CHAPTER 1

Signals and Amplifiers

IN THIS CHAPTER YOU WILL LEARN

1. That electronic circuits process signals, and thus understanding electrical signals is essential to appreciating the material in this book.
2. The Thévenin and Norton representations of signal sources.
3. The representation of a signal as the sum of sine waves.
4. The analog and digital representations of a signal.
5. The most basic and pervasive signal-processing function: signal amplification, and correspondingly, the signal amplifier.
6. How amplifiers are characterized (modeled) as circuit building blocks independent of their internal circuitry.
7. How the frequency response of an amplifier is measured, and how it is calculated, especially in the simple but common case of a single-time-constant (STC) type response.

Introduction

The subject of this book is modern electronics, a field that has come to be known as **microelectronics**. **Microelectronics** refers to the integrated-circuit (IC) technology that at the time of this writing is capable of producing circuits that contain hundreds of millions of components in a small piece of silicon (known as a **silicon chip**) whose area is on the order of 100 mm^2. One such microelectronic circuit, for example, is a complete digital computer, which accordingly is known as a **microcomputer** or, more generally, a **microprocessor**.

In this book we shall study electronic devices that can be used singly (in the design of **discrete circuits**) or as components of an **integrated-circuit (IC)** chip. We shall study the design and analysis of interconnections of these devices, which form discrete and integrated circuits of varying complexity and perform a wide variety of functions. We shall also learn about available IC chips and their application in the design of electronic systems.

The purpose of this first chapter is to introduce some basic concepts and terminology. In particular, we shall learn about signals and about one of the most important signal-processing functions electronic circuits are designed to perform, namely, signal amplification. We shall then look at circuit representations or models for linear amplifiers. These models will be employed in subsequent chapters in the design and analysis of actual amplifier circuits.

In addition to motivating the study of electronics, this chapter serves as a bridge between the study of linear circuits and that of the subject of this book: the design and analysis of electronic circuits.

1.1 Signals

Signals contain information about a variety of things and activities in our physical world. Examples abound: Information about the weather is contained in signals that represent the air temperature, pressure, wind speed, etc. The voice of a radio announcer reading the news into a microphone provides an acoustic signal that contains information about world affairs. To monitor the status of a nuclear reactor, instruments are used to measure a multitude of relevant parameters, each instrument producing a signal.

To extract required information from a set of signals, the observer (be it a human or a machine) invariably needs to **process** the signals in some predetermined manner. This **signal processing** is usually most conveniently performed by electronic systems. For this to be possible, however, the signal must first be converted into an electrical signal, that is, a voltage or a current. This process is accomplished by devices known as **transducers**. A variety of transducers exist, each suitable for one of the various forms of physical signals. For instance, the sound waves generated by a human can be converted into electrical signals by using a microphone, which is in effect a pressure transducer. It is not our purpose here to study transducers; rather, we shall assume that the signals of interest already exist in the electrical domain and represent them by one of the two equivalent forms shown in Fig. 1.1. In Fig. 1.1(a) the signal is represented by a voltage source $v_s(t)$ having a source resistance R_s. In the alternate representation of Fig. 1.1(b) the signal is represented by a current source $i_s(t)$ having a source resistance R_s. Although the two representations are equivalent, that in Fig. 1.1(a) (known as the Thévenin form) is preferred when R_s is low. The representation of Fig. 1.1(b) (known as the Norton form) is preferred when R_s is high. The reader will come to appreciate this point later in this chapter when we study the different types of amplifiers. For the time being, it is important to be familiar with Thévenin's and Norton's theorems (for a brief review, see Appendix D) and to note that for the two representations in Fig. 1.1 to be equivalent, their parameters are related by

$$v_s(t) = R_s i_s(t)$$

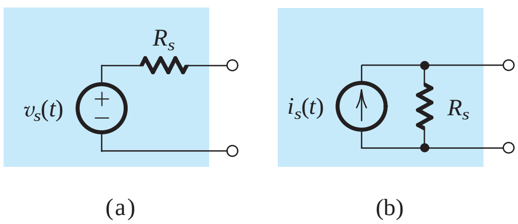


Figure 1.1 Two alternative representations of a signal source: **(a)** the Thévenin form; **(b)** the Norton form.

Example 1.1

The output resistance of a signal source, although inevitable, is an imperfection that limits the ability of the source to deliver its full signal strength to a **load**. To see this point more clearly, consider the signal source when connected to a load resistance R_L as shown in Fig. 1.2. For the case in which the source is represented by its Thévenin equivalent form, find the voltage v_o that appears across R_L, and hence the condition that R_s must satisfy for v_o to be close to the value of v_s. Repeat for the Norton-represented source; in this case finding the current i_o that flows through R_L and hence the condition that R_s must satisfy for i_o to be close to the value of i_s.

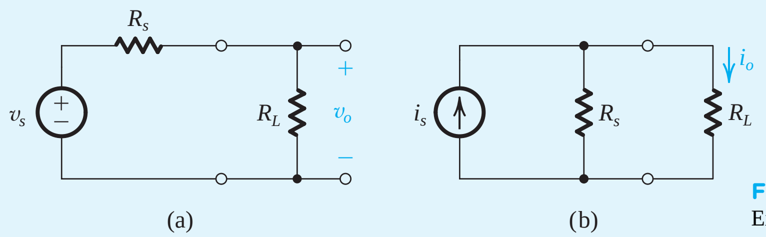


Figure 1.2 Circuits for Example 1.1.

Solution

For the Thévenin-represented signal source shown in Fig. 1.2(a), the output voltage v_o that appears across the load resistance R_L can be found from the ratio of the voltage divider formed by R_s and R_L,

$$v_o = v_s \frac{R_L}{R_L + R_s}$$

From this equation we see that for

$$v_o \simeq v_s$$

the source resistance R_s must be much lower than the load resistance R_L,

$$R_s \ll R_L$$

Thus, for a source represented by its Thévenin equivalent, ideally $R_s = 0$, and as R_s is increased, relative to the load resistance R_L with which this source is intended to operate, the voltage v_o that appears across the load becomes smaller, not a desirable outcome.

Next, we consider the Norton-represented signal source in Fig. 1.2(b). To obtain the current i_o that flows through the load resistance R_L, we utilize the ratio of the current divider formed by R_s and R_L,

$$i_o = i_s \frac{R_s}{R_s + R_L}$$

From this relationship we see that for

$$i_o \simeq i_s$$

the source resistance R_s must be much larger that R_L,

$$R_s \gg R_L$$

Thus for a signal source represented by its Norton equivalent, ideally $R_s = \infty$, and as R_s is reduced, relative to the load resistance R_L with which this source is intended to operate, the current i_o that flows through the load becomes smaller, not a desirable outcome.

Finally, we note that although circuit designers cannot usually do much about the value of R_s; they may have to devise a circuit solution that minimizes or eliminates the loss of signal strength that results when the source is connected to the load.

EXERCISES

1.1 For the signal-source representations shown in Figs. 1.1(a) and 1.1(b), what are the open-circuit output voltages that would be observed? If, for each, the output terminals are short-circuited (i.e., wired together), what current would flow? For the representations to be equivalent, what must the relationship be between v_s, i_s, and R_s?
Ans. For (a), $v_{oc} = v_s(t)$; for (b), $v_{oc} = R_s i_s(t)$; for (a), $i_{sc} = v_s(t)/R_s$; for (b), $i_{sc} = i_s(t)$; for equivalency, $v_s(t) = R_s i_s(t)$

1.2 A signal source has an open-circuit voltage of 10 mV and a short-circuit current of 10 µA. What is the source resistance?
Ans. 1 kΩ

1.3 A signal source that is most conveniently represented by its Thévenin equivalent has v_s = 10 mV and R_s = 1 kΩ. If the source feeds a load resistance R_L, find the voltage v_o that appears across the load for R_L = 100 kΩ, 10 kΩ, 1 kΩ, and 100 Ω. Also, find the lowest permissible value of R_L for which the output voltage is at least 80% of the source voltage.
Ans. 9.9 mV; 9.1 mV; 5 mV; 0.9 mV; 4 kΩ

1.4 A signal source that is most conveniently represented by its Norton equivalent form has i_s = 10 µA and R_s = 100 kΩ. If the source feeds a load resistance R_L, find the current i_o that flows through the load for R_L = 1 kΩ, 10 kΩ, 100 kΩ, and 1 MΩ. Also, find the largest permissible value of R_L for which the load current is at least 80% of the source current.
Ans. 9.9 µA; 9.1 µA; 5 µA; 0.9 µA; 25 kΩ

From the discussion above, it should be apparent that a signal is a time-varying quantity that can be represented by a graph such as that shown in Fig. 1.3. In fact, the information content of the signal is represented by the changes in its magnitude as time progresses; that is, the information is contained in the "wiggles" in the signal waveform. In general, such waveforms are difficult to characterize mathematically. In other words, it is not easy to describe succinctly an arbitrary-looking waveform such as that of Fig. 1.3. Of course, such a description is of great importance for the purpose of designing appropriate signal-processing circuits that perform desired functions on the given signal. An effective approach to signal characterization is studied in the next section.

