CHAPTER 16

Filters and Tuned Amplifiers

IN THIS CHAPTER YOU WILL LEARN

1. How filters are characterized by their signal-transmission properties and how they are classified into different types based on the relative location of their passband(s) and stopband(s).
2. How filters are specified and how to obtain a filter transfer function that meets the given specifications, including the use of popular special functions such as the Butterworth and the Chebyshev.
3. The various first-order and second-order filter functions and their realization using op amps and RC circuits.
4. The basic second-order LCR resonator and how it can be used to realize the various second-order filter functions.
5. The best op amp–RC circuit for realizing an inductance and how it can be used as the basis for realizing the various second-order filter functions.
6. That connecting two op-amp integrators, one inverting and one noninverting, in a feedback loop realizes a second-order resonance circuit and can be used to obtain circuit realizations of the various second-order filter functions.
7. How second-order filter functions can be realized using a single op amp and an RC circuit, and the performance limitations of these minimal realizations.
8. How the powerful concept of circuit sensitivity can be applied to assess the performance of filter circuits in the face of finite component tolerances.
9. The basis for the most popular approach to the realization of filter functions in IC form; the switched-capacitor technique.
10. The design of tuned transistor amplifiers for radio-frequency (RF) applications.

Introduction

In this chapter, we study the design of an important building block of communications and instrumentation systems, the electronic filter. Filter design is one of the very few areas of engineering for which a complete design theory exists, starting from specification and ending with a circuit realization. A detailed study of filter design requires an entire book, and indeed such textbooks exist. In the limited space available here, we shall concentrate on a selection of topics that provide an introduction to the subject as well as a useful arsenal of filter circuits and design methods.

The oldest technology for realizing filters makes use of inductors and capacitors, and the resulting circuits are called **passive LC filters**. Such filters work well at high frequencies; however, in low-frequency applications (dc to 100 kHz) the required inductors are large and physically bulky, and their characteristics are quite nonideal. Furthermore, such inductors are impossible to fabricate in monolithic form and are incompatible with any of the modern techniques for assembling electronic systems. Therefore, there has been considerable interest in finding filter realizations that do not require inductors. Of the various possible types of **inductorless filters**, we shall study **active-RC filters** and **switched-capacitor filters**.

Active-RC filters utilize op amps together with resistors and capacitors and are fabricated using discrete, hybrid thick-film, or hybrid thin-film technology. However, for large-volume production, such technologies do not yield the economies achieved by monolithic (IC) fabrication. At the present time, the most viable approach for realizing fully integrated monolithic filters is the switched-capacitor technique.

The last topic studied in this chapter is the tuned amplifier commonly employed in the design of radio and TV receivers. Although tuned amplifiers are in effect bandpass filters, they are studied separately because their design is based on somewhat different techniques.

The material in this chapter requires a thorough familiarity with op-amp circuit applications. Thus the study of Chapter 2 is a prerequisite.

16.1 Filter Transmission, Types, and Specification

16.1.1 Filter Transmission

The filters we are about to study are linear circuits that can be represented by the general two-port network shown in Fig. 16.1. The filter **transfer function** $T(s)$ is the ratio of the output voltage $V_o(s)$ to the input voltage $V_i(s)$,

$$T(s) \equiv \frac{V_o(s)}{V_i(s)} \tag{16.1}$$

The filter **transmission** is found by evaluating $T(s)$ for physical frequencies, $s = j\omega$, and can be expressed in terms of its magnitude and phase as

$$T(j\omega) = |T(j\omega)|e^{j\phi(\omega)} \tag{16.2}$$

The magnitude of transmission is often expressed in decibels in terms of the **gain function**

$$G(\omega) \equiv 20 \log|T(j\omega)|, \text{ dB} \tag{16.3}$$

or, alternatively, in terms of the **attenuation function**

$$A(\omega) \equiv -20 \log|T(j\omega)|, \text{ dB} \tag{16.4}$$

A filter shapes the frequency spectrum of the input signal, $|V_i(j\omega)|$, according to the magnitude of the transfer function $|T(j\omega)|$, thus providing an output $V_o(j\omega)$ with a spectrum

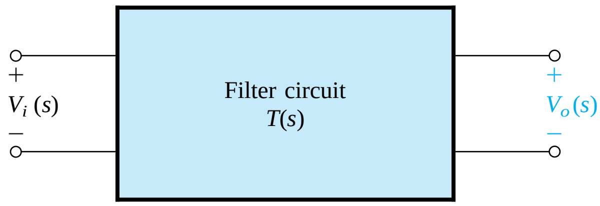


Figure 16.1 The filters studied in this chapter are linear circuits represented by the general two-port network shown. The filter transfer function $T(s) \equiv V_o(s)/V_i(s)$.

$$|V_o(j\omega)| = |T(j\omega)||V_i(j\omega)| \tag{16.5}$$

Also, the phase characteristics of the signal are modified as it passes through the filter according to the filter phase function $\phi(\omega)$.

16.1.2 Filter Types

We are specifically interested here in filters that perform a **frequency-selection** function: **passing** signals whose frequency spectrum lies within a specified range, and **stopping** signals whose frequency spectrum falls outside this range. Such a filter has ideally a frequency band (or bands) over which the magnitude of transmission is unity (the filter **passband**) and a frequency band (or bands) over which the transmission is zero (the filter **stopband**). Figure 16.2 depicts the ideal transmission characteristics of the four major filter types: **low-pass** (LP) in Fig. 16.2(a), **high-pass** (HP) in Fig. 16.2(b), **bandpass** (BP) in Fig. 16.2(c), and **bandstop** (BS) or **band-reject** in Fig. 16.2(d). These idealized characteristics, by virtue of their vertical edges, are known as **brick-wall** responses.

16.1.3 Filter Specification

The filter-design process begins with the filter user specifying the transmission characteristics required of the filter. Such a specification cannot be of the form shown in Fig. 16.2 because physical circuits cannot realize these idealized characteristics. Figure 16.3 shows realistic specifications for the transmission characteristics of a low-pass filter. Observe that since a physical circuit cannot provide constant transmission at all passband frequencies, the specifications allow for deviation of the passband transmission from the ideal 0 dB, but place an upper bound, A_{max} (dB), on this deviation. Depending on the application, A_{max} typically ranges from 0.05 dB to 3 dB. Also, since a physical circuit cannot provide zero transmission at all stopband frequencies, the specifications in Fig. 16.3 allow for some transmission over the stopband. However, the specifications require the stopband signals to be attenuated by at least A_{min} (dB) relative to the passband signals. Depending on the filter application, A_{min} can range from 20 dB to 100 dB.

Since the transmission of a physical circuit cannot change abruptly at the edge of the passband, the specifications of Fig. 16.3 provide for a band of frequencies over which the attenuation increases from near 0 dB to A_{min}. This **transition band** extends from the passband edge ω_p to the stopband edge ω_s. The ratio ω_s/ω_p is usually used as a measure of the sharpness of the low-pass filter response and is called the **selectivity factor**. Finally, observe that for convenience the passband transmission is specified to be 0 dB. The final filter, however, can be given a passband gain, if desired, without changing its selectivity characteristics.

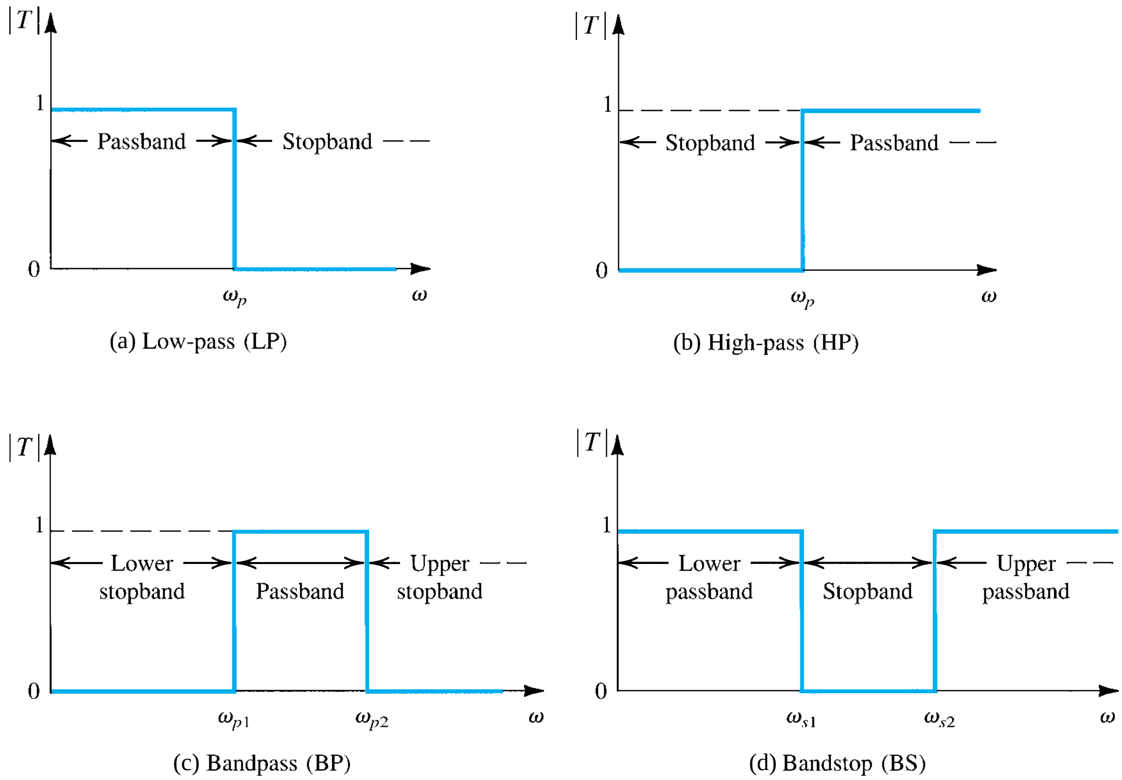


Figure 16.2 Ideal transmission characteristics of the four major filter types: **(a)** low-pass (LP), **(b)** high-pass (HP), **(c)** bandpass (BP), and **(d)** bandstop (BS).

To summarize, the transmission of a low-pass filter is specified by four parameters:

1. The passband edge ω_p
2. The maximum allowed variation in passband transmission A_{max}
3. The stopband edge ω_s
4. The minimum required stopband attenuation A_{min}

The more tightly one specifies a filter—that is, lower A_{max}, higher A_{min}, and/or a selectivity ratio ω_s/ω_p closer to unity—the closer the response of the resulting filter will be to the ideal. However, the resulting filter circuit must be of higher order and thus more complex and expensive.

In addition to specifying the magnitude of transmission, there are applications in which the phase response of the filter is also of interest. The filter-design problem, however, is considerably complicated when both magnitude and phase are specified.

Once the filter specifications have been decided upon, the next step in the design is to find a transfer function whose magnitude meets the specification. To meet specification, the magnitude-response curve must lie in the unshaded area in Fig. 16.3. The curve shown in the figure is for a filter that *just* meets specifications. Observe that for this particular filter, the magnitude response *ripples* throughout the passband, and the ripple peaks are all equal. Since the peak ripple is equal to A_{max} it is usual to refer to A_{max} as the **passband ripple** and to

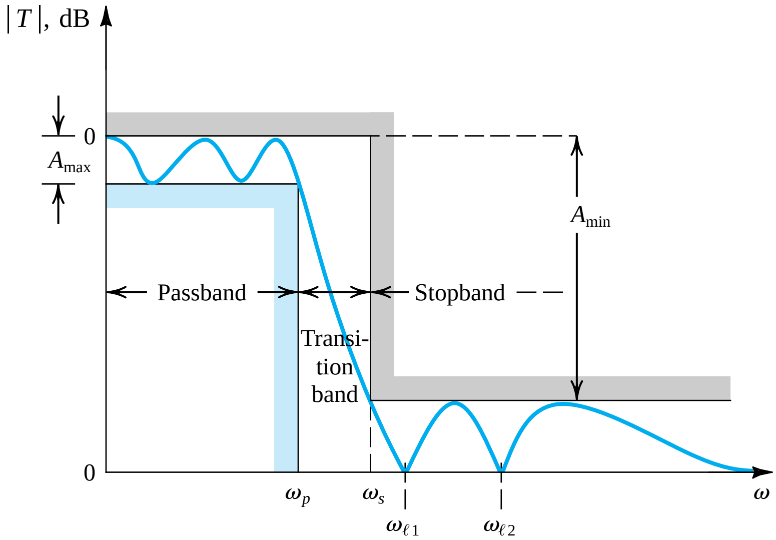


Figure 16.3 Specification of the transmission characteristics of a low-pass filter. The magnitude response of a filter that just meets specifications is also shown.

ω_p as the **ripple bandwidth**. The particular filter response shows ripples also in the stopband, again with the ripple peaks all equal and of such a value that the minimum stopband attenuation achieved is equal to the specified value, A_{min}. Thus this particular response is said to be **equiripple** in both the passband and the stopband.

The process of obtaining a transfer function that meets given specifications is known as **filter approximation**. Filter approximation is usually performed using computer programs (Snelgrove, 1982; Ouslis and Sedra, 1995) or filter design tables (Zverev, 1967). In simpler cases, filter approximation can be performed using closed-form expressions, as will be seen in Section 16.3.

Finally, Fig. 16.4 shows transmission specifications for a bandpass filter and the response of a filter that meets these specifications. For this example we have chosen an approximation function that does not ripple in the passband; rather, the transmission decreases monotonically on both sides of the center frequency, attaining the maximum allowable deviation at the two edges of the passband.

EXERCISES

16.1 Find approximate values of attenuation (in dB) corresponding to filter transmissions of 1, 0.99, 0.9, 0.8, 0.7, 0.5, 0.1, 0.
Ans. 0, 0.1, 1, 2, 3, 6, 20, ∞ (dB)

16.2 If the magnitude of passband transmission is to remain constant to within ±5%, and if the stopband transmission is to be no greater than 1% of the passband transmission, find A_{max} and A_{min}.
Ans. 0.9 dB; 40 dB

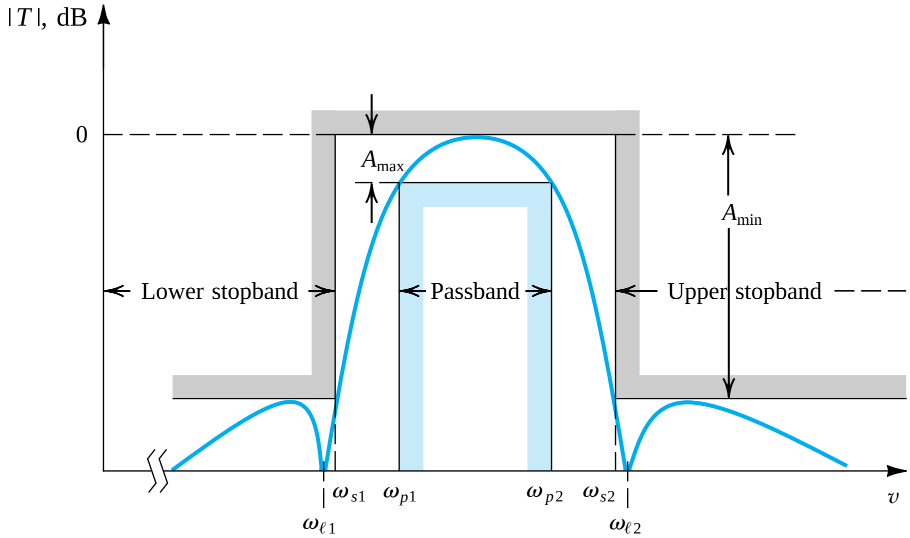


Figure 16.4 Transmission specifications for a bandpass filter. The magnitude response of a filter that just meets specifications is also shown. Note that this particular filter has a monotonically decreasing transmission in the passband on both sides of the peak frequency.

16.2 The Filter Transfer Function

The filter transfer function $T(s)$ can be written as the ratio of two polynomials as

$$T(s) = \frac{a_M s^M + a_{M-1} s^{M-1} + \cdots + a_0}{s^N + b_{N-1} s^{N-1} + \cdots + b_0} \tag{16.6}$$

The degree of the denominator, N, is the **filter order**. For the filter circuit to be stable, the degree of the numerator must be less than or equal to that of the denominator; $M \leq N$. The numerator and denominator coefficients, $a_0, a_1, \ldots, a_M$ and $b_0, b_1, \ldots, b_{N-1}$, are real numbers. The polynomials in the numerator and denominator can be factored, and $T(s)$ can be expressed in the form

$$T(s) = \frac{a_M(s - z_1)(s - z_2) \cdots (s - z_M)}{(s - p_1)(s - p_2) \cdots (s - p_N)} \tag{16.7}$$

The numerator roots, $z_1, z_2, \ldots, z_M$, are the **transfer function zeros**, or **transmission zeros**; and the denominator roots, $p_1, p_2, \ldots, p_N$, are the **transfer function poles**, or the **natural modes**.[1] Each transmission zero or pole can be either a real or a complex number. Complex zeros and poles, however, must occur in conjugate pairs. Thus, if $-1 + j2$ happens to be a zero, then $-1 - j2$ also must be a zero.

Since in the filter stopband the transmission is required to be zero or small, the filter transmission zeros are usually placed on the $j\omega$ axis at stopband frequencies. This indeed is the case for the filter whose transmission function is sketched in Fig. 16.3. This particular filter can be seen to have infinite attenuation (zero transmission) at two stopband frequencies: ω_{l1} and ω_{l2}. The filter then must have transmission zeros at $s = +j\omega_{l1}$ and $s = +j\omega_{l2}$.

[1] Throughout this chapter, we use the names *poles* and *natural modes* interchangeably.

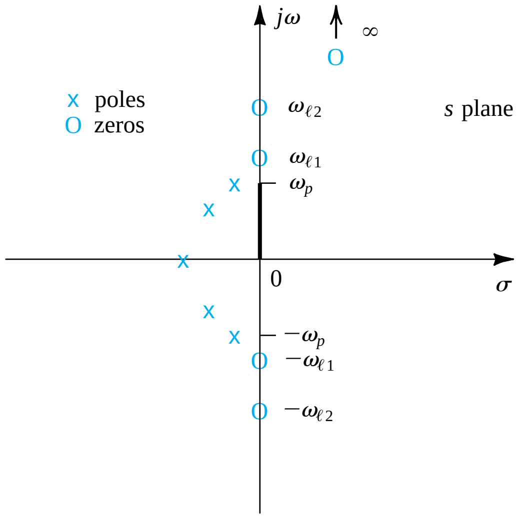


Figure 16.5 Pole–zero pattern for the low-pass filter whose transmission is sketched in Fig. 16.3. This is a fifth-order filter ($N = 5$).

However, since complex zeros occur in conjugate pairs, there must also be transmission zeros at $s = -j\omega_{l1}$ and $s = -j\omega_{l2}$. Thus the numerator polynomial of this filter will have the factors $(s + j\omega_{l1})(s - j\omega_{l1})(s + j\omega_{l2})(s - j\omega_{l2})$, which can be written as $(s^2 + \omega_{l1}^2)(s^2 + \omega_{l2}^2)$. For $s = j\omega$ (physical frequencies) the numerator becomes $(-\omega^2 + \omega_{l1}^2)(-\omega^2 + \omega_{l2}^2)$, which indeed is zero at $\omega = \omega_{l1}$ and $\omega = \omega_{l2}$.

Continuing with the example in Fig. 16.3, we observe that the transmission decreases toward $-\infty$ as ω approaches ∞. Thus the filter must have one or more transmission zeros at $s = \infty$. In general, the number of transmission zeros at $s = \infty$ is the difference between the degree of the numerator polynomial, M, and the degree of the denominator polynomial, N, of the transfer function in Eq. (16.6). This is because as s approaches ∞, $T(s)$ approaches a_M/s^{N-M} and thus is said to have $N - M$ zeros at $s = \infty$.

For a filter circuit to be stable, all its poles must lie in the left half of the s plane, and thus p_1, $p_2, \ldots, p_N$ must all have negative real parts. Figure 16.5 shows typical pole and zero locations for the low-pass filter whose transmission function is depicted in Fig. 16.3. We have assumed that this filter is of fifth order ($N = 5$). It has two pairs of complex-conjugate poles and one real-axis pole, for a total of five poles. All the poles lie in the vicinity of the passband, which is what gives the filter its high transmission at passband frequencies. The five transmission zeros are at $s = \pm j\omega_{l1}$, $s = \pm j\omega_{l2}$, and $s = \infty$. Thus, the transfer function for this filter is of the form

$$T(s) = \frac{a_4(s^2 + \omega_{l1}^2)(s^2 + \omega_{l2}^2)}{s^5 + b_4 s^4 + b_3 s^3 + b_2 s^2 + b_1 s + b_0} \tag{16.8}$$

As another example, consider the bandpass filter whose magnitude response is shown in Fig. 16.4. This filter has transmission zeros at $s = \pm j\omega_{l1}$ and $s = \pm j\omega_{l2}$. It also has one or more zeros at $s = 0$ and one or more zeros at $s = \infty$ (because the transmission decreases toward 0 as ω approaches 0 and ∞). Assuming that only one zero exists at each of $s = 0$ and $s = \infty$, the filter must be of sixth order, and its transfer function takes the form

$$T(s) = \frac{a_5 s(s^2 + \omega_{l1}^2)(s^2 + \omega_{l2}^2)}{s^6 + b_5 s^5 + \cdots + b_0} \tag{16.9}$$

A typical pole–zero plot for such a filter is shown in Fig. 16.6.

As a third and final example, consider the low-pass filter whose transmission function is depicted in Fig. 16.7(a). We observe that in this case there are no finite values of ω at which

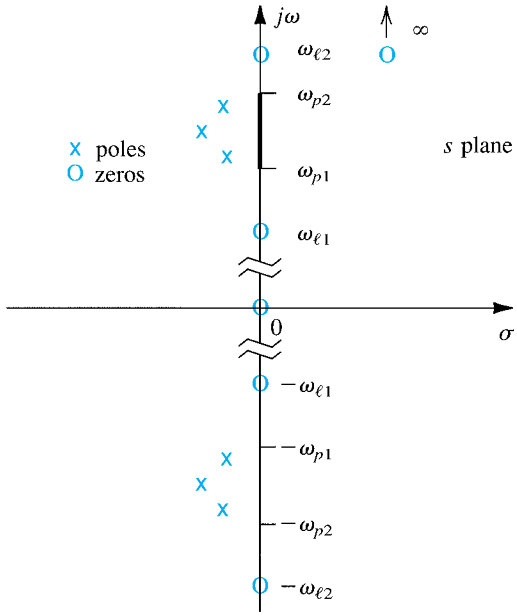


Figure 16.6 Pole–zero pattern for the bandpass filter whose transmission function is shown in Fig. 16.4. This is a sixth-order filter ($N = 6$).

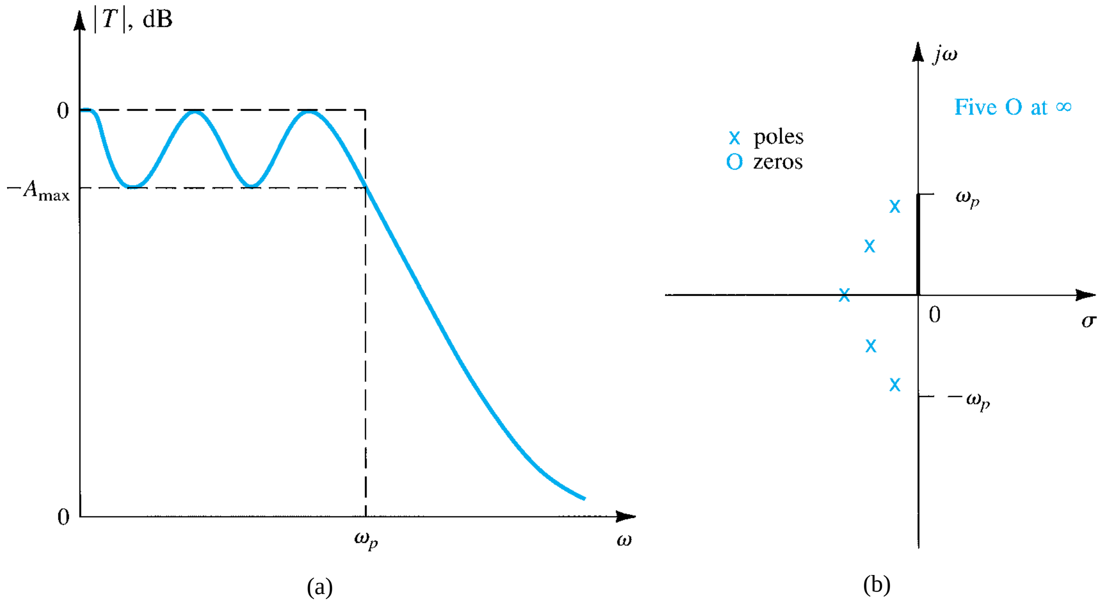


Figure 16.7 **(a)** Transmission characteristics of a fifth-order low-pass filter having all transmission zeros at infinity. **(b)** Pole–zero pattern for the filter in **(a)**.

the attenuation is infinite (zero transmission). Thus it is possible that all the transmission zeros of this filter are at $s = \infty$. If this is the case, the filter transfer function takes the form

$$T(s) = \frac{a_0}{s^N + b_{N-1}s^{N-1} + \cdots + b_0} \tag{16.10}$$

Such a filter is known as an **all-pole filter**. Typical pole–zero locations for a fifth-order all-pole low-pass filter are shown in Fig. 16.7(b).

Almost all the filters studied in this chapter have all their transmission zeros on the $j\omega$ axis, in the filter stopband(s), including[2] $\omega = 0$ and $\omega = \infty$. Also, to obtain high selectivity, all the natural modes will be complex conjugate (except for the case of odd-order filters, where one natural mode must be on the real axis). Finally we note that the more selective the required filter response is, the higher its order must be, and the closer its natural modes are to the $j\omega$ axis.

EXERCISES

16.3 A second-order filter has its poles at $s = -(1/2) \pm j(\sqrt{3}/2)$. The transmission is zero at $\omega = 2$ rad/s and is unity at dc ($\omega = 0$). Find the transfer function.

Ans. $T(s) = \dfrac{1}{4}\,\dfrac{s^2+4}{s^2+s+1}$

16.4 A fourth-order filter has zero transmission at $\omega = 0$, $\omega = 2$ rad/s, and $\omega = \infty$. The natural modes are $-0.1 \pm j0.8$ and $-0.1 \pm j1.2$. Find $T(s)$.

Ans. $T(s) = \dfrac{a_3 s(s^2+4)}{(s^2+0.2s+0.65)(s^2+0.2s+1.45)}$

16.5 Find the transfer function $T(s)$ of a third-order all-pole low-pass filter whose poles are at a radial distance of 1 rad/s from the origin and whose complex poles are at 30° angles from the $j\omega$ axis. The dc gain is unity. Show that $|T(j\omega)| = 1/\sqrt{1+\omega^6}$. Find ω_{3dB} and the attenuation at $\omega = 3$ rad/s.

Ans. $T(s) = 1/(s+1)(s^2+s+1)$; 1 rad/s; 28.6 dB

16.3 Butterworth and Chebyshev Filters

In this section, we present two functions that are frequently used in approximating the transmission characteristics of low-pass filters. Closed-form expressions are available for the parameters of these functions, and thus one can use them in filter design without the need for computers or filter-design tables. Their utility, however, is limited to relatively simple applications.

Although in this section we discuss the design of low-pass filters only, the approximation functions presented can be applied to the design of other filter types through the use of frequency transformations (see Sedra and Brackett, 1978).

16.3.1 The Butterworth Filter

Figure 16.8 shows a sketch of the magnitude response of a Butterworth[3] filter. This filter exhibits a monotonically decreasing transmission with all the transmission zeros at $\omega = \infty$, making it an all-pole filter. The magnitude function for an Nth-order Butterworth filter with a passband edge ω_p is given by

---

[2] Obviously, a low-pass filter should *not* have a transmission zero at $\omega = 0$, and, similarly, a high-pass filter should not have a transmission zero at $\omega = \infty$.

[3] The Butterworth filter approximation is named after S. Butterworth, a British engineer who in 1930 was among the first to employ it.

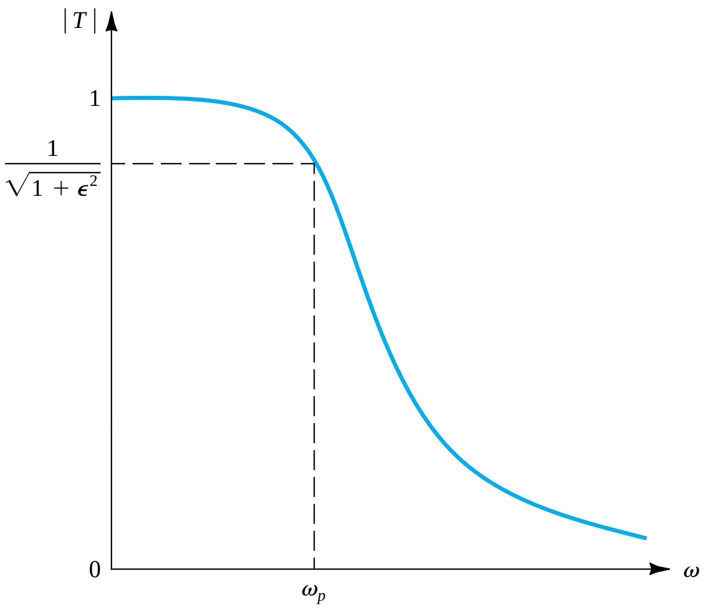


Figure 16.8 The magnitude response of a Butterworth filter.

$$|T(j\omega)| = \frac{1}{\sqrt{1+\epsilon^2\left(\frac{\omega}{\omega_p}\right)^{2N}}} \tag{16.11}$$

At $\omega = \omega_p$,

$$|T(j\omega_p)| = \frac{1}{\sqrt{1+\epsilon^2}} \tag{16.12}$$

Thus, the parameter ϵ determines the maximum variation in passband transmission, A_{max}, according to

$$A_{max} = 20 \log \sqrt{1+\epsilon^2} \tag{16.13}$$

Conversely, given A_{max}, the value of ϵ can be determined from

$$\epsilon = \sqrt{10^{A_{max}/10} - 1} \tag{16.14}$$

Observe that in the Butterworth response the maximum deviation in passband transmission (from the ideal value of unity) occurs at the passband edge only. It can be shown that the first $2N-1$ derivatives of $|T|$ relative to ω are zero at $\omega = 0$ [see Van Valkenburg (1980)]. This property makes the Butterworth response very flat near $\omega = 0$ and gives the response the name **maximally flat** response. The degree of passband flatness increases as the order N is increased, as can be seen from Fig. 16.9. This figure indicates also that, as should be expected, as the order N is increased the filter response approaches the ideal brick-wall type of response.

At the edge of the stopband, $\omega = \omega_s$, the attenuation of the Butterworth filter can be obtained by substituting $\omega = \omega_s$ in Eq. (16.11). The result is given by

$$\begin{aligned} A(\omega_s) &= -20 \log[1/\sqrt{1+\epsilon^2(\omega_s/\omega_p)^{2N}}] \\ &= 10 \log[1+\epsilon^2(\omega_s/\omega_p)^{2N}] \end{aligned} \tag{16.15}$$

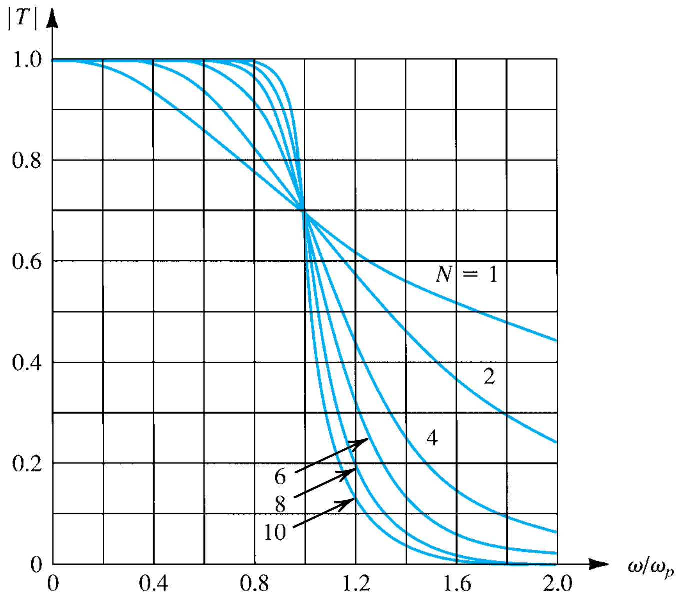


Figure 16.9 Magnitude response for Butterworth filters of various order with $\epsilon = 1$. Note that as the order increases, the response approaches the ideal brick-wall type of transmission.

