this band, called the middle-frequency band or **midband**. The amplifier is designed so that its midband coincides with the frequency spectrum of the signals it is required to amplify. If this were not the case, the amplifier would distort the frequency spectrum of the input signal, with different components of the input signal being amplified by different amounts.

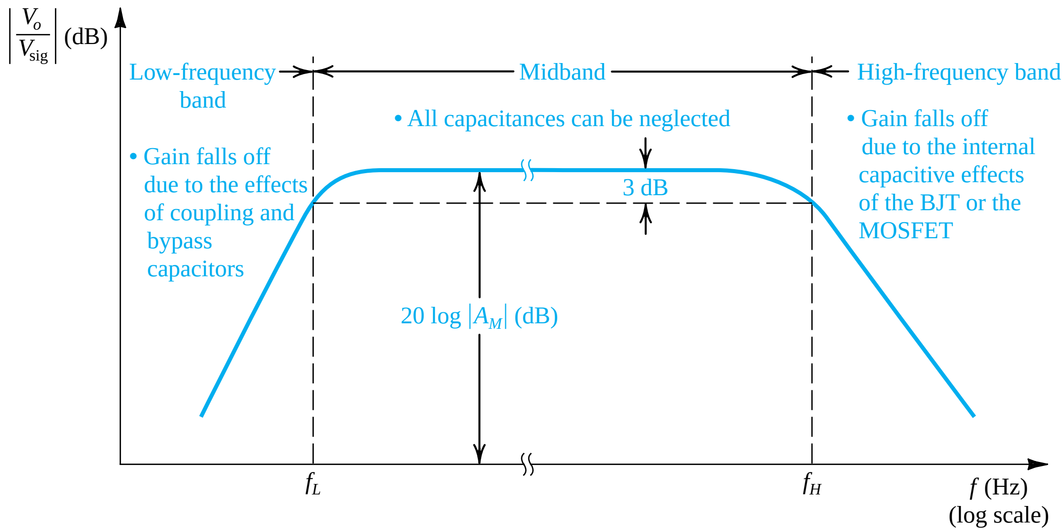



**Figure 9.1** Sketch of the magnitude of the gain of a discrete-circuit BJT or MOS amplifier versus frequency. The graph delineates the three frequency bands relevant to frequency-response determination.

Figure 9.1 indicates that at lower frequencies, the magnitude of the amplifier gain falls off. This occurs because the coupling and bypass capacitors no longer have low impedances. Recall that we assumed that their impedances were small enough to act as short circuits. Although this can be true at midband frequencies, as the frequency of the input signal is lowered, the reactance $1/j\omega C$ of each of these capacitors becomes significant and, as will be shown in Section 9.1, this results in a decrease in the overall voltage gain of the amplifier. In the analysis of the low-frequency response of discrete-circuit amplifiers in Section 9.1 we will be particularly interested in the determination of the frequency $f_L$, which defines the lower end of the midband. It is usually defined as the frequency at which the gain drops by 3 dB below its value in midband. Integrated-circuit amplifiers do not utilize coupling and bypass capacitors, and thus their midband extends down to zero frequency (dc).

Figure 9.1 indicates also that the gain of the amplifier falls off at the high-frequency end. This is due to internal capacitive effects in the BJT and in the MOSFET. We shall study these effects in Section 9.2 and model them with capacitances that we will add to the hybrid-$\pi$ model of the BJT and the MOSFET. The resulting high-frequency device models will be utilized in Section 9.3 in the analysis of the high-frequency response of the CS and CE amplifiers, both discrete and integrated. We will be specifically interested in the determination of the frequency $f_H$, which defines the upper end of the midband. It is defined as the frequency at which the gain drops by 3 dB below its midband value. Thus, the amplifier bandwidth is defined by $f_L$ and $f_H$ (0 and $f_H$ for IC amplifiers).

The remainder of this chapter will be concerned with the frequency response analysis of a variety of amplifier configurations of varying degrees of complexity. Of particular interest to us are ways to extend the amplifier bandwidth (i.e., increase $f_H$) either by adding specific circuit components, such as source and emitter degeneration resistances, or by changing the circuit configuration altogether.

Before embarking on the study of this chapter, the reader is urged to review Section 1.6, which introduces the subject of amplifier frequency response and the extremely important topic of single-time-constant (STC) circuits. More details on STC circuits can be found in

Appendix E. As well, Appendix F provides a review of important tools from circuit and system theory: poles, zeros, and Bode plots.

# 9.1 Low-Frequency Response of the Common-Source and Common-Emitter Amplifiers

## 9.1.1 The CS Amplifier

Figure 9.2(a) shows a discrete-circuit, common-source amplifier utilizing coupling capacitors $C_{C1}$ and $C_{C2}$, and bypass capacitor $C_S$. We wish to determine the effect of these capacitances on the gain $V_o/V_{\text{sig}}$ of the amplifier. As mentioned before, at midband frequencies, these capacitances have negligibly small impedances and can be assumed to be perfect short circuits for the purpose of calculating the midband gain. At low frequencies, however, the reactance $1/j\omega C$ of each of the three capacitances increases and the amplifier gain decreases, as we shall now show.

**Determining $V_o/V_{\text{sig}}$** To determine the low-frequency gain or transfer function of the common-source amplifier, we show in Fig. 9.2(b) the circuit with the dc sources eliminated (current source $I$ open-circuited and voltage source $V_{DD}$ short-circuited). We shall perform the small-signal analysis directly on this circuit. However, we will ignore $r_o$. This is done in order to keep the analysis simple and thus focus attention on significant issues. The effect of $r_o$ on the low-frequency operation of this amplifier is minor, as can be verified by a SPICE simulation.

To determine the gain $V_o/V_{\text{sig}}$, we start at the signal source and work our way through the circuit, determining $V_g$, $I_d$, $I_o$, and $V_o$, in this order.[1] To find the fraction of $V_{\text{sig}}$ that appears at the transistor gate, $V_g$, we use the voltage divider rule at the input to write

$$V_g = V_{\text{sig}} \frac{R_G}{R_G + \dfrac{1}{sC_{C1}} + R_{\text{sig}}}$$

which can be written in the alternate form

$$V_g = V_{\text{sig}} \frac{R_G}{R_G + R_{\text{sig}}} \frac{s}{s + \dfrac{1}{C_{C1}(R_G + R_{\text{sig}})}} \qquad (9.1)$$

Thus we see that the expression for the signal transmission from signal generator to amplifier input has acquired a frequency-dependent factor. From our study of frequency response in Section 1.6 (see also Appendix E), we recognize this factor as the transfer function of an STC circuit of the high-pass type with a break or corner frequency $\omega_0 = 1/C_{C1}(R_G + R_{\text{sig}})$. Thus the effect of the coupling capacitor $C_{C1}$ is to introduce a high-pass STC response with a

[1] Note that since we are now dealing with quantities that are functions of frequency, or, equivalently, the Laplace variable $s$, we are using capital letters with lowercase subscripts for our symbols. This conforms with the symbol notation introduced in Chapter 1.

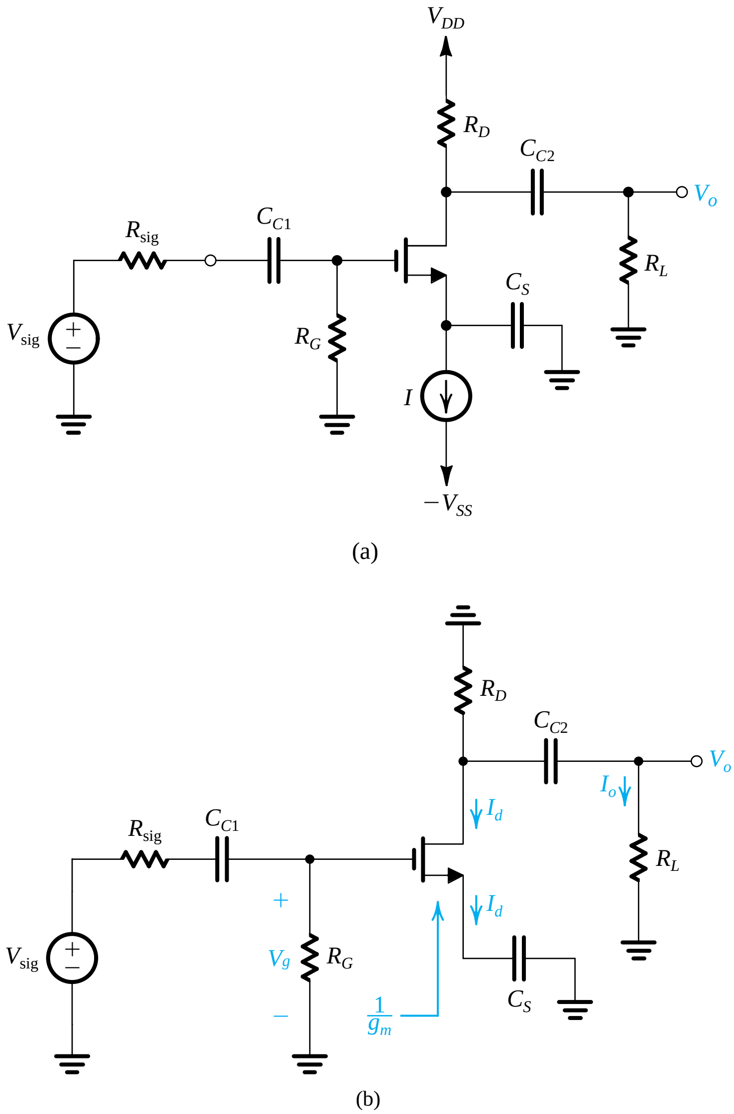



**Figure 9.2** **(a)** Capacitively coupled common-source amplifier. **(b)** Analysis of the CS amplifier to determine its low-frequency transfer function. For simplicity, $r_o$ is neglected.

break frequency that we shall denote $\omega_{P1}$,

$$\omega_{P1} = \omega_0 = \frac{1}{C_{C1}(R_G + R_{\text{sig}})} \tag{9.2}$$

Continuing with the analysis, we next determine the drain current $I_d$ by dividing $V_g$ by the total impedance in the source circuit, which is $[(1/g_m) + (1/sC_S)]$ to obtain

$$I_d = \frac{V_g}{\dfrac{1}{g_m} + \dfrac{1}{sC_S}}$$

which can be written in the alternate form

$$I_d = g_m V_g \frac{s}{s + \dfrac{g_m}{C_S}} \tag{9.3}$$

We observe that $C_S$ introduces a frequency-dependent factor, which is also of the STC high-pass type. Thus the amplifier acquires another break frequency,

$$\omega_{P2} = \frac{g_m}{C_S} \tag{9.4}$$

To complete the analysis, we find $V_o$ by first using the current divider rule to determine the fraction of $I_d$ that flows through $R_L$,

$$I_o = -I_d \frac{R_D}{R_D + \dfrac{1}{sC_{C2}} + R_L}$$

and then multiplying $I_o$ by $R_L$ to obtain

$$V_o = I_o R_L = -I_d \frac{R_D R_L}{R_D + R_L} \frac{s}{s + \dfrac{1}{C_{C2}(R_D + R_L)}} \tag{9.5}$$

from which we see that $C_{C2}$ introduces a third STC high-pass factor, giving the amplifier a third break frequency at

$$\omega_{P3} = \frac{1}{C_{C2}(R_D + R_L)} \tag{9.6}$$

The overall low-frequency transfer function of the amplifier can be found by combining Eqs. (9.1), (9.3), and (9.5) and replacing the break frequencies by their symbols from Eqs. (9.2), (9.4), and (9.6):

$$\frac{V_o}{V_{sig}} = -\left(\frac{R_G}{R_G + R_{sig}}\right)[g_m(R_D \parallel R_L)]\left(\frac{s}{s + \omega_{P1}}\right)\left(\frac{s}{s + \omega_{P2}}\right)\left(\frac{s}{s + \omega_{P3}}\right) \tag{9.7}$$

which can be expressed in the form

$$\frac{V_o}{V_{sig}} = A_M\left(\frac{s}{s + \omega_{P1}}\right)\left(\frac{s}{s + \omega_{P2}}\right)\left(\frac{s}{s + \omega_{P3}}\right) \tag{9.8}$$

where $A_M$, the midband gain, is given by

$$A_M = -\frac{R_G}{R_G + R_{sig}}[g_m(R_D \| R_L)] \tag{9.9}$$

which is the value we would have obtained, had we assumed that $C_{C1}$, $C_{C2}$, and $C_S$ were acting as perfect short circuits. In this regard, note that at midband frequencies—that is, at frequencies $s = j\omega$ much higher than $\omega_{P1}$, $\omega_{P2}$, and $\omega_{P3}$—Eq. (9.8) shows that $V_o/V_{sig}$ approaches $-A_M$, as should be the case.

**Determining the Lower 3-dB Frequency, $f_L$** The magnitude of the amplifier gain, $|V_o/V_{sig}|$ at frequency $\omega$ can be obtained by substituting $s = j\omega$ in Eq. (9.8) and evaluating the magnitude of the transfer function. In this way, the frequency response of the amplifier can be plotted versus frequency, and the lower 3-dB frequency $f_L$ can be determined as the frequency at which $|V_o/V_{sig}|$ drops to $|A_M|/\sqrt{2}$. A simpler approach, however, is possible if the break frequencies $\omega_{P1}$, $\omega_{P2}$, and $\omega_{P3}$ are sufficiently separated. In this case, we can employ the Bode plot rules (see Appendix F) to sketch a Bode plot for the gain magnitude. Such a plot is shown in Fig. 9.3. Observe that since the break frequencies are sufficiently separated, their effects appear distinct. At each break frequency, the slope of the asymptote to the gain function increases by 20 dB/decade. Readers familiar with poles and zeros will recognize $f_{P1}$, $f_{P2}$, and $f_{P3}$ as the frequencies of the three real-axis, low-frequency poles of the amplifier. (For a brief review of poles and zeros, refer to Appendix F.)

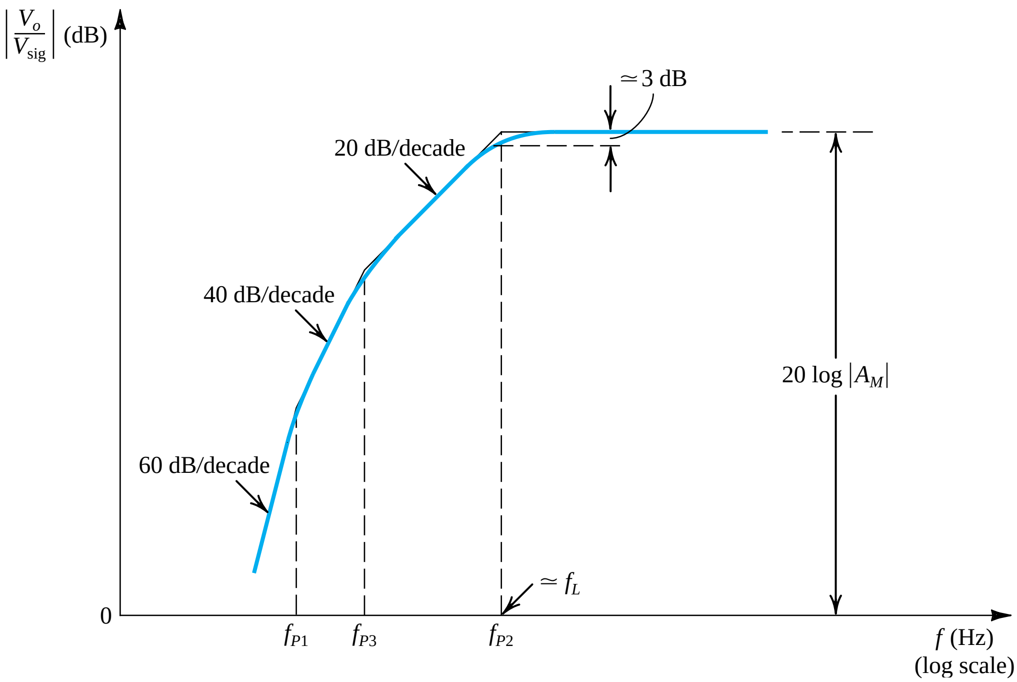



**Figure 9.3** Sketch of the low-frequency magnitude response of a CS amplifier for which the three pole frequencies are sufficiently separated for their effects to appear distinct.

A quick way for estimating the 3-dB frequency $f_L$ is possible if the highest-frequency pole (here, $f_{P2}$) is separated from the nearest pole (here, $f_{P3}$) by at least a factor of 4 (two octaves). In such a case, $f_L$ is approximately equal to the highest of the pole frequencies,

$$f_L \simeq f_{P2}$$

Usually, the highest-frequency pole is the one caused by $C_S$. This is because $C_S$ interacts with $1/g_m$, which is relatively low (see Eq. 9.4).

**Determining the Pole Frequencies by Inspection** Before leaving this section, we present a simple method for finding the time constant and hence the pole frequency associated with each of the three capacitors. The procedure is simple:

1. Reduce $V_{sig}$ to zero.
2. Consider each capacitor separately; that is, assume that the other two capacitors are acting as perfect short circuits.
3. For each capacitor, find the total resistance seen between its terminals. This is the resistance that determines the time constant associated with this capacitor.

The reader is encouraged to apply this procedure to $C_{C1}$, $C_S$, and $C_{C2}$ and thus see that Eqs. (9.2), (9.4), and (9.6) can be written by inspection.

**Selecting Values for the Coupling and Bypass Capacitors** We now address the design issue of selecting appropriate values for $C_{C1}$, $C_S$, and $C_{C2}$. The design objective is to place the lower 3-dB frequency $f_L$ at a specified value while minimizing the capacitor values. Since as mentioned above $C_S$ results in the highest of the three pole frequencies, the total capacitance is minimized by selecting $C_S$ so that its pole frequency $f_{P2} = f_L$. We then decide on the location of the other two pole frequencies, say 5 to 10 times lower than the frequency of the dominant pole, $f_{P2}$. However, the values selected for $f_{P1}$ and $f_{P3}$ should not be too low, for that would require larger values for $C_{C1}$ and $C_{C2}$ than may be necessary. The design procedure will be illustrated by an example.

## Example 9.1

We wish to select appropriate values for the coupling capacitors $C_{C1}$ and $C_{C2}$ and the bypass capacitor $C_S$ for a CS amplifier for which $R_G = 4.7\ \text{M}\Omega$, $R_D = R_L = 15\ \text{k}\Omega$, $R_{sig} = 100\ \text{k}\Omega$, and $g_m = 1$ mA/V. It is required to have $f_L$ at 100 Hz and that the nearest break frequency be at least a decade lower.

### Solution

We select $C_S$ so that

$$f_{P2} = \frac{1}{2\pi(C_S/g_m)} = f_L$$

Thus,

$$C_S = \frac{g_m}{2\pi f_L} = \frac{1 \times 10^{-3}}{2\pi \times 100} = 1.6\ \mu\text{F}$$

For $f_{P1} = f_{P3} = 10$ Hz, we obtain

$$10 = \frac{1}{2\pi C_{C1}(0.1 + 4.7) \times 10^6}$$

**Example 9.1** *continued*

which yields

$$C_{C1} = 3.3 \text{ nF}$$

and

$$10 = \frac{1}{2\pi C_{C2}(15+15)\times 10^3}$$

which results in

$$C_{C2} = 0.53 \ \mu\text{F}$$

**EXERCISE**

**9.1** A CS amplifier has $C_{C1} = C_S = C_{C2} = 1\ \mu$F, $R_G = 10$ MΩ, $R_{sig} = 100$ kΩ, $g_m = 2$ mA/V, $R_D = R_L = 10$ kΩ. Find $A_M$, $f_{P1}$, $f_{P2}$, $f_{P3}$, and $f_L$.
**Ans.** –9.9 V/V; 0.016 Hz; 318.3 Hz; 8 Hz; 318.3 Hz

## 9.1.2 The CE Amplifier

Figure 9.4 shows a common-emitter amplifier that utilizes coupling capacitors $C_{C1}$ and $C_{C2}$ and emitter bypass capacitor $C_E$. As in the case of the MOS amplifier, the effect of these capacitors is felt only at low frequencies. Our objective is to determine the amplifier gain or transfer function $V_o/V_{sig}$ with these three capacitances taken into account. Toward that end, we show in Fig. 9.4(b) the circuit with the dc sources eliminated. We shall perform the small-signal analysis directly on the circuit. To keep the analysis simple, we shall neglect the effect of $r_o$, as we have done in the MOS case.

The analysis of the circuit in Fig. 9.4(b) is somewhat more complicated than that for the CS case. This is a result of the finite $\beta$ of the BJT, which causes the input impedance at the base to be a function of $C_E$. Thus the effects of $C_{C1}$ and $C_E$ are no longer separable. Although one can certainly still derive an expression for the overall transfer function, the result will be quite complicated, making it difficult to obtain design insight. Therefore we shall pursue an approximate alternative approach.

**Considering the Effect of Each of the Three Capacitors Separately** Our first cut at the analysis of the circuit in Fig. 9.4(b) is to consider the effect of the three capacitors $C_{C1}$, $C_E$, and $C_{C2}$ *one at a time*. That is, when finding the effect of $C_{C1}$, we shall assume that $C_E$ and $C_{C2}$ are acting as perfect short circuits, and when considering $C_E$, we assume that $C_{C1}$ and $C_{C2}$ are perfect short circuits, and so on. This is obviously a major simplifying assumption—and one that might not be justified. However, it should serve as a first cut at the analysis, enabling us to gain insight into the effect of these capacitances.

Figure 9.5(a) shows the circuit with $C_E$ and $C_{C2}$ replaced with short circuits. The voltage $V_\pi$ at the base of the transistor can be written as

$$V_\pi = V_{sig}\frac{R_B \| r_\pi}{(R_B \| r_\pi) + R_{sig} + \dfrac{1}{sC_{C1}}}$$

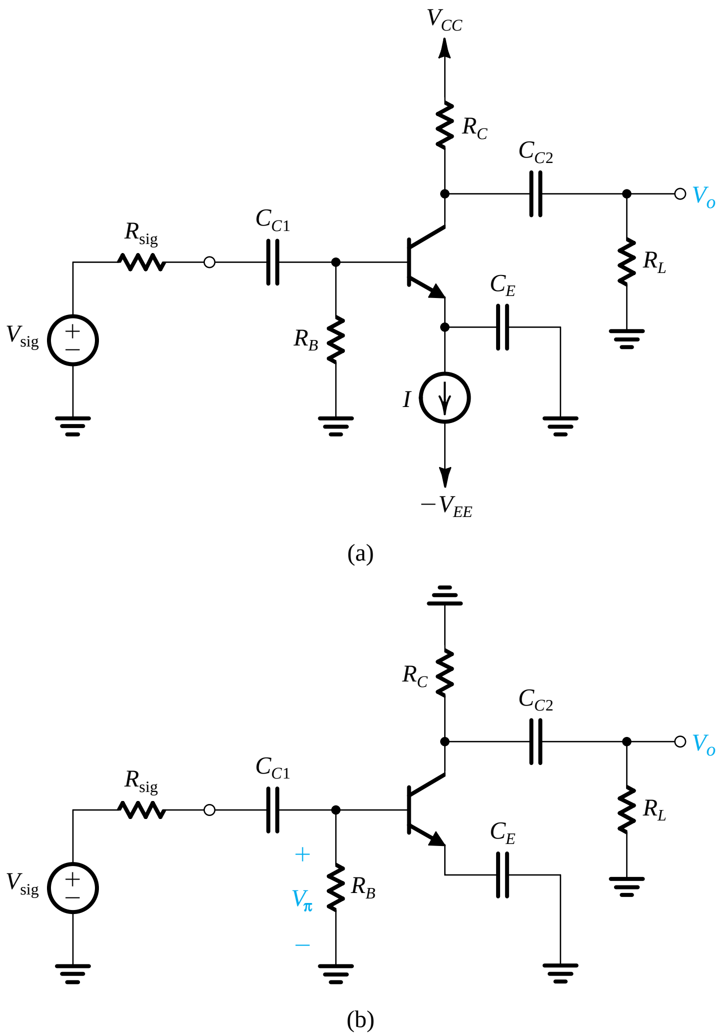



**Figure 9.4** **(a)** A capacitively coupled common-emitter amplifier. **(b)** The circuit prepared for small-signal analysis.

and the output voltage is obtained as

$$V_o = -g_m V_\pi (R_C \| R_L)$$

These two equations can be combined to obtain the voltage gain $V_o/V_{sig}$ including the effect of $C_{C1}$ as

$$\frac{V_o}{V_{sig}} = -\frac{(R_B \| r_\pi)}{(R_B \| r_\pi) + R_{sig}} g_m (R_C \| R_L) \left[ \frac{s}{s + \dfrac{1}{C_{C1}[(R_B \| r_\pi) + R_{sig}]}} \right] \tag{9.10}$$

from which we observe that the effect of $C_{C1}$ is to introduce the frequency-dependent factor between the square brackets on the right-hand side of Eq. (9.10). We recognize this factor as the transfer fraction of a single-time-constant (STC) circuit of the high-pass type (see Section 1.6 and Appendix E) with a corner (or break or pole) frequency $\omega_{P1}$,

$$\omega_{P1} = \frac{1}{C_{C1}[(R_B \| r_\pi) + R_{\text{sig}}]} \tag{9.11}$$

Note that $[(R_B \| r_\pi) + R_{\text{sig}}]$ is the *resistance seen between the terminals of* $C_{C1}$ *when* $V_{\text{sig}}$ *is set to zero*. The STC high-pass factor introduced by $C_{C1}$ will cause the amplifier gain to roll off at low frequencies at the rate of 6 dB/octave (20 dB/decade) with a 3-dB frequency at $f_{P1} = \omega_{P1}/2\pi$, as indicated in Fig. 9.5(a). Also note that we have denoted the midband gain $A_M$,

$$A_M = -\frac{(R_B \| r_\pi)}{(R_B \| r_\pi) + R_{\text{sig}}} g_m (R_C \| R_L) \tag{9.12}$$

Next, we consider the effect of $C_E$. For this purpose we assume that $C_{C1}$ and $C_{C2}$ are acting as perfect short circuits and thus obtain the circuit in Fig. 9.5(b). Reflecting $r_e$ and $C_E$ into the base circuit and utilizing the Thévenin theorem enables us to obtain the base current as

$$I_b = V_{\text{sig}} \frac{R_B}{R_B + R_{\text{sig}}} \frac{1}{(R_B \| R_{\text{sig}}) + (\beta + 1)\left(r_e + \dfrac{1}{sC_E}\right)}$$

The collector current can then be found as $\beta I_b$ and the output voltage as

$$\begin{aligned} V_o &= -\beta I_b (R_C \| R_L) \\ &= -\frac{R_B}{R_B + R_{\text{sig}}} \frac{\beta (R_C \| R_L)}{(R_B \| R_{\text{sig}}) + (\beta + 1)\left(r_e + \dfrac{1}{sC_E}\right)} V_{\text{sig}} \end{aligned}$$

Thus the voltage gain including the effect of $C_E$ can be expressed as[2]

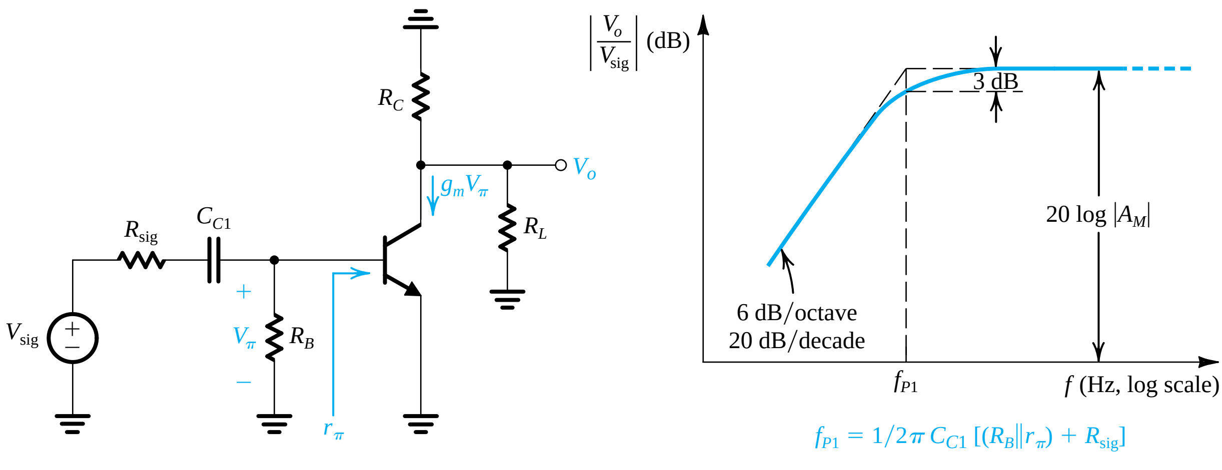
$$\frac{V_o}{V_{\text{sig}}} = -\frac{R_B}{R_B + R_{\text{sig}}} \frac{\beta (R_C \| R_L)}{(R_B \| R_{\text{sig}}) + (\beta + 1) r_e} \frac{s}{s + \left[1/C_E\left(r_e + \dfrac{R_B \| R_{\text{sig}}}{\beta + 1}\right)\right]} \tag{9.13}$$

We observe that $C_E$ introduces the STC high-pass factor on the extreme right-hand side. Thus $C_E$ causes the gain to fall off at low frequency at the rate of 6 dB/octave with a 3-dB frequency equal to the corner (or pole) frequency of the high-pass STC function; that is,

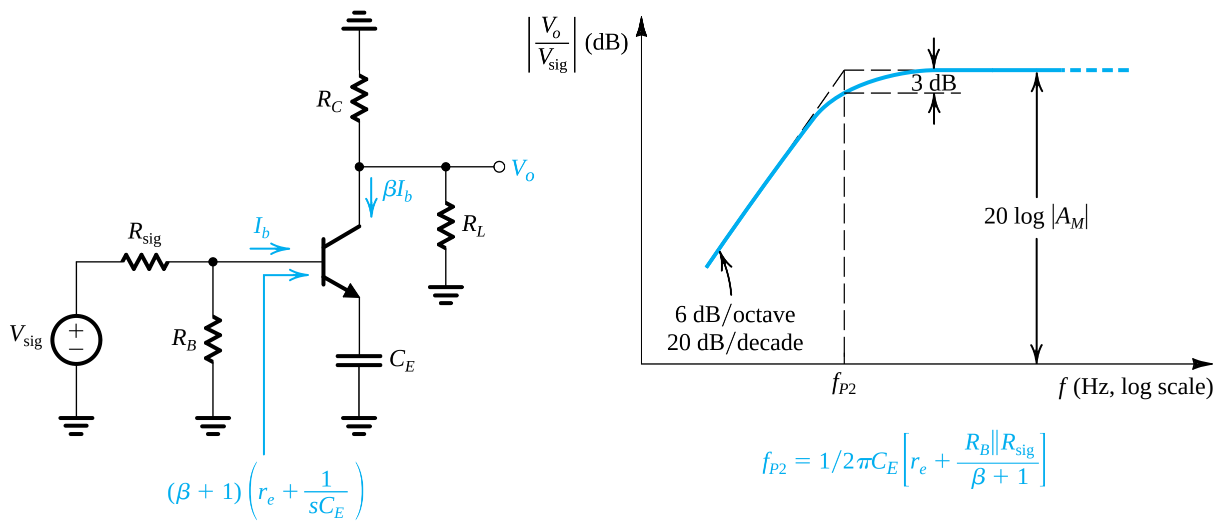
$$\omega_{P2} = \frac{1}{C_E\left[r_e + \dfrac{R_B \| R_{\text{sig}}}{\beta + 1}\right]} \tag{9.14}$$

Observe that $[r_e + ((R_B \| R_{\text{sig}})/(\beta + 1))]$ *is the resistance seen between the two terminals of* $C_E$ *when* $V_{\text{sig}}$ *is set to zero*. The effect of $C_E$ on the amplifier frequency response is illustrated by the sketch in Fig. 9.5(b).

[2] It can be shown that the factor multiplying the high-pass transfer function in Eq. (9.13) is equal to $A_M$ of Eq. (9.12).




(a)




(b)

**Figure 9.5** Analysis of the low-frequency response of the CE amplifier of Fig. 9.4: **(a)** the effect of $C_{C1}$ is determined with $C_E$ and $C_{C2}$ assumed to be acting as perfect short circuits; **(b)** the effect of $C_E$ is determined with $C_{C1}$ and $C_{C2}$ assumed to be acting as perfect short circuits;

Finally, we consider the effect of $C_{C2}$. The circuit with $C_{C1}$ and $C_E$ assumed to be acting as perfect short circuits is shown in Fig. 9.5(c), for which we can write

$$V_\pi = V_{sig}\frac{R_B \| r_\pi}{(R_B \| r_\pi) + R_{sig}}$$

and

$$V_o = -g_m V_\pi \frac{R_C}{R_C + \dfrac{1}{sC_{C2}} + R_L} R_L$$

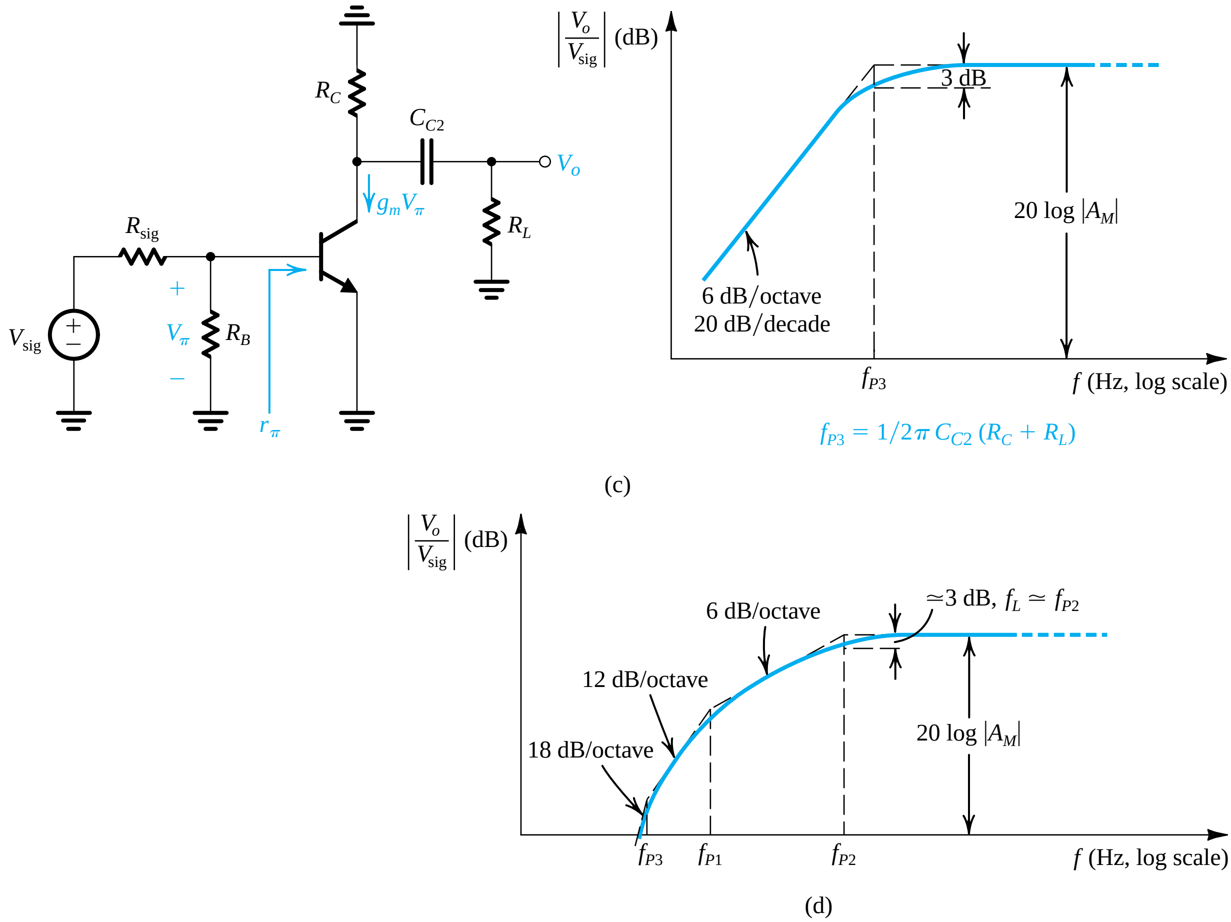



**Figure 9.5** (*continued*) **(c)** the effect of $C_{C2}$ is determined with $C_{C1}$ and $C_E$ assumed to be acting as perfect short circuits; **(d)** sketch of the low-frequency gain under the assumptions that $C_{C1}$, $C_E$, and $C_{C2}$ do not interact and that their break (or pole) frequencies are widely separated.

These two equations can be combined to obtain the low-frequency gain including the effect of $C_{C2}$ as

$$\frac{V_o}{V_{sig}} = -\frac{R_B \| r_\pi}{(R_B \| r_\pi) + R_{sig}} g_m (R_C \| R_L) \left[ \frac{s}{s + \dfrac{1}{C_{C2}(R_C + R_L)}} \right] \tag{9.15}$$

We observe that $C_{C2}$ introduces the frequency-dependent factor between the square brackets, which we recognize as the transfer function of a high-pass STC circuit with a pole frequency $\omega_{P3}$,

$$\omega_{P3} = \frac{1}{C_{C2}(R_C + R_L)} \tag{9.16}$$

Here we note that as expected, $(R_C + R_L)$ *is the resistance seen between the terminals of* $C_{C2}$ *when* $V_{sig}$ *is set to zero*. Thus capacitor $C_{C2}$ causes the low-frequency gain of the amplifier to decrease at the rate of 6 dB/octave with a 3-dB frequency at $f_{P3} = \omega_{P3}/2\pi$, as illustrated by the sketch in Fig. 9.5(c).

**Determining the Lower 3-dB Frequency, $f_L$** Now that we have determined the effects of each of $C_{C1}$, $C_E$, and $C_{C2}$ acting alone, the question becomes what will happen when all three are present at the same time. This question has two parts: First, what happens when all three capacitors are present but do not interact? The answer is that the amplifier low-frequency gain can be expressed as

$$\frac{V_o}{V_{sig}} = -A_M\left(\frac{s}{s+\omega_{P1}}\right)\left(\frac{s}{s+\omega_{P2}}\right)\left(\frac{s}{s+\omega_{P3}}\right) \qquad (9.17)$$

from which we see that it acquires three poles with frequencies $f_{P1}$, $f_{P2}$, and $f_{P3}$, all in the low-frequency band. If the three frequencies are widely separated, their effects will be distinct, as indicated by the sketch in Fig. 9.5(d). The important point to note here is that the 3-dB frequency $f_L$ *is determined by the highest of the three pole frequencies*. This is usually the pole caused by the bypass capacitor $C_E$, simply because the resistance that it sees is usually quite small. Thus, even if one uses a large value for $C_E$, $f_{P2}$ is usually the highest of the three pole frequencies.

If $f_{P1}$, $f_{P2}$, and $f_{P3}$ are close together, none of the three dominates, and to determine $f_L$, we have to evaluate $|V_o/V_{sig}|$ in Eq. (9.17) and calculate the frequency at which it drops to $|A_M|/\sqrt{2}$. The work involved in doing this, however, is usually too great and is rarely justified in practice, particularly because in any case, Eq. (9.17) is an approximation based on the assumption that the three capacitors do not interact. This leads to the second part of the question: What happens when all three capacitors are present and interact? We do know that $C_{C1}$ and $C_E$ usually interact and that their combined effect is two poles at frequencies that will differ somewhat from $\omega_{P1}$ and $\omega_{P2}$. Of course, one can derive the overall transfer function taking this interaction into account and find more precisely the low-frequency response. This, however, will be too complicated to yield additional insight. As an alternative, for hand calculations, we can obtain a reasonably good estimate for $f_L$ using the following formula (which we will not derive here)[3]:

$$f_L \simeq \frac{1}{2\pi}\left[\frac{1}{C_{C1}R_{C1}} + \frac{1}{C_E R_E} + \frac{1}{C_{C2}R_{C2}}\right] \qquad (9.18)$$

or equivalently,

$$f_L = f_{P1} + f_{P2} + f_{P3} \qquad (9.19)$$

where $R_{C1}$, $R_E$, and $R_{C2}$ are the resistances seen by $C_{C1}$, $C_E$, and $C_{C2}$, respectively, when $V_{sig}$ is set to zero and the other two capacitances are replaced with short circuits. Equations (9.18) and (9.19) provide insight regarding the relative contributions of the three capacitors to $f_L$. Finally, we note that a far more precise determination of the low-frequency gain and the 3-dB frequency $f_L$ can be obtained using SPICE.

**Selecting Values for $C_{C1}$, $C_E$, and $C_{C2}$** We now address the design issue of selecting appropriate values for $C_{C1}$, $C_E$, and $C_{C2}$. The design objective is to place the lower 3-dB frequency $f_L$ at a specified location while minimizing the capacitor values. Since, as mentioned above, $C_E$ usually sees the lowest of the three resistances, the total capacitance is minimized

[3] The interested reader can refer to Appendix F.

by selecting $C_E$ so that its contribution to $f_L$ is dominant. That is, by reference to Eq. (9.18), we may select $C_E$ such that $1/(C_E R_E)$ is, say, 80% of $\omega_L = 2\pi f_L$, leaving each of the other capacitors to contribute 10% to the value of $\omega_L$. Example 9.2 should help to illustrate this process.

## Example 9.2

We wish to select appropriate values for $C_{C1}$, $C_{C2}$, and $C_E$ for the common-emitter amplifier, which has $R_B = 100$ kΩ, $R_C = 8$ kΩ, $R_L = 5$ kΩ, $R_{sig} = 5$ kΩ, $\beta = 100$, $g_m = 40$ mA/V, and $r_\pi = 2.5$ kΩ. It is required to have $f_L = 100$ Hz.

### Solution

We first determine the resistances seen by the three capacitors $C_{C1}$, $C_E$, and $C_{C2}$ as follows:

$$R_{C1} = (R_B \| r_\pi) + R_{sig}$$

$$= (100 \| 2.5) + 5 = 7.44 \text{ k}\Omega$$

$$R_E = r_e + \frac{R_B \| R_{sig}}{\beta + 1}$$

$$= 0.025 + \frac{100 \| 5}{101} = 0.072 \text{ k}\Omega = 72 \ \Omega$$

$$R_{C2} = R_C + R_L = 8 + 5 = 13 \text{ k}\Omega$$

Now, selecting $C_E$ so that it contributes 80% of the value of $\omega_L$ gives

$$\frac{1}{C_E \times 72} = 0.8 \times 2\pi \times 100$$

$$C_E = 27.6 \ \mu\text{F}$$

Next, if $C_{C1}$ is to contribute 10% of $f_L$,

$$\frac{1}{C_{C1} \times 7.44 \times 10^3} = 0.1 \times 2\pi \times 100$$

$$C_{C1} = 2.1 \ \mu\text{F}$$

Similarly, if $C_{C2}$ is to contribute 10% of $f_L$, its value should be selected as follows:

$$\frac{1}{C_{C2} \times 13 \times 10^3} = 0.1 \times 2\pi \times 100$$

$$C_{C2} = 1.2 \ \mu\text{F}$$

In practice, we would select the nearest standard values for the three capacitors while ensuring that $f_L \leq 100$ Hz.

**EXERCISE**

**9.2** A common-emitter amplifier has $C_{C1} = C_E = C_{C2} = 1\ \mu\text{F}$, $R_B = 100\ \text{k}\Omega$, $R_{\text{sig}} = 5\ \text{k}\Omega$, $g_m = 40\ \text{mA/V}$, $r_\pi = 2.5\ \text{k}\Omega$, $R_C = 8\ \text{k}\Omega$, and $R_L = 5\ \text{k}\Omega$. Assuming that the three capacitors do not interact, find $f_{P1}$, $f_{P2}$, and $f_{P3}$, and hence estimate $f_L$.
**Ans.** 21.4 Hz; 2.21 kHz; 12.2 Hz; since $f_{P2} \gg f_{P1}$ and $f_{P3}$, $f_L \simeq f_{P2} = 2.21$ kHz; using Eq. (9.19), a somewhat better estimate for $f_L$ is obtained: 2.24 kHz

# 9.2 Internal Capacitive Effects and the High-Frequency Model of the MOSFET and the BJT

While coupling and bypass capacitors cause the gain of transistor amplifiers to fall off at the low-frequency end, the gain falloff at high frequencies is caused by the capacitive effects internal to the transistors. In this section we shall briefly consider these effects and, more importantly, show how the device small-signal model can be augmented to take these effects into account.

## 9.2.1 The MOSFET

From our study of the physical operation of the MOSFET in Section 5.1, we know that the device has internal capacitances. In fact, we used one of these, the gate-to-channel capacitance, in our derivation of the MOSFET $i$–$v$ characteristics. We did, however, implicitly assume that the steady-state charges on these capacitances are acquired instantaneously. In other words, we did not account for the finite time required to charge and discharge the various internal capacitances. As a result, the device models we derived, such as the small-signal model, do not include any capacitances. The use of these models would predict constant amplifier gains independent of frequency. We know, however, that this (unfortunately) does not happen; in fact, the gain of every MOSFET amplifier falls off at some high frequency. Similarly, the MOSFET digital logic inverter (Chapter 13) exhibits a finite nonzero propagation delay. To be able to predict these results, the MOSFET model must be augmented by including internal capacitances. This is the subject of this section.

To visualize the physical origin of the various internal capacitances, the reader is referred to Fig. 5.1. There are basically two types of internal capacitance in the MOSFET.

1. *The gate capacitive effect:* The gate electrode (polysilicon) forms a parallel-plate capacitor with the channel, with the oxide layer serving as the capacitor dielectric. We discussed the gate (or oxide) capacitance in Section 5.1 and denoted its value per unit area as $C_{ox}$.
2. *The source-body and drain-body depletion-layer capacitances:* These are the capacitances of the reverse-biased *pn* junctions formed by the $n^+$ source region (also called the **source diffusion**) and the *p*-type substrate and by the $n^+$ drain region (the **drain diffusion**) and the substrate. Evaluation of these capacitances will utilize the material studied in Chapter 3.

These two capacitive effects can be modeled by including capacitances in the MOSFET model between its four terminals, G, D, S, and B. There will be five capacitances in total:

$C_{gs}$, $C_{gd}$, $C_{gb}$, $C_{sb}$, and $C_{db}$, where the subscripts indicate the location of the capacitances in the model. In the following, we show how the values of the five model capacitances can be determined. We will do so by considering each of the two capacitive effects separately.

### The Gate Capacitive Effect

The gate capacitive effect can be modeled by the three capacitances $C_{gs}$, $C_{gd}$, and $C_{gb}$. The values of these capacitances can be determined as follows:

1. When the MOSFET is operating in the triode region at small $v_{DS}$, the channel will be of uniform depth. The gate-channel capacitance will be $WL\,C_{ox}$ and can be modeled by dividing it equally between the source and drain ends; thus,

$$C_{gs} = C_{gd} = \tfrac{1}{2}WL\,C_{ox} \quad \text{(triode region)} \tag{9.20}$$

   This is obviously an approximation (as all modeling is), but it works well for triode-region operation even when $v_{DS}$ is not small.