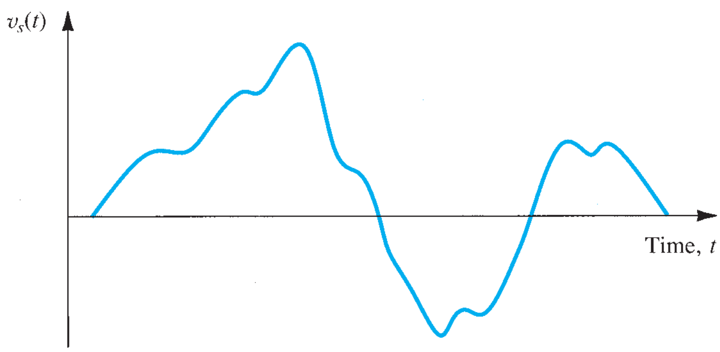


Figure 1.3 An arbitrary voltage signal $v_s(t)$.

1.2 Frequency Spectrum of Signals

An extremely useful characterization of a signal, and for that matter of any arbitrary function of time, is in terms of its **frequency spectrum**. Such a description of signals is obtained through the mathematical tools of **Fourier series** and **Fourier transform**.[1] We are not interested here in the details of these transformations; suffice it to say that they provide the means for representing a voltage signal $v_s(t)$ or a current signal $i_s(t)$ as the sum of sine-wave signals of different frequencies and amplitudes. This makes the sine wave a very important signal in the analysis, design, and testing of electronic circuits. Therefore, we shall briefly review the properties of the sinusoid.

Figure 1.4 shows a sine-wave voltage signal $v_a(t)$,

$$v_a(t) = V_a \sin \omega t \tag{1.1}$$

where V_a denotes the peak value or amplitude in volts and ω denotes the angular frequency in radians per second; that is, $\omega = 2\pi f$ rad/s, where f is the frequency in hertz, $f = 1/T$ Hz, and T is the period in seconds.

The sine-wave signal is completely characterized by its peak value V_a, its frequency ω, and its phase with respect to an arbitrary reference time. In the case depicted in Fig. 1.4, the time origin has been chosen so that the phase angle is 0. It should be mentioned that it is common to express the amplitude of a sine-wave signal in terms of its root-mean-square (rms) value, which is equal to the peak value divided by $\sqrt{2}$. Thus the rms value of the sinusoid $v_a(t)$ of Fig. 1.4 is $V_a/\sqrt{2}$. For instance, when we speak of the wall power supply in our homes as being 120 V, we mean that it has a sine waveform of $120\sqrt{2}$ volts peak value.

Returning now to the representation of signals as the sum of sinusoids, we note that the Fourier series is utilized to accomplish this task for the special case of a signal that is a periodic function of time. On the other hand, the Fourier transform is more general and can be used to obtain the frequency spectrum of a signal whose waveform is an arbitrary function of time.

The Fourier series allows us to express a given periodic function of time as the sum of an infinite number of sinusoids whose frequencies are harmonically related. For instance, the symmetrical square-wave signal in Fig. 1.5 can be expressed as

$$v(t) = \frac{4V}{\pi}\left(\sin \omega_0 t + \tfrac{1}{3}\sin 3\omega_0 t + \tfrac{1}{5}\sin 5\omega_0 t + \cdots\right) \tag{1.2}$$

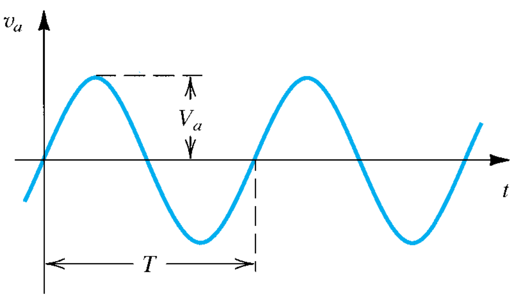


Figure 1.4 Sine-wave voltage signal of amplitude V_a and frequency $f = 1/T$ Hz. The angular frequency $\omega = 2\pi f$ rad/s.

[1]The reader who has not yet studied these topics should not be alarmed. No detailed application of this material will be made until Chapter 9. Nevertheless, a general understanding of Section 1.2 should be very helpful in studying early parts of this book.

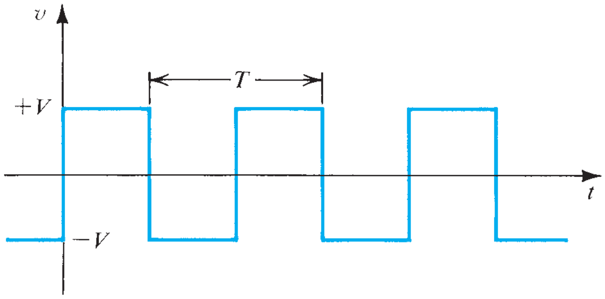


Figure 1.5 A symmetrical square-wave signal of amplitude V.

where V is the amplitude of the square wave and $\omega_0 = 2\pi/T$ (T is the period of the square wave) is called the **fundamental frequency**. Note that because the amplitudes of the harmonics progressively decrease, the infinite series can be truncated, with the truncated series providing an approximation to the square waveform.

The sinusoidal components in the series of Eq. (1.2) constitute the frequency spectrum of the square-wave signal. Such a spectrum can be graphically represented as in Fig. 1.6, where the horizontal axis represents the angular frequency ω in radians per second.

The Fourier transform can be applied to a nonperiodic function of time, such as that depicted in Fig. 1.3, and provides its frequency spectrum as a continuous function of frequency, as indicated in Fig. 1.7. Unlike the case of periodic signals, where the spectrum consists of discrete frequencies (at ω_0 and its harmonics), the spectrum of a nonperiodic signal contains in general all possible frequencies. Nevertheless, the essential parts of the spectra of practical signals are usually confined to relatively short segments of the frequency (ω) axis—an observation that is very useful in the processing of such signals. For instance, the spectrum of audible sounds such as speech and music extends from about 20 Hz to about 20 kHz—a frequency range known as the **audio band**. Here we should note that although some musical tones have frequencies above 20 kHz, the human ear is incapable of hearing frequencies that are much above 20 kHz. As another example, analog video signals have their spectra in the range of 0 MHz to 4.5 MHz.

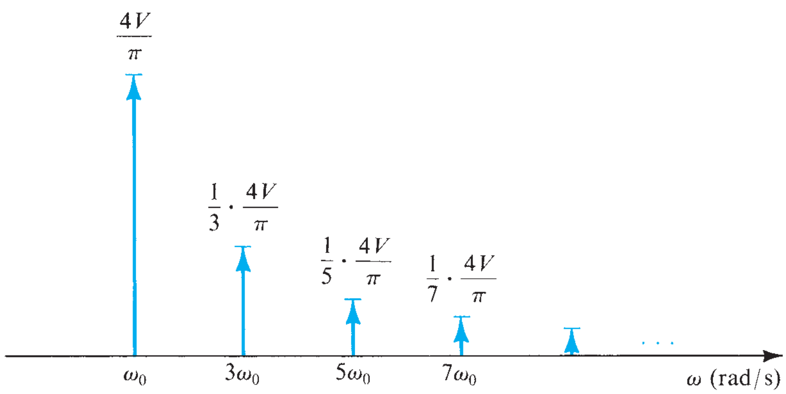


Figure 1.6 The frequency spectrum (also known as the **line spectrum**) of the periodic square wave of Fig. 1.5.

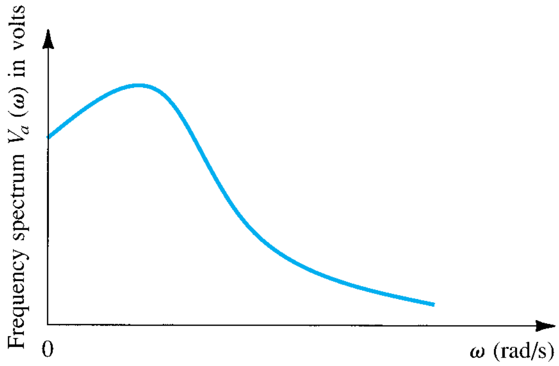


Figure 1.7 The frequency spectrum of an arbitrary waveform such as that in Fig. 1.3.

We conclude this section by noting that a signal can be represented either by the manner in which its waveform varies with time, as for the voltage signal $v_a(t)$ shown in Fig. 1.3, or in terms of its frequency spectrum, as in Fig. 1.7. The two alternative representations are known as the time-domain representation and the frequency-domain representation, respectively. The frequency-domain representation of $v_a(t)$ will be denoted by the symbol $V_a(\omega)$.