This equation can be used to determine the filter order required, which is the lowest integer value of N that yields $A(\omega_s) \geq A_{\min}$.

The natural modes of an Nth-order Butterworth filter can be determined from the graphical construction shown in Fig. 16.10(a). Observe that the natural modes lie on a circle of radius $\omega_p(1/\epsilon)^{1/N}$ and are spaced by equal angles of π/N, with the first mode at an angle $\pi/2N$ from the $+j\omega$ axis. Since the natural modes all have equal radial distance from the origin, they all have the same frequency $\omega_0 = \omega_p(1/\epsilon)^{1/N}$. Figure 16.10(b), (c), and (d) shows the natural modes of Butterworth filters of order N = 2, 3, and 4, respectively. Once the N natural modes $p_1, p_2, \ldots, p_N$ have been found, the transfer function can be written as

$$T(s) = \frac{K\omega_0^N}{(s-p_1)(s-p_2)\cdots(s-p_N)} \tag{16.16}$$

where K is a constant equal to the required dc gain of the filter.

To summarize, to find a Butterworth transfer function that meets transmission specifications of the form in Fig. 16.3 we perform the following procedure:

1. Determine ϵ from Eq. (16.14).
2. Use Eq. (16.15) to determine the required filter order as the lowest integer value of N that results in $A(\omega_s) \geq A_{\min}$.
3. Use Fig. 16.10(a) to determine the N natural modes.
4. Use Eq. (16.16) to determine $T(s)$.

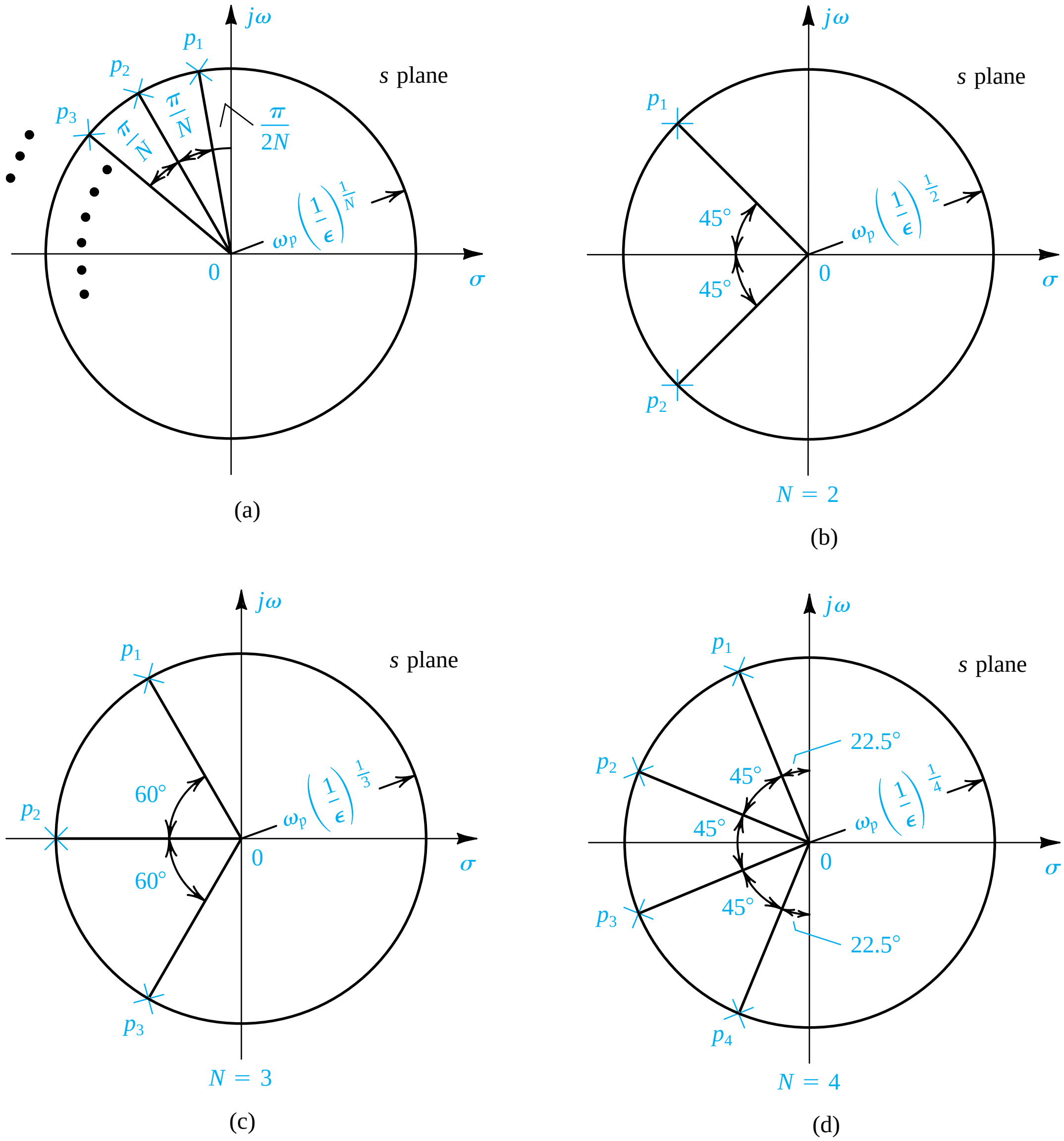


Figure 16.10 Graphical construction for determining the poles of a Butterworth filter of order N. All the poles lie in the left half of the s plane on a circle of radius $\omega_0 = \omega_p(1/\epsilon)^{1/N}$, where ϵ is the passband deviation parameter ($\epsilon = \sqrt{10^{A_{\max}/10} - 1}$): **(a)** the general case; **(b)** $N = 2$; **(c)** $N = 3$; **(d)** $N = 4$.

Example 16.1

Find the Butterworth transfer function that meets the following low-pass filter specifications: $f_p = 10$ kHz, $A_{\max} = 1$ dB, $f_s = 15$ kHz, $A_{\min} = 25$ dB, dc gain = 1.

Solution

Substituting $A_{\max} = 1$ dB into Eq. (16.14) yields $\epsilon = 0.5088$. Equation (16.15) is then used to determine the filter order by trying various values for N. We find that $N = 8$ yields $A(\omega_s) = 22.3$ dB and $N = 9$ gives 25.8 dB. We thus select $N = 9$.

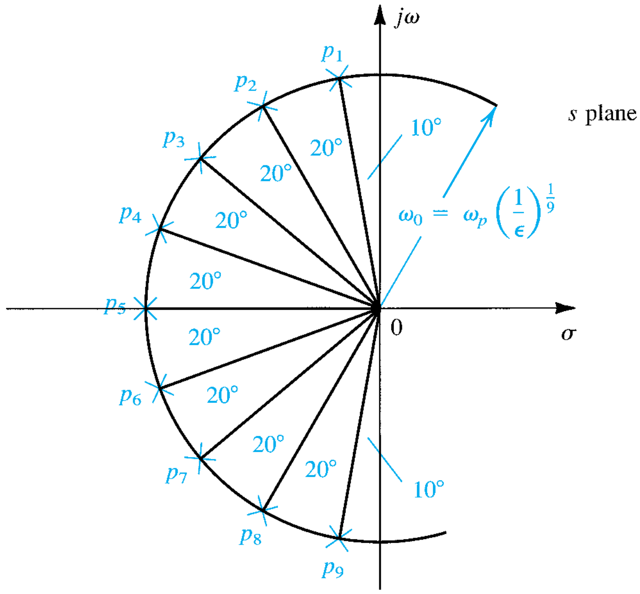


Figure 16.11 Poles of the ninth-order Butterworth filter of Example 16.1.

Figure 16.11 shows the graphical construction for determining the poles. The poles all have the same frequency $\omega_0 = \omega_p(1/\epsilon)^{1/N} = 2\pi \times 10 \times 10^3(1/0.5088)^{1/9} = 6.773 \times 10^4$ rad/s. The first pole p_1 is given by

$$p_1 = \omega_0(-\cos 80° + j\sin 80°) = \omega_0(-0.1736 + j0.9848)$$

Combining p_1 with its complex conjugate p_9 yields the factor $(s^2 + s0.3472\,\omega_0 + \omega_0^2)$ in the denominator of the transfer function. The same can be done for the other complex poles, and the complete transfer function is obtained using Eq. (16.16),

$$T(s) = \frac{\omega_0^9}{(s+\omega_0)(s^2 + s1.8794\,\omega_0 + \omega_0^2)(s^2 + s1.5321\,\omega_0 + \omega_0^2)} \tag{16.17}$$

$$\times \frac{1}{(s^2 + s\omega_0 + \omega_0^2)(s^2 + s0.3472\,\omega_0 + \omega_0^2)}$$

16.3.2 The Chebyshev Filter

Figure 16.12 shows representative transmission functions for Chebyshev[4] filters of even and odd orders. The Chebyshev filter exhibits an equiripple response in the passband and a monotonically decreasing transmission in the stopband. While the odd-order filter has $|T(0)| = 1$, the even-order filter exhibits its maximum magnitude deviation at $\omega = 0$. In both

[4] Named after the Russian mathematician P. L. Chebyshev, who in 1899 used these functions in studying the construction of steam engines.

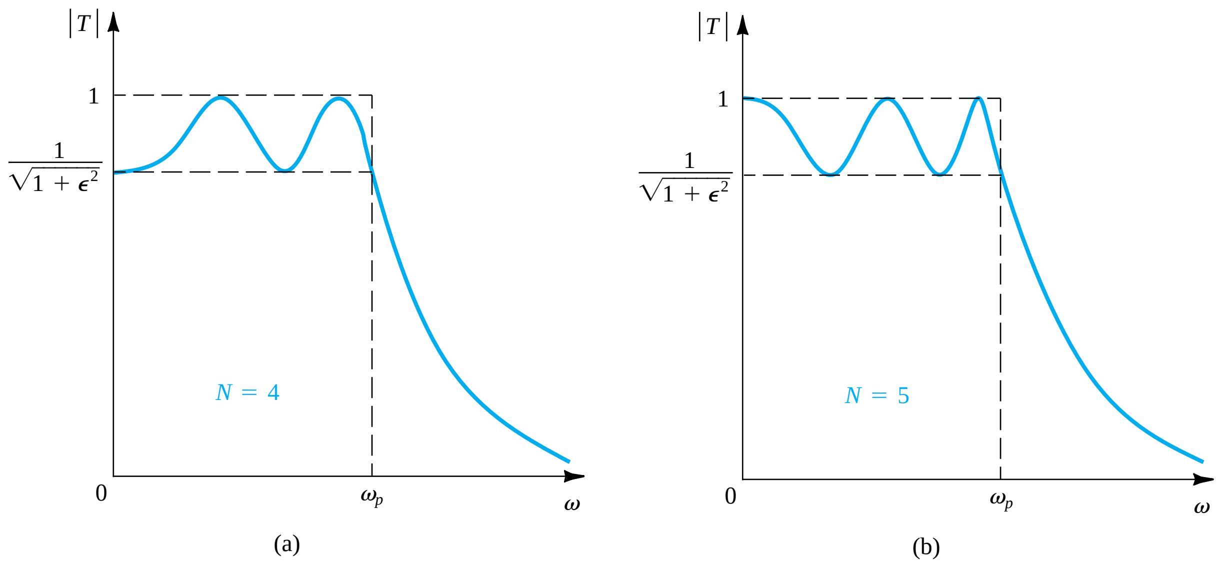


Figure 16.12 Sketches of the transmission characteristics of representative **(a)** even-order and **(b)** odd-order Chebyshev filters.

cases the total number of passband maxima and minima equals the order of the filter, N. All the transmission zeros of the Chebyshev filter are at $\omega = \infty$, making it an all-pole filter.

The magnitude of the transfer function of an Nth-order Chebyshev filter with a passband edge (ripple bandwidth) ω_p is given by

$$|T(j\omega)| = \frac{1}{\sqrt{1+\epsilon^2\cos^2[N\cos^{-1}(\omega/\omega_p)]}} \qquad \text{for } \omega \le \omega_p \tag{16.18}$$

and

$$|T(j\omega)| = \frac{1}{\sqrt{1+\epsilon^2\cosh^2[N\cosh^{-1}(\omega/\omega_p)]}} \qquad \text{for } \omega \ge \omega_p \tag{16.19}$$

At the passband edge, $\omega = \omega_p$, the magnitude function is given by

$$|T(j\omega_p)| = \frac{1}{\sqrt{1+\epsilon^2}}$$

Thus, the parameter determines the passband ripple according to

$$A_{max} = 10\log(1+\epsilon^2) \tag{16.20}$$

Conversely, given A_{max}, the value of ϵ is determined from

$$\epsilon = \sqrt{10^{A_{max}/10}-1} \tag{16.21}$$

The attenuation achieved by the Chebyshev filter at the stopband edge ($\omega = \omega_s$) is found using Eq. (16.19) as

$$A(\omega_s) = 10\log[1+\epsilon^2\cosh^2(N\cosh^{-1}(\omega_s/\omega_p))] \tag{16.22}$$

With the aid of a calculator, this equation can be used to determine the order N required to obtain a specified A_{min} by finding the lowest integer value of N that yields $A(\omega_s) \geq A_{min}$. As in the case of the Butterworth filter, increasing the order N of the Chebyshev filter causes its magnitude function to approach the ideal brick-wall low-pass response.

The poles of the Chebyshev filter are given by

$$p_k = -\omega_p \sin\left(\frac{2k-1}{N}\frac{\pi}{2}\right)\sinh\left(\frac{1}{N}\sinh^{-1}\frac{1}{\epsilon}\right) \tag{16.23}$$

$$+j\omega_p \cos\left(\frac{2k-1}{N}\frac{\pi}{2}\right)\cosh\left(\frac{1}{N}\sinh^{-1}\frac{1}{\epsilon}\right) \qquad k = 1, 2, \ldots, N$$

Finally, the transfer function of the Chebyshev filter can be written as

$$T(s) = \frac{K\omega_p^N}{\epsilon\, 2^{N-1}(s-p_1)(s-p_2)\cdots(s-p_N)} \tag{16.24}$$

where K is the dc gain that the filter is required to have.

To summarize, given low-pass transmission specifications of the type shown in Fig. 16.3, the transfer function of a Chebyshev filter that meets these specifications can be found as follows:

1. Determine ϵ from Eq. (16.21).
2. Use Eq. (16.22) to determine the order required.
3. Determine the poles using Eq. (16.23).
4. Determine the transfer function using Eq. (16.24).

The Chebyshev filter provides a more efficient approximation than the Butterworth filter. Thus, for the same order and the same A_{max}, the Chebyshev filter provides greater stopband attenuation than the Butterworth filter. Alternatively, to meet identical specifications, one requires a lower order for the Chebyshev than for the Butterworth filter. This point will be illustrated by the following example.

Example 16.2

Find the Chebyshev transfer function that meets the same low-pass filter specifications given in Example 16.1: namely, $f_p = 10$ kHz, $A_{max} = 1$ dB, $f_s = 15$ kHz, $A_{min} = 25$ dB, dc gain = 1.

Solution

Substituting $A_{max} = 1$ dB into Eq. (16.21) yields $\epsilon = 0.5088$. By trying various values for N in Eq. (16.22) we find that $N = 4$ yields $A(\omega_s) = 21.6$ dB and $N = 5$ provides 29.9 dB. We thus select $N = 5$. Recall that we required a ninth-order Butterworth filter to meet the same specifications in Example 16.1.

The poles are obtained by substituting in Eq. (16.23) as

$$p_1, p_5 = \omega_p(-0.0895 \pm j0.9901)$$

$$p_2, p_4 = \omega_p(-0.2342 \pm j0.6119)$$

Example 16.2 *continued*

$$p_5 = \omega_p(-0.2895)$$

The transfer function is obtained by substituting these values in Eq. (16.24) as

$$T(s) = \frac{\omega_p^5}{8.1408(s+0.2895\omega_p)(s^2+s0.4684\omega_p+0.4293\omega_p^2)} \qquad (16.25)$$

$$\times \frac{1}{s^2+s0.1789\omega_p+0.9883\omega_p^2}$$

where $\omega_p = 2\pi \times 10^4$ rad/s.

EXERCISES

D16.6 Determine the order N of a Butterworth filter for which $A_{max} = 1$ dB, $\omega_s/\omega_p = 1.5$, and $A_{min} = 30$ dB. What is the actual value of minimum stopband attenuation realized? If A_{min} is to be exactly 30 dB, to what value can A_{max} be reduced?
Ans. $N = 11$; $A_{min} = 32.87$ dB; 0.54 dB

16.7 Find the natural modes and the transfer function of a Butterworth filter with $\omega_p = 1$ rad/s, $A_{max} = 3$ dB ($\epsilon \simeq 1$), and $N = 3$.
Ans. $-0.5 \pm j\sqrt{3}/2$ and -1; $T(s) = 1/(s+1)(s^2+s+1)$

16.8 Observe that Eq. (16.18) can be used to find the frequencies in the passband at which $|T|$ is at its peaks and at its valleys. (The peaks are reached when the $\cos^2[\ ]$ term is zero, and the valleys correspond to the $\cos^2[\ ]$ term equal to unity.) Find these frequencies for a fifth-order filter.
Ans. Peaks at $\omega = 0$, $0.59\omega_p$, and $0.95\omega_p$; the valleys at $\omega = 0.31\omega_p$ and $0.81\omega_p$

D16.9 Find the attenuation provided at $\omega = 2\omega_p$ by a seventh-order Chebyshev filter with a 0.5-dB passband ripple. If the passband ripple is allowed to increase to 1 dB, by how much does the stopband attenuation increase?
Ans. 64.9 dB; 3.3 dB

D16.10 It is required to design a low-pass filter having $f_p = 1$ kHz, $A_{max} = 1$ dB, $f_s = 1.5$ kHz, $A_{min} = 50$ dB. (a) Find the required order of a Chebyshev filter. What is the excess stopband attenuation obtained? (b) Repeat for a Butterworth filter.
Ans. (a) $N = 8$, 5 dB; (b) $N = 16$, 0.5 dB

16.4 First-Order and Second-Order Filter Functions

In this section, we shall study the simplest filter transfer functions, those of first and second order. These functions are useful in their own right in the design of simple filters. First- and second-order filters can also be cascaded to realize a high-order filter. Cascade design is in fact one of the most popular methods for the design of active filters (those utilizing op amps and RC circuits). Because the filter poles occur in complex-conjugate pairs, a high-order transfer function $T(s)$ is factored into the product of second-order functions. If $T(s)$ is odd, there will also be a first-order function in the factorization. Each of the second-order functions [and the first-order function when $T(s)$ is odd] is then realized using one of the op amp–RC circuits that will be studied in this chapter, and the resulting blocks are placed in cascade. If the output of each block is taken at the output terminal of an op amp where the impedance level is low (ideally zero), cascading does not change the transfer functions of the individual blocks. Thus the overall transfer function of the cascade is simply the product of the transfer functions of the individual blocks, which is the original $T(s)$.

16.4.1 First-Order Filters

The general first-order transfer function is given by

$$T(s) = \frac{a_1 s + a_0}{s + \omega_0} \tag{16.26}$$

This **bilinear transfer function** characterizes a first-order filter with a natural mode at $s = -\omega_0$, a transmission zero at $s = -a_0/a_1$, and a high-frequency gain that approaches a_1. The numerator coefficients, a_0 and a_1, determine the type of filter (e.g., low pass, high pass, etc.). Some special cases together with passive (RC) and active (op amp–RC) realizations are shown in Fig. 16.13. Note that the active realizations provide considerably more versatility than their passive counterparts; in many cases the gain can be set to a desired value, and some transfer function parameters can be adjusted without affecting others. The output impedance of the active circuit is also very low, making cascading easily possible. The op amp, however, limits the high-frequency operation of the active circuits.

An important special case of the first-order filter function is the **all-pass filter** shown in Fig. 16.14. Here, the transmission zero and the natural mode are symmetrically located relative to the $j\omega$ axis. (They are said to display mirror-image symmetry with respect to the $j\omega$ axis.) Observe that although the transmission of the all-pass filter is (ideally) constant at all frequencies, its phase shows frequency selectivity. All-pass filters are used as phase shifters and in systems that require phase shaping (e.g., in the design of circuits called *delay equalizers*, which cause the overall time delay of a transmission system to be constant with frequency).

EXERCISES

D16.11 Using $R_1 = 10\ \text{k}\Omega$, design the op amp–RC circuit of Fig. 16.13(b) to realize a high-pass filter with a corner frequency of 10^4 rad/s and a high-frequency gain of 10.
Ans. $R_2 = 100\ \text{k}\Omega$; $C = 0.01\ \mu\text{F}$

D16.12 Design the op amp–RC circuit of Fig. 16.14 to realize an all-pass filter with a 90° phase shift at 10^3 rad/s. Select suitable component values.
Ans. Possible choices: $R = R_1 = R_2 = 10\ \text{k}\Omega$; $C = 0.1\ \mu\text{F}$

16.4.2 Second-Order Filter Functions

The general second-order (or **biquadratic**) filter transfer function is usually expressed in the standard form

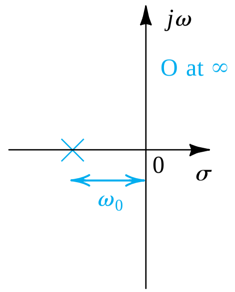
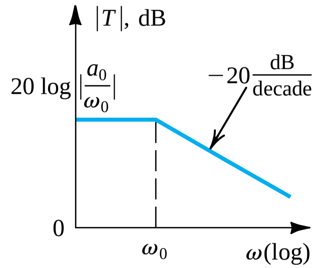
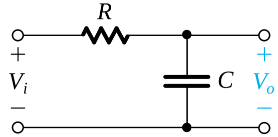
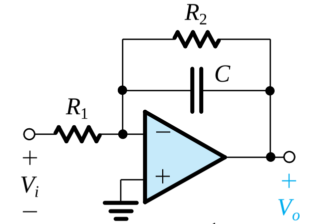
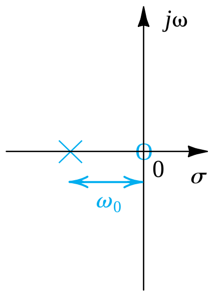
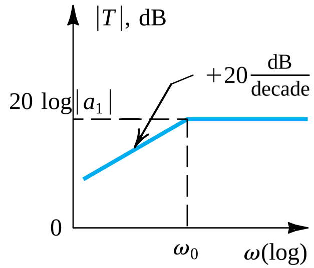
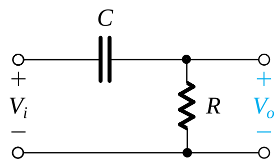
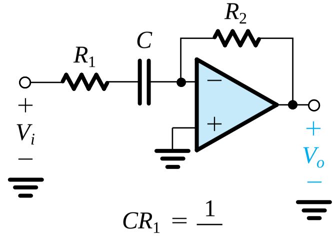
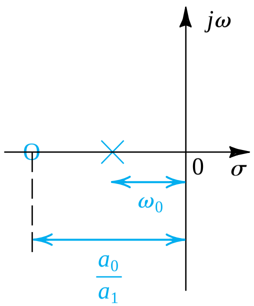
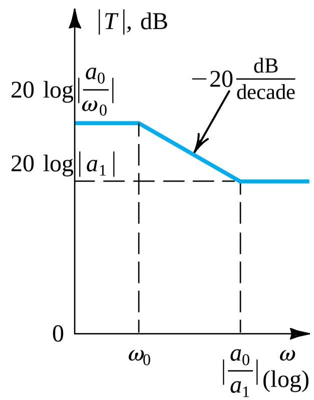
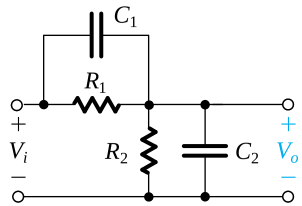
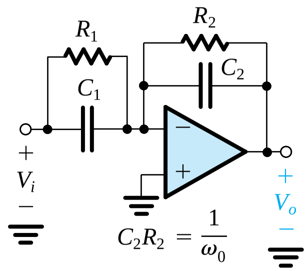
Filter Type and $T(s)$	s-Plane Singularities	Bode Plot for $\lvert T\rvert$	Passive Realization	Op Amp–RC Realization
(a) Low pass (LP) $T(s) = \frac{a_0}{s+\omega_0}$			 $CR = \frac{1}{\omega_0}$ DC gain $= 1$	 $CR_2 = \frac{1}{\omega_0}$ DC gain $= -\frac{R_2}{R_1}$
(b) High pass (HP) $T(s) = \frac{a_1 s}{s+\omega_0}$			 $CR = \frac{1}{\omega_0}$ High-frequency gain $= 1$	 $CR_1 = \frac{1}{\omega_0}$ High-frequency gain $= -\frac{R_2}{R_1}$
(c) General $T(s) = \frac{a_1 s + a_0}{s+\omega_0}$			 $(C_1 + C_2)(R_1 // R_2) = \frac{1}{\omega_0}$ $C_1R_1 = \frac{a_1}{a_0}$ DC gain $= \frac{R_2}{R_1 + R_2}$ HF gain $= \frac{C_1}{C_1 + C_2}$	 $C_2R_2 = \frac{1}{\omega_0}$ $C_1R_1 = \frac{a_1}{a_0}$ DC gain $= -\frac{R_2}{R_1}$ HF gain $= -\frac{C_1}{C_2}$

Figure 16.13 First-order filters.

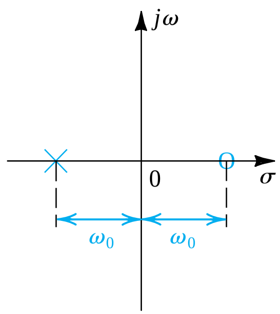
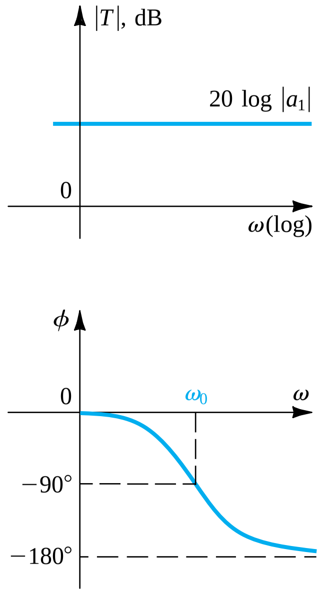
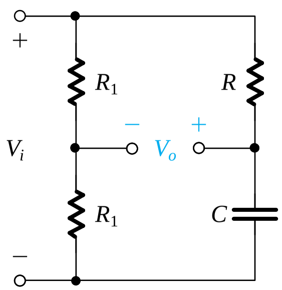
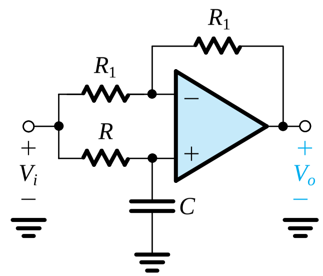
$T(s)$	Singularities	$\|T\|$ and ϕ	Passive Realization	Op Amp–RC Realization
All pass (AP) $T(s) = -a_1 \dfrac{s - \omega_0}{s + \omega_0}$ $a_1 > 0$			 $CR = 1/\omega_0$ Flat gain $(a_1) = 0.5$	 $CR = 1/\omega_0$ Flat gain $(a_1) = 1$

Figure 16.14 First-order all-pass filter.

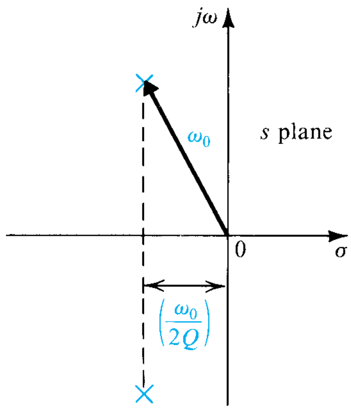


Figure 16.15 Definition of the parameters ω_0 and Q of a pair of complex-conjugate poles.

$$T(s) = \frac{a_2 s^2 + a_1 s + a_0}{s^2 + (\omega_0 / Q)s + \omega_0^2} \tag{16.27}$$

where ω_0 and Q determine the natural modes (poles) according to

$$p_1, p_2 = -\frac{\omega_0}{2Q} \pm j\omega_0\sqrt{1 - (1/4Q^2)} \tag{16.28}$$

We are usually interested in the case of complex-conjugate natural modes, obtained for $Q > 0.5$. Figure 16.15 shows the location of the pair of complex-conjugate poles in the s plane. Observe that the radial distance of the natural modes (from the origin) is equal to ω_0, which is known as the **pole frequency**. The parameter Q determines the distance of the poles from the $j\omega$ axis: the higher the value of Q, the closer the poles are to the $j\omega$ axis, and the more selective the filter response becomes. An infinite value for Q locates the poles on the $j\omega$ axis and can yield sustained oscillations in the circuit realization. A negative value of Q implies that the poles are in the right half of the s plane, which certainly produces oscillations. The parameter Q is called the **pole quality factor**, or simply, **pole *Q***.

The transmission zeros of the second-order filter are determined by the numerator coefficients, a_0, a_1, and a_2. It follows that the numerator coefficients determine the type of second-order filter function (i.e., LP, HP, etc.). Seven special cases of interest are illustrated in Fig. 16.16. For each case we give the transfer function, the s-plane locations of the transfer function singularities, and the magnitude response. Circuit realizations for the various second-order filter functions will be given in subsequent sections.

All seven special second-order filters have a pair of complex-conjugate natural modes characterized by a frequency ω_0 and a quality factor Q.

In the low-pass (LP) case, shown in Fig. 16.16(a), the two transmission zeros are at $s = \infty$. The magnitude response can exhibit a peak with the details indicated. It can be shown that the peak occurs only for $Q > 1/\sqrt{2}$. The response obtained for $Q = 1/\sqrt{2}$ is the Butterworth, or maximally flat, response.

The high-pass (HP) function shown in Fig. 16.16(b) has both transmission zeros at $s = 0$ (dc). The magnitude response shows a peak for $Q > 1/\sqrt{2}$, with the details of the response as indicated. Observe the duality between the LP and HP responses.

Next consider the bandpass (BP) filter function shown in Fig. 16.16(c). Here, one transmission zero is at $s = 0$ (dc), and the other is at $s = \infty$. The magnitude response peaks at $\omega = \omega_0$. Thus the **center frequency** of the bandpass filter is equal to the pole frequency ω_0. The selectivity of the second-order bandpass filter is usually measured by its *3-dB bandwidth*. This is the difference between the two frequencies ω_1 and ω_2 at which the magnitude response is 3 dB below its maximum value (at ω_0). It can be shown that

$$\omega_1, \omega_2 = \omega_0\sqrt{1 + (1/4Q^2)} \pm \frac{\omega_0}{2Q} \tag{16.29}$$

Thus,

$$BW \equiv \omega_2 - \omega_1 = \omega_0/Q \tag{16.30}$$

Observe that as Q increases, the bandwidth decreases and the bandpass filter becomes more selective.

If the transmission zeros are located on the $j\omega$ axis, at the complex-conjugate locations $\pm j\omega_n$, then the magnitude response exhibits zero transmission at $\omega = \omega_n$. Thus a **notch** in the magnitude response occurs at $\omega = \omega_n$, and ω_n is known as the **notch frequency**. Three cases of the second-order notch filter are possible: the regular notch, obtained when $\omega_n = \omega_0$ (Fig. 16.16d); the low-pass notch, obtained when $\omega_n > \omega_0$ (Fig. 16.16e); and the high-pass notch, obtained when $\omega_n < \omega_0$ (Fig. 16.16f). The reader is urged to verify the response details given in these figures (a rather tedious task, though!). Observe that in all notch cases, the transmission at dc and at $s = \infty$ is finite. This is so because there are no transmission zeros at either $s = 0$ or $s = \infty$.

The last special case of interest is the all-pass (AP) filter whose characteristics are illustrated in Fig. 16.16(g). Here the two transmission zeros are in the right half of the s plane, at the mirror-image locations of the poles. (This is the case for all-pass functions of any order.) The magnitude response of the all-pass function is constant over all frequencies; the **flat gain**, as it is called, is in our case equal to $|a_2|$. The frequency selectivity of the all-pass function is in its phase response.

EXERCISES

16.13 For a maximally flat second-order low-pass filter $(Q = 1/\sqrt{2})$, show that at $\omega = \omega_0$ the magnitude response is 3 dB below the value at dc.

16.14 Give the transfer function of a second-order bandpass filter with a center frequency of 10^5 rad/s, a center-frequency gain of 10, and a 3-dB bandwidth of 10^3 rad/s.