2. When the MOSFET operates in saturation, the channel has a tapered shape and is pinched off at or near the drain end. It can be shown that the gate-to-channel capacitance in this case is approximately $\tfrac{2}{3}WL\,C_{ox}$ and can be modeled by assigning this entire amount to $C_{gs}$, and a zero amount to $C_{gd}$ (because the channel is pinched off at the drain); thus,

$$\left.\begin{aligned} C_{gs} &= \tfrac{2}{3}WL\,C_{ox} \quad (9.21)\\ C_{gd} &= 0 \quad (9.22)\end{aligned}\right\} \quad \text{(saturation region)}$$

3. When the MOSFET is cut off, the channel disappears, and thus $C_{gs} = C_{gd} = 0$. However, we can (after some rather complex reasoning) model the gate capacitive effect by assigning a capacitance $WL\,C_{ox}$ to the gate-body model capacitance; thus,

$$\left.\begin{aligned} C_{gs} &= C_{gd} = 0 \quad (9.23)\\ C_{gb} &= WL\,C_{ox} \quad (9.24)\end{aligned}\right\} \quad \text{(cutoff)}$$

4. There is an additional small capacitive component that should be added to $C_{gs}$ and $C_{gd}$ in all the preceding formulas. This is the capacitance that results from the fact that the source and drain diffusions extend slightly under the gate oxide (refer to Fig. 5.1). If the *overlap* length is denoted $L_{ov}$, we see that the **overlap capacitance** component is

$$C_{ov} = WL_{ov}\,C_{ox} \tag{9.25}$$

   Typically, $L_{ov} = 0.05$ to $0.1\,L$.

**The Junction Capacitances** The depletion-layer capacitances of the two reverse-biased *pn* junctions formed between each of the source and the drain diffusions and the body can be determined using the formula developed in Section 3.6 (Eq. 3.47). Thus, for the source diffusion, we have the source-body capacitance, $C_{sb}$,

$$C_{sb} = \frac{C_{sb0}}{\sqrt{1 + \dfrac{V_{SB}}{V_0}}} \tag{9.26}$$

where $C_{sb0}$ is the value of $C_{sb}$ at zero body-source bias, $V_{SB}$ is the magnitude of the reverse-bias voltage, and $V_0$ is the junction built-in voltage (0.6 V to 0.8 V). Similarly, for the drain diffusion, we have the drain-body capacitance $C_{db}$,

$$C_{db} = \frac{C_{db0}}{\sqrt{1 + \frac{V_{DB}}{V_0}}} \tag{9.27}$$

where $C_{db0}$ is the capacitance value at zero reverse-bias voltage, and $V_{DB}$ is the magnitude of this reverse-bias voltage. Note that we have assumed that for both junctions, the grading coefficient $m = \frac{1}{2}$.

It should be noted also that each of these junction capacitances includes a component arising from the bottom side of the diffusion and a component arising from the *side walls* of the diffusion. In this regard, observe that each diffusion has three side walls that are in contact with the substrate and thus contribute to the junction capacitance (the fourth wall is in contact with the channel). In more advanced MOSFET modeling, the two components of each of the junction capacitances are calculated separately.

The formulas for the junction capacitances in Eqs. (9.26) and (9.27) assume small-signal operation. These formulas, however, can be modified to obtain approximate average values for the capacitances when the transistor is operating under large-signal conditions such as in logic circuits. Finally, typical values for the various capacitances exhibited by an *n*-channel MOSFET in a 0.5-µm CMOS process are given in the following exercise.

## EXERCISE

**9.3** For an *n*-channel MOSFET with $t_{ox} = 10$ nm, $L = 1.0$ µm, $W = 10$ µm, $L_{ov} = 0.05$ µm, $C_{sb0} = C_{db0} = 10$ fF, $V_0 = 0.6$ V, $V_{SB} = 1$ V, and $V_{DS} = 2$ V, calculate the following capacitances when the transistor is operating in saturation: $C_{ox}$, $C_{ov}$, $C_{gs}$, $C_{gd}$, $C_{sb}$, and $C_{db}$.
**Ans.** 3.45 fF/µm$^2$; 1.72 fF; 24.7 fF; 1.72 fF; 6.1 fF; 4.1 fF

**The High-Frequency MOSFET Model** Figure 9.6(a) shows the small-signal model of the MOSFET, including the four capacitances $C_{gs}$, $C_{gd}$, $C_{sb}$, and $C_{db}$. This model can be used to predict the high-frequency response of MOSFET amplifiers. It is, however, quite complex for manual analysis, and its use is limited to computer simulation using, for example, SPICE. Fortunately, when the source is connected to the body, the model simplifies considerably, as shown in Fig. 9.6(b). In this model, $C_{gd}$, although small, plays a significant role in determining the high-frequency response of amplifiers and thus must be kept in the model. Capacitance $C_{db}$, on the other hand, can usually be neglected, resulting in significant simplification of manual analysis. The resulting circuit is shown in Fig. 9.6(c).

**The MOSFET Unity-Gain Frequency ($f_T$)** A figure of merit for the high-frequency operation of the MOSFET as an amplifier is the unity-gain frequency, $f_T$, also known as the **transition frequency**, which gives rise to the subscript *T*. This is defined as *the frequency at which the short-circuit current-gain of the common-source configuration becomes unity*. Figure 9.7

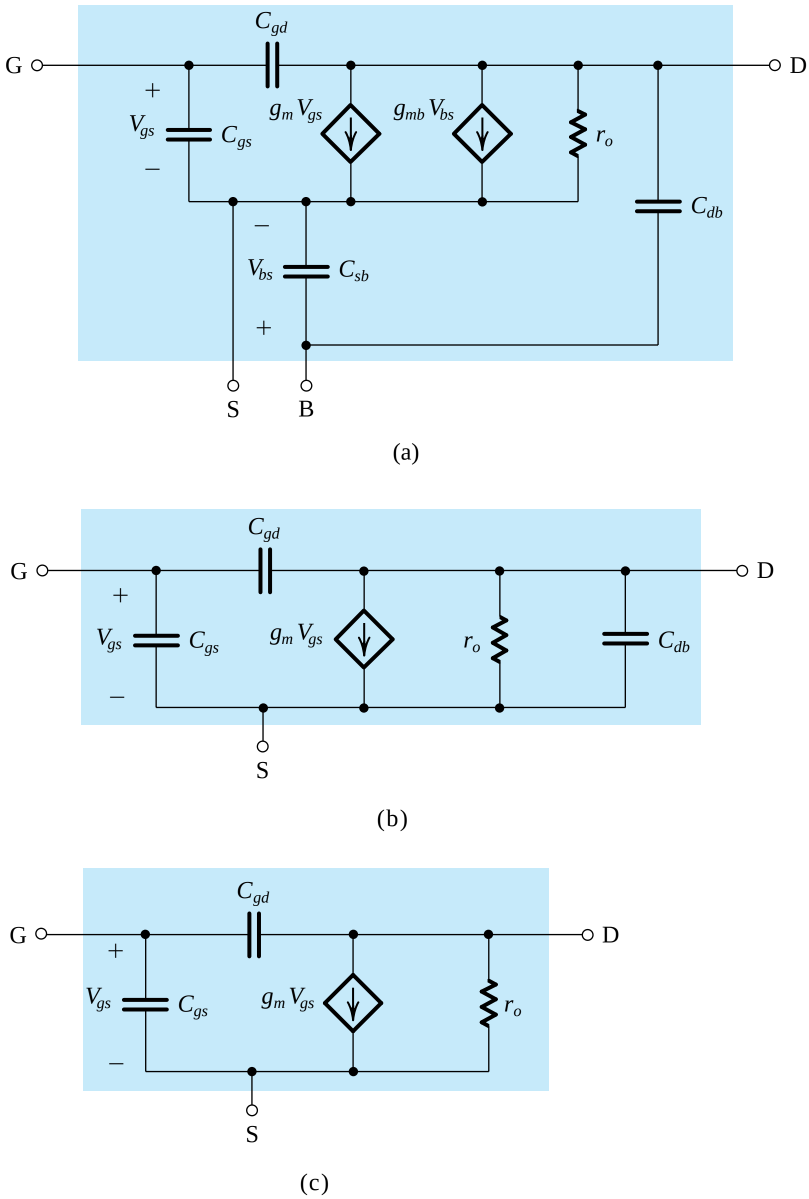



**Figure 9.6** **(a)** High-frequency, equivalent-circuit model for the MOSFET. **(b)** The equivalent circuit for the case in which the source is connected to the substrate (body). **(c)** The equivalent-circuit model of **(b)** with $C_{db}$ neglected (to simplify analysis).

shows the MOSFET hybrid-$\pi$ model with the source as the common terminal between the input and output ports. To determine the short-circuit current gain, the input is fed with a current-source signal $I_i$ and the output terminals are short-circuited. It can be seen that the current in the short circuit is given by

$$I_o = g_m V_{gs} - sC_{gd} V_{gs}$$

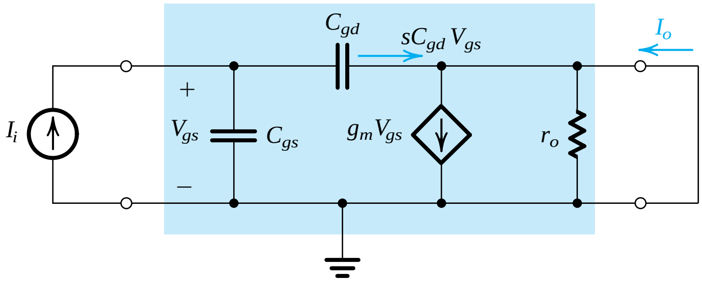



**Figure 9.7** Determining the short-circuit current gain $I_o/I_i$.

Recalling that $C_{gd}$ is small, at the frequencies of interest the second term in this equation can be neglected,

$$I_o \simeq g_m V_{gs} \tag{9.28}$$

From Fig. 9.7, we can express $V_{gs}$ in terms of the input current $I_i$ as

$$V_{gs} = I_i/s(C_{gs}+C_{gd}) \tag{9.29}$$

Equations (9.28) and (9.29) can be combined to obtain the short-circuit current gain,

$$\frac{I_o}{I_i} = \frac{g_m}{s(C_{gs}+C_{gd})} \tag{9.30}$$

For physical frequencies $s = j\omega$, it can be seen that the magnitude of the current gain becomes unity at the frequency

$$\omega_T = g_m/(C_{gs}+C_{gd})$$

Thus the unity-gain frequency $f_T = \omega_T/2\pi$ is

$$f_T = \frac{g_m}{2\pi(C_{gs}+C_{gd})} \tag{9.31}$$

Since $f_T$ is proportional to $g_m$ and inversely proportional to the MOSFET internal capacitances, the higher the value of $f_T$, the more effective the MOSFET becomes as an amplifier. Substituting for $g_m$ using Eq. (5.56), we can express $f_T$ in terms of the bias current $I_D$ (see Problem 9.18). Alternatively, we can substitute for $g_m$ from Eq. (5.55) to express $f_T$ in terms of the overdrive voltage $V_{OV}$ (see Problem 9.19). Both expressions yield additional insight into the high-frequency operation of the MOSFET. The reader is also referred to Chapter 7, Appendix 7.A for a further discussion of $f_T$.

Typically, $f_T$ ranges from about 100 MHz for the older technologies (e.g., a 5-µm CMOS process) to many GHz for newer high-speed technologies (e.g., a 0.13-µm CMOS process).

## EXERCISE

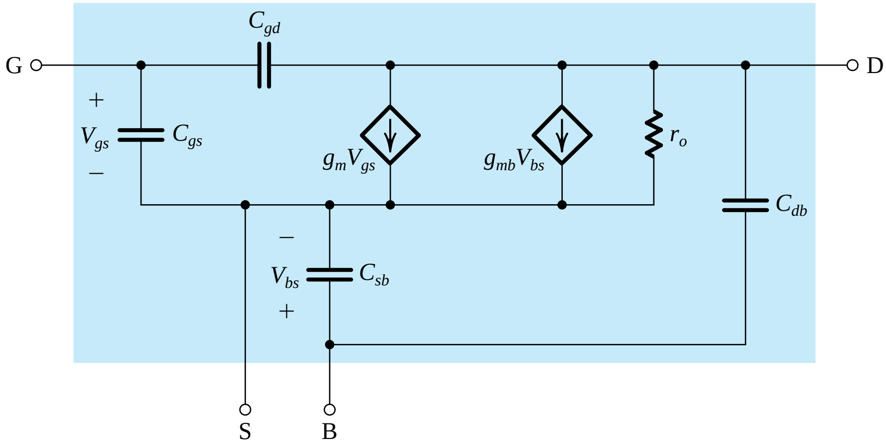
**9.4** Calculate $f_T$ for the $n$-channel MOSFET whose capacitances were found in Exercise 9.3. Assume operation at 100 µA, and that $k_n' = 160\ \mu A/V^2$.
**Ans.** 3.7 GHz.

## Summary

We conclude this section by presenting a summary in Table 9.1.

**Table 9.1** The MOSFET High-Frequency Model

**Model**




**Model Parameters**

$$g_m = \mu_n C_{ox}\frac{W}{L}|V_{OV}| = \sqrt{2\mu_n C_{ox}\frac{W}{L}I_D} = \frac{2I_D}{|V_{OV}|}$$

$$g_{mb} = \chi g_m, \quad \chi = 0.1 \text{ to } 0.2$$

$$r_o = |V_A|/I_D$$

$$C_{gs} = \tfrac{2}{3}WLC_{ox} + WL_{ov}C_{ox}$$

$$C_{gd} = WL_{ov}C_{ox}$$

$$C_{sb} = \frac{C_{sb0}}{\sqrt{1+\dfrac{|V_{SB}|}{V_0}}}$$

$$C_{db} = \frac{C_{db0}}{\sqrt{1+\dfrac{|V_{DB}|}{V_0}}}$$

$$f_T = \frac{g_m}{2\pi(C_{gs}+C_{gd})}$$

## 9.2.2 The BJT

In our study of the physical operation of the BJT in Section 6.1, we assumed transistor action to be instantaneous, and as a result the transistor models we developed do not include any elements (i.e., capacitors or inductors) that would cause time or frequency dependence. Actual transistors, however, exhibit charge-storage phenomena that limit the speed and frequency of their operation. We have already encountered such effects in our study of the *pn* junction in Chapter 3, and learned that they can be modeled using capacitances. In the following we study the charge-storage effects that take place in the BJT and take them into account by adding capacitances to the hybrid-$\pi$ model. The resulting augmented BJT model will be able to predict the observed dependence of amplifier gain on frequency, and the time delays that transistor switches and logic gates exhibit.

**The Base-Charging or Diffusion Capacitance $C_{de}$** When the transistor is operating in the active mode, minority carrier charge is stored in the base region. For an *npn* transistor, the stored electron charge in the base, $Q_n$, can be expressed in terms of the collector current $i_C$ as

$$Q_n = \tau_F i_C \tag{9.32}$$

where $\tau_F$ is a device constant with the dimension of time. It is known as the **forward base-transit time** and represents the average time a charge carrier (electron) spends in crossing the base. Typically, $\tau_F$ is in the range of 10 ps to 100 ps.

Equation (9.32) applies for large signals and, since $i_C$ is exponentially related to $v_{BE}$, $Q_n$ will similarly depend on $v_{BE}$. Thus this charge-storage mechanism represents a nonlinear capacitive effect. However, for small signals we can define the **small-signal diffusion capacitance $C_{de}$**,

$$C_{de} \equiv \frac{dQ_n}{dv_{BE}} \tag{9.33}$$

$$= \tau_F \frac{di_C}{dv_{BE}}$$

resulting in

$$C_{de} = \tau_F g_m = \tau_F \frac{I_C}{V_T} \tag{9.34}$$

Thus, whenever $v_{BE}$ changes by $v_{be}$, the collector current changes by $g_m v_{be}$ and the charge stored in the base changes by $C_{de}\, v_{be} = (\tau_F\, g_m)\, v_{be}$.

**The Base–Emitter Junction Capacitance $C_{je}$** A change in $v_{BE}$ not only changes the charge stored in the base region but also the charge stored in the base–emitter depletion layer. This distinct charge-storage effect is represented by the EBJ depletion-layer capacitance, $C_{je}$. From the development in Chapter 3, we know that for a forward-biased junction, which the EBJ is, the depletion-layer capacitance is given approximately by

$$C_{je} \simeq 2C_{je0} \tag{9.35}$$

where $C_{je0}$ is the value of $C_{je}$ at zero EBJ voltage.

**The Collector–Base Junction Capacitance $C_\mu$** In active-mode operation, the CBJ is reverse biased, and its junction or **depletion capacitance**, usually denoted $C_\mu$, can be found from

$$C_\mu = \frac{C_{\mu 0}}{\left(1 + \dfrac{V_{CB}}{V_{0c}}\right)^m} \tag{9.36}$$

where $C_{\mu 0}$ is the value of $C_\mu$ at zero voltage; $V_{CB}$ is the magnitude of the CBJ reverse-bias voltage, $V_{0c}$ is the CBJ built-in voltage (typically, 0.75 V), and $m$ is its grading coefficient (typically, 0.2–0.5).

**The High-Frequency Hybrid-$\pi$ Model** Figure 9.8 shows the hybrid-$\pi$ model of the BJT, including capacitive effects. Specifically, there are two capacitances: the emitter–base capacitance $C_\pi = C_{de} + C_{je}$ and the collector–base capacitance $C_\mu$. Typically, $C_\pi$ is in the

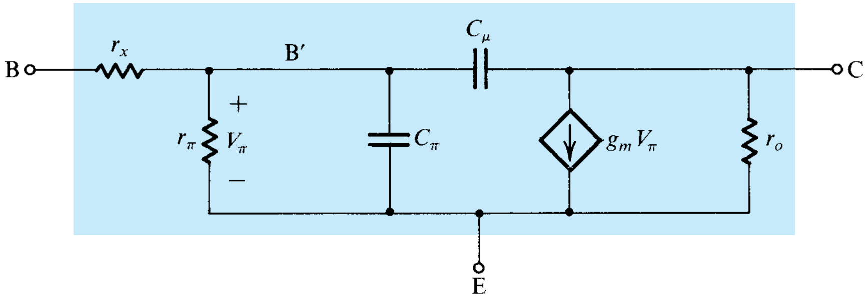



**Figure 9.8** The high-frequency hybrid-$\pi$ model.

range of a few picofarads to a few tens of picofarads, and $C_\mu$ is in the range of a fraction of a picofarad to a few picofarads.[4] Note that we have also added a resistor $r_x$ to model the resistance of the silicon material of the base region between the base terminal B and a fictitious internal, or intrinsic, base terminal B′ that is right under the emitter region (refer to Fig. 6.6). Typically, $r_x$ is a few tens of ohms, and its value depends on the current level in a rather complicated manner. Since (usually) $r_x \ll r_\pi$, its effect is negligible at low frequencies. Its presence is felt, however, at high frequencies, as will become apparent later.

The values of the hybrid-$\pi$, equivalent-circuit parameters can be determined at a given bias point using the formulas presented in this section and in Chapter 6. They can also be found from the terminal measurements specified on the BJT data sheets. For computer simulation, SPICE uses the parameters of the given IC technology to evaluate the BJT model parameters (see Appendix B).

**The Cutoff Frequency** The transistor data sheets do not usually specify the value of $C_\pi$. Rather, the behavior of $\beta$ (or $h_{fe}$) versus frequency is normally given. In order to determine $C_\pi$ and $C_\mu$, we shall derive an expression for $h_{fe}$, the CE short-circuit current gain, as a function of frequency in terms of the hybrid-$\pi$ components. For this purpose consider the circuit shown in Fig. 9.9, in which the collector is shorted to the emitter. A node equation at C provides the short-circuit collector current $I_c$ as

$$I_c = (g_m - sC_\mu)V_\pi \tag{9.37}$$

A relationship between $V_\pi$ and $I_b$ can be established by multiplying $I_b$ by the impedance seen between B′ and E:

$$V_\pi = I_b(r_\pi \,\|\, C_\pi \| \, C_\mu) = \frac{I_b}{1/r_\pi + sC_\pi + sC_\mu} \tag{9.38}$$

Thus $h_{fe}$ can be obtained by combining Eqs. (9.37) and (9.38):

$$h_{fe} \equiv \frac{I_c}{I_b} = \frac{g_m - sC_\mu}{1/r_\pi + s(C_\pi + C_\mu)}$$

[4] These values apply for discrete devices and devices fabricated with a relatively old IC process technology (the so-called high-voltage process, see Appendix 7.A). For modern IC fabrication processes, $C_\pi$ and $C_\mu$ are in the range of tens of femtofarads (fF).

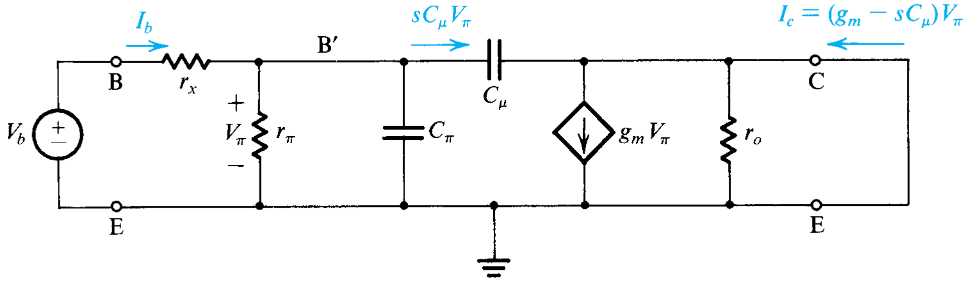



**Figure 9.9** Circuit for deriving an expression for $h_{fe}(s) \equiv I_c/I_b$.

At the frequencies for which this model is valid, $\omega C_\mu \ll g_m$; thus we can neglect the $sC_\mu$ term in the numerator and write

$$h_{fe} \simeq \frac{g_m r_\pi}{1 + s(C_\pi + C_\mu)r_\pi}$$

Thus,

$$h_{fe} = \frac{\beta_0}{1 + s(C_\pi + C_\mu)r_\pi} \tag{9.39}$$

where $\beta_0$ is the low-frequency value of $\beta$. Thus $h_{fe}$ has a single-pole (or STC) response with a 3-dB frequency at $\omega = \omega_\beta$, where

$$\omega_\beta = \frac{1}{(C_\pi + C_\mu)r_\pi} \tag{9.40}$$

Figure 9.10 shows a Bode plot for $|h_{fe}|$. From the –6-dB/octave slope, it follows that the frequency at which $|h_{fe}|$ drops to unity, which is called the **unity-gain bandwidth** $\omega_T$, is given by

$$\omega_T = \beta_0 \omega_\beta \tag{9.41}$$

Thus,

$$\omega_T = \frac{g_m}{C_\pi + C_\mu} \tag{9.42}$$

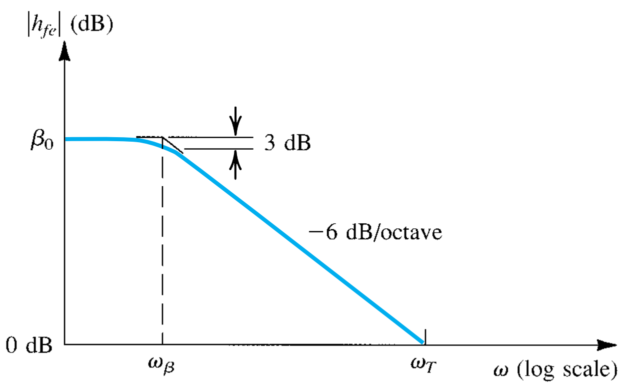



**Figure 9.10** Bode plot for $|h_{fe}|$.

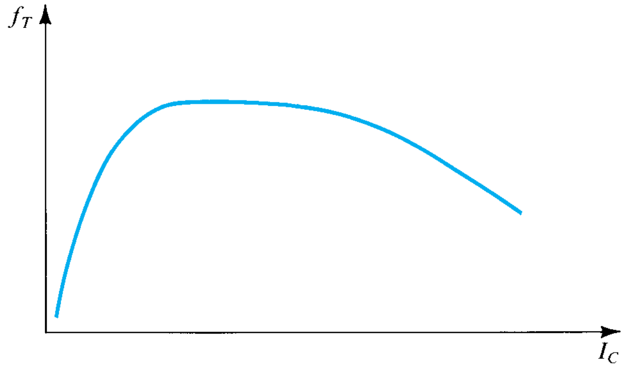



**Figure 9.11** Variation of $f_T$ with $I_C$.

and

$$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)} \tag{9.43}$$

This expression is very similar to that of $f_T$ for the MOSFET (Eq. 9.31) with $C_\pi$ replacing $C_{gs}$ and $C_\mu$ replacing $C_{gd}$.

The unity-gain bandwidth $f_T$, also known as the **transition frequency**, which gives rise to the subscript $T$, is usually specified on the data sheets of a transistor. In some cases $f_T$ is given as a function of $I_C$ and $V_{CE}$. To see how $f_T$ changes with $I_C$, recall that $g_m$ is directly proportional to $I_C$, but only part of $C_\pi$ (the diffusion capacitance $C_{de}$) is directly proportional to $I_C$. It follows that $f_T$ decreases at low currents, as shown in Fig. 9.11. However, the decrease in $f_T$ at high currents, also shown in Fig. 9.11, cannot be explained by this argument; rather, it is due to the same phenomenon that causes $\beta_0$ to decrease at high currents (Section 6.9.2). In the region where $f_T$ is almost constant, $C_\pi$ is dominated by the diffusion part.

Typically, $f_T$ is in the range of 100 MHz to tens of gigahertz. The value of $f_T$ can be used in Eq. (9.43) to determine $C_\pi + C_\mu$. The capacitance $C_\mu$ is usually determined separately by measuring the capacitance between base and collector at the desired reverse-bias voltage $V_{CB}$.

Before leaving this section, we should mention that the hybrid-$\pi$ model of Fig. 9.8 characterizes transistor operation fairly accurately up to a frequency of about $0.2\,f_T$. At higher frequencies one has to add other parasitic elements to the model as well as refine the model to account for the fact that the transistor is in fact a distributed-parameter network that we are trying to model with a lumped-component circuit. One such refinement consists of splitting $r_x$ into a number of parts and replacing $C_\mu$ by a number of capacitors, each connected between the collector and one of the taps of $r_x$. This topic is beyond the scope of this book.

An important observation to make from the high-frequency model of Fig. 9.8 is that at frequencies above 5 to 10 $f_\beta$, one may ignore the resistance $r_\pi$. It can be seen then that $r_x$ becomes the only resistive part of the input impedance at high frequencies. Thus $r_x$ plays an important role in determining the frequency response of transistor circuits at high frequencies. It follows that an accurate determination of $r_x$ can be made only from a high-frequency measurement.

## EXERCISES

**9.5** Find $C_{de}$, $C_{je}$, $C_\pi$, $C_\mu$, and $f_T$ for a BJT operating at a dc collector current $I_C = 1$ mA and a CBJ reverse bias of 2 V. The device has $\tau_F = 20$ ps, $C_{je0} = 20$ fF, $C_{\mu 0} = 20$ fF, $V_{0e} = 0.9$ V, $V_{0c} = 0.5$ V, and $m_{CBJ} = 0.33$.
**Ans.** 0.8 pF; 40 fF; 0.84 pF; 12 fF; 7.47 GHz

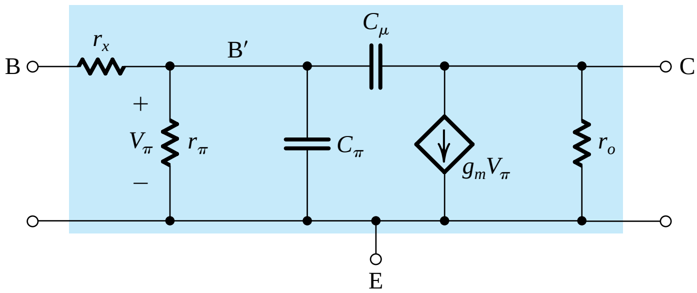
**9.6** For a BJT operated at $I_C = 1$ mA, determine $f_T$ and $C_\pi$ if $C_\mu = 2$ pF and $|h_{fe}| = 10$ at 50 MHz.
**Ans.** 500 MHz; 10.7 pF

**9.7** If $C_\pi$ of the BJT in Exercise 9.6 includes a relatively constant depletion-layer capacitance of 2 pF, find $f_T$ of the BJT when operated at $I_C = 0.1$ mA.
**Ans.** 130.7 MHz

### Summary

For convenient reference, Table 9.2 provides a summary of the relationships used to determine the values of the parameters of the BJT high-frequency model.

**Table 9.2** The BJT High-Frequency Model




$g_m = I_C/V_T$ $\qquad r_o = |V_A|/I_C$ $\qquad r_\pi = \beta_0/g_m$

$C_\pi + C_\mu = \dfrac{g_m}{2\pi f_T}$ $\qquad C_\pi = C_{de} + C_{je}$ $\qquad C_{de} = \tau_F g_m$ $\qquad C_{je} \simeq 2C_{je0}$

$C_\mu = C_{jc0}\Big/\left(1 + \dfrac{|V_{CB}|}{V_{0c}}\right)^m$, $\qquad m = 0.3–0.5$

# 9.3 High-Frequency Response of the CS and CE Amplifiers

Equipped with equivalent-circuit models that represent the high-frequency operation of the MOSFET and the BJT, we now address the question of the high-frequency performance of the CS and CE amplifiers. Our objective is to identify the mechanism that limits the high-frequency performance of these important amplifier configurations. As well, we need to find a simple approach to estimate the frequency $f_H$ at which the gain falls by 3 dB below its value at midband frequencies, $|A_M|$.

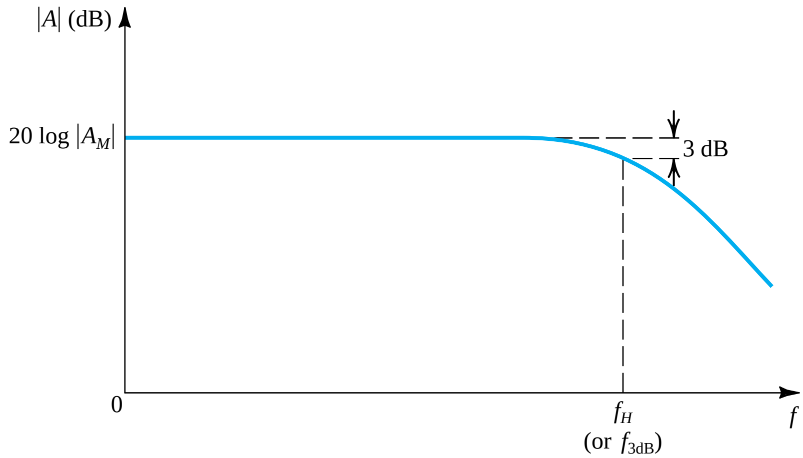



**Figure 9.12** Frequency response of a direct-coupled (dc) amplifier. Observe that the gain does *not* fall off at low frequencies, and the midband gain $A_M$ extends down to zero frequency.

The analysis presented here applies equally well to discrete-circuit, capacitively coupled amplifiers and to IC amplifiers. The frequency response of the first was shown in Figs. 5.61 and 6.69 and that of the latter is shown in Fig. 9.12. At the frequencies of interest to us here (the high-frequency band), all coupling and bypass capacitors behave as perfect short circuits and amplifiers of both types have identical high-frequency equivalent circuits.

### 9.3.1 The Common-Source Amplifier

Figure 9.13(a) shows the high-frequency, equivalent-circuit model of a CS amplifier. It is obtained by replacing the MOSFET in an amplifier circuit such as that in Fig. 9.2 by its high-frequency, equivalent-circuit model of Fig. 9.6(c), while as always eliminating dc sources. Observe that the circuit in Fig. 9.13(a) is general; for instance, it includes a resistance $R_G$, which arises only in the case of a discrete-circuit amplifier. Also, $R_D$ can be either a passive resistance or the output resistance of a current-source load, and similarly for $R_L$.

The equivalent circuit of Fig. 9.13(a) can be simplified by utilizing Thévenin theorem at the input side and by combining the three parallel resistances at the output side. The resulting simplified circuit is shown in Fig. 9.13(b). The midband gain $A_M$ can be found from this circuit by setting $C_{gs}$ and $C_{gd}$ to zero. The result is

$$A_M = \frac{V_o}{V_{sig}} = -\frac{R_G}{R_G + R_{sig}}(g_m R_L') \tag{9.44}$$

The equivalent circuit in Fig. 9.13(b) can be further simplified if we can find a way to deal with the bridging capacitor $C_{gd}$ that connects the output node to the input side. Toward that end, consider first the output node. It can be seen that the load current is $(g_m V_{gs} - I_{gd})$, where $(g_m V_{gs})$ is the output current of the transistor and $I_{gd}$ is the current supplied through the very small capacitance $C_{gd}$. At frequencies in the vicinity of $f_H$, which defines the edge of the midband, it is reasonable to assume that $I_{gd}$ is still much smaller than $(g_m V_{gs})$, with the result that $V_o$ can be given approximately by

$$V_o \simeq -(g_m V_{gs})R_L' = -g_m R_L' V_{gs} \tag{9.45}$$

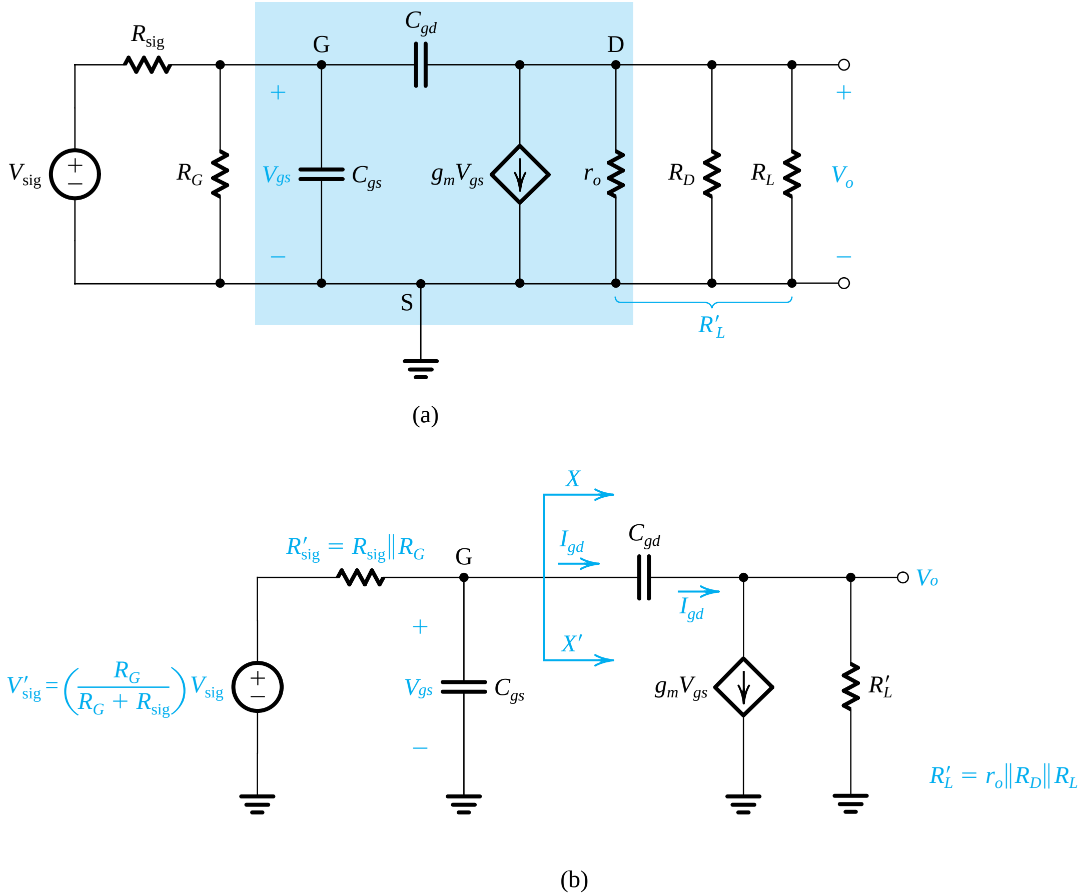



**Figure 9.13** Determining the high-frequency response of the CS amplifier: **(a)** equivalent circuit; **(b)** the circuit of **(a)** simplified at the input and the output; (*Continued*)

where

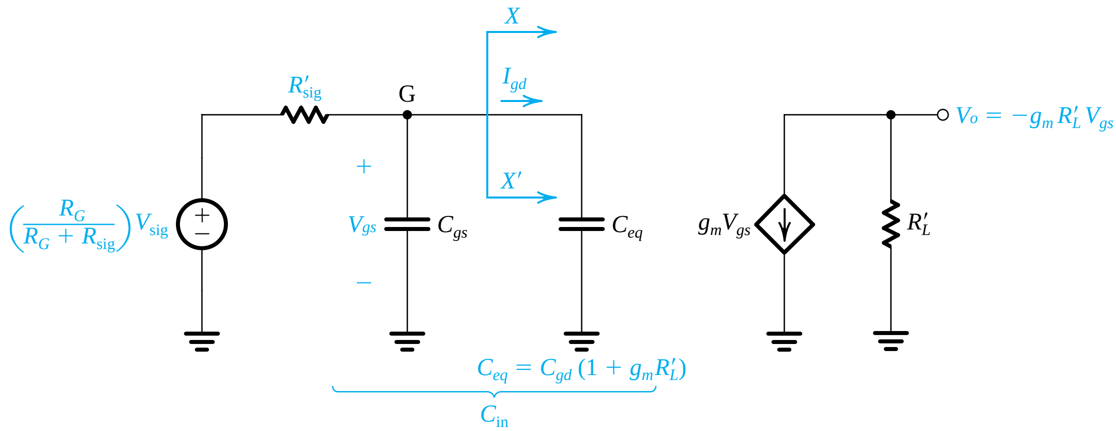
$$R'_L = r_o \,\|\, R_D \,\|\, R_L$$

Since $V_o = V_{ds}$, Eq. (9.45) indicates that the gain from gate to drain is $-g_m R'_L$, the same value as in the midband. The current $I_{gd}$ can now be found as

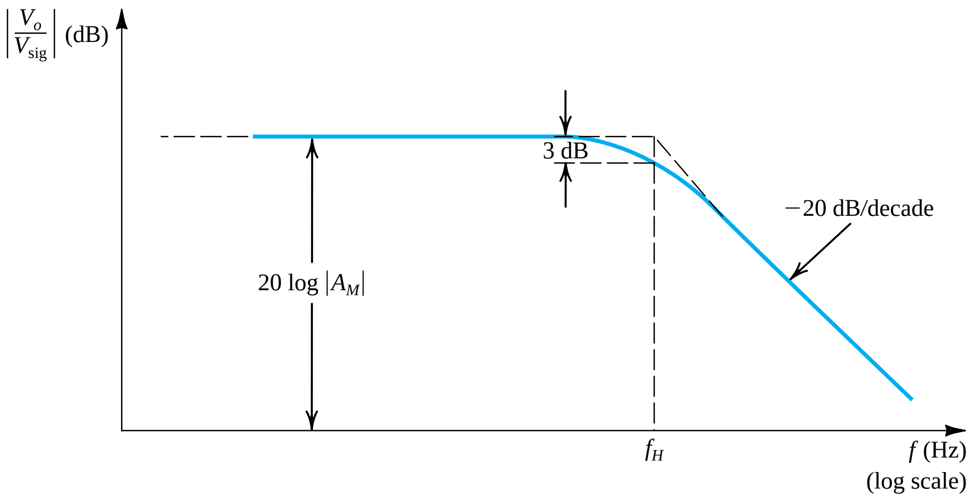
$$\begin{aligned} I_{gd} &= sC_{gd}(V_{gs} - V_o) \\ &= sC_{gd}[V_{gs} - (-g_m R'_L V_{gs})] \\ &= sC_{gd}(1 + g_m R'_L)V_{gs} \end{aligned}$$

Now, the left-hand side of the circuit in Fig. 9.13(b), at $XX'$, knows of the existence of $C_{gd}$ only through the current $I_{gd}$. Therefore, we can replace $C_{gd}$ by an equivalent capacitance $C_{eq}$ between the gate and ground as long as $C_{eq}$ draws a current equal to $I_{gd}$. That is,

$$sC_{eq}V_{gs} = sC_{gd}(1 + g_m R'_L)V_{gs}$$




(c)




(d)

**Figure 9.13** (*Continued*) **(c)** the equivalent circuit with $C_{gd}$ replaced at the input side with the equivalent capacitance $C_{eq}$; **(d)** the frequency response plot, which is that of a low-pass, single-time-constant circuit.

which results in

$$C_{eq} = C_{gd}(1 + g_m R'_L) \tag{9.46}$$

Thus $C_{gd}$ gives rise to a much larger capacitance $C_{eq}$, which appears at the amplifier input. The multiplication effect that $C_{gd}$ undergoes comes about because it is connected between circuit nodes $g$ and $d$, whose voltages are related by a large negative gain $(-g_m R'_L)$. This effect is known as the **Miller effect**, and $(1 + g_m R'_L)$ is known as the **Miller multiplier**. We will study Miller's theorem more formally in Section 9.4.

Using $C_{eq}$ enables us to simplify the equivalent circuit at the input side to that shown in Fig. 9.13(c). We recognize the circuit of Fig. 9.13(c) as a single-time-constant (STC) circuit of the low-pass type (Section 1.6 and Appendix E). Reference to Table 1.2 enables us to express the output voltage $V_{gs}$ of the STC circuit in the form

$$V_{gs} = \left(\frac{R_G}{R_G + R_{\text{sig}}} V_{\text{sig}}\right) \frac{1}{1 + \dfrac{s}{\omega_0}} \tag{9.47}$$

where $\omega_0$ is the corner frequency, the break frequency, or the pole frequency of the STC circuit,

$$\omega_0 = 1/C_{\text{in}} R'_{\text{sig}} \tag{9.48}$$

with

$$C_{\text{in}} = C_{gs} + C_{eq} = C_{gs} + C_{gd}(1 + g_m R'_L) \tag{9.49}$$

and

$$R'_{\text{sig}} = R_{\text{sig}} \parallel R_G \tag{9.50}$$

Combining Eqs. (9.45) and (9.47) results in the following expression for the high-frequency gain of the CS amplifier,

$$\frac{V_o}{V_{\text{sig}}} = -\left(\frac{R_G}{R_G + R_{\text{sig}}}\right)(g_m R'_L) \frac{1}{1 + \dfrac{s}{\omega_0}} \tag{9.51}$$

which can be expressed in the form

$$\frac{V_o}{V_{\text{sig}}} = \frac{A_M}{1 + \dfrac{s}{\omega_H}} \tag{9.52}$$

where the midband gain $A_M$ is given by Eq. (9.44) and $\omega_H$ is the upper 3-dB frequency,

$$\omega_H = \omega_0 = \frac{1}{C_{\text{in}} R'_{\text{sig}}} \tag{9.53}$$

and

$$f_H = \frac{\omega_H}{2\pi} = \frac{1}{2\pi C_{\text{in}} R'_{\text{sig}}} \tag{9.54}$$

We thus see that the high-frequency response will be that of a low-pass STC network with a 3-dB frequency $f_H$ determined by the time constant $C_{\text{in}} R'_{\text{sig}}$. Figure 9.13(d) shows a sketch of the magnitude of the high-frequency gain.

Before leaving this section we wish to make a number of observations:

1. The upper 3-dB frequency is determined by the interaction of $R'_{\text{sig}} = R_{\text{sig}} \parallel R_G$ and $C_{\text{in}} = C_{gs} + C_{gd}(1 + g_m R'_L)$. Since the bias resistance $R_G$ is usually very large, it can be neglected, resulting in $R'_{\text{sig}} \simeq R_{\text{sig}}$, the resistance of the signal source. It follows that a large value of $R_{\text{sig}}$ will cause $f_H$ to be lowered.
2. The total input capacitance $C_{\text{in}}$ is usually dominated by $C_{eq}$, which in turn is made large by the multiplication effect that $C_{gd}$ undergoes. Thus, although $C_{gd}$ is usually a very small capacitance, its effect on the amplifier frequency response can be very significant as a result of its multiplication by the factor $(1 + g_m R'_L)$, which is approximately equal

to the midband gain of the amplifier. This is the Miller effect, which causes the CS amplifier to have a large total input capacitance $C_{in}$ and hence a low $f_H$.

3. To extend the high-frequency response of a MOSFET amplifier, we have to find configurations in which the Miller effect is absent or at least reduced. We shall return to this subject at great length in Section 9.6 and beyond.
4. The above analysis, resulting in an STC or a single-pole response, is approximate. Specifically, it is based on neglecting $I_{gd}$ relative to $g_m V_{gs}$, an assumption that applies well at frequencies not too much higher than $f_H$. An exact analysis of the circuit in Fig. 9.13(a) will be carried out in Section 9.5. The results above, however, are more than sufficient for a quick estimate of $f_H$. As well, the approximate approach helps to reveal the primary limitation on the high-frequency response: the Miller effect.

## Example 9.3

Find the midband gain $A_M$ and the upper 3-dB frequency $f_H$ of a CS amplifier fed with a signal source having an internal resistance $R_{sig} = 100\ \text{k}\Omega$. The amplifier has $R_G = 4.7\ \text{M}\Omega$, $R_D = R_L = 15\ \text{k}\Omega$, $g_m = 1\ \text{mA/V}$, $r_o = 150\ \text{k}\Omega$, $C_{gs} = 1\ \text{pF}$, and $C_{gd} = 0.4\ \text{pF}$.

### Solution

$$A_M = -\frac{R_G}{R_G + R_{sig}} g_m R_L'$$

where

$$R_L' = r_o \parallel R_D \parallel R_L = 150 \parallel 15 \parallel 15 = 7.14\ \text{k}\Omega.$$

$$g_m R_L' = 1 \times 7.14 = 7.14\ \text{V/V}$$

Thus,

$$A_M = -\frac{4.7}{4.7 + 0.1} \times 7.14 = -7\ \text{V/V}$$

The equivalent capacitance, $C_{eq}$, is found as

$$\begin{aligned} C_{eq} &= (1 + g_m R_L')C_{gd} \\ &= (1 + 7.14) \times 0.4 = 3.26\ \text{pF} \end{aligned}$$

The total input capacitance $C_{in}$ can be now obtained as

$$C_{in} = C_{gs} + C_{eq} = 1 + 3.26 = 4.26\ \text{pF}$$

The upper 3-dB frequency $f_H$ is found from

$$\begin{aligned} f_H &= \frac{1}{2\pi C_{in}(R_{sig} \parallel R_G)} \\ &= \frac{1}{2\pi \times 4.26 \times 10^{-12}(0.1 \parallel 4.7) \times 10^6} \\ &= 382\ \text{kHz} \end{aligned}$$

## EXERCISES

**9.8** For the CS amplifier specified in Example 9.3, find the values of $A_M$ and $f_H$ that result when the signal-source resistance is reduced to 10 kΩ.
**Ans.** –7.12 V/V; 3.7 MHz

**9.9** If it is possible to replace the MOSFET used in the amplifier in Example 9.3 with another having the same $C_{gs}$ but a smaller $C_{gd}$, what is the maximum value that its $C_{gd}$ can be in order to obtain an $f_H$ of at least 1 MHz?
**Ans.** 0.08 pF

### 9.3.2 The Common-Emitter Amplifier

Figure 9.14(a) shows the high-frequency equivalent circuit of a CE amplifier. It is obtained by replacing the BJT in a circuit such as that in Fig. 9.4(a) with its high-frequency, equivalent-circuit model of Fig. 9.8, and, as usual, eliminating all dc sources. Observe that the circuit in Fig. 9.14(a) is general and applies to both discrete and IC amplifiers. Thus, it includes $R_B$, which is usually present in discrete circuits. Also $R_C$ can be either a passive resistance or the output resistance of a current-source load, and similarly for $R_L$.

The equivalent circuit of Fig. 9.14(a) can be simplified by utilizing Thévenin theorem at the input side and by combining the three parallel resistances at the output side. Specifically, the reader should be able to show that applying Thévenin theorem *twice* simplifies the resistive network at the input side to a signal generator $V'_{sig}$ and a resistance $R'_{sig}$, with the values indicated in the figure.