EXERCISES

1.5 Find the frequencies f and ω of a sine-wave signal with a period of 1 ms.
Ans. $f = 1000$ Hz; $\omega = 2\pi \times 10^3$ rad/s

1.6 What is the period T of sine waveforms characterized by frequencies of (a) $f = 60$ Hz? (b) $f = 10^{-3}$ Hz? (c) $f = 1$ MHz?
Ans. 16.7 ms; 1000 s; 1 μs

1.7 The UHF (ultra high frequency) television broadcast band begins with channel 14 and extends from 470 MHz to 806 MHz. If 6 MHz is allocated for each channel, how many channels can this band accommodate?
Ans. 56; channels 14 to 69

1.8 When the square-wave signal of Fig. 1.5, whose Fourier series is given in Eq. (1.2), is applied to a resistor, the total power dissipated may be calculated directly using the relationship $P = 1/T \int_0^T (v^2/R)\, dt$ or indirectly by summing the contribution of each of the harmonic components, that is, $P = P_1 + P_3 + P_5 + \dots$, which may be found directly from rms values. Verify that the two approaches are equivalent. What fraction of the energy of a square wave is in its fundamental? In its first five harmonics? In its first seven? First nine? In what number of harmonics is 90% of the energy? (Note that in counting harmonics, the fundamental at ω_0 is the first, the one at $2\omega_0$ is the second, etc.)
Ans. 0.81; 0.93; 0.95; 0.96; 3

1.3 Analog and Digital Signals

The voltage signal depicted in Fig. 1.3 is called an **analog signal**. The name derives from the fact that such a signal is analogous to the physical signal that it represents. The magnitude of an analog signal can take on any value; that is, the amplitude of an analog signal exhibits a continuous variation over its range of activity. The vast majority of signals in the world around us are analog. Electronic circuits that process such signals are known as **analog circuits**. A variety of analog circuits will be studied in this book.

An alternative form of signal representation is that of a sequence of numbers, each number representing the signal magnitude at an instant of time. The resulting signal is called a **digital signal**. To see how a signal can be represented in this form—that is, how signals can be converted from analog to digital form—consider Fig. 1.8(a). Here the curve represents a voltage signal, identical to that in Fig. 1.3. At equal intervals along the time axis, we have marked the time instants t_0, t_1, t_2, and so on. At each of these time instants, the magnitude of the signal is measured, a process known as **sampling**. Figure 1.8(b) shows a representation of the signal of Fig. 1.8(a) in terms of its samples. The signal of Fig. 1.8(b) is defined only at the sampling instants; it no longer is a continuous function of time; rather, it is a **discrete-time signal**. However, since the magnitude of each sample can take any value in a continuous range, the signal in Fig. 1.8(b) is still an analog signal.

Now if we represent the magnitude of each of the signal samples in Fig. 1.8(b) by a number having a finite number of digits, then the signal amplitude will no longer be continuous; rather, it is said to be **quantized**, **discretized**, or **digitized**. The resulting digital signal then is simply a sequence of numbers that represent the magnitudes of the successive signal samples.

The choice of number system to represent the signal samples affects the type of digital signal produced, and has a profound effect on the complexity of the digital circuits required to process the signals. It turns out that the **binary** number system results in the simplest possible digital signals and circuits. In a binary system, each digit in the number takes on one of

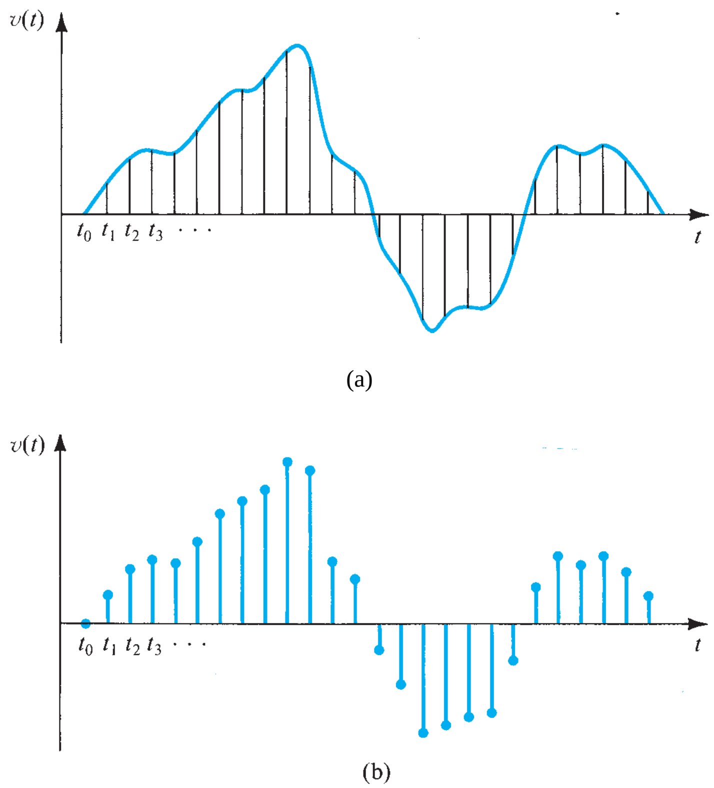


Figure 1.8 Sampling the continuous-time analog signal in **(a)** results in the discrete-time signal in **(b).**

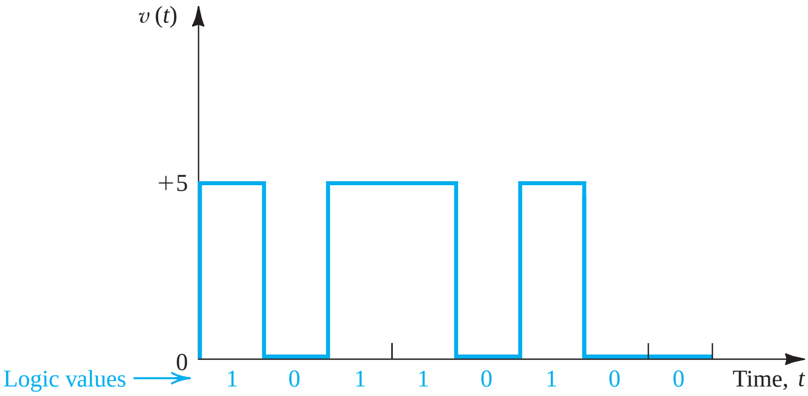


Figure 1.9 Variation of a particular binary digital signal with time.

only two possible values, denoted 0 and 1. Correspondingly, the digital signals in binary systems need have only two voltage levels, which can be labeled low and high. As an example, in some of the digital circuits studied in this book, the levels are 0 V and +5 V. Figure 1.9 shows the time variation of such a digital signal. Observe that the waveform is a pulse train with 0 V representing a 0 signal, or logic 0, and +5 V representing logic 1.

If we use N *b*inary dig*its* (bits) to represent each sample of the analog signal, then the digitized sample value can be expressed as

$$D = b_0 2^0 + b_1 2^1 + b_2 2^2 + \cdots + b_{N-1} 2^{N-1} \tag{1.3}$$

where b_0, b_1, …, b_{N-1}, denote the N bits and have values of 0 or 1. Here bit b_0 is the **least significant bit** (**LSB**), and bit b_{N-1} is the **most significant bit** (**MSB**). Conventionally, this binary number is written as $b_{N-1}\, b_{N-2} \ldots b_0$. We observe that such a representation quantizes the analog sample into one of 2^N levels. Obviously the greater the number of bits (i.e., the larger the N), the closer the digital word D approximates the magnitude of the analog sample. That is, increasing the number of bits reduces the *quantization error* and increases the resolution of the analog-to-digital conversion. This improvement is, however, usually obtained at the expense of more complex and hence more costly circuit implementations. It is not our purpose here to delve into this topic any deeper; we merely want the reader to appreciate the nature of analog and digital signals. Nevertheless, it is an opportune time to introduce a very important circuit building block of modern electronic systems: the **analog-to-digital converter** (**A/D** or **ADC**) shown in block form in Fig. 1.10 The ADC accepts at its input the samples of an analog signal and provides for each input sample the corresponding N-bit digital representation (according to Eq. 1.3) at its N output terminals. Thus although the voltage at the input might be, say, 6.51 V, at each of the output terminals (say, at the ith terminal), the voltage will be either low (0 V) or high (5 V) if b_i is supposed

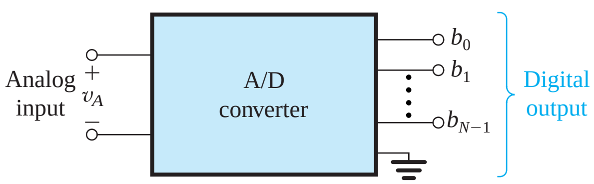


Figure 1.10 Block-diagram representation of the analog-to-digital converter (ADC).

to be 0 or 1, respectively. The dual circuit of the ADC is the **digital-to-analog converter** (**D/A** or **DAC**). It converts an N-bit digital input to an analog output voltage.

Once the signal is in digital form, it can be processed using **digital circuits.** Of course digital circuits can deal also with signals that do not have an analog origin, such as the signals that represent the various instructions of a digital computer.