Ans. $T(s) = \dfrac{10^4 s}{s^2 + 10^3 s + 10^{10}}$

16.15 (a) For the second-order notch function with $\omega_n = \omega_0$, show that for the attenuation to be greater than A dB over a frequency band BW_a, the value of Q is given by

$$Q \le \frac{\omega_0}{BW_a\sqrt{10^{A/10} - 1}}$$

(*Hint:* First, show that any two frequencies, ω_1 and ω_2, at which $|T|$ is the same, are related by $\omega_1\omega_2 = \omega_0^2$.) (b) Use the result of (a) to show that the 3-dB bandwidth is ω_0/Q, as indicated in Fig. 16.16(d).

16.16 Consider a low-pass notch with $\omega_0 = 1$ rad/s, $Q = 10$, $\omega_n = 1.2$ rad/s, and a dc gain of unity. Find the frequency and magnitude of the transmission peak. Also find the high-frequency transmission.

Ans. 0.986 rad/s; 3.17; 0.69

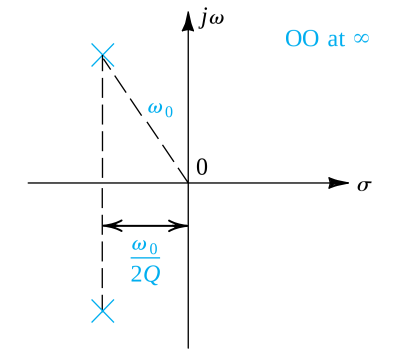
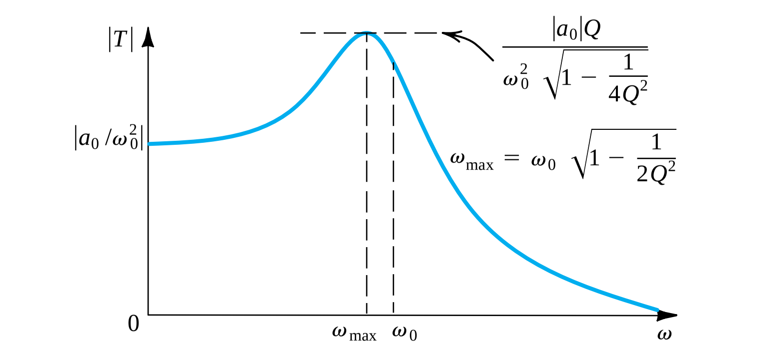
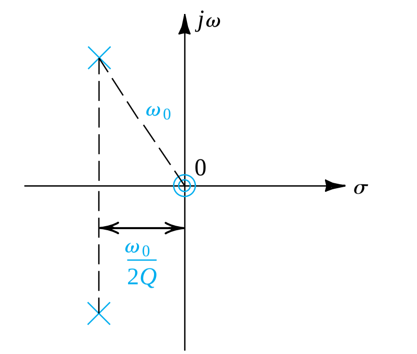
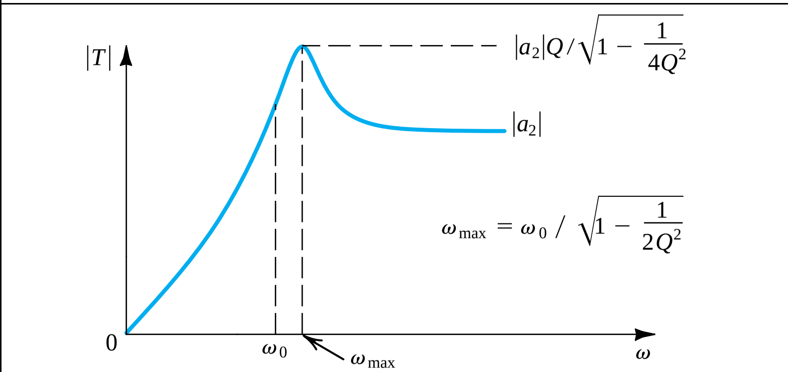
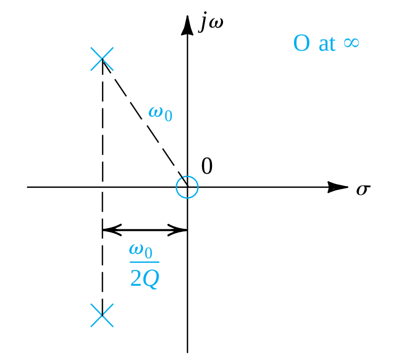
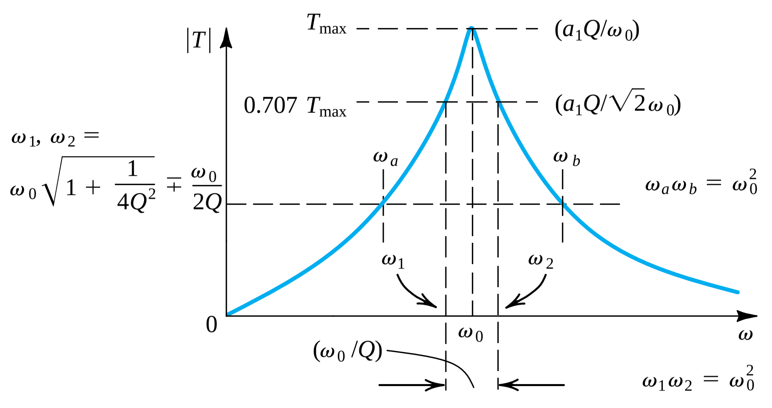
Filter Type and $T(s)$	s-Plane Singularities	$\|T\|$
(a) Low pass (LP) $T(s) = \dfrac{a_0}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$ DC gain $= \dfrac{a_0}{\omega_0^2}$		
(b) High pass (HP) $T(s) = \dfrac{a_2 s^2}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$ High-frequency gain $= a_2$		
(c) Bandpass (BP) $T(s) = \dfrac{a_1 s}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$ Center-frequency gain $= \dfrac{a_1 Q}{\omega_0}$		

Figure 16.16 Second-order filtering functions.

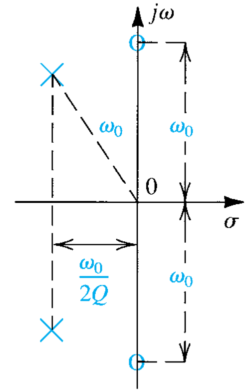
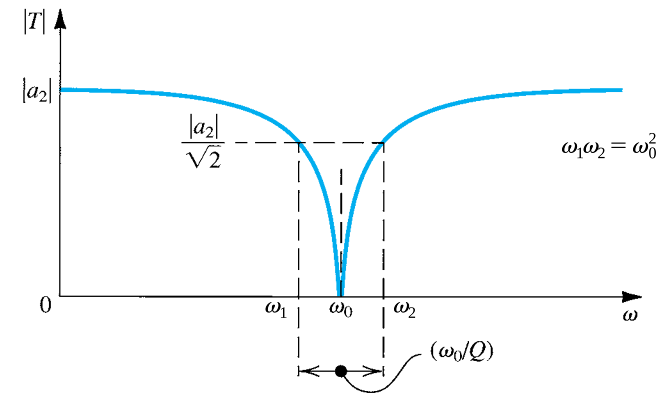
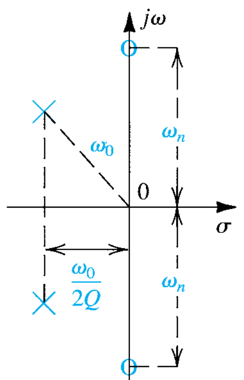
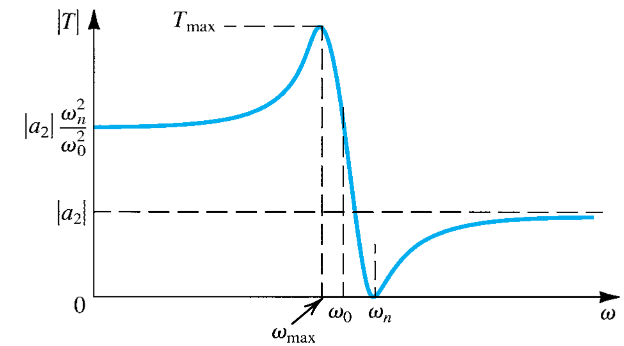
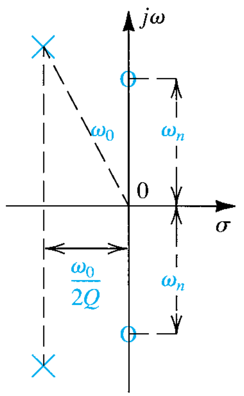
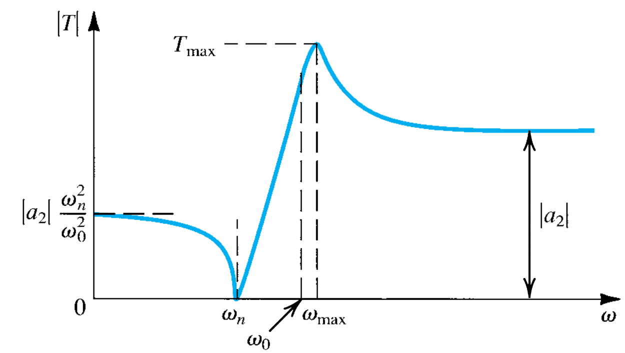
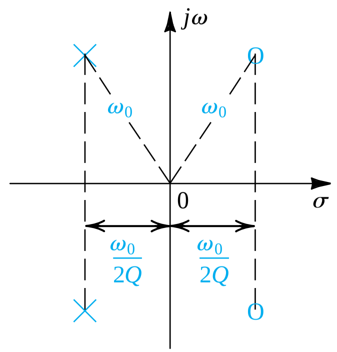
Filter Type and $T(s)$	s-Plane Singularities	$\lvert T\rvert$
(d) Notch $T(s) = a_2 \dfrac{s^2 + \omega_0^2}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$ DC gain = High-frequency gain = a_2		
(e) Low-pass notch (LPN) $T(s) = a_2 \dfrac{s^2 + \omega_n^2}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$ $\omega_n \geq \omega_0$ DC gain $= a_2 \dfrac{\omega_n^2}{\omega_0^2}$ High-frequency gain = a_2		 $$\omega_{max} = \omega_0 \sqrt{\frac{\left(\frac{\omega_n^2}{\omega_0^2}\right)\left(1 - \frac{1}{2Q^2}\right) - 1}{\frac{\omega_n^2}{\omega_0^2} + \frac{1}{2Q^2} - 1}}$$
(f) High-pass notch (HPN) $T(s) = a_2 \dfrac{s^2 + \omega_n^2}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$ $\omega_n \leq \omega_0$ DC gain $= a_2 \dfrac{\omega_n^2}{\omega_0^2}$ High-frequency gain = a_2		 $$T_{max} = \frac{\lvert a_2\rvert \, \lvert\omega_n^2 - \omega_{max}^2\rvert}{\sqrt{(\omega_0^2 - \omega_{max}^2)^2 + \left(\frac{\omega_0}{Q}\right)^2 \omega_{max}^2}}$$

Figure 16.16 *(continued)*

(g) All pass (AP)

$$T(s) = a_2 \frac{s^2 - s\frac{\omega_0}{Q} + \omega_0^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

Flat gain $= a_2$



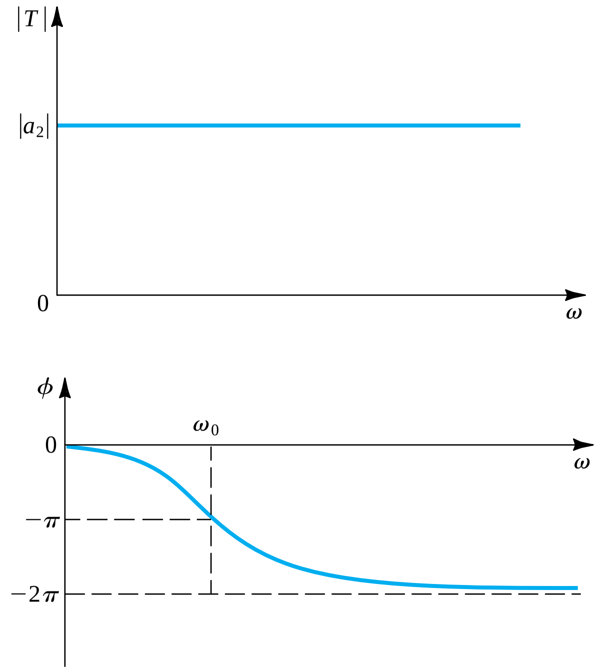


Figure 16.16 (*continued*)

16.5 The Second-Order LCR Resonator

In this section we shall study the second-order LCR resonator shown in Fig. 16.17(a). The use of this resonator to derive circuit realizations for the various second-order filter functions will be demonstrated. It will be shown in the next section that replacing the inductor L by a simulated inductance obtained using an op amp–RC circuit results in an op amp–RC resonator. The latter forms the basis of an important class of active-RC filters to be studied in Section 16.6.

16.5.1 The Resonator Natural Modes

The natural modes of the parallel resonance circuit of Fig. 16.17(a) can be determined by applying *an excitation that does not change the natural structure of the circuit.* Two possible ways of exciting the circuit are shown in Fig. 16.17(b) and (c). In Fig. 16.17(b) the resonator is excited with a current source I connected in parallel. Since, as far as the natural response of a circuit is concerned, an independent ideal current source is equivalent to an open circuit, the excitation of Fig. 16.17(b) does not alter the natural structure of the resonator. Thus the circuit in Fig. 16.17(b) can be used to determine the natural modes of the resonator by simply finding the poles of any response function. We can for instance take the voltage V_o across the resonator as the response and thus obtain the response function $V_o/I = Z$, where Z is the impedance of the parallel resonance circuit. It is obviously more convenient, however, to work in terms of the admittance Y; thus,

$$\frac{V_o}{I} = \frac{1}{Y} = \frac{1}{(1/sL)+sC+(1/R)} \tag{16.31}$$

$$= \frac{s/C}{s^2+s(1/CR)+(1/LC)}$$

Equating the denominator to the standard form $[s^2+s(\omega_0/Q)+\omega_0^2]$ leads to

$$\omega_0^2 = 1/LC \tag{16.32}$$

and

$$\omega_0/Q = 1/CR \tag{16.33}$$

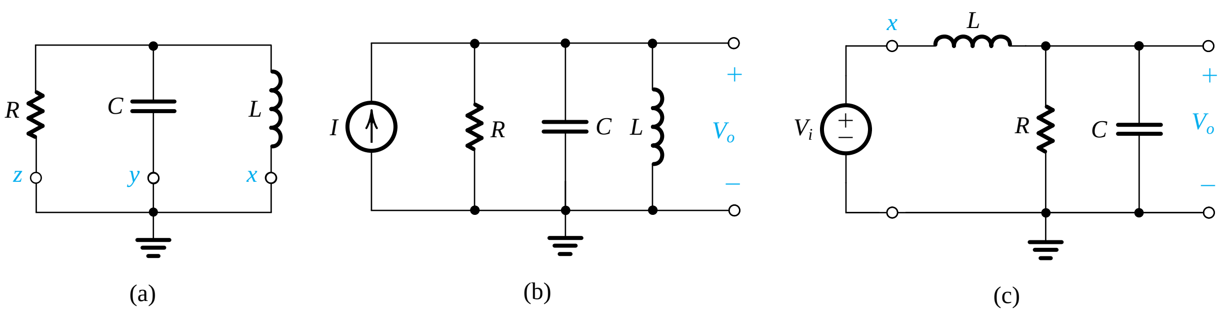


Figure 16.17 **(a)** The second-order parallel LCR resonator. **(b, c)** Two ways of exciting the resonator of **(a)** without changing its *natural structure:* resonator poles are those poles of V_o/I and V_o/V_i.

Thus,

$$\omega_0 = 1/\sqrt{LC} \tag{16.34}$$

$$Q = \omega_0 CR \tag{16.35}$$

These expressions should be familiar to the reader from studies of parallel resonance circuits in introductory courses on circuit theory.

An alternative way of exciting the parallel LCR resonator for the purpose of determining its natural modes is shown in Fig. 16.17(c). Here, node x of inductor L has been disconnected from ground and connected to an ideal voltage source V_i. Now, since as far as the natural response of a circuit is concerned, an ideal independent voltage source is equivalent to a short circuit, the excitation of Fig. 16.17(c) does not alter the natural structure of the resonator. Thus we can use the circuit in Fig. 16.17(c) to determine the natural modes of the resonator. These are the poles of any response function. For instance, we can select V_o as the response variable and find the transfer function V_o/V_i. The reader can easily verify that this will lead to the natural modes determined earlier.

In a design problem, we will be given ω_0 and Q and will be asked to determine L, C, and R. Equations (16.34) and (16.35) are two equations in the three unknowns. The one available degree of freedom can be utilized to set the impedance level of the circuit to a value that results in practical component values.

16.5.2 Realization of Transmission Zeros

Having selected the component values of the LCR resonator to realize a given pair of complex-conjugate natural modes, we now consider the use of the resonator to realize a desired filter type (e.g., LP, HP, etc.). Specifically, we wish to find out where to inject the input voltage signal V_i so that the transfer function V_o/V_i is the desired one. Toward that end, note that in the resonator circuit in Fig. 16.17(a), any of the nodes labeled x, y, or z can be disconnected from ground and connected to V_i without altering the circuit's natural modes. When this is done, the circuit takes the form of a voltage divider, as shown in Fig. 16.18(a). Thus the transfer function realized is

$$T(s) = \frac{V_o(s)}{V_i(s)} = \frac{Z_2(s)}{Z_1(s)+Z_2(s)} \tag{16.36}$$

We observe that *the transmission zeros are the values of s at which $Z_2(s)$ is zero, provided $Z_1(s)$ is not simultaneously zero, and the values of s at which $Z_1(s)$ is infinite, provided $Z_2(s)$ is not simultaneously infinite*. This statement makes physical sense: The output will be zero either when $Z_2(s)$ behaves as a short circuit or when $Z_1(s)$ behaves as an open circuit. If there is a value of s at which both Z_1 and Z_2 are zero, then V_o/V_i will be finite and no transmission zero is obtained. Similarly, if there is a value of s at which both Z_1 and Z_2 are infinite, then V_o/V_i will be finite and no transmission zero is realized.

16.5.3 Realization of the Low-Pass Function

Using the scheme just outlined, we see that to realize a low-pass function, node x is disconnected from ground and connected to V_i, as shown in Fig. 16.18(b). The transmission zeros of this circuit will be at the value of s for which the series impedance becomes infinite (sL becomes infinite at $s = \infty$) and the value of s at which the shunt impedance becomes zero ($1/[sC+(1/R)]$ becomes zero at $s = \infty$). Thus this circuit has two transmission zeros

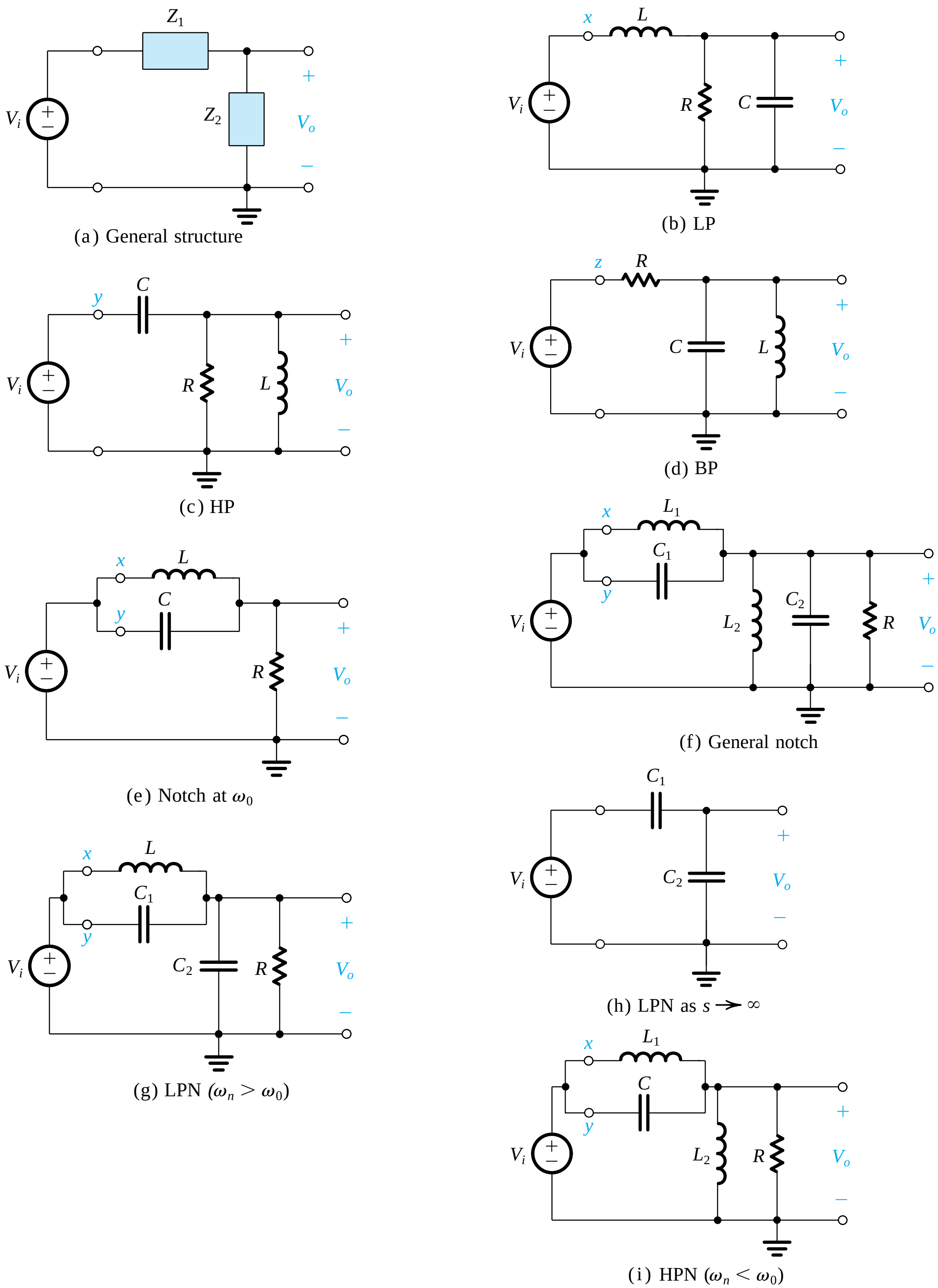


Figure 16.18 Realization of various second-order filter functions using the LCR resonator of Fig. 16.17(b): **(a)** general structure, **(b)** LP, **(c)** HP, **(d)** BP, **(e)** notch at ω_0, **(f)** general notch, **(g)** LPN ($\omega_n \geq \omega_0$), **(h)** LPN as $s \rightarrow \infty$, **(i)** HPN ($\omega_n < \omega_0$).

at $s = \infty$, as an LP is supposed to. The transfer function can be written either by inspection or by using the voltage divider rule. Following the latter approach, we obtain

$$T(s) \equiv \frac{V_o}{V_i} = \frac{Z_2}{Z_1 + Z_2} = \frac{Y_1}{Y_1 + Y_2} = \frac{1/sL}{(1/sL) + sC + (1/R)} \tag{16.37}$$

$$= \frac{1/LC}{s^2 + s(1/CR) + (1/LC)}$$

16.5.4 Realization of the High-Pass Function

To realize the second-order high-pass function, node y is disconnected from ground and connected to V_i, as shown in Fig. 16.18(c). Here the series capacitor introduces a transmission zero at $s = 0$ (dc), and the shunt inductor introduces another transmission zero at $s = 0$ (dc). Thus, by inspection, the transfer function may be written as

$$T(s) \equiv \frac{V_o}{V_i} = \frac{a_2 s^2}{s^2 + s(\omega_0/Q) + \omega_0^2} \tag{16.38}$$

where ω_0 and Q are the natural mode parameters given by Eqs. (16.34) and (16.35) and a_2 is the high-frequency transmission. The value of a_2 can be determined from the circuit by observing that as s approaches ∞, the capacitor approaches a short circuit and V_o approaches V_i, resulting in $a_2 = 1$.

16.5.5 Realization of the Bandpass Function

The bandpass function is realized by disconnecting node z from ground and connecting it to V_i, as shown in Fig. 16.18(d). Here the series impedance is resistive and thus does not introduce any transmission zeros. These are obtained as follows: One zero at $s = 0$ is realized by the shunt inductor, and one zero at $s = \infty$ is realized by the shunt capacitor. At the center frequency ω_0, the parallel LC-tuned circuit exhibits an infinite impedance, and thus no current flows in the circuit. It follows that at $\omega = \omega_0$, $V_o = V_i$. In other words, the center-frequency gain of the bandpass filter is unity. Its transfer function can be obtained as follows:

$$T(s) = \frac{Y_R}{Y_R + Y_L + Y_C} = \frac{1/R}{(1/R) + (1/sL) + sC} \tag{16.39}$$

$$= \frac{s(1/CR)}{s^2 + s(1/CR) + (1/LC)}$$

16.5.6 Realization of the Notch Functions

To obtain a pair of transmission zeros on the $j\omega$ axis, we use a parallel resonance circuit in the series arm, as shown in Fig. 16.18(e). Observe that this circuit is obtained by disconnecting both nodes x and y from ground and connecting them together to V_i. The impedance of the LC circuit becomes infinite at $\omega = \omega_0 = 1/\sqrt{LC}$, thus causing zero transmission at this frequency. The shunt impedance is resistive and thus does not introduce transmission zeros. It follows that the circuit in Fig. 16.18(e) will realize the notch transfer function

$$T(s) = a_2 \frac{s^2 + \omega_0^2}{s^2 + s(\omega_0/Q) + \omega_0^2} \tag{16.40}$$

The value of the high-frequency gain a_2 can be found from the circuit to be unity.

To obtain a notch-filter realization in which the notch frequency ω_n is arbitrarily placed relative to ω_0, we adopt a variation on the scheme above. We still use a parallel LC circuit in the series branch, as shown in Fig. 16.18(f) where L_1 and C_1 are selected so that

$$L_1 C_1 = 1/\omega_n^2 \tag{16.41}$$

Thus the L_1C_1 tank circuit will introduce a pair of transmission zeros at $\pm j\omega_n$, provided the L_2C_2 tank is not resonant at ω_n. Apart from this restriction, the values of L_2 and C_2 must be selected to ensure that the natural modes have not been altered; thus,

$$C_1 + C_2 = C \tag{16.42}$$

$$L_1 \| L_2 = L \tag{16.43}$$

In other words, when V_i is replaced by a short circuit, the circuit should reduce to the original LCR resonator. Another way of thinking about the circuit of Fig. 16.18(f) is that it is obtained from the original LCR resonator by lifting part of L and part of C off ground and connecting them to V_i.

It should be noted that in the circuit of Fig. 16.18(f), L_2 does *not* introduce a zero at $s = 0$ because at $s = 0$, the L_1C_1 circuit also has a zero. In fact, at $s = 0$ the circuit reduces to an inductive voltage divider with the dc transmission being $L_2/(L_1 + L_2)$. Similar comments can be made about C_2 and the fact that it does *not* introduce a zero at $s = \infty$.

The LPN and HPN filter realizations are special cases of the general notch circuit of Fig. 16.18(f). Specifically, for the LPN,

$$\omega_n > \omega_0$$

and thus

$$L_1 C_1 < (L_1 \| L_2)(C_1 + C_2)$$

This condition can be satisfied with L_2 eliminated (i.e., $L_2 = \infty$ and $L_1 = L$), resulting in the LPN circuit in Fig. 16.18(g). The transfer function can be written by inspection as

$$T(s) \equiv \frac{V_o}{V_i} = a_2 \frac{s^2 + \omega_n^2}{s^2 + s(\omega_0/Q) + \omega_0^2} \tag{16.44}$$

where $\omega_n^2 = 1/LC_1$, $\omega_0^2 = 1/L(C_1 + C_2)$, $\omega_0/Q = 1/CR$, and a_2 is the high-frequency gain. From the circuit we see that as $s \to \infty$, the circuit reduces to that in Fig. 16.18(h), for which

$$\frac{V_o}{V_i} = \frac{C_1}{C_1 + C_2}$$

Thus,

$$a_2 = \frac{C_1}{C_1 + C_2} \tag{16.45}$$

To obtain an HPN realization we start with the circuit of Fig. 16.18(f) and use the fact that $\omega_n < \omega_0$ to obtain

$$L_1C_1 > (L_1 \| L_2)(C_1 + C_2)$$

which can be satisfied while selecting $C_2 = 0$ (i.e., $C_1 = C$). Thus we obtain the reduced circuit shown in Fig. 16.18(i). Observe that as $s \to \infty$, V_o approaches V_i and thus the high-frequency gain is unity. Thus, the transfer function can be expressed as

$$T(s) \equiv \frac{V_o}{V_i} = \frac{s^2 + (1/L_1C)}{s^2 + s(1/CR) + [1/(L_1 \| L_2)C]} \tag{16.46}$$

16.5.7 Realization of the All-Pass Function

The all-pass transfer function

$$T(s) = \frac{s^2 - s(\omega_0/Q) + \omega_0^2}{s^2 + s(\omega_0/Q) + \omega_0^2} \tag{16.47}$$

can be written as

$$T(s) = 1 - \frac{s2(\omega_0/Q)}{s^2 + s(\omega_0/Q) + \omega_0^2} \tag{16.48}$$

The second term on the right-hand side is a bandpass function with a center-frequency gain of 2. We already have a bandpass circuit (Fig. 16.18d), but with a center-frequency gain of unity. We shall therefore attempt an all-pass realization with a flat gain of 0.5, that is,

$$T(s) = 0.5 - \frac{s(\omega_0/Q)}{s^2 + s(\omega_0/Q) + \omega_0^2}$$

This function can be realized using a voltage divider with a transmission ratio of 0.5 together with the bandpass circuit of Fig. 16.18(d). To effect the subtraction, the output of the all-pass circuit is taken between the output terminal of the voltage divider and that of the bandpass filter, as shown in Fig. 16.19. Unfortunately this circuit has the disadvantage of lacking a common ground terminal between the input and the output. An op amp–RC realization of the all-pass function will be presented in the next section.

EXERCISES

16.17 Use the circuit of Fig. 16.18(b) to realize a second-order low-pass function of the maximally flat type with a 3-dB frequency of 100 kHz.
Ans. Selecting $R = 1$ kΩ, we obtain $C = 1125$ pF and $L = 2.25$ mH.

16.18 Use the circuit of Fig. 16.18(e) to design a notch filter to eliminate a bothersome power-supply hum at a 60-Hz frequency. The filter is to have a 3-dB bandwidth of 10 Hz (i.e., the attenuation is greater than 3 dB over a 10-Hz band around the 60-Hz center frequency; see Exercise 16.15 and Fig. 16.16d). Use $R = 10$ kΩ.
Ans. $C = 1.6$ μF and $L = 4.42$ H (Note the large inductor required. This is the reason passive filters are not practical in low-frequency applications.)

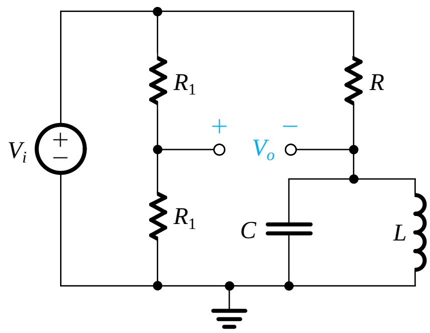


Figure 16.19 Realization of the second-order all-pass transfer function using a voltage divider and an LCR resonator.

16.6 Second-Order Active Filters Based on Inductor Replacement

In this section, we study a family of op amp–RC circuits that realize the various second-order filter functions. The circuits are based on an op amp–RC resonator obtained by replacing the inductor L in the LCR resonator with an op amp–RC circuit that has an inductive input impedance.

16.6.1 The Antoniou Inductance-Simulation Circuit

Over the years, many op amp–RC circuits have been proposed for simulating the operation of an inductor. Of these, one circuit invented by A. Antoniou[5] (see Antoniou, 1969) has proved to be the "best." By "best" we mean that the operation of the circuit is very tolerant of the nonideal properties of the op amps, in particular their finite gain and bandwidth. Figure 16.20(a) shows the Antoniou inductance-simulation circuit. If the circuit is fed at its input (node 1) with a voltage source V_1 and the input current is denoted I_1, then for ideal op amps the input impedance can be shown to be

$$Z_{in} \equiv V_1/I_1 = sC_4R_1R_3R_5/R_2 \tag{16.49}$$

which is that of an inductance L given by

$$L = C_4R_1R_3R_5/R_2 \tag{16.50}$$

Figure 16.20(b) shows the analysis of the circuit assuming that the op amps are ideal and thus that a virtual short circuit appears between the two input terminals of each op amp, and assuming also that the input currents of the op amps are zero. The analysis begins at node 1, which is assumed to be fed by a voltage source V_1, and proceeds step by step, with the order of the steps indicated by the circled numbers. The result of the analysis is the expression shown for the input current I_1 from which Z_{in} is found.