The equivalent circuit in Fig. 9.14(b) can be used to obtain the midband gain $A_M$ by setting $C_\pi$ and $C_\mu$ to zero. The result is

$$A_M = \frac{V_o}{V_{sig}} = -\frac{R_B}{R_B + R_{sig}} \frac{r_\pi}{r_\pi + r_x + (R_{sig} \| R_B)}(g_m R'_L) \tag{9.55}$$

where

$$R'_L = r_o \| R_C \| R_L \tag{9.56}$$

Next we observe that the circuit in Fig. 9.14(b) is identical to that of the CS amplifier in Fig. 9.13(b). Thus the analysis can follow the same process we used for the CS case. The analysis is illustrated in Fig. 9.13(c) and (d). The final result is that the CE amplifier gain at high frequencies is given approximately by

$$\frac{V_o}{V_{sig}} = \frac{A_M}{1 + \dfrac{s}{\omega_H}} \tag{9.57}$$

where $A_M$ is given by Eq. (9.55) and the 3-dB frequency $f_H$ is given by

$$f_H = \frac{\omega_H}{2\pi} = \frac{1}{2\pi C_{in} R'_{sig}} \tag{9.58}$$

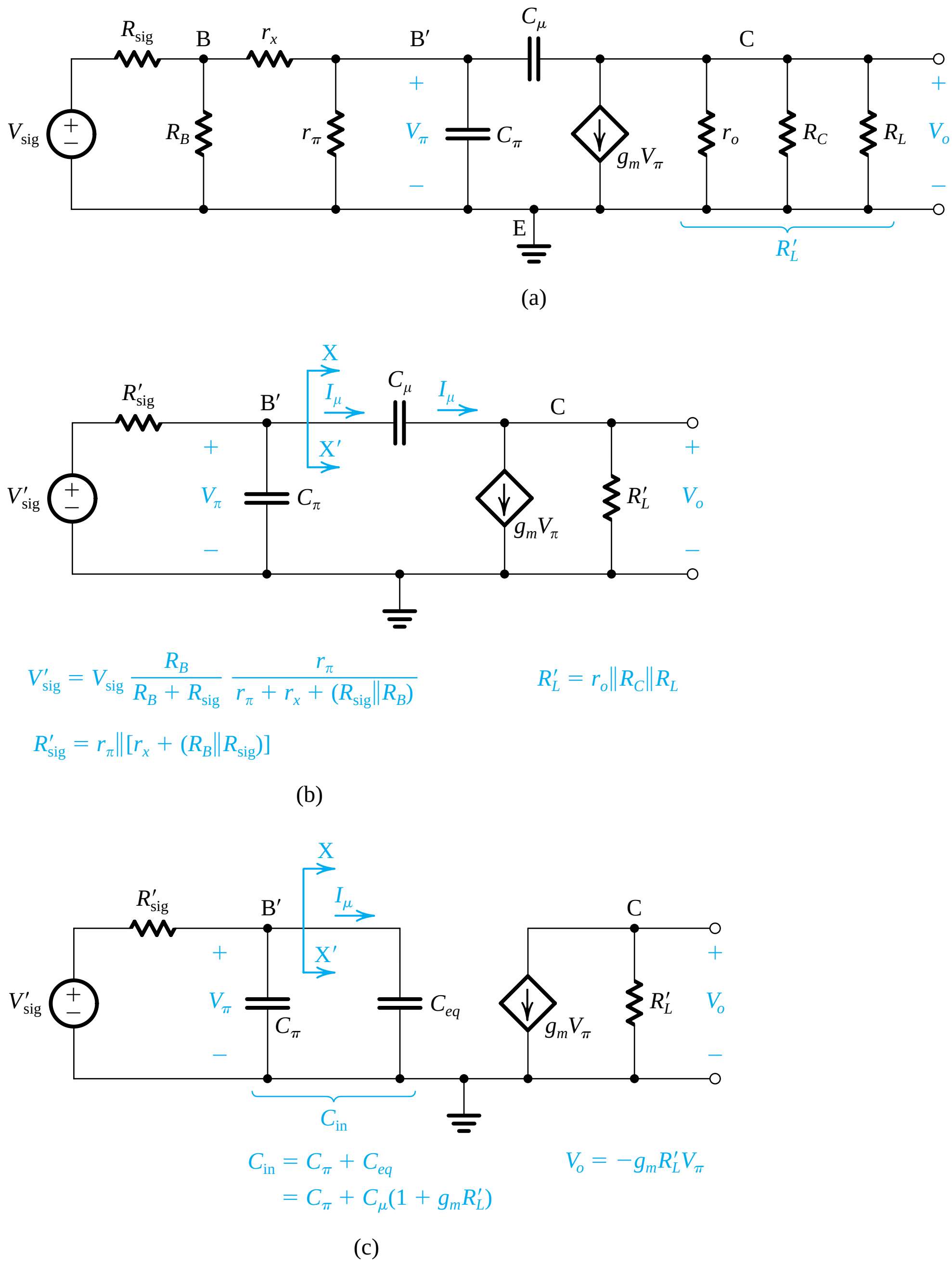
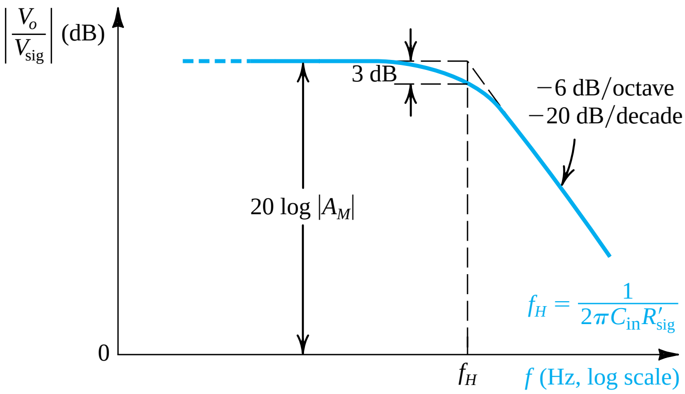



**Figure 9.14** Determining the high-frequency response of the CE amplifier: **(a)** equivalent circuit; **(b)** the circuit of **(a)** simplified at both the input side and the output side; **(c)** equivalent circuit with $C_\mu$ replaced at the input side with the equivalent capacitance $C_{eq}$; (*continued*)




(d)

**Figure 9.14** (*Continued*) **(d)** sketch of the frequency-response plot, which is that of a low-pass STC circuit.

where

$$C_{in} = C_\pi + C_\mu(1 + g_m R'_L) \tag{9.59}$$

and

$$R'_{sig} = r_\pi \| [r_x + (R_B \| R_{sig})] \tag{9.60}$$

Observe that $C_{in}$ is simply the sum of $C_\pi$ and the Miller capacitance $C_\mu(1 + g_m R'_L)$ . The resistance $R'_{sig}$ seen by $C_{in}$ can be easily found from the circuit in Fig. 9.14(a) as follows: Reduce $V_{sig}$ to zero, "grab hold" of the terminals B′ and E and look back (to the left). You will see $r_\pi$ in parallel with $r_x$, which is in series with $(R_B \| R_{sig})$. This way of finding the resistance "seen by a capacitance" is very useful and spares one from tedious work!

Finally, comments very similar to those made on the high-frequency response of the CS amplifier can be made here as well.

## Example 9.4

It is required to find the midband gain and the upper 3-dB frequency of the common-emitter amplifier of Fig. 9.4(a) for the following case: $V_{CC} = V_{EE} = 10$ V, $I = 1$ mA, $R_B = 100$ kΩ, $R_C = 8$ kΩ, $R_{sig} =$ 5 kΩ, $R_L = 5$ kΩ, $\beta_0 = 100$, $V_A = 100$ V, $C_\mu = 1$ pF, $f_T = 800$ MHz, and $r_x = 50$ Ω.

### Solution

The transistor is biased at $I_C \simeq 1$ mA. Thus the values of its hybrid-$\pi$ model parameters are

$$g_m = \frac{I_C}{V_T} = \frac{1\text{ mA}}{25\text{ mV}} = 40\text{ mA/V}$$

$$r_\pi = \frac{\beta_0}{g_m} = \frac{100}{40\text{ mA/V}} = 2.5\text{ k}\Omega$$

$$r_o = \frac{V_A}{I_C} = \frac{100\text{ V}}{1\text{ mA}} = 100\text{ k}\Omega$$

**Example 9.4** *continued*

$$C_\pi + C_\mu = \frac{g_m}{\omega_T} = \frac{40 \times 10^{-3}}{2\pi \times 800 \times 10^6} = 8 \text{ pF}$$

$$C_\mu = 1 \text{ pF}$$

$$C_\pi = 7 \text{ pF}$$

$$r_x = 50 \ \Omega$$

The midband voltage gain is

$$A_M = -\frac{R_B}{R_B + R_{sig}} \frac{r_\pi}{r_\pi + r_x + (R_B \| R_{sig})} g_m R_L'$$

where

$$R_L' = r_o \| R_C \| R_L$$
$$= (100 \| 8 \| 5) \text{ k}\Omega = 3 \text{ k}\Omega$$

Thus,

$$g_m R_L' = 40 \times 3 = 120 \text{ V/V}$$

and

$$A_M = -\frac{100}{100+5} \times \frac{2.5}{2.5 + 0.05 + (100 \| 5)} \times 120$$
$$= -39 \text{ V/V}$$

and

$$20 \log |A_M| = 32 \text{ dB}$$

To determine $f_H$ we first find $C_{in}$,

$$C_{in} = C_\pi + C_\mu(1 + g_m R_L')$$
$$= 7 + 1(1 + 120) = 128 \text{ pF}$$

and the effective source resistance $R_{sig}'$ ,

$$R_{sig}' = r_\pi \| [r_x + (R_B \| R_{sig})]$$
$$= 2.5 \| [0.05 + (100 \| 5)]$$
$$= 1.65 \text{ k}\Omega$$

Thus,

$$f_H = \frac{1}{2\pi C_{in} R_{sig}'} = \frac{1}{2\pi \times 128 \times 10^{-12} \times 1.65 \times 10^3} = 754 \text{ kHz}$$

## EXERCISE

**9.10** For the amplifier in Example 9.4, find the value of $R_L$ that reduces the midband gain to half the value found. What value of $f_H$ results? Note the trade-off between gain and bandwidth.
**Ans.** 1.9 kΩ; 1.42 MHz

# 9.4 Useful Tools for the Analysis of the High-Frequency Response of Amplifiers

The approximate method used in the previous section to analyze the high-frequency response of the CS and CE amplifiers provides a reasonably accurate estimate of $f_H$ and, equally important, considerable insight into the mechanism that limits high-frequency operation. Unfortunately, however, this method is not easily extendable to more complex amplifier circuits. For this reason, we will digress briefly in this section to equip ourselves with a number of tools that will prove useful in the analysis of more complex circuits such as the cascode amplifier. We will begin by stepping back and more generally considering the amplifier high-frequency transfer function.

## 9.4.1 The High-Frequency Gain Function

The amplifier gain, taking into account the internal transistor capacitances, can be expressed as a function of the complex-frequency variable *s* in the general form

$$A(s) = A_M F_H(s) \tag{9.61}$$

where $A_M$ is the midband gain, which for IC amplifiers is also equal to the low-frequency or dc gain (refer to Fig. 9.12). The value of $A_M$ can be determined by analyzing the amplifier equivalent circuit while neglecting the effect of the transistor internal capacitances—that is, by assuming that they act as perfect open circuits. By taking these capacitances into account, we see that the gain acquires the factor $F_H(s)$, which can be expressed in terms of its poles and zeros, which are usually real, as follows:

$$F_H(s) = \frac{(1+s/\omega_{Z1})(1+s/\omega_{Z2})\ldots(1+s/\omega_{Zn})}{(1+s/\omega_{P1})(1+s/\omega_{P2})\ldots(1+s/\omega_{Pn})} \tag{9.62}$$

where $\omega_{P1}, \omega_{P2}, \ldots, \omega_{Pn}$ are positive numbers representing the frequencies of the *n* real poles and $\omega_{Z1}, \omega_{Z2}, \ldots, \omega_{Zn}$ are positive, negative, or infinite numbers representing the frequencies of the *n* real transmission zeros. Note from Eq. (9.62) that, as should be expected, as *s* approaches 0, $F_H(s)$ approaches unity and the gain approaches $A_M$.

## 9.4.2 Determining the 3-dB Frequency $f_H$

The amplifier designer usually is particularly interested in the part of the high-frequency band that is close to the midband. This is because the designer needs to estimate—and if need be modify—the value of the upper 3-dB frequency $f_H$ (or $\omega_H$; $f_H = \omega_H/2\pi$). Toward that end it should be mentioned that in many cases the zeros are either at infinity or such high frequencies as to be of little significance to the determination of $\omega_H$. If in addition one of the poles, say $\omega_{P1}$, is of much lower frequency than any of the other poles, then this pole will have the greatest effect on the value of the amplifier $\omega_H$. In other words, this pole will *dominate* the high-frequency response of the amplifier, and the amplifier is said to have a **dominant-pole response**. In such cases, the function $F_H(s)$ can be approximated by

$$F_H(s) \simeq \frac{1}{1+s/\omega_{P1}} \tag{9.63}$$

which is the transfer function of a first-order (or STC) low-pass network (Appendix E). It follows that if a dominant pole exists, then the determination of $\omega_H$ is greatly simplified;

$$\omega_H \simeq \omega_{P1} \tag{9.64}$$

This is the situation we encountered in the cases of the common-source and common-emitter amplifiers analyzed in Section 9.3. As a rule of thumb, *a dominant pole exists if the lowest-frequency pole is at least two octaves (a factor of 4) away from the nearest pole or zero*.

If a dominant pole does not exist, the 3-dB frequency $\omega_H$ can be determined from a plot of $|F_H(j\omega)|$. Alternatively, an approximate formula for $\omega_H$ can be derived as follows: Consider, for simplicity, the case of a circuit having two poles and two zeros in the high-frequency band; that is,

$$F_H(s) = \frac{(1+s/\omega_{Z1})(1+s/\omega_{Z2})}{(1+s/\omega_{P1})(1+s/\omega_{P2})} \tag{9.65}$$

Substituting $s = j\omega$ and taking the squared magnitude gives

$$|F_H(j\omega)|^2 = \frac{(1+\omega^2/\omega_{Z1}^2)(1+\omega^2/\omega_{Z2}^2)}{(1+\omega^2/\omega_{P1}^2)(1+\omega^2/\omega_{P2}^2)}$$

By definition, at $\omega = \omega_H$, $|F_H|^2 = \frac{1}{2}$; thus,

$$\begin{aligned}\frac{1}{2} &= \frac{(1+\omega_H^2/\omega_{Z1}^2)(1+\omega_H^2/\omega_{Z2}^2)}{(1+\omega_H^2/\omega_{P1}^2)(1+\omega_H^2/\omega_{P2}^2)} \\ &= \frac{1+\omega_H^2\left(\dfrac{1}{\omega_{Z1}^2}+\dfrac{1}{\omega_{Z2}^2}\right)+\omega_H^4/\omega_{Z1}^2\omega_{Z2}^2}{1+\omega_H^2\left(\dfrac{1}{\omega_{P1}^2}+\dfrac{1}{\omega_{P2}^2}\right)+\omega_H^4/\omega_{P1}^2\omega_{P2}^2}\end{aligned} \tag{9.66}$$

Since $\omega_H$ is usually smaller than the frequencies of all the poles and zeros, we may neglect the terms containing $\omega_H^4$ and solve for $\omega_H$ to obtain

$$\omega_H \simeq 1\Bigg/\sqrt{\frac{1}{\omega_{P1}^2}+\frac{1}{\omega_{P2}^2}-\frac{2}{\omega_{Z1}^2}-\frac{2}{\omega_{Z2}^2}} \tag{9.67}$$

This relationship can be extended to any number of poles and zeros as

$$\omega_H \simeq 1\Bigg/\sqrt{\left(\frac{1}{\omega_{P1}^2}+\frac{1}{\omega_{P2}^2}+\cdots\right)-2\left(\frac{1}{\omega_{Z1}^2}+\frac{1}{\omega_{Z2}^2}+\cdots\right)} \tag{9.68}$$

Note that if one of the poles, say $P_1$, is dominant, then $\omega_{P1} \ll \omega_{P2}, \omega_{P3}, \ldots, \omega_{Z1}, \omega_{Z2}, \ldots,$ and Eq. (9.68) reduces to Eq. (9.69).

## Example 9.5

The high-frequency response of an amplifier is characterized by the transfer function

$$F_H(s) = \frac{1-s/10^5}{(1+s/10^4)(1+s/4\times 10^4)}$$

Determine the 3-dB frequency approximately and exactly.

### Solution

Noting that the lowest-frequency pole at $10^4$ rad/s is two octaves lower than the second pole and a decade lower than the zero, we find that a dominant-pole situation almost exists and $\omega_H \simeq 10^4$ rad/s. A better estimate of $\omega_H$ can be obtained using Eq. (9.68), as follows:

$$\omega_H = 1 \bigg/ \sqrt{\frac{1}{10^8} + \frac{1}{16 \times 10^8} - \frac{2}{10^{10}}}$$

$$= 9800 \text{ rad/s}$$

The exact value of $\omega_H$ can be determined from the given transfer function as 9537 rad/s. Finally, we show in Fig. 9.15 a Bode plot and an exact plot for the given transfer function. Note that this is a plot of the high-frequency response of the amplifier normalized relative to its midband gain. That is, if the midband gain is, say, 100 dB, then the entire plot should be shifted upward by 100 dB.

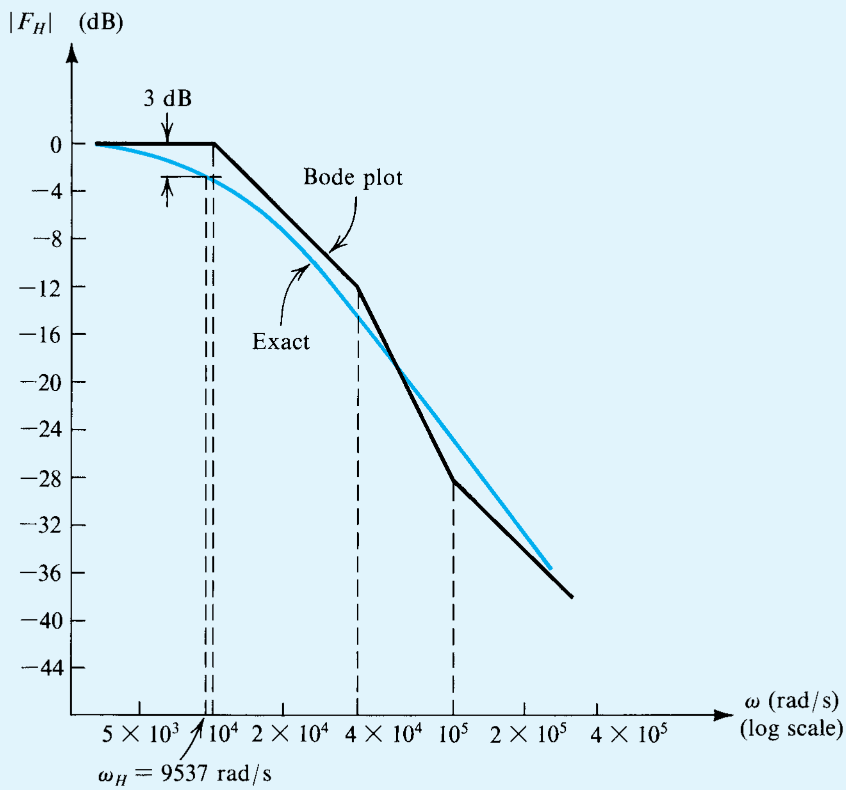



**Figure 9.15** Normalized high-frequency response of the amplifier in Example 9.5.

### 9.4.3 Using Open-Circuit Time Constants for the Approximate Determination of $f_H$

If the poles and zeros of the amplifier transfer function can be determined easily, then we can determine $f_H$ using the techniques above. In many cases, however, it is not a simple matter to determine the poles and zeros by quick hand analysis. In such cases an approximate value for $f_H$ can be obtained using the following method.

Consider the function $F_H(s)$ (Eq. 9.62), which determines the high-frequency response of the amplifier. The numerator and denominator factors can be multiplied out and $F_H(s)$ expressed in the alternative form

$$F_H(s) = \frac{1 + a_1 s + a_2 s^2 + \cdots + a_n s^n}{1 + b_1 s + b_2 s^2 + \cdots + b_n s^n} \tag{9.69}$$

where the coefficients $a$ and $b$ are related to the frequencies of the zeros and poles, respectively. Specifically, the coefficient $b_1$ is given by

$$b_1 = \frac{1}{\omega_{P1}} + \frac{1}{\omega_{P2}} + \cdots + \frac{1}{\omega_{Pn}} \tag{9.70}$$

It can be shown [see Gray and Searle (1969)] that the value of $b_1$ can be obtained by considering the various capacitances in the high-frequency equivalent circuit one at a time while reducing all other capacitors to zero (or, equivalently, replacing them with open circuits). That is, to obtain the contribution of capacitance $C_i$ we reduce all other capacitances to zero, reduce the input signal source to zero, and determine the resistance $R_i$ seen by $C_i$. This process is then repeated for all other capacitors in the circuit. The value of $b_1$ is computed by summing the individual time constants, called **open-circuit time constants**,

$$b_1 = \sum_{i=1}^{n} C_i R_i \tag{9.71}$$

where we have assumed that there are $n$ capacitors in the high-frequency equivalent circuit.

This method for determining $b_1$ is *exact*; the approximation comes about in using the value of $b_1$ to determine $\omega_H$. Specifically, if the zeros are not dominant and if one of the poles, say $P_1$, is dominant, then from Eq. (9.70),

$$b_1 \simeq \frac{1}{\omega_{P1}} \tag{9.72}$$

But, also, the upper 3-dB frequency will be approximately equal to $\omega_{P1}$, leading to the approximation

$$\omega_H \simeq \frac{1}{b_1} = \frac{1}{\sum_i C_i R_i} \tag{9.73}$$

Here it should be pointed out that in complex circuits we usually do not know whether a dominant pole exists. Nevertheless, using Eq. (9.73) to determine $\omega_H$ normally yields remarkably good results[5] even if a dominant pole does not exist. The method will be illustrated by an example.

[5] The method of open-circuit time constants yields good results only when all the poles are real, as is the case in this chapter.

## Example 9.6

Figure 9.16(a) shows the high-frequency equivalent circuit of a common-source MOSFET amplifier. The amplifier is fed with a signal generator $V_{sig}$ having a resistance $R_{sig}$. Resistance $R_G$ is due to the biasing network. Resistance $R_L'$ is the parallel equivalent of the load resistance $R_L$, the drain bias resistance $R_D$, and the FET output resistance $r_o$. Capacitors $C_{gs}$ and $C_{gd}$ are the MOSFET internal capacitances. For $R_{sig} = 100$ kΩ, $R_G = 420$ kΩ, $C_{gs} = C_{gd} = 1$ pF, $g_m = 4$ mA/V, and $R_L' = 3.33$ kΩ, find the midband voltage gain, $A_M = V_o/V_{sig}$ and the upper 3-dB frequency, $f_H$.

### Solution

The midband voltage gain is determined by assuming that the capacitors in the MOSFET model are perfect open circuits. This results in the midband equivalent circuit shown in Fig. 9.16(b),

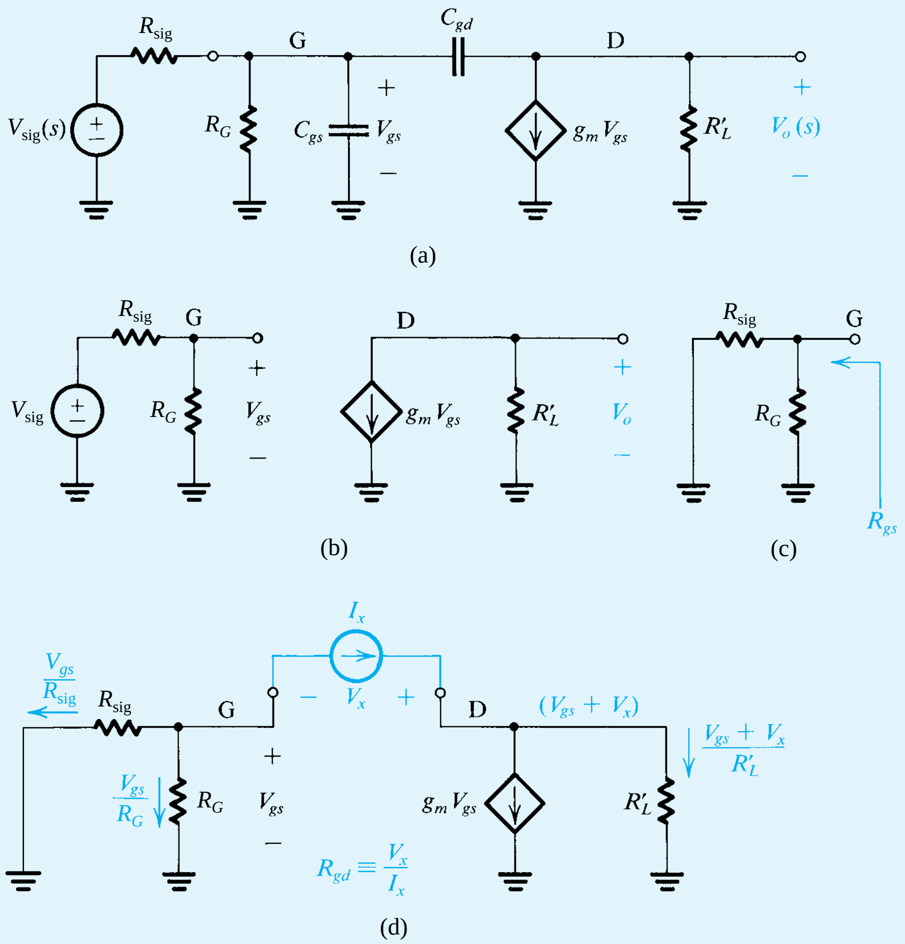



**Figure 9.16** Circuits for Example 9.6: **(a)** high-frequency equivalent circuit of a MOSFET amplifier; **(b)** the equivalent circuit at midband frequencies; **(c)** circuit for determining the resistance seen by $C_{gs}$; **(d)** circuit for determining the resistance seen by $C_{gd}$.

**Example 9.6** *continued*

from which we find

$$A_M \equiv \frac{V_o}{V_{sig}} = -\frac{R_G}{R_G + R_{sig}}(g_m R_L')$$

$$= -\frac{420}{420+100} \times 4 \times 3.33 = -10.8 \text{ V/V}$$

We shall determine $\omega_H$ using the method of open-circuit time constants. The resistance $R_{gs}$ seen by $C_{gs}$ is found by setting $C_{gd} = 0$ and short-circuiting the signal generator $V_{sig}$. This results in the circuit of Fig. 9.16(c), from which we find that

$$R_{gs} = R_G \| R_{sig} = 420 \text{ k}\Omega \| 100 \text{ k}\Omega = 80.8 \text{ k}\Omega$$

Thus the open-circuit time constant of $C_{gs}$ is

$$\tau_{gs} \equiv C_{gs}R_{gs} = 1 \times 10^{-12} \times 80.8 \times 10^3 = 80.8 \text{ ns}$$

The resistance $R_{gd}$ seen by $C_{gd}$ is found by setting $C_{gs} = 0$ and short-circuiting $V_{sig}$. The result is the circuit in Fig. 9.16(d), to which we apply a test current $I_x$. Writing a node equation at G gives

$$I_x = -\frac{V_{gs}}{R_G} - \frac{V_{gs}}{R_{sig}}$$

Thus,

$$V_{gs} = -I_x R_{sig}' \tag{9.74}$$

where $R_{sig}' = R_G \| R_{sig}$. A node equation at D provides

$$I_x = g_m V_{gs} + \frac{V_{gs} + V_x}{R_L'}$$

Substituting for $V_{gs}$ from Eq. (9.74) and rearranging terms yields

$$R_{gd} \equiv \frac{V_x}{I_x} = R_{sig}' + R_L' + g_m R_L' R_{sig}' = 1.16 \text{ M}\Omega$$

Thus the open-circuit time constant of $C_{gd}$ is

$$\begin{aligned} \tau_{gd} &\equiv C_{gd}R_{gd} \\ &= 1 \times 10^{-12} \times 1.16 \times 10^6 = 1160 \text{ ns} \end{aligned}$$

The upper 3-dB frequency $\omega_H$ can now be determined from

$$\begin{aligned} \omega_H &\simeq \frac{1}{\tau_{gs} + \tau_{gd}} \\ &= \frac{1}{(80.8 + 1160) \times 10^{-9}} = 806 \text{ krad/s} \end{aligned}$$

Thus,

$$f_H = \frac{\omega_H}{2\pi} = 128.3 \text{ kHz}$$

The method of open-circuit time constants has an important advantage in that it tells the circuit designer which of the various capacitances is significant in determining the amplifier frequency response. Specifically, the relative contribution of the various capacitances to the effective time constant $b_1$ is immediately obvious. For instance, in the above example we see that $C_{gd}$ is the dominant capacitance in determining $f_H$. We also note that, in effect to increase $f_H$ either we use a MOSFET with smaller $C_{gd}$ or, for a given MOSFET, we reduce $R_{gd}$ by using a smaller $R'_{\text{sig}}$ or $R'_L$. If $R'_{\text{sig}}$ is fixed, then for a given MOSFET the only way to increase bandwidth is by reducing the load resistance. Unfortunately, this also decreases the midband gain. This is an example of the usual trade-off between gain and bandwidth, a common circumstance which was mentioned earlier.

## 9.4.4 Miller's Theorem

In our analysis of the high-frequency response of the common-source and common-emitter amplifiers (Section 9.3), we employed a technique for replacing the bridging capacitance $(C_{gs}$ or $C_\mu)$ by an equivalent input capacitance. This very useful and effective technique is based on a general theorem known as **Miller's theorem**, which we now present.

Consider the situation in Fig. 9.17(a). As part of a larger circuit that is not shown, we have isolated two circuit nodes, labeled 1 and 2, between which an impedance $Z$ is connected. Nodes 1 and 2 are also connected to other parts of the circuit, as signified by the broken lines emanating from the two nodes. Furthermore, it is assumed that somehow it has been determined that the voltage at node 2 is related to that at node 1 by

$$V_2 = KV_1 \tag{9.75}$$

In typical situations $K$ is a gain factor that can be positive or negative and that has a magnitude usually larger than unity. This, however, is not an assumption for Miller's theorem.

Miller's theorem states that impedance $Z$ can be replaced by two impedances: $Z_1$ connected between node 1 and ground and $Z_2$ connected between node 2 and ground, where

$$Z_1 = Z/(1-K) \tag{9.76a}$$

and

$$Z_2 = Z\Big/\left(1-\frac{1}{K}\right) \tag{9.76b}$$

to obtain the equivalent circuit shown in Fig. 9.17(b).

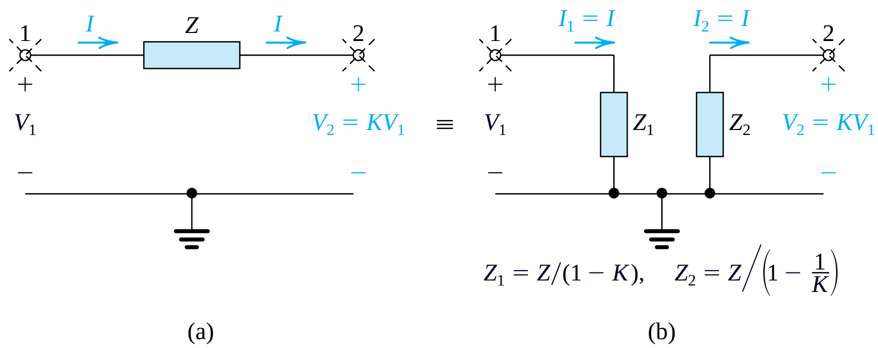



**Figure 9.17** The Miller equivalent circuit.

The proof of Miller's theorem is achieved by deriving Eq. (9.76) as follows: In the original circuit of Fig. 9.17(a), the only way that node 1 "feels the existence" of impedance $Z$ is through the current $I$ that $Z$ draws away from node 1. Therefore, to keep this current unchanged in the equivalent circuit, we must choose the value of $Z_1$ so that it draws an equal current,

$$I_1 = \frac{V_1}{Z_1} = I = \frac{V_1 - KV_1}{Z}$$

which yields the value of $Z_1$ in Eq. (9.76a). Similarly, to keep the current into node 2 unchanged, we must choose the value of $Z_2$ so that

$$I_2 = \frac{0 - V_2}{Z_2} = \frac{0 - KV_1}{Z_2} = I = \frac{V_1 - KV_1}{Z}$$

which yields the expression[6] for $Z_2$ in Eq. (9.76b).

## Example 9.7

Figure 9.18(a) shows an ideal voltage amplifier having a gain of –100 V/V with an impedance $Z$ connected between its output and input terminals. Find the Miller equivalent circuit when $Z$ is (a) a 1-MΩ resistance and (b) a 1-pF capacitance. In each case, use the equivalent circuit to determine $V_o/V_{sig}$.

### Solution

(a) For $Z$ = 1 MΩ, employing Miller's theorem results in the equivalent circuit in Fig. 9.18(b), where

$$Z_1 = \frac{Z}{1-K} = \frac{1000 \text{ k}\Omega}{1+100} = 9.9 \text{ k}\Omega$$

$$Z_2 = \frac{Z}{1-\dfrac{1}{K}} = \frac{1 \text{ M}\Omega}{1+\dfrac{1}{100}} = 0.99 \text{ M}\Omega$$

The voltage gain can be found as follows:

$$\frac{V_o}{V_{sig}} = \frac{V_o}{V_i}\frac{V_i}{V_{sig}} = -100 \times \frac{Z_1}{Z_1 + R_{sig}}$$

$$= -100 \times \frac{9.9}{9.9 + 10} = -49.7 \text{ V/V}$$

[6] Although not highlighted, the Miller equivalent circuit derived above is valid only as long as the rest of the circuit remains unchanged; otherwise the ratio of $V_2$ to $V_1$ might change. It follows that the Miller equivalent circuit *cannot* be used directly to determine the output resistance of an amplifier. This is because in determining output resistances it is implicitly assumed that the source signal is reduced to zero and that a test-signal source (voltage or current) is applied to the output terminals—obviously a major change in the circuit, rendering the Miller equivalent circuit no longer valid.

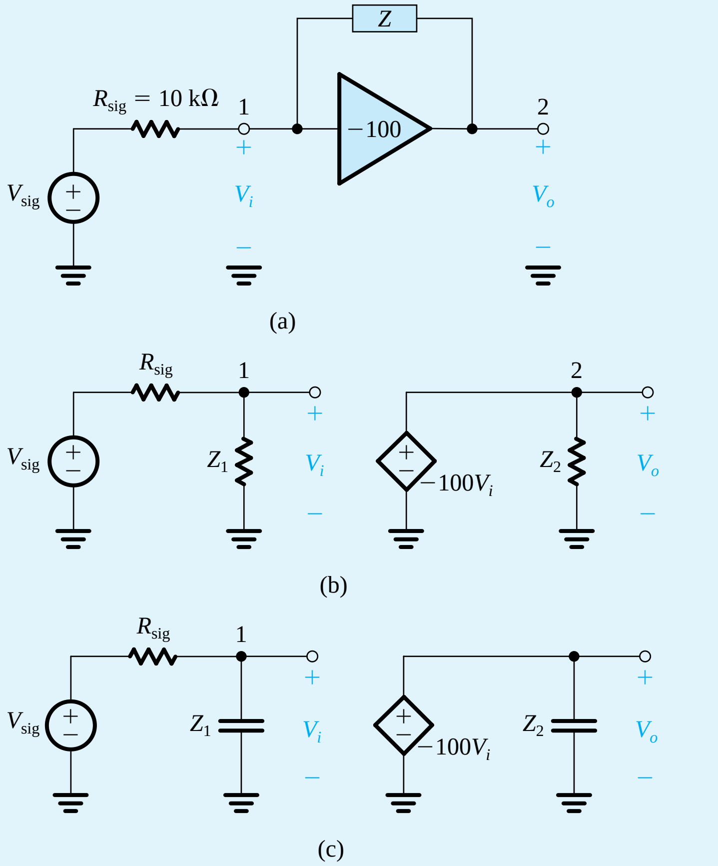



**Figure 9.18** Circuits for Example 9.7.

(b) For $Z$ as a 1-pF capacitance—that is, $Z = 1/sC = 1/s \times 1 \times 10^{-12}$—applying Miller's theorem allows us to replace $Z$ by $Z_1$ and $Z_2$, where

$$Z_1 = \frac{Z}{1-K} = \frac{1/sC}{1+100} = 1/s(101C)$$

$$Z_2 = \frac{Z}{1-\frac{1}{K}} = \frac{1}{1.01}\frac{1}{sC} = \frac{1}{s(1.01C)}$$

It follows that $Z_1$ is a capacitance $101C = 101$ pF and that $Z_2$ is a capacitance $1.01C = 1.01$ pF. The resulting equivalent circuit is shown in Fig. 9.18(c), from which the voltage gain can be found as follows:

$$\frac{V_o}{V_{sig}} = \frac{V_o}{V_i}\frac{V_i}{V_{sig}} = -100\frac{1/sC_1}{1/(sC_1)+R_{sig}}$$

$$= \frac{-100}{1+sC_1R_{sig}}$$

**Example 9.7** *continued*

$$= \frac{-100}{1 + s \times 101 \times 1 \times 10^{-12} \times 10 \times 10^{3}}$$

$$= \frac{-100}{1 + s \times 1.01 \times 10^{-6}}$$

This is the transfer function of a first-order low-pass network with a dc gain of −100 and a 3-dB frequency $f_{3dB}$ of

$$f_{3dB} = \frac{1}{2\pi \times 1.01 \times 10^{-6}} = 157.6 \text{ kHz}$$

From Example 9.7, we observe that the Miller replacement of a feedback or bridging resistance results, for a negative $K$, in a smaller resistance [by a factor $(1-K)$] at the input. If the feedback element is a capacitance, its value is multiplied by $(1-K)$ to obtain the equivalent capacitance at the input side. The multiplication of a feedback capacitance by $(1-K)$ is referred to as **Miller multiplication** or **Miller effect**. We have encountered the Miller effect in the analysis of the CS and CE amplifiers in Section 9.3.

## EXERCISES

**9.11** A direct-coupled amplifier has a dc gain of 1000 V/V and an upper 3-dB frequency of 100 kHz. Find the transfer function and the gain–bandwidth product in hertz.

**Ans.** $\dfrac{1000}{1 + \dfrac{s}{2\pi \times 10^{5}}}$; $10^{8}$ Hz

**9.12** The high-frequency response of an amplifier is characterized by two zeros at $s = \infty$ and two poles at $\omega_{P1}$ and $\omega_{P2}$. For $\omega_{P2} = k\omega_{P1}$, find the value of $k$ that results in the exact value of $\omega_H$ being $0.9\,\omega_{P1}$. Repeat for $\omega_H = 0.99\,\omega_{P1}$.

**Ans.** 2.78; 9.88

**9.13** For the amplifier described in Exercise 9.12, find the exact and approximate values (using Eq. 9.68) of $\omega_H$ (as a function of $\omega_{P1}$) for the cases $k = 1$, 2, and 4.

**Ans.** 0.64, 0.71; 0.84, 0.89; 0.95, 0.97

**9.14** For the amplifier in Example 9.6, find the gain–bandwidth product in megahertz. Find the value of $R_L'$ that will result in $f_H = 180$ kHz. Find the new values of the midband gain and of the gain–bandwidth product.

**Ans.** 1.39 MHz; 2.23 kΩ; −7.2 V/V; 1.30 MHz

**9.15** Use Miller's theorem to investigate the performance of the inverting op-amp circuit shown in Fig. E9.15. Assume the op amp to be ideal except for having a finite differential gain, $A$. Without using any knowledge of op-amp circuit analysis, find $R_{in}$, $V_i$, $V_o$, and $V_o/V_{sig}$, for each of the following values of $A$: 10 V/V, 100 V/V, 1000 V/V, and 10,000 V/V. Assume $V_{sig} = 1$ V.

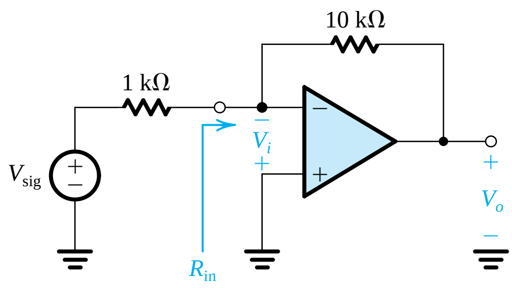



Figure E9.15

**Ans.**

| $A$ | $R_{in}$ | $V_i$ | $V_o$ | $V_o/V_{sig}$ |
|---|---|---|---|---|
| 10 V/V | 909 Ω | 476 mV | − 4.76 V | − 4.76 V/V |
| 100 V/V | 99 Ω | 90 mV | − 9 V | − 9 V/V |
| 1000 V/V | 9.99 Ω | 9.9 mV | − 9.9 V | − 9.9 V/V |
| 10,000 V/V | 1 Ω | 0.999 mV | − 9.99 V | − 9.99 V/V |

# 9.5 A Closer Look at the High-Frequency Response of the CS and CE Amplifiers

In Section 9.3 we utilized the Miller approximation to obtain an estimate of the high-frequency 3-dB frequency $f_H$ of the CS and CE amplifiers. We shall now use the powerful tools we studied in the last section to revisit this subject. Specifically, we will first employ Miller's theorem to refine the Miller approximation, thus obtaining a better estimate of $f_H$. Then we will use the method of open-circuit time constants to obtain another estimate of $f_H$. In order to assess how good these various estimates are, the exact transfer function will be derived and analyzed. Finally, we will consider the case of low source resistance $R_{sig}$ with the limitation on the high-frequency response determined by the capacitance at the output node, a situation that is not uncommon in IC amplifiers.

## 9.5.1 The Equivalent Circuit

Figure 9.19 shows a generalized high-frequency equivalent circuit for the common-source amplifier. Here, $V'_{sig}$ and $R'_{sig}$ are the Thévenin equivalent of the signal generator together with whatever bias circuit may be present at the amplifier input (e.g., $R_G$ in the circuit of Fig. 9.2a). Resistance $R'_L$ represents the total resistance between the output (drain) node and ground and includes $R_D$, $r_o$, and $R_L$ (if one is present). Similarly, $C_L$ represents the total capacitance between the drain node and ground and includes the MOSFET's drain-to-body capacitance ($C_{db}$), the capacitance introduced by a current-source load, the input capacitance of a succeeding amplifier stage (if one is present), and in some cases, as we will see in later chapters, a deliberately introduced capacitance. In IC MOS amplifiers, $C_L$ can be substantial.

The equivalent circuit in Fig. 9.19 can also be used to represent the CE amplifier. Thus, we will not need to repeat the analysis, rather we will adapt the CS results to the CE case by simply renaming the components (i.e., replacing $C_{gs}$ by $C_\pi$ and $C_{gd}$ by $C_\mu$).

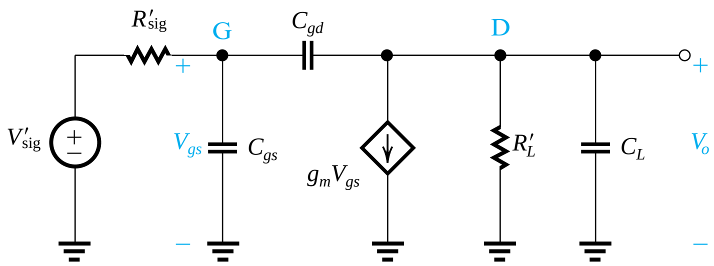



**Figure 9.19** Generalized high-frequency equivalent circuit for the CS amplifier.

## 9.5.2 Analysis Using Miller's Theorem

Miller's theorem allows us to replace the bridging capacitor $C_{gd}$ by two capacitors: $C_1$ between the input node and ground and $C_2$ between the output node and ground, as shown in Fig. 9.20. The value of $C_1$ and $C_2$ can be determined using Eqs. (9.76a) and (9.76b),

$$C_1 = C_{gd}(1 - K)$$

$$C_2 = C_{gd}\left(1 - \frac{1}{K}\right)$$

where

$$K = \frac{V_o}{V_{gs}}$$

Obviously, $K$ will depend on the value of $C_2$, which in turn depends on the value of $K$. To simplify matters, we shall adopt an iterative procedure: First, we will neglect $C_2$ and $C_L$ in

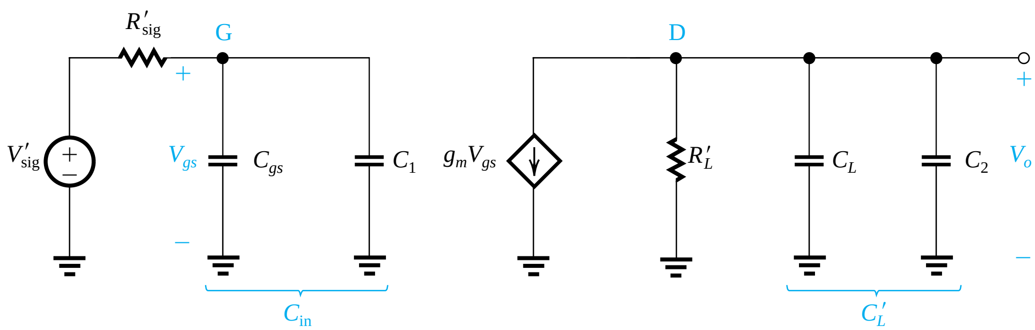



**Figure 9.20** The high-frequency equivalent circuit model of the CS amplifier after the application of Miller's theorem to replace the bridging capacitor $C_{gd}$ by two capacitors: $C_1 = C_{gd}(1 - K)$ and $C_2 = C_{gd}(1 - 1/K)$, where $K = V_o/V_{gs}$.

determining $V_o$, resulting in

$$V_o \simeq -g_m V_{gs} R'_L \tag{9.77}$$

That is, $K$ is given by

$$K \simeq -g_m R'_L$$

Then we will use this value to determine $C_1$ and $C_2$ as

$$C_1 = C_{gd}(1 + g_m R'_L) \tag{9.78}$$

$$C_2 = C_{gd}\left(1 + \frac{1}{g_m R'_L}\right) \tag{9.79}$$

Next, we use $C_1$ and $C_2$ to determine the overall transfer function $V_o/V'_{\text{sig}}$. At the input side, we see that the input capacitance $C_{\text{in}} = C_{gs} + C_1$ together with $R'_{\text{sig}}$ form an STC low-pass circuit with a pole frequency $f_{Pi}$:

$$f_{Pi} = \frac{1}{2\pi C_{\text{in}} R'_{\text{sig}}} \tag{9.80}$$

At the output sides we see that $C'_L = C_L + C_2$ together with $R'_L$ form an STC low-pass circuit with a pole frequency $f_{Po}$:

$$f_{Po} = \frac{1}{2\pi C'_L R'_L} \tag{9.81}$$

At this point we note that in Section 9.3 we neglected both $C_2$ and $C_L$ and thus $f_{Po}$. Thus the estimate of $f_H$ in Section 9.3 was based on the assumption that $V_o$ is given by Eq. (9.77), and thus the frequency limitation is caused entirely by the interaction of $C_{\text{in}}$ with $R'_{\text{sig}}$, that is, by the input pole $f_{Pi}$:

$$f_H \simeq f_{Pi} \tag{9.82}$$

A somewhat better estimate of $f_H$ can be obtained by considering both $f_{Pi}$ and $f_{Po}$, that is, by using the approximate transfer function[7]

$$\frac{V_o}{V'_{\text{sig}}} = \frac{-g_m R'_L}{\left(1 + \frac{s}{\omega_{Pi}}\right)\left(1 + \frac{s}{\omega_{Po}}\right)}$$

[7] This transfer function is approximate because we obtained it using an iterative process with in fact only one iteration!