Since digital circuits deal exclusively with binary signals, their design is simpler than that of analog circuits. Furthermore, digital systems can be designed using a relatively few different kinds of digital circuit blocks. However, a large number (e.g., hundreds of thousands or even millions) of each of these blocks are usually needed. Thus the design of digital circuits poses its own set of challenges to the designer but provides reliable and economic implementations of a great variety of signal-processing functions, many of which are not possible with analog circuits. At the present time, more and more of the signal-processing functions are being performed digitally. Examples around us abound: from the digital watch and the calculator to digital audio systems, digital cameras and, more recently, digital television. Moreover, some longstanding analog systems such as the telephone communication system are now almost entirely digital. And we should not forget the most important of all digital systems, the digital computer.

The basic building blocks of digital systems are logic circuits and memory circuits. We shall study both in this book, beginning in Chapter 13.

One final remark: Although the digital processing of signals is at present all-pervasive, there remain many signal-processing functions that are best performed by analog circuits. Indeed, many electronic systems include both analog and digital parts. It follows that a good electronics engineer must be proficient in the design of both analog and digital circuits, or **mixed-signal** or **mixed-mode** design as it is currently known. Such is the aim of this book.

EXERCISE

1.9 Consider a 4-bit digital word $D = b_3b_2b_1b_0$ (see Eq. 1.3) used to represent an analog signal v_A that varies between 0 V and +15 V.
(a) Give D corresponding to v_A = 0 V, 1 V, 2 V, and 15 V.
(b) What change in v_A causes a change from 0 to 1 in (i) b_0, (ii) b_1, (iii) b_2, and (iv) b_3?
(c) If v_A = 5.2 V, what do you expect D to be? What is the resulting error in representation?
Ans. (a) 0000, 0001, 0010, 1111; (b) +1 V, +2 V, +4 V, +8 V; (c) 0101, –4%

1.4 Amplifiers

In this section, we shall introduce the most fundamental signal-processing function, one that is employed in some form in almost every electronic system, namely, signal amplification. We shall study the amplifier as a circuit building-block; that is, we shall consider its external characteristics and leave the design of its internal circuit to later chapters.

1.4.1 Signal Amplification

From a conceptual point of view the simplest signal-processing task is that of **signal amplification**. The need for amplification arises because transducers provide signals that are said to be "weak," that is, in the microvolt (μV) or millivolt (mV) range and possessing little energy. Such signals are too small for reliable processing, and processing is much easier if the signal magnitude is made larger. The functional block that accomplishes this task is the **signal amplifier**.

It is appropriate at this point to discuss the need for **linearity** in amplifiers. Care must be exercised in the amplification of a signal, so that the information contained in the signal is not changed and no new information is introduced. Thus when we feed the signal shown in Fig. 1.3 to an amplifier, we want the output signal of the amplifier to be an exact replica of that at the input, except of course for having larger magnitude. In other words, the "wiggles" in the output waveform must be identical to those in the input waveform. Any change in waveform is considered to be **distortion** and is obviously undesirable.

An amplifier that preserves the details of the signal waveform is characterized by the relationship

$$v_o(t) = Av_i(t) \tag{1.4}$$

where v_i and v_o are the input and output signals, respectively, and A is a constant representing the magnitude of amplification, known as **amplifier gain**. Equation (1.4) is a linear relationship; hence the amplifier it describes is a **linear amplifier**. It should be easy to see that if the relationship between v_o and v_i contains higher powers of v_i, then the waveform of v_o will no longer be identical to that of v_i. The amplifier is then said to exhibit **nonlinear distortion**.

The amplifiers discussed so far are primarily intended to operate on very small input signals. Their purpose is to make the signal magnitude larger and therefore are thought of as **voltage amplifiers**. The **preamplifier** in the home stereo system is an example of a voltage amplifier.

At this time we wish to mention another type of amplifier, namely, the **power amplifier**. Such an amplifier may provide only a modest amount of voltage gain but substantial current gain. Thus while absorbing little power from the input signal source to which it is connected, often a preamplifier, it delivers large amounts of power to its load. An example is found in the power amplifier of the home stereo system, whose purpose is to provide sufficient power to drive the loudspeaker, which is the amplifier load. Here we should note that the loudspeaker is the output transducer of the stereo system; it converts the electric output signal of the system into an acoustic signal. A further appreciation of the need for linearity can be acquired by reflecting on the power amplifier. A linear power amplifier causes both soft and loud music passages to be reproduced without distortion.

1.4.2 Amplifier Circuit Symbol

The signal amplifier is obviously a two-port network. Its function is conveniently represented by the circuit symbol of Fig. 1.11(a). This symbol clearly distinguishes the input and output ports and indicates the direction of signal flow. Thus, in subsequent diagrams it will not be necessary to label the two ports "input" and "output." For generality we have shown the amplifier to have two input terminals that are distinct from the two output terminals. A more common situation is illustrated in Fig. 1.11(b), where a common terminal exists between the input and output ports of the amplifier. This common terminal is used as a reference point and is called the **circuit ground**.

1.4.3 Voltage Gain

A linear amplifier accepts an input signal $v_I(t)$ and provides at the output, across a load resistance R_L (see Fig. 1.12(a)), an output signal $v_O(t)$ that is a magnified replica of $v_I(t)$. The **voltage gain** of the amplifier is defined by

$$\text{Voltage gain } (A_v) \equiv \frac{v_O}{v_I} \tag{1.5}$$

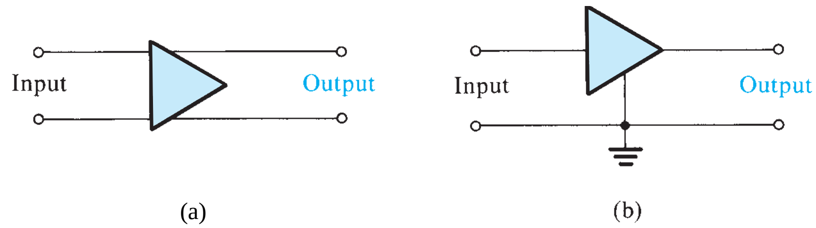


Figure 1.11 **(a)** Circuit symbol for amplifier. **(b)** An amplifier with a common terminal (ground) between the input and output ports.

Fig. 1.12(b) shows the **transfer characteristic** of a linear amplifier. If we apply to the input of this amplifier a sinusoidal voltage of amplitude $\hat{V}$, we obtain at the output a sinusoid of amplitude $A_v\hat{V}$.

1.4.4 Power Gain and Current Gain

An amplifier increases the signal power, an important feature that distinguishes an amplifier from a transformer. In the case of a transformer, although the voltage delivered to the load could be greater than the voltage feeding the input side (the primary), the power delivered to the load (from the secondary side of the transformer) is less than or at most equal to the power supplied by the signal source. On the other hand, an amplifier provides the load with power greater than that obtained from the signal source. That is, amplifiers have power gain. The **power gain** of the amplifier in Fig. 1.12(a) is defined as

$$\text{Power gain } (A_p) \equiv \frac{\text{load power } (P_L)}{\text{input power } (P_I)} \tag{1.6}$$

$$= \frac{v_O i_O}{v_I i_I} \tag{1.7}$$

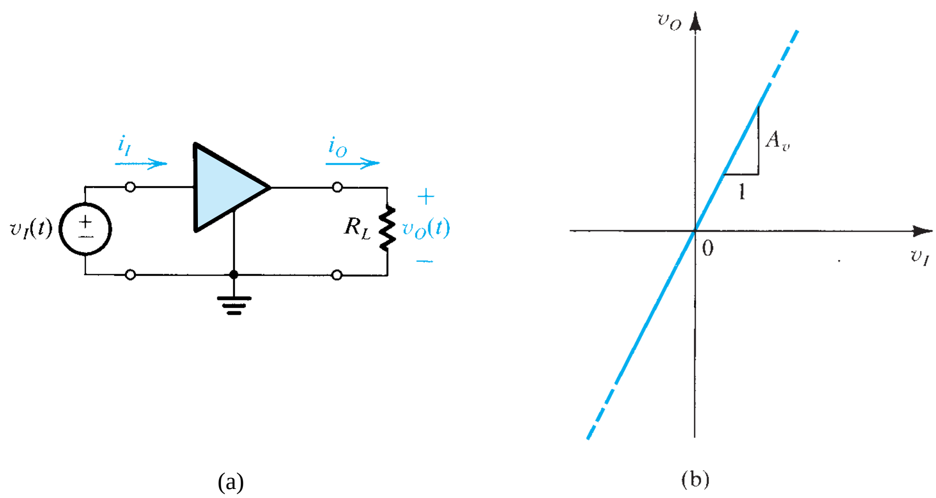


Figure 1.12 **(a)** A voltage amplifier fed with a signal $v_I(t)$ and connected to a load resistance R_L. **(b)** Transfer characteristic of a linear voltage amplifier with voltage gain A_v.

where i_O is the current that the amplifier delivers to the load (R_L), $i_O = v_O/R_L$, and i_I is the current the amplifier draws from the signal source. The **current gain** of the amplifier is defined as

$$\text{Current gain } (A_i) \equiv \frac{i_O}{i_I} \tag{1.8}$$

From Eqs. (1.5) to (1.8) we note that

$$A_p = A_v A_i \tag{1.9}$$

1.4.5 Expressing Gain in Decibels

The amplifier gains defined above are ratios of similarly dimensioned quantities. Thus they will be expressed either as dimensionless numbers or, for emphasis, as V/V for the voltage gain, A/A for the current gain, and W/W for the power gain. Alternatively, for a number of reasons, some of them historic, electronics engineers express amplifier gain with a logarithmic measure. Specifically the voltage gain A_v can be expressed as

$$\text{Voltage gain in decibels} = 20 \log|A_v| \quad \text{dB}$$

and the current gain A_i can be expressed as

$$\text{Current gain in decibels} = 20 \log|A_i| \quad \text{dB}$$

Since power is related to voltage (or current) squared, the power gain A_p can be expressed in decibels as

$$\text{Power gain in decibels} = 10 \log A_p \quad \text{dB}$$

The absolute values of the voltage and current gains are used because in some cases A_v or A_i will be a negative number. A negative gain A_v simply means that there is a 180° phase difference between input and output signals; it does not imply that the amplifier is **attenuating** the signal. On the other hand, an amplifier whose voltage gain is, say, –20 dB is in fact attenuating the input signal by a factor of 10 (i.e., $A_v = 0.1$ V/V).