The design of this circuit is usually based on selecting $R_1 = R_2 = R_3 = R_5 = R$ and $C_4 = C$, which leads to $L = CR^2$. Convenient values are then selected for C and R to yield the

[5] Andreas Antoniou is a Canadian academic, currently (2009) a member of the faculty of the University of Victoria, British Columbia.

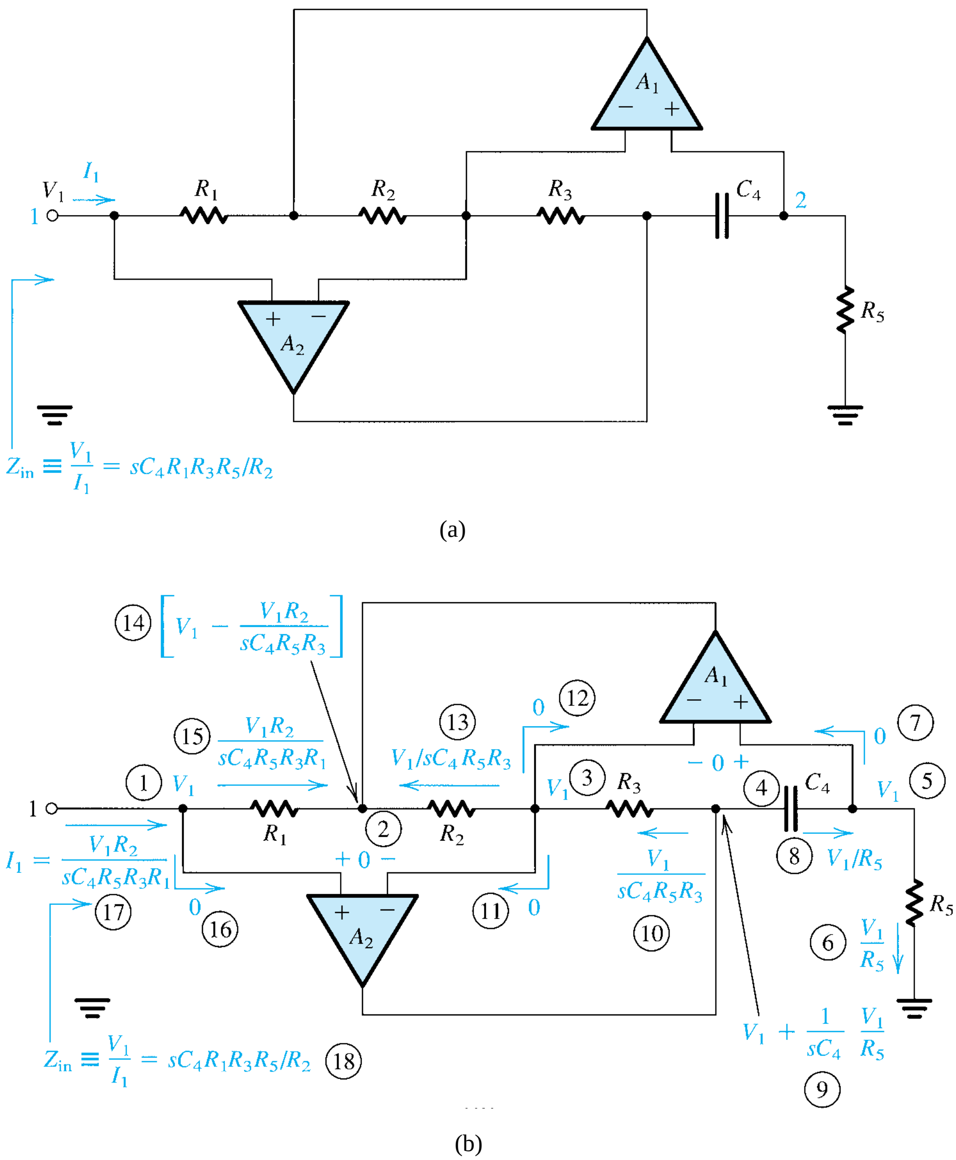


Figure 16.20 **(a)** The Antoniou inductance-simulation circuit. **(b)** Analysis of the circuit assuming ideal op amps. The order of the analysis steps is indicated by the circled numbers.

desired inductance value L. More details on this circuit and the effect of the nonidealities of the op amps on its performance can be found in Sedra and Brackett (1978).

16.6.2 The Op Amp–RC Resonator

Figure 16.21(a) shows the LCR resonator we studied in detail in Section 16.5. Replacing the inductor L with a simulated inductance realized by the Antoniou circuit of Fig. 16.20(a) results in the op amp–RC resonator of Fig. 16.21(b). (Ignore for the moment the additional

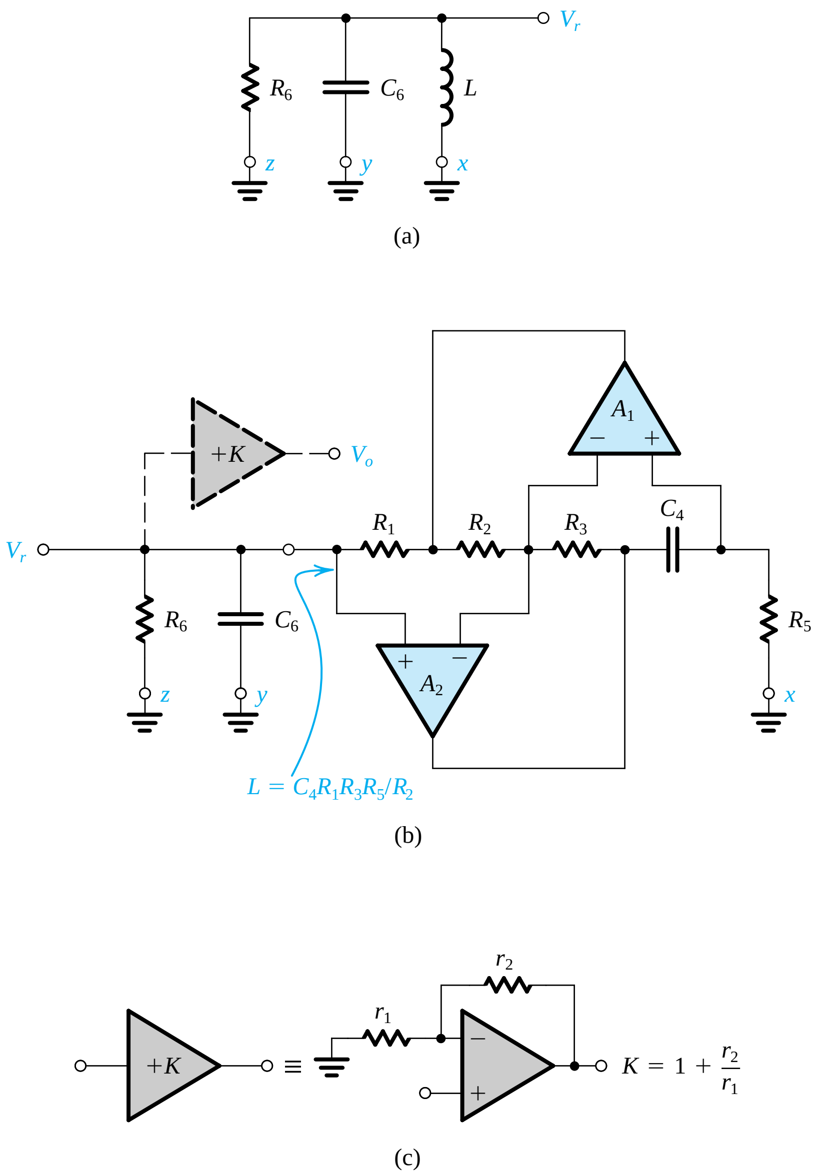


Figure 16.21 **(a)** An LCR resonator. **(b)** An op amp–RC resonator obtained by replacing the inductor L in the LCR resonator of **(a)** with a simulated inductance realized by the Antoniou circuit of Fig. 16.20(a). **(c)** Implementation of the buffer amplifier K.

amplifier drawn with broken lines.) The circuit of Fig. 16.21(b) is a second-order resonator having a pole frequency

$$\omega_0 = 1/\sqrt{LC_6} = 1/\sqrt{C_4C_6R_1R_3R_5/R_2} \tag{16.51}$$

where we have used the expression for L given in Eq. (16.50). The pole Q factor can be obtained using the expressent in Eq. (16.35) with $C = C_6$ and $R = R_6$; thus, $Q = \omega_0 C_6 R_6$. Replacing ω_0 by the expression in Eq. (16.51) gives

$$Q = \omega_0 C_6 R_6 = R_6 \sqrt{\frac{C_6}{C_4} \frac{R_2}{R_1 R_3 R_5}} \tag{16.52}$$

Usually one selects $C_4 = C_6 = C$ and $R_1 = R_2 = R_3 = R_5 = R$, which results in

$$\omega_0 = 1/CR \tag{16.53}$$

$$Q = R_6/R \tag{16.54}$$

Thus, if we select a practically convenient value for C, we can use Eq. (16.53) to determine the value of R to realize a given ω_0, and then use Eq. (16.54) to determine the value of R_6 to realize a given Q.

16.6.3 Realization of the Various Filter Types

The op amp–RC resonator of Fig. 16.21(b) can be used to generate circuit realizations for the various second-order filter functions by following the approach described in detail in Section 16.5 in connection with the LCR resonator. Thus to obtain a bandpass function, we disconnect node z from ground and connect it to the signal source V_i. A high-pass function is obtained by injecting V_i to node y. To realize a low-pass function using the LCR resonator, the inductor terminal x is disconnected from ground and connected to V_i. The corresponding node in the active resonator is the node at which R_5 is connected to ground,[6] labeled as node x in Fig. 16.21(b). A regular notch function ($\omega_n = \omega_0$) is obtained by feeding V_i to nodes x and y. In all cases the output can be taken as the voltage across the resonance circuit, V_r. However, this is not a convenient node to use as the filter output terminal because connecting a load there would change the filter characteristics. The problem can be solved easily by utilizing a buffer amplifier. This is the amplifier of gain K, drawn with broken lines in Fig. 16.21(b). Figure 16.21(c) shows how this amplifier can be simply implemented using an op amp connected in the noninverting configuration. Note that not only does the amplifier K buffer the output of the filter, but it also allows the designer to set the filter gain to any desired value by appropriately selecting the value of K.

Figure 16.22 shows the various second-order filter circuits obtained from the resonator of Fig. 16.21(b). The transfer functions and design equations for these circuits are given in Table 16.1. Note that the transfer functions can be written by analogy to those of the LCR resonator. We have already commented on the LP, HP, BP, and regular-notch circuits given in Fig. 16.22(a) to (d). The LPN and HPN circuits in Fig. 16.22(e) and (f) are obtained by

[6] This point might not be obvious! The reader, however, can show by direct analysis that when V_i is fed to this node, the function V_r/V_i is indeed low pass.

direct analogy to their LCR counterparts in Fig. 16.18(g) and (i), respectively. The all-pass circuit in Fig. 16.22(g), however, deserves some explanation.

16.6.4 The All-Pass Circuit

From Eq. (16.48) we see that an all-pass function with a flat gain of unity can be written as

$$\text{AP} = 1 - (\text{BP with a center-frequency gain of 2}) \tag{16.55}$$

Two circuits whose transfer functions are related in this fashion are said to be **complementary**.[7] Thus the all-pass circuit with unity flat gain is the complement of the bandpass circuit with a center-frequency gain of 2. A simple procedure exists for obtaining the complement of a given linear circuit: Disconnect all the circuit nodes that are connected to ground and connect them to V_i, and disconnect all the nodes that are connected to V_i and connect them to ground. That is, *interchanging input and ground in a linear circuit generates a circuit whose transfer function is the complement of that of the original circuit*.

Returning to the problem at hand, we first use the circuit of Fig. 16.22(c) to realize a BP with a gain of 2 by simply selecting $K = 2$ and implementing the buffer amplifier with the circuit of Fig. 16.21(c) with $r_1 = r_2$. We then interchange input and ground and thus obtain the all-pass circuit of Fig. 16.22(g).

Finally, in addition to being simple to design, the circuits in Fig. 16.22 exhibit excellent performance. They can be used on their own to realize second-order filter functions, or they can be cascaded to implement high-order filters.

EXERCISES

D16.19 Use the circuit of Fig. 16.22(c) to design a second-order bandpass filter with a center frequency of 10 kHz, a 3-dB bandwidth of 500 Hz, and a center-frequency gain of 10. Use $C = 1.2$ nF.
Ans. $R_1 = R_2 = R_3 = R_5 = 13.26$ kΩ; $R_6 = 265$ kΩ; $C_4 = C_6 = 1.2$ nF; $K = 10$, $r_1 = 10$ kΩ, $r_2 = 90$ kΩ

D16.20 Realize the Chebyshev filter of Example 16.2, whose transfer function is given in Eq. (16.25), as the cascade connection of three circuits: two of the type shown in Fig. 16.22(a) and one first-order op amp–RC circuit of the type shown in Fig. 16.13(a). Note that you can make the dc gain of all sections equal to unity. Do so. Use as many 10-kΩ resistors as possible.
Ans. First-order section: $R_1 = R_2 = 10$ kΩ, $C = 5.5$ nF; second-order section with $\omega_0 = 4.117 \times 10^4$ rad/s and $Q = 1.4$: $R_1 = R_2 = R_3 = R_5 = 10$ kΩ, $R_6 = 14$ kΩ, $C_4 = C_6 = 2.43$ nF, $r_1 = \infty$, $r_2 = 0$; second-order section with $\omega_0 = 6.246 \times 10^4$ rad/s and $Q = 5.56$: $R_1 = R_2 = R_3 = R_5 = 10$ kΩ, $R_6 = 55.6$ kΩ, $C_4 = C_6 = 1.6$ nF, $r_1 = \infty$, $r_2 = 0$

[7] More about complementary circuits will be presented later in conjunction with Fig. 16.31.

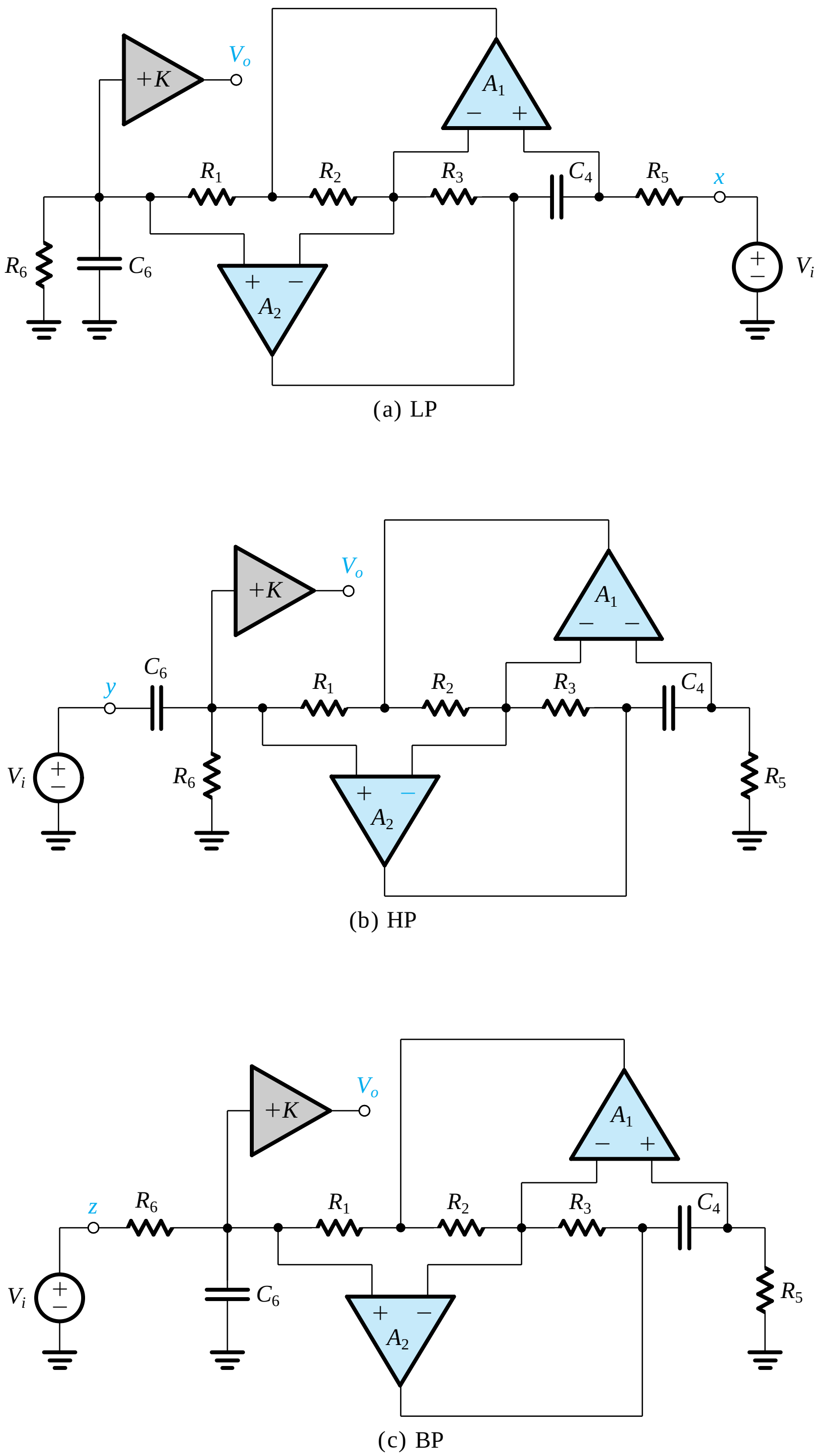
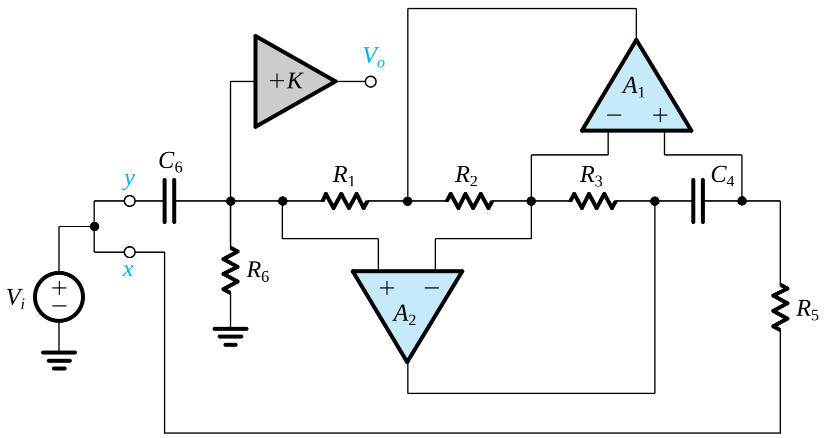
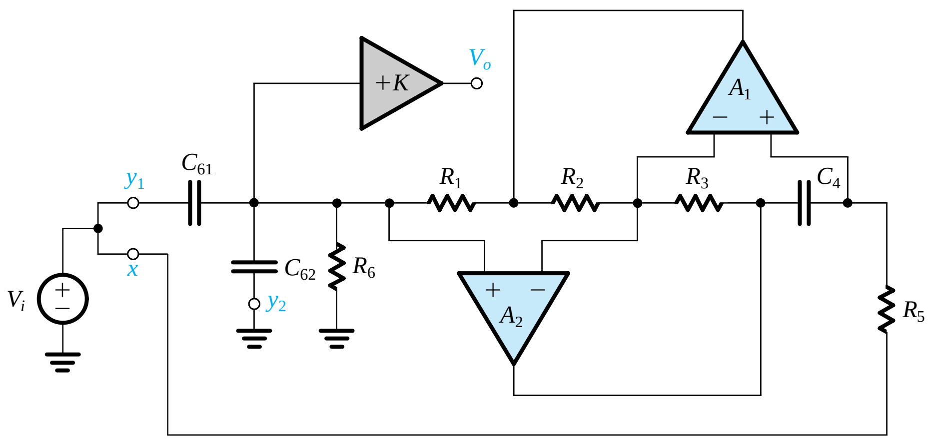


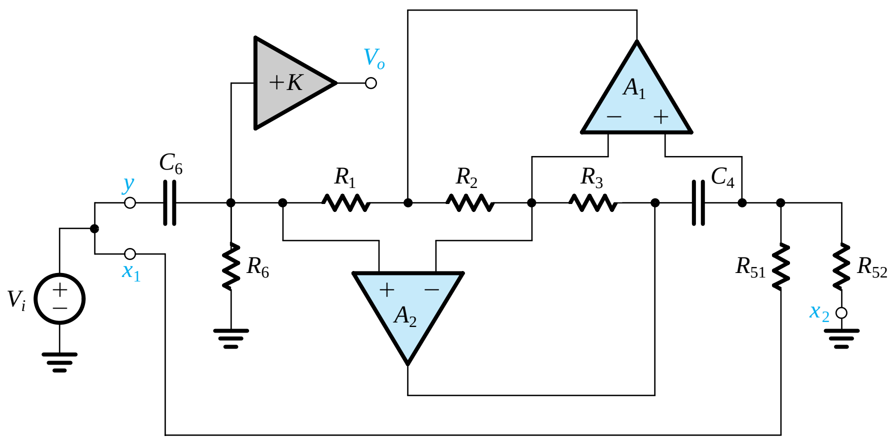
Figure 16.22 Realizations for the various second-order filter functions using the op amp–RC resonator of Fig. 16.21(b): **(a)** LP, **(b)** HP, **(c)** BP. The circuits are based on the LCR circuit in Fig. 16.18. Design considerations are given in Table 16.1.



(d) Notch at ω_0



(e) LPN, $\omega_n \geq \omega_0$



(f) HPN, $\omega_n \leq \omega_0$

Figure 16.22 (*continued*) **(d)** Notch at ω_0, **(e)** LPN, $\omega_n \geq \omega_0$, **(f)** HPN, $\omega_n \leq \omega_0$.

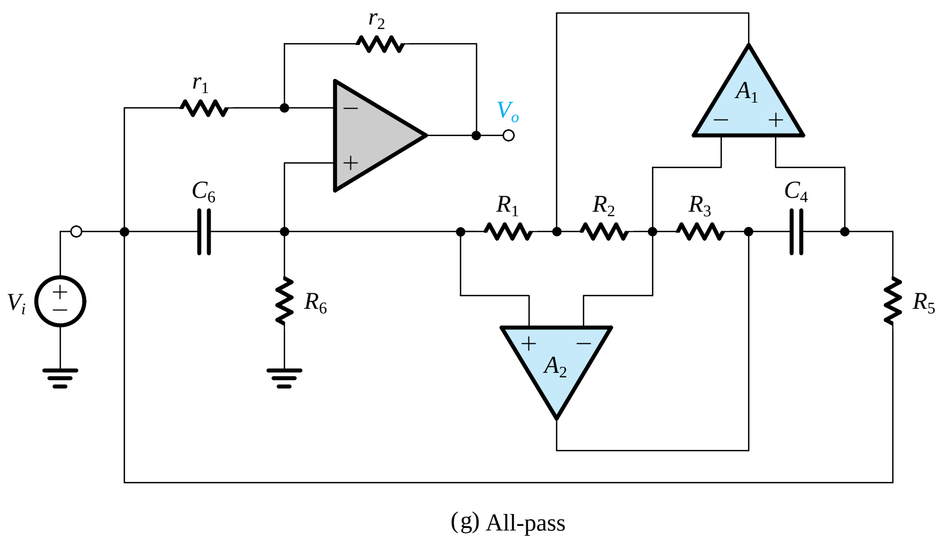


Figure 16.22 (*continued*) (g) All pass.

Table 16.1 Design Data for the Circuits of Fig. 16.22

Circuit	Transfer Function and Other Parameters	Design Equations
Resonator Fig. 16.21(b)	$\omega_0 = 1/\sqrt{C_4C_6R_1R_3R_5/R_2}$ $Q = R_6\sqrt{\frac{C_6}{C_4}\frac{R_2}{R_1R_3R_5}}$	$C_4 = C_6 = C$ (practical value) $R_1 = R_2 = R_3 = R_5 = 1/\omega_0 C$ $R_6 = Q/\omega_0 C$
Low-pass (LP) Fig. 16.22(a)	$T(s) = \dfrac{KR_2/C_4C_6R_1R_3R_5}{s^2 + s\frac{1}{C_6R_6} + \frac{R_2}{C_4C_6R_1R_3R_5}}$	K = DC gain
High-pass (HP) Fig. 16.22(b)	$T(s) = \dfrac{Ks^2}{s^2 + s\frac{1}{C_6R_6} + \frac{R_2}{C_4C_6R_1R_3R_5}}$	K = High-frequency gain
Bandpass (BP) Fig. 16.22(c)	$T(s) = \dfrac{Ks/C_6R_6}{s^2 + s\frac{1}{C_6R_6} + \frac{R_2}{C_4C_6R_1R_3R_5}}$	K = Center-frequency gain
Regular notch (N) Fig. 16.22(d)	$T(s) = \dfrac{K[s^2 + (R_2/C_4C_6R_1R_3R_5)]}{s^2 + s\frac{1}{C_6R_6} + \frac{R_2}{C_4C_6R_1R_3R_5}}$	K = Low- and high-frequency gain

Low-pass notch (LPN) Fig. 16.22(e)	$T(s) = K\dfrac{C_{61}}{C_{61}+C_{62}} \times \dfrac{s^2 + (R_2/C_4C_{61}R_1R_3R_5)}{s^2 + s\dfrac{1}{(C_{61}+C_{62})R_6} + \dfrac{R_2}{C_4(C_{61}+C_{62})R_1R_3R_5}}$ $\omega_n = 1/\sqrt{C_4C_{61}R_1R_3R_5/R_2}$ $\omega_0 = 1/\sqrt{C_4(C_{61}+C_{62})R_1R_3R_5/R_2}$ $Q = R_6\sqrt{\dfrac{C_{61}+C_{62}}{C_4}\dfrac{R_2}{R_1R_3R_5}}$	K = DC gain $C_{61}+C_{62} = C_6 = C$ $C_{61} = C(\omega_0/\omega_n)^2$ $C_{62} = C - C_{61}$
High-pass notch (HPN) Fig. 16.22(f)	$T(s) = K\dfrac{s^2 + (R_2/C_4C_6R_1R_3R_{51})}{s^2 + s\dfrac{1}{C_6R_6} + \dfrac{R_2}{C_4C_6R_1R_3}\left(\dfrac{1}{R_{51}}+\dfrac{1}{R_{52}}\right)}$ $\omega_n = 1/\sqrt{C_4C_6R_1R_3R_{51}/R_2}$ $\omega_0 = \sqrt{\dfrac{R_2}{C_4C_6R_1R_3}\left(\dfrac{1}{R_{51}}+\dfrac{1}{R_{52}}\right)}$ $Q = R_6\sqrt{\dfrac{C_6}{C_4}\dfrac{R_2}{R_1R_3}\left(\dfrac{1}{R_{51}}+\dfrac{1}{R_{52}}\right)}$	K = High-frequency gain $\dfrac{1}{R_{51}}+\dfrac{1}{R_{52}} = \dfrac{1}{R_5} = \omega_0 C$ $R_{51} = R_5(\omega_0/\omega_n)^2$ $R_{52} = R_5/[1-(\omega_n/\omega_0)^2]$
All-pass (AP) Fig. 16.22(g)	$T(s) = \dfrac{s^2 - s\dfrac{1}{C_6R_6}\dfrac{r_2}{r_1} + \dfrac{R_2}{C_4C_6R_1R_3R_5}}{s^2 + s\dfrac{1}{C_6R_6} + \dfrac{R_2}{C_4C_6R_1R_3R_5}}$ $\omega_z = \omega_0 \quad Q_z = Q(r_1/r_2) \quad$ Flat gain $= 1$	$r_1 = r_2 = r$ (arbitrary) Adjust r_2 to make $Q_z = Q$

16.7 Second-Order Active Filters Based on the Two-Integrator-Loop Topology

In this section, we study another family of op amp–RC circuits that realize second-order filter functions. The circuits are based on the use of two integrators connected in cascade in an overall feedback loop and are thus known as two-integrator-loop circuits.

16.7.1 Derivation of the Two-Integrator-Loop Biquad

To derive the two-integrator-loop biquadratic circuit, or **biquad** as it is commonly known,[8] consider the second-order high-pass transfer function

$$\frac{V_{hp}}{V_i} = \frac{Ks^2}{s^2 + s(\omega_0/Q) + \omega_0^2} \tag{16.56}$$

[8] The name biquad stems from the fact that this circuit in its most general form is capable of realizing a biquadratic transfer function, that is, one that is the ratio of two quadratic polynomials.

where K is the high-frequency gain. Cross-multiplying Eq. (16.56) and dividing both sides of the resulting equation by s^2 (to get all the terms involving s in the form $1/s$, which is the transfer function of an integrator) gives

$$V_{hp} + \frac{1}{Q}\left(\frac{\omega_0}{s}V_{hp}\right) + \left(\frac{\omega_0^2}{s^2}V_{hp}\right) = KV_i \tag{16.57}$$

In this equation we observe that the signal $(\omega_0/s)V_{hp}$ can be obtained by passing V_{hp} through an integrator with a time constant equal to $1/\omega_0$. Furthermore, passing the resulting signal through another identical integrator results in the third signal involving V_{hp} in Eq. (16.57)—namely, $(\omega_0^2/s^2)V_{hp}$. Figure 16.23(a) shows a block diagram for such a two-integrator arrangement. Note that in anticipation of the use of the inverting op-amp Miller integrator circuit (Section 2.5.2) to implement each integrator, the integrator blocks in Fig. 16.23(a) have been assigned negative signs.

The problem still remains, however, of how to form V_{hp}, the input signal feeding the two cascaded integrators. Toward that end, we rearrange Eq. (16.57), expressing V_{hp} in terms of its single- and double-integrated versions and of V_i as

$$V_{hp} = KV_i - \frac{1}{Q}\frac{\omega_0}{s}V_{hp} - \frac{\omega_0^2}{s^2}V_{hp} \tag{16.58}$$

which suggests that V_{hp} can be obtained by using the weighted summer of Fig. 16.23(b). Now it should be easy to see that a complete block diagram realization can be obtained by combining the integrator blocks of Fig. 16.23(a) with the summer block of Fig. 16.23(b), as shown in Fig. 16.23(c).

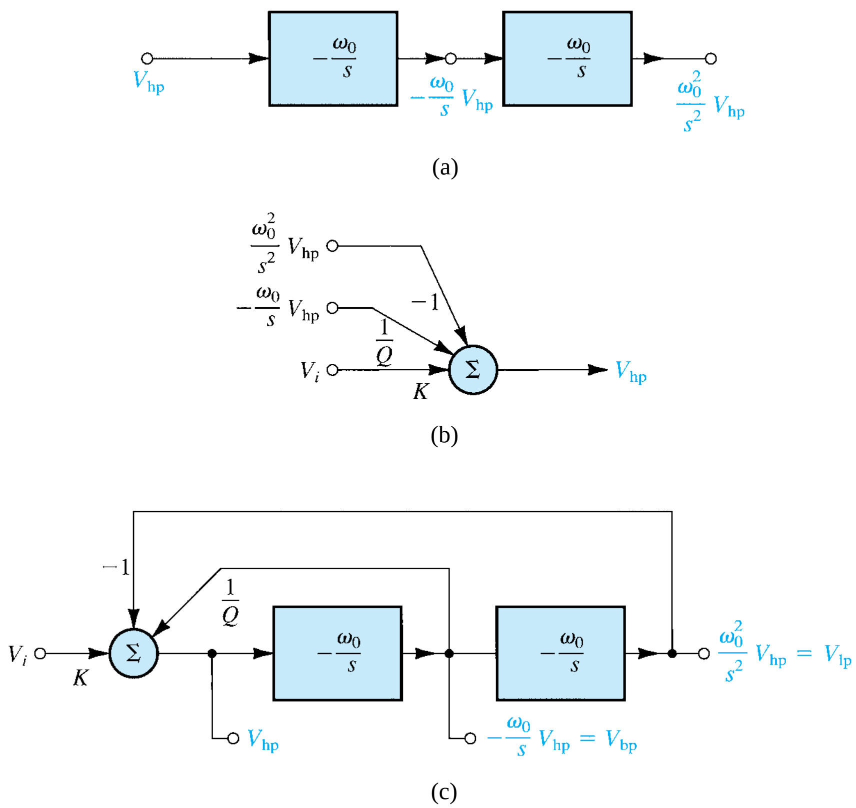


Figure 16.23 Derivation of a block diagram realization of the two-integrator-loop biquad.