An estimate of $f_H$ can then be found using Eq. (9.68) as

$$f_H = 1 \Bigg/ \sqrt{\frac{1}{f_{Pi}^2} + \frac{1}{f_{Po}^2}}$$

$$= \frac{f_{Pi}}{\sqrt{1 + \left(\frac{f_{Pi}}{f_{Po}}\right)^2}} \tag{9.83}$$

This estimate will diverge from that in Eq. (9.82) in situations for which $f_{Po}$ is not much higher than $f_{Pi}$. This will be the case when $R'_{\text{sig}}$ is not very high and $C_L$ is relatively large.

## Example 9.8

Consider an IC CS amplifier for which $g_m = 1.25\ \text{mA/V}^2$, $C_{gs} = 20\ \text{fF}$, $C_{gd} = 5\ \text{fF}$, $C_L = 25\ \text{fF}$, $R'_{\text{sig}} = 10\ \text{k}\Omega$, and $R'_L = 10\ \text{k}\Omega$. Assume that $C_L$ includes $C_{db}$. Determine $f_H$ using (a) the Miller approximation and (b) Miller's theorem.

### Solution

(a) The Miller approximation assumes $V_o = -g_m R'_L V_{gs}$ and thus neglects the effect of $C_L$ and $C_2$. In this case,

$$f_H \simeq f_{Pi} = \frac{1}{2\pi C_{\text{in}} R'_{\text{sig}}}$$

where

$$C_{\text{in}} = C_{gs} + C_1 = C_{gs} + C_{gd}(1 + g_m R'_L)$$

Thus,

$$C_{\text{in}} = 20 + 5(1 + 1.25 \times 10)$$

$$= 87.5\ \text{fF}$$

and $f_{Pi}$ will be

$$f_{Pi} = \frac{1}{2\pi \times 87.5 \times 10^{-15} \times 10 \times 10^3}$$

$$= 181.9\ \text{MHz}$$

Thus,

$$f_H \simeq 181.9\ \text{MHz}$$

(b) Using Miller's theorem, we obtain the same $f_{Pi}$ as above:

$$f_{Pi} = 181.9\ \text{MHz}$$

But now we can take $C_2$ and $C_L$ into account. Capacitance $C_2$ can be determined as

$$C_2 = C_{gd}\left(1 + \frac{1}{g_m R_L'}\right) = 5\left(1 + \frac{1}{12.5}\right) = 5.4 \text{ fF}$$

The frequency of the output pole can now be determined as

$$f_{Po} = \frac{1}{2\pi(C_L + C_2)R_L'}$$

$$f_{Po} = \frac{1}{2\pi(25 + 5.4) \times 10^{-15} \times 10 \times 10^{3}}$$

$$= 523.5 \text{ MHz}$$

An estimate of $f_H$ can now be found from Eq. (9.83):

$$f_H = \frac{181.9}{\sqrt{1 + \left(\frac{181.9}{523.5}\right)^2}} = 171.8 \text{ MHz}$$

## 9.5.3 Analysis Using Open-Circuit Time Constants

The method of open-circuit time constants presented in Section 9.4.3 can be directly applied to the CS equivalent circuit of Fig. 9.19, as illustrated in Fig. 9.21, from which we see that the resistance seen by $C_{gs}$, $R_{gs} = R'_{\text{sig}}$ and that seen by $C_L$ is $R'_L$. The resistance $R_{gd}$ seen by $C_{gd}$ can be found by analyzing the circuit in Fig. 9.21(b) with the result that

$$R_{gd} = R'_{\text{sig}}(1 + g_m R'_L) + R'_L \tag{9.84}$$

Thus the effective time constant $b_1$ or $\tau_H$ can be found as

$$\begin{aligned}\tau_H &= C_{gs}R_{gs} + C_{gd}R_{gd} + C_L R_{C_L} \\ &= C_{gs}R'_{\text{sig}} + C_{gd}[R'_{\text{sig}}(1 + g_m R'_L) + R'_L] + C_L R'_L\end{aligned} \tag{9.85}$$

and the 3-dB frequency $f_H$ is

$$f_H \simeq \frac{1}{2\pi\tau_H} \tag{9.86}$$

For situations in which $C_L$ is substantial, this approach yields a better estimate of $f_H$ than that obtained using the Miller approximation (simply because in the latter case we completely neglected $C_L$).

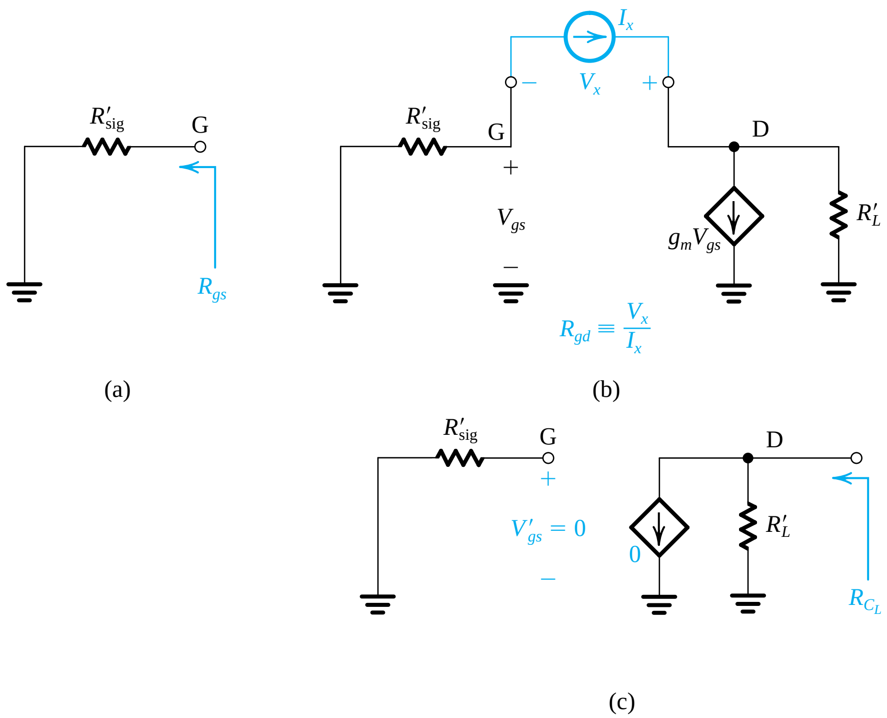



**Figure 9.21** Application of the open-circuit time-constants method to the CS equivalent circuit of Fig. 9.19.

It is interesting and useful, however, to note that applying the open-circuit time-constants method to the Miller equivalent circuit shown in Fig. 9.20 results in a very close approximation to the value of $\tau_H$ in Eq. (9.85).

## Example 9.9

Use the method of open-circuit time constants to obtain another estimate of $f_H$ for the CS amplifier of Example 9.8.

### Solution

$$R_{gs} = R'_{sig} = 10 \text{ k}\Omega$$

$$R_{gd} = R'_{sig}(1 + g_m R'_L) + R'_L$$

$$= 10(1 + 1.25 \times 10) + 10 = 145 \text{ k}\Omega$$

$$R'_L = 10 \text{ k}\Omega$$

Thus,

$$\begin{aligned}\tau_H &= C_{gs}R_{gs} + C_{gd}R_{gd} + C_L R_L' \\ &= 20 \times 10^{-15} \times 10 \times 10^3 + 5 \times 10^{-15} \times 145 \times 10^3 + 25 \times 10^{-15} \times 10 \times 10^3 \\ &= 1175 \text{ ps}\end{aligned}$$

and the 3-dB frequency $f_H$ can be estimated at

$$f_H \simeq \frac{1}{2\pi\, \tau_H} = \frac{1}{2\pi \times 1175 \times 10^{-12}} = 135.5 \text{ MHz}$$

We note that this estimate is considerably lower than both estimates found in Example 9.8. Which one is closer to the exact value will be determined next.

## 9.5.4 Exact Analysis

The approximate analysis presented above provides insight regarding the mechanism by which and the extent to which the various capacitances limit the high-frequency gain of the CS amplifier. Nevertheless, given that the circuit of Fig. 9.19 is relatively simple, it is instructive to also perform an exact analysis.[8] This is illustrated in Fig. 9.22. A node equation at the drain provides

$$sC_{gd}(V_{gs} - V_o) = g_m V_{gs} + \frac{V_o}{R_L'} + sC_L V_o$$

which can be manipulated to the form

$$V_{gs} = \frac{-V_o}{g_m R_L'} \frac{1 + s(C_L + C_{gd})R_L'}{1 - sC_{gd}/g_m} \tag{9.87}$$

A loop equation at the input yields

$$V_{\text{sig}}' = I_i R_{\text{sig}}' + V_{gs}$$

in which we can substitute for $I_i$ from a node equation at G,

$$I_i = sC_{gs}V_{gs} + sC_{gd}(V_{gs} - V_o)$$

to obtain

$$V_{\text{sig}}' = V_{gs}[1 + s(C_{gs} + C_{gd})R_{\text{sig}}'] - sC_{gd}R_{\text{sig}}' V_o$$

[8] "Exact" only in the sense that we are not making approximations in the circuit-analysis process. The reader is reminded, however, that the high-frequency model itself represents an approximation of the device performance.

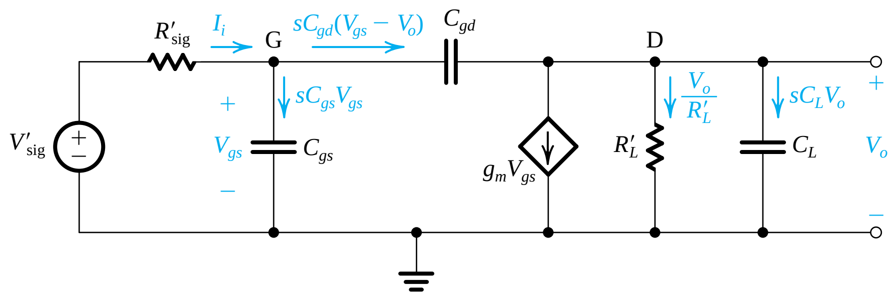



**Figure 9.22** Analysis of the CS high-frequency equivalent circuit.

We can now substitute in this equation for $V_{gs}$ from Eq. (9.87) to obtain an equation in $V_o$ and $V'_{sig}$ that can be arranged to yield the amplifier gain as

$$\frac{V_o}{V'_{sig}} = \frac{-(g_m R'_L)[1 - s(C_{gd}/g_m)]}{1 + s\{[C_{gs} + C_{gd}(1 + g_m R'_L)]R'_{sig} + (C_L + C_{gd})R'_L\} + s^2[(C_L + C_{gd})C_{gs} + C_L C_{gd}]R'_{sig}R'_L} \tag{9.88}$$

The transfer function in Eq. (9.88) has a second-order denominator, and thus the amplifier has two poles. Now, since the numerator is of the first order, it follows that one of the two transmission zeros is at infinite frequency. This is readily verifiable by noting that as $s$ approaches $\infty$, $(V_o/V'_{sig})$ approaches zero. The second zero is at

$$s = s_Z = \frac{g_m}{C_{gd}} \tag{9.89}$$

That is, it is on the positive *real axis* of the *s*-plane[9] and has a frequency $\omega_Z$,

$$\omega_Z = g_m/C_{gd} \tag{9.90}$$

Since $g_m$ is usually large and $C_{gd}$ is usually small, $f_Z$ is normally a very high frequency and thus has negligible effect on the value of $f_H$.

It is useful at this point to show a simple method for finding the value of $s$ at which $V_o = 0$—that is, $s_Z$. Figure 9.23 shows the circuit at $s = s_Z$. By definition, $V_o = 0$ and a node equation at D yields

$$s_Z C_{gd} V_{gs} = g_m V_{gs}$$

Now, since $V_{gs}$ is *not* zero (why not?), we can divide both sides by $V_{gs}$ to obtain

$$s_Z = \frac{g_m}{C_{gd}} \tag{9.91}$$

Before considering the poles, we should note that in Eq. (9.88), as $s$ goes toward zero, $V_o/V'_{sig}$ approaches the dc gain $(-g_m R'_L)$, as should be the case. Let's now take a closer look at the denominator polynomial. First, we observe that the coefficient of the $s$ term is equal to the effective time constant $\tau_H$ obtained using the open-circuit time-constants method as given by Eq. (9.85). Again, this should have been expected, since it is the basis for the open-circuit

[9] Because the transmission zero is on the real axis, there is no physical frequency $\omega$ at which the transmission is actually zero (except $\omega = \infty$).

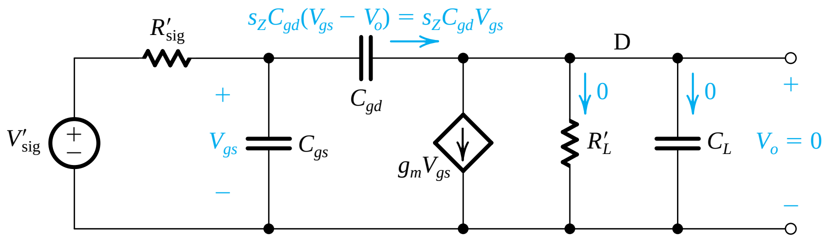



**Figure 9.23** The CS circuit at $s = s_Z$. The output voltage $V_o = 0$, enabling us to determine $s_Z$ from a node equation at D.

time-constants method (Section 9.4). Next, denoting the frequencies of the two poles $\omega_{P1}$ and $\omega_{P2}$, we can express the denominator polynomial $D(s)$ as

$$D(s) = \left(1 + \frac{s}{\omega_{P1}}\right)\left(1 + \frac{s}{\omega_{P2}}\right)$$

$$= 1 + s\left(\frac{1}{\omega_{P1}} + \frac{1}{\omega_{P2}}\right) + \frac{s^2}{\omega_{P1}\omega_{P2}} \tag{9.92}$$

Now, if $\omega_{P2} \gg \omega_{P1}$ —that is, the pole at $\omega_{P1}$ is dominant—we can approximate $D(s)$ as

$$D(s) \simeq 1 + \frac{s}{\omega_{P1}} + \frac{s^2}{\omega_{P1}\omega_{P2}} \tag{9.93}$$

Equating the coefficients of the $s$ term in denominator polynomial of Eq. (9.88) to that of the $s$ term in Eq. (9.93) gives

$$\omega_{P1} \simeq \frac{1}{[C_{gs} + C_{gd}(1 + g_m R'_L)]\, R'_{sig} + (C_L + C_{gd})R'_L} \tag{9.94}$$

where the approximation is that involved in Eq. (9.93). Note that the expression in Eq. (9.94) is identical to the value of $\omega_H$ obtained using open-circuit time constants. Equating the coefficients of $s^2$ in Eqs. (9.88) and (9.93) and using Eq. (9.94) gives the frequency of the second pole:

$$\omega_{P2} = \frac{[C_{gs} + C_{gd}(1 + g_m R'_L)]\, R'_{sig} + (C_L + C_{gd})R'_L}{[(C_L + C_{gd})C_{gs} + C_L C_{gd}]R'_L R'_{sig}} \tag{9.95}$$

## Example 9.10

For the CS amplifier considered in Examples 9.8 and 9.9, use the exact transfer function in Eq. (9.88) to determine the frequencies of the two poles and the zero and hence the 3-dB frequency $f_H$. Compare to the approximate values for $f_H$ obtained in Examples 9.8 and 9.9.

**Example 9.10** *continued*

## Solution

The frequency of the zero is determined using Eq. (9.90),

$$f_Z = \frac{g_m}{2\pi C_{gd}} = \frac{1.25 \times 10^{-3}}{2\pi \times 5 \times 10^{-15}} = 40 \text{ GHz}$$

The frequencies of the two poles, $\omega_{P1}$ and $\omega_{P2}$, are found as the roots of the equation obtained by equating the denominator polynomial of Eq. (9.88) to zero:

$$1 + 1.175 \times 10^{-9} s + 7.25 \times 10^{-20} s^2 = 0$$

The result is

$$f_{P1} = 143.4 \text{ MHz}$$

and

$$f_{P2} = 2.44 \text{ GHz}$$

Since $f_Z, f_{P2} \gg f_{P1}$, a good estimate for $f_H$ is

$$f_H \simeq f_{P1} = 143.4 \text{ MHz}$$

Finally, we note that the estimate of $f_{P1}$ obtained using Eq. (9.94) is 135.5 MHz, which is about 5.5% lower than the exact value. Thus, the method of open-circuit time constants underestimates $f_H$ by about 5.5%. The estimate from the Miller approximation is 181.9 MHz, which is about 27% higher than the exact value, and that using the refined application of Miller theorem is 171.8 MHz, which is about 20% higher than the exact value. We conclude that the estimate obtained using open-circuit time constants is remarkably good!

## EXERCISES

**9.16** For the CS amplifier in Example 9.10, using the value of $f_H$ determined by the exact analysis, find the gain–bandwidth product. Recall that $g_m = 1.25$ mA/V and $R_L' = 10$ kΩ. Also, convince yourself that this is the frequency at which the gain magnitude reduces to unity, that is, $f_t$.
**Ans.** GBW = 1.79 GHz; since this is lower than $f_{P2}$, then $f_t = 1.79$ GHz

**9.17** As a way to trade gain for bandwidth, the designer of the CS amplifier in Example 9.10 connects a load resistor at the output that results in halving the value of $R_L'$. Find the new values of $|A_M|$, $f_H$ (using $f_H \simeq f_{P1}$ of Eq. 9.94), and $f_t$.
**Ans.** 6.25 V/V; 223 MHz; 1.4 GHz

**9.18** As another way to trade dc gain for bandwidth, the designer of the CS amplifier in Example 9.10 decides to operate the amplifying transistor at double the value of $V_{OV}$ by increasing the bias current fourfold. Find the new values of $g_m$, $R_L'$, $|A_M|$, $f_{P1}$, $f_H$, and $f_t$. Assume that $R_L'$ is the parallel equivalent of $r_o$ of the amplifying transistor and that of the current-source load. Use the approximate formula for $f_{P1}$ given in Eq. (9.94).
**Ans.** 2.5 mA/V; 2.5 kΩ; 6.25 V/V; 250 MHz; 250 MHz; 1.56 GHz

## 9.5.5 Adapting the Formulas for the Case of the CE Amplifier

Adapting the formulas presented above to the case of the CE amplifier is straightforward. First, note from Fig. 9.24 how $V'_{sig}$ and $R'_{sig}$ relate to $V_{sig}$, $R_{sig}$, and the other equivalent-circuit parameters:

$$V'_{sig} = V_{sig}\frac{r_\pi}{R_{sig}+r_x+r_\pi} \tag{9.96}$$

$$R'_{sig} = r_\pi \,\|\, (R_{sig}+r_x) \tag{9.97}$$

Thus the dc gain is now given by

$$A_M = -\frac{r_\pi}{R_{sig}+r_x+r_\pi}(g_m R'_L) \tag{9.98}$$

Using the Miller approximation, we obtain

$$C_{in} = C_\pi + C_\mu(1+g_m R'_L) \tag{9.99}$$

Correspondingly, the 3-dB frequency $f_H$ can be estimated from

$$f_H \simeq \frac{1}{2\pi C_{in} R'_{sig}} \tag{9.100}$$

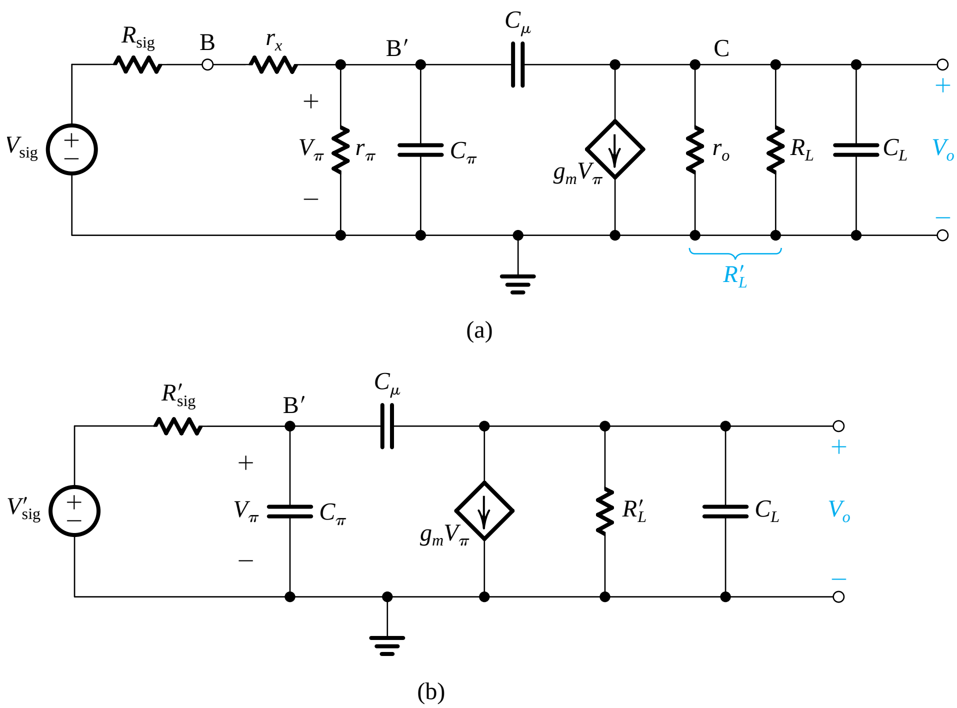



**Figure 9.24** **(a)** High-frequency equivalent circuit of the common-emitter amplifier. **(b)** Equivalent circuit obtained after Thévenin theorem has been employed to simplify the resistive circuit at the input.

Alternatively, using the method of open-circuit time constants yields

$$\begin{aligned}\tau_H &= C_\pi R_\pi + C_\mu R_\mu + C_L R_{C_L} \\ &= C_\pi R'_{\text{sig}} + C_\mu[(1+g_m R'_L)R'_{\text{sig}} + R'_L] + C_L R'_L\end{aligned} \quad (9.101)$$

from which $f_H$ can be estimated as

$$f_H \simeq \frac{1}{2\pi\tau_H} \quad (9.102)$$

The exact analysis yields the following zero frequency:

$$f_Z = \frac{1}{2\pi}\frac{g_m}{C_\mu} \quad (9.103)$$

and, assuming that a dominant pole exists,

$$f_{P1} \simeq \frac{1}{2\pi}\frac{1}{[C_\pi + C_\mu(1+g_m R'_L)]R'_{\text{sig}} + (C_L + C_\mu)R'_L} \quad (9.104)$$

$$f_{P2} \simeq \frac{1}{2\pi}\frac{[C_\pi + C_\mu(1+g_m R'_L)]R'_{\text{sig}} + (C_L + C_\mu)R'_L}{[C_\pi(C_L + C_\mu) + C_L C_\mu]R'_{\text{sig}}R'_L} \quad (9.105)$$

For $f_Z$, $f_{P2} \gg f_{P1}$,

$$f_H \simeq f_{P1}$$

## EXERCISE

**9.19** Consider a bipolar active-loaded CE amplifier having the load current source implemented with a *pnp* transistor. Let the circuit be operating at a 1-mA bias current. The transistors are specified as follows: $\beta(npn) = 200$, $V_{An} = 130$ V, $|V_{Ap}| = 50$ V, $C_\pi = 16$ pF, $C_\mu = 0.3$ pF, $C_L = 5$ pF, and $r_x = 200\ \Omega$. The amplifier is fed with a signal source having a resistance of 36 k$\Omega$. Determine: (a) $A_M$; (b) $C_{\text{in}}$ and $f_H$ using the Miller approximation; (c) $f_H$ using open-circuit time constants; (d) $f_Z$, $f_{P1}$, $f_{P2}$, and hence $f_H$ (use the approximate expressions in Eqs. 9.105 and 9.104); and (e) the gain–bandwidth product.

**Ans.** (a) –175 V/V; (b) 448 pF, 82.6 kHz; (c) 75.1 kHz; (d) 21.2 GHz, 75.1 kHz, 25.2 MHz, 75.1 kHz; (e) 13.1 MHz

### 9.5.6 The Situation When $R_{\text{sig}}$ Is Low

There are applications in which the CS amplifier is fed with a low-resistance signal source. Obviously, in such a case, the high-frequency gain will no longer be limited by the interaction of the source resistance and the input capacitance. Rather, the high-frequency limitation happens at the amplifier output, as we shall now show.

Figure 9.25(a) shows the high-frequency equivalent circuit of the common-source amplifier in the limiting case when $R_{\text{sig}}$ is zero. The voltage transfer function $V_o/V_{\text{sig}} = V_o/V_{gs}$ can be

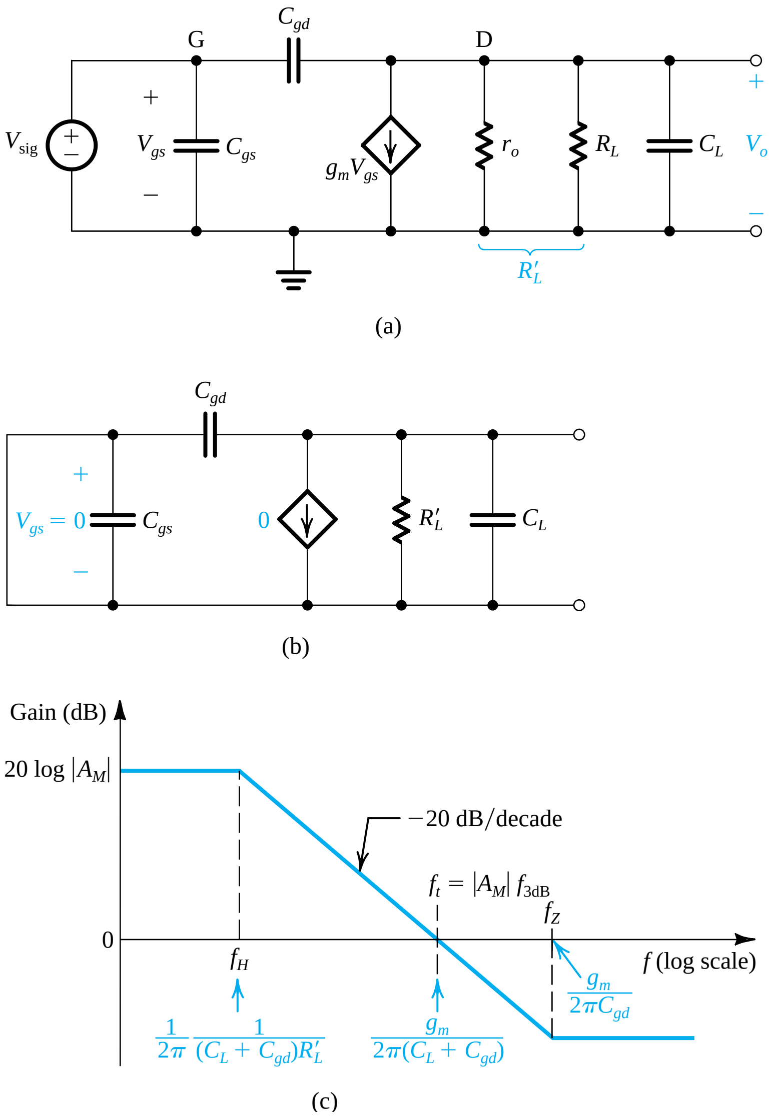



**Figure 9.25** **(a)** High-frequency equivalent circuit of a CS amplifier fed with a signal source having a very low (effectively zero) resistance. **(b)** The circuit with $V_{sig}$ reduced to zero. **(c)** Bode plot for the gain of the circuit in **(a)**.

found by setting $R_{sig} = 0$ in Eq. (9.88). The result is

$$\frac{V_o}{V_{sig}} = \frac{(-g_m R_L')[1 - s(C_{gd}/g_m)]}{1 + s(C_L + C_{gd})R_L'} \tag{9.106}$$

Thus, while the dc gain and the frequency of the zero do not change, the high-frequency response is now determined by a pole formed by $C_L + C_{gd}$ together with $R_L'$. Thus the 3-dB frequency is now given by

$$f_H = \frac{1}{2\pi(C_L + C_{gd})R_L'} \tag{9.107}$$

To see how this pole is formed, refer to Fig. 9.25(b), which shows the equivalent circuit with the input signal source reduced to zero. Observe that the circuit reduces to a capacitance $(C_L + C_{gd})$ in parallel with a resistance $R_L'$.

As we have seen above, the transfer-function zero is usually at a very high frequency and thus does not play a significant role in shaping the high-frequency response. The gain of the CS amplifier will therefore fall off at a rate of –6 dB/octave (–20 dB/decade), reaching unity (0 dB) at a frequency $f_t$, which is equal to the **gain–bandwidth product**,

$$\begin{aligned} f_t &= |A_M|\, f_H \\ &= g_m R_L' \frac{1}{2\pi(C_L + C_{gd})R_L'} \end{aligned}$$

Thus,

$$f_t = \frac{g_m}{2\pi(C_L + C_{gd})} \tag{9.108}$$

Figure 9.25(c) shows a sketch of the high-frequency gain of the CS amplifier.

## Example 9.11

Consider the CS amplifier specified in Example 9.8 when fed with a signal source having a negligible resistance (i.e., $R_{sig} = 0$). Find $A_M$, $f_{3dB}$, $f_t$, and $f_Z$. If the amplifying transistor is to be operated at twice the original overdrive voltage while $W$ and $L$ remain unchanged, by what factor must the bias current be changed? What are the new values of $A_M$, $f_{3dB}$, $f_t$, and $f_Z$? Assume that $R_L'$ is the parallel equivalent of $r_o$ of the amplifying transistor and that of the current-source load.

### Solution

In Example 9.8 we found that

$$A_M = -g_m R_L' = -12.5 \text{ V/V}$$

The 3-dB frequency can be found using Eq. (9.107),

$$\begin{aligned} f_H &= \frac{1}{2\pi(C_L + C_{gd})R_L'} \\ &= \frac{1}{2\pi(25+5)\times 10^{-15} \times 10 \times 10^3} \\ &= 530.5 \text{ MHz} \end{aligned}$$

and the unity-gain frequency, which is equal to the gain–bandwidth product, can be determined as

$$f_t = |A_M|\, f_H = 12.5 \times 530.5 = 6.63 \text{ GHz}$$

The frequency of the zero is

$$f_Z = \frac{1}{2\pi}\frac{g_m}{C_{gd}}$$

$$= \frac{1}{2\pi}\frac{1.25\times10^{-3}}{5\times10^{-15}} \simeq 40 \text{ GHz}$$

Now, to double $V_{OV}$, $I_D$ must be quadrupled. The new values of $g_m$ and $R_L'$ can be found as follows:

$$g_m = \frac{I_D}{V_{OV}/2} = 2.5 \text{ mA/V}$$

$$R_L' = \frac{1}{4}\times 10 = 2.5\text{k}\Omega$$

Thus the new value of $A_M$ becomes

$$A_M = -g_m R_L' = -2.5\times 2.5 = -6.25 \text{ V/V}$$

That of $f_H$ becomes

$$f_H = \frac{1}{2\pi(C_L + C_{gd})R_L'}$$

$$= \frac{1}{2\pi(25+5)\times10^{-15}\times 2.5\times10^{3}}$$

$$= 2.12 \text{ GHz}$$

and the unity-gain frequency (i.e., the gain–bandwidth product) becomes

$$f_t = 6.25\times 2.12 = 13.3 \text{ GHz}$$

We note that doubling $V_{OV}$ results in reducing the dc gain by a factor of 2 and increasing the bandwidth by a factor of 4. Thus, the gain–bandwidth product is doubled—a good bargain!

## EXERCISES

**9.20** For the CS amplifier considered in Example 9.11 operating at the original values of $V_{OV}$ and $I_D$, find the value to which $C_L$ should be increased to place $f_t$ at 2 GHz.
**Ans.** 94.4 fF

**9.21** Show that the CS amplifier when fed with $R_{\text{sig}} = 0$ has a transfer-function zero whose frequency is related to $f_t$ by

$$\frac{f_Z}{f_t} = 1 + \frac{C_L}{C_{gd}}$$

# 9.6 High-Frequency Response of the Common-Gate and Cascode Amplifiers

Although common-source and common-emitter amplifiers provide substantial gain at midband frequencies, their gain falls off in the high-frequency band at a relatively low frequency. This is primarily due to the large input capacitance $C_{in}$, whose value is significantly increased by the Miller component. The latter is large because of the Miller multiplication effect. It follows that the key to obtaining wideband operation, that is, high $f_H$, is to use circuit configurations that do not suffer from the Miller effect. One such configuration is the common-gate circuit.

## 9.6.1 High-Frequency Response of the CG Amplifier

Figure 9.26(a) shows the CG amplifier with the MOSFET internal capacitances $C_{gs}$ and $C_{gd}$ indicated. For generality, a capacitance $C_L$ is included at the output node to represent the combination of the output capacitance of a current-source load and the input capacitance of a succeeding amplifier stage. Capacitance $C_L$ also includes the MOSFET capacitance $C_{db}$. Note the $C_L$ appears in effect in parallel with $C_{gd}$; therefore, in the following discussion we will lump the two capacitances together.

It is important to note at the outset that each of the three capacitances in the circuit of Fig. 9.26(a) has a grounded node. Therefore none of the capacitances undergoes the Miller multiplication effect observed in the CS stage. It follows that the CG circuit can be designed to have a much wider bandwidth than that of the CS circuit, especially when the resistance of the signal generator is large.

Analysis of the circuit in Fig. 9.26(a) is greatly simplified if $r_o$ can be neglected. In such a case the input side is isolated from the output side, and the high-frequency equivalent circuit takes the form shown in Fig. 9.26(b). We immediately observe that there are two poles: one at the input side with a frequency $f_{P1}$,

$$f_{P1} = \frac{1}{2\pi C_{gs}\left(R_{sig} \| \frac{1}{g_m}\right)} \tag{9.109}$$

and the other at the output side with a frequency $f_{P2}$,

$$f_{P2} = \frac{1}{2\pi(C_{gd} + C_L)R_L} \tag{9.110}$$

The relative locations of the two poles will depend on the specific situation. However, $f_{P2}$ is usually lower than $f_{P1}$; thus $f_{P2}$ can be dominant. The important point to note is that both $f_{P1}$ and $f_{P2}$ are usually much higher than the frequency of the dominant input pole in the CS stage.

In IC amplifiers, $r_o$ has to be taken into account. In these cases, the method of open-circuit time constants can be employed to obtain an estimate for the 3-dB frequency $f_H$. Figure 9.27 shows the circuits for determining the resistances $R_{gs}$ and $R_{gd}$ seen by $C_{gs}$ and $(C_{gd} + C_L)$, respectively. By inspection we obtain

$$R_{gs} = R_{sig} \| R_{in} \tag{9.111}$$

and

$$R_{gd} = R_L \| R_o \tag{9.112}$$

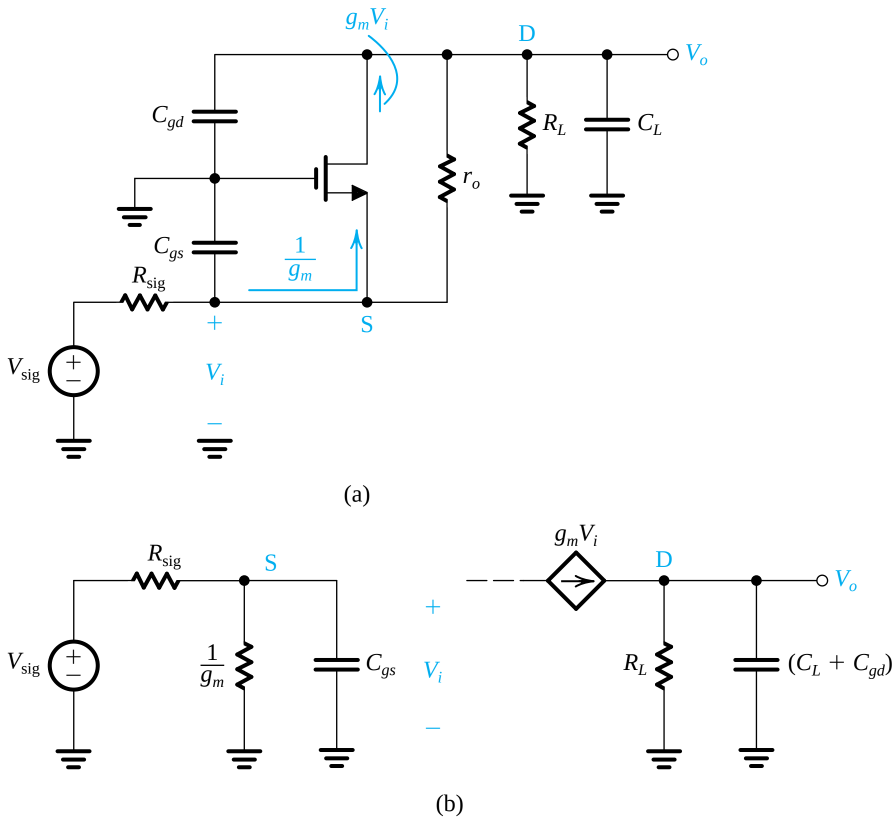



**Figure 9.26** **(a)** The common-gate amplifier with the transistor internal capacitances shown. A load capacitance $C_L$ is also included. **(b)** Equivalent circuit for the case in which $r_o$ is neglected.

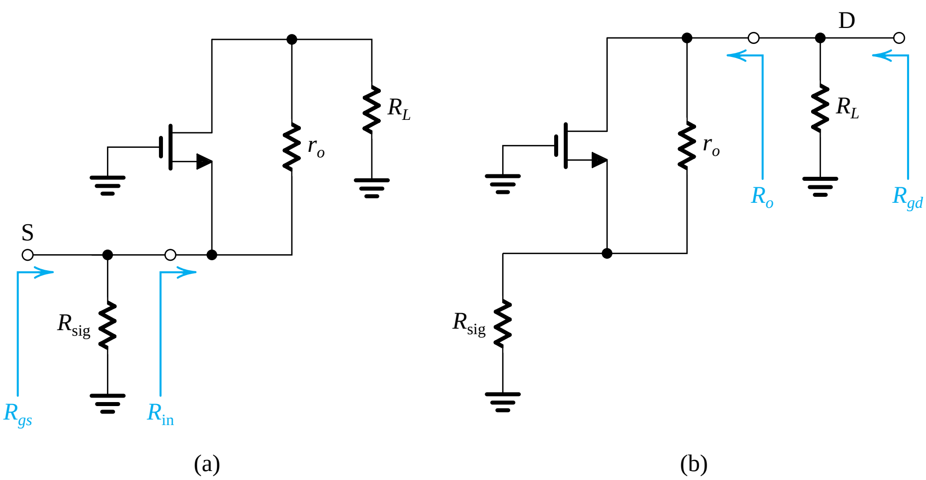



**Figure 9.27** Circuits for determining $R_{gs}$ and $R_{gd}$.

which can be used to obtain $f_H$,

$$f_H = \frac{1}{2\pi[C_{gs}R_{gs} + (C_{gd} + C_L)R_{gd}]} \tag{9.113}$$

Finally note that the input resistance $R_{in}$ and output resistance $R_o$ of the CG amplifier were derived in Section 7.3 and are summarized in Fig. 7.13, from which we obtain

$$R_{in} = \frac{r_o + R_L}{1 + g_m r_o} \tag{9.114}$$

and

$$R_o = r_o + R_{sig} + (g_m r_o)R_{sig} \tag{9.115}$$

## Example 9.12

Consider a common-gate amplifier with $g_m = 1.25$ mA/V, $r_o = 20\ \text{k}\Omega$, $C_{gs} = 20$ fF, $C_{gd} = 5$ fF, $C_L = 15$ fF, $R_{sig} = 10\ \text{k}\Omega$, and $R_L = 20\ \text{k}\Omega$. Assume that $C_L$ includes $C_{db}$. Determine the input resistance, the midband gain, and the upper 3-dB frequency $f_H$.

### Solution

Figure 9.28 shows the CG amplifier circuit at midband frequencies. We note that

$$v_o = iR_L$$

$$v_{sig} = i(R_{sig} + R_{in})$$

Thus, the overall voltage gain is given by

$$G_v = \frac{v_o}{v_{sig}} = \frac{R_L}{R_{sig} + R_{in}}$$

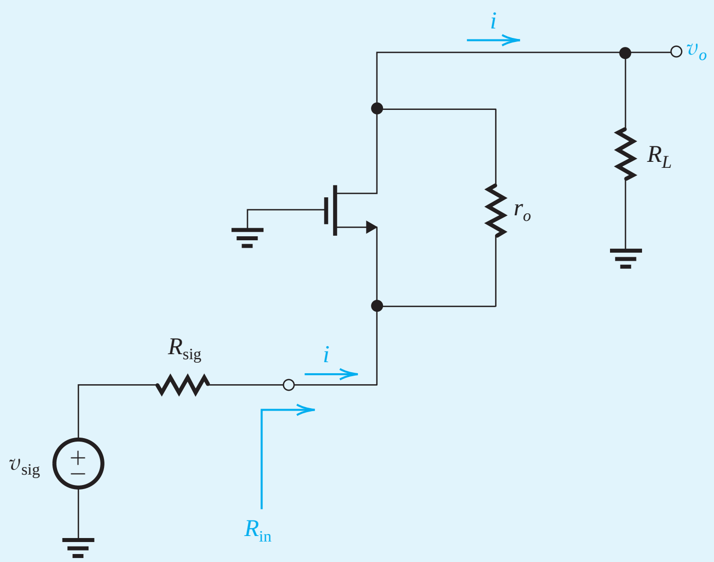



**Figure 9.28** The CG amplifier circuit at midband.

The value of $R_{\text{in}}$ is found from Eq. (9.114) as

$$R_{\text{in}} = \frac{r_o + R_L}{1 + g_m r_o}$$

$$= \frac{20 + 20}{1 + (1.25 \times 20)} = 1.54 \text{ k}\Omega$$

Thus, $G_v$ can now be determined as

$$G_v = \frac{20}{10 + 1.54} = 1.73 \text{ V/V}$$

Observe that as expected $G_v$ is very low. This is due to the low input resistance of the CG amplifier.

To obtain an estimate of the 3-dB frequency $f_H$, we first determine $R_{gs}$ and $R_{gd}$ using Eqs. (9.111) and (9.112),

$$R_{gs} = R_{\text{sig}} \| R_{\text{in}} = 10 \| 1.54 = 1.33 \text{ k}\Omega$$
$$R_{gd} = R_L \| R_o$$

where $R_o$ is given by Eq. (9.115),

$$R_o = r_o + R_{\text{sig}} + (g_m r_o) R_{\text{sig}}$$

$$= 20 + 10 + 25 \times 10 = 280 \text{ k}\Omega$$

Thus,

$$R_{gd} = 20 \| 280 = 18.7 \text{ k}\Omega$$

Now we can compute the sum of the open-circuit time constants, $\tau_H$,

$$\tau_H = C_{gs} R_{gs} + (C_{gd} + C_L) R_{gd}$$
$$\tau_H = 20 \times 10^{-15} \times 1.33 \times 10^3 + (5 + 15) \times 10^{-15} \times 18.7 \times 10^3$$
$$= 26.6 \times 10^{-12} + 374 \times 10^{-12}$$
$$= 400.6 \text{ ps}$$

and the upper 3-dB frequency $f_H$ can be obtained as

$$f_H = \frac{1}{2\pi\tau_H} = \frac{1}{2\pi \times 400.6 \times 10^{-12}} = 397.3 \text{ MHz}$$

Observe that $f_H$ is indeed much higher (more than twice) the corresponding value for the CS amplifier found in Example 9.9. Another important observation can be made by examining the two components of $\tau_H$: The contribution of the input circuit is 26.6 ps, while that of the output circuit is 374 ps; thus the limitation on the high-frequency response is posed by the output circuit.

**EXERCISE**

**9.22** In order to raise the midband gain of the CG amplifier in Example 9.12, the circuit designer decides to use a cascode current source for the load device, thus raising $R_L$ by a factor of $g_m r_o = 25$; that is, $R_L$ becomes 500 k$\Omega$. Find $R_{\text{in}}$, the midband gain, and $f_H$. Comment on the results.
**Ans.** 20 k$\Omega$; 16.7 V/V; 42.7 MHz. While the midband gain has been increased substantially (by a factor of 9.7), the bandwidth $f_H$ has been substantially lowered (by a factor of about 9.3). Thus, the high-frequency advantage of the CG amplifier is completely lost!

We conclude this section by noting that a properly designed CG circuit can have a wide bandwidth. However, the input resistance will be low and the overall midband gain can be very low. It follows that the CG circuit alone will not do the job! However, combining the CG with the CS amplifier in the cascode configuration can result in a circuit having the high input resistance and gain of the CS amplifier together with the wide bandwidth of the CG amplifier, as we shall now see.

## 9.6.2 High-Frequency Response of the MOS Cascode Amplifier

In Section 7.3 we studied the cascode amplifier and analyzed its performance at midband frequencies. There we learned that by combining the CS and CG configurations, the cascode amplifier exhibits a very high input resistance and a voltage gain that can be as high as $A_0^2$, where $A_0 = g_m r_o$ is the intrinsic gain of the MOSFET. For our purposes here, we shall see that the versatility of the cascode circuit allows us to trade off some of this high midband gain in return for a wider bandwidth.

Figure 9.29 shows the cascode amplifier with all transistor internal capacitances indicated. Also included is a capacitance $C_L$ at the output node to represent the combination of $C_{db2}$, the output capacitance of a current-source load, and the input capacitance of a succeeding

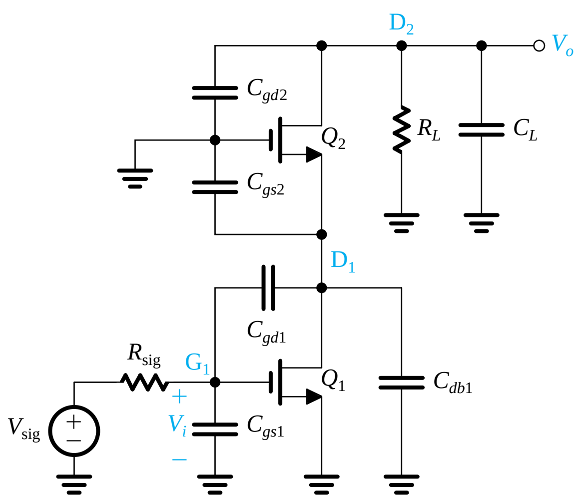



**Figure 9.29** The cascode circuit with the various transistor capacitances indicated.

amplifier stage (if any). Note that $C_{db1}$ and $C_{gs2}$ appear in parallel, and we shall combine them in the following analysis. Similarly, $C_L$ and $C_{gd2}$ appear in parallel and will be combined.

The easiest and, in fact, quite insightful approach to determining the 3-dB frequency $f_H$ is to employ the open-circuit time-constants method:

1. Capacitance $C_{gs1}$ sees a resistance $R_{sig}$.
2. Capacitance $C_{gd1}$ sees a resistance $R_{gd1}$, which can be obtained by adapting the formula in Eq. (9.84) to

$$R_{gd1} = (1+g_{m1}R_{d1})R_{sig} + R_{d1} \tag{9.116}$$

   where $R_{d1}$, the total resistance at $D_1$, is given by

$$R_{d1} = r_{o1} \| R_{in2} = r_{o1} \| \frac{r_{o2}+R_L}{g_{m2}r_{o2}} \tag{9.117}$$

3. Capacitance $(C_{db1}+C_{gs2})$ sees a resistance $R_{d1}$.
4. Capacitance $(C_L + C_{gd2})$ sees a resistance $(R_L \| R_o)$ where $R_o$ is given by

$$R_o = r_{o2} + r_{o1} + (g_{m2}r_{o2})r_{o1}$$

With the resistances determined, the effective time constant $\tau_H$ can be computed as

$$\begin{aligned}\tau_H = {} & C_{gs1}R_{sig} + C_{gd1}[(1+g_{m1}R_{d1})R_{sig} + R_{d1}] \\ & + (C_{db1}+C_{gs2})R_{d1} + (C_L + C_{gd2})(R_L \| R_o)\end{aligned} \tag{9.118}$$

and the 3-dB frequency $f_H$ as

$$f_H \simeq \frac{1}{2\pi\tau_H}$$

To gain insight regarding what limits the high-frequency gain of the MOS cascode amplifier, we rewrite Eq. (9.118) in the form

$$\begin{aligned}\tau_H = {} & R_{sig}[C_{gs1} + C_{gd1}(1+g_{m1}R_{d1})] + R_{d1}(C_{gd1}+C_{db1}+C_{gs2}) \\ & + (R_L \| R_o)(C_L + C_{gd2})\end{aligned} \tag{9.119}$$

In the case of a large $R_{sig}$, the first term can dominate, especially if the Miller multiplier $(1+g_{m1}R_{d1})$ is large. This in turn happens when the load resistance $R_L$ is large (on the order of $A_0 r_o$), causing $R_{in2}$ to be large and requiring the first stage, $Q_1$, to provide a large proportion of the gain (see Section 7.3.3). It follows that when $R_{sig}$ is large, to extend the bandwidth we have to lower $R_L$ to the order of $r_o$. This in turn lowers $R_{in2}$ and hence $R_{d1}$ and renders the Miller effect in $Q_1$ insignificant. Note, however, that the dc gain of the cascode will then be $A_0$. Thus, while the dc gain will be the same as (or a little higher than) that achieved in a CS amplifier, the bandwidth will be greater.