1.4.6 The Amplifier Power Supplies

Since the power delivered to the load is greater than the power drawn from the signal source, the question arises as to the source of this additional power. The answer is found by observing that amplifiers need dc power supplies for their operation. These dc sources supply the extra power delivered to the load as well as any power that might be dissipated in the internal circuit of the amplifier (such power is converted to heat). In Fig. 1.12(a) we have not explicitly shown these dc sources.

Figure 1.13(a) shows an amplifier that requires two dc sources: one positive of value V_{CC} and one negative of value V_{EE}. The amplifier has two terminals, labeled V^+ and V^-, for connection to the dc supplies. For the amplifier to operate, the terminal labeled V^+ has to be connected to the positive side of a dc source whose voltage is V_{CC} and whose negative side is connected to the circuit ground. Also, the terminal labeled V^- has to be connected to the negative side of a dc source whose voltage is V_{EE} and whose positive side is connected to the circuit ground. Now, if the current drawn from the positive supply is denoted I_{CC} and that from the negative supply is I_{EE} (see Fig. 1.13a), then the dc power delivered to the amplifier is

$$P_{dc} = V_{CC}I_{CC} + V_{EE}I_{EE}$$

If the power dissipated in the amplifier circuit is denoted $P_{\text{dissipated}}$, the power-balance equation for the amplifier can be written as

$$P_{\text{dc}} + P_I = P_L + P_{\text{dissipated}}$$

where P_I is the power drawn from the signal source and P_L is the power delivered to the load. Since the power drawn from the signal source is usually small, the amplifier power **efficiency** is defined as

$$\eta \equiv \frac{P_L}{P_{\text{dc}}} \times 100 \tag{1.10}$$

The power efficiency is an important performance parameter for amplifiers that handle large amounts of power. Such amplifiers, called power amplifiers, are used, for example, as output amplifiers of stereo systems.

In order to simplify circuit diagrams, we shall adopt the convention illustrated in Fig. 1.13(b). Here the V^+ terminal is shown connected to an arrowhead pointing upward and the V^- terminal to an arrowhead pointing downward. The corresponding voltage is indicated next to each arrowhead. Note that in many cases we will not explicitly show the connections of the amplifier to the dc power sources. Finally, we note that some amplifiers require only one power supply.

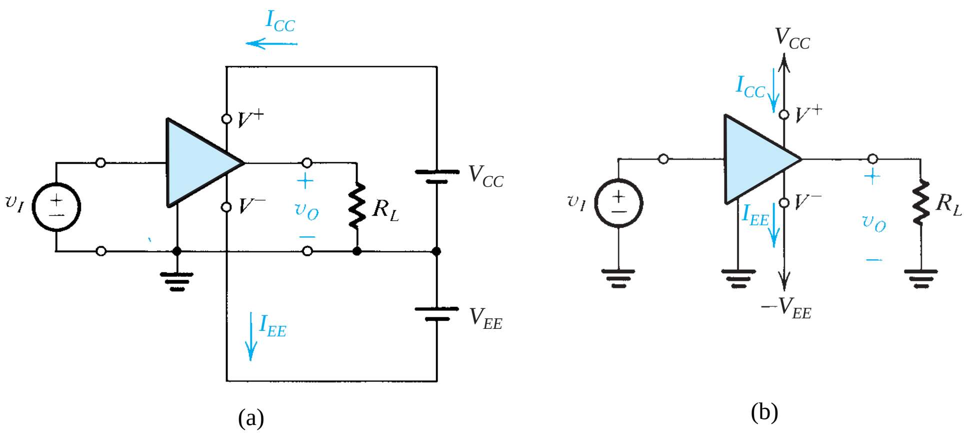


Figure 1.13 An amplifier that requires two dc supplies (shown as batteries) for operation.

Example 1.2

Consider an amplifier operating from ±10-V power supplies. It is fed with a sinusoidal voltage having 1 V peak and delivers a sinusoidal voltage output of 9 V peak to a 1-kΩ load. The amplifier draws a current of 9.5 mA from each of its two power supplies. The input current of the amplifier is found to be sinusoidal with 0.1 mA peak. Find the voltage gain, the current gain, the power gain, the power drawn from the dc supplies, the power dissipated in the amplifier, and the amplifier efficiency.

Solution

$$A_v = \frac{9}{1} = 9 \text{ V/V}$$

or

$$A_v = 20 \log 9 = 19.1 \text{ dB}$$

$$\hat{I}_o = \frac{9 \text{ V}}{1 \text{ k}\Omega} = 9 \text{ mA}$$

$$A_i = \frac{\hat{I}_o}{\hat{I}_i} = \frac{9}{0.1} = 90 \text{ A/A}$$

or

$$A_i = 20 \log 90 = 39.1 \text{ dB}$$

$$P_L = V_{o_{\text{rms}}} I_{o_{\text{rms}}} = \frac{9}{\sqrt{2}} \frac{9}{\sqrt{2}} = 40.5 \text{ mW}$$

$$P_I = V_{i_{\text{rms}}} I_{i_{\text{rms}}} = \frac{1}{\sqrt{2}} \frac{0.1}{\sqrt{2}} = 0.05 \text{ mW}$$

$$A_p = \frac{P_L}{P_I} = \frac{40.5}{0.05} = 810 \text{ W/W}$$

or

$$A_p = 10 \log 810 = 29.1 \text{ dB}$$

$$P_{\text{dc}} = 10 \times 9.5 + 10 \times 9.5 = 190 \text{ mW}$$

$$\begin{aligned} P_{\text{dissipated}} &= P_{\text{dc}} + P_I - P_L \\ &= 190 + 0.05 - 40.5 = 149.6 \text{ mW} \end{aligned}$$

$$\eta = \frac{P_L}{P_{\text{dc}}} \times 100 = 21.3\%$$

From the above example we observe that the amplifier converts some of the dc power it draws from the power supplies to signal power that it delivers to the load.

1.4.7 Amplifier Saturation

Practically speaking, the amplifier transfer characteristic remains linear over only a limited range of input and output voltages. For an amplifier operated from two power supplies the output voltage cannot exceed a specified positive limit and cannot decrease below a specified negative limit. The resulting transfer characteristic is shown in Fig. 1.14, with the positive and negative saturation levels denoted L_+ and L_-, respectively. Each of the two saturation levels is usually within a fraction of a volt of the voltage of the corresponding power supply.

Obviously, in order to avoid distorting the output signal waveform, the input signal swing must be kept within the linear range of operation,

$$\frac{L_-}{A_v} \leq v_I \leq \frac{L_+}{A_v}$$

In Fig. 1.14, which shows two input waveforms and the corresponding output waveforms, the peaks of the larger waveform have been clipped off because of amplifier saturation.