In the realization of Fig. 16.23(c), V_{hp}, obtained at the output of the summer, realizes the high-pass transfer function $T_{hp} \equiv V_{hp}/V_i$ of Eq. (16.56). The signal at the output of the first integrator is $-(\omega_0/s)V_{hp}$, which is a bandpass function,

$$\frac{(-\omega_0/s)V_{hp}}{V_i} = -\frac{K\omega_0 s}{s^2 + s(\omega_0/Q) + \omega_0^2} = T_{bp}(s) \tag{16.59}$$

Therefore the signal at the output of the first integrator is labeled V_{bp}. Note that the center-frequency gain of the bandpass filter realized is equal to $-KQ$.

In a similar fashion, we can show that the transfer function realized at the output of the second integrator is the low-pass function,

$$\frac{(\omega_0^2/s^2)V_{hp}}{V_i} = \frac{K\omega_0^2}{s^2 + s(\omega_0/Q) + \omega_0^2} = T_{lp}(s) \tag{16.60}$$

Thus the output of the second integrator is labeled V_{lp}. Note that the dc gain of the low-pass filter realized is equal to K.

We conclude that the two-integrator-loop biquad shown in block diagram form in Fig. 16.23(c) realizes the three basic second-order filtering functions, LP, BP, and HP, *simultaneously*. This versatility has made the circuit very popular and has given it the name *universal active filter*.

16.7.2 Circuit Implementation

To obtain an op-amp circuit implementation of the two-integrator-loop biquad of Fig. 16.23(c), we replace each integrator with a Miller integrator circuit having $CR = 1/\omega_0$, and we replace the summer block with an op-amp summing circuit that is capable of assigning both positive and negative weights to its inputs. The resulting circuit, known as the Kerwin–Huelsman–Newcomb or **KHN biquad**, after its inventors, is shown in Fig. 16.24(a). Given values for ω_0, Q, and K, the design of the circuit is straightforward: We select suitably practical values for the components of the integrators C and R so that $CR = 1/\omega_0$. To determine the values of the resistors associated with the summer, we first use *superposition* to express the output of the summer V_{hp} in terms of its inputs, V_i, V_{bp} and V_{lp} as

$$V_{hp} = V_i\frac{R_3}{R_2 + R_3}\left(1 + \frac{R_f}{R_1}\right) + V_{bp}\frac{R_2}{R_2 + R_3}\left(1 + \frac{R_f}{R_1}\right) - V_{lp}\frac{R_f}{R_1}$$

Substituting $V_{bp} = -(\omega_0/s)V_{hp}$ and $V_{lp} = (\omega_0^2/s^2)V_{hp}$ gives

$$V_{hp} = \frac{R_3}{R_2 + R_3}\left(1 + \frac{R_f}{R_1}\right)V_i + \frac{R_2}{R_2 + R_3}\left(1 + \frac{R_f}{R_1}\right)\left(-\frac{\omega_0}{s}V_{hp}\right) - \frac{R_f}{R_1}\left(\frac{\omega_0^2}{s^2}V_{hp}\right) \tag{16.61}$$

Equating the last right-hand-side terms of Eqs. (16.61) and (16.58) gives

$$R_f/R_1 = 1 \tag{16.62}$$

which implies that we can select arbitrary but practically convenient equal values for R_1 and R_f. Then, equating the second-to-last terms on the right-hand side of Eqs. (16.61) and (16.58) and setting $R_1 = R_f$ yields the ratio R_3/R_2 required to realize a given Q as

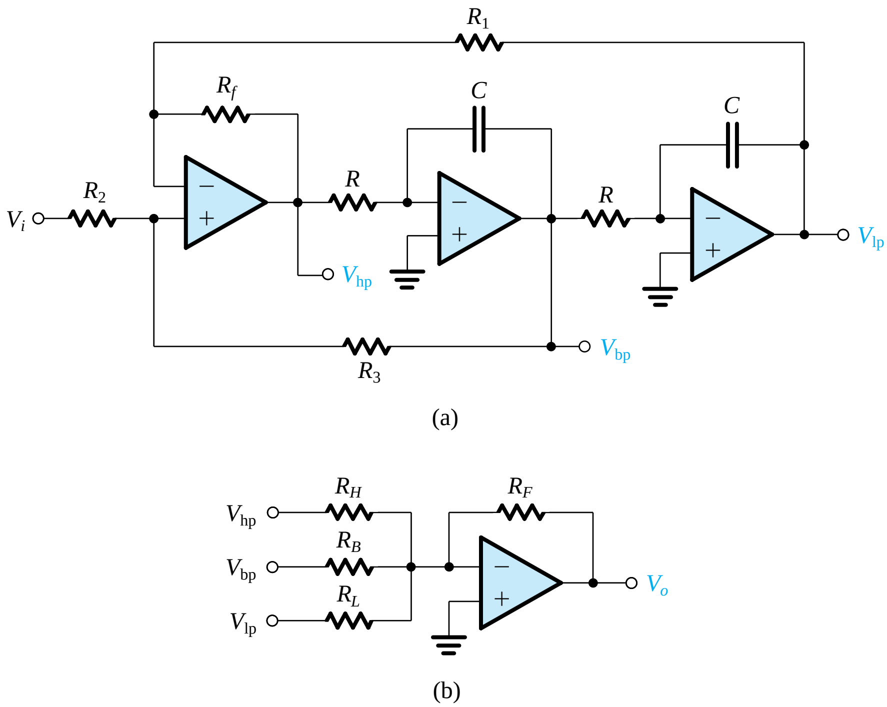


Figure 16.24 **(a)** The KHN biquad circuit, obtained as a direct implementation of the block diagram of Fig. 16.23(c). The three basic filtering functions, HP, BP, and LP, are simultaneously realized. **(b)** To obtain notch and all-pass functions, the three outputs are summed with appropriate weights using this op-amp summer.

$$R_3/R_2 = 2Q - 1 \tag{16.63}$$

Thus an arbitrary but convenient value can be selected for either R_2 or R_3, and the value of the other resistance can be determined using Eq. (16.63). Finally, equating the coefficients of V_i in Eqs. (16.61) and (16.58) and substituting $R_f = R_1$ and for R_3/R_2 from Eq. (16.63) results in

$$K = 2 - (1/Q) \tag{16.64}$$

Thus the gain parameter K is fixed to this value.

The KHN biquad can be used to realize notch and all-pass functions by summing weighted versions of the three outputs, LP, BP, and HP. Such an op-amp summer is shown in Fig. 16.24(b); for this summer we can write

$$\begin{aligned} V_o &= -\left(\frac{R_F}{R_H}V_{\text{hp}} + \frac{R_F}{R_B}V_{\text{bp}} + \frac{R_F}{R_L}V_{\text{lp}}\right) \\ &= -V_i\left(\frac{R_F}{R_H}T_{\text{hp}} + \frac{R_F}{R_B}T_{\text{bp}} + \frac{R_F}{R_L}T_{\text{lp}}\right) \end{aligned} \tag{16.65}$$

Substituting for T_{hp}, T_{bp}, and T_{lp} from Eqs. (16.56), (16.59), and (16.60), respectively, gives the overall transfer function

$$\frac{V_o}{V_i} = -K\frac{(R_F/R_H)s^2 - s(R_F/R_B)\omega_0 + (R_F/R_L)\omega_0^2}{s^2 + s(\omega_0/Q) + \omega_0^2} \tag{16.66}$$

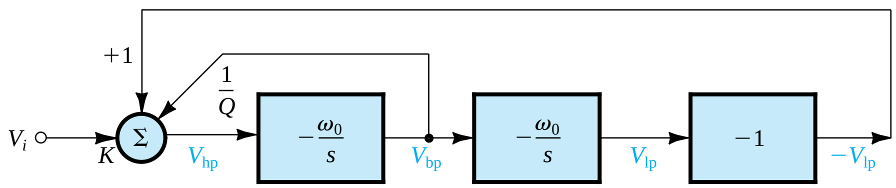
from which we can see that different transmission zeros can be obtained by the appropriate selection of the values of the summing resistors. For instance, a notch is obtained by selecting $R_B = \infty$ and

$$\frac{R_H}{R_L} = \left(\frac{\omega_n}{\omega_0}\right)^2 \tag{16.67}$$

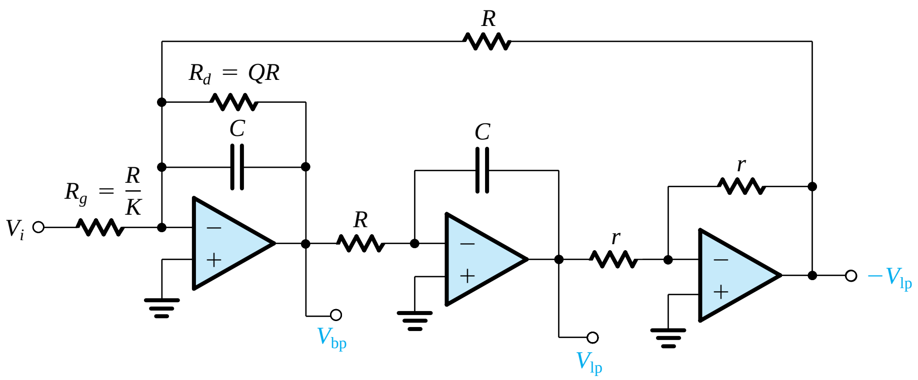
16.7.3 An Alternative Two-Integrator-Loop Biquad Circuit

An alternative two-integrator-loop biquad circuit in which all three op amps are used in a single-ended mode can be developed as follows: Rather than using the input summer to add signals with positive and negative coefficients, we can introduce an additional inverter, as shown in Fig. 16.25(a). Now all the coefficients of the summer have the same sign, and we can dispense with the summing amplifier altogether and perform the summation at the virtual-ground input of the first integrator. Observe that the summing weights of 1, $1/Q$, and K are realized by using resistances of R, QR, and R/K, respectively. The resulting circuit is shown in Fig. 16.25(b), from which we observe that the high-pass function is no longer available! This is the price paid for obtaining a circuit that utilizes all op amps in a single-ended mode. The circuit of Fig. 16.25(b) is known as the **Tow–Thomas biquad**, after its originators.

Rather than using a fourth op amp to realize the finite transmission zeros required for the notch and all-pass functions, as was done with the KHN biquad, an economical *feedforward* scheme can be employed with the Tow–Thomas circuit. Specifically, the virtual ground available at the input of each of the three op amps in the Tow–Thomas circuit permits the input signal to be fed to all three op amps, as shown in Fig. 16.26. If V_o is taken at the output of the damped integrator, straightforward analysis yields the filter transfer function



(a)



(b)

Figure 16.25 **(a)** Derivation of an alternative two-integrator-loop biquad in which all op amps are used in a single-ended fashion. **(b)** The resulting circuit, known as the Tow–Thomas biquad.

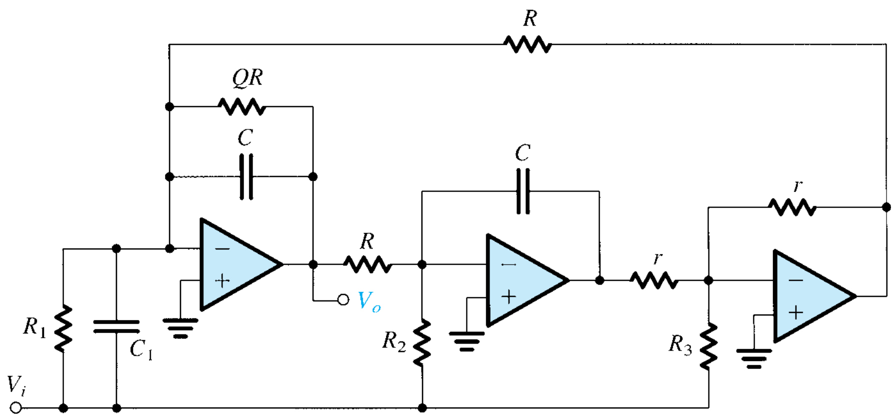


Figure 16.26 The Tow–Thomas biquad with feedforward. The transfer function of Eq. (16.68) is realized by feeding the input signal through appropriate components to the inputs of the three op amps. This circuit can realize all special second-order functions. The design equations are given in Table 16.2.

Table 16.2 Design Data for the Circuit in Fig. 16.26

All cases	$C =$ arbitrary, $R = 1/\omega_0 C$, $r =$ arbitrary
LP	$C_1 = 0$, $R_1 = \infty$, $R_2 = R$/dc gain, $R_3 = \infty$
Positive BP	$C_1 = 0$, $R_1 = \infty$, $R_2 = \infty$, $R_3 = Qr$/center-frequency gain
Negative BP	$C_1 = 0$, $R_1 = QR$/center-frequency gain, $R_2 = \infty$, $R_3 = \infty$
HP	$C_1 = C \times$ high-frequency gain, $R_1 = \infty$, $R_2 = \infty$, $R_3 = \infty$
Notch	$C_1 = C \times$ high-frequency gain, $R_1 = \infty$,
(all types)	$R_2 = R(\omega_0/\omega_n)^2$/high-frequency gain, $R_3 = \infty$
AP	$C_1 = C \times$ flat gain, $R_1 = \infty$, $R_2 = R$/gain, $R_3 = Qr$/gain

$$\frac{V_o}{V_i} = -\frac{s^2\left(\frac{C_1}{C}\right) + s\frac{1}{C}\left(\frac{1}{R_1} - \frac{r}{RR_3}\right) + \frac{1}{C^2 RR_2}}{s^2 + s\frac{1}{QCR} + \frac{1}{C^2R^2}} \tag{16.68}$$

which can be used to obtain the design data given in Table 16.2.

16.7.4 Final Remarks

Two-integrator-loop biquads are extremely versatile and easy to design. However, their performance is adversely affected by the finite bandwidth of the op amps. Special techniques exist for compensating the circuit for such effects [see the SPICE simulation example on the CD and the website, and Sedra and Brackett (1978)].

EXERCISES

D16.21 Design the KHN circuit to realize a high-pass function with $f_0 = 10$ kHz and $Q = 2$. Choose $C = 1$ nF. What is the value of high-frequency gain obtained? What is the center-frequency gain of the bandpass function that is simultaneously available at the output of the first integrator?
Ans. $R = 15.9$ kΩ; $R_1 = R_f = R_2 = 10$ kΩ (arbitrary); $R_3 = 30$ kΩ; 1.5; 3

D16.22 Use the KHN circuit together with an output summing amplifier to design a low-pass notch filter with $f_0 = 5$ kHz, $f_n = 8$ kHz, $Q = 5$, and a dc gain of 3. Select $C = 1$ nF and $R_L = 10$ kΩ.
Ans. $R = 31.83$ kΩ; $R_1 = R_f = R_2 = 10$ kΩ (arbitrary); $R_3 = 90$ kΩ; $R_H = 25.6$ kΩ; $R_F = 16.7$ kΩ; $R_B = \infty$

D16.23 Use the Tow–Thomas biquad (Fig. 16.25b) to design a second-order bandpass filter with $f_0 = 10$ kHz, $Q = 20$, and unity center-frequency gain. If $R = 10$ kΩ, give the values of C, R_d, and R_g.
Ans. 1.59 nF; 200 kΩ; 200 kΩ

D16.24 Use the data of Table 16.2 to design the biquad circuit of Fig. 16.26 to realize an all-pass filter with $\omega_0 = 10^4$ rad/s, $Q = 5$, and flat gain = 1. Use $C = 10$ nF and $r = 10$ kΩ.
Ans. $R = 10$ kΩ; Q-determining resistor = 50 kΩ; $C_1 = 10$ nF; $R_1 = \infty$; $R_2 = 10$ kΩ; $R_3 = 50$ kΩ

16.8 Single-Amplifier Biquadratic Active Filters

The op amp–RC biquadratic circuits studied in the two preceding sections provide good performance, are versatile, and are easy to design and to adjust (tune) after final assembly. Unfortunately, however, they are not economic in their use of op amps, requiring three or four amplifiers per second-order section. This can be a problem, especially in applications where power-supply current is to be conserved: for instance, in a battery-operated instrument. In this section we shall study a class of second-order filter circuits that requires only one op amp per biquad. These minimal realizations, however, suffer a greater dependence on the limited gain and bandwidth of the op amp and can also be more sensitive to the unavoidable tolerances in the values of resistors and capacitors than the multiple-op-amp biquads of the preceding sections. The **single-amplifier biquads** (SABs) are therefore limited to the less stringent filter specifications—for example, pole Q factors less than about 10.

The synthesis of SAB circuits is based on the use of feedback to move the poles of an RC circuit from the negative real axis, where they naturally lie, to the complex-conjugate locations required to provide selective filter response. The synthesis of SABs follows a two-step process:

1. Synthesis of a feedback loop that realizes a pair of complex-conjugate poles characterized by a frequency ω_0 and a Q factor Q.
2. Injecting the input signal in a way that realizes the desired transmission zeros.

16.8.1 Synthesis of the Feedback Loop

Consider the circuit shown in Fig. 16.27(a), which consists of a two-port RC network n placed in the negative-feedback path of an op amp. We shall assume that, except for having a finite gain A, the op amp is ideal. We shall denote by $t(s)$ the open-circuit voltage transfer

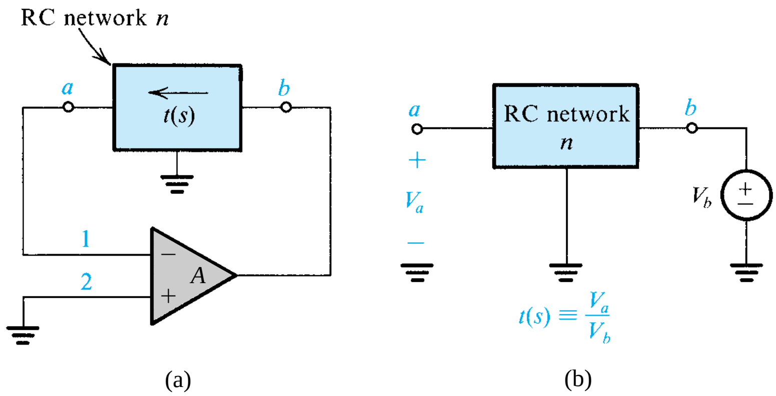


Figure 16.27 **(a)** Feedback loop obtained by placing a two-port RC network n in the feedback path of an op amp. **(b)** Definition of the open-circuit transfer function $t(s)$ of the RC network.

function of the RC network n, where the definition of $t(s)$ is illustrated in Fig. 16.27(b). The transfer function $t(s)$ can in general be written as the ratio of two polynomials $N(s)$ and $D(s)$:

$$t(s) = \frac{N(s)}{D(s)}$$

The roots of $N(s)$ are the transmission zeros of the RC network, and the roots of $D(s)$ are its poles. Study of circuit theory shows that while the poles of an RC network are restricted to lie on the negative real axis, the zeros can in general lie anywhere in the s plane.

The loop gain $L(s)$ of the feedback circuit in Fig. 16.27(a) can be determined using the method of Section 10.9. It is simply the product of the op-amp gain A and the transfer function $t(s)$,

$$L(s) = At(s) = \frac{AN(s)}{D(s)} \tag{16.69}$$

Substituting for $L(s)$ into the characteristic equation

$$1 + L(s) = 0 \tag{16.70}$$

results in the poles s_P of the closed-loop circuit obtained as solutions to the equation

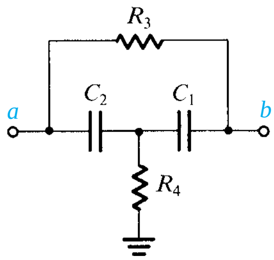
$$t(s_P) = -\frac{1}{A} \tag{16.71}$$

In the ideal case, $A = \infty$ and the poles are obtained from

$$N(s_P) = 0 \tag{16.72}$$

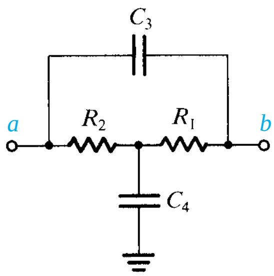
That is, *the filter poles are identical to the zeros of the RC network.*

Since our objective is to realize a pair of complex-conjugate poles, we should select an RC network that can have complex-conjugate transmission zeros. The simplest such networks are the bridged-T networks shown in Fig. 16.28 together with their transfer functions $t(s)$ from b to a, with a open-circuited. As an example, consider the circuit generated by placing the bridged-T network of Fig. 16.28(a) in the negative-feedback path of an op amp, as shown in Fig. 16.29. The pole polynomial of the active-filter circuit will be equal to the numerator polynomial of the bridged-T network; thus,



$$t(s) = \frac{s^2 + s\left(\frac{1}{C_1} + \frac{1}{C_2}\right)\frac{1}{R_3} + \frac{1}{C_1 C_2 R_3 R_4}}{s^2 + s\left(\frac{1}{C_1 R_3} + \frac{1}{C_2 R_3} + \frac{1}{C_1 R_4}\right) + \frac{1}{C_1 C_2 R_3 R_4}}$$

(a)



$$t(s) = \frac{s^2 + s\left(\frac{1}{R_1} + \frac{1}{R_2}\right)\frac{1}{C_4} + \frac{1}{C_3 C_4 R_1 R_2}}{s^2 + s\left(\frac{1}{C_4 R_1} + \frac{1}{C_4 R_2} + \frac{1}{C_3 R_2}\right) + \frac{1}{C_3 C_4 R_1 R_2}}$$

(b)

Figure 16.28 Two RC networks (called bridged-T networks) that can have complex transmission zeros. The transfer functions given are from b to a, with a open-circuited.

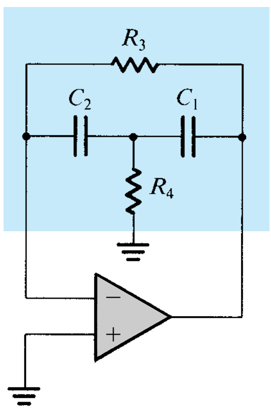


Figure 16.29 An active-filter feedback loop generated using the bridged-T network of Fig. 16.28(a).

$$s^2 + s\frac{\omega_0}{Q} + \omega_0^2 = s^2 + s\left(\frac{1}{C_1} + \frac{1}{C_2}\right)\frac{1}{R_3} + \frac{1}{C_1 C_2 R_3 R_4}$$

which enables us to obtain ω_0 and Q as

$$\omega_0 = \frac{1}{\sqrt{C_1 C_2 R_3 R_4}} \tag{16.73}$$

$$Q = \left[\frac{\sqrt{C_1 C_2 R_3 R_4}}{R_3}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)\right]^{-1} \tag{16.74}$$

If we are designing this circuit, ω_0 and Q are given and Eqs. (16.73) and (16.74) can be used to determine C_1, C_2, R_3, and R_4. It follows that there are two degrees of freedom. Let us exhaust one of these by selecting $C_1 = C_2 = C$. Let us also denote $R_3 = R$ and $R_4 = R/m$. By substituting in Eqs. (16.73) and (16.74), and with some manipulation, we obtain

$$m = 4Q^2 \tag{16.75}$$

$$CR = \frac{2Q}{\omega_0} \tag{16.76}$$

Thus if we are given the value of Q, Eq. (16.75) can be used to determine the ratio of the two resistances R_3 and R_4. Then the given values of ω_0 and Q can be substituted in Eq. (16.76) to determine the time constant CR. There remains one degree of freedom—the value of C or R can be arbitrarily chosen. In an actual design, this value, which sets the *impedance level* of the circuit, should be chosen so that the resulting component values are practical.

EXERCISES

D16.25 Design the circuit of Fig. 16.29 to realize a pair of poles with $\omega_0 = 10^4$ rad/s and $Q = 1$. Select $C_1 = C_2 = 1$ nF.
Ans. $R_3 = 200$ kΩ; $R_4 = 50$ kΩ

16.26 For the circuit designed in Exercise 16.25, find the location of the poles of the RC network in the feedback loop.
Ans. -0.382×10^4 and -2.618×10^4 rad/s

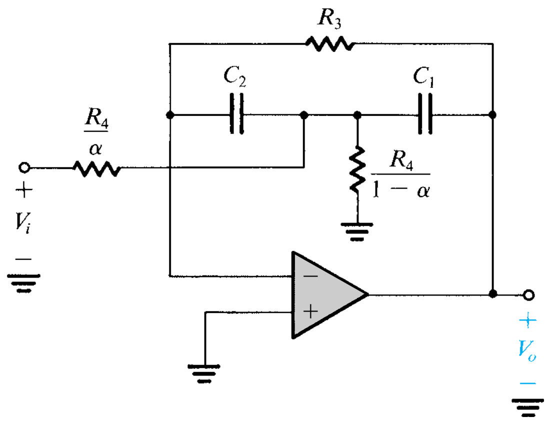
16.8.2 Injecting the Input Signal

Having synthesized a feedback loop that realizes a given pair of poles, we now consider connecting the input signal source to the circuit. We wish to do this, of course, without altering the poles.

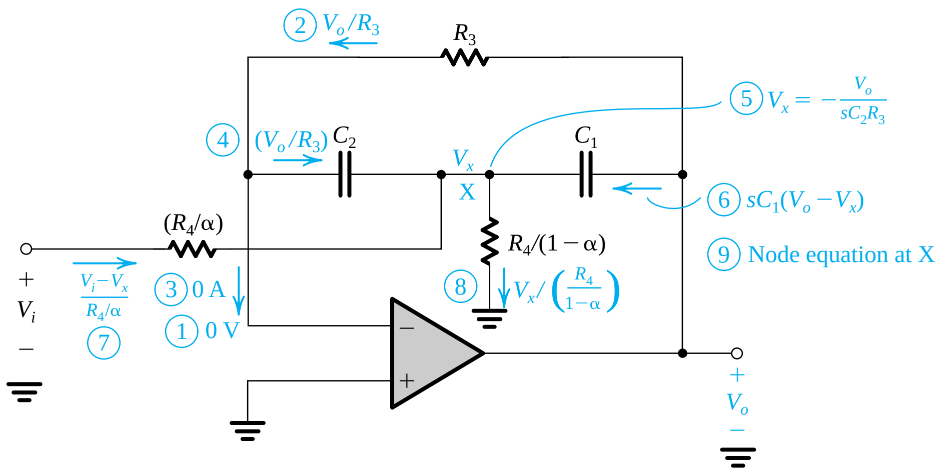
Since, for the purpose of finding the poles of a circuit, an ideal voltage source is equivalent to a short circuit, it follows that any circuit node that is connected to ground can instead be connected to the input voltage source without causing the poles to change. Thus the method of injecting the input voltage signal into the feedback loop is simply to disconnect a component (or several components) that is (are) connected to ground and connect it (them) to the input source. Depending on the component(s) through which the input signal is injected, different transmission zeros are obtained. This is, of course, the same method we used in Section 16.5 with the LCR resonator and in Section 16.6 with the biquads based on the LCR resonator.

As an example, consider the feedback loop of Fig. 16.29. Here we have two grounded nodes (one terminal of R_4 and the positive input terminal of the op amp) that can serve for injecting the input signal. Figure 16.30(a) shows the circuit with the input signal injected through part of the resistance R_4. Note that the two resistances R_4/α and $R_4/(1-\alpha)$ have a parallel equivalent of R_4.

Analysis of the circuit to determine its voltage transfer function $T(s) \equiv V_o(s)/V_i(s)$ is illustrated in Fig. 16.30(b). Note that we have assumed the op amp to be ideal, and have



(a)



(b)

Figure 16.30 **(a)** The feedback loop of Fig. 16.29 with the input signal injected through part of resistance R_4. This circuit realizes the bandpass function. **(b)** Analysis of the circuit in **(a)** to determine its voltage transfer function $T(s)$ with the order of the analysis steps indicated by the circled numbers.

indicated the order of the analysis steps by the circled numbers. The final step, number 9, consists of writing a node equation at X and substituting for V_x by the value determined in step 5. The result is the transfer function

$$\frac{V_o}{V_i} = \frac{-s(\alpha/C_1R_4)}{s^2 + s\left(\frac{1}{C_1} + \frac{1}{C_2}\right)\frac{1}{R_3} + \frac{1}{C_1C_2R_3R_4}}$$

We recognize this as a bandpass function whose center-frequency gain can be controlled by the value of α. As expected, the denominator polynomial is identical to the numerator polynomial of $t(s)$ given in Fig. 16.28(a).

EXERCISE

16.27 Use the component values obtained in Exercise 16.25 to design the bandpass circuit of Fig. 16.30(a). Determine the values of (R_4/α) and $R_4/(1-\alpha)$ to obtain a center-frequency gain of unity.
Ans. 100 kΩ; 100 kΩ

16.8.3 Generation of Equivalent Feedback Loops

The **complementary transformation** of feedback loops is based on the property of linear networks illustrated in Fig. 16.31 for the two-port (three-terminal) network *n*. In Fig. 16.31(a), terminal *c* is grounded and a signal V_b is applied to terminal *b*. The transfer function from *b* to *a* with *c* grounded is denoted *t*. Then, in Fig. 16.31(b), terminal *b* is grounded and the input signal is applied to terminal *c*. The transfer function from *c* to *a* with *b* grounded can be shown to be the complement of *t*—that is, $1-t$. (Recall that we used this property in generating a circuit realization for the all-pass function in Section 16.6.)

Application of the complementary transformation to a feedback loop to generate an equivalent feedback loop is a two-step process:

1. Nodes of the feedback network and any of the op-amp inputs that are connected to ground should be disconnected from ground and connected to the op-amp output. Conversely, those nodes that were connected to the op-amp output should be now connected to ground. That is, we simply interchange the op-amp output terminal with ground.
2. The two input terminals of the op amp should be interchanged.

The feedback loop generated by this transformation has the same characteristic equation, and hence the same poles, as the original loop.

To illustrate, we show in Fig. 16.32(a) the feedback loop formed by connecting a two-port RC network in the negative-feedback path of an op amp. Application of the complementary transformation to this loop results in the feedback loop of Fig. 16.32(b). Note that in the latter loop the op amp is used in the unity-gain follower configuration. We shall now show that the two loops of Fig. 16.32 are equivalent.

If the op amp has an open-loop gain *A*, the follower in the circuit of Fig. 16.32(b) will have a gain of $A/(A+1)$. This, together with the fact that the transfer function of network *n* from *c* to *a* is $1-t$ (see Fig. 16.31), enables us to write for the circuit in Fig. 16.32(b) the characteristic equation

$$1-\frac{A}{A+1}(1-t) = 0$$

This equation can be manipulated to the form

$$1+At = 0$$

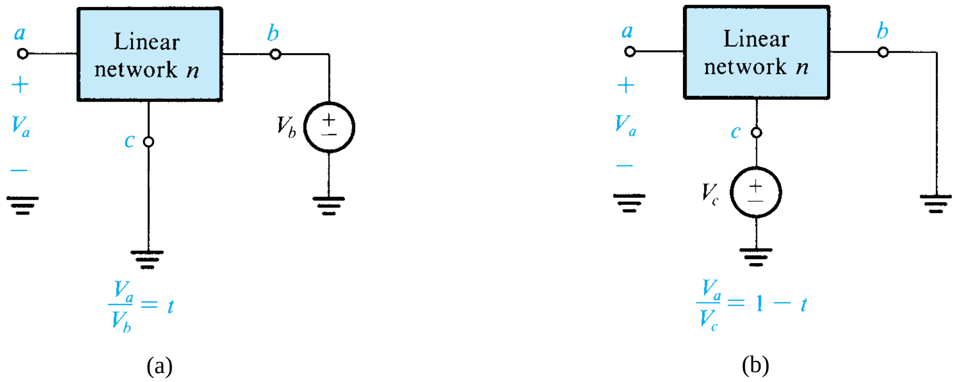


Figure 16.31 Interchanging input and ground results in the complement of the transfer function.