In the case when $R_{sig}$ is small, the Miller effect in $Q_1$ will not be of concern. A large value of $R_L$ (on the order of $A_0 r_o$) can then be used to realize the large dc gain possible with a cascode amplifier—that is, a dc gain on the order of $A_0^2$. Equation (9.119) indicates that in this case the third term will usually be dominant. To pursue this point a little further, consider the case $R_{sig} = 0$, and assume that the middle term is much smaller than the third term. It follows that

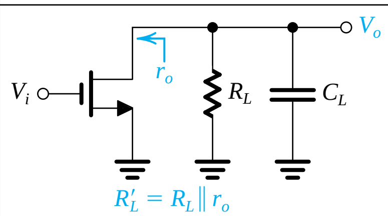
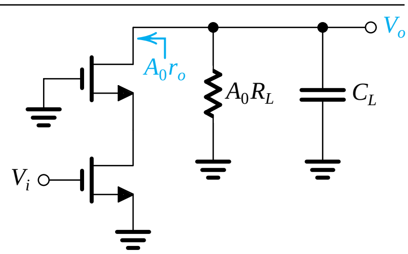
$$\tau_H \simeq (C_L + C_{gd2})(R_L \| R_o)$$

and the 3-dB frequency becomes

$$f_H = \frac{1}{2\pi(C_L + C_{gd2})(R_L \parallel R_o)} \tag{9.120}$$

which is of the same form as the formula for the CS amplifier with $R_{\text{sig}} = 0$ (Eq. 9.107). Here, however, $(R_L \parallel R_o)$ is larger that $R'_L$ by a factor of about $A_0$. Thus the $f_H$ of the cascode will be lower than that of the CS amplifier by the same factor $A_0$. Figure 9.30 shows a sketch of the frequency response of the cascode and of the corresponding common-source amplifier. We observe that in this case, cascoding increases the dc gain by a factor $A_0$ while keeping the unity-gain frequency unchanged at

$$f_t \simeq \frac{1}{2\pi}\frac{g_m}{C_L + C_{gd2}} \tag{9.121}$$

| | Common Source | Cascode |
|---|---|---|
| Circuit |   |   |
| DC Gain | $-g_m R'_L$ | $-A_0 g_m R'_L$ |
| $f_{3\text{dB}}$ | $\dfrac{1}{2\pi(C_L + C_{gd})R'_L}$ | $\dfrac{1}{2\pi(C_L + C_{gd})A_0 R'_L}$ |
| $f_t$ | $\dfrac{g_m}{2\pi(C_L + C_{gd})}$ | $\dfrac{g_m}{2\pi(C_L + C_{gd})}$ |

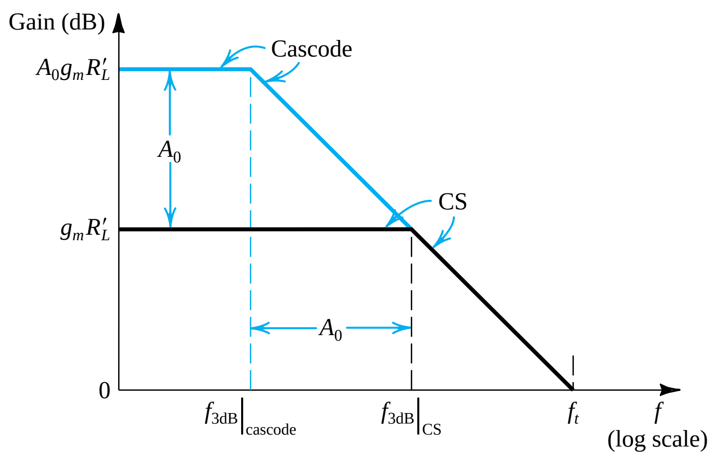



**Figure 9.30** Effect of cascoding on gain and bandwidth in the case $R_{\text{sig}} = 0$. Cascoding can increase the dc gain by the factor $A_0$ while keeping the unity-gain frequency constant. Note that to achieve the high gain, the load resistance must be increased by the factor $A_0$.

## Example 9.13

This example illustrates the advantages of cascoding by comparing the performance of a cascode amplifier with that of a common-source amplifier in two cases:

(a) The resistance of the signal source is significant, $R_{sig} = 10\ k\Omega$.

(b) $R_{sig}$ is negligibly small.

Assume all MOSFETs have $g_m = 1.25$ mA/V, $r_o = 20\ k\Omega$, $C_{gs} = 20$ fF, $C_{gd} = 5$ fF, $C_{db} = 5$ fF, and $C_L$ (excluding $C_{db}$) = 10 fF. For case (a), let $R_L = r_o = 20\ k\Omega$ for both amplifiers. For case (b), let $R_L = r_o = 20\ k\Omega$ for the CS amplifier and $R_L = R_o$ for the cascode amplifier. For all cases, determine $A_v$, $f_H$, and $f_t$.

### Solution

(a) For the CS amplifier:

$$A_0 = g_m r_o = 1.25 \times 20 = 25 \text{ V/V}$$
$$A_v = -g_m(R_L \| r_o) = -g_m(r_o \| r_o)$$
$$= -\tfrac{1}{2}A_0 = -12.5 \text{ V/V}$$
$$\tau_H = C_{gs}R_{sig} + C_{gd}[(1 + g_m R_L')R_{sig} + R_L'] + (C_L + C_{db})R_L'$$

where

$$R_L' = r_o \| R_L = r_o \| r_o = 10\ k\Omega$$
$$\tau_H = 20 \times 10 + 5[(1 + 12.5)10 + 10] + (10 + 5)10$$
$$= 200 + 725 + 150 = 1075 \text{ ps}$$

Thus,

$$f_H = \frac{1}{2\pi \times 1075 \times 10^{-12}} = 148 \text{ MHz}$$
$$f_t = |A_v| f_H = 12.5 \times 148 = 1.85 \text{ GHz}$$

For the cascode amplifier:

$$R_o = 2r_o + (g_m r_o)r_o = (2 \times 20) + (25 \times 20) = 540\ k\Omega$$
$$A_v = -g_m(R_o \| R_L)$$
$$= -1.25(540 \| 20) = -24.1 \text{ V/V}$$
$$R_{in2} = \frac{r_o + R_L}{g_m r_o} = \frac{r_o + r_o}{g_m r_o} = \frac{2}{g_m} = \frac{2}{1.25} = 1.6\ k\Omega$$
$$R_{d1} = r_o \| R_{in2} = 20 \| 1.6 = 1.48\ k\Omega$$
$$\tau_H = R_{sig}[C_{gs1} + C_{gd1}(1 + g_{m1}R_{d1})]$$
$$+ R_{d1}(C_{gd1} + C_{db1} + C_{gs2})$$
$$+ (R_L \| R_o)(C_L + C_{db2} + C_{gd2})$$

**Example 9.13** *continued*

$$
\begin{aligned}
&= 10[20 + 5(1 + 1.25 \times 1.48)] \\
&\quad + 1.48(5 + 5 + 20) \\
&\quad + (20 \,\|\, 540)(10 + 5 + 5) \\
&= 342.5 + 44.4 + 385.7 \\
&= 772.6 \text{ ps}
\end{aligned}
$$

$$f_H = \frac{1}{2\pi \times 772.6 \times 10^{-12}} = 206 \text{ MHz}$$

$$f_t = 24.1 \times 206 = 4.96 \text{ GHz}$$

Thus cascoding has increased $f_t$ by a factor of 2.7.

(b) For the CS amplifier:

$$A_v = -12.5 \text{ V/V}$$

$$
\begin{aligned}
\tau_H &= (C_{gd} + C_L + C_{db})R_L' \\
&= (5 + 10 + 5)10 = 200 \text{ ps}
\end{aligned}
$$

$$f_H = \frac{1}{2\pi \times 200 \times 10^{-12}} = 796 \text{ MHz}$$

$$f_t = 12.5 \times 796 = 9.95 \text{ GHz}$$

For the cascode amplifier:

$$R_L = R_o = 540 \text{ k}\Omega$$

$$
\begin{aligned}
A_v &= -g_m(R_o \,\|\, R_L) \\
&= -1.25(540 \,\|\, 540) = -337.5 \text{ V/V}
\end{aligned}
$$

$$R_{\text{in}2} = \frac{r_o + R_L}{g_m r_o} = \frac{20 + 540}{1.25 \times 20} = 22.4 \text{ k}\Omega$$

$$R_{d1} = r_{o1} \,\|\, R_{\text{in}2} = 20 \,\|\, 22.4 = 10.6 \text{ k}\Omega$$

$$
\begin{aligned}
\tau_H &= R_{d1}(C_{gd1} + C_{db1} + C_{gs2}) + (R_L \,\|\, R_o)(C_L + C_{gd2} + C_{db2}) \\
&= 10.6(5 + 5 + 20) + (540 \,\|\, 540)(10 + 5 + 5) \\
&= 318 + 5400 = 5718 \text{ ps}
\end{aligned}
$$

$$f_H = \frac{1}{2\pi \times 5718 \times 10^{-12}} = 27.8 \text{MHz}$$

$$f_t = 337.5 \times 27.8 = 9.39 \text{ GHz}$$

Thus cascoding increases the dc gain from 12.5 V/V to 337.5 V/V. The unity-gain frequency (which, in this case, is equal to the gain–bandwidth product), however, remains nearly constant.

**EXERCISE**

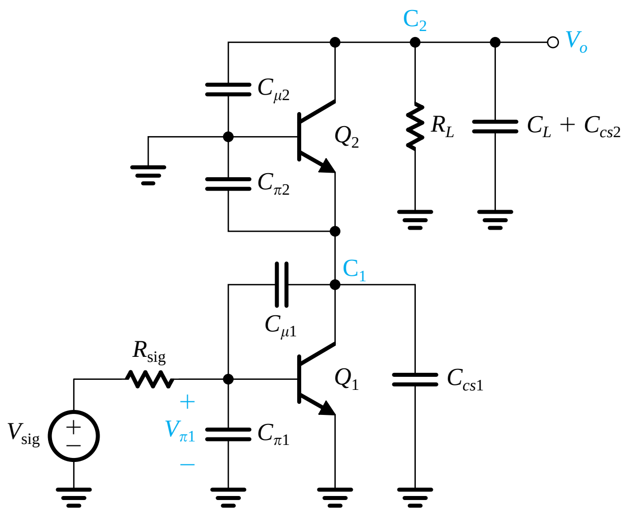
**9.23** In this exercise we wish to contrast the gain of a CS amplifier and a cascode amplifier. Assume that both are fed with a large source resistance $R_{sig}$ that effectively determines the high-frequency response. Thus, neglect components of $\tau_H$ that do not include $R_{sig}$. Also assume that all transistors are operated at the same conditions and thus corresponding small-signal parameters are equal. Also, both amplifiers have equal $R_L = r_o$, and $g_m r_o = 40$.

(a) Find the ratio of the low-frequency gain of the cascode amplifier to that of the CS amplifier.
(b) For the case of $C_{gd} = 0.25C_{gs}$, find the ratio of $f_H$ of the cascode to that of the CS amplifier.
(c) Use (a) and (b) to find the ratio of $f_t$ of the cascode to that of the CS.

**Ans.** 2; 3.6; 7.2

## 9.6.3 High-Frequency Response of the Bipolar Cascode Amplifier

The analysis method studied in the previous section can be directly applied to the BJT cascode amplifier. Figure 9.31 presents the circuits and the formulas for determining the high-frequency response of the bipolar cascode.




$$R'_{sig} = r_{\pi1} \| (r_{x1} + R_{sig})$$
$$R_{\pi1} = R'_{sig}$$
$$R_{\mu1} = R'_{sig}(1 + g_{m1}R_{c1}) + R_{c1}$$
$$R_{c1} = r_{o1} \left\| \left[ r_{e2}\left( \frac{r_{o2} + R_L}{r_{o2} + R_L/(\beta_2 + 1)} \right) \right] \right.$$
$$\tau_H = C_{\pi1}R_{\pi1} + C_{\mu1}R_{\mu1} + (C_{cs1} + C_{\pi2})R_{c1} + (C_L + C_{cs2} + C_{\mu2})(R_L \| R_o)$$
$$R_o \simeq \beta_2 r_{o2}$$
$$f_H \simeq \frac{1}{2\pi\tau_H}$$
$$A_M = -\frac{r_\pi}{r_\pi + r_x + R_{sig}} g_m(\beta r_o \| R_L)$$

**Figure 9.31** Determining the frequency response of the BJT cascode amplifier. Note that in addition to the BJT capacitances $C_\pi$ and $C_\mu$, the capacitance between the collector and the substrate $C_{cs}$ for each transistor are included.

EXERCISE

**9.24** The objective of this exercise is to evaluate the effect of cascoding on the performance of the CE amplifier of Exercise 9.19. The specifications are as follows: $I = 1$ mA, $\beta = 200$, $r_o = 130$ kΩ, $C_\pi = 16$ pF, $C_\mu = 0.3$ pF, $r_x = 200$ Ω, $C_{cs1} = C_{cs2} = 0$, $C_L = 5$ pF, $R_{sig} = 36$ kΩ, $R_L = 50$ kΩ. Find $R_{in}$, $A_0$, $R_{o1}$, $R_{in2}$, $R_o$, $A_M$, $f_H$, and $f_t$. Compare $A_M$, $f_H$, and $f_t$ with the corresponding values obtained in Exercise 9.19 for the CE amplifier. What should $C_L$ be reduced to in order to have $f_H = 1$ MHz?

**Ans.** 5.2 kΩ; 5200 V/V; 130 kΩ; 35 Ω; 26 MΩ; –238 V/V; 469 kHz; 111.6 MHz. $|A_M|$ has increased from 175 V/V to 238 V/V; $f_H$ has increased from 75 kHz to 469 kHz; $f_t$ has increased from 13.1 MHz to 111.6 MHz. $C_L$ must be reduced to 1.6 pF.

## 9.7 High-Frequency Response of the Source and Emitter Followers

In this section we study the high-frequency response of two important circuit building blocks: the source follower and the emitter follower. Both have voltage gain that is less than but close to unity. Their advantage lies in their high input resistance and low output resistance. They find application as the output stage of a multistage amplifier. As we will see shortly, both exhibit wide bandwidth.

### 9.7.1 The Source Follower

A major advantage of the source follower is its excellent high-frequency response. This comes about because, as we shall now see, none of the internal capacitances suffers from the Miller effect. Figure 9.32(a) shows the high-frequency equivalent circuit of a source follower fed with a signal $V_{sig}$ from a source having a resistance $R_{sig}$. In addition to the MOSFET capacitances $C_{gs}$ and $C_{gd}$, a capacitance $C_L$ is included between the output node and ground to account for the source-to-body capacitance $C_{sb}$ as well as any actual load capacitance.

To obtain the low-frequency gain $A_M$ and the output resistance $R_o$, we set all capacitances to zero. The results are

$$A_M = \frac{(R_L \parallel r_o)}{(R_L \parallel r_o) + (1/g_m)} \tag{9.122}$$

$$R_o = \frac{1}{g_m} \parallel r_o \tag{9.123}$$

Combining $R_L$ and $r_o$ into a single resistance $R_L'$, we can redraw the circuit in the simplified from shown in Fig. 9.32(b). Although one can derive the transfer function of this circuit, the resulting expression will be too complicated to yield insight regarding the role that each of the three capacitances plays. Rather, we shall first determine the location of the transmission zeros and then use the method of open-circuit time constants to estimate the 3-dB frequency, $f_{3dB}$.

Although there are three capacitances in the circuit of Fig. 9.32(b), the transfer function is of the second order. This is because the three capacitances form a continuous loop. To determine the location of the two transmission zeros, refer to the circuit in Fig. 9.32(b), and note that $V_o$ is zero at the frequency at which $C_L$ has a zero impedance and thus acts as a short circuit across the output, which is $\omega$ or $s = \infty$. Also, $V_o$ will be zero at the value of $s$ that causes

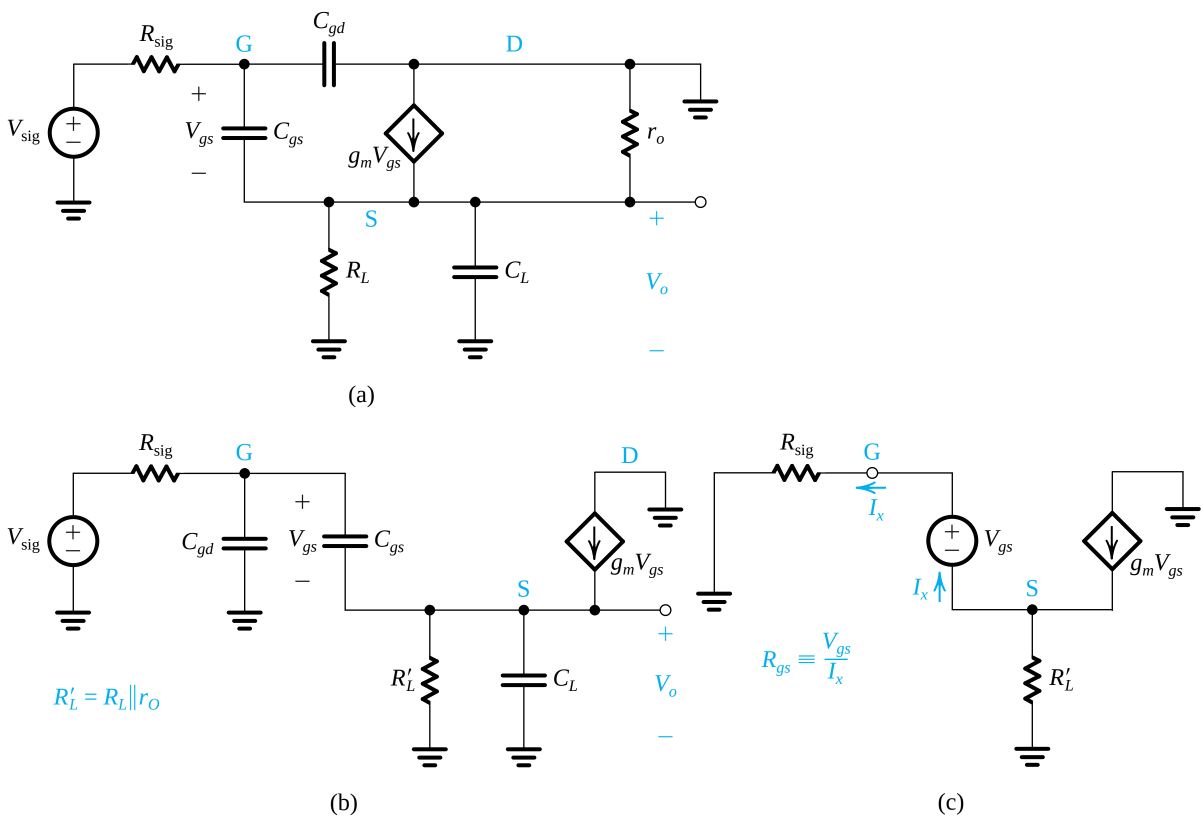



**Figure 9.32** Analysis of the high-frequency response of the source follower: **(a)** equivalent circuit; **(b)** simplified equivalent circuit; **(c)** determining the resistance $R_{gs}$ seen by $C_{gs}$.

the current into the impedance $R'_L \| C_L$ to be zero. Since this current is $(g_m + sC_{gs})V_{gs}$, the transmission zero will be at $s = s_Z$, where

$$s_Z = -\frac{g_m}{C_{gs}} \tag{9.124}$$

That is, the zero will be on the negative real axis of the $s$-plane with a frequency

$$\omega_Z = \frac{g_m}{C_{gs}} \tag{9.125}$$

Recalling that the MOSFET's $\omega_T = g_m/(C_{gs} + C_{gd})$ and that $C_{gd} \ll C_{gs}$, we see that $\omega_Z$ will be very close to $\omega_T$,

$$f_Z \simeq f_T \tag{9.126}$$

Since the zero is at such a high frequency, we can employ the method of open-circuit time constants to obtain an estimate of $f_H$. Specifically, we will find the resistance seen by each of three capacitances $C_{gd}$, $C_{gs}$, and $C_L$ and then compute the time constant associated with each. With $V_{sig}$ set to zero and $C_{gs}$ and $C_L$ assumed to be open circuited, we find by inspection that the resistance $R_{gd}$ seen by $C_{gd}$ is given by

$$R_{gd} = R_{sig} \tag{9.127}$$

Next, we consider the effect of $C_{gs}$. The resistance $R_{gs}$ seen by $C_{gs}$ can be determined by straightforward analysis of the circuit in Fig. 9.32(c) to obtain

$$R_{gs} = \frac{R_{\text{sig}} + R_L'}{1 + g_m R_L'} \tag{9.128}$$

We note that the factor $(1 + g_m R_L')$ in the denominator will result in reducing the effective resistance with which $C_{gs}$ interacts.

Finally, it is easy to see from the circuit in Fig. 9.32(b) that $C_L$ interacts with $R_L' \| 1/g_m$; that is,

$$R_{C_L} = R_L \| r_o \| \frac{1}{g_m}$$

which is usually low because of $1/g_m$. Thus the effect of $C_L$ will be small. Adding all three time constants, we obtain $\tau_H$ and hence $f_H$,

$$f_H = \frac{1}{2\pi\tau_H} = 1/2\pi(C_{gd} R_{\text{sig}} + C_{gs} R_{gs} + C_L R_{C_L}) \tag{9.129}$$

## EXERCISE

**9.25** Consider a source follower specified as follows: $g_m = 1.25$ mA/V, $r_o = 20\ \text{k}\Omega$, $R_{\text{sig}} = 10\ \text{k}\Omega$, $R_L = 20\ \text{k}\Omega$, $C_{gs} = 20$ fF, $C_{gd} = 5$ fF, and $C_L = 15$ fF. Find $A_M$, $f_T$, and $f_Z$. Also, find $R_{gd}$, $R_{gs}$, $R_{C_L}$, and hence the time constant associated with each of the three capacitances $C_{gd}$, $C_{gs}$, and $C_L$. Find $\tau_H$ and the percentage contribution to it from each of three capacitances. Find $f_H$.
**Ans.** 0.93 V/V; 8 GHz; 10 GHz; 10 kΩ; 1.48 kΩ; 0.74 kΩ; 50 ps; 30 ps; 11 ps; 91 ps; 55%; 33%; 12%; 1.75 GHz

### 9.7.2 The Emitter Follower

Figure 9.33(a) shows an emitter follower suitable for IC fabrication. It is biased by a constant-current source $I$. However, the circuit that sets the dc voltage at the base is not shown. The emitter follower is fed with a signal $V_{\text{sig}}$ from a source with resistance $R_{\text{sig}}$. The resistance $R_L$, shown at the output, includes the output resistance of current source $I$ as well as any actual load resistance.

Analysis of the emitter follower of Fig. 9.32(a) to determine its low-frequency gain, input resistance, and output resistance is identical to that performed in Section 6.6.6. We shall concentrate here on the analysis of the high-frequency response of the circuit.

Figure 9.33(b) shows the high-frequency equivalent circuit. Lumping $r_o$ together with $R_L$, and $r_x$ together with $R_{\text{sig}}$ and making a slight change in the way the circuit is drawn results in the simplified equivalent circuit shown in Fig. 9.33(c). We will follow a procedure for the analysis of this circuit similar to that used above for the source follower. Specifically, to obtain the location of the transmission zero, note that $V_o$ will be zero at the frequency $s_Z$ for which the current fed to $R_L'$ is zero:

$$g_m V_\pi + \frac{V_\pi}{r_\pi} + s_Z C_\pi V_\pi = 0$$

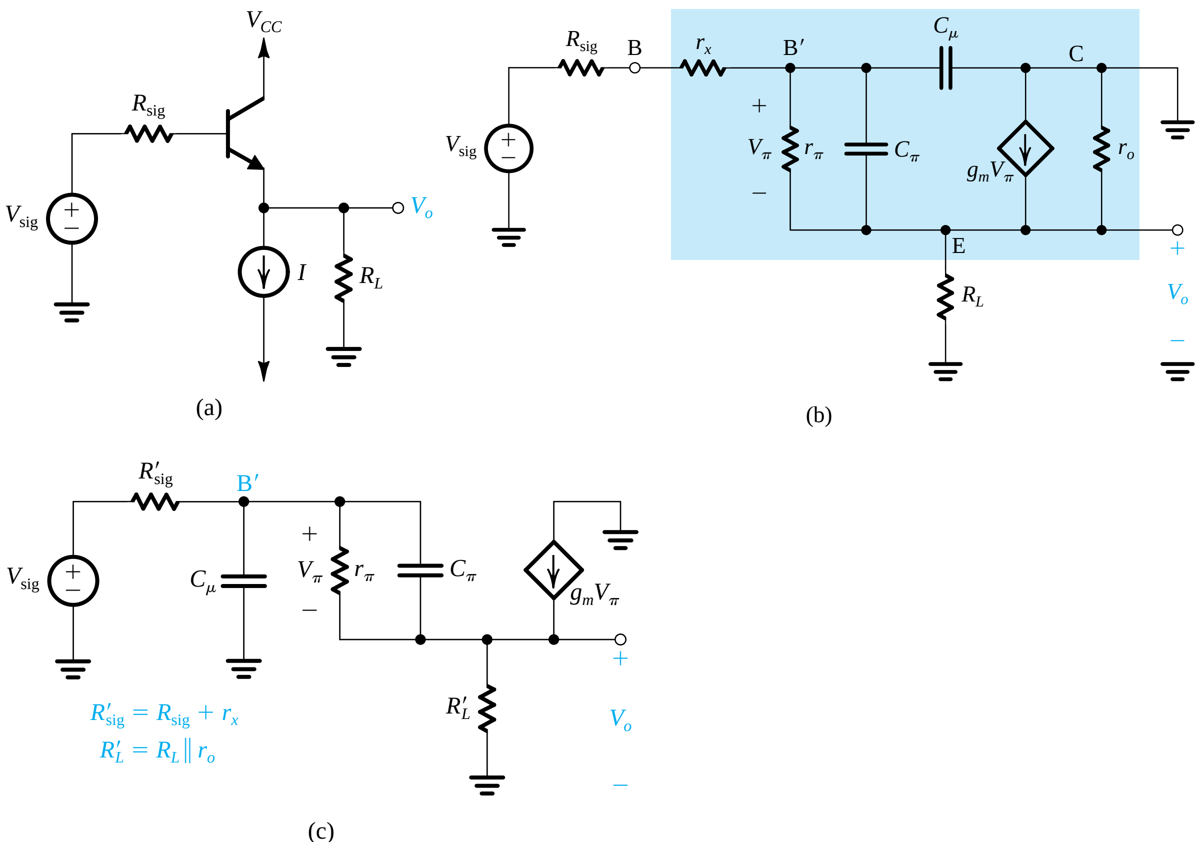



**Figure 9.33** **(a)** Emitter follower. **(b)** High-frequency equivalent circuit. **(c)** Simplified equivalent circuit.

Thus,

$$s_Z = -\frac{g_m + (1/r_\pi)}{C_\pi} = -\frac{1}{C_\pi r_e} \tag{9.130}$$

which is on the negative real-axis of the $s$-plane and has a frequency

$$\omega_Z = \frac{1}{C_\pi r_e} \tag{9.131}$$

This frequency is very close to the unity-gain frequency $\omega_T$ of the transistor. The other transmission zero is at $s = \infty$. This is because at this frequency, $C_\mu$ acts as a short circuit, making $V_\pi$ zero, and hence $V_o$ will be zero.

Next, we determine the resistances seen by $C_\mu$ and $C_\pi$. For $C_\mu$ the reader should be able to show that the resistance it sees, $R_\mu$, is the parallel equivalent of $R'_{sig}$ and the input resistance looking into B′; that is,

$$R_\mu = R'_{sig} \| [r_\pi + (\beta + 1)R'_L] \tag{9.132}$$

Equation (9.132) indicates that $R_\mu$ will be smaller than $R'_{sig}$, and since $C_\mu$ is usually very small, the time constant $C_\mu R_\mu$ will be correspondingly small.

The resistance $R_\pi$ seen by $C_\pi$ can be determined using an analysis similar to that employed for the determination of $R_{gs}$ in the MOSFET case. The result is

$$R_\pi = \frac{R'_{\text{sig}} + R'_L}{1 + \frac{R'_{\text{sig}}}{r_\pi} + \frac{R'_L}{r_e}} \tag{9.133}$$

We observe that the term $R'_L/r_e$ will usually make the denominator much greater than unity, thus rendering $R_\pi$ rather low. Thus, the time constant $C_\pi R_\pi$ will be small. The end result is that the 3-dB frequency $f_H$ of the emitter follower,

$$f_H = 1/2\pi[C_\mu R_\mu + C_\pi R_\pi] \tag{9.134}$$

will usually be very high. We urge the reader to solve the following exercise to gain familiarity with typical values of the various parameters that determine $f_H$.

## EXERCISE

**9.26** For an emitter follower biased at $I_C = 1$ mA and having $R_{\text{sig}} = R_L = 1\text{ k}\Omega$, $r_o = 100\text{ k}\Omega$, $\beta = 100$, $C_\mu = 2$ pF, and $f_T = 400$ MHz, find the low-frequency gain, $f_Z$, $R_\mu$, $R_\pi$, and $f_H$.
**Ans.** 0.965 V/V; 458 MHz; 1.09 kΩ; 51 Ω; 55 MHz

# 9.8 High-Frequency Response of Differential Amplifiers

In this section we study the high-frequency response of the differential amplifier. We will consider the variation with frequency of both the differential gain and the common-mode gain and hence of the CMRR. We will rely heavily on the study of frequency response of single-ended amplifiers presented in the sections above. Also, we will only consider MOS circuits; the bipolar case is a straightforward extension, as we saw above on a number of occasions.

## 9.8.1 Analysis of the Resistively Loaded MOS Amplifier

We begin with the basic, resistively loaded MOS differential pair shown in Fig. 9.34(a). Note that we have explicitly shown transistor $Q_S$ that supplies the bias current $I$. Although we are showing a dc bias voltage $V_{\text{BIAS}}$ at its gate, usually $Q_S$ is part of a current mirror. This detail, however, is of no consequence to our present needs. Most importantly, we are interested in the total impedance between node $S$ and ground, $Z_{SS}$. As we shall shortly see, this impedance plays a significant role in determining the common-mode gain and the CMRR of the differential amplifier. Resistance $R_{SS}$ is simply the output resistance of current source $Q_S$. Capacitance $C_{SS}$ is the total capacitance between node $S$ and ground and includes $C_{db}$ and $C_{gd}$ of $Q_S$, as well as $C_{sb1}$, and $C_{sb2}$. This capacitance can be significant, especially if wide transistors are used for $Q_S$, $Q_1$, and $Q_2$.

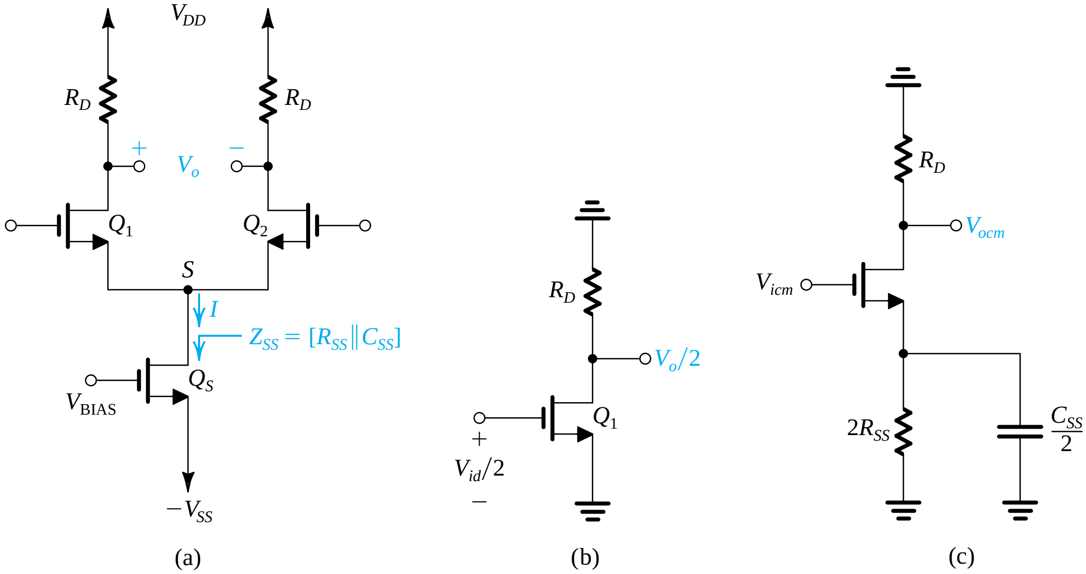



**Figure 9.34** **(a)** A resistively loaded MOS differential pair; the transistor supplying the bias current is explicitly shown. It is assumed that the total impedance between node $S$ and ground, $Z_{SS}$, consists of a resistance $R_{SS}$ in parallel with a capacitance $C_{SS}$. **(b)** Differential half-circuit. **(c)** Common-mode half-circuit.

The differential half-circuit shown in Fig. 9.34(b) can be used to determine the frequency dependence of the differential gain $V_o/V_{id}$. Indeed the gain function $A_d(s)$ of the differential amplifier will be identical to the transfer function of this common-source amplifier. We studied the frequency response of the common-source amplifier at great length in Sections 9.3 and 9.5 and will not repeat this material here.

## EXERCISE

**9.27** A MOSFET differential amplifier such as that in Fig. 9.34(a) is biased with a current $I = 0.8$ mA. The transistors have $W/L = 100$, $k_n' = 0.2$ mA/V$^2$, $V_A = 20$ V, $C_{gs} = 50$ fF, $C_{gd} = 10$ fF, and $C_{db} = 10$ fF. The drain resistors are 5 kΩ each. Also, there is a 100-fF capacitive load between each drain and ground.

(a) Find $V_{OV}$ and $g_m$ for each transistor.
(b) Find the differential gain $A_d$.
(c) If the input signal source has a small resistance $R_{sig}$ and thus the frequency response is determined primarily by the output pole, estimate the 3-dB frequency $f_H$. (*Hint:* Refer to Section 9.5.6 and specifically to Eq. 9.107.)
(d) If, in a different situation, the amplifier is fed symmetrically with a signal source of 20 kΩ resistance (i.e., 10 kΩ in series with each gate terminal), use the open-circuit time-constants method to estimate $f_H$. (*Hint:* Refer to Section 9.5.3 and specifically to Eqs. [(9.85) and (9.86)].)

**Ans.** (a) 0.2 V, 4 mA/V; (b) 18.2 V/V; (c) 291 MHz; (d) 53.7 MHz

The common-mode half-circuit is shown in Fig. 9.34(c). Although this circuit has other capacitances, namely $C_{gs}$, $C_{gd}$, and $C_{db}$ of the transistor in addition to other stray capacitances, we have chosen to show only $C_{SS}/2$. This is because $(C_{SS}/2)$ together with $(2R_{SS})$ forms a real-axis zero in the common-mode gain function at a frequency much lower than those of the other poles and zeros of the circuit. This zero then dominates the frequency dependence of $A_{cm}$ and CMRR.

If the output of the differential amplifier is taken single-endedly, then the common-mode gain of interest is $V_{ocm}/V_{icm}$. More typically, the output is taken differentially. Nevertheless, as we have seen in Section 8.2, $V_{ocm}/V_{icm}$ still plays a major role in determining the common-mode gain. To be specific, consider what happens when the output is taken differentially and there is a mismatch $\Delta R_D$ between the two drain resistances. The resulting common-mode gain was found in Section 8.2 to be (Eq. 8.49′)

$$A_{cm} = -\left(\frac{R_D}{2R_{SS}}\right)\frac{\Delta R_D}{R_D} \tag{9.135}$$

which is simply the product of $V_{ocm}/V_{icm}$ and the per-unit mismatch $(\Delta R_D/R_D)$. Similar expressions can be found for the effects of other circuit mismatches. The important point to note is that the factor $R_D/2R_{SS}$ is always present in these expressions. Thus, the frequency dependence of $A_{cm}$ can be obtained by simply replacing $R_{SS}$ by $Z_{SS}$ in this factor. Doing so for the expression in Eq. (9.135) gives

$$\begin{aligned} A_{cm}(s) &= -\frac{R_D}{2Z_{SS}}\left(\frac{\Delta R_D}{R_D}\right) \\ &= -\frac{1}{2}R_D\left(\frac{\Delta R_D}{R_D}\right)Y_{SS} \\ &= -\frac{1}{2}R_D\left(\frac{\Delta R_D}{R_D}\right)\left(\frac{1}{R_{SS}}+sC_{SS}\right) \\ &= -\frac{R_D}{2R_{SS}}\left(\frac{\Delta R_D}{R_D}\right)(1+sC_{SS}R_{SS}) \end{aligned} \tag{9.136}$$

from which we see that $A_{cm}$ acquires a zero on the negative real axis of the $s$-plane with frequency $\omega_Z$,

$$\omega_Z = \frac{1}{C_{SS}R_{SS}} \tag{9.137}$$

or in hertz,

$$f_Z = \frac{\omega_Z}{2\pi} = \frac{1}{2\pi C_{SS}R_{SS}} \tag{9.138}$$

As mentioned above, usually $f_Z$ is much lower than the frequencies of the other poles and zeros. As a result, the common-mode gain increases at the rate of +6 dB/octave (20 dB/decade) starting at a relatively low frequency, as indicated in Fig. 9.35(a). Of course, $A_{cm}$ drops off at high frequencies because of the other poles of the common-mode half-circuit. It is, however, $f_Z$ that is significant, for it is the frequency at which the CMRR of

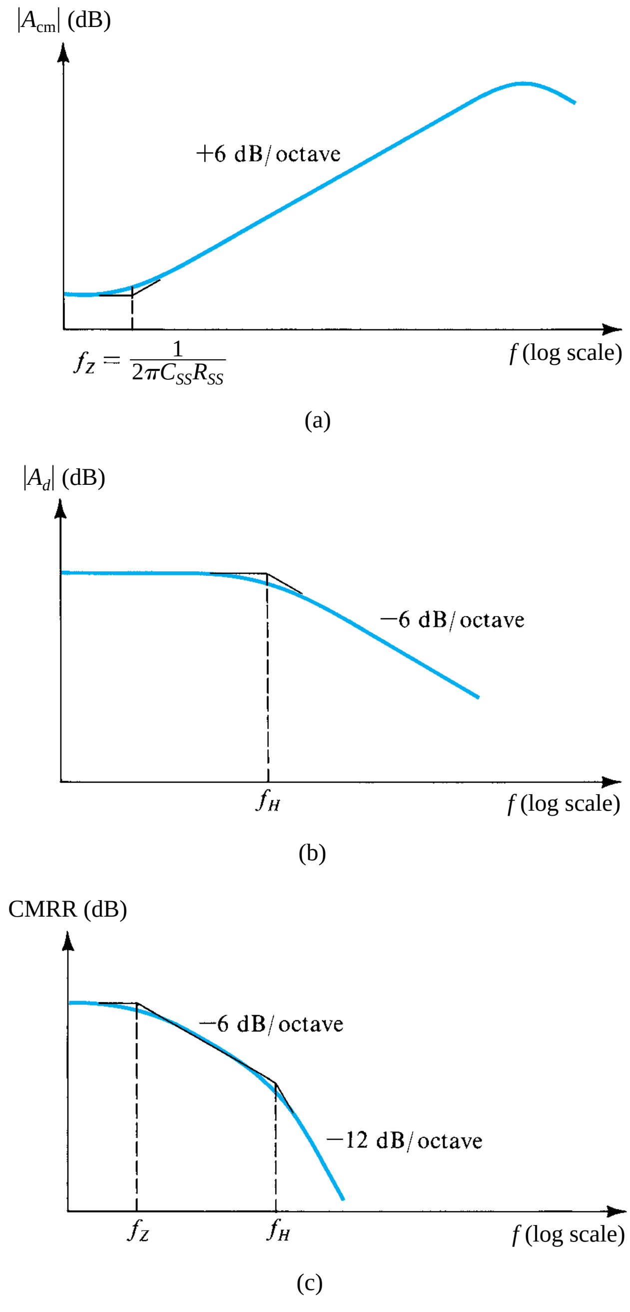



**Figure 9.35** Variation of **(a)** common-mode gain, **(b)** differential gain, and **(c)** common-mode rejection ratio with frequency.

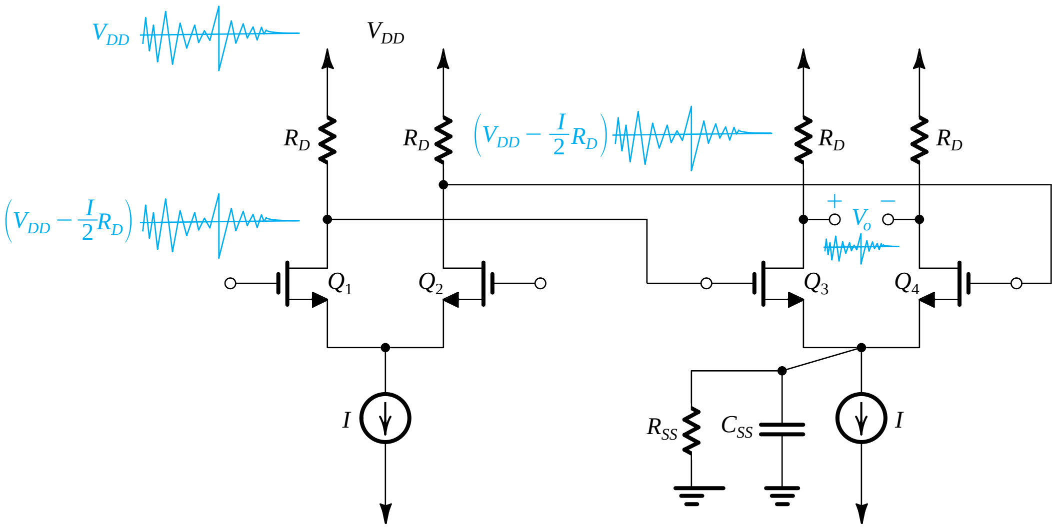



**Figure 9.36** The second stage in a differential amplifier, which is relied on to suppress high-frequency noise injected by the power supply of the first stage, and therefore must maintain a high CMRR at higher frequencies.

the differential amplifier begins to decrease, as indicated in Fig. 9.35(c). Note that if both $A_d$ and $A_{cm}$ are expressed and plotted in dB, then CMRR in dB is simply the difference between $A_d$ and $A_{cm}$.

Although in the foregoing we considered only the common-mode gain resulting from an $R_D$ mismatch, it should be obvious that the results apply to the common-mode gain resulting from any other mismatch. For instance, it applies equally well to the case of a $g_m$ mismatch, modifying Eq. (8.63) by replacing $R_{SS}$ by $Z_{SS}$, and so on.

Before leaving this section, it is interesting to point out an important trade-off found in the design of the current-source transistor $Q_S$: In order to operate this current source with a small $V_{DS}$ (to conserve the already low $V_{DD}$), we desire to operate the transistor at a low overdrive voltage $V_{OV}$. For a given value of the current $I$, however, this means using a large $W/L$ ratio (i.e., a wide transistor). This in turn increases $C_{SS}$ and hence lowers $f_Z$ with the result that the CMRR deteriorates (i.e., decreases) at a relatively low frequency. Thus there is a trade-off between the need to reduce the dc voltage across $Q_S$ and the need to keep the CMRR reasonably high at higher frequencies.

To appreciate the need for high CMRR at higher frequencies, consider the situation illustrated in Fig. 9.36: We show two stages of a differential amplifier whose power-supply voltage $V_{DD}$ is corrupted with high-frequency noise. Since the quiescent voltage at each of the drains of $Q_1$ and $Q_2$ is $[V_{DD} - (I/2)R_D]$, we see that $v_{D1}$ and $v_{D2}$ will have the same high-frequency noise as $V_{DD}$. This high-frequency noise then constitutes a common-mode input signal to the second differential stage, formed by $Q_3$ and $Q_4$. If the second differential stage is perfectly matched, its differential output voltage $V_o$ should be free of high-frequency noise. However, in practice there is no such thing as perfect matching, and the second stage will have a finite common-mode gain. Furthermore, because of the zero formed by $R_{SS}$ and $C_{SS}$ of the second stage, the common-mode gain will increase with frequency, causing some of the noise to make its way to $V_o$. With careful design, this undesirable component of $V_o$ can be kept small.

## EXERCISE

**9.28** The differential amplifier specified in Exercise 9.27 has $R_{SS} = 25\ \text{k}\Omega$ and $C_{SS} = 0.4$ pF. Find the 3-dB frequency of the CMRR.
**Ans.** 15.9 MHz

### 9.8.2 Analysis of the Active-Loaded MOS Amplifier

We next consider the frequency response of the current-mirror-loaded MOS differential-pair circuit studied in Section 8.5. The circuit is shown in Fig. 9.37(a) with two capacitances indicated: $C_m$, which is the total capacitance at the input node of the current mirror, and $C_L$, which is the total capacitance at the output node. Capacitance $C_m$ is mainly formed by $C_{gs3}$ and $C_{gs4}$ but also includes $C_{gd1}$, $C_{db1}$, and $C_{db3}$,

$$C_m = C_{gd1} + C_{db1} + C_{db3} + C_{gs3} + C_{gs4} \tag{9.139}$$

Capacitance $C_L$ includes $C_{gd2}$, $C_{db2}$, $C_{db4}$, $C_{gd4}$ as well as an actual load capacitance and/or the input capacitance of a subsequent stage ($C_x$),

$$C_L = C_{gd2} + C_{db2} + C_{gd4} + C_{db4} + C_x \tag{9.140}$$

These two capacitances primarily determine the dependence of the differential gain of this amplifier on frequency.

As indicated in Fig. 9.37(a) the input differential signal $V_{id}$ is applied in a balanced fashion and the output node is short-circuited to ground in order to determine the transconductance $G_m$.