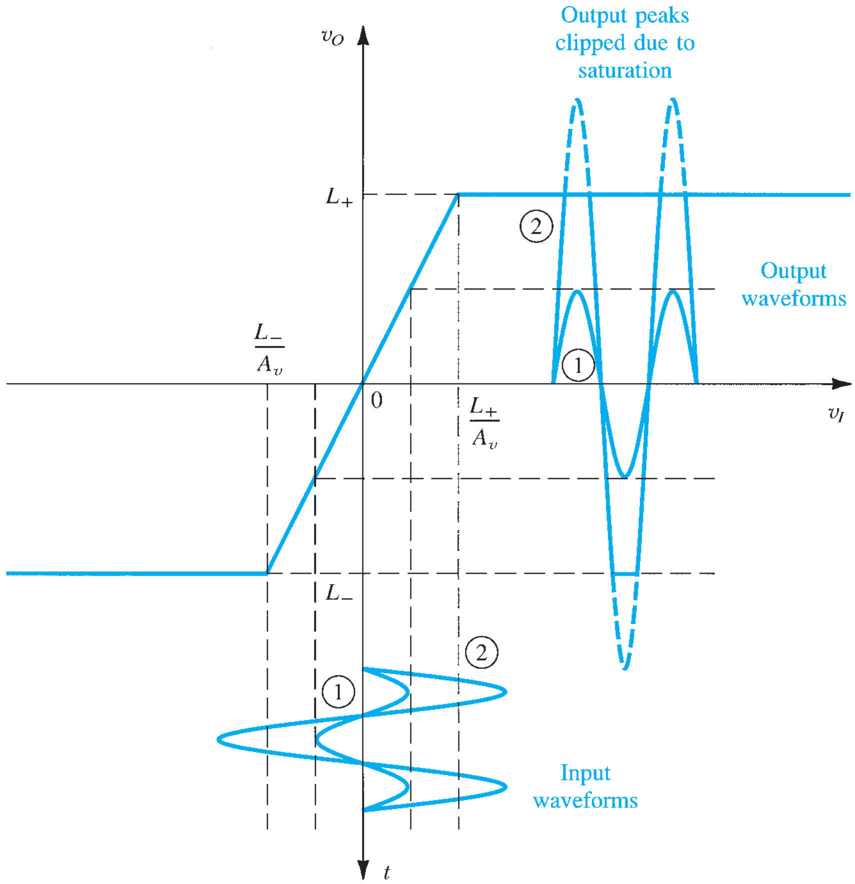


Figure 1.14 An amplifier transfer characteristic that is linear except for output saturation.

1.4.8 Symbol Convention

At this point, we draw the reader's attention to the terminology we shall employ throughout the book. To illustrate the terminology, Fig. 1.15 shows the waveform of a current $i_C(t)$ that is flowing through a branch in a particular circuit. The current $i_C(t)$ consists of a dc component I_C on which is superimposed a sinusoidal component $i_c(t)$ whose peak amplitude is I_c. Observe that at a time t, the **total instantaneous** current $i_C(t)$ is the sum of the dc current I_C and the signal current $i_c(t)$,

$$i_C(t) = I_C + i_c(t) \tag{1.11}$$

where the signal current is given by

$$i_c(t) = I_c \sin \omega t$$

Thus, we state some conventions: Total instantaneous quantities are denoted by a lowercase symbol with uppercase subscript(s), for example, $i_C(t)$, $v_{DS}(t)$. Direct-current (dc) quantities are denoted by an uppercase symbol with uppercase subscript(s), for example I_C, V_{DS}. Incremental

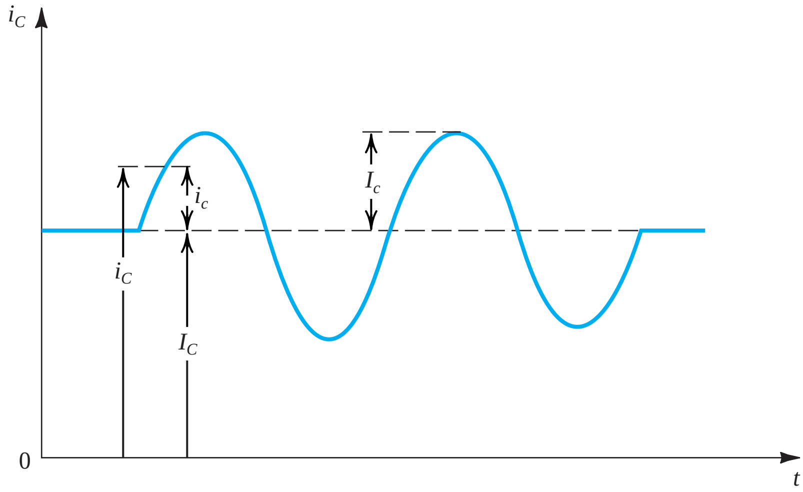


Figure 1.15 Symbol convention employed throughout the book.

signal quantities are denoted by a lowercase symbol with lowercase subscript(s), for example, $i_c(t)$, $v_{gs}(t)$. If the signal is a sine wave, then its amplitude is denoted by an uppercase symbol with lowercase subscript(s), for example I_c, V_{gs}. Finally, although not shown in Fig. 1.15, dc power supplies are denoted by an uppercase letter with a double-letter uppercase subscript, for example, V_{CC}, V_{DD}. A similar notation is used for the dc current drawn from the power supply, for example, I_{CC}, I_{DD}.

EXERCISES

1.10 An amplifier has a voltage gain of 100 V/V and a current gain of 1000 A/A. Express the voltage and current gains in decibels and find the power gain.
Ans. 40 dB; 60 dB; 50 dB

1.11 An amplifier operating from a single 15-V supply provides a 12-V peak-to-peak sine-wave signal to a 1-kΩ load and draws negligible input current from the signal source. The dc current drawn from the 15-V supply is 8 mA. What is the power dissipated in the amplifier, and what is the amplifier efficiency?
Ans. 102 mW; 15%

1.5 Circuit Models for Amplifiers

A substantial part of this book is concerned with the design of amplifier circuits that use transistors of various types. Such circuits will vary in complexity from those using a single transistor to those with 20 or more devices. In order to be able to apply the resulting amplifier circuit as a building block in a system, one must be able to characterize, or **model**, its terminal behavior. In this section, we study simple but effective amplifier models. These models apply irrespective of the complexity of the internal circuit of the amplifier. The values of the model parameters can be found either by analyzing the amplifier circuit or by performing measurements at the amplifier terminals.

1.5.1 Voltage Amplifiers

Figure 1.16(a) shows a circuit model for the voltage amplifier. The model consists of a voltage-controlled voltage source having a gain factor A_{vo}, an input resistance R_i that accounts for the fact that the amplifier draws an input current from the signal source, and an output resistance R_o that accounts for the change in output voltage as the amplifier is called upon to supply output current to a load. To be specific, we show in Fig. 1.16(b) the amplifier model fed with a signal voltage source v_s having a resistance R_s and connected at the output to a load resistance R_L. The nonzero output resistance R_o causes only a fraction of $A_{vo}v_i$ to appear across the output. Using the voltage-divider rule we obtain

$$v_o = A_{vo}v_i \frac{R_L}{R_L + R_o}$$

Thus the voltage gain is given by

$$A_v \equiv \frac{v_o}{v_i} = A_{vo}\frac{R_L}{R_L + R_o} \tag{1.12}$$

It follows that in order not to lose gain in coupling the amplifier output to a load, the output resistance R_o should be much smaller than the load resistance R_L. In other words, for a given R_L one must design the amplifier so that its R_o is much smaller than R_L. Furthermore, there are applications in which R_L is known to vary over a certain range. In order to keep the output voltage v_o as constant as possible, the amplifier is designed with R_o much smaller than the lowest value of R_L. An ideal voltage amplifier is one with $R_o = 0$. Equation (1.12) indicates also that for $R_L = \infty$, $A_v = A_{vo}$. Thus A_{vo} is the voltage gain of the unloaded amplifier, or the **open-circuit voltage gain**. It should also be clear that in specifying the voltage gain of an amplifier, one must also specify the value of load resistance

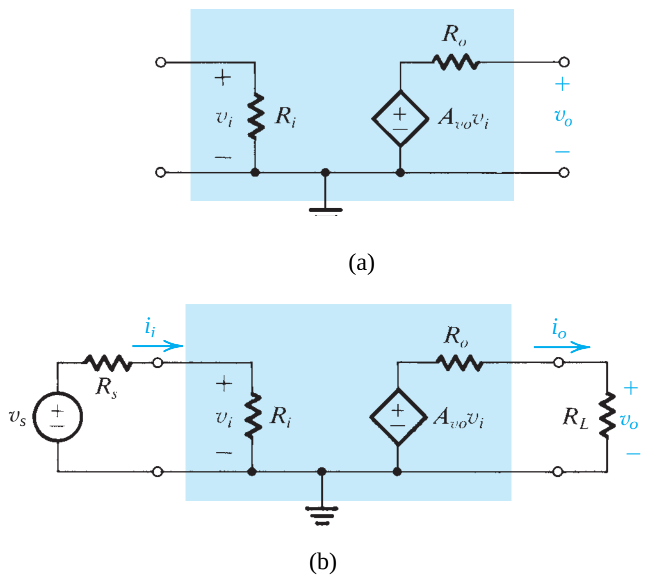


Figure 1.16 **(a)** Circuit model for the voltage amplifier. **(b)** The voltage amplifier with input signal source and load.

at which this gain is measured or calculated. If a load resistance is not specified, it is normally assumed that the given voltage gain is the open-circuit gain A_{vo}.