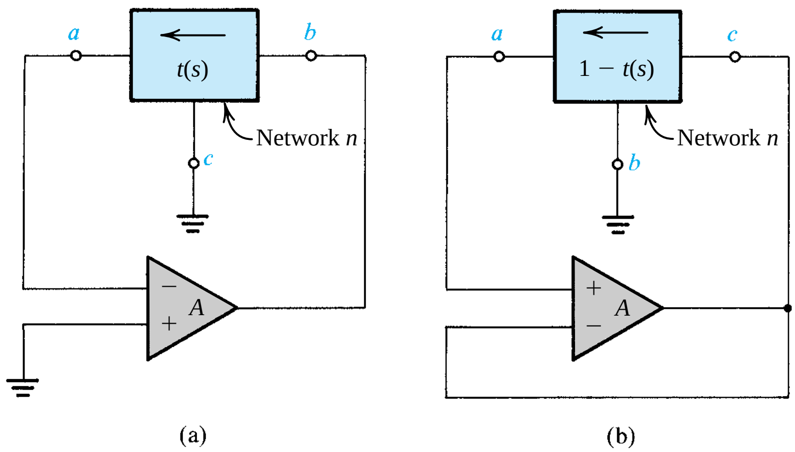
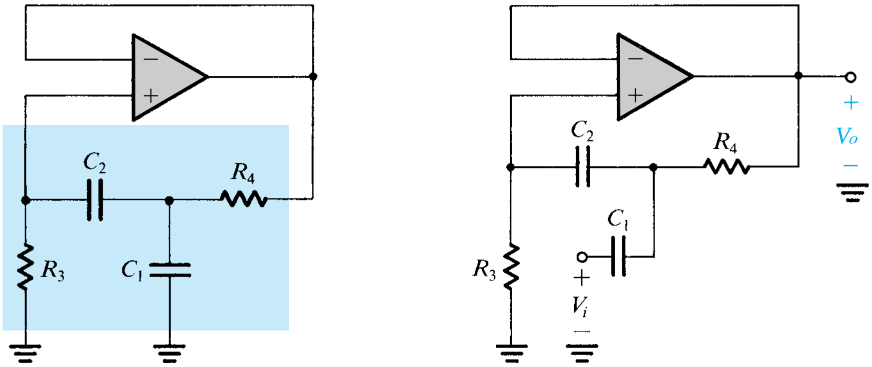


Figure 16.32 Application of the complementary transformation to the feedback loop in **(a)** results in the equivalent loop (same poles) shown in **(b)**.

which is the characteristic equation of the loop in Fig. 16.32(a). As an example, consider the application of the complementary transformation to the feedback loop of Fig. 16.29: The feedback loop of Fig. 16.33(a) results. Injecting the input signal through C_1 results in the circuit in Fig. 16.33(b), which can be shown (by direct analysis) to realize a second-order high-pass function. This circuit is one of a family of SABs known as the **Sallen-and-Key circuits**, after their originators. The design of the circuit in Fig. 16.33(b) is based on Eqs. (16.73) through (16.76): namely, $R_3 = R$, $R_4 = R/4Q^2$, $C_1 = C_2 = C$, $CR = 2Q/\omega_0$, and the value of C is arbitrarily chosen to be practically convenient.

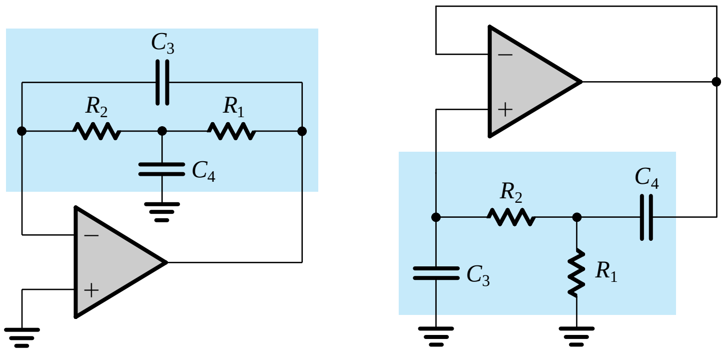
As another example, Fig. 16.34(a) shows the feedback loop generated by placing the two-port RC network of Fig. 16.28(b) in the negative-feedback path of an op amp. For an ideal op amp, this feedback loop realizes a pair of complex-conjugate natural modes having the same location as the zeros of $t(s)$ of the RC network. Thus, using the expression for $t(s)$ given in Fig. 16.28(b), we can write for the active-filter poles

$$\omega_0 = 1/\sqrt{C_3 C_4 R_1 R_2} \tag{16.77}$$

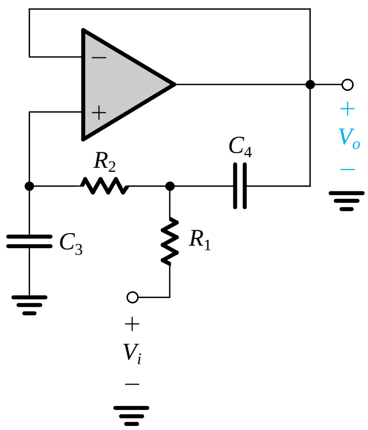


(a) (b)

Figure 16.33 **(a)** Feedback loop obtained by applying the complementary transformation to the loop in Fig. 16.29. **(b)** Injecting the input signal through C_1 realizes the high-pass function. This is one of the Sallen-and-Key family of circuits.



(a) (b)



(c)

Figure 16.34 **(a)** Feedback loop obtained by placing the bridged-T network of Fig. 16.28(b) in the negative-feedback path of an op amp. **(b)** Equivalent feedback loop generated by applying the complementary transformation to the loop in **(a)**. **(c)** A low-pass filter obtained by injecting V_i through R_1 into the loop in **(b)**.

$$Q = \left[\frac{\sqrt{C_3C_4R_1R_2}}{C_4}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)\right]^{-1} \tag{16.78}$$

Normally the design of this circuit is based on selecting $R_1 = R_2 = R$, $C_4 = C$, and $C_3 = C/m$. When substituted in Eqs. (16.77) and (16.78), these yield

$$m = 4Q^2 \tag{16.79}$$

$$CR = 2Q/\omega_0 \tag{16.80}$$

with the remaining degree of freedom (the value of C or R) left to the designer to choose.

Injecting the input signal to the C_4 terminal that is connected to ground can be shown to result in a bandpass realization. If, however, we apply the complementary transformation to the feedback loop in Fig. 16.34(a), we obtain the equivalent loop in Fig. 16.34(b). The loop equivalence means that the circuit of Fig. 16.34(b) has the same poles and thus the same ω_0 and Q and the same design equations (Eqs. 16.77 through 16.80). The new loop in Fig. 16.34(b) can be used to realize a low-pass function by injecting the input signal as shown in Fig. 16.34(c).

EXERCISES

16.28 Analyze the circuit in Fig. 16.34(c) to determine its transfer function $V_o(s)/V_i(s)$ and thus show that ω_0 and Q are indeed those in Eqs. (16.77) and (16.78). Also show that the dc gain is unity.

D16.29 Design the circuit in Fig. 16.34(c) to realize a low-pass filter with $f_0 = 4$ kHz and $Q = 1/\sqrt{2}$. Use 10-kΩ resistors.
Ans. $R_1 = R_2 = 10$ kΩ; $C_3 = 2.81$ nF; $C_4 = 5.63$ nF

16.9 Sensitivity

Because of the tolerances in component values and because of the finite op-amp gain, the response of the actual assembled filter will deviate from the ideal response. As a means for predicting such deviations, the filter designer employs the concept of **sensitivity**. Specifically, for second-order filters one is usually interested in finding how *sensitive* their poles are relative to variations (both initial tolerances and future drifts) in RC component values and amplifier gain. These sensitivities can be quantified using the **classical sensitivity function** S_x^y, defined as

$$S_x^y \equiv \lim_{\Delta x \to 0} \frac{\Delta y/y}{\Delta x/x} \tag{16.81}$$

Thus,

$$S_x^y = \frac{\partial y}{\partial x}\frac{x}{y} \tag{16.82}$$

Here, x denotes the value of a component (a resistor, a capacitor, or an amplifier gain) and y denotes a circuit parameter of interest (say, ω_0 or Q). For small changes

$$S_x^y \simeq \frac{\Delta y/y}{\Delta x/x} \tag{16.83}$$

Thus we can use the value of S_x^y to determine the per-unit change in y due to a given per-unit change in x. For instance, if the sensitivity of Q relative to a particular resistance R_1 is 5, then a 1% increase in R_1 results in a 5% increase in the value of Q.

Example 16.3

For the feedback loop of Fig. 16.29, find the sensitivities of ω_0 and Q relative to all the passive components and the op-amp gain. Evaluate these sensitivities for the design considered in the preceding section for which $C_1 = C_2$.

Solution

To find the sensitivities with respect to the passive components, called **passive sensitivities**, we assume that the op-amp gain is infinite. In this case, ω_0 and Q are given by Eqs. (16.73) and (16.74). Thus for ω_0 we have

$$\omega_0 = \frac{1}{\sqrt{C_1 C_2 R_3 R_4}}$$

which can be used together with the sensitivity definition of Eq. (16.82) to obtain

$$S_{C_1}^{\omega_0} = S_{C_2}^{\omega_0} = S_{R_3}^{\omega_0} = S_{R_4}^{\omega_0} = -\tfrac{1}{2}$$

For Q we have

$$Q = \left[\sqrt{C_1 C_2 R_3 R_4}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)\frac{1}{R_3}\right]^{-1}$$

to which we apply the sensitivity definition to obtain

$$S_{C_1}^{Q} = \frac{1}{2}\left(\sqrt{\frac{C_2}{C_1}} - \sqrt{\frac{C_1}{C_2}}\right)\left(\sqrt{\frac{C_2}{C_1}} + \sqrt{\frac{C_1}{C_2}}\right)^{-1}$$

For the design with $C_1 = C_2$ we see that $S_{C_1}^{Q} = 0$. Similarly, we can show that

$$S_{C_2}^{Q} = 0, \qquad S_{R_3}^{Q} = \tfrac{1}{2}, \qquad S_{R_4}^{Q} = -\tfrac{1}{2}$$

It is important to remember that the sensitivity expression should be derived *before* values corresponding to a particular design are substituted.

Next we consider the sensitivities relative to the amplifier gain. If we assume the op amp to have a finite gain A, the characteristic equation for the loop becomes

$$1 + At(s) = 0 \tag{16.84}$$

where $t(s)$ is given in Fig. 16.28(a). To simplify matters we can substitute for the passive components by their design values. This causes no errors in evaluating sensitivities, since we are now finding the sensitivity with respect to the amplifier gain. Using the design values obtained earlier—namely, $C_1 = C_2 = C$, $R_3 = R, R_4 = R/4Q^2$, and $CR = 2Q/\omega_0$ —we get

$$t(s) = \frac{s^2 + s(\omega_0/Q) + \omega_0^2}{s^2 + s(\omega_0/Q)(2Q^2+1) + \omega_0^2} \tag{16.85}$$

where ω_0 and Q denote the nominal or design values of the pole frequency and Q factor. The actual values are obtained by substituting for $t(s)$ in Eq. (16.84):

$$s^2 + s\frac{\omega_0}{Q}(2Q^2+1) + \omega_0^2 + A\left(s^2 + s\frac{\omega_0}{Q} + \omega_0^2\right) = 0$$

Assuming the gain A to be real and dividing both sides by $A + 1$, we get

$$s^2 + s\frac{\omega_0}{Q}\left(1 + \frac{2Q^2}{A+1}\right) + \omega_0^2 = 0 \tag{16.86}$$

From this equation we see that the actual pole frequency, ω_{0a}, and the pole Q, Q_a, are

$$\omega_{0a} = \omega_0 \tag{16.87}$$

$$Q_a = \frac{Q}{1 + 2Q^2/(A+1)} \tag{16.88}$$

Thus

$$S_A^{\omega_{0a}} = 0$$

$$S_A^{Q_a} = \frac{A}{A+1}\frac{2Q^2/(A+1)}{1 + 2Q^2/(A+1)}$$

For $A \gg 2Q^2$ and $A \gg 1$ we obtain

$$S_A^{Q_a} \simeq \frac{2Q^2}{A}$$

It is usual to drop the subscript a in this expression and write

$$S_A^Q \simeq \frac{2Q^2}{A} \tag{16.89}$$

Note that if Q is high $(Q \geq 5)$, its sensitivity relative to the amplifier gain can be quite high.[9]

16.9.1 A Concluding Remark

The results of Example 16.3 indicate a serious disadvantage of single-amplifier biquads—the sensitivity of Q relative to the amplifier gain is quite high. Although a technique exists for reducing S_A^Q in SABs (see Sedra et al., 1980), this is done at the expense of increased passive sensitivities. Nevertheless, the resulting SABs are used extensively in many applications. However, for filters with Q factors greater than about 10, one usually opts for one of the multiamplifier biquads studied in Sections 16.6 and 16.7. For these circuits S_A^Q is proportional to Q, rather than to Q^2 as in the SAB case (Eq. 16.89).

[9] Because the open-loop gain A of op amps usually has wide tolerance, it is important to keep $S_A^{\omega_0}$ and S_A^Q very small.

EXERCISE

16.30 In a particular filter utilizing the feedback loop of Fig. 16.29, with $C_1 = C_2$, use the results of Example 16.3 to find the expected percentage change in ω_0 and Q under the conditions that (a) R_3 is 2% high, (b) R_4 is 2% high, (c) both R_3 and R_4 are 2% high, and (d) both capacitors are 2% low and both resistors are 2% high.
Ans. (a) −1%, +1%; (b) −1%, −1%; (c) −2%, 0%; (d) 0%, 0%

16.10 Switched-Capacitor Filters

The active-RC filter circuits presented above have two properties that make their production in monolithic IC form difficult, if not practically impossible; these are the need for large-valued capacitors and the requirement of accurate RC time constants. The search therefore has continued for a method of filter design that would lend itself more naturally to IC implementation. In this section we shall introduce one such method.

16.10.1 The Basic Principle

The switched-capacitor filter technique is based on the realization that a capacitor switched between two circuit nodes at a sufficiently high rate is equivalent to a resistor connecting these two nodes. To be specific, consider the active-RC integrator of Fig. 16.35(a). This is the familiar Miller integrator, which we used in the two-integrator-loop biquad in Section 16.7. In Fig. 16.35(b) we have replaced the input resistor R_1 by a grounded capacitor C_1 together with two MOS transistors acting as switches. In some circuits, more elaborate switch configurations are used, but such details are beyond our present need.

The two MOS switches in Fig. 16.35(b) are driven by a *nonoverlapping* two-phase clock. Figure 16.35(c) shows the clock waveforms. We shall assume in this introductory exposition that the clock frequency f_c ($f_c = 1/T_c$) is much higher than the frequency of the input signal v_i. Thus during clock phase ϕ_1, when C_1 is connected across the input signal source v_i, the variations in the input signal are negligibly small. It follows that during ϕ_1, capacitor C_1 charges up to the voltage v_i,

$$q_{C1} = C_1 v_i$$

Then, during clock phase ϕ_2, capacitor C_1 is connected to the virtual-ground input of the op amp, as indicated in Fig. 16.35(d). Capacitor C_1 is thus forced to discharge, and its previous charge q_{C1} is transferred to C_2, in the direction indicated in Fig. 16.35(d).

From the description above we see that during each clock period T_c an amount of charge $q_{C1} = C_1 v_i$ is extracted from the input source and supplied to the integrator capacitor C_2. Thus the average current flowing between the input node (IN) and the virtual-ground node (VG) is

$$i_{\text{av}} = \frac{C_1 v_i}{T_c}$$

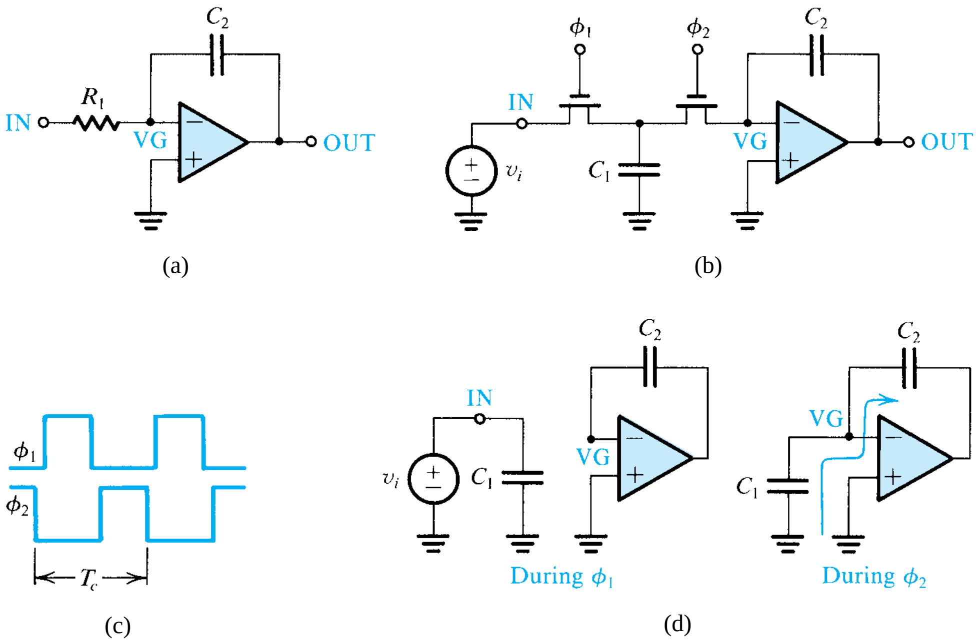


Figure 16.35 Basic principle of the switched-capacitor filter technique. **(a)** Active-RC integrator. **(b)** Switched-capacitor integrator. **(c)** Two-phase clock (nonoverlapping). **(d)** During ϕ_1, C_1 charges up to the current value of v_i and then, during ϕ_2, discharges into C_2.

If T_c is sufficiently short, one can think of this process as almost continuous and define an equivalent resistance R_{eq} that is in effect present between nodes IN and VG:

$$R_{eq} \equiv v_i / i_{av}$$

Thus,

$$R_{eq} = T_c / C_1 \tag{16.90}$$

Using R_{eq} we obtain an equivalent time constant for the integrator:

$$\text{Time constant} = C_2 R_{eq} = T_c \frac{C_2}{C_1} \tag{16.91}$$

Thus the time constant that determines the frequency response of the filter is established by the clock period T_c and the capacitor ratio C_2/C_1. Both these parameters can be well controlled in an IC process. Specifically, note the dependence on capacitor ratios rather than on absolute values of capacitors. The accuracy of capacitor ratios in MOS technology can be controlled to within 0.1%.

Another point worth observing is that with a reasonable clocking frequency (such as 100 kHz) and not-too-large capacitor ratios (say, 10), one can obtain reasonably large time constants (such as 10^{-4} s) suitable for audio applications. Since capacitors typically occupy relatively large areas on the IC chip, one attempts to minimize their values. In this context, it is important to note that the ratio accuracies quoted earlier are obtainable with the smaller capacitor value as low as 0.1 pF.

16.10.2 Practical Circuits

The switched-capacitor (SC) circuit in Fig. 16.35(b) realizes an inverting integrator (note the direction of charge flow through C_2 in Fig. 16.35d). As we saw in Section 16.7, a two-integrator-loop active filter is composed of one inverting and one noninverting integrator.[10] To realize a switched-capacitor biquad filter, we therefore need a pair of complementary switched-capacitor integrators. Figure 16.36(a) shows a noninverting, or positive, integrator circuit. The reader is urged to follow the operation of this circuit during the two clock phases and thus show that it operates in much the same way as the basic circuit of Fig. 16.35(b), except for a sign reversal.

In addition to realizing a noninverting integrator function, the circuit in Fig. 16.36(a) is insensitive to stray capacitances; however, we shall not explore this point any further. The interested reader is referred to Schaumann, Ghausi, and Laker (1990). By reversal of the clock phases on two of the switches, the circuit in Fig. 16.36(b) is obtained. This circuit realizes the inverting integrator function, like the circuit of Fig. 16.35(b), but is insensitive to stray capacitances (which the original circuit of Fig. 16.35b is not). The complementary integrators of Fig. 16.36 have become the standard building blocks in the design of switched-capacitor filters.

Let us now consider the realization of a complete biquad circuit. Figure 16.37(a) shows the active-RC, two-integrator-loop circuit studied earlier. By considering the cascade of

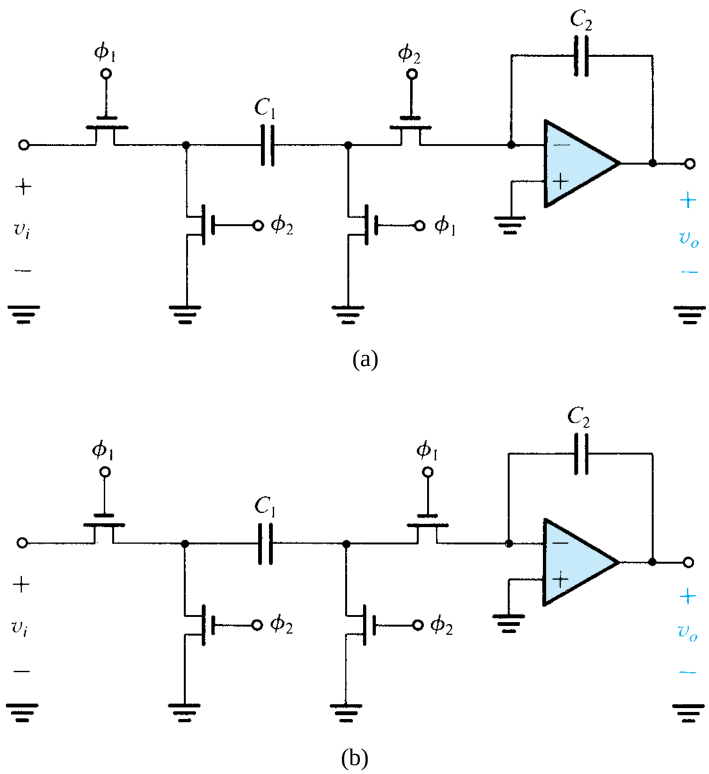


Figure 16.36 A pair of complementary stray-insensitive, switched-capacitor integrators. **(a)** Noninverting switched-capacitor integrator. **(b)** Inverting switched-capacitor integrator.

[10] In the two-integrator loop of Fig. 16.25(b), the noninverting integrator is realized by the cascade of a Miller integrator and an inverting amplifier.

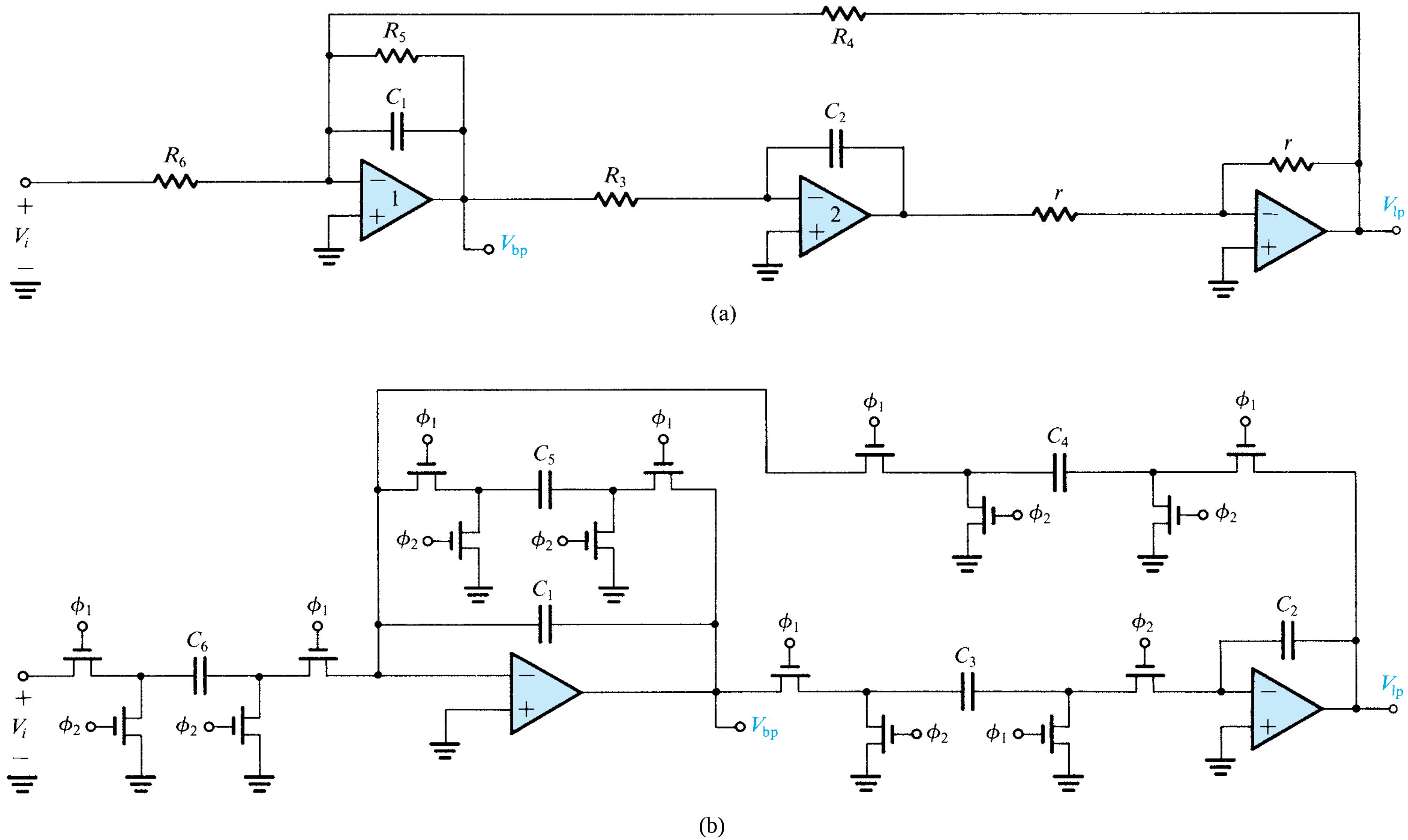


Figure 16.37 **(a)** A two-integrator-loop, active-RC biquad **(b)** its switched-capacitor counterpart.

integrator 2 and the inverter as a positive integrator, and then simply replacing each resistor by its switched-capacitor equivalent, we obtain the circuit in Fig. 16.37(b). Ignore the damping around the first integrator (i.e., the switched capacitor C_5) for the time being and note that the feedback loop indeed consists of one inverting and one noninverting integrator. Then note the phasing of the switched capacitor used for damping. Reversing the phases here would convert the feedback to positive and move the poles to the right half of the s plane. On the other hand, the phasing of the feed-in switched capacitor (C_6) is not that important; a reversal of phases would result only in an inversion in the sign of the function realized.

Having identified the correspondences between the active-RC biquad and the switched-capacitor biquad, we can now derive design equations. Analysis of the circuit in Fig. 16.37(a) yields

$$\omega_0 = \frac{1}{\sqrt{C_1 C_2 R_3 R_4}} \tag{16.92}$$

Replacing R_2 and R_4 with their switched-capacitor equivalent values, that is,

$$R_3 = T_c/C_3 \quad \text{and} \quad R_4 = T_c/C_4$$

gives ω_0 of the switched-capacitor biquad as

$$\omega_0 = \frac{1}{T_c}\sqrt{\frac{C_3 C_4}{C_2 C_1}} \tag{16.93}$$

It is usual to select the time constants of the two integrators to be equal; that is,

$$\frac{T_c}{C_3}C_2 = \frac{T_c}{C_4}C_1 \tag{16.94}$$

If, further, we select the two integrating capacitors C_1 and C_2 to be equal,

$$C_1 = C_2 = C \tag{16.95}$$

then

$$C_3 = C_4 = KC \tag{16.96}$$

where from Eq. (16.93)

$$K = \omega_0 T_c \tag{16.97}$$

For the case of equal time constants, the Q factor of the circuit in Fig. 16.37(a) is given by R_5/R_4. Thus the Q factor of the corresponding switched-capacitor circuit in Fig. 16.37(b) is given by

$$Q = \frac{T_c/C_5}{T_c/C_4} \tag{16.98}$$

Thus C_5 should be selected from

$$C_5 = \frac{C_4}{Q} = \frac{KC}{Q} = \omega_0 T_c \frac{C}{Q} \tag{16.99}$$

Finally, the center-frequency gain of the bandpass function is given by

$$\text{Center-frequency gain} = \frac{C_6}{C_5} = Q\frac{C_6}{\omega_0 T_c C} \tag{16.100}$$

EXERCISE

D16.31 Use $C_1 = C_2 = 20$ pF and design the circuit in Fig. 16.37(b) to realize a bandpass function with $f_0 = 10$ kHz, $Q = 20$, and unity center-frequency gain. Use a clock frequency $f_c = 200$ kHz. Find the values of C_3, C_4, C_5, and C_6.
Ans. 6.283 pF; 6.283 pF; 0.314 pF; 0.314 pF

16.10.3 A Final Remark

We have attempted to provide only an introduction to switched-capacitor filters. We have made many simplifying assumptions, the most important being the switched-capacitor–resistor equivalence (Eq. 16.90). This equivalence is correct only at $f_c = \infty$ and is approximately correct for $f_c \gg f$. Switched-capacitor filters are, in fact, sampled-data networks whose analysis and design can be carried out exactly using z-transform techniques. The interested reader is referred to the bibliography in Appendix G.

16.11 Tuned Amplifiers

In this section, we study a special kind of frequency-selective network, the LC-tuned amplifier. Figure 16.38 shows the general shape of the frequency response of a tuned amplifier. The techniques discussed apply to amplifiers with center frequencies in the range of a few hundred kilohertz to a few hundred megahertz. Tuned amplifiers find application in the radio-frequency (RF) and intermediate-frequency (IF) sections of communications receivers and in a variety of other systems. It should be noted that the tuned-amplifier response of Fig. 16.38 is similar to that of the bandpass filter discussed in earlier sections.

As indicated in Fig. 16.38, the response is characterized by the center frequency ω_0, the 3-dB bandwidth B, and the *skirt selectivity*, which is usually measured as the ratio of the 30-dB bandwidth to the 3-dB bandwidth. In many applications, the 3-dB bandwidth is less than 5% of ω_0. This **narrow-band** property makes possible certain approximations that can simplify the design process, as will be explained later.

The tuned amplifiers studied in this section are small-signal voltage amplifiers in which the transistors operate in the “class A” mode; that is, the transistors conduct at all times. Tuned power amplifiers based on class C and other switching modes of operation are not studied in this book. (For a discussion on the classification of amplifiers, refer to Section 11.1.)

16.11.1 The Basic Principle

The basic principle underlying the design of tuned amplifiers is the use of a parallel LCR circuit as the load, or at the input, of a BJT or a FET amplifier. This is illustrated in Fig. 16.39 with a MOSFET amplifier having a tuned-circuit load. For simplicity, the bias details are not included. Since this circuit uses a single tuned circuit, it is known as a **single-tuned**

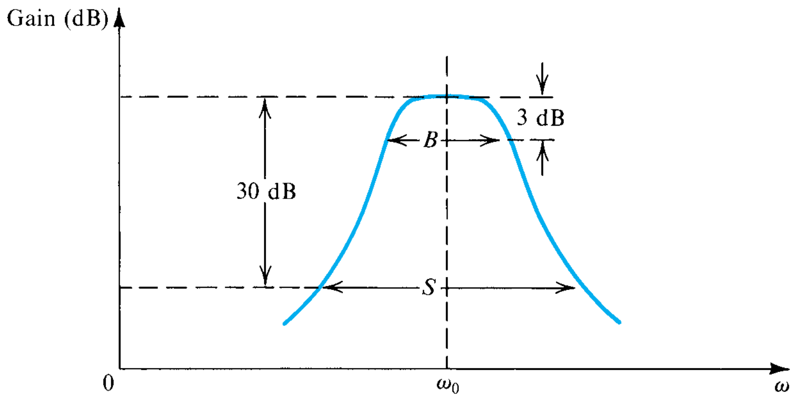


Figure 16.38 Frequency response of a tuned amplifier.

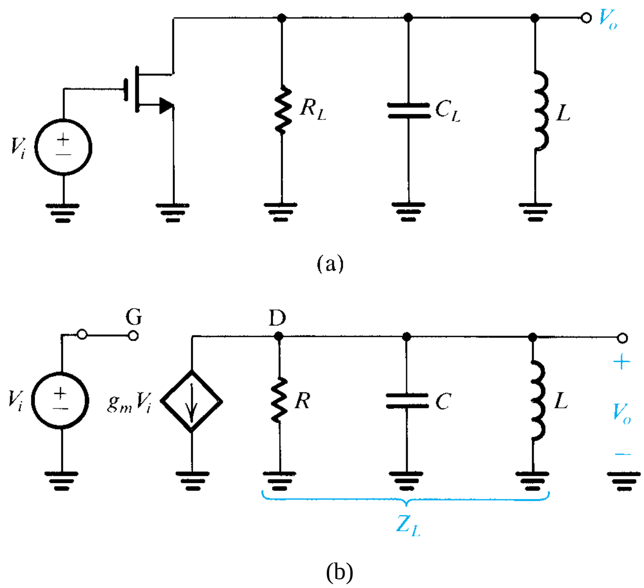


Figure 16.39 The basic principle of tuned amplifiers is illustrated using a MOSFET with a tuned-circuit load. Bias details are not shown.

amplifier. The amplifier equivalent circuit is shown in Fig. 16.39(b). Here R denotes the parallel equivalent of R_L and the output resistance r_o of the FET, and C is the parallel equivalent of C_L and the FET output capacitance (usually very small). From the equivalent circuit we can write

$$V_o = \frac{-g_m V_i}{Y_L} = \frac{-g_m V_i}{sC + 1/R + 1/sL}$$

Thus the voltage gain can be expressed as

$$\frac{V_o}{V_i} = -\frac{g_m}{C} \frac{s}{s^2 + s(1/CR) + 1/LC} \tag{16.101}$$

which is a second-order bandpass function. Thus the tuned amplifier has a center frequency of

$$\omega_0 = 1/\sqrt{LC} \tag{16.102}$$

a 3-dB bandwidth of

$$B = \frac{1}{CR} \tag{16.103}$$

a Q factor of

$$Q \equiv \omega_0/B = \omega_0 CR \tag{16.104}$$

and a center-frequency gain of

$$\frac{V_o(j\omega_0)}{V_i(j\omega_0)} = -g_m R \tag{16.105}$$

Note that the expression for the center-frequency gain could have been written by inspection; at resonance, the reactances of L and C cancel out and the impedance of the parallel LCR circuit reduces to R.