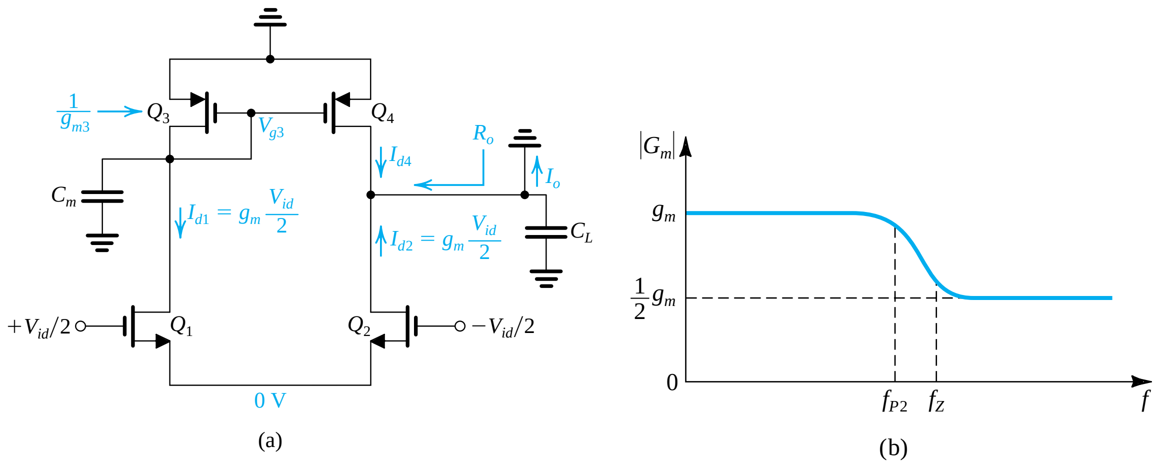



**Figure 9.37** **(a)** Frequency–response analysis of the active-loaded MOS differential amplifier. **(b)** The overall transconductance $G_m$ as a function of frequency.

Obviously, because of the output short circuit, $C_L$ will have no effect on $G_m$. Transistor $Q_1$ will conduct a drain current signal of $g_m V_{id}/2$, which flows through the diode-connected transistor $Q_3$ and thus through the parallel combination of $(1/g_{m3})$ and $C_m$, where we have neglected the resistances $r_{o1}$ and $r_{o3}$ which are much larger than $(1/g_{m3})$, thus

$$V_{g3} = -\frac{g_m V_{id}/2}{g_{m3} + sC_m} \tag{9.141}$$

In response to $V_{g3}$, transistor $Q_4$ conducts a drain current $I_{d4}$,

$$I_{d4} = -g_{m4} V_{g3} = \frac{g_{m4}\, g_m V_{id}/2}{g_{m3} + sC_m}$$

Since $g_{m3} = g_{m4}$, this equation reduces to

$$I_{d4} = \frac{g_m V_{id}/2}{1 + s\dfrac{C_m}{g_{m3}}} \tag{9.142}$$

Now, at the output node the total output current that flows through the short circuit is

$$\begin{aligned} I_o &= I_{d4} + I_{d2} \\ &= \frac{g_m V_{id}/2}{1 + s\dfrac{C_m}{g_{m3}}} + g_m (V_{id}/2) \end{aligned} \tag{9.143}$$

We can thus obtain $G_m$ as

$$G_m \equiv \frac{I_o}{V_{id}} = g_m \frac{1 + s\dfrac{C_m}{2g_{m3}}}{1 + s\dfrac{C_m}{g_{m3}}} \tag{9.144}$$

Thus, as expected, the low-frequency value of $G_m$ is equal to $g_m$ of $Q_1$ and $Q_2$. At high frequencies, $G_m$ acquires a pole and a zero, the frequencies of which are

$$f_{P2} = \frac{g_{m3}}{2\pi C_m} \tag{9.145}$$

and

$$f_Z = \frac{2g_{m3}}{2\pi C_m} \tag{9.146}$$

That is, the zero frequency is twice that of the pole. Since $C_m$ is approximately equal to $C_{gs2} + C_{gs4} = 2C_{gs}$, we also have

$$f_{P2} = \frac{g_{m3}}{2\pi C_m} \simeq \frac{g_{m3}}{2\pi(2C_{gs})} \simeq f_T/2 \tag{9.147}$$

and

$$f_Z \simeq f_T \tag{9.148}$$

where $f_T$ is the unity-gain frequency of the MOSFET $Q_3$. Thus, the **mirror pole and zero** occur at very high frequencies. Nevertheless, their effect can be significant.

Figure 9.37(b) shows a sketch of the magnitude of $G_m$ versus frequency. It is interesting and useful to observe that the path of the signal current produced by $Q_1$ has a transfer function different from that of the signal current produced by $Q_2$. It is the first signal that encounters $C_m$ and experiences the mirror pole. This observation leads to an interesting view of the effect of $C_m$ on the overall transconductance $G_m$ of the differential amplifier. As we learned in Section 8.5, at low frequencies $I_{d1}$ is replicated by the mirror $Q_3 - Q_4$ in the drain of $Q_4$ as $I_{d4}$, which adds to $I_{d2}$ to provide a factor-of-2 increase in $G_m$ (thus making $G_m$ equal to $g_m$, which is double the value available without the current mirror). Now, at high frequencies $C_m$ acts as a short circuit causing $V_{g3}$ to be zero, and hence $I_{d4}$ will be zero, reducing $G_m$ to $g_m/2$, as borne out by the sketch in Fig. 9.37(b).

Having determined the short-circuit output current $I_o$, we now multiply it by the total impedance between the output node and ground to determine the output voltage $V_o$,

$$V_o = I_o \frac{1}{\frac{1}{R_o} + sC_L}$$

$$= G_m V_{id} \frac{R_o}{1 + sC_L R_o}$$

Thus,

$$\frac{V_o}{V_{id}} = (g_m R_o) \left[ \frac{1 + s\dfrac{C_m}{2g_{m3}}}{1 + s\dfrac{C_m}{g_{m3}}} \right] \left( \frac{1}{1 + sC_L R_o} \right) \tag{9.149}$$

where

$$R_o = r_{o2} \parallel r_{o4}$$

Thus, in addition to the pole and zero of $G_m$, the gain of the differential amplifier will have a pole with frequency $f_{P1}$,

$$f_{P1} = \frac{1}{2\pi C_L R_o} \tag{9.150}$$

This, of course, is entirely expected, and in fact this output pole is often dominant, especially when a large load capacitance is present.

## Example 9.14

Consider an active-loaded MOS differential amplifier of the type shown in Fig. 9.37(a). Assume that for all transistors, $W/L = 7.2\ \mu\text{m}/0.36\ \mu\text{m}$, $C_{gs} = 20$ fF, $C_{gd} = 5$ fF, and $C_{db} = 5$ fF. Also, let $\mu_n C_{ox} = 387\ \mu\text{A/V}^2$, $\mu_p C_{ox} = 86\ \mu\text{A/V}^2$, $V'_{An} = 5$ V/µm, $|V'_{Ap}| = 6$ V/µm. The bias current $I = 0.2$ mA, and the bias current source has an output resistance $R_{SS} = 25$ kΩ and an output capacitance $C_{SS} = 0.2$ pF. In addition to the capacitances introduced by the transistors at the output node, there is a capacitance $C_x$ of 25 fF. It is required to determine the low-frequency values of $A_d$, $A_{cm}$, and CMRR. It is also required to find the poles and zero of $A_d$ and the dominant pole of CMRR.

Example 9.14 *continued*

## Solution

Since $I = 0.2$ mA, each of the four transistors is operating at a bias current of 100 μA. Thus, for $Q_1$ and $Q_2$,

$$100 = \frac{1}{2} \times 387 \times \frac{7.2}{0.36} \times V_{OV}^2$$

which leads to

$$V_{OV} = 0.16 \text{ V}$$

Thus,

$$g_m = g_{m1} = g_{m2} = \frac{2 \times 0.1}{0.16} = 1.25 \text{ mA/V}$$

$$r_{o1} = r_{o2} = \frac{5 \times 0.36}{0.1} = 18 \text{ k}\Omega$$

For $Q_3$ and $Q_4$ we have

$$100 = \frac{1}{2} \times 86 \times \frac{7.2}{0.36} \times V_{OV3,4}^2$$

Thus,

$$V_{OV3,4} = 0.34 \text{ V},$$

and

$$g_{m3} = g_{m4} = \frac{2 \times 0.1}{0.34} = 0.6 \text{ mA/V}$$

$$r_{o3} = r_{o4} = \frac{6 \times 0.36}{0.1} = 21.6 \text{ k}\Omega$$

The low-frequency value of the differential gain can be determined from

$$\begin{aligned} A_d &= g_m(r_{o2} \| r_{o4}) \\ &= 1.25(18 \| 21.6) = 12.3 \text{ V/V} \end{aligned}$$

The low-frequency value of the common-mode gain can be determined from Eq. (8.146′) as

$$\begin{aligned} A_{cm} &= -\frac{1}{2g_{m3}R_{SS}} \\ &= -\frac{1}{2 \times 0.6 \times 25} = -0.033 \text{ V/V} \end{aligned}$$

The low-frequency value of the CMRR can now be determined as

$$\text{CMRR} = \frac{|A_d|}{|A_{cm}|} = \frac{12.3}{0.033} = 369$$

or,

$$20 \log 369 = 51.3 \text{ dB}$$

To determine the poles and zero of $A_d$ we first compute the values of the two pertinent capacitances $C_m$ and $C_L$. Using Eq. (9.139),

$$\begin{aligned} C_m &= C_{gd1} + C_{db1} + C_{db3} + C_{gs3} + C_{gs4} \\ &= 5 + 5 + 5 + 20 + 20 = 55 \text{ fF} \end{aligned}$$

Capacitance $C_L$ is found using Eq. (9.140) as

$$\begin{aligned} C_L &= C_{gd2} + C_{db2} + C_{gd4} + C_{db4} + C_x \\ &= 5 + 5 + 5 + 5 + 25 = 45 \text{ fF} \end{aligned}$$

Now, the poles and zero of $A_d$ can be found from Eqs. (9.150), (9.145), and (9.146) as

$$\begin{aligned} f_{P1} &= \frac{1}{2\pi C_L R_o} \\ &= \frac{1}{2\pi \times C_L(r_{o2} \| r_{o4})} \\ &= \frac{1}{2\pi \times 45 \times 10^{-15}(18 \| 21.6)10^3} \\ &= 360 \text{ MHz} \end{aligned}$$

$$f_{P2} = \frac{g_{m3}}{2\pi C_m} = \frac{0.6 \times 10^{-3}}{2\pi \times 55 \times 10^{-15}} = 1.74 \text{ GHz}$$

$$f_Z = 2f_{P2} = 3.5 \text{ GHz}$$

Thus the dominant pole is that produced by $C_L$ at the output node. As expected, the pole and zero of the mirror are at much higher frequencies.

The dominant pole of the CMRR is at the location of the common-mode-gain zero introduced by $C_{SS}$ and $R_{SS}$, that is,

$$\begin{aligned} f_Z &= \frac{1}{2\pi C_{SS} R_{SS}} \\ &= \frac{1}{2\pi \times 0.2 \times 10^{-12} \times 25 \times 10^3} \\ &= 31.8 \text{ MHz} \end{aligned}$$

Thus, the CMRR begins to decrease at 31.8 MHz, which is much lower than $f_{P1}$.

## EXERCISE

**9.29** A bipolar current-mirror-loaded differential amplifier is biased with a current source $I = 1$ mA. The transistors are specified to have $|V_A| = 100$ V. The total capacitance at the output node is 2 pF. Find the dc value and the frequency of the dominant high-frequency pole of the differential voltage gain.

**Ans.** 2000 V/V; 0.8 MHz

# 9.9 Other Wideband Amplifier Configurations

Thus far, we have studied one wideband amplifier configuration: the cascode amplifier. Cascoding can, of course, be applied to differential amplifiers to obtain wideband differential amplification. In this section we discuss a number of other circuit configurations that are capable of achieving wide bandwidths.

## 9.9.1 Obtaining Wideband Amplification by Source and Emitter Degeneration

As we discussed in Chapters 5 and 6, adding a resistance in the source (emitter) lead of a CS (CE) amplifier can result in a number of performance improvements at the expense of a reduction in voltage gain. Extension of the amplifier bandwidth, which is the topic of interest to us in this section, is among those improvements.

Figure 9.38(a) shows a common-source amplifier with a source-degeneration resistance $R_s$. As indicated in Fig. 9.38(b), the output of the amplifier can be modeled at low frequencies by a controlled current-source $G_m V_i$ and an output resistance $R_o$, where the transconductance $G_m$ is given by

$$G_m \simeq \frac{g_m}{1 + g_m R_s} \tag{9.151}$$

and the output resistance is given by

$$R_o \simeq r_o(1 + g_m R_s) \tag{9.152}$$

Thus, source degeneration reduces the transconductance and increases the output resistance by the same factor, $(1 + g_m R_s)$. The low-frequency voltage gain can be obtained as

$$A_M = \frac{V_o}{V_{sig}} = -G_m(R_o \parallel R_L) = -G_m R_L' \tag{9.153}$$

Let's now consider the high-frequency response of the source-degenerated amplifier. Figure 9.38(c) shows the amplifier, indicating the capacitances $C_{gs}$ and $C_{gd}$. A capacitance $C_L$ that *includes* the MOSFET capacitance $C_{db}$ is also shown at the output. The method of open-circuit time constants can be employed to obtain an estimate of the 3-dB frequency $f_H$. Toward that end, we show in Fig. 9.38(d) the circuit for determining $R_{gd}$, which is the resistance seen by $C_{gd}$. We observe that $R_{gd}$ can be determined by simply adapting the formula in Eq. (9.84) to the case with source degeneration as follows:

$$R_{gd} = R_{sig}(1 + G_m R_L') + R_L' \tag{9.154}$$

where

$$R_L' = R_L \parallel R_o \tag{9.155}$$

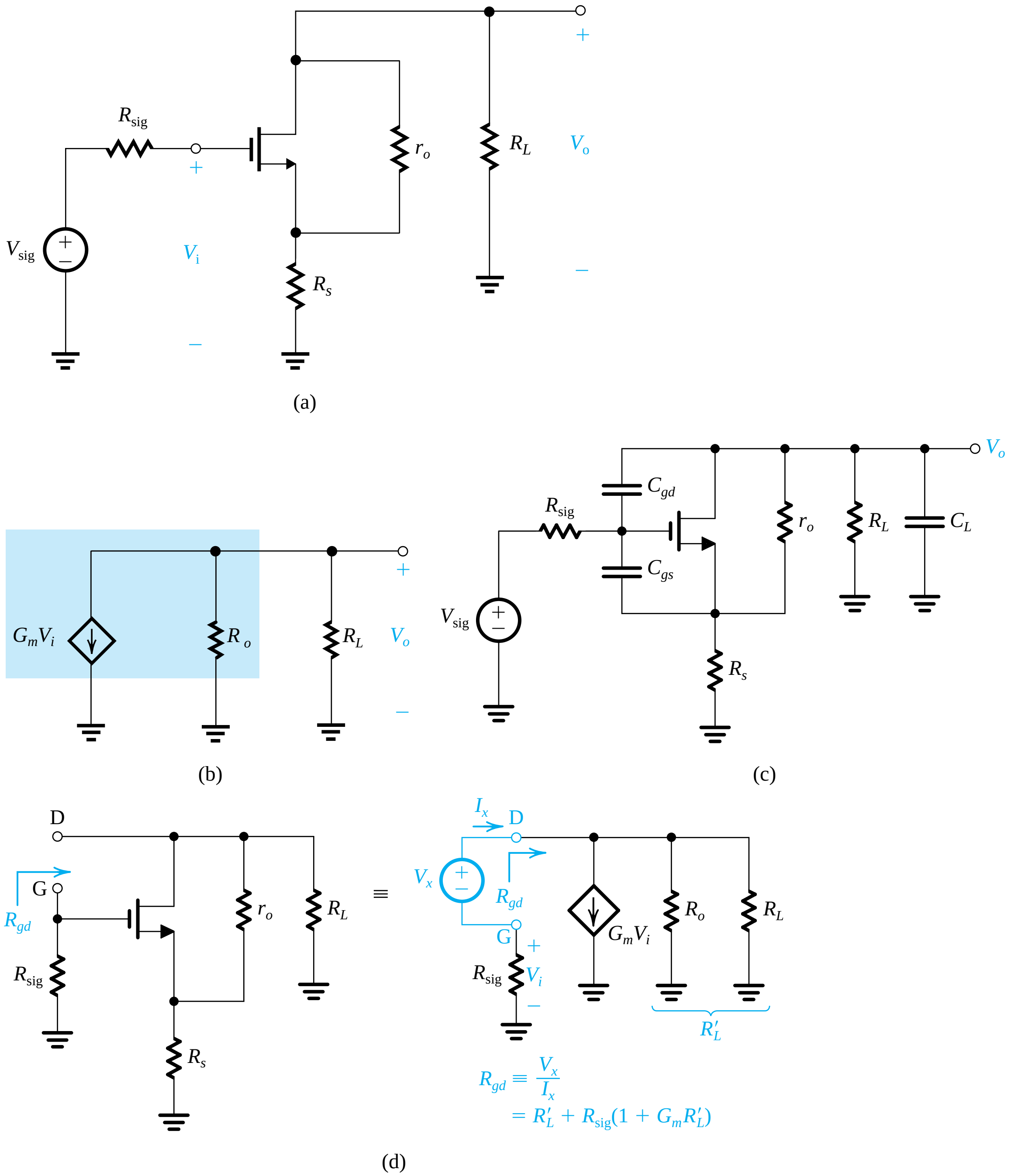



**Figure 9.38** **(a)** The CS amplifier circuit, with a source resistance $R_s$. **(b)** Equivalent-circuit representation of the amplifier output. **(c)** The circuit prepared for frequency-response analysis. **(d)** Determining the resistance $R_{gd}$ seen by the capacitance $C_{gd}$.

The formula for $R_{C_L}$ can be seen to be simply

$$R_{C_L} = R_L \parallel R_o = R_L' \tag{9.156}$$

The formula for $R_{gs}$ is the most difficult to derive, and the derivation should be performed with the hybrid-$\pi$ model explicitly utilized. The result is

$$R_{gs} \simeq \frac{R_{sig} + R_s}{1 + g_m R_s\left(\dfrac{r_o}{r_o + R_L}\right)} \tag{9.157}$$

When $R_{sig}$ is relatively large, the frequency response will be dominated by the Miller multiplication of $C_{gd}$. Another way for saying this is that $C_{gd} R_{gd}$ will be the largest of the three open-circuit time constants that make up $\tau_H$,

$$\tau_H = C_{gs} R_{gs} + C_{gd} R_{gd} + C_L R_{C_L} \tag{9.158}$$

enabling us to approximate $\tau_H$ as

$$\tau_H \simeq C_{gd} R_{gd} \tag{9.159}$$

and correspondingly to obtain $f_H$ as

$$f_H \simeq \frac{1}{2\pi C_{gd} R_{gd}} \tag{9.160}$$

Now, as $R_s$ is increased, the gain magnitude, $|A_M| = G_m R_L'$, will decrease, causing $R_{gd}$ to decrease (Eq. 9.154), which in turn causes $f_H$ to increase (Eq. 9.160). To highlight the trade-off between gain and bandwidth that $R_s$ affords the designer, let us simplify the expression for $R_{gd}$ in Eq. (9.154) by assuming that $G_m R_L' \gg 1$ and $G_m R_{sig} \gg 1$,

$$R_{gd} \simeq G_m R_L' R_{sig} = |A_M| R_{sig}$$

which can be substituted in Eq. (9.160) to obtain

$$f_H = \frac{1}{2\pi C_{gd} R_{sig} |A_M|} \tag{9.161}$$

which very clearly shows the gain–bandwidth trade-off. The gain–bandwidth product remains constant at

$$\text{Gain–bandwidth product} = |A_M|\, f_H = \frac{1}{2\pi C_{gd} R_{sig}} \tag{9.162}$$

In practice, however, the other capacitances will play a role in determining $f_H$, and the gain–bandwidth product will decrease somewhat as $R_s$ is increased.

**EXERCISE**

**9.30** Consider a CS amplifier having $g_m = 2$ mA/V, $r_o = 20$ kΩ, $R_L = 20$ kΩ, $R_{sig} = 20$ kΩ, $C_{gs} = 20$ fF, $C_{gd} = 5$ fF, and $C_L = 5$ fF. (a) Find the voltage gain $A_M$ and the 3-dB frequency $f_H$ (using the method of open-circuit time constants) and hence the gain–bandwidth product. (b) Repeat (a) for the case in which a resistance $R_s$ is connected in series with the source terminal with a value selected so that $g_m R_s = 2$.

**Ans.** (a) –20 V/V, 61.2 MHz, 1.22 GHz; (b) –10 V/V, 109 MHz, 1.1 GHz

## 9.9.2 The CD–CS, CC–CE and CD–CE Configurations

In Section 7.6.1 we discussed the performance improvements obtained by preceding the CS and CE amplifiers by a buffer implemented by a CD or a CC amplifier, as in the circuits shown in Fig. 9.39. A major advantage of each of these circuits is wider bandwidth than that obtained in the CS or CE stage alone. To see how this comes about, consider as an example the CD–CS amplifier in Fig 9.39(a) and note that the CS transistor $Q_2$ will still exhibit a Miller effect that results in a large input capacitance, $C_{in2}$, between its gate and ground. However, the resistance that this capacitance interacts with will be much lower than $R_{sig}$; the buffering action of the source follower causes a relatively low resistance, approximately equal to a $1/g_{m1}$, to appear between the source of $Q_1$ and ground across $C_{in2}$.

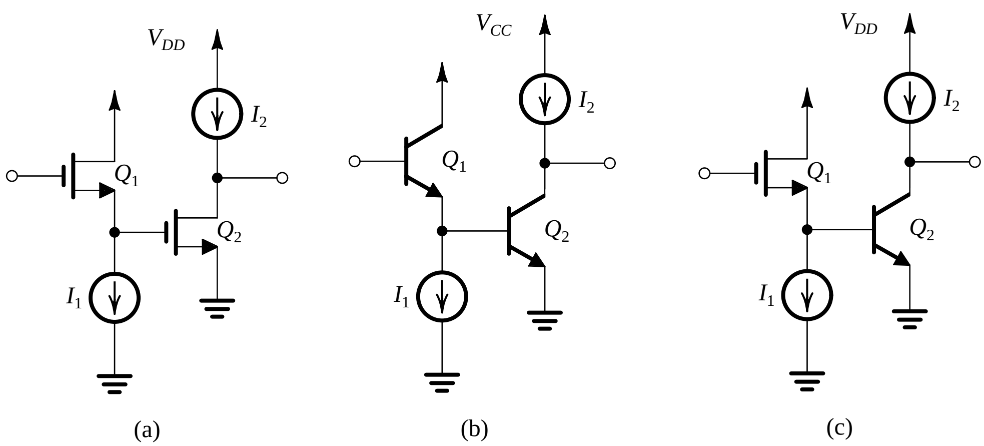



**Figure 9.39** **(a)** CD–CS amplifier. **(b)** CC–CE amplifier. **(c)** CD–CE amplifier.

## Example 9.15

Consider a CC–CE amplifier such as that in Fig. 9.39(b) with the following specifications: $I_1 = I_2 =$ 1 mA and identical transistors with $\beta = 100$, $f_T = 400$ MHz, and $C_\mu = 2$ pF. Let the amplifier be fed with a source $V_{sig}$ having a resistance $R_{sig} = 4$ kΩ, and assume a load resistance of 4 kΩ. Find the voltage gain $A_M$, and estimate the 3-dB frequency, $f_H$. Compare the results with those obtained with a CE amplifier operating under the same conditions. For simplicity, neglect $r_o$ and $r_x$.

### Solution

At an emitter bias current of 1 mA, $Q_1$ and $Q_2$ have

$$g_m = 40 \text{ mA/V}$$

$$r_e = 25\ \Omega$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{40} = 2.5 \text{ k}\Omega$$

$$C_\pi + C_\mu = \frac{g_m}{\omega_T} = \frac{g_m}{2\pi f_T}$$

$$= \frac{40 \times 10^{-3}}{2\pi \times 400 \times 10^6} = 15.9 \text{ pF}$$

$$C_\mu = 2 \text{ pF}$$

$$C_\pi = 13.9 \text{ pF}$$

The voltage gain $A_M$ can be determined from the circuit shown in Fig. 9.40(a) as follows:

$$R_{in2} = r_{\pi 2} = 2.5 \text{ k}\Omega$$

$$R_{in} = (\beta_1 + 1)(r_{e1} + R_{in2})$$

$$= 101(0.025 + 2.5) = 255 \text{ k}\Omega$$

$$\frac{V_{b1}}{V_{sig}} = \frac{R_{in}}{R_{in} + R_{sig}} = \frac{255}{255 + 4} = 0.98 \text{ V/V}$$

$$\frac{V_{b2}}{V_{b1}} = \frac{R_{in2}}{R_{in2} + r_{e1}} = \frac{2.5}{2.5 + 0.025} = 0.99 \text{ V/V}$$

$$\frac{V_o}{V_{b2}} = -g_{m2} R_L = -40 \times 4 = -160 \text{ V/V}$$

Thus,

$$A_M = \frac{V_o}{V_{sig}} = -160 \times 0.99 \times 0.98 = -155 \text{ V/V}$$

To determine $f_H$ we use the method of open-circuit time constants. Figure 9.40(b) shows the circuit with $V_{sig}$ set to zero and the four capacitances indicated. Capacitance $C_{\mu 1}$ sees a resistance $R_{\mu 1}$,

$$R_{\mu 1} = R_{sig} \,\|\, R_{in}$$
$$= 4 \,\|\, 255 = 3.94 \text{ k}\Omega$$

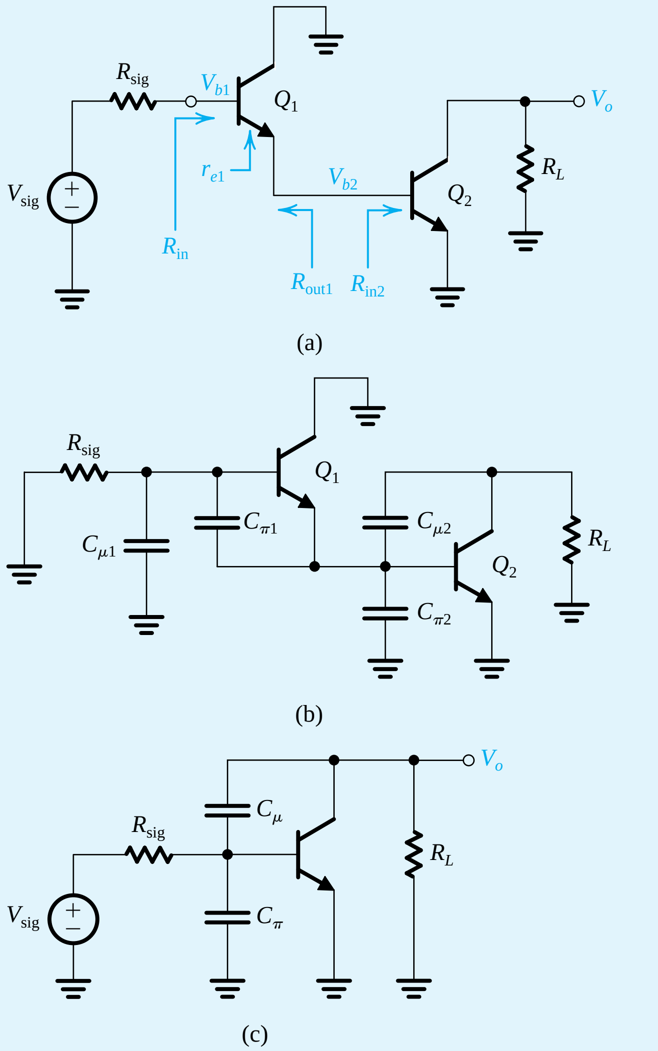



**Figure 9.40** Circuits for Example 9.14: **(a)** the CC–CE circuit prepared for low-frequency, small-signal analysis; **(b)** the circuit at high frequencies, with $V_{sig}$ set to zero to enable determination of the open-circuit time constants; **(c)** a CE amplifier for comparison.

To find the resistance $R_{\pi 1}$ seen by capacitance $C_{\pi 1}$ we refer to the analysis of the high-frequency response of the emitter follower in Section 9.7.2. Specifically, we adapt Eq. (9.133) to the situation here as follows:

$$R_{\pi 1} = \frac{R_{sig} + R_{in2}}{1 + \dfrac{R_{sig}}{r_{\pi 1}} + \dfrac{R_{in2}}{r_{e1}}}$$

$$= \frac{4000 + 2500}{1 + \dfrac{4000}{2500} + \dfrac{2500}{25}} = 63.4\ \Omega$$

**Example 9.15** *continued*

Capacitance $C_{\pi 2}$ sees a resistance $R_{\pi 2}$,

$$\begin{aligned} R_{\pi 2} &= R_{\text{in2}} \,\|\, R_{\text{out1}} \\ &= r_{\pi 2} \,\|\, \left[ r_{e1} + \frac{R_{\text{sig}}}{\beta_1 + 1} \right] \\ &= 2500 \,\|\, \left[ 25 + \frac{4000}{101} \right] = 63 \ \Omega \end{aligned}$$

Capacitance $C_{\mu 2}$ sees a resistance $R_{\mu 2}$. To determine $R_{\mu 2}$ we refer to the analysis of the frequency response of the CE amplifier in Section 9.5 to obtain

$$\begin{aligned} R_{\mu 2} &= (1 + g_{m2} R_L)(R_{\text{in}2} \,\|\, R_{\text{out1}}) + R_L \\ &= (1 + 40 \times 4)\left[ 2500 \,\|\, \left( 25 + \frac{4000}{101} \right) \right] + 4000 \\ &= 14{,}143 \ \Omega \simeq 14.1 \text{ k}\Omega \end{aligned}$$

We now can determine $\tau_H$ from

$$\begin{aligned} \tau_H &= C_{\mu 1} R_{\mu 1} + C_{\pi 1} R_{\pi 1} + C_{\mu 2} R_{\mu 2} + C_{\pi 2} R_{\pi 2} \\ &= 2 \times 3.94 + 13.9 \times 0.0634 + 2 \times 14.1 + 13.9 \times 0.063 \\ &= 7.88 + 0.88 + 28.2 + 0.88 = 37.8 \text{ ns} \end{aligned}$$

We observe that $C_{\pi 1}$ and $C_{\pi 2}$ play a very minor role in determining the high-frequency response. As expected, $C_{\mu 2}$ through the Miller effect plays the most significant role. Also, $C_{\mu 1}$, which interacts directly with $(R_{\text{sig}} \,\|\, R_{\text{in}})$, also plays an important role. The 3-dB frequency $f_H$ can be found as follows:

$$f_H = \frac{1}{2\pi \tau_H} = \frac{1}{2\pi \times 37.8 \times 10^{-9}} = 4.2 \text{ MHz}$$

For comparison, we evaluate $A_M$ and $f_H$ of a CE amplifier operating under the same conditions. Refer to Fig. 9.40(c). The voltage gain $A_M$ is given by

$$\begin{aligned} A_M &= \frac{R_{\text{in}}}{R_{\text{in}} + R_{\text{sig}}}(-g_m R_L) \\ &= \frac{r_\pi}{r_\pi + R_{\text{sig}}}(-g_m R_L) \\ &= \frac{2.5}{2.5 + 4}(-40 \times 4) \\ &= -61.5 \text{ V/V} \end{aligned}$$

$$R_\pi = r_\pi \,\|\, R_{\text{sig}} = 2.5 \,\|\, 4 = 1.54 \text{ k}\Omega$$

$$\begin{aligned} R_\mu &= (1 + g_m R_L)(R_{\text{sig}} \,\|\, r_\pi) + R_L \\ &= (1 + 40 \times 4)(4 \,\|\, 2.5) + 4 \\ &= 251.7 \text{ k}\Omega \end{aligned}$$

Thus,

$$\tau_H = C_\pi R_\pi + C_\mu R_\mu$$
$$= 13.9 \times 1.54 + 2 \times 251.7$$
$$= 21.4 + 503.4 = 524.8 \text{ ns}$$

Observe the dominant role played by $C_\mu$. The 3-dB frequency $f_H$ is

$$f_H = \frac{1}{2\pi\tau_H} = \frac{1}{2\pi \times 524.8 \times 10^{-9}} = 303 \text{ kHz}$$

Thus, including the buffering transistor $Q_1$ increases the gain, $|A_M|$, from 61.5 V/V to 155 V/V—a factor of 2.5—and increases the bandwidth from 303 kHz to 4.2 MHz—a factor of 13.9! The gain–bandwidth product is increased from 18.63 MHz to 651 MHz—a factor of 35!

### 9.9.3 The CC–CB and CD–CG Configurations

In Section 7.6.2 we showed that preceding a CB or CG transistor with a buffer implemented with a CC or a CD transistor solves the low-input-resistance problem of the CB and CG amplifiers. Examples of the resulting compound-transistor amplifiers are shown in Fig. 9.41. Since in each of these circuits, neither of the two transistors suffers from the Miller effect, the resulting amplifiers have even wider bandwidths than those achieved in the compound amplifier stages of the last section. To illustrate, consider as an example the circuit in

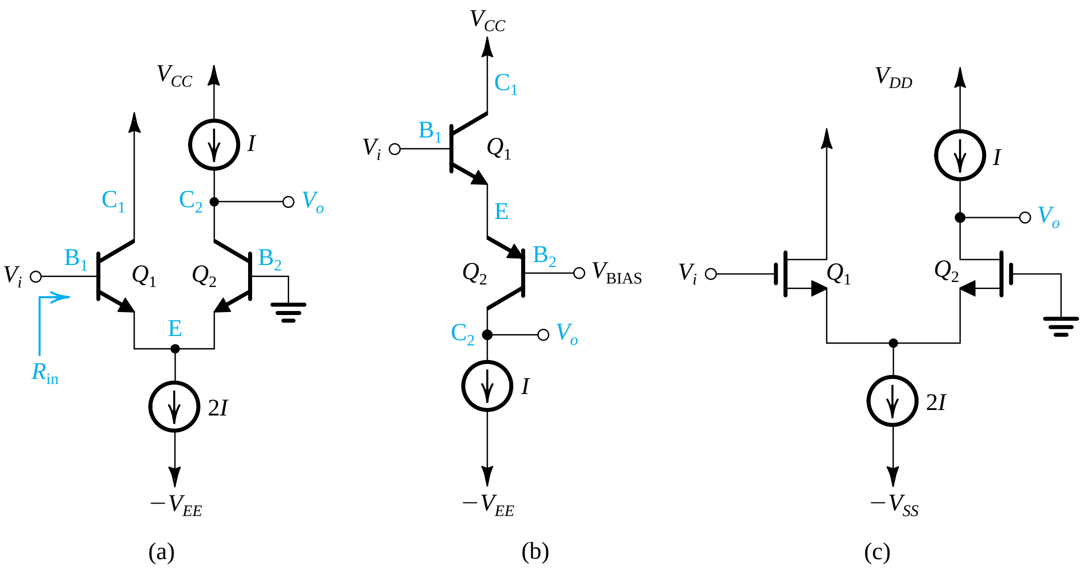



**Figure 9.41** **(a)** A CC–CB amplifier. **(b)** Another version of the CC–CB circuit with $Q_2$ implemented using a *pnp* transistor. **(c)** The MOSFET version of the circuit in **(a)**.

Fig. 9.41(a).[10] The low-frequency analysis of this circuit in Section 7.6.2 provides for the input resistance,

$$R_{in} = (\beta_1 + 1)(r_{e1} + r_{e2}) \tag{9.163}$$

which for $r_{e1} = r_{e2} = r_e$ and $\beta_1 = \beta_2 = \beta$ becomes

$$R_{in} = 2r_\pi \tag{9.164}$$

If a load resistance $R_L$ is connected at the output, the voltage gain $V_o/V_i$ will be

$$\frac{V_o}{V_i} = \frac{\alpha_2 R_L}{r_{e1} + r_{e2}} = \frac{1}{2} g_m R_L \tag{9.165}$$

Now, if the amplifier is fed with a voltage signal $V_{sig}$ from a source with a resistance $R_{sig}$, the overall voltage gain will be

$$\frac{V_o}{V_{sig}} = \frac{1}{2}\left(\frac{R_{in}}{R_{in} + R_{sig}}\right)(g_m R_L) \tag{9.166}$$

The high-frequency analysis is illustrated in Fig. 9.42(a). Here we have drawn the hybrid-$\pi$ equivalent circuit for each of $Q_1$ and $Q_2$. Recalling that the two transistors are operating at equal bias currents, their corresponding model components will be equal (i.e., $r_{\pi 1} = r_{\pi 2}$, $C_{\pi 1} = C_{\pi 2}$, etc.). With this in mind the reader should be able to see that $V_{\pi 1} = -V_{\pi 2}$ and the horizontal line through the node labeled E in Fig. 9.42(a) can be deleted. Thus the circuit reduces to that in Fig. 9.42(b). This is a very attractive outcome because the circuit shows clearly the two poles that determine the high-frequency response: The pole at the input, with a frequency $f_{P1}$, is

$$f_{P1} = \frac{1}{2\pi\left(\frac{C_\pi}{2} + C_\mu\right)(R_{sig} \| 2r_\pi)} \tag{9.167}$$

and the pole at the output, with a frequency $f_{P2}$, is

$$f_{P2} = \frac{1}{2\pi C_\mu R_L} \tag{9.168}$$

This result is also intuitively obvious: The input impedance at $B_1$ of the circuit in Fig. 9.42(a) consists of the series connection of $r_{\pi 1}$ and $r_{\pi 2}$ in parallel with the series connection of $C_{\pi 1}$ and $C_{\pi 2}$. Then there is $C_{\mu 1}$ in parallel. At the output, we simply have $R_L$ in parallel with $C_\mu$.

Whether one of the two poles is dominant will depend on the relative values of $R_{sig}$ and $R_L$. If the two poles are close to each other, then the 3-dB frequency $f_H$ can be determined either by exact analysis—that is, finding the frequency at which the gain is down by 3 dB—or by using the approximate formula in Eq. (9.68),

$$f_H \simeq 1 \bigg/ \sqrt{\frac{1}{f_{P1}^2} + \frac{1}{f_{P2}^2}} \tag{9.169}$$

[10] The results derived for the circuit in Fig. 9.41(a) apply directly to the circuit of Fig. 9.41(b) and with appropriate change of variables to the MOS circuit of Fig. 9.41(c).

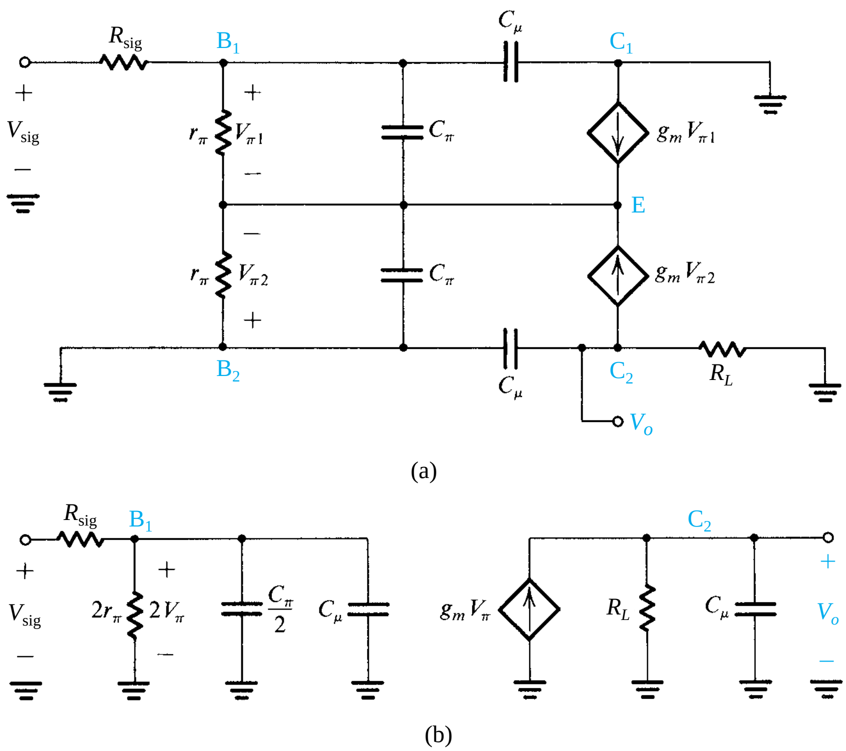



**Figure 9.42** **(a)** Equivalent circuit for the amplifier in Fig. 9.41(a). **(b)** Simplified equivalent circuit. Note that the equivalent circuits in **(a)** and **(b)** also apply to the circuit shown in Fig. 9.41(b). In addition, they can be easily adapted for the MOSFET circuit in Fig. 9.41(c), with $2r_\pi$ eliminated, $C_\pi$ replaced with $C_{gs}$, $C_\mu$ replaced with $C_{gd}$, and $V_\pi$ replaced with $V_{gs}$.

**EXERCISE**

**9.31** For the CC–CB amplifier of Fig. 9.41(a), let $I = 0.5$ mA, $\beta = 100$, $C_\pi = 6$ pF, $C_\mu = 2$ pF, $R_{sig} = 10$ kΩ, and $R_L = 10$ kΩ. Find the low-frequency overall voltage gain $A_M$, the frequencies of the poles, and the 3-dB frequency $f_H$. Find $f_H$ both exactly and using the approximate formula in Eq. (9.169).
**Ans.** 50 V/V; 6.4 MHz and 8 MHz; $f_H$ by exact evaluation = 4.6 MHz; $f_H$ using Eq. (9.169) = 5 MHz.

## 9.10 Multistage Amplifier Examples

We conclude this chapter with the frequency-response analysis of the two multistage amplifiers we studied in Section 8.6. As we shall see, these are relatively complex circuits: Simply replacing each transistor with its high-frequency, equivalent-circuit model will make it exceedingly difficult for pencil-and-paper analysis, and will most certainly not lead to any analysis and design insight. Rather, we will use the knowledge and experience we have gained throughout this chapter to decide on ways to simplify the analysis. Our objective is multifold: to be able to pinpoint the part or parts of a circuit that limit its high-frequency performance, to understand how this limitation comes about, to obtain an estimate of the 3-dB bandwidth $f_H$, and finally and most importantly, to find ways to improve the design of the circuit so as to extend its high-frequency operation.

It is useful at this juncture to point out that computer simulation using PSpice and Multisim is a very valuable tool for the circuit designer, especially when frequency-response analysis is under consideration. Nevertheless, it is a tool that has to be used judiciously and certainly not as a replacement for a first-cut pencil-and-paper analysis. Circuit simulation, by utilizing sophisticated device models, will enable the designer to obtain a reasonably accurate prediction of what to expect after the circuit has been fabricated. If the expected performance is unsatisfactory, the designer will then have the opportunity to alter the design to meet specifications.

### 9.10.1 Frequency Response of the Two-Stage CMOS Op Amp

Figure 9.43 shows the two-stage CMOS amplifier we studied in Section 8.6.1. Before continuing with this section, we urge the reader to review Section 8.6.1 and Example 8.5. To analyze the frequency response of the two-stage op amp, consider its simplified small-signal equivalent circuit shown in Fig. 9.44. Here $G_{m1}$ is the transconductance of the input stage ($G_{m1} = g_{m1} = g_{m2}$), $R_1$ is the output resistance of the first stage ($R_1 = r_{o2} \| r_{o4}$), and $C_1$ is the total capacitance at the interface between the first and second stages

$$C_1 = C_{gd4} + C_{db4} + C_{gd2} + C_{db2} + C_{gs6}$$

$G_{m2}$ is the transconductance of the second stage ($G_{m2} = g_{m6}$), $R_2$ is the output resistance of the second stage ($R_2 = r_{o6} \| r_{o7}$), and $C_2$ is the total capacitance at the output node of the op amp

$$C_2 = C_{db6} + C_{db7} + C_{gd7} + C_L$$

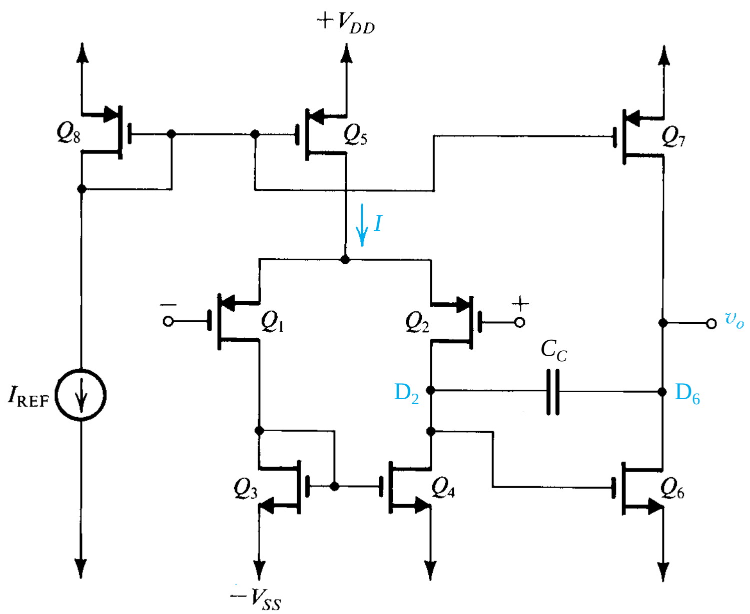



**Figure 9.43** Two-stage CMOS op-amp configuration.

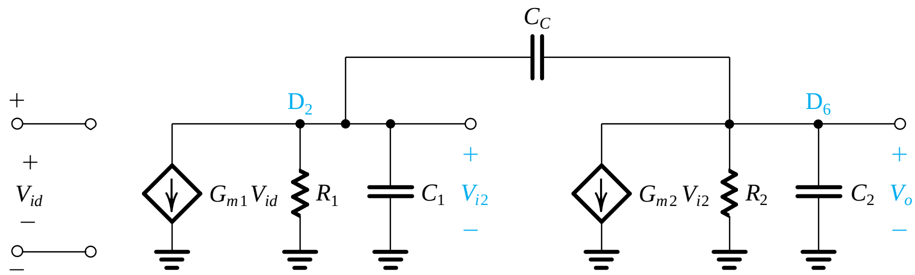



**Figure 9.44** Equivalent circuit of the op amp in Fig. 9.43.

where $C_L$ is the load capacitance. Usually $C_L$ is much larger than the transistor capacitances, with the result that $C_2$ is much larger than $C_1$. Capacitor $C_C$ is deliberately included for the purpose of equipping the op amp with a uniform −6-dB/octave frequency response. In the following, we shall see how this is possible and how to select a value for $C_C$. Finally, note that in the equivalent circuit of Fig. 9.44 we should have included $C_{gd6}$ in parallel with $C_C$. Usually, however, $C_C \gg C_{gd6}$, which is the reason we have neglected $C_{gd6}$.