The finite input resistance R_i introduces another voltage-divider action at the input, with the result that only a fraction of the source signal v_s actually reaches the input terminals of the amplifier; that is,

$$v_i = v_s \frac{R_i}{R_i + R_s} \tag{1.13}$$

It follows that in order not to lose a significant portion of the input signal in coupling the signal source to the amplifier input, the amplifier must be designed to have an input resistance R_i much greater than the resistance of the signal source, $R_i \gg R_s$. Furthermore, there are applications in which the source resistance is known to vary over a certain range. To minimize the effect of this variation on the value of the signal that appears at the input of the amplifier, the design ensures that R_i is much greater than the largest value of R_s. An ideal voltage amplifier is one with $R_i = \infty$. In this ideal case both the current gain and power gain become infinite.

The overall voltage gain (v_o/v_s) can be found by combining Eqs. (1.12) and (1.13),

$$\frac{v_o}{v_s} = A_{vo} \frac{R_i}{R_i + R_s} \frac{R_L}{R_L + R_o}$$

There are situations in which one is interested not in voltage gain but only in a significant power gain. For instance, the source signal can have a respectable voltage but a source resistance that is much greater than the load resistance. Connecting the source directly to the load would result in significant signal attenuation. In such a case, one requires an amplifier with a high input resistance (much greater than the source resistance) and a low output resistance (much smaller than the load resistance) but with a modest voltage gain (or even unity gain). Such an amplifier is referred to as a **buffer amplifier**. We shall encounter buffer amplifiers often throughout this book.

EXERCISES

1.12 A transducer characterized by a voltage of 1 V rms and a resistance of 1 MΩ is available to drive a 10-Ω load. If connected directly, what voltage and power levels result at the load? If a unity-gain (i.e., $A_{vo} = 1$) buffer amplifier with 1-MΩ input resistance and 10-Ω output resistance is interposed between source and load, what do the output voltage and power levels become? For the new arrangement, find the voltage gain from source to load, and the power gain (both expressed in decibels).
Ans. 10 μV rms; 10^{-11} W; 0.25 V; 6.25 mW; –12 dB; 44 dB

1.13 The output voltage of a voltage amplifier has been found to decrease by 20% when a load resistance of 1 kΩ is connected. What is the value of the amplifier output resistance?
Ans. 250 Ω

1.14 An amplifier with a voltage gain of +40 dB, an input resistance of 10 kΩ, and an output resistance of 1 kΩ is used to drive a 1-kΩ load. What is the value of A_{vo}? Find the value of the power gain in decibels.
Ans. 100 V/V; 44 dB

1.5.2 Cascaded Amplifiers

To meet given amplifier specifications, we often need to design the amplifier as a cascade of two or more stages. The stages are usually not identical; rather, each is designed to serve a specific purpose. For instance, in order to provide the overall amplifier with a large input resistance, the first stage is usually required to have a large input resistance. Also, in order to equip the overall amplifier with a low output resistance, the final stage in the cascade is usually designed to have a low output resistance. To illustrate the analysis and design of cascaded amplifiers, we consider a practical example.

Example 1.3

Figure 1.17 depicts an amplifier composed of a cascade of three stages. The amplifier is fed by a signal source with a source resistance of 100 kΩ and delivers its output into a load resistance of 100 Ω. The first stage has a relatively high input resistance and a modest gain factor of 10. The second stage has a higher gain factor but lower input resistance. Finally, the last, or output, stage has unity gain but a low output resistance. We wish to evaluate the overall voltage gain, that is, v_L/v_s, the current gain, and the power gain.

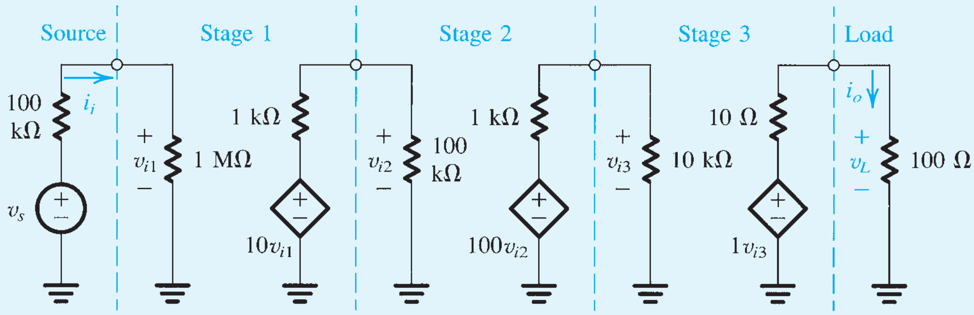


Figure 1.17 Three-stage amplifier for Example 1.3.

Solution

The fraction of source signal appearing at the input terminals of the amplifier is obtained using the voltage-divider rule at the input, as follows:

$$\frac{v_{i1}}{v_s} = \frac{1\ \text{M}\Omega}{1\ \text{M}\Omega + 100\ \text{k}\Omega} = 0.909\ \text{V/V}$$

The voltage gain of the first stage is obtained by considering the input resistance of the second stage to be the load of the first stage; that is,

$$A_{v1} \equiv \frac{v_{i2}}{v_{i1}} = 10\frac{100\ \text{k}\Omega}{100\ \text{k}\Omega + 1\ \text{k}\Omega} = 9.9\ \text{V/V}$$

Similarly, the voltage gain of the second stage is obtained by considering the input resistance of the third stage to be the load of the second stage,

$$A_{v2} \equiv \frac{v_{i3}}{v_{i2}} = 100\frac{10\ \text{k}\Omega}{10\ \text{k}\Omega + 1\ \text{k}\Omega} = 90.9\ \text{V/V}$$

Finally, the voltage gain of the output stage is as follows:

$$A_{v3} \equiv \frac{v_L}{v_{i3}} = 1\frac{100\ \Omega}{100\ \Omega + 10\ \Omega} = 0.909\ \text{V/V}$$

The total gain of the three stages in cascade can be now found from

$$A_v \equiv \frac{v_L}{v_{i1}} = A_{v1}A_{v2}A_{v3} = 818\ \text{V/V}$$

or 58.3 dB.

To find the voltage gain from source to load, we multiply A_v by the factor representing the loss of gain at the input; that is,

$$\begin{aligned}\frac{v_L}{v_s} &= \frac{v_L}{v_{i1}}\frac{v_{i1}}{v_s} = A_v\frac{v_{i1}}{v_s}\\ &= 818 \times 0.909 = 743.6\ \text{V/V}\end{aligned}$$

or 57.4 dB.

The current gain is found as follows:

$$\begin{aligned}A_i \equiv \frac{i_o}{i_i} &= \frac{v_L/100\ \Omega}{v_{i1}/1\ \text{M}\Omega}\\ &= 10^4 \times A_v = 8.18 \times 10^6\ \text{A/A}\end{aligned}$$

or 138.3 dB.

The power gain is found from

$$\begin{aligned}A_p \equiv \frac{P_L}{P_I} &= \frac{v_L i_o}{v_{i1} i_i}\\ &= A_v A_i = 818 \times 8.18 \times 10^6 = 66.9 \times 10^8\ \text{W/W}\end{aligned}$$

or 98.3 dB. Note that

$$A_p(\text{dB}) = \tfrac{1}{2}[A_v(\text{dB}) + A_i(\text{dB})]$$

A few comments on the cascade amplifier in the above example are in order. To avoid losing signal strength at the amplifier input where the signal is usually very small, the first stage is designed to have a relatively large input resistance (1 MΩ), which is much larger than the source resistance. The trade-off appears to be a moderate voltage gain (10 V/V). The second stage does not need to have such a high input resistance; rather, here we need to realize the bulk of the required voltage gain. The third and final, or output, stage is not asked to provide any voltage gain; rather, it functions as a buffer amplifier, providing a relatively large input resistance and a low output resistance, much lower than R_L. It is this stage that enables connecting the amplifier to the 10-Ω load. These points can be made more concrete by solving the following exercises. In so doing, observe that in finding the gain of an amplifier stage in a cascade amplifier, the loading effect of the succeeding amplifier stage must be taken into account as we have done in the above example.