Example 16.4

It is required to design a tuned amplifier of the type shown in Fig. 16.39, having $f_0 = 1$ MHz, 3-dB bandwidth = 10 kHz, and center-frequency gain = –10 V/V. The FET available has at the bias point $g_m = 5$ mA/V and $r_o = 10$ kΩ. The output capacitance is negligibly small. Determine the values of R_L, C_L, and L.

Solution

Center-frequency gain = –10 = –5R. Thus $R = 2$ kΩ. Since $R = R_L \| r_o$, then $R_L = 2.5$ kΩ.

$$B = 2\pi \times 10^4 = \frac{1}{CR}$$

Thus

$$C = \frac{1}{2\pi \times 10^4 \times 2 \times 10^3} = 7958 \text{ pF}$$

Since $\omega_0 = 2\pi \times 10^6 = 1/\sqrt{LC}$, we obtain

$$L = \frac{1}{4\pi^2 \times 10^{12} \times 7958 \times 10^{-12}} = 3.18 \ \mu\text{H}$$

16.11.2 Inductor Losses

The power loss in the inductor is usually represented by a series resistance r_s as shown in Fig. 16.40(a). However, rather than specifying the value of r_s, the usual practice is to specify the inductor Q factor at the frequency of interest,

$$Q_0 \equiv \frac{\omega_0 L}{r_s} \tag{16.106}$$

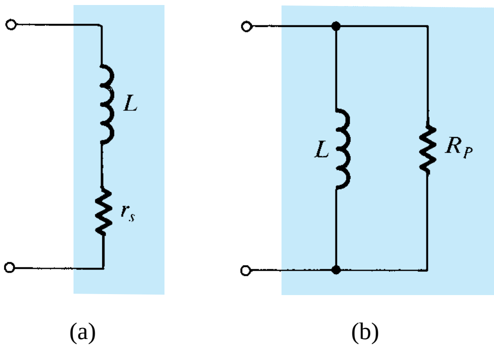


Figure 16.40 Inductor equivalent circuits.

Typically, Q_0 is in the range of 50 to 200.

The analysis of a tuned amplifier is greatly simplified by representing the inductor loss by a parallel resistance R_p, as shown in Fig. 16.40(b). The relationship between R_p and Q_0 can be found by writing, for the admittance of the circuit in Fig. 16.40(a),

$$Y(j\omega_0) = \frac{1}{r_s + j\omega_0 L}$$

$$= \frac{1}{j\omega_0 L}\,\frac{1}{1 - j(1/Q_0)} = \frac{1}{j\omega_0 L}\,\frac{1 + j(1/Q_0)}{1 + (1/Q_0^2)}$$

For $Q_0 \gg 1$,

$$Y(j\omega_0) \simeq \frac{1}{j\omega_0 L}\left(1 + j\frac{1}{Q_0}\right) \tag{16.107}$$

Equating this to the admittance of the circuit in Fig. 16.40(b) gives

$$Q_0 = \frac{R_p}{\omega_0 L} \tag{16.108}$$

or, equivalently,

$$R_p = \omega_0 L Q_0 \tag{16.109}$$

Finally, it should be noted that the coil Q factor poses an upper limit on the value of Q achieved by the tuned circuit.

EXERCISE

16.32 If the inductor in Example 16.4 has $Q_0 = 150$, find R_p and then find the value to which R_L should be changed to keep the overall Q, and hence the bandwidth, unchanged.
Ans. 3 kΩ; 15 kΩ

16.11.3 Use of Transformers

In many cases it is found that the required value of inductance is not practical, in the sense that coils with the required inductance might not be available with the required high values of Q_0. A simple solution is to use a transformer to effect an impedance change. Alternatively, a tapped coil, known as an **autotransformer**, can be used, as shown in Fig. 16.41. Provided the two parts of the inductor are tightly coupled, which can be achieved by winding on a ferrite core, the transformation relationships shown hold. The result is that the tuned circuit seen between terminals 1 and 1′ is equivalent to that in Fig. 16.39(b). For example, if a turns ratio $n = 3$ is used in the amplifier of Example 16.4, then a coil with inductance $L' = 9 \times 3.18 = 28.6\ \mu\text{H}$ and a capacitance $C' = 7958/9 = 884$ pF will be required. Both these values are more practical than the original ones.

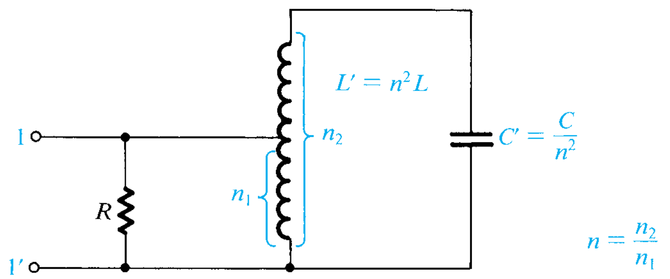


Figure 16.41 A tapped inductor is used as an impedance transformer to allow using a higher inductance, L', and a smaller capacitance, C'.

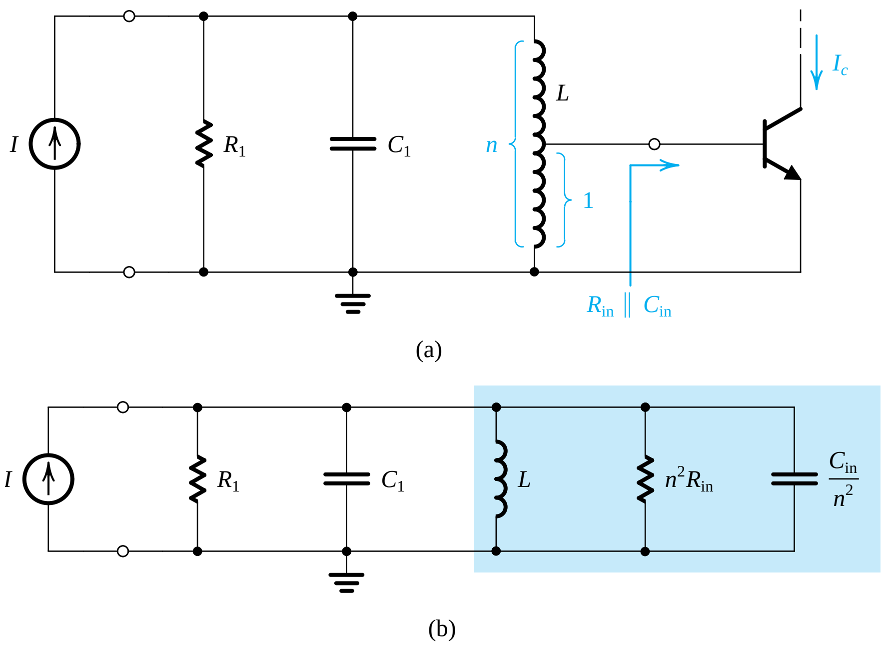


Figure 16.42 **(a)** The output of a tuned amplifier is coupled to the input of another amplifier via a tapped coil. **(b)** An equivalent circuit. Note that the use of a tapped coil increases the effective input impedance of the second amplifier stage.

In applications that involve coupling the output of a tuned amplifier to the input of another amplifier, the tapped coil can be used to raise the effective input resistance of the latter amplifier stage. In this way, one can avoid reduction of the overall Q. This point is illustrated in Fig. 16.42 and in the following exercises.

EXERCISES

D16.33 Consider the circuit in Fig. 16.42(a), first without tapping the coil. Let $L = 5\ \mu\text{H}$ and assume that R_1 is fixed at 1 kΩ. We wish to design a tuned amplifier with $f_0 = 455$ kHz and a 3-dB bandwidth of 10 kHz [this is the intermediate frequency (IF) amplifier of an AM radio]. If the BJT has $R_{in} = 1$ kΩ and $C_{in} = 200$ pF, find the actual bandwidth obtained and the required value of C_1.
Ans. 13 kHz; 24.27 nF

D16.34 Since the bandwidth realized in Exercise 16.33 is greater than desired, find an alternative design utilizing a tapped coil as in Fig. 16.42(a). Find the value of n that allows the specifications to be just met. Also find the new required value of C_1 and the current gain I_c/I at resonance. Assume that at the bias point the BJT has $g_m = 40$ mA/V.
Ans. 1.36; 24.36 nF; 19.1 A/A

16.11.4 Amplifiers with Multiple Tuned Circuits

The selectivity achieved with the single tuned circuit of Fig. 16.39 is not sufficient in many applications—for instance, in the IF amplifier of a radio or a TV receiver. Greater selectivity is obtained by using additional tuned stages. Figure 16.43 shows a BJT with tuned circuits at both the input and the output.[11] In this circuit the bias details are shown, from which we note that biasing is quite similar to the classical arrangement employed in low-frequency, discrete-circuit design. However, to avoid the loading effect of the bias resistors R_{B1} and R_{B2} on the input tuned circuit, a **radio frequency choke** (RFC) is inserted in series with each resistor. Such chokes have high impedances at the frequencies of interest. The use of RFCs in biasing tuned RF amplifiers is common practice.

The analysis and design of the double-tuned amplifier of Fig. 16.43 is complicated by the Miller effect[12] due to capacitance C_μ. Since the load is not simply resistive, as was the case in the amplifiers studied in Section 9.5.2, the Miller impedance at the input will be complex. This reflected impedance will cause detuning of the input circuit as well as "skewing" of the response of the input circuit. Needless to say, the coupling introduced by C_μ makes tuning (or aligning) the amplifier quite difficult. Worse still, the capacitor C_μ can cause oscillations to occur [see Gray and Searle (1969) and Problem 16.75].

Methods exist for **neutralizing** the effect of C_μ, using additional circuits arranged to feed back a current equal and opposite to that through C_μ. An alternative, and preferred, approach is to use circuit configurations that do not suffer from the Miller effect. These are discussed later. Before leaving this section, however, we wish to point out that circuits of the type shown in Fig. 16.43 are usually designed utilizing the y-parameter model of the BJT

[11] Note that because the input circuit is a parallel resonant circuit, an input current source (rather than voltage source) signal is utilized.
[12] Here we use "Miller effect" to refer to the effect of the feedback capacitance C_μ in reflecting back an input impedance that is a function of the amplifier load impedance.

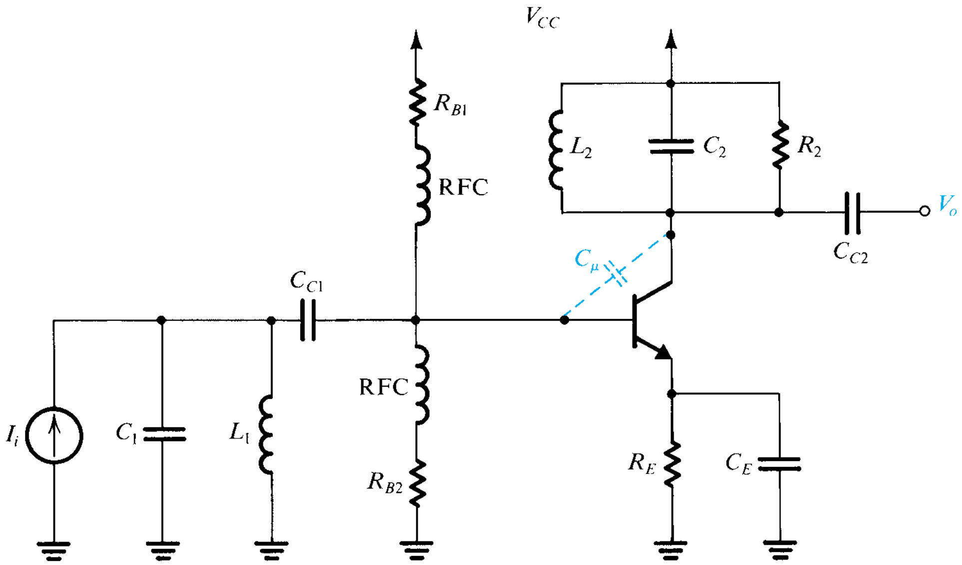


Figure 16.43 A BJT amplifier with tuned circuits at the input and the output.

(see Appendix C). This is done because here, in view of the fact that C_μ plays a significant role, the y-parameter model makes the analysis simpler (in comparison to that using the hybrid-π model). Also, the y parameters can easily be measured at the particular frequency of interest, ω_0. For narrow-band amplifiers, the assumption is usually made that the y parameters remain approximately constant over the passband.

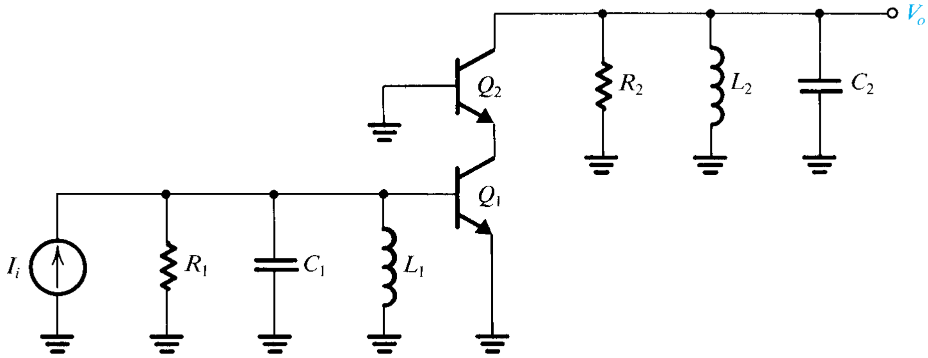
16.11.5 The Cascode and the CC–CB Cascade

From our study of amplifier frequency response in Chapter 9, we know that two amplifier configurations do not suffer from the Miller effect. These are the cascode configuration and the common-collector, common-base cascade. Figure 16.44 shows tuned amplifiers based on these two configurations. The CC–CB cascade is usually preferred in IC implementations because its differential structure makes it suitable for IC biasing techniques. (Note that the biasing details of the cascode circuit are not shown in Fig. 16.44a. Biasing can be done using arrangements similar to those discussed in earlier chapters.)

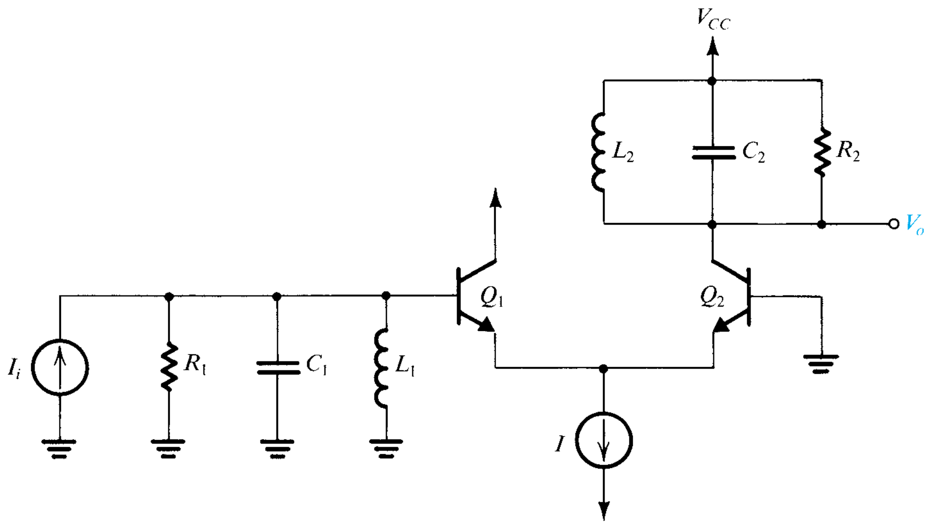
16.11.6 Synchronous Tuning

In the design of a tuned amplifier with multiple tuned circuits, the question of the frequency to which each circuit should be tuned arises. The objective, of course, is for the overall response to exhibit high passband flatness and skirt selectivity. To investigate this question, we shall assume that the overall response is the product of the individual responses: in other words, that the stages do not interact. This can easily be achieved using circuits such as those in Fig. 16.44.

Consider first the case of N identical resonant circuits, known as the **synchronously tuned** case. Figure 16.45 shows the response of an individual stage and that of the cascade. Observe the bandwidth "shrinkage" of the overall response. The 3-dB bandwidth B of the overall amplifier is related to that of the individual tuned circuits, ω_0/Q, by (see Problem 16.77)



(a)



(b)

Figure 16.44 Two tuned-amplifier configurations that do not suffer from the Miller effect: **(a)** cascode and **(b)** common-collector, common-base cascade. (Note that bias details of the cascode circuit are not shown.)

$$B = \frac{\omega_0}{Q}\sqrt{2^{1/N} - 1} \tag{16.110}$$

The factor $\sqrt{2^{1/N} - 1}$ is known as the **bandwidth-shrinkage factor**. Given B and N, we can use Eq. (16.110) to determine the bandwidth required of the individual stages, ω_0/Q.

EXERCISE

D16.35 Consider the design of an IF amplifier for an FM radio receiver. Using two synchronously tuned stages with $f_0 = 10.7$ MHz, find the 3-dB bandwidth of each stage so that the overall bandwidth is 200 kHz. Using 3-μH inductors find C and R for each stage.

Ans. 310.8 kHz; 73.7 pF; 6.95 kΩ

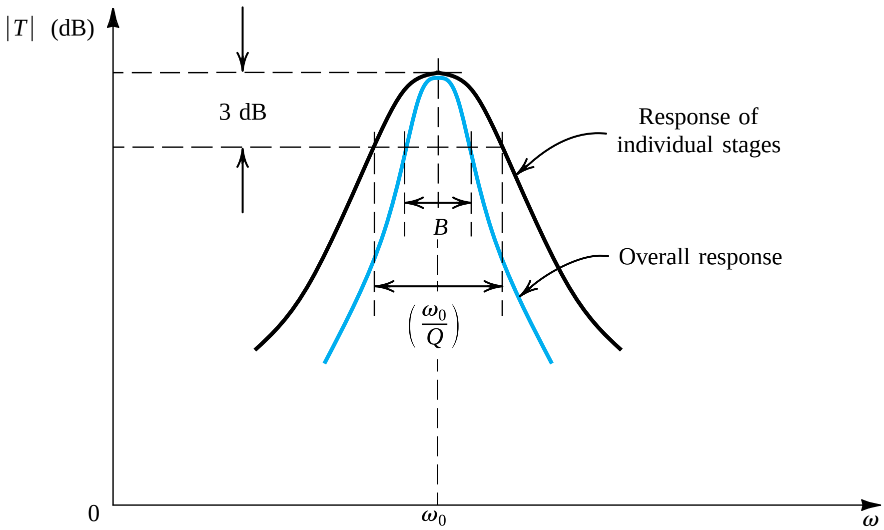


Figure 16.45 Frequency response of a synchronously tuned amplifier.

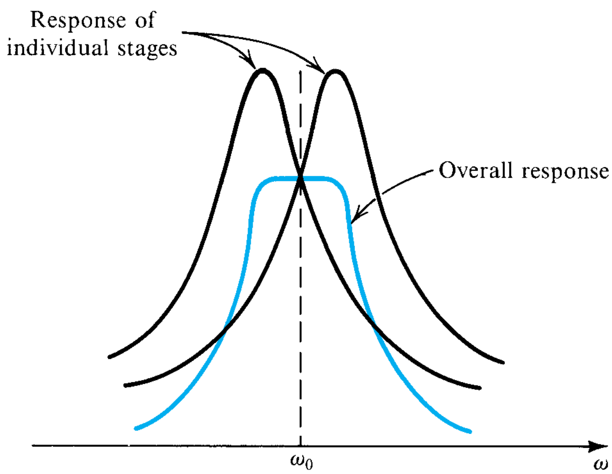


Figure 16.46 Stagger-tuning the individual resonant circuits can result in an overall response with a passband flatter than that obtained with synchronous tuning (Fig. 16.45).

16.11.7 Stagger-Tuning

A much better overall response is obtained by stagger-tuning the individual stages, as illustrated in Fig. 16.46. Stagger-tuned amplifiers are usually designed so that the overall response exhibits *maximal flatness* around the center frequency f_0. Such a response can be obtained by transforming the response of a maximally flat (Butterworth) low-pass filter up the frequency axis to ω_0. We show here how this can be done.

The transfer function of a second-order bandpass filter can be expressed in terms of its poles as

$$T(s) = \frac{a_1 s}{\left(s+\frac{\omega_0}{2Q}-j\omega_0\sqrt{1-\frac{1}{4Q^2}}\right)\left(s+\frac{\omega_0}{2Q}+j\omega_0\sqrt{1-\frac{1}{4Q^2}}\right)} \tag{16.111}$$

For a narrow-band filter, $Q \gg 1$, and for values of s in the neighborhood of $+j\omega_0$ (see Fig. 16.47b), the second factor in the denominator is approximately $(s+j\omega_0 \simeq 2s)$. Hence Eq. (16.111) can be approximated in the neighborhood of $j\omega_0$ by

$$T(s) \simeq \frac{a_1/2}{s+\omega_0/2Q - j\omega_0} = \frac{a_1/2}{(s-j\omega_0)+\omega_0/2Q} \tag{16.112}$$

This is known as the **narrow-band approximation**.[13] Note that the magnitude response, for $s = j\omega$, has a peak value of $a_1 Q/\omega_0$ at $\omega = \omega_0$, as expected.

Now consider a first-order low-pass network with a single pole at $p = -\omega_0/2Q$ (we use p to denote the complex frequency variable for the low-pass filter). Its transfer function is

$$T(p) = \frac{K}{p+\omega_0/2Q} \tag{16.113}$$

where K is a constant. Comparing Eqs. (16.112) and (16.113) we note that they are identical for $p = s - j\omega_0$ or, equivalently,

$$s = p + j\omega_0 \tag{16.114}$$

This result implies that the response of the second-order bandpass filter *in the neighborhood of its center frequency* $s = j\omega_0$ is identical to the response of a first-order low-pass filter with a pole at $(-\omega_0/2Q)$ *in the neighborhood of* $p = 0$. Thus the bandpass response can be obtained by shifting the pole of the low-pass prototype and adding the complex-conjugate pole, as illustrated in Fig. 16.47(b). This is called a **lowpass-to-bandpass transformation** for *narrow-band* filters.

The transformation $p = s - j\omega_0$ can be applied to low-pass filters of order greater than one. For instance, we can transform a maximally flat, second-order low-pass filter $(Q = 1/\sqrt{2})$ to obtain a maximally flat bandpass filter. If the 3-dB bandwidth of the bandpass filter is to be B rad/s, then the low-pass filter should have a 3-dB frequency (and thus a pole frequency) of $(B/2)$ rad/s, as illustrated in Fig. 16.48. The resulting fourth-order bandpass filter will be a stagger-tuned one, with its two tuned circuits (refer to Fig. 16.48) having

$$\omega_{01} = \omega_0 + \frac{B}{2\sqrt{2}} \qquad B_1 = \frac{B}{\sqrt{2}} \qquad Q_1 \simeq \frac{\sqrt{2}\,\omega_0}{B} \tag{16.115}$$

[13] The bandpass response is *geometrically symmetrical* around the center frequency ω_0. That is, each pair of frequencies ω_1 and ω_2 at which the magnitude response is equal are related by $\omega_1\omega_2 = \omega_0^2$. For high Q, the symmetry becomes almost *arithmetic* for frequencies close to ω_0. That is, two frequencies with the same magnitude response are almost equally spaced from ω_0. The same is true for higher-order bandpass filters designed using the transformation presented in this section.

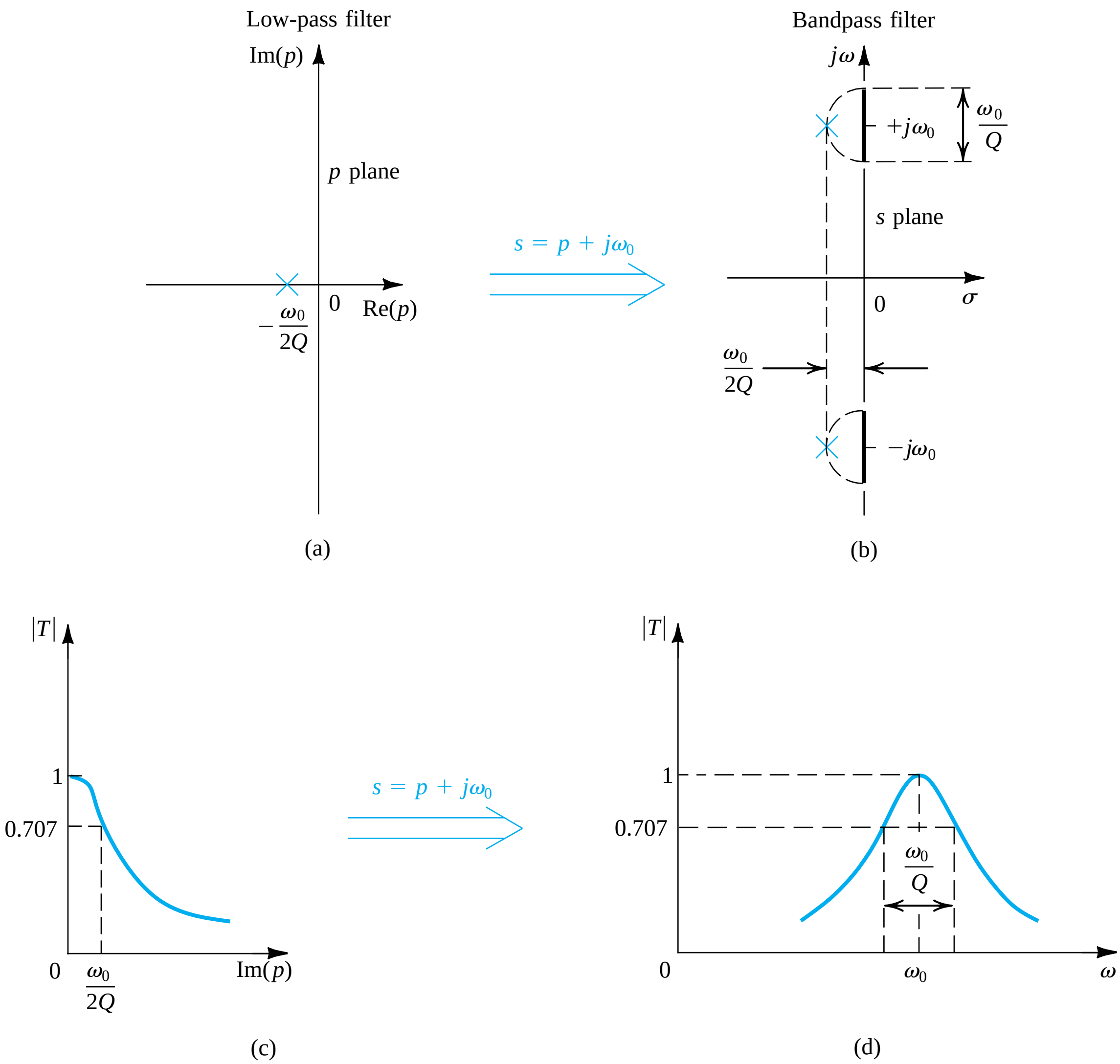


Figure 16.47 Obtaining a second-order narrow-band bandpass filter by transforming a first-order low-pass filter. **(a)** Pole of the first-order filter in the p plane. **(b)** Applying the transformation $s = p + j\omega_0$ and adding a complex-conjugate pole results in the poles of the second-order bandpass filter. **(c)** Magnitude response of the first-order low-pass filter. **(d)** Magnitude response of the second-order bandpass filter.

$$\omega_{02} = \omega_0 - \frac{B}{2\sqrt{2}} \qquad B_2 = \frac{B}{\sqrt{2}} \qquad Q_2 = \frac{\sqrt{2}\,\omega_0}{B} \tag{16.116}$$

Note that for the overall response to have a normalized center-frequency gain of unity, the individual responses have to have equal center-frequency gains of $\sqrt{2}$, as shown in Fig. 16.48(d).

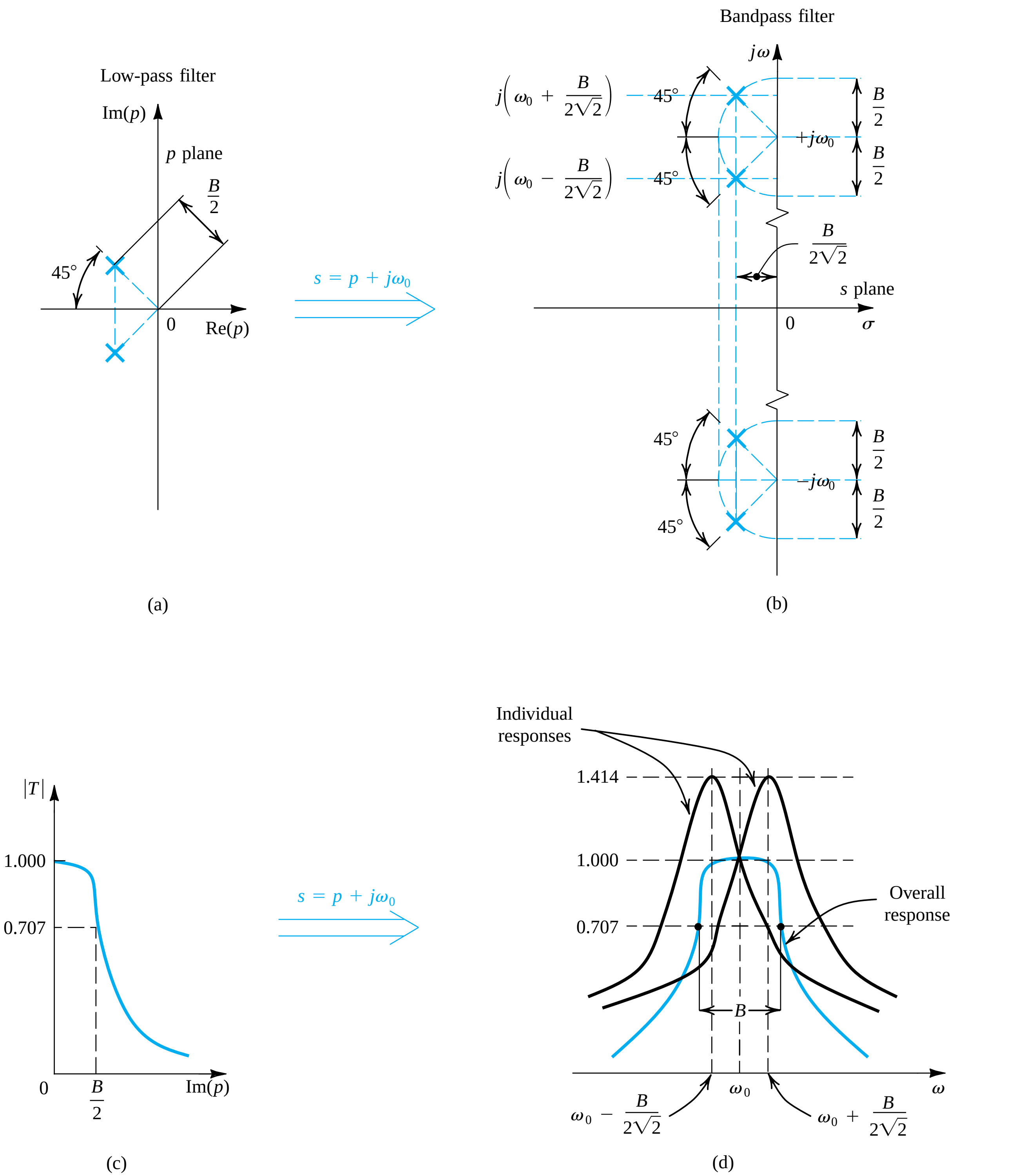


Figure 16.48 Obtaining the poles and the frequency response of a fourth-order stagger-tuned, narrow-band bandpass amplifier by transforming a second-order low-pass, maximally flat response.