To determine $V_o$, analysis of the circuit in Fig. 9.44 proceeds as follows. Writing a node equation at node D$_2$ yields

$$G_{m1}V_{id} + \frac{V_{i2}}{R_1} + sC_1V_{i2} + sC_C(V_{i2} - V_o) = 0 \tag{9.170}$$

Writing a node equation at node D$_6$ yields

$$G_{m2}V_{i2} + \frac{V_o}{R_2} + sC_2V_o + sC_C(V_o - V_{i2}) = 0 \tag{9.171}$$

To eliminate $V_{i2}$ and thus determine $V_o$ in terms of $V_{id}$, we use Eq. (9.171) to express $V_{i2}$ in terms of $V_o$ and substitute the result into Eq. (9.170). After some straightforward manipulations we obtain the amplifier transfer function

$$\frac{V_o}{V_{id}} = \frac{G_{m1}(G_{m2} - sC_C)R_1R_2}{1 + s[C_1R_1 + C_2R_2 + C_C(G_{m2}R_1R_2 + R_1 + R_2)] + s^2[C_1C_2 + C_C(C_1 + C_2)]R_1R_2} \tag{9.172}$$

First we note that for $s = 0$ (i.e., dc), Eq. (9.172) gives $V_o/V_{id} = (G_{m1}R_1)(G_{m2}R_2)$, which is what we should have expected. Second, the transfer function in Eq. (9.172) indicates that the amplifier has a transmission zero at $s = s_Z$, which is determined from

$$G_{m2} - s_ZC_C = 0$$

Thus,

$$s_Z = \frac{G_{m2}}{C_C} \tag{9.173}$$

In other words, the zero is on the positive real axis with a frequency $\omega_Z$ of

$$\omega_Z = \frac{G_{m2}}{C_C} \tag{9.174}$$

Also, the amplifier has two poles that are the roots of the denominator polynomial of Eq. (9.172). If the frequencies of the two poles are denoted $\omega_{P1}$ and $\omega_{P2}$, then the denominator polynomial can be expressed as

$$D(s) = \left(1+\frac{s}{\omega_{P1}}\right)\left(1+\frac{s}{\omega_{P2}}\right) = 1+s\left(\frac{1}{\omega_{P1}}+\frac{1}{\omega_{P2}}\right)+\frac{s^2}{\omega_{P1}\omega_{P2}}$$

Now if one of the poles is dominant, say with frequency $\omega_{P1}$, then $\omega_{P1} \ll \omega_{P2}$, and $D(s)$ can be approximated by

$$D(s) \simeq 1+\frac{s}{\omega_{P1}}+\frac{s^2}{\omega_{P1}\omega_{P2}} \tag{9.175}$$

The frequency of the dominant pole, $\omega_{P1}$, can now be determined by equating the coefficients of the $s$ terms in the denominator in Eq. (9.172) and in Eq. (9.175),

$$\begin{aligned}\omega_{P1} &= \frac{1}{C_1R_1+C_2R_2+C_C(G_{m2}R_2R_1+R_1+R_2)}\\ &= \frac{1}{R_1[C_1+C_C(1+G_{m2}R_2)]+R_2(C_2+C_C)}\end{aligned} \tag{9.176}$$

We recognize the first term in the denominator as arising at the interface between the first and second stages. Here, $R_1$, the output resistance of the first stage, is interacting with the total capacitance at the interface. The latter is the sum of $C_1$ and the Miller capacitance $C_C(1+G_{m2}R_2)$, which results from connecting $C_C$ in the negative feedback path of the second stage whose gain is $G_{m2}R_2$. Now, since $R_1$ and $R_2$ are usually of comparable value, we see that the first term in the denominator will be much larger than the second and we can approximate $\omega_{P1}$ as

$$\omega_{P1} \simeq \frac{1}{R_1[C_1+C_C(1+G_{m2}R_2)]}$$

A further approximation is possible because $C_1$ is usually much smaller than the Miller capacitance and $G_{m2}R_2 \gg 1$, thus

$$\omega_{P1} \simeq \frac{1}{R_1C_CG_{m2}R_2} \tag{9.177}$$

The frequency of the second, nondominant pole can be found by equating the coefficients of the $s^2$ terms in the denominator of Eq. (9.172) and in Eq. (9.175) and substituting for $\omega_{P1}$ from Eq. (9.176). The result is

$$\omega_{P2} = \frac{G_{m2}C_C}{C_1C_2+C_C(C_1+C_2)}$$

Since $C_1 \ll C_2$ and $C_1 \ll C_C$, $\omega_{P2}$ can be approximated as

$$\omega_{P2} \simeq \frac{G_{m2}}{C_2} \tag{9.178}$$

In order to provide the op amp with a uniform gain rolloff of –20 dB/decade down to 0 dB, the value of the compensation capacitor $C_C$ is selected so that the resulting value of $\omega_{P1}$ (Eq. 9.177), when multiplied by the dc gain ($G_{m1}R_1G_{m2}R_2$), results in a unity-gain frequency $\omega_t$ lower than $\omega_Z$ and $\omega_{P2}$. Specifically

$$\omega_t = (G_{m1}R_1G_{m2}R_2)\,\omega_{P1} = \frac{G_{m1}}{C_C} \tag{9.179}$$

which must be lower than $\omega_Z = \dfrac{G_{m2}}{C_C}$ and $\omega_{P2} \simeq \dfrac{G_{m2}}{C_2}$. We will have more to say about this point in Section 12.1.

**EXERCISE**

**D9.32** Consider the frequency response of the op amp analyzed in Example 8.5. Let $C_1 = 0.1$ pF and $C_2 =$ 2 pF. Find the value of $C_C$ that results in $f_t = 10$ MHz and verify that $f_t$ is lower than $f_Z$ and $f_{P2}$. Recall from the results of Example 8.5, that $G_{m1} = 0.3$ mA/V and $G_{m2} = 0.6$ mA/V.
**Ans.** $C_C = 4.8$ pF; $f_Z = 20$ MHz; $f_{P2} = 48$ MHz

## 9.10.2 Frequency Response of the Bipolar Op Amp of Section 8.6.2

We urge the reader to review Section 8.6.2 and Examples 8.6 and 8.7 before studying this section. The bipolar op-amp circuit shown earlier in Fig. 8.43 is rather complex. Nevertheless, it is possible to obtain an approximate estimate of its high-frequency response. Figure 9.45(a) shows an approximate equivalent circuit for this purpose. Note that we have utilized the equivalent differential half-circuit concept, with $Q_2$ representing the input stage and $Q_5$ representing the second stage. We observe, of course, that the second stage is not symmetrical, and strictly speaking the equivalent half-circuit does not apply. Nevertheless, we use it as an approximation so as to obtain a quick pencil-and-paper estimate of the dominant high-frequency pole of the amplifier. More precise results can of course be obtained using computer simulation with SPICE.

Examination of the equivalent circuit in Fig. 9.45(a) reveals that if the resistance of the source of signal $V_i$ is small, the high-frequency limitation will not occur at the input but rather at the interface between the first and the second stages. This is because the total capacitance at node A will be high as a result of the Miller multiplication of $C_{\mu 5}$. Also, the third stage, formed by transistor $Q_7$, should exhibit good high-frequency response, since $Q_7$ has a large emitter-degeneration resistance, $R_3$. The same is also true for the emitter-follower stage, $Q_8$.

To determine the frequency of the dominant pole that is formed at the interface between $Q_2$ and $Q_5$ we show in Fig. 9.45(b) the pertinent equivalent circuit. The total resistance between node A and ground can now be found as

$$R_{\text{eq}} = R_2 \,\|\, r_{o2} \,\|\, r_{\pi 5}$$

and the total capacitance is

$$C_{\text{eq}} = C_{\mu 2} + C_{\pi 5} + C_{\mu 5}(1 + g_{m5}R_{L5})$$

where

$$R_{L5} = R_3 \,\|\, r_{o5} \,\|\, R_{i3}$$

The frequency of the pole can be calculated from $R_{\text{eq}}$ and $C_{\text{eq}}$ as

$$f_P = \frac{1}{2\pi R_{\text{eq}} C_{\text{eq}}}$$

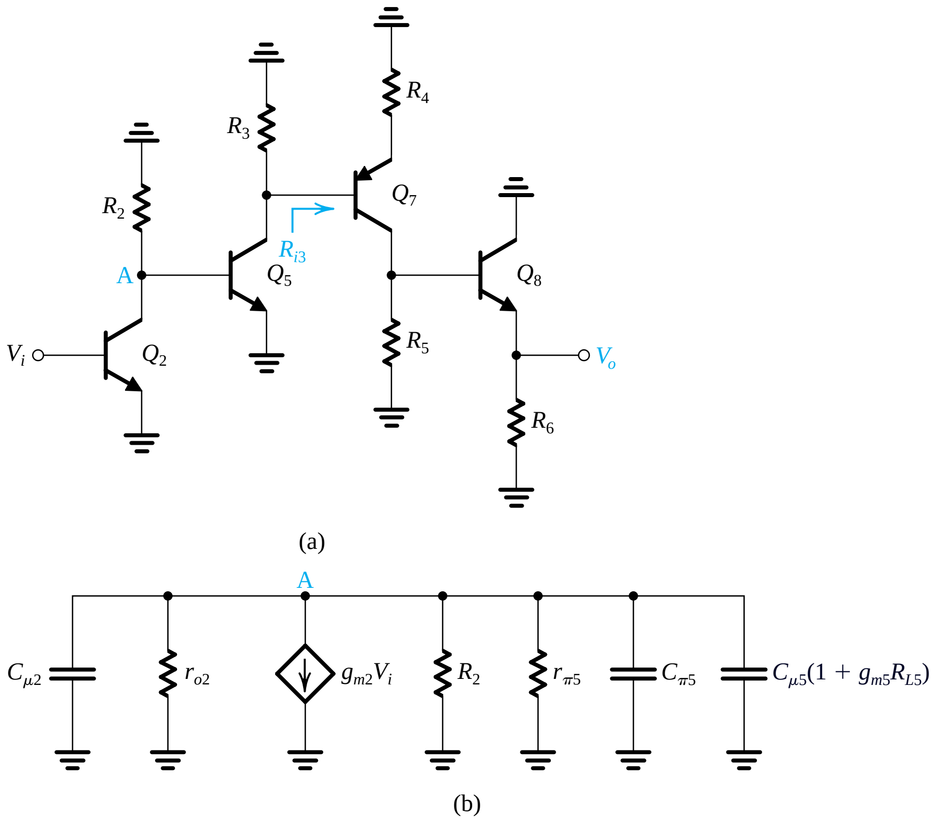



**Figure 9.45** **(a)** Approximate equivalent circuit for determining the high-frequency response of the op amp of Fig. 8.43. **(b)** Equivalent circuit of the interface between the output of $Q_2$ and the input of $Q_5$.

## EXERCISE

**9.33** Determine $R_{eq}$, $C_{eq}$, and $f_P$ for the amplifier in Fig. 8.43, utilizing the facts that $Q_2$ is biased at 0.25 mA and $Q_5$ at 1 mA. Assume $\beta = 100$, $V_A = 100$ V, $f_T = 400$ MHz, and $C_\mu = 2$ pF. Assume $R_{L5} \simeq R_3$.
**Ans.** 2.21 kΩ; 258 pF; 280 kHz

# Summary

- The coupling and bypass capacitors utilized in discrete-circuit amplifiers cause the amplifier gain to fall off at low frequencies. The frequencies of the low-frequency poles can be estimated by considering each of these capacitors separately and determining the resistance seen by the capacitor. The highest-frequency pole is the one that determines the lower 3-dB frequency $f_L$.

- Both the MOSFET and the BJT have internal capacitive effects that can be modeled by augmenting the device hybrid-$\pi$ model with capacitances. Usually at least two capacitances are needed: $C_{gs}$ and $C_{gd}$ ($C_\pi$ and $C_\mu$ for the BJT). A figure-of-merit for the high-frequency operation of the transistor is the frequency $f_T$ at which the short-circuit current gain of the CS (CE) transistor reduces to unity. For the MOSFET, $f_T = g_m/2\pi(C_{gs} + C_{gd})$, and for the BJT, $f_T = g_m/2\pi(C_\pi + C_\mu)$.

- The internal capacitances of the MOSFET and the BJT cause the amplifier gain to fall off at high frequencies. An estimate of the amplifier bandwidth is provided by the frequency $f_H$ at which the gain drops 3 dB below its value at midband, $A_M$. A figure-of-merit for the amplifier is the gain–bandwidth product $GB = A_M f_H$. Usually, it is possible to trade off gain for increased bandwidth, with *GB* remaining nearly constant. For amplifiers with a dominant pole with frequency $f_H$, the gain falls off at a uniform 6-dB/octave (20-dB/decade) rate, reaching 0 dB at $f_t = GB$.

- The high-frequency response of the CS and CE amplifiers is severely limited by the Miller effect: The small capacitance $C_{gd}$ $(C_\mu)$ is multiplied by a factor approximately equal to the gain from gate to drain (base to collector) $g_m R'_L$ and thus gives rise to a large capacitance at the amplifier input. The increased $C_{in}$ interacts with the effective signal-source resistance $R'_{sig}$ and causes the amplifier gain to have a 3-dB frequency $f_H = 1/2\pi R'_{sig} C_{in}$.

- The method of open-circuit time constants provides a simple and powerful way to obtain a reasonably good estimate of the upper 3-dB frequency $f_H$. The capacitors that limit the high-frequency response are considered one at time with $V_{sig} = 0$ and all the other capacitances set to zero (open circuited). The resistance seen by each capacitance is determined, and the overall time constant $\tau_H$ is obtained by summing the individual time constants. Then $f_H$ is found as $1/2\,\pi\tau_H$.

- The CG and CB amplifiers do *not* suffer from the Miller effect. Thus the cascode amplifier, which consists of a cascade of a CS and CG stages (CE and CB stages), can be designed to obtain wider bandwidth than that achieved in the CS (CE) amplifier alone. The key, however, is to design the cascode so that the gain obtained in the CS (CE) stage is minimized.

- The source and emitter followers do not suffer from the Miller effect and thus feature wide bandwidths.

- The high-frequency response of the differential amplifier can be obtained by considering the differential and common-mode half-circuits. The CMRR falls off at a relatively low frequency determined by the output impedance of the bias current source.

- The high-frequency response of the current-mirror-loaded differential amplifier is complicated by the fact that there are two signal paths between input and output: a direct path and one through the current mirror.

- Combining two transistors in a way that eliminates or minimizes the Miller effect can result in a much wider bandwidth. Some such configurations are presented in Section 9.9.

- The key to the analysis of the high-frequency response of a multistage amplifier is to use simple macro models to estimate the frequencies of the poles formed at the interface between each two stages, in addition to the input and output poles. The pole with the lowest frequency dominates and determines $f_H$.

# PROBLEMS

## Computer Simulation Problems

SIM Problems identified by this icon are intended to demonstrate the value of using SPICE simulation to verify hand analysis and design, and to investigate important issues such as gain–bandwidth tradeoff. Instructions to assist in setting up PSpice and Multisim simulations for all the indicated problems can be found in the corresponding files on the disc. Note that if a particular parameter value is not specified in the problem statement, you are to make a reasonable assumption. *difficult problem; ** more difficult; *** very challenging and/or time-consuming; D: design problem.

## Section 9.1: Low-Frequency Response of the CS and CE Amplifiers

**D 9.1** The amplifier in Fig. P9.1 is biased to operate at $g_m = 1$ mA/V. Neglecting $r_o$, find the midband gain. Find the value of $C_S$ that places $f_L$ at 20 Hz.

**9.2** Consider the amplifier of Fig. 9.2(a). Let $R_D = 10$ kΩ, $r_o = 100$ kΩ, and $R_L = 10$ kΩ. Find the value of $C_{C2}$, specified to one significant digit, to ensure that the associated break frequency is at, or below, 10 Hz. If a higher-power design results in doubling $I_D$, with both $R_D$ and $r_o$ reduced by a factor of 2, what does the corner frequency (due to $C_{C2}$) become? For

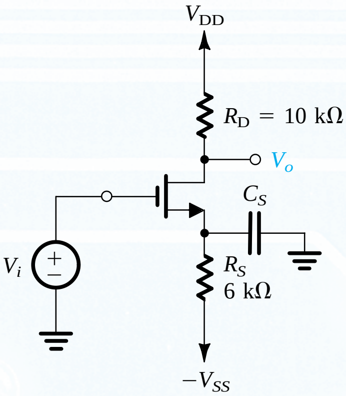



**Figure P9.1**

increasingly higher-power designs, what is the highest corner frequency that can be associated with $C_{C2}$?

**9.3** The NMOS transistor in the discrete CS amplifier circuit of Fig. P9.3 is biased to have $g_m = 5$ mA/V. Find $A_M$, $f_{P1}$, $f_{P2}$, $f_{P3}$, and $f_L$.

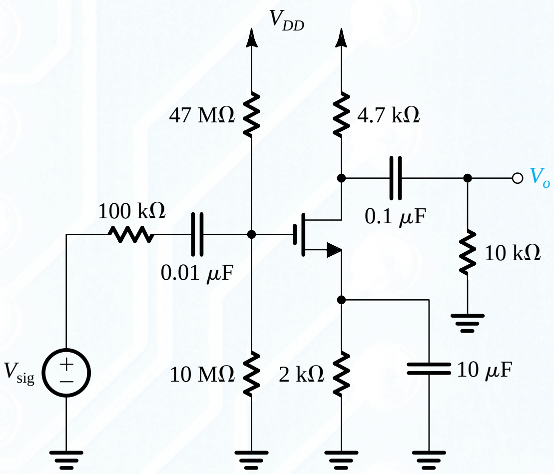



**Figure P9.3**

**D 9.4** Consider the low-frequency response of the CS amplifier of Fig. 9.2(a). Let $R_{\text{sig}} = 0.5$ MΩ, $R_G = 2$ MΩ, $g_m = 3$ mA/V, $R_D = 20$ kΩ, and $R_L = 10$ kΩ. Find $A_M$. Also, design the coupling and bypass capacitors to locate the three low-frequency poles at 50 Hz, 10 Hz, and 3 Hz. Use a minimum total capacitance, with capacitors specified only to a single significant digit. What value of $f_L$ results?

**D 9.5** A particular version of the CS amplifier in Fig. 9.2 uses a transistor biased to operate with $g_m = 5$ mA/V. Resistances $R_{\text{sig}} = 200\ \text{k}\Omega$, $R_G = 10\ \text{M}\Omega$, $R_D = 3\ \text{k}\Omega$, and $R_L = 5\ \text{k}\Omega$. As an initial design, the circuit designer selects $C_{C1} = C_{C2} = C_S = 1\ \mu\text{F}$. Find the frequencies $f_{P1}$, $f_{P2}$, and $f_{P3}$ and rank them in order of frequency, highest first. Calculate the ratios of the first to second, and second to third. The final design requires that the first pole dominate at 10 Hz with the second a factor of 4 lower, and the third another a factor of 4 lower. Find the values of all the capacitances and the total capacitance needed. If the separation factor were 10, what capacitor values and total capacitance would be needed? (*Note*: You can see that the total capacitance need not be much larger to spread the poles, as is desired in certain applications.)

**D 9.6** Repeat Example 9.1 to find $C_S$, $C_{C1}$, and $C_{C2}$ that provide $f_L = 20$ Hz and the other pole frequencies at 4 Hz and 1 Hz. Design to keep the total capacitance to a minimum.

**D 9.7** Reconsider Exercise 9.1 with the aim of finding a better-performing design using the same total capacitance, that is, 3 μF. Prepare a design in which the break frequencies are separated by a factor of 5 (i.e., $f$, $f/5$, and $f/25$). What are the three capacitor values, the three break frequencies, and $f_L$ that you achieve?

**9.8** Repeat Exercise 9.2 for the situation in which $C_E = 50$ μF and $C_{C1} = C_{C2} = 2$ μF. Find the three break frequencies and estimate $f_L$.

**D 9.9** Repeat Example 9.2 for a related CE amplifier whose supply voltages and bias current are each reduced to half their original value but $R_B$, $R_C$, $R_{\text{sig}}$, and $R_L$ are left unchanged. Find $C_{C1}$, $C_E$, and $C_{C2}$ for $f_L = 100$ Hz. Minimize the total capacitance used, under the following conditions. Arrange that the contributions of $C_E$, $C_{C1}$, and $C_{C2}$ are 80%, 10%, and 10%, respectively. Specify capacitors to two significant digits, choosing the next highest value, in general, for a conservative design, but realizing that for $C_E$, this may represent a larger capacitance increment. Check the value of $f_L$ that results. [*Note*: An attractive approach can be to select $C_E$ on the small side, allowing it to contribute more than 80% to $f_L$, while making $C_{C1}$ and $C_{C2}$ larger, since they must contribute less to $f_L$.)

**D 9.10** A particular current-biased CE amplifier operating at 100 μA from ±3-V power supplies employs $R_C = 20\ \text{k}\Omega$, $R_B = 200\ \text{k}\Omega$; it operates between a 20-kΩ source and a 10-kΩ load. The transistor $\beta = 100$. Select $C_E$ first for a minimum value specified to one significant digit and providing up to 90% of $f_L$. Then choose $C_{C1}$ and $C_{C2}$, each specified to one significant digit, with the goal of minimizing the total capacitance used. What $f_L$ results? What total capacitance is needed?

**9.11** Consider the common-emitter amplifier of Fig. P9.11 under the following conditions: $R_{\text{sig}} = 5$ kΩ, $R_1 = 33$ kΩ, $R_2 = 22$ kΩ, $R_E = 3.9$ kΩ, $R_C = 4.7$ kΩ, $R_L = 5.6$ kΩ, $V_{CC} = 5$ V. The dc emitter current can be shown to be $I_E \simeq 0.3$ mA, at which $\beta = 120$. Find the input resistance $R_{\text{in}}$ and the midband gain $A_M$. If $C_{C1} = C_{C2} = 1$ μF and $C_E = 20$ μF, find the three break frequencies $f_{P1}$, $f_{P2}$, and $f_{P3}$ and an estimate for $f_L$. Note that $R_E$ has to be taken into account in evaluating $f_{P2}$.

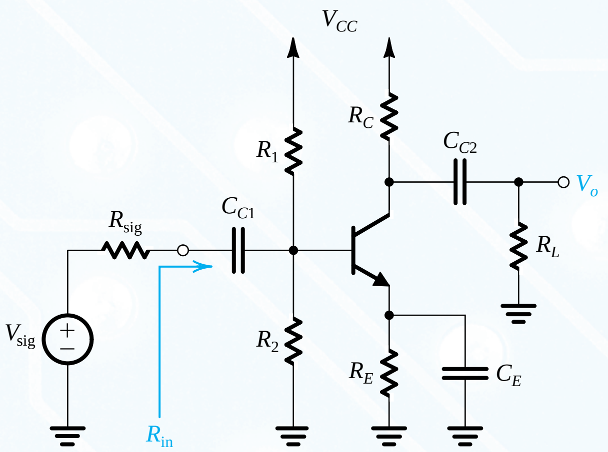



**Figure P9.11**

**D 9.12** For the amplifier described in Problem 9.11, design the coupling and bypass capacitors for a lower 3-dB frequency of 100 Hz. Design so that the contribution of each of $C_{C1}$ and $C_{C2}$ to determining $f_L$ is only 5%.

**9.13** Consider the circuit of Fig. P9.11. For $R_{sig}$ = 10 kΩ, $R_B \equiv R_1 || R_2 =$ 10 kΩ, $r_\pi = 1$ kΩ, $\beta_0 = 100$, and $R_E = 1$ kΩ, what is the ratio $C_E / C_{C1}$ that makes their contributions to the determination of $f_L$ equal?

**D *9.14** For the common-emitter amplifier of Fig. P9.14, neglect $r_o$ and assume the current source to be ideal.

(a) Derive an expression for the midband gain.

(b) Derive expressions for the break frequencies caused by $C_E$ and $C_C$.

(c) Give an expression for the amplifier voltage gain $A(s)$.

(d) For $R_{sig} = R_C = R_L = 10$ kΩ, $\beta = 100$, and $I = 1$ mA, find the value of the midband gain.

(e) Select values for $C_E$ and $C_C$ to place the two break frequencies a decade apart and to obtain a lower 3-dB frequency of 100 Hz while minimizing the total capacitance.

(f) Sketch a Bode plot for the gain magnitude, and estimate the frequency at which the gain becomes unity.

(g) Find the phase shift at 100 Hz.

**9.15** The BJT common-emitter amplifier of Fig. P9.15 includes an emitter degeneration resistance $R_e$.

(a) Assuming $\alpha \simeq 1$, neglecting $r_o$, and assuming the current source to be ideal, derive an expression for the small-signal voltage gain $A(s) \equiv V_o / V_{sig}$ that applies in the midband and the low-frequency band. Hence find the midband gain $A_M$ and the lower 3-dB frequency $f_L$.

(b) Show that including $R_e$ reduces the magnitude of $A_M$ by a certain factor. What is this factor?

(c) Show that including $R_e$ reduces $f_L$ by the same factor as in (b) and thus one can use $R_e$ to trade-off gain for bandwidth.

(d) For $I = 0.25$ mA, $R_C = 10$ kΩ, and $C_E = 10$ µF, find $|A_M|$ and $f_L$ with $R_e = 0$. Now find the value of $R_e$ that lowers $f_L$ by a factor of 5. What will the gain become? Sketch on the same diagram a Bode plot for the gain magnitude for both cases.

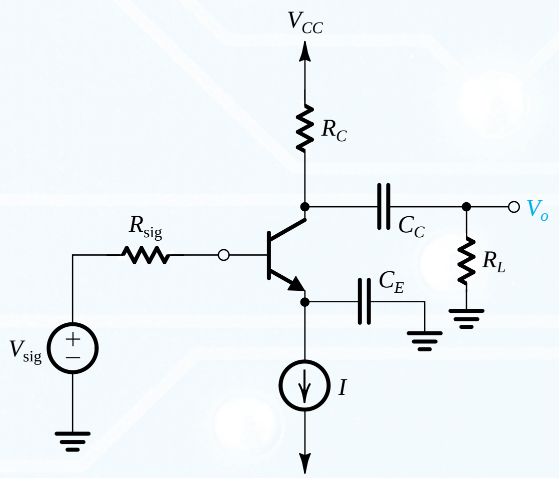



**Figure P9.14**

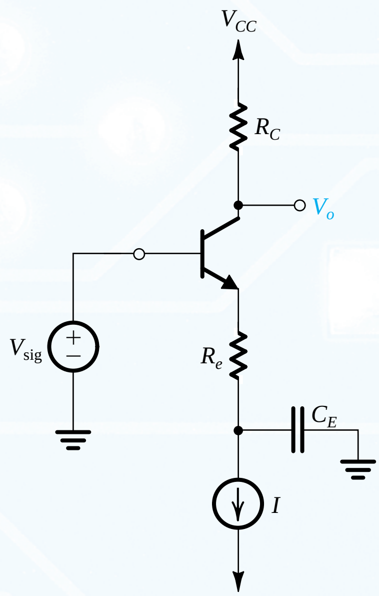



**Figure P9.15**

## Section 9.2: Internal Capacitative Effects and the High-Frequency Model of the MOSFET and the BJT

**9.16** Refer to the MOSFET high-frequency model in Fig. 9.6(a). Evaluate the model parameters for an NMOS transistor operating at $I_D = 100\ \mu\text{A}$, $V_{SB} = 1$ V, and $V_{DS} = 1.5$ V. The MOSFET has $W = 20\ \mu\text{m}$, $L = 1\ \mu\text{m}$, $t_{ox} = 8$ nm, $\mu_n = 450\ \text{cm}^2/\text{Vs}$, $\gamma = 0.5\ \text{V}^{1/2}$, $2\phi_f = 0.65$ V, $\lambda = 0.05\ \text{V}^{-1}$, $V_0 = 0.7$ V, $C_{sb0} = C_{db0} = 15$ fF, and $L_{ov} = 0.05\ \mu\text{m}$. (Recall that $g_{mb} = \chi g_m$, where $\chi = \gamma/(2\sqrt{2\phi_f + V_{SB}})$.)

**9.17** Find $f_T$ for a MOSFET operating at $I_D = 100\ \mu\text{A}$ and $V_{OV} = 0.2$ V. The MOSFET has $C_{gs} = 20$ fF and $C_{gd} = 5$ fF.

**9.18** Starting from the expression of $f_T$ for a MOSFET,

$$f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd})}$$

and making the approximation that $C_{gs} \gg C_{gd}$ and that the overlap component of $C_{gs}$ is negligibly small, show that

$$f_T \simeq \frac{1.5}{\pi L}\sqrt{\frac{\mu_n I_D}{2C_{ox}WL}}$$

Thus note that to obtain a high $f_T$ from a given device, it must be operated at a high current. Also note that faster operation is obtained from smaller devices.

**9.19** Starting from the expression for the MOSFET unity-gain frequency,

$$f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd})}$$

and making the approximation that $C_{gs} \gg C_{gd}$ and that the overlap component of $C_{gs}$ is negligibly small, show that for an $n$-channel device

$$f_T \simeq \frac{3\mu_n V_{OV}}{4\pi L^2}$$

Observe that for a given channel length, $f_T$ can be increased by operating the MOSFET at a higher overdrive voltage. Evaluate $f_T$ for devices with $L = 1.0\ \mu\text{m}$ operated at overdrive voltages of 0.25 V and 0.5 V. Use $\mu_n = 450\ \text{cm}^2/\text{Vs}$.

**9.20** It is required to calculate the intrinsic gain $A_0$ and the unity-gain frequency $f_T$ of an $n$-channel transistor fabricated in a 0.18-μm CMOS process for which $L_{ov} = 0.1\ L$, $\mu_n = 450\ \text{cm}^2/\text{V.s}$, and $V_A' = 5\ \text{V}/\mu\text{m}$. The device is operated at $V_{OV} = 0.2$ V. Find $A_0$ and $f_T$ for devices with $L = L_{\min}$, $2L_{\min}$, $3L_{\min}$, $4L_{\min}$, and $5L_{\min}$. Present your results in a table.

**9.21** A particular BJT operating at $I_C = 1$ mA has $C_\mu = 1$ pF, $C_\pi = 10$ pF, and $\beta = 100$. What are $f_T$ and $f_\beta$ for this situation?

**9.22** For the transistor described in Problem 9.21, $C_\pi$ includes a relatively constant depletion-layer capacitance of 2 pF. If the device is operated at $I_C = 0.2$ mA, what does its $f_T$ become?

**9.23** An *npn* transistor is operated at $I_C = 0.5$ mA and $V_{CB} = 2$ V. It has $\beta_0 = 100$, $V_A = 50$ V, $\tau_F = 30$ ps, $C_{je0} = 20$ fF, $C_{\mu 0} = 30$ fF, $V_{0c} = 0.75$ V, $m_{CBJ} = 0.5$, and $r_x = 100\ \Omega$. Sketch the complete hybrid-$\pi$ model, and specify the values of all its components. Also, find $f_T$.

**9.24** Measurement of $h_{fe}$ of an *npn* transistor at 50 MHz shows that $|h_{fe}| = 10$ at $I_C = 0.2$ mA and 12 at $I_C = 1.0$ mA. Furthermore, $C_\mu$ was measured and found to be 0.1 pF. Find $f_T$ at each of the two collector currents used. What must $\tau_F$ and $C_{je}$ be?

**9.25** A particular small-geometry BJT has $f_T$ of 8 GHz and $C_\mu = 0.1$ pF when operated at $I_C = 1.0$ mA. What is $C_\pi$ in this situation? Also, find $g_m$. For $\beta = 160$, find $r_\pi$ and $f_\beta$.

**9.26** For a BJT whose unity-gain bandwidth is 2 GHz and $\beta_0 = 200$, at what frequency does the magnitude of $h_{fe}$ become 20? What is $f_\beta$?

***9.27** For a sufficiently high frequency, measurement of the complex input impedance of a BJT having (ac) grounded emitter and collector yields a real part approximating $r_x$. For what frequency, defined in terms of $\omega_\beta$, is such an estimate of $r_x$ good to within 10% under the condition that $r_x \le r_\pi/10$?

***9.28** Complete the table entries below for transistors (a) through (g), under the conditions indicated. Neglect $r_x$.

| Transistor | $I_E$(mA) | $r_e$(Ω) | $g_m$ (mA/V) | $r_p$k(Ω) | $\beta_0$ | $f_T$(MHz) | $C_m$ (pF) | $C_\pi$ (pF) | $f_\beta$(MHz) |
|---|---|---|---|---|---|---|---|---|---|
| (a) | 1 | | | | 100 | 400 | 2 | | |
| (b) | | 25 | | | | | 2 | 10.7 | 4 |
| (c) | | | | 2.525 | | 400 | | 13.84 | |
| (d) | 10 | | | | 100 | 400 | 2 | | |
| (e) | 0.1 | | | | 100 | 100 | 2 | | |
| (f) | 1 | | | | 10 | 400 | 2 | | |
| (g) | | | | | | 800 | 1 | 9 | 80 |

## Section 9.3: High-Frequency Response of the CS and CE Amplifiers

**9.29** In a particular common-source amplifier for which the midband voltage gain between gate and drain (i.e., $-g_m R_L'$ ) is $-29$ V/V, the NMOS transistor has $C_{gs}$ = 0.5 pF and $C_{gd}$ = 0.1 pF. What input capacitance would you expect? For what range of signal-source resistances can you expect the 3-dB frequency to exceed 10 MHz? Neglect the effect of $R_G$.

**D 9.30** A design is required for a CS amplifier for which the MOSFET is operated at $g_m$ = 5 mA/V and has $C_{gs}$ = 5 pF and $C_{gd}$ = 1 pF. The amplifier is fed with a signal source having $R_{sig}$ = 1 kΩ, and $R_G$ is very large. What is the largest value of $R_L'$ for which the upper 3-dB frequency is at least 10 MHz? What is the corresponding value of midband gain and gain–bandwidth product? If the specification on the upper 3-dB frequency can be relaxed by a factor of 3, that is, to (10/3) MHz, what can $A_M$ and GB become?

**9.31** Reconsider Example 9.3 for the situation in which the transistor is replaced by one whose width $W$ is half that of the original transistor while the bias current remains unchanged. Find modified values for all the device parameters along with $A_M$, $f_H$, and the gain–bandwidth product, GB. Contrast this with the original design by calculating the ratios of new value to old for $W$, $V_{OV}$, $g_m$, $C_{gs}$, $C_{gd}$, $C_{in}$, $A_M$, $f_H$, and GB.

**D 9.32** In a CS amplifier, such as that in Fig. 9.2(a), the resistance of the source $R_{sig}$ = 100 kΩ, amplifier input resistance (which is due to the biasing network) $R_{in}$ = 100 kΩ, $C_{gs}$ = 1 pF, $C_{gd}$ = 0.2 pF, $g_m$ = 3 mA/V, $r_o$ = 50 kΩ, $R_D$ = 8 kΩ, and $R_L$ = 10 kΩ. Determine the expected 3-dB cutoff frequency $f_H$ and the midband gain. In evaluating ways to double $f_H$, a designer considers the alternatives of changing either $R_L$ or $R_{in}$. To raise $f_H$ as described, what separate change in each would be required? What midband voltage gain results in each case?

**9.33** A discrete MOSFET common-source amplifier has $R_G$ = 1 MΩ, $g_m$ = 5 mA/V, $r_o$ = 100 kΩ, $R_D$ = 10 kΩ, $C_{gs}$ = 2 pF, and $C_{gd}$ = 0.4 pF. The amplifier is fed from a voltage source with an internal resistance of 500 kΩ and is connected to a 10-kΩ load. Find:

(a) the overall midband gain $A_M$
(b) the upper 3-dB frequency $f_H$

**9.34** The analysis of the high-frequency response of the common-source amplifier, presented in the text, is based on the assumption that the resistance of the signal source, $R_{sig}$, is large and, thus, that its interaction with the input capacitance $C_{in}$ produces the "dominant pole" that determines the upper 3-dB frequency $f_H$. In some situations, however, the CS amplifier is fed with a very low $R_{sig}$. To investigate the high-frequency response of the amplifier in such a case, Fig. P9.34 shows the equivalent circuit when the CS amplifier is fed with an ideal voltage source $V_{sig}$ having $R_{sig}$ = 0. Note that $C_L$ denotes the total capacitance at the output node. By writing a node equation at the output, show that the transfer function $V_o/V_{sig}$ is given by

$$\frac{V_o}{V_{sig}} = -g_m R_L' \frac{1 - s(C_{gd}/g_m)}{1 + s(C_L + C_{gd})R_L'}$$

At frequencies $\omega \ll (g_m/C_{gd})$, the $s$ term in the numerator can be neglected. In such case, what is the upper 3-dB frequency resulting? Compute the values of $A_M$ and $f_H$ for the case: $C_{gd}$ = 0.4 pF, $C_L$ = 2 pF, $g_m$ = 5 mA/V, and $R_L'$ = 5 kΩ.

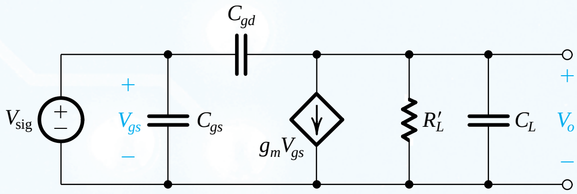



**Figure P9.34**

**9.35** The NMOS transistor in the discrete CS amplifier circuit of Fig. P9.3 is biased to have $g_m$ = 1 mA/V and $r_o$ = 100 kΩ. Find $A_M$. If $C_{gs}$ = 1 pF and $C_{gd}$ = 0.2 pF, find $f_H$.

**9.36** A designer wishes to investigate the effect of changing the bias current $I$ on the midband gain and high-frequency response of the CE amplifier considered in Example 9.4. Let $I$ be doubled to 2 mA, and assume that $\beta_0$ and $f_T$ remain unchanged at 100 and 800 MHz, respectively. To keep the node voltages nearly unchanged, the designer reduces $R_B$ and $R_C$ by a factor of 2, to 50 kΩ and 4 kΩ, respectively. Assume $r_x$ = 50 Ω, and recall that $V_A$ = 100 V and that $C_\mu$ remains constant at 1 pF. As before, the amplifier is fed with a source having $R_{sig}$ = 5 kΩ and feeds a load $R_L$ = 5 kΩ. Find the new values of $A_M$, $f_H$, and the gain–bandwidth product, $|A_M| f_H$. Comment on the results. Note that the price paid for whatever improvement in performance is achieved is an increase in power. By what factor does the power dissipation increase?

***9.37** The purpose of this problem is to investigate the high-frequency response of the CE amplifier when it is fed with a relatively large source resistance $R_{sig}$. Refer to the amplifier in Fig. 9.4 (a) and to its high-frequency, equivalent-circuit model and the analysis shown in Fig. 9.14. Let $R_B \gg R_{sig}$, $r_x \ll R_{sig}$, $R_{sig} \gg r_\pi$, $g_m R_L' \gg 1$, and $g_m R_L' C_\mu \gg C_\pi$.

Under these conditions, show that:

(a) the midband gain $A_M \simeq -\beta R_L'/R_{sig}$
(b) the upper 3-dB frequency $f_H \simeq 1/2\pi C_\mu \beta R_L'$
(c) the gain–bandwidth product $A_M f_H \simeq 1/2\pi C_\mu R_{sig}$

Evaluate this approximate value of the gain–bandwidth product for the case $R_{sig}$ = 25 kΩ and $C_\mu$ = 1 pF. Now, if the transistor is biased at $I_C$ = 1 mA and has $\beta$ = 100, find the midband gain and $f_H$ for the two cases $R_L'$ = 25 kΩ and $R_L'$ = 2.5 kΩ. On the same coordinates, sketch Bode plots for the gain magnitude versus frequency for the two cases. What $f_H$ is obtained when the gain is unity? What value of $R_L'$ corresponds?

**9.38** For a version of the CE amplifier circuit in Fig. P9.11, $R_{sig} = 10$ kΩ, $R_1 = 68$ kΩ, $R_2 = 27$ kΩ, $R_E = 2.2$ kΩ, $R_C = 4.7$ kΩ, and $R_L = 10$ kΩ. The collector current is 0.8 mA, $\beta = 200$, $f_T = 1$ GHz, and $C_\mu = 0.8$ pF. Neglecting the effect of $r_x$ and $r_o$, find the midband voltage gain and the upper 3-dB frequency $f_H$.

**9.39** A particular BJT operating at 2 mA is specified to have $f_T = 2$ GHz, $C_\mu = 1$ pF, $r_x = 100\ \Omega$, and $\beta = 120$. The device is used in a CE amplifier operating from a very-low-resistance voltage source.

(a) If the midband gain obtained is $-10$ V/V, what is the value of $f_H$?
(b) If the midband gain is reduced to $-1$ V/V (by changing $R_L'$), what $f_H$ is obtained?

**9.40** Repeat Example 9.4 for the situation in which the power supplies are reduced to $\pm 5$ V and the bias current is reduced to 0.5 mA. Assume that all other component values and transistor parameter values remain unchanged. Find $A_M$, $f_H$, and the gain–bandwidth product and compare to the values obtained in Example 9.4.

***9.41** The amplifier shown in Fig. P9.41 has $R_{sig} = R_L = 1$ kΩ, $R_C = 1$ kΩ, $R_B = 47$ kΩ, $\beta = 100$, $C_\mu = 0.8$ pF, and $f_T = 600$ MHz. Assume the coupling capacitors to be very large.

(a) Find the dc collector current of the transistor.
(b) Find $g_m$ and $r_\pi$.
(c) Neglecting $r_o$, find the midband voltage gain from base to collector (neglect the effect of $R_B$).
(d) Use the gain obtained in (c) to find the component of $R_{in}$ that arises as a result of $R_B$. Hence find $R_{in}$.
(e) Find the overall gain at midband.
(f) Find $C_{in}$.
(g) Find $f_H$.

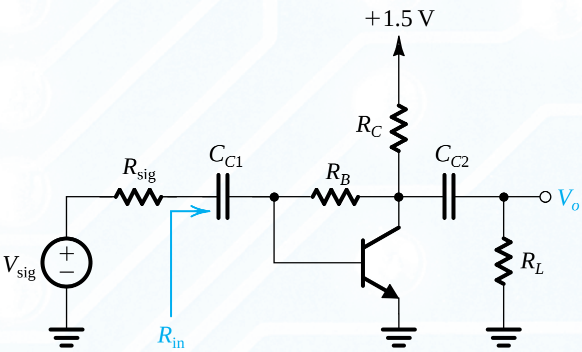



**Figure P9.41**

***9.42** Figure P9.42 shows a diode-connected transistor with the bias circuit omitted. Utilizing the BJT high-frequency, hybrid-$\pi$ model with $r_x = 0$ and $r_o = \infty$, derive an expression for $Z_i(s)$ as a function of $r_e$ and $C_\pi$. Find the frequency at which the impedance has a phase angle of 45° for the case in which the BJT has $f_T = 400$ MHz and the bias current is relatively high. What is the frequency when the bias current is reduced so that $C_\pi \simeq C_\mu$? Assume $\alpha = 1$.

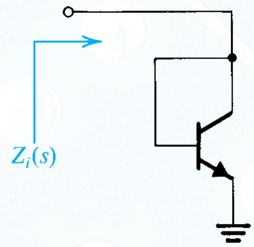



**Figure P9.42**

## Section 9.4: Useful Tools for the Analysis of the High-Frequency Response of Amplifiers

**9.43** A direct-coupled amplifier has a low-frequency gain of 40 dB, poles at 1 MHz and 10 MHz, a zero on the negative real axis at 100 MHz, and another zero at infinite frequency. Express the amplifier gain function in the form of Eqs. (9.61) and (9.62), and sketch a Bode plot for the gain magnitude. What do you estimate the 3-dB frequency $f_H$ to be?

**9.44** An amplifier with a dc gain of 60 dB has a single-pole high-frequency response with a 3-dB frequency of 10 kHz.

(a) Give an expression for the gain function $A(s)$.
(b) Sketch Bode diagrams for the gain magnitude and phase.
(c) What is the gain–bandwidth product?
(d) What is the unity-gain frequency?
(e) If a change in the amplifier circuit causes its transfer function to acquire another pole at 100 kHz, sketch the resulting gain magnitude and specify the unity-gain frequency. Note that this is an example of an amplifier with a unity-gain bandwidth that is different from its gain–bandwidth product.

**9.45** Consider an amplifier whose $F_H(s)$ is given by

$$F_H(s) = \frac{1}{\left(1 + \dfrac{s}{\omega_{P1}}\right)\left(1 + \dfrac{s}{\omega_{P2}}\right)}$$

with $\omega_{P1} < \omega_{P2}$. Find the ratio $\omega_{P2}/\omega_{P1}$ for which the value of the 3-dB frequency $\omega_H$ calculated using the dominant-pole approximation differs from that calculated using the root-sum-of-squares formula (Eq. 9.68) by:

(a) 10%
(b) 1%

**9.46** The high-frequency response of a direct-coupled amplifier having a dc gain of –1000 V/V incorporates zeros at ∞ and $10^5$ rad/s (one at each frequency) and poles at $10^4$ rad/s and $10^6$ rad/s (one at each frequency). Write an expression for the amplifier transfer function. Find $\omega_H$ using

(a) the dominant-pole approximation
(b) the root-sum-of-squares approximation (Eq. 9.68).

If a way is found to lower the frequency of the finite zero to $10^4$ rad/s, what does the transfer function become? What is the 3-dB frequency of the resulting amplifier?

**9.47** A direct-coupled amplifier has a dominant pole at 1000 rad/s and three coincident poles at a much higher frequency. These nondominant poles cause the phase lag of the amplifier at high frequencies to exceed the 90° angle due to the dominant pole. It is required to limit the excess phase at $\omega = 10^7$ rad/s to 30° (i.e., to limit the total phase angle to –120°). Find the corresponding frequency of the nondominant poles.

**D 9.48** Refer to Example 9.6. Give an expression for $\omega_H$ in terms of $C_{gs}$, $R'_{sig}$ (note that $R'_{sig} = R_G \| R_{sig}$), $C_{gd}$, $R'_L$, and $g_m$. If all component values except for the generator resistance $R_{sig}$ are left unchanged, to what value must $R_{sig}$ be reduced in order to raise $f_H$ to 200 kHz?

**9.49** (a) For the amplifier circuit in Example 9.6, find the expression for $\tau_H$ using symbols (as opposed to numbers).
(b) For the same circuit, use the approximate method of the previous section to determine an expression for $C_{in}$ and hence the effective time constant $\tau = C_{in} R'_{sig}$ that can be used to find $\omega_H$ as $1/\tau$. Compare this expression of $\tau$ with that of $\tau_H$ in (a). What is the difference? Compute the value of the difference and express it as a percentage of $\tau$.

**9.50** If a capacitor $C_L = 20$ pF is connected across the output terminals of the amplifier in Example 9.6, find the resulting increase in $\tau_H$ and hence the new value of $f_H$.

**9.51** A FET amplifier resembling that in Example 9.6, when operated at lower currents in a higher-impedance application, has $R_{sig} = 100\ \text{k}\Omega$, $R_{in} = 1.0\ \text{M}\Omega$, $g_m = 2$ mA/V, $R'_L = 15\ \text{k}\Omega$, and $C_{gs} = C_{gd} = 1$ pF. Find the midband voltage gain $A_M$ and the 3-dB frequency $f_H$.

***9.52** Figure P9.52 shows the high-frequency equivalent circuit of a CS amplifier with a resistance $R_s$ connected in the source lead. The purpose of this problem is to show that the value of $R_s$ can be used to control the gain and bandwidth of the amplifier, specifically to allow the designer to trade gain for increased bandwidth.

(a) Derive an expression for the low-frequency voltage gain (set $C_{gs}$ and $C_{gd}$ to zero).
(b) To be able to determine $\omega_H$ using the open-circuit time-constants method, derive expressions for $R_{gs}$ and $R_{gd}$.
(c) Let $R_{sig} = 100\ \text{k}\Omega$, $g_m = 4$ mA/V, $R'_L = 5\ \text{k}\Omega$, and $C_{gs} = C_{gd} = 1$ pF. Use the expressions found in (a) and (b) to determine the low-frequency gain and the 3-dB frequency $f_H$ for three cases: $R_s = 0\ \Omega$, 100 Ω, and 250 Ω. In each case also evaluate the gain–bandwidth product. Comment.

**9.53** A common-source MOS amplifier, whose equivalent circuit resembles that in Fig. 9.16(a), is to be evaluated for its high-frequency response. For this particular design, $R_{sig} = 1\ \text{M}\Omega$, $R_G = 4\ \text{M}\Omega$, $R'_L = 100\ \text{k}\Omega$, $C_{gs} = 0.2$ pF, $C_{gd} = 0.1$ pF, and $g_m = 0.5$ mA/V. Estimate the midband gain and the 3-dB frequency.

**9.54** For a particular amplifier modeled by the circuit of Fig. 9.16(a), $g_m = 5$ mA/V, $R_{sig} = 150\ \text{k}\Omega$, $R_G = 0.65\ \text{M}\Omega$, $R'_L = 10\ \text{k}\Omega$, $C_{gs} = 2$ pF, and $C_{gd} = 0.5$ pF. There is also a load capacitance of 30 pF. Find the corresponding midband voltage gain, the open-circuit time constants, and an estimate of the 3-dB frequency.

**9.55** Consider the high-frequency response of an amplifier consisting of two identical stages in cascade, each with an input resistance of 10 kΩ and an output resistance of 2 kΩ. The two-stage amplifier is driven from a 5-kΩ source and drives a

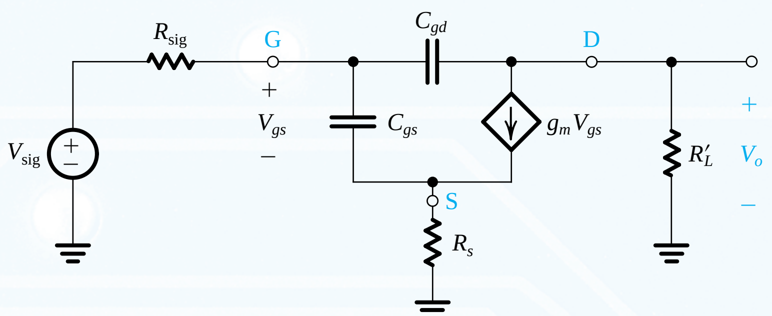



**Figure P9.52**

1-kΩ load. Associated with each stage is a parasitic input capacitance (to ground) of 10 pF and a parasitic output capacitance (to ground) of 2 pF. Parasitic capacitances of 5 pF and 7 pF also are associated with the signal-source and load connections, respectively. For this arrangement, find the three poles and estimate the 3-dB frequency $f_H$.

**9.56** Consider an ideal voltage amplifier with a gain of 0.9 V/V and a resistance $R = 100$ kΩ connected in the feedback path—that is, between the output and input terminals. Use Miller's theorem to find the input resistance of this circuit.

**9.57** An ideal voltage amplifier with a voltage gain of –1000 V/V has a 0.2-pF capacitance connected between its output and input terminals. What is the input capacitance of the amplifier? If the amplifier is fed from a voltage source $V_{sig}$ having a resistance $R_{sig}$ = 1 kΩ, find the transfer function $V_o/V_{sig}$ as a function of the complex-frequency variable $s$ and hence the 3-dB frequency $f_H$ and the unity-gain frequency $f_t$.

**9.58** The amplifiers listed below are characterized by the descriptor ($A$, $C$), where $A$ is the voltage gain from input to output and $C$ is an internal capacitor connected between input and output. For each, find the equivalent capacitances at the input and at the output as provided by the use of Miller's theorem:

(a) −1000 V/V, 1 pF
(b) −10 V/V, 10 pF
(c) −1 V/V, 10 pF
(d) +1 V/V, 10 pF
(e) +10 V/V, 10 pF

Note that the input capacitance found in case (e) can be used to cancel the effect of other capacitance connected from input to ground. In (e), what capacitance can be canceled?

**9.59** Figure P9.59 shows an ideal voltage amplifier with a gain of +2 V/V (usually implemented with an op amp connected in the noninverting configuration) and a resistance $R$ connected between output and input.

(a) Using Miller's theorem, show that the input resistance $R_{in} = -R$.

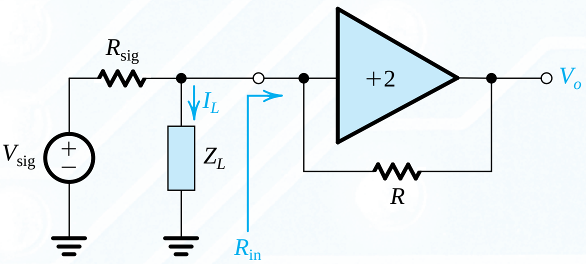



**Figure P9.59**

(b) Use Norton's theorem to replace $V_{sig}$, $R_{sig}$, and $R_{in}$ with a signal current source and an equivalent parallel resistance. Show that by selecting $R_{sig} = R$, the equivalent parallel resistance becomes infinite and the current $I_L$ into the load impedance $Z_L$ becomes $V_{sig}/R$. The circuit then functions as an ideal voltage-controlled current source with an output current $I_L$.
(c) If $Z_L$ is a capacitor $C$, find the transfer function $V_o/V_{sig}$ and show it is that of an ideal noninverting integrator.

## Section 9.5: A Closer Look at the High-Frequency Response of the CS and CE Amplifiers

**9.60** A CS amplifier that can be represented by the equivalent circuit of Fig. 9.19 has $C_{gs} = 2$ pF, $C_{gd} = 0.1$ pF, $C_L = 2$ pF, $g_m = 4$ mA/V, and $R'_{sig} = R'_L = 20$ kΩ. Find the midband gain $A_M$, the input capacitance $C_{in}$ using the Miller approximation, and hence an estimate of the 3-dB frequency $f_H$. Also, obtain a better estimate of $f_H$ using Miller's theorem.

**9.61** A CS amplifier that can be represented by the equivalent circuit of Fig. 9.19 has $C_{gs} = 2$ pF, $C_{gd} = 0.1$ pF, $C_L = 2$ pF, $g_m = 4$ mA/V, and $R'_{sig} = R'_L = 20$ kΩ. Find the midband $A_M$ gain, and estimate the 3-dB frequency $f_H$ using the method of open-circuit time constants. Also, give the percentage contribution to $\tau_H$ by each of three capacitances. (Note that this is the same amplifier considered in Problem 9.60; if you have solved Problem 9.60, compare your results.)

**9.62** A CS amplifier represented by the equivalent circuit of Fig. 9.19 has $C_{gs} = 2$ pF, $C_{gd} = 0.1$, pF, $C_L = 2$ pF, $g_m = 4$ mA/V, and $R'_{sig} = R'_L = 20$ kΩ. Find the exact values of $f_Z$, $f_{P1}$, and $f_{P2}$ using Eq. (9.88), and hence estimate $f_H$. Compare the values of $f_{P1}$ and $f_{P2}$ to the approximate values obtained using Eqs. (9.94) and (9.95). (Note that this is the same amplifier considered in Problems 9.60 and 9.61; if you have solved either or both of these problems, compare your results.)

**9.63** A CS amplifier represented by the equivalent circuit of Fig. 9.19 has $C_{gs} = 2$ pF, $C_{gd} = 0.1$ pF, $C_L = 2$ pF, $g_m = 4$ mA/V, and $R'_{sig} = 20$ kΩ. It is required to find $A_M$, $f_H$, and the gain–bandwidth product for each of the following values of $R'_L$: 5 kΩ, 10 kΩ, and 20 kΩ. Use the approximate expression for $f_{P1}$ in Eq. (9.94). However, in each case, also evaluate $f_{P2}$ and $f_Z$ to ensure that a dominant pole exists, and in each case, state whether the unity-gain frequency is equal to the gain–bandwidth product. Present your results in tabular form, and comment on the gain–bandwidth trade-off.

**9.64** A common-emitter amplifier that can be represented by the equivalent circuit of Fig. 9.24(a) has $C_\pi = 10$ pF, $C_\mu = 0.3$ pF, $C_L = 3$ pF, $g_m = 40$ mA/V, $\beta = 100$,

$r_x = 100\ \Omega$, $R_L' = 5\ \text{k}\Omega$, and $R_{\text{sig}} = 1\ \text{k}\Omega$. Find the midband gain $A_M$, and an estimate of the 3-dB frequency $f_H$ using the Miller approximation. Also, obtain a better estimate of $f_H$ using Miller's theorem.

**9.65** A common-emitter amplifier that can be represented by the equivalent circuit of Fig. 9.24(a) has $C_\pi = 10$ pF, $C_\mu = 0.3$ pF, $C_L = 3$ pF, $g_m = 40$ mA/V, $\beta = 100$, $r_x = 100\ \Omega$, $R_L = 5\ \text{k}\Omega$, and $R_{\text{sig}} = 1\ \text{k}\Omega$. Find the midband gain $A_M$, and estimate the 3-dB frequency $f_H$ using the method of open-circuit time constants. Also give the percentage contribution to $\tau_H$ of each of the three capacitances. (Note that this is the same amplifier considered in Problem 9.64; if you have solved this problem, compare your results.)

**9.66** A common-emitter amplifier that can be represented by the equivalent circuit of Fig. 9.24(a) has $C_\pi = 10$ pF, $C_\mu = 0.3$ pF, $C_L = 3$ pF, $g_m = 40$ mA/V, $\beta = 100$, $r_x = 100\ \Omega$, $R_L' = 5\ \text{k}\Omega$, and $R_{\text{sig}} = 1\ \text{k}\Omega$. Find the midband gain $A_M$, the frequency of the zero $f_Z$, and the values of the pole frequencies $f_{P1}$ and $f_{P2}$. Hence, estimate the 3-dB frequency $f_H$. (Note that this is the same amplifier considered in Problems 6.64 and 9.65; if you have solved these problems, compare your results.)

***9.67** For the current mirror in Fig. P9.67, derive an expression for the current transfer function $I_o(s)/I_i(s)$ taking into account the BJT internal capacitances and neglecting $r_x$ and $r_o$. Assume the BJTs to be identical. Observe that a signal ground appears at the collector of $Q_2$. If the mirror is biased at 1 mA and the BJTs at this operating point are characterized by $f_T = 400$ MHz, $C_\mu = 2$ pF, and $\beta_0 = \infty$, find the frequencies of the pole and zero of the transfer function.

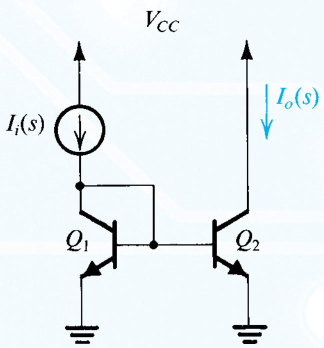



**Figure P9.67**

**9.68** A CS amplifier modeled with the equivalent circuit of Fig 9.25(a) is specified to have $C_{gs} = 2$ pF, $C_{gd} = 0.1$ pF, $g_m = 4$ mA/V, $C_L = 2$ pF, and $R_L' = 20\ \text{k}\Omega$. Find $A_M$, $f_{3\text{dB}}$, and $f_t$.

***9.69** It is required to analyze the high-frequency response of the CMOS amplifier shown in Fig. P9.69. The dc bias current is 100 µA. For $Q_1$, $\mu_n C_{ox} = 90\ \mu\text{A/V}^2$, $V_A = 12.8$ V, $W/L = 100\ \mu\text{m}/1.6\ \mu\text{m}$, $C_{gs} = 0.2$ pF, $C_{gd} = 0.015$ pF, and $C_{db} = 20$ fF. For $Q_2$, $C_{gd} = 0.015$ pF, $C_{db} = 36$ fF, and $|V_A| = 19.2$ V. Assume that the resistance of the input signal generator is negligibly small. Also, for simplicity, assume that the signal voltage at the gate of $Q_2$ is zero. Find the low-frequency gain, the frequency of the pole, and the frequency of the zero.

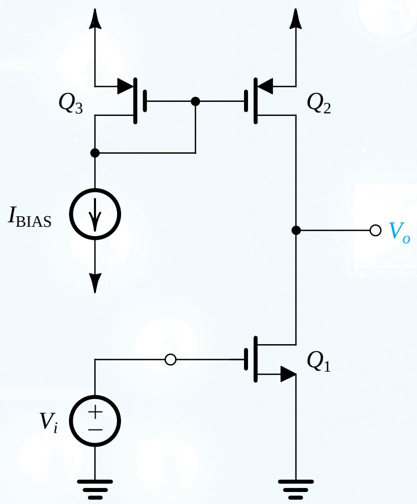



**Figure P9.69**

****9.70** This problem investigates the use of MOSFETs in the design of wideband amplifiers (Steininger, 1990). Such amplifiers can be realized by cascading low-gain stages.

(a) Show that for the case $C_{gd} \ll C_{gs}$ and the gain of the common-source amplifier is low so that the Miller effect is negligible, the MOSFET can be modeled by the approximate equivalent circuit shown in Fig. P9.70(a), where $\omega_T$ is the unity-gain frequency of the MOSFET.

(b) Figure P9.70(b) shows an amplifier stage suitable for the realization of low gain and wide bandwidth. Transistors $Q_1$ and $Q_2$ have the same channel length $L$ but different widths $W_1$ and $W_2$. They are biased at the same $V_{GS}$ and have the same $f_T$. Use the MOSFET equivalent circuit of Fig. P9.70(a) to model this amplifier stage assuming that its output is connected to the input of an identical stage. Show that the voltage gain $V_o/V_i$ is given by

$$\frac{V_o}{V_i} = -\frac{G_0}{1 + \dfrac{s}{\omega_T/(G_0+1)}}$$

where

$$G_0 = \frac{g_{m1}}{g_{m2}} = \frac{W_1}{W_2}$$

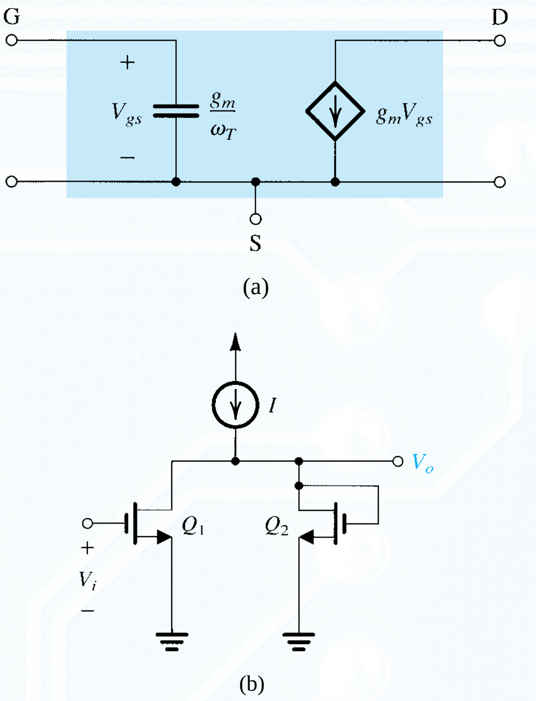



**Figure P9.70**

(c) For $L = 0.5$ μm, $W_2 = 25$ μm, $f_T = 12$ GHz, and $\mu_n C_{ox} = 200$ μA/V$^2$, design the circuit to obtain a gain of 3 V/V per stage. Bias the MOSFETs at $V_{OV} = 0.3$ V. Specify the required values of $W_1$ and $I$. What is the 3-dB frequency achieved?

**9.71** Consider an active-loaded common-emitter amplifier. Let the amplifier be fed with an ideal voltage source $V_i$, and neglect the effect of $r_x$. Assume that the bias current source has a very high resistance and that there is a capacitance $C_L$ present between the output node and ground. This capacitance represents the sum of the input capacitance of the subsequent stage and the inevitable parasitic capacitance between collector and ground. Show that the voltage gain is given by

$$\frac{V_o}{V_i} = -g_m r_o \frac{1 - s(C_\mu/g_m)}{1 + s(C_L + C_\mu)r_o}$$

If the transistor is biased at $I_C = 200$ μA and $V_A = 100$ V, $C_\mu = 0.2$ pF, and $C_L = 1$ pF, find the dc gain, the 3-dB frequency, the frequency of the zero, and the frequency at which the gain reduces to unity. Sketch a Bode plot for the gain magnitude.

**9.72** A common-source amplifier fed with a low-resistance signal source and operating with $g_m = 2$ mA/V has a unity-gain frequency of 2 GHz. What additional capacitance must be connected to the drain node to reduce $f_t$ to 1 GHz?

**9.73** Consider a CS amplifier loaded in a current source with an output resistance equal to $r_o$ of the amplifying transistor. The amplifier is fed from a signal source with $R_{\text{sig}} = r_o/2$. The transistor is biased to operate at $g_m = 2$ mA/V and $r_o = 20$ kΩ; $C_{gs} = C_{gd} = 0.1$ pF. Use the Miller approximation to determine an estimate of $f_H$. Repeat for the following two cases: (i) the bias current $I$ in the entire system is reduced by a factor of 4, and (ii) the bias current $I$ in the entire system is increased by a factor of 4. Remember that both $R_{\text{sig}}$ and $R_L$ will change as $r_o$ changes.

**9.74** Use the method of open-circuit time constants to find $f_H$ for a CS amplifier for which $g_m = 1.5$ mA/V, $C_{gs} = C_{gd} = 0.2$ pF, $r_o = 20$ kΩ, $R_L = 12$ kΩ, and $R_{\text{sig}} = 100$ kΩ for the following cases: (a) $C_L = 0$, (b) $C_L = 10$ pF, and (c) $C_L = 50$ pF. Compare with the value of $f_H$ obtained using the Miller approximation.

## Section 9.6: High-Frequency Response of the Common-Gate and Cascode Amplifiers

**9.75** A CG amplifier is specified to have $C_{gs} = 2$ pF, $C_{gd} = 0.1$ pF, $C_L = 2$ pF, $g_m = 4$ mA/V, $R_{\text{sig}} = 1$ kΩ, and $R_L' = 20$ kΩ. Neglecting the effects of $r_o$, find the low-frequency gain $v_o/v_{\text{sig}}$, the frequencies of the poles $f_{P1}$ and $f_{P2}$, and hence an estimate of the 3-dB frequency $f_H$.

***9.76** Sketch the high-frequency equivalent circuit of a CB amplifier fed from a signal generator characterized by $V_{\text{sig}}$ and $R_{\text{sig}}$ and feeding a load resistance $R_L$ in parallel with a capacitance $C_L$.

(a) Show that for $r_o = \infty$ the circuit can be separated into two parts: an input part that produces a pole at

$$f_{P1} = \frac{1}{2\pi C_\pi (R_{\text{sig}} \| r_e)}$$

and an output part that forms a pole at

$$f_{P2} = \frac{1}{2\pi (C_\mu + C_L) R_L}$$

Note that these are the bipolar counterparts of the MOS expressions in Eqs. (9.109) and (9.110).

(b) Evaluate $f_{P1}$ and $f_{P2}$ and hence obtain an estimate for $f_H$ for the case $C_\pi = 14$ pF, $C_\mu = 2$ pF, $C_L = 1$ pF, $I_C = 1$ mA, $R_{\text{sig}} = 1$ kΩ, and $R_L = 10$ kΩ. Also, find $f_T$ of the transistor.

***9.77** Consider a CG amplifier loaded in a resistance $R_L = r_o$ and fed with a signal source having a resistance $R_{\text{sig}} = r_o/2$. Also let $C_L = C_{gs}$. Use the method of open-circuit time constants to show that for $g_m r_o \gg 1$, the upper 3-dB frequency is related to the MOSFET $f_T$ by the approximate expression

$$f_H = f_T/(g_m r_o)$$

**9.78** For the CG amplifier in Example 9.12, how much additional capacitance should be connected between the output node and ground to reduce $f_H$ to 300 MHz?

**9.79** Find the dc gain and the 3-dB frequency of a MOS cascode amplifier operated at $g_m = 1$ mA/V and $r_o = 50\,\text{k}\Omega$. The MOSFETs have $C_{gs} = 30$ fF, $C_{gd} = 10$ fF, and $C_{db} = 10$ fF. The amplifier is fed from a signal source with $R_{\text{sig}} = 100\ \text{k}\Omega$ and is connected to a load resistance of 2 M$\Omega$. There is also a load capacitance $C_L$ of 40 fF.

***9.80** (a) Consider a CS amplifier having $C_{gd} = 0.2$ pF, $R_{\text{sig}} = R_L = 20\ \text{k}\Omega$, $g_m = 4$ mA/V, $C_{gs} = 2$ pF, $C_L$ (including $C_{db}$) = 1 pF, $C_{db} = 0.2$ pF, and $r_o = 20\ \text{k}\Omega$. Find the low-frequency gain $A_M$, and estimate $f_H$ using open-circuit time constants. Hence determine the gain–bandwidth product.
(b) If a CG stage is cascaded with the CS transistor in (a) to create a cascode amplifier, determine the new values of $A_M$, $f_H$, and gain–bandwidth product. Assume $R_L$ remains unchanged.

**D 9.81** It is required to design a cascode amplifier to provide a dc gain of 74 dB when driven with a low-resistance generator and utilizing NMOS transistors for which $V_A = 10$ V, $\mu_n C_{ox} = 200\ \mu\text{A/V}^2$, $W/L = 50$, $C_{gd} = 0.1$ pF, and $C_L = 1$ pF. Assuming that $R_L = R_o$, determine the overdrive voltage and the drain current at which the MOSFETs should be operated. Find the unity-gain frequency and the 3-dB frequency. If the cascode transistor is removed and $R_L$ remains unchanged, what will the dc gain become?

**9.82** Consider a bipolar cascode amplifier biased at a current of 1 mA. The transistors used have $\beta = 100$, $r_o = 100\,\text{k}\Omega$, $C_\pi = 14$ pF, $C_\mu = 2$ pF, $C_{cs} = 0$, and $r_x = 50\ \Omega$. The amplifier is fed with a signal source having $R_{\text{sig}} = 4\ \text{k}\Omega$. The load resistance $R_L = 2.4\ \text{k}\Omega$. Find the low-frequency gain $A_M$, and estimate the value of the 3-dB frequency $f_H$.

***9.83** In this problem we consider the frequency response of the bipolar cascode amplifier in the case that $r_o$ can be neglected.

(a) Refer to the circuit in Fig. 9.31, and note that the total resistance between the collector of $Q_1$ and ground will be equal to $r_{e2}$, which is usually very small. It follows that the pole introduced at this node will typically be at a very high frequency and thus will have negligible effect on $f_H$. It also follows that at the frequencies of interest the gain from the base to the collector of $Q_1$ will be $-g_{m1}r_{e2} \simeq -1$. Use this to find the capacitance at the input of $Q_1$ and hence show that the pole introduced at the input node will have a frequency

$$f_{P1} \simeq \frac{1}{2\pi R'_{\text{sig}}(C_{\pi 1} + 2C_{\mu 1})}$$

Then show that the pole introduced at the output node will have a frequency

$$f_{P2} \simeq \frac{1}{2\pi R_L(C_L + C_{cs2} + C_{\mu 2})}$$

(b) Evaluate $f_{P1}$ and $f_{P2}$, and use the sum-of-the-squares formula to estimate $f_H$ for the amplifier with $I = 1$ mA, $C_\pi = 5$ pF, $C_\mu = 5$ pF, $C_{cs} = C_L = 0$, $\beta = 100$, and $r_x = 0$ in the following two cases:

(i) $R_{\text{sig}} = 1\ \text{k}\Omega$

(ii) $R_{\text{sig}} = 10\ \text{k}\Omega$

**9.84** A BJT cascode amplifier uses transistors for which $\beta = 100$, $V_A = 100$ V, $f_T = 1$ GHz, and $C_\mu = 0.1$ pF. It operates at a bias current of 0.1 mA between a source with $R_{\text{sig}} = r_\pi$ and a load $R_L = \beta r_o$. Let $C_L = C_{cs} = 0$ and find the overall voltage gain at dc, $f_H$, and $f_t$.

## Section 9.7: High-Frequency Response of the Source and Emitter Followers

**9.85** A source follower has $g_m = 5$ mA/V, $r_o = 20\ \text{k}\Omega$, $R_{\text{sig}} = 20\ \text{k}\Omega$, $R_L = 2\ \text{k}\Omega$, $C_{gs} = 2$ pF, $C_{gd} = 0.1$ pF, and $C_L = 1$ pF. Find $A_M$, $R_o$, $f_Z$, and $f_H$. Also, find the percentage contribution of each of the three capacitances to the time-constant $\tau_H$.

**9.86** Using the expression for the source follower $f_H$ in Eq. (9.129) show that for situations in which $R_{\text{sig}}$ is large and $R_L$ is small,

$$f_H \simeq \frac{1}{2\pi R_{\text{sig}}\left[C_{gd} + \dfrac{C_{gs}}{1 + g_m R'_L}\right]}$$

Find $f_H$ for the case $R_{\text{sig}} = 100\ \text{k}\Omega$, $R_L = 1\ \text{k}\Omega$, $r_o = 20\ \text{k}\Omega$, $g_m = 5$ mA/V, $C_{gd} = 10$ fF, and $C_{gs} = 30$ fF.

**9.87** Refer to Fig. 9.32(b). In situations in which $R_{\text{sig}}$ is large, the high-frequency response of the source follower is determined by the low-pass circuit formed by $R_{\text{sig}}$ and the input capacitance. An estimate of $C_{\text{in}}$ can be obtained by using the Miller approximation to replace $C_{gs}$ with an input capacitance $C_{eq} = C_{gs}(1 - K)$ where $K$ is the gain from gate to source. Using the low-frequency value of $K = g_m R'_L/(1 + g_m R'_L)$ find $C_{eq}$ and hence $C_{\text{in}}$ and an estimate of $f_H$. Is this estimate higher or lower than that obtained by the method of open-circuit time constants?

**9.88** For an emitter follower biased at $I_C = 1$ mA and having $R_{\text{sig}} = R_L = 1\,\text{k}\Omega$, and using a transistor specified to have $f_T = 2$ GHz, $C_\mu = 0.1$ pF, $r_x = 100\ \Omega$, $\beta = 100$, and $V_A = 20$ V, evaluate the low-frequency gain $A_M$ and the 3-dB frequency $f_H$.

***9.89** For the emitter follower shown in Fig. P9.89, find the low-frequency gain and the 3-dB frequency $f_H$ for the following three cases:

(a) $R_{sig} = 1\ k\Omega$
(b) $R_{sig} = 10\ k\Omega$
(c) $R_{sig} = 100\ k\Omega$

Let $\beta = 100$, $f_T = 400$ MHz, and $C_\mu = 2$ pF.

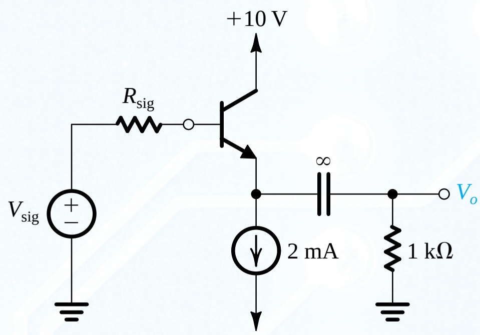



**Figure P9.89**

## Section 9.8: High-Frequency Response of Differential Amplifiers

**9.90** A MOSFET differential amplifier such as that shown in Fig. 9.34(a) is biased with a current source $I$ = 200 μA. The transistors have $W/L = 25$, $k'_n = 200\ \mu A/V^2$, $V_A = 200$ V, $C_{gs} = 40$ fF, $C_{gd} = 5$ fF, and $C_{db} = 5$ fF. The drain resistors are 20 kΩ each. Also, there is a 100-fF capacitive load between each drain and ground.

(a) Find $V_{OV}$ and $g_m$ for each transistor.
(b) Find the differential gain $A_d$.
(c) If the input signal source has a small resistance $R_{sig}$ and thus the frequency response is determined primarily by the output pole, estimate the 3-dB frequency $f_H$.
(d) If, in a different situation, the amplifier is fed symmetrically with a signal source of 40 kΩ resistance (i.e., 20 kΩ in series with each gate terminal), use the open-circuit time-constants method to estimate $f_H$.

**9.91** The amplifier specified in Problem 9.90 has $R_{SS}$ = 80 kΩ and $C_{SS}$ = 0.1 pF. Find the 3-dB frequency of the CMRR.

**9.92** In a particular MOS differential amplifier design, the bias current $I$ = 100 μA is provided by a single transistor operating at $V_{OV}$ = 0.5 V with $V_A$ = 30 V and output capacitance $C_{SS}$ of 100 fF. What is the frequency of the common-mode gain zero ($f_Z$) at which $A_{cm}$ begins to rise above its low-frequency value? To meet a requirement for reduced power supply, consideration is given to reducing $V_{OV}$ to 0.2 V while keeping $I$ unchanged. Assuming the current-source capacitance to be directly proportional to the device width, what is the impact on $f_Z$ of this proposed change?

**9.93** Repeat Exercise 9.27 for the situation in which the bias current is reduced to 80 μA and $R_D$ is raised to 20 kΩ. For (d), let $R_{sig}$ be raised from 20 kΩ to 100 kΩ.
(*Note*: This is a low-voltage, low-power design.)

**9.94** A BJT differential amplifier operating with a 1-mA current source uses transistors for which $\beta = 100$, $f_T = 600$ MHz, $C_\mu = 0.5$ pF, and $r_x = 100\ \Omega$. Each of the collector resistances is 10 kΩ, and $r_o$ is very large. The amplifier is fed in a symmetrical fashion with a source resistance of 10 kΩ in series with each of the two input terminals.

(a) Sketch the differential half-circuit and its high-frequency equivalent circuit.
(b) Determine the low-frequency value of the overall differential gain.
(c) Use the Miller approximation to determine the input capacitance and hence estimate the 3-dB frequency $f_H$ and the gain–bandwidth product.

**9.95** A differential amplifier is biased by a current source having an output resistance of 1 MΩ and an output capacitance of 1 pF. The differential gain exhibits a dominant pole at 2 MHz. What are the poles of the CMRR?

**9.96** A current-mirror-loaded MOS differential amplifier is biased with a current source $I$ = 0.2 mA. The two NMOS transistors of the differential pair are operating at $V_{OV}$ = 0.2 V, and the PMOS devices of the mirror are operating at $|V_{OV}|$ = 0.2 V. The Early voltage $V_{An} = |V_{Ap}| = 10$ V. The total capacitance at the input node of the mirror is 0.1 pF and that at the output node of the amplifier is 0.2 pF. Find the dc value and the frequencies of the poles and zero of the differential voltage gain.

**9.97** Consider the active-loaded CMOS differential amplifier of Fig. 9.37(a) for the case of all transistors operated at the same $|V_{OV}|$ and having the same $|V_A|$. Also let the total capacitance at the output node ($C_L$) be four times the total capacitance at the input node of the current mirror $C_m$, and show that the unity-gain frequency of $A_d$ is $g_m/2\pi C_L$. For $V_A$ = 20 V, $V_{OV}$ = 0.2 V, $I$ = 0.2 mA, $C_L$ = 100 fF, and $C_m$ = 25 fF, find the dc value of $A_d$, and the value of $f_{P1}$, $f_t$, $f_{P2}$, and $f_Z$ and sketch a Bode plot for $|A_d|$.

## Section 9.9: Other Wideband Amplifier Configurations

**9.98** A CS amplifier is specified to have $g_m$ = 5 mA/V, $r_o$ = 40 kΩ, $C_{gs}$ = 2 pF, $C_{gd}$ = 0.1 pF, $C_L$ = 1 pF, $R_{sig}$ = 20 kΩ, and $R_L$ = 40 kΩ.

(a) Find the low-frequency gain $A_M$, and use open-circuit time constants to estimate the 3-dB frequency $f_H$. Hence determine the gain–bandwidth product.
(b) If a 500-Ω resistance is connected in the source lead, find the new values of $|A_M|$, $f_H$, and the gain–bandwidth product.

**D 9.99** (a) Use the approximate expression in Eq. (9.161) to determine the gain–bandwidth product of a CS amplifier with a source-degeneration resistance. Assume $C_{gd} = 0.1$ pF and $R_{sig} = 10$ kΩ.
(b) If a low-frequency gain of 20 V/V is required, what $f_H$ corresponds?
(c) For $g_m = 5$ mA/V, $A_0 = 100$ V/V, and $R_L = 20$ kΩ, find the required value of $R_s$.

**9.100** For the CS amplifier with a source-degeneration resistance $R_s$, show for $R_{sig} \gg R_s$ and $R_L = r_o$ that

$$\tau_H \simeq \frac{C_{gs}R_{sig}}{1+(k/2)} + C_{gd}R_{sig}\left(1+\frac{A_0}{2+k}\right) + (C_L + C_{gd})r_o\left(\frac{1+k}{2+k}\right)$$

where $k \equiv g_m R_s$

**D *9.101** It is required to generate a table of $|A_M|$, $f_H$, and $f_t$ versus $k \equiv g_m R_s$ for a CS amplifier with a source-degeneration resistance $R_s$. The table should have entries for $k = 0, 1, 2, \ldots, 15$. The amplifier is specified to have $g_m = 5$ mA/V, $r_o = 40$ kΩ, $R_L = 40$ kΩ, $R_{sig} = 20$ kΩ, $C_{gs} = 2$ pF, $C_{gd} = 0.1$ pF, and $C_L = 1$ pF. Use the formula for $\tau_H$ given in the statement for Problem 9.100. If $f_H = 2$ MHz is required, find the value needed for $R_s$ and the corresponding value of $|A_M|$.

***9.102** In this problem we investigate the bandwidth extension obtained by placing a source follower between the signal source and the input of the CS amplifier.

(a) First consider the CS amplifier of Fig. P9.102(a). Show that

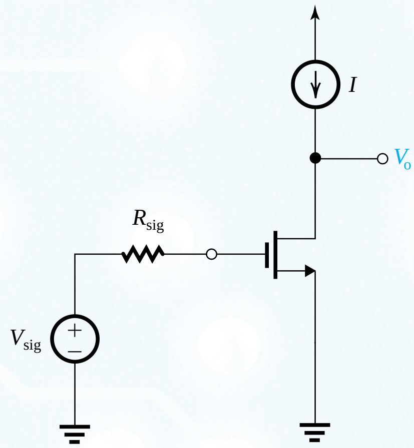
$$A_M = -g_m r_o$$

$$\tau_H = C_{gs}R_{sig} + C_{gd}[R_{sig}(1 + g_m r_o) + r_o] + C_L r_o$$

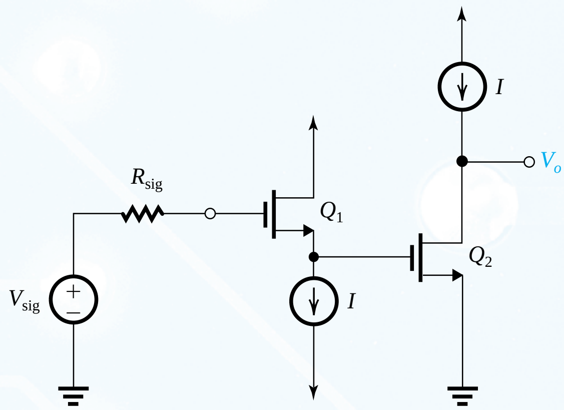
where $C_L$ is the total capacitance between the output node and ground. Calculate the value of $A_M$, $f_H$, and the gain–bandwidth product for the case $g_m = 1$ mA/V, $r_o = 20$ kΩ, $R_{sig} = 20$ kΩ, $C_{gs} = 20$ fF, $C_{gd} = 5$ fF, and $C_L = 10$ fF.

(b) For the CD-CS amplifier in Fig. P9.102(b), show that

$$A_M = -\frac{r_{o1}}{1/g_{m1} + r_{o1}}(g_{m2}r_{o2})$$




(a)




(b)

**Figure P9.102**

$$\tau_H = C_{gd1}R_{sig} + C_{gs1}\frac{R_{sig} + r_{o1}}{1 + g_{m1}r_{o1}} + C_{gs2}\left(\frac{1}{g_{m1}} \| r_{o1}\right)$$
$$+ C_{gd2}\left[\left(\frac{1}{g_{m1}} \| r_{o1}\right)(1 + g_{m2}r_{o2}) + r_{o2}\right]$$
$$+ C_L r_{o2}$$

Calculate the values of $A_M$, $f_H$, and the gain–bandwidth product for the same parameter values used in (a). Compare with the results of (a).

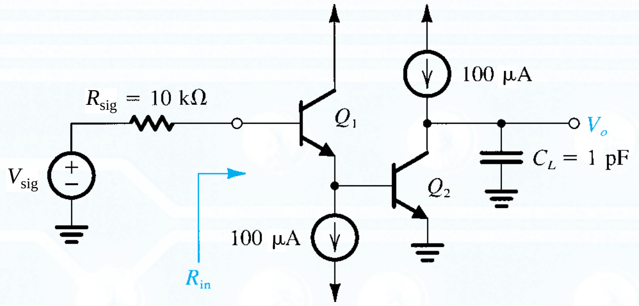



**Figure P9.103**

**D *9.103** The transistor in the circuit of Fig. P9.103 have $\beta_0 = 100$, $V_A = 100$ V, $C_\mu = 0.2$ pF, and $C_{je} = 0.8$ pF. At a bias current of 100 μA, $f_T = 400$ MHz. (Note that the bias details are not shown.)

(a) Find $R_{in}$ and the midband gain.
(b) Find an estimate of the upper 3-dB frequency $f_H$. Which capacitor dominates? Which one is the second most significant?
(*Hint*. Use the formulas in Example 9.15.)

**D **9.104** Consider the BiCMOS amplifier shown in Fig. P9.104. The BJT has $|V_{BE}| = 0.7$ V, $\beta = 200$, $C_\mu = 0.8$ pF, and $f_T = 600$ MHz. The NMOS transistor has $V_t = 1$ V, $k_n' W/L = 2$ mA/V$^2$, and $C_{gs} = C_{gd} = 1$ pF.

(a) Consider the dc bias circuit. Neglect the base current of $Q_2$ in determining the current in $Q_1$. Find the dc bias currents in $Q_1$ and $Q_2$, and show that they are approximately 100 μA and 1 mA, respectively.

(b) Evaluate the small-signal parameters of $Q_1$ and $Q_2$ at their bias points.
(c) Consider the circuit at midband frequencies. First, determine the small-signal voltage gain $V_o/V_i$. (Note that $R_G$ can be neglected in this process.) Then use Miller's theorem on $R_G$ to determine the amplifier input resistance $R_{in}$. Finally, determine the overall voltage gain $V_o/V_{sig}$.
(d) Consider the circuit at low frequencies. Determine the frequency of the poles due to $C_1$ and $C_2$, and hence estimate the lower 3-dB frequency, $f_L$.
(e) Consider the circuit at higher frequencies. Use Miller's theorem to replace $R_G$ with a resistance at the input. (The one at the output will be too large to matter.) Use open-circuit time constants to estimate $f_H$.
(f) To considerably reduce the effect of $R_G$ on $R_{in}$ and hence on amplifier performance, consider the effect of adding another 10-MΩ resistor in series with the existing one and placing a large bypass capacitor between their joint node and ground. What will $R_{in}$, $A_M$, and $f_H$ become?

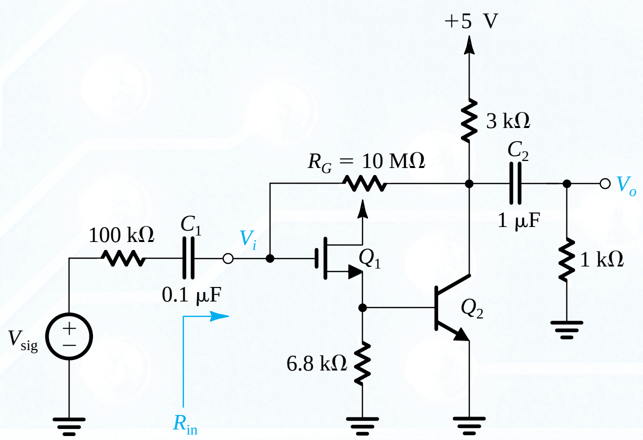



**Figure P9.104**

**9.105** Consider the circuit of Fig. P9.105 for the case: $I = 200\ \mu A$ and $V_{OV} = 0.2$ V, $R_{sig} = 200$ kΩ, $R_D = 50$ kΩ, $C_{gs} = C_{gd} = 1$ pF. Find the dc gain, the high-frequency poles, and an estimate of $f_H$.

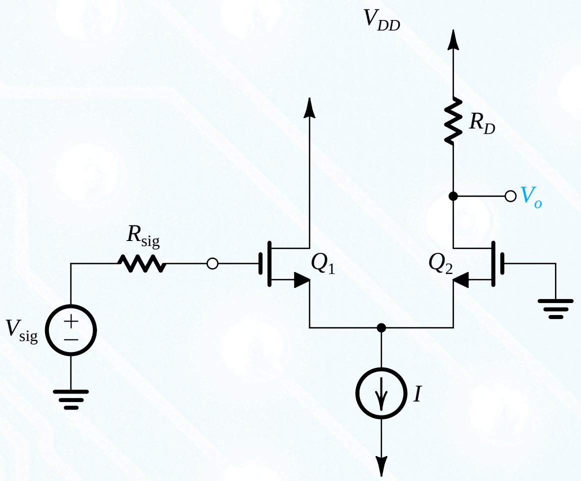



**Figure P9.105**

**9.106** For the amplifier in Fig. 9.41(a), let $I = 1$ mA, $\beta = 120$, $f_T = 700$ MHz, and $C_\mu = 0.5$ pF, and neglect $r_x$ and $r_o$. Assume that a load resistance of 10 kΩ is connected to the output terminal. If the amplifier is fed with a signal $V_{sig}$ having a source resistance $R_{sig} = 20$ kΩ, find $A_M$ and $f_H$.

**9.107** Consider the CD–CG amplifier of Fig. 9.41(c) for the case $g_m = 5$ mA/V, $C_{gs} = 2$ pF, $C_{gd} = 0.1$ pF, $C_L$ (at the output node) = 1 pF, and $R_{sig} = R_L = 20$ kΩ. Neglecting $r_o$, find $A_M$, and $f_H$.

*** **9.108** In each of the six circuits in Fig. P9.108 (p. 800), let $\beta = 100$, $C_\mu = 2$ pF, and $f_T = 400$ MHz, and neglect $r_x$ and $r_o$. Calculate the midband gain $A_M$ and the 3-dB frequency $f_H$.

## Section 9.10: Multistage Amplifier Examples

**9.109** Use open-circuit time constants to obtain an expression for $\omega_H$ of the amplifier in Fig. 9.44. Compare to the expression in Eq. (9.176).

**9.110** For the CMOS amplifier in Fig. 9.43, whose equivalent circuit is shown in Fig. 9.44, let $G_{m1} = 1$ mA/V, $R_1 = 100$ kΩ, $C_1 = 0.1$ pF, $G_{m2} = 2$ mA/V, $R_2 = 50$ kΩ, and $C_2 = 2$ pF.

(a) Find the dc gain.
(b) Without $C_C$ connected, find the frequencies of the two poles in radians per seconds and sketch a Bode plot for the gain magnitude.
(c) With $C_C$ connected, find $\omega_{P2}$. Then find the value of $C_C$ that will result in a unity-gain frequency $\omega_t$ at least two octaves below $\omega_{P2}$. For this value of $C_C$, find $\omega_{P1}$ and $\omega_Z$ and sketch a Bode plot for the gain magnitude.

**9.111** A CMOS op amp with the topology in Fig. 9.43 has $g_{m1} = g_{m2} = 1$ mA/V, $g_{m6} = 3$ mA/V, the total capacitance between node $D_2$ and ground is 0.2 pF, and the total capacitance between the output node and ground is 3 pF. Find the value of $C_C$ that results in $f_t = 50$ MHz and verify that $f_t$ is lower than $f_Z$ and $f_{P2}$.

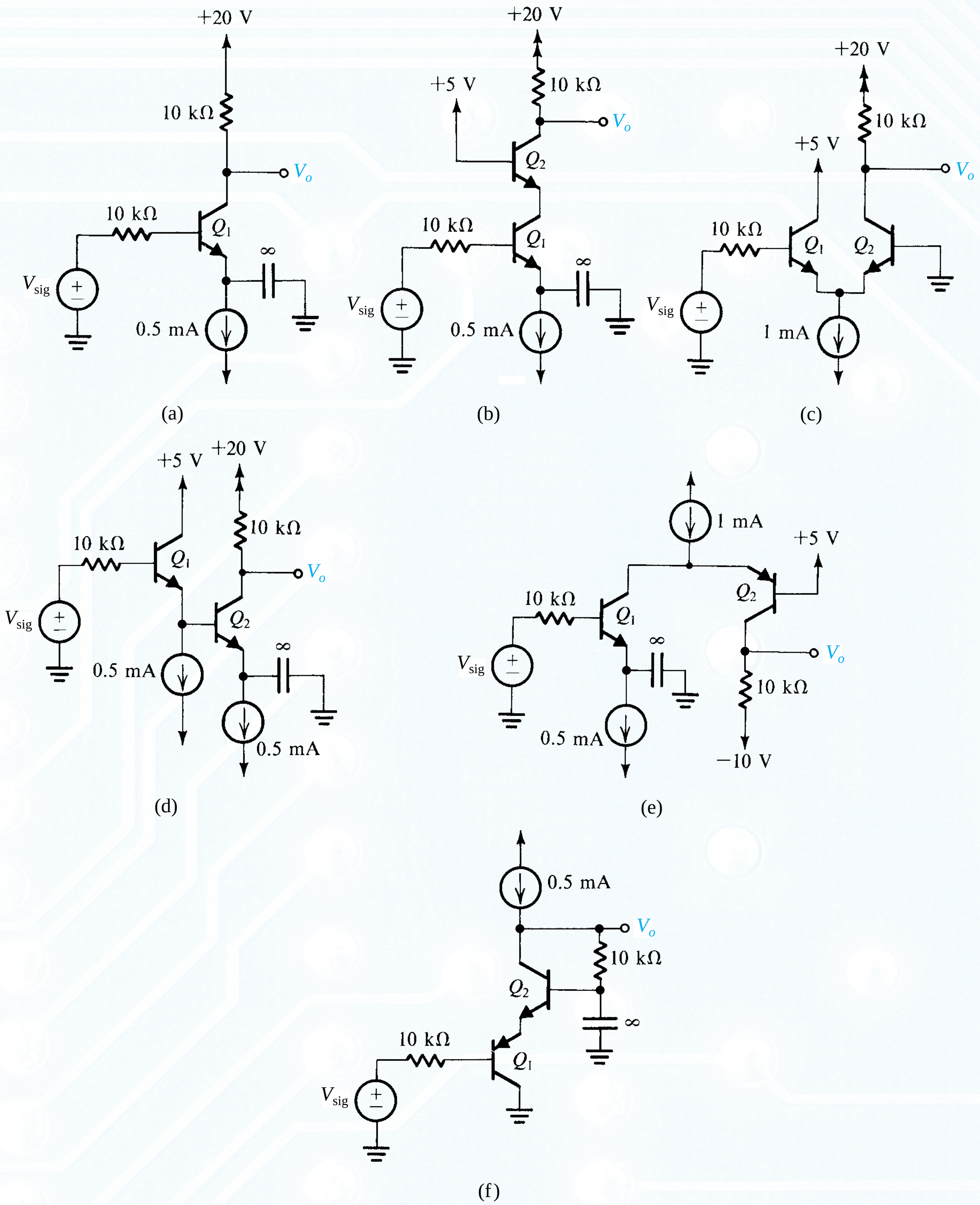



Figure P9.108

**9.112** Figure P9.112 shows an amplifier formed by cascading two CS stages. Note that the input bias voltage is not shown. Each of $Q_1$ and $Q_2$ is operated at an overdrive voltage of 0.2 V, and $|V_A| = 10$ V. The transistor capacitances are as follows: $C_{gs} = 20$ fF, $C_{gd} = 5$ fF, and $C_{db} = 5$ fF.

(a) Find the dc voltage gain.
(b) Find the input capacitance at the gate of $Q_1$, using the Miller approximation.
(c) Use the capacitance in (b) to determine the frequency of the pole formed at the amplifier input. Let $R_{sig} = 10$ kΩ.
(d) Use the Miller approximation to find the input capacitance of $Q_2$ and hence determine the total capacitance at the drain of $Q_1$.
(e) Use the capacitance found in (d) to obtain the frequency of the pole formed at the interface between the two stages.
(f) Determine the total capacitance at the output node and hence estimate the frequency of the pole formed at the output node.
(g) Does the amplifier have a dominant pole? If so, at what frequency

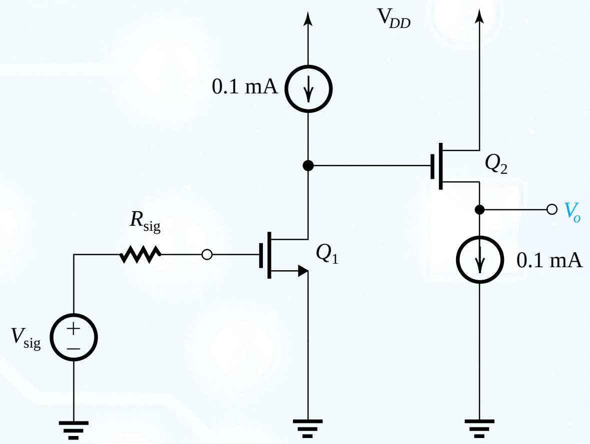



Figure P9.112