EXERCISES

D16.36 A stagger-tuned design for the IF amplifier specified in Exercise 16.35 is required. Find f_{01}, B_1, f_{02}, and B_2. Also give the value of C and R for each of the two stages. (Recall that 3-µH inductors are to be used.)
Ans. 10.77 MHz; 141.4 kHz; 10.63 MHz; 141.4 kHz; 72.8 pF; 15.5 kΩ; 74.7 pF; 15.1 kΩ

16.37 Using the fact that the voltage gain at resonance is proportional to the value of R, find the ratio of the gain at 10.7 MHz of the stagger-tuned amplifier designed in Exercise 16.36 and the synchronously tuned amplifier designed in Exercise 16.35. (*Hint:* For the stagger-tuned amplifier, note that the gain at ω_0 is equal to the product of the gains of the individual stages at their 3-dB frequencies.)
Ans. 2.42

Summary

- A filter is a linear two-port network with a transfer function $T(s) = V_o(s)/V_i(s)$. For physical frequencies, the filter transmission is expressed as $T(j\omega) = |T(j\omega)|e^{j\phi(\omega)}$. The magnitude of transmission can be expressed in decibels using either the gain function $G(\omega) \equiv 20 \log|T|$ or the attenuation function $A(\omega) \equiv -20 \log|T|$.

- The transmission characteristics of a filter are specified in terms of the edges of the passband(s) and the stopband(s); the maximum allowed variation in passband transmission, A_{max} (dB); and the minimum attenuation required in the stopband, A_{min} (dB). In some applications, the phase characteristics are also specified.

- The filter transfer function can be expressed as the ratio of two polynomials in s; the degree of the denominator polynomial, N, is the filter order. The N roots of the denominator polynomial are the poles (natural modes).

- To obtain a highly selective response, the poles are complex and occur in conjugate pairs (except for one real pole when N is odd). The zeros are placed on the $j\omega$ axis in the stopband(s) including $\omega = 0$ and $\omega = \infty$.

- The Butterworth filter approximation provides a low-pass response that is maximally flat at $\omega = 0$. The transmission decreases monotonically as ω increases, reaching 0 (infinite attenuation) at $\omega = \infty$, where all N transmission zeros lie. Eq. (16.11) gives $|T|$, where ϵ is given by Eq. (16.14) and the order N is determined using Eq. (16.15). The poles are found using the graphical construction of Fig. 16.10, and the transfer function is given by Eq. (16.16).

- The Chebyshev filter approximation provides a low-pass response that is equiripple in the passband with the transmission decreasing monotonically in the stopband. All the transmission zeros are at $s = \infty$. Eq. (16.18) gives $|T|$ in the passband and Eq. (16.19) gives $|T|$ in the stopband, where ϵ is given by Eq. (16.21). The order N can be determined using Eq. (16.22). The poles are given by Eq. (16.23) and the transfer function by Eq. (16.24).

- Figures 16.13 and 16.14 provide a summary of first-order filter functions and their realizations.

- Figure 16.16 provides the characteristics of seven special second-order filtering functions.

- The second-order LCR resonator of Fig. 16.17(a) realizes a pair of complex-conjugate poles with $\omega_0 = 1/\sqrt{LC}$ and $Q = \omega_0 CR$. This resonator can be used to realize the various special second-order filtering functions, as shown in Fig. 16.18.

- By replacing the inductor of an LCR resonator with a simulated inductance obtained using the Antoniou circuit of Fig. 16.20(a), the op amp–RC resonator of Fig. 16.21(b) is obtained. This resonator can be used to realize the various second-order filter functions as shown in Fig. 16.22. The design equations for these circuits are given in Table 16.1.

- Biquads based on the two-integrator-loop topology are the most versatile and popular second-order filter realizations. There are two varieties: the KHN circuit of Fig. 16.24(a), which realizes the LP, BP, and HP functions simultaneously and can be combined with the output summing amplifier of Fig. 16.28(b) to realize the notch and all-pass functions; and the Tow–Thomas circuit of Fig. 16.25(b), which realizes the BP and LP functions simultaneously. Feedforward can be applied to the Tow–Thomas circuit to obtain the circuit of Fig. 16.26, which can be designed to realize any of the second-order functions (see Table 16.2).

- Single-amplifier biquads (SABs) are obtained by placing a bridged-T network in the negative-feedback path of an op amp. If the op amp is ideal, the poles realized are at the same locations as the zeros of the RC network. The complementary transformation can be applied to the feedback loop to obtain another feedback loop having identical poles. Different transmission zeros are realized by feeding the input signal to circuit nodes that are connected to ground. SABs are economic in their use of op amps but are sensitive to the op-amp nonidealities and are thus limited to low-Q applications ($Q \leq 10$).

- The classical sensitivity function

$$S_x^y = \frac{\partial y/y}{\partial x/x}$$

is a very useful tool in investigating how tolerant a filter circuit is to the unavoidable inaccuracies in component values and to the nonidealities of the op amps.

- Switched-capacitor (SC) filters are based on the principle that a capacitor C, periodically switched between two circuit nodes at a high rate, f_c, is equivalent to a resistance $R = 1/Cf_c$ connecting the two circuit nodes. SC filters can be fabricated in monolithic form using CMOS IC technology.

- Tuned amplifiers utilize LC-tuned circuits as loads, or at the input, of transistor amplifiers. They are used in the design of the RF tuner and the IF amplifier of communication receivers. The cascode and the CC–CB cascade configurations are frequently used in the design of tuned amplifiers. Stagger-tuning the individual tuned circuits results in a flatter passband response (in comparison to that obtained with all the tuned circuits synchronously tuned).

PROBLEMS

Computer Simulation Problems

SIM Problems involving design are marked with D throughout the text. As well, problems are marked with asterisks to describe their degree of difficulty. Difficult problems are marked with an asterisk (*); more difficult problems with two asterisks (**); and very challenging and/or time-consuming problems with three asterisks (***).

Section 16.1: Filter Transmission, Types and Specification

16.1 The transfer function of a first-order low-pass filter (such as that realized by an RC circuit) can be expressed as $T(s) = \omega_0/(s+\omega_0)$, where ω_0 is the 3-dB frequency of the filter. Give in table form the values of $|T|$, ϕ, G, and A at $\omega = 0, 0.5\omega_0, \omega_0, 2\omega_0, 5\omega_0, 10\omega_0$, and $100\omega_0$.

***16.2** A filter has the transfer function $T(s) = 1/[(s+1)(s^2+s+1)]$. Show that $|T| = \sqrt{1+\omega^6}$ and find an expression for its phase response $\phi(\omega)$. Calculate the values of $|T|$ and ϕ for $\omega = 0.1$, 1, and 10 rad/s and then find the output corresponding to each of the following input signals:

(a) $2 \sin 0.1t$ (volts)
(b) $2 \sin t$ (volts)
(c) $2 \sin 10t$ (volts)

16.3 For the filter whose magnitude response is sketched (as the colored curve) in Fig. 16.3, find $|T|$ at $\omega = 0$, $\omega = \omega_p$, and $\omega = \omega_s$. $A_{max} = 0.5$ dB, and $A_{min} = 40$ dB.

D 16.4 A low-pass filter is required to pass all signals within its passband, extending from 0 to 4 kHz, with a transmission variation of at most 10% (i.e., the ratio of the maximum to minimum transmission in the passband should not exceed 1.1). The transmission in the stopband, which extends from 5 kHz to ∞, should not exceed 0.1% of the maximum passband transmission. What are the values of A_{max}, A_{min}, and the selectivity factor for this filter?

16.5 A low-pass filter is specified to have $A_{max} = 1$ dB and $A_{min} = 10$ dB. It is found that these specifications can be just met with a single-time-constant RC circuit having a time constant of 1 s and a dc transmission of unity. What must ω_p and ω_s of this filter be? What is the selectivity factor?

16.6 Sketch transmission specifications for a high-pass filter having a passband defined by $f \geq 2$ kHz and a stopband defined by $f \leq 1$ kHz. $A_{max} = 0.5$ dB, and $A_{min} = 50$ dB.

16.7 Sketch transmission specifications for a bandstop filter that is required to pass signals over the bands $0 \leq f \leq$ 10 kHz and 20 kHz $\leq f \leq \infty$ with A_{max} of 1 dB. The stopband extends from f = 12 kHz to f = 16 kHz, with a minimum required attenuation of 40 dB.

Section 16.2: The Filter Transfer Function

16.8 Consider a fifth-order filter whose poles are all at a radial distance from the origin of 10^3 rad/s. One pair of complex conjugate poles is at 18° angles from the $j\omega$ axis, and the other pair is at 54° angles. Give the transfer function in each of the following cases:

(a) The transmission zeros are all at $s = \infty$ and the dc gain is unity.
(b) The transmission zeros are all at $s = 0$ and the high-frequency gain is unity.

What type of filter results in each case?

16.9 A third-order low-pass filter has transmission zeros at $\omega = 2$ rad/s and $\omega = \infty$. Its natural modes are at $s = -1$ and $s = -0.5 \pm j0.8$. The dc gain is unity. Find $T(s)$.

16.10 Find the order N and the form of $T(s)$ of a bandpass filter having transmission zeros as follows: one at $\omega = 0$, one at $\omega = 10^3$ rad/s, one at 3×10^3 rad/s, one at 6×10^3 rad/s, and one at $\omega = \infty$. If this filter has a monotonically decreasing passband transmission with a peak at the center frequency of 2×10^3 rad/s, and equiripple response in the stopbands, sketch the shape of its $|T|$.

***16.11** Analyze the RLC network of Fig. P16.11 to determine its transfer function $V_o(s)/V_i(s)$ and hence its poles and zeros. (*Hint:* Begin the analysis at the output and work your way back to the input.)

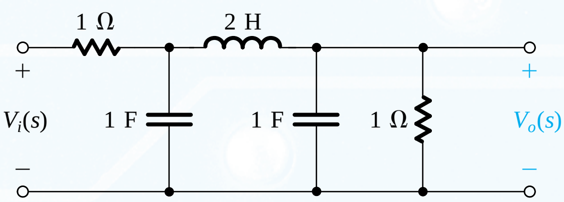


Figure P16.11

Section 16.3: Butterworth and Chebyshev Filters

D 16.12 Determine the order N of the Butterworth filter for which $A_{max} = 1$ dB, $A_{min} \geq 20$ dB, and the selectivity ratio $\omega_s/\omega_p = 1.3$. What is the actual value of minimum stopband attenuation realized? If A_{min} is to be exactly 20 dB, to what value can A_{max} be reduced?

16.13 Calculate the value of attenuation obtained at a frequency 1.6 times the 3-dB frequency of a seventh-order Butterworth filter.

16.14 Find the natural modes of a Butterworth filter with a 1-dB bandwidth of 10^3 rad/s and $N = 5$.

D 16.15 Design a Butterworth filter that meets the following low-pass specifications: $f_p = 10$ kHz, $A_{max} = 2$ dB, $f_s = 15$ kHz, and $A_{min} = 15$ dB. Find N, the natural modes, and $T(s)$. What is the attenuation provided at 20 kHz?

***16.16** Sketch $|T|$ for a seventh-order low-pass Chebyshev filter with $\omega_p = 1$ rad/s and $A_{max} = 1$ dB. Use Eq. (16.18) to determine the values of ω at which $|T| = 1$ and the values of ω at which $|T| = 1/\sqrt{1+\epsilon^2}$. Indicate these values on your sketch. Use Eq. (16.19) to determine $|T|$ at $\omega = 2$ rad/s, and indicate this point on your sketch. For large values of ω, at what rate (in dB/octave) does the transmission decrease?

16.17 Contrast the attenuation provided by a fifth-order Chebyshev filter at $\omega_s = 2\omega_p$ to that provided by a Butterworth filter of equal order. For both, $A_{max} = 1$ dB. Sketch $|T|$ for both filters on the same axes.

D *16.18 It is required to design a low-pass filter to meet the following specifications: $f_p = 3.4$ kHz, $A_{max} = 1$ dB, $f_s = 4$ kHz, $A_{min} = 35$ dB.

(a) Find the required order of Chebyshev filter. What is the excess (above 35 dB) stopband attenuation obtained?
(b) Find the poles and the transfer function.

Section 16.4: First-Order and Second-Order Filter Functions

D 16.19 Use the information displayed in Fig. 16.13 to design a first-order op amp–RC low-pass filter having a 3-dB frequency of 10 kHz, a dc gain magnitude of 10, and an input resistance of 10 kΩ.

D 16.20 Use the information given in Fig. 16.13 to design a first-order op amp–RC high-pass filter with a 3-dB frequency of 100 Hz, a high-frequency input resistance of 100 kΩ, and a high-frequency gain magnitude of unity.

D *16.21 Use the information given in Fig. 16.13 to design a first-order op amp–RC spectrum-shaping network with a transmission zero frequency of 1 kHz, a pole frequency of 100 kHz, and a dc gain magnitude of unity. The low-frequency input resistance is to be 1 kΩ. What is the high-frequency gain that results? Sketch the magnitude of the transfer function versus frequency.

D *16.22 By cascading a first-order op amp–RC low-pass circuit with a first-order op amp–RC high-pass circuit, one can design a wideband bandpass filter. Provide such a design for the case in which the midband gain is 12 dB and the 3-dB bandwidth extends from 100 Hz to 10 kHz. Select appropriate component values under the constraint that no resistors higher than 100 kΩ are to be used and that the input resistance is to be as high as possible.

D 16.23 Derive $T(s)$ for the op amp–RC circuit in Fig. 16.14. We wish to use this circuit as a variable phase shifter by adjusting R. If the input signal frequency is 10^4 rad/s and if $C = 10$ nF, find the values of R required to obtain phase shifts of –30°, –60°, –90°, –120°, and –150°.

16.24 Show that by interchanging R and C in the op amp–RC circuit of Fig. 16.14, the resulting phase shift covers the range 0 to 180° (with 0° at high frequencies and 180° at low frequencies).

16.25 Use the information in Fig. 16.16(a) to obtain the transfer function of a second-order low-pass filter with $\omega_0 = 10^3$ rad/s, $Q = 1$, and dc gain = 1. At what frequency does $|T|$ peak? What is the peak transmission?

D *16.26 Use the information in Fig. 16.16(a) to obtain the transfer function of a second-order low-pass filter that just meets the specifications defined in Fig. 16.3 with $\omega_p = 1$ rad/s and $A_{max} = 3$ dB. Note that there are two possible solutions. For each, find ω_0 and Q. Also, if $\omega_s = 2$ rad/s, find the value of A_{min} obtained in each case.

D **16.27 Use two first-order op amp–RC all-pass circuits in cascade to design a circuit that provides a set of three-phase 60-Hz voltages, each separated by 120° and equal in magnitude, as shown in the phasor diagram of Fig. P16.27. These voltages simulate those used in three-phase power transmission systems. Use 1-µF capacitors.

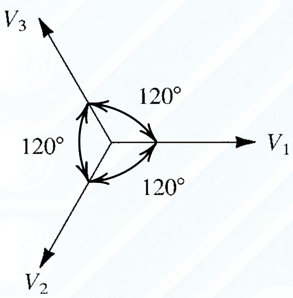


Figure P16.27

16.28 Use the information given in Fig. 16.16(b) to find the transfer function of a second-order high-pass filter with natural modes at $-0.5 \pm j\sqrt{3}/2$ and a high-frequency gain of unity.

D **16.29 (a) Show that $|T|$ of a second-order bandpass function is geometrically symmetrical around the center frequency ω_0. That is, the members of each pair of frequencies ω_1 and ω_2 for which $|T(j\omega_1)| = |T(j\omega_2)|$ are related by $\omega_1\omega_2 = \omega_0^2$.

(b) Find the transfer function of the second-order bandpass filter that meets specifications of the form in Fig. 16.4 where $\omega_{p1} = 8100$ rad/s, $\omega_{p2} = 10{,}000$ rad/s, and $A_{max} = 1$ dB. If $\omega_{s1} = 3000$ rad/s find A_{min} and ω_{s2}.

D *16.30 Use the result of Exercise 16.15 to find the transfer function of a notch filter that is required to eliminate a bothersome interference of 60-Hz frequency. Since the frequency of the interference is not stable, the filter should be designed to provide attenuation ≥20 dB over a 6-Hz band centered around 60 Hz. The dc transmission of the filter is to be unity.

16.31 Consider a second-order all-pass circuit in which errors in the component values result in the frequency of the zeros being slightly lower than that of the poles. Roughly sketch the expected $|T|$. Repeat for the case of the frequency of the zeros slightly higher than the frequency of the poles.

16.32 Consider a second-order all-pass filter in which errors in the component values result in the Q factor of the zeros being greater than the Q factor of the poles. Roughly sketch the expected $|T|$. Repeat for the case of the Q factor of the zeros lower than the Q factor of the poles.

Section 16.5: The Second-Order LCR Resonator

D 16.33 Design the LCR resonator of Fig. 16.17(a) to obtain natural modes with $\omega_0 = 10^4$ rad/s and $Q = 2$. Use $R = 10$ kΩ.

16.34 For the LCR resonator of Fig. 16.17(a), find the change in ω_0 that results from

(a) increasing L by 1%
(b) increasing C by 1%
(c) increasing R by 1%

16.35 Derive an expression for $V_o(s)/V_i(s)$ of the high-pass circuit in Fig. 16.18(c).

D 16.36 Use the circuit of Fig. 16.18(b) to design a low-pass filter with $\omega_0 = 10^5$ rad/s and $Q = 1/\sqrt{2}$. Utilize a 0.01-µF capacitor.

D 16.37 Modify the bandpass circuit of Fig. 16.18(d) to change its center-frequency gain from 1 to 0.5 without changing ω_0 or Q.

16.38 Consider the LCR resonator of Fig. 16.17(a) with node x disconnected from ground and connected to an input signal source V_x, node y disconnected from ground and connected to another input signal source V_y, and node z disconnected from ground and connected to a third input signal source V_z. Use superposition to find the voltage that develops across the resonator, V_o, in terms of V_x, V_y, and V_z.

16.39 Consider the notch circuit shown in Fig. 16.18(i). For what ratio of L_1 to L_2 does the notch occur at $0.9\omega_0$? For this case, what is the magnitude of the transmission at frequencies $\ll\omega_0$? At frequencies $\gg\omega_0$?

Section 16.6: Second-Order Active Filters Based on Inductor Replacement

D 16.40 Design the circuit of Fig. 16.20 (utilizing suitable component values) to realize an inductance of (a) 10 H, (b) 1 H, and (c) 0.1 H.

***16.41** Starting from first principles and assuming ideal op amps, derive the transfer function of the circuit in Fig. 16.22(a).

D *16.42 It is required to design a fifth-order Butterworth filter having a 3-dB bandwidth of 10^4 rad/s and a unity dc gain. Use a cascade of two circuits of the type shown in Fig. 16.22(a) and a first-order op amp–RC circuit of the type shown in Fig. 16.13(a). Select appropriate component values.

D 16.43 Design the circuit of Fig. 16.22(e) to realize an LPN function with $f_0 = 4$ kHz, $f_n = 5$ kHz, $Q = 10$, and a unity dc gain. Select $C_4 = 10$ nF.

D 16.44 Design the all-pass circuit of Fig. 16.22(g) to provide a phase shift of 180° at $f = 1$ kHz and to have $Q = 1$. Use 1-nF capacitors.

16.45 Consider the Antoniou circuit of Fig. 16.20(a) with R_5 eliminated, a capacitor C_6 connected between node 1 and ground, and a voltage source V_2 connected to node 2. Show that the input impedance seen by V_2 is $R_2/s^2C_4C_6R_1R_3$. How does this impedance behave for physical frequencies ($s = j\omega$)? (This impedance is known as a **frequency-dependent negative resistance**, or FDNR.)

D 16.46 Using the transfer function of the LPN filter, given in Table 16.1, derive the design equations also given.

D 16.47 Using the transfer function of the HPN filter, given in Table 16.1, derive the design equations also given.

D **16.48 It is required to design a third-order low-pass filter whose $|T|$ is equiripple in both the passband and the stopband (in the manner shown in Fig. 16.3, except that the response shown is for $N = 5$). The filter passband extends from $\omega = 0$ to $\omega = 1$ rad/s, and the passband transmission varies between 1 and 0.9. The stopband edge is at $\omega = 1.2$ rad/s. The following transfer function was obtained using filter design tables:

$$T(s) = \frac{0.4508(s^2 + 1.6996)}{(s + 0.7294)(s^2 + s0.2786 + 1.0504)}$$

The actual filter realized is to have $\omega_p = 10^4$ rad/s.

(a) Obtain the transfer function of the actual filter by replacing s by $s/10^4$.

(b) Realize this filter as the cascade connection of a first-order LP op amp–RC circuit of the type shown in Fig. 16.13(a) and a second-order LPN circuit of the type shown in Fig. 16.22(e). Each section is to have a dc gain of unity. Select appropriate component values. (*Note:* A filter with an equiripple response in both the passband and the stopband is known as an **elliptic filter**.)

Section 16.7: Second-Order Active Filters Based on the Two-Integrator-Loop Topology

D 16.49 Design the KHN circuit of Fig. 16.24(a) to realize a bandpass filter with a center frequency of 1 kHz and a 3-dB bandwidth of 50 Hz. Use 10-nF capacitors. Give the complete circuit and specify all component values. What value of center-frequency gain is obtained?

D 16.50 (a) Using the KHN biquad with the output summing amplifier of Fig. 16.24(b), show that an all-pass function is realized by selecting $R_L = R_H = R_B/Q$. Also show that the flat gain obtained is KR_F/R_H.
(b) Design the all-pass circuit to obtain $\omega_0 = 10^4$ rad/s, $Q = 2$, and flat gain = 10. Select appropriate component values.

D 16.51 Consider a notch filter with $\omega_n = \omega_0$ realized by using the KHN biquad with an output summing amplifier. If the summing resistors used have 1% tolerances, what is the worst-case percentage deviation between ω_n and ω_0?

D 16.52 Design the circuit of Fig. 16.26 to realize a low-pass notch filter with $\omega_0 = 10^4$ rad/s, $Q = 10$, dc gain = 1, and $\omega_n = 1.2 \times 10^4$ rad/s. Use $C = 10$ nF and $r = 20$ kΩ.

D 16.53 In the all-pass realization using the circuit of Fig. 16.26, which component(s) does one need to trim to adjust (a) only ω_z and (b) only Q_z?

D **16.54 Repeat Problem 16.48 using the Tow–Thomas biquad of Fig. 16.26 to realize the second-order section in the cascade.

Section 16.8: Single-Amplifier Biquadratic Active Filters

D 16.55 Design the circuit of Fig. 16.29 to realize a pair of poles with $\omega_0 = 10^4$ rad/s and $Q = 1/\sqrt{2}$. Use $C_1 = C_2 = 1$ nF.

16.56 Consider the bridged-T network of Fig. 16.28(a) with $R_3 = R_4 = R$ and $C_1 = C_2 = C$, and denote $CR = \tau$. Find the zeros and poles of the bridged-T network. If the network is placed in the negative-feedback path of an ideal infinite-gain op amp, as in Fig. 16.29, find the poles of the closed-loop amplifier.

***16.57** Consider the bridged-T network of Fig. 16.28(b) with $R_1 = R_2 = R$, $C_4 = C$, and $C_3 = C/16$. Let the network be placed in the negative-feedback path of an infinite-gain op amp and let C_4 be disconnected from ground and connected to the input signal source V_i. Analyze the resulting circuit to determine its transfer function $V_o(s)/V_i(s)$, where $V_o(s)$ is the voltage at the op-amp output. Show that the circuit realized is a bandpass filter and find its ω_0, Q, and the center-frequency gain.

D **16.58 Consider the bandpass circuit shown in Fig. 16.30a. Let $C_1 = C_2 = C$, $R_3 = R$, $R_4 = R/4Q^2$, $CR = 2Q/\omega_0$, and $\alpha = 1$. Disconnect the positive input terminal of the op amp from ground and apply V_i through a voltage divider R_1, R_2 to the positive input terminal as well as through R_4/α as before. Analyze the circuit to find its transfer function V_o/V_i. Find the ratio R_2/R_1 so that the circuit realizes (a) an all-pass function and (b) a notch function. Assume the op amp to be ideal.

D *16.59 Derive the transfer function of the circuit in Fig. 16.33(b) assuming the op amp to be ideal. Thus show that the circuit realizes a high-pass function. What is the high-frequency gain of the circuit? Design the circuit for a maximally flat response with a 3-dB frequency of 10^3 rad/s. Use $C_1 = C_2 = 10$ nF. (*Hint:* For a maximally flat response, $Q = 1/\sqrt{2}$ and $\omega_{3dB} = \omega_0$.)

D *16.60 Design a fifth-order Butterworth low-pass filter with a 3-dB bandwidth of 5 kHz and a dc gain of unity using the cascade connection of two Sallen-and-Key circuits (Fig. 16.34c) and a first-order section (Fig. 16.13a). Use a 10-kΩ value for all resistors.

16.61 The process of obtaining the complement of a transfer function by interchanging input and ground, as illustrated in Fig. 16.31, applies to any general network (not just RC networks as shown). Show that if the network n is a bandpass with a center-frequency gain of unity, then the complement obtained is a notch. Verify this by using the RLC circuits of Fig. 16.18(d) and (e).

Section 16.9: Sensitivity

16.62 Evaluate the sensitivities of ω_0 and Q relative to R, L, and C of the bandpass circuit in Fig. 16.18(d).

***16.63** Verify the following sensitivity identities:

(a) If $y = uv$, then $S_x^y = S_x^u + S_x^v$.
(b) If $y = u/v$, then $S_x^y = S_x^u - S_x^v$.
(c) If $y = ku$, where k is a constant, then $S_x^y = S_x^u$.
(d) If $y = u^n$, where n is a constant, then $S_x^y = nS_x^u$.
(e) If $y = f_1(u)$ and $u = f_2(x)$, then $S_x^y = S_u^y \cdot S_x^u$.

***16.64** For the high-pass filter of Fig. 16.33(b), what are the sensitivities of ω_0 and Q to amplifier gain A?

***16.65** For the feedback loop of Fig. 16.34(a), use the expressions in Eqs. (16.77) and (16.78) to determine the sensitivities of ω_0 and Q relative to all passive components for the design in which $R_1 = R_2$.

16.66 For the op amp–RC resonator of Fig. 16.21(b), use the expressions for ω_0 and Q given in the top row of Table 16.1 to determine the sensitivities of ω_0 and Q to all resistors and capacitors.

Section 16.10: Switched-Capacitor Filters

16.67 For the switched-capacitor input circuit of Fig. 16.35(b), in which a clock frequency of 100 kHz is used, what input resistances correspond to capacitance C_1 values of 1 pF and 10 pF?

16.68 For a dc voltage of 1 V applied to the input of the circuit of Fig. 16.35(b), in which C_1 is 1 pF, what charge is transferred for each cycle of the two-phase clock? For a 100-kHz clock, what is the average current drawn from the input source? For a feedback capacitance of 10 pF, what change would you expect in the output for each cycle of the clock? For an amplifier that saturates at ±10 V and the feedback capacitor initially discharged, how many clock cycles would it take to saturate the amplifier? What is the average slope of the staircase output voltage produced?

D 16.69 Repeat Exercise 16.31 for a clock frequency of 400 kHz.

D 16.70 Repeat Exercise 16.31 for $Q = 40$.

D 16.71 Design the circuit of Fig. 16.37(b) to realize, at the output of the second (noninverting) integrator, a maximally flat low-pass function with $\omega_{3dB} = 10^4$ rad/s and unity dc gain. Use a clock frequency $f_c = 100$ kHz and select $C_1 = C_2 = 10$ pF. Give the values of C_3, C_4, C_5, and C_6. (*Hint:* For a maximally flat response, $Q = 1/\sqrt{2}$ and $\omega_{3dB} = \omega_0$.)

Section 16.11: Tuned Amplifiers

***16.72** A voltage signal source with a resistance $R_s = 10$ kΩ is connected to the input of a common-emitter BJT amplifier. Between base and emitter is connected a tuned circuit with

$L = 1\ \mu H$ and $C = 200$ pF. The transistor is biased at 1 mA and has $\beta = 200$, $C_\pi = 10$ pF, and $C_\mu = 1$ pF. The transistor load is a resistance of 5 kΩ. Find ω_0, Q, the 3-dB bandwidth, and the center-frequency gain of this single-tuned amplifier.

16.73 A coil having an inductance of 10 μH is intended for applications around 1-MHz frequency. Its Q is specified to be 200. Find the equivalent parallel resistance R_p. What is the value of the capacitor required to produce resonance at 1 MHz? What additional parallel resistance is required to produce a 3-dB bandwidth of 10 kHz?

16.74 An inductance of 36 μH is resonated with a 1000-pF capacitor. If the inductor is tapped at one-third of its turns and a 1-kΩ resistor is connected across the one-third part, find f_0 and Q of the resonator.

***16.75** Consider a common-emitter transistor amplifier loaded with an inductance L. Ignoring r_o and r_x, show that for $\omega C_\mu \ll 1/\omega L$, the amplifier input admittance is given by

$$Y_{in} \simeq \left(\frac{1}{r_\pi} - \omega^2 C_\mu L g_m\right) + j\omega(C_\pi + C_\mu)$$

(*Note:* The real part of the input admittance can be negative. This can lead to oscillations.)

***16.76** (a) Substituting $s = j\omega$ in the transfer function $T(s)$ of a second-order bandpass filter (see Fig. 16.16c), find $|T(j\omega)|$. For ω in the vicinity of ω_0 [i.e., $\omega = \omega_0 + \delta\omega = \omega_0(1 + \delta\omega/\omega_0)$, where $\delta\omega/\omega_0 \ll 1$ so that $\omega^2 \simeq \omega_0^2(1 + 2\delta\omega/\omega_0)$], show that, for $Q \gg 1$,

$$|T(j\omega)| \simeq \frac{|T(j\omega_0)|}{\sqrt{1 + 4Q^2(\delta\omega/\omega_0)^2}}$$

(b) Use the result obtained in (a) to show that the 3-dB bandwidth B, of N synchronously tuned sections connected in cascade, is

$$B = (\omega_0/Q)\sqrt{2^{1/N} - 1}$$

****16.77** (a) Using the fact that for $Q \gg 1$ the second-order bandpass response in the neighborhood of ω_0 is the same as the response of a first-order low-pass with 3-dB frequency of $(\omega_0/2Q)$, show that the bandpass response at $\omega = \omega_0 + \delta\omega$, for $\delta\omega \ll \omega_0$, is given by

$$|T(j\omega)| \simeq \frac{|T(j\omega_0)|}{\sqrt{1 + 4Q^2(\delta\omega/\omega_0)^2}}$$

(b) Use the relationship derived in (a) together with Eq. (16.110) to show that a bandpass amplifier with a 3-dB bandwidth B, designed using N synchronously tuned stages, has an overall transfer function given by

$$|T(j\omega)|_{\text{overall}} = \frac{|T(j\omega_0)|_{\text{overall}}}{[1 + 4(2^{1/N} - 1)(\delta\omega/B)^2]^{N/2}}$$

(c) Use the relationship derived in (b) to find the attenuation (in decibels) obtained at a bandwidth $2B$ for $N = 1$ to 5. Also find the ratio of the 30-dB bandwidth to the 3-dB bandwidth for $N = 1$ to 5.

***16.78** This problem investigates the selectivity of maximally flat stagger-tuned amplifiers derived in the manner illustrated in Fig. 16.48.

(a) The low-pass maximally flat (Butterworth) filter having a 3-dB bandwidth $B/2$ and order N has the magnitude response

$$|T| = 1\Big/\sqrt{1 + \left(\frac{\Omega}{B/2}\right)^{2N}}$$

where $\Omega = \text{Im}(p)$ is the frequency in the low-pass domain. (This relationship can be obtained using the information provided in Section 16.3 on Butterworth filters.) Use this expression to obtain for the corresponding bandpass filter at $\omega = \omega_0 + \delta\omega$, where $\delta\omega \ll \omega_0$, the relationship

$$|T| = 1\Big/\sqrt{1 + \left(\frac{\delta\omega}{B/2}\right)^{2N}}$$

(b) Use the transfer function in (a) to find the attenuation (in decibels) obtained at a bandwidth of $2B$ for $N = 1$ to 5. Also find the ratio of the 30-dB bandwidth to the 3-dB bandwidth for $N = 1$ to 5.

****16.79** Consider a sixth-order, stagger-tuned bandpass amplifier with center frequency ω_0 and 3-dB bandwidth B. The poles are to be obtained by shifting those of the third-order maximally flat low-pass filter, given in Fig. 16.10(c). For each of the three resonant circuits, find ω_0, the 3-dB bandwidth, and Q.