EXERCISES

1.15 What would the overall voltage gain of the cascade amplifier in Example 1.3 be without stage 3?
Ans. 81.8 V/V

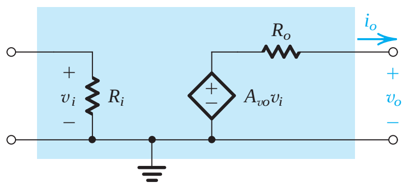
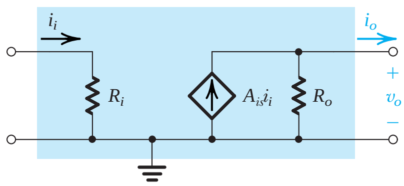
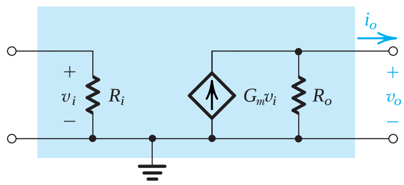
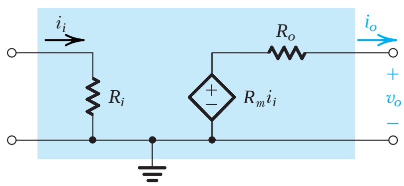
1.16 For the cascade amplifier of Example 1.3, let v_s be 1 mV. Find v_{i1}, v_{i2}, v_{i3}, and v_L.
Ans. 0.91 mV; 9 mV; 818 mV; 744 mV

1.17 (a) Model the three-stage amplifier of Example 1.3 (without the source and load), using the voltage amplifier model. What are the values of R_i, A_{vo}, and R_o?
(b) If R_L varies in the range 10 Ω to 1000 Ω, find the corresponding range of the overall voltage gain, v_o/v_s.
Ans. 1 MΩ, 900 V/V, 10 Ω; 409 V/V to 810 V/V

1.5.3 Other Amplifier Types

In the design of an electronic system, the signal of interest—whether at the system input, at an intermediate stage, or at the output—can be either a voltage or a current. For instance, some transducers have very high output resistances and can be more appropriately modeled as current sources. Similarly, there are applications in which the output current rather than the voltage is of

Table 1.1 The Four Amplifier Types

Type	Circuit Model	Gain Parameter	Ideal Characteristics
Voltage Amplifier		Open-Circuit Voltage Gain $A_{vo} \equiv \left.\frac{v_o}{v_i}\right\rvert_{i_o=0}$ (V/V)	$R_i = \infty$ $R_o = 0$
Current Amplifier		Short-Circuit Current Gain $A_{is} \equiv \left.\frac{i_o}{i_i}\right\rvert_{v_o=0}$ (A/A)	$R_i = 0$ $R_o = \infty$
Transconductance Amplifier		Short-Circuit Transconductance $G_m \equiv \left.\frac{i_o}{v_i}\right\rvert_{v_o=0}$ (A/V)	$R_i = \infty$ $R_o = \infty$
Transresistance Amplifier		Open-Circuit Transresistance $R_m \equiv \left.\frac{v_o}{i_i}\right\rvert_{i_o=0}$ (V/A)	$R_i = 0$ $R_o = 0$

interest. Thus, although it is the most popular, the voltage amplifier considered above is just one of four possible amplifier types. The other three are the current amplifier, the transconductance amplifier, and the transresistance amplifier. Table 1.1 shows the four amplifier types, their circuit models, the definition of their gain parameters, and the ideal values of their input and output resistances.

1.5.4 Relationships between the Four Amplifier Models

Although for a given amplifier a particular one of the four models in Table 1.1 is most preferable, *any of the four can be used to model any amplifier*. In fact, simple relationships can be derived to relate the parameters of the various models. For instance, the open-circuit voltage gain A_{vo} can be related to the short-circuit current gain A_{is} as follows: The open-circuit output voltage given by the voltage amplifier model of Table 1.1 is $A_{vo}v_i$. The current amplifier model in the same table gives an open-circuit output voltage of $A_{is}i_iR_o$. Equating these two values and noting that $i_i = v_i/R_i$ gives

$$A_{vo} = A_{is}\left(\frac{R_o}{R_i}\right) \tag{1.14}$$

Similarly, we can show that

$$A_{vo} = G_m R_o \tag{1.15}$$

and

$$A_{vo} = \frac{R_m}{R_i} \tag{1.16}$$

The expressions in Eqs. (1.14) to (1.16) can be used to relate any two of the gain parameters A_{vo}, A_{is}, G_m, and R_m.

1.5.5 Determining R_i and R_o

From the amplifier circuit models given in Table 1.1, we observe that the input resistance R_i of the amplifier can be determined by applying an input voltage v_i and measuring (or calculating) the input current i_i; that is, $R_i = v_i/i_i$. The output resistance is found as the ratio of the open-circuit output voltage to the short-circuit output current. Alternatively, the output resistance can be found by eliminating the input signal source (then i_i and v_i will both be zero) and applying a voltage signal v_x to the output of the amplifier, as shown in Fig. 1.18. If we denote the current drawn from v_x *into* the output terminals as i_x (note that i_x is opposite in direction to i_o), then $R_o = v_x/i_x$. Although these techniques are conceptually correct, in actual practice more refined methods are employed in measuring R_i and R_o.

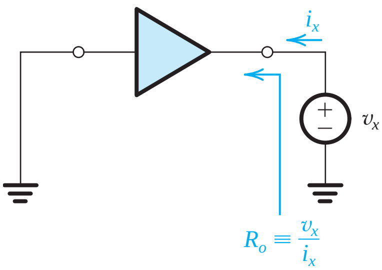


Figure 1.18 Determining the output resistance.

1.5.6 Unilateral Models

The amplifier models considered above are **unilateral**; that is, signal flow is unidirectional, from input to output. Most real amplifiers show some reverse transmission, which is usually undesirable but must nonetheless be modeled. We shall not pursue this point further at this time except to mention that more complete models for linear two-port networks are given in Appendix C. Also, in later chapters, we will find it necessary in certain cases to augment the models of Table 1.1 to take into account the nonunilateral nature of some transistor amplifiers.

Example 1.4

The **bipolar junction transistor (BJT)**, which will be studied in Chapter 6, is a three-terminal device that when powered-up by a dc source (battery) and operated with small signals can be modeled by the linear circuit shown in Fig. 1.19(a). The three terminals are the **base (B)**, the **emitter (E)**, and the **collector (C)**. The heart of the model is a transconductance amplifier represented by an input resistance between B and E (denoted r_π), a short-circuit transconductance g_m, and an output resistance r_o.

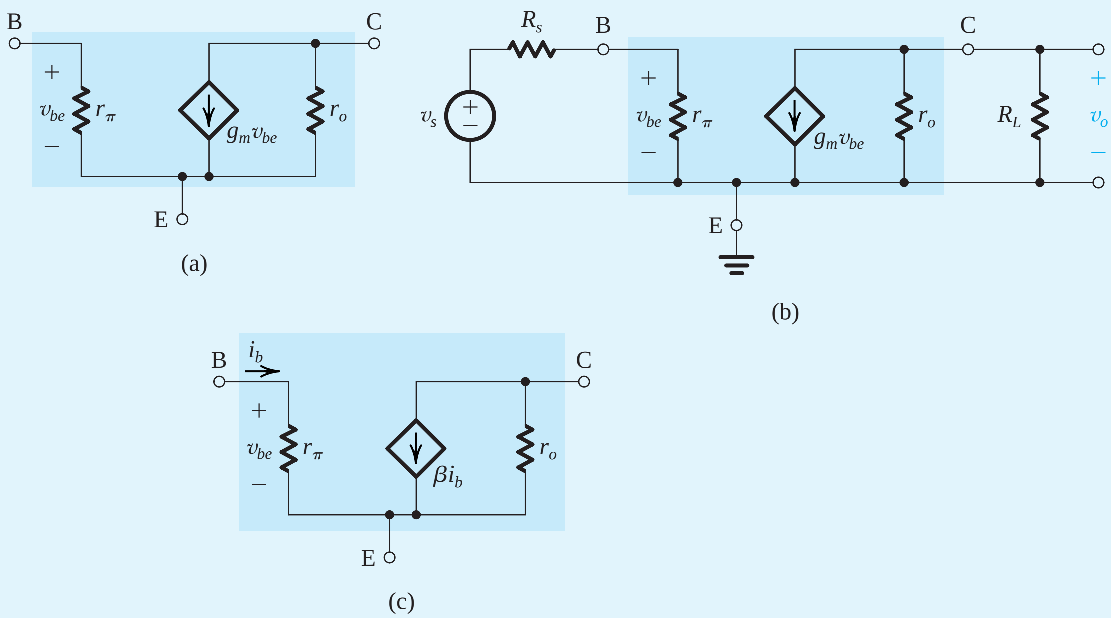


Figure 1.19 **(a)** Small-signal circuit model for a bipolar junction transistor (BJT). **(b)** The BJT connected as an amplifier with the emitter as a common terminal between input and output (called a common-emitter amplifier). **(c)** An alternative small-signal circuit model for the BJT.

(a) With the emitter used as a common terminal between input and output, Fig. 1.19(b) shows a transistor amplifier known as a **common-emitter** or **grounded-emitter** circuit. Derive an expression for the voltage gain v_o/v_s, and evaluate its magnitude for the case $R_s = 5\ \text{k}\Omega$, $r_\pi = 2.5\ \text{k}\Omega$, $g_m = 40\ \text{mA/V}$, $r_o = 100\ \text{k}\Omega$, and $R_L = 5\ \text{k}\Omega$. What would the gain value be if the effect of r_o were neglected?

(b) An alternative model for the transistor in which a current amplifier rather than a transconductance amplifier is utilized is shown in Fig. 1.19(c). What must the short-circuit current gain β be? Give both an expression and a value.

Solution

(a) Refer to Fig. 1.19(b). We use the voltage-divider rule to determine the fraction of input signal that appears at the amplifier input as

$$v_{be} = v_s \frac{r_\pi}{r_\pi + R_s} \tag{1.17}$$

Next we determine the output voltage v_o by multiplying the current $(g_m v_{be})$ by the resistance $(R_L \parallel r_o)$,

$$v_o = -g_m v_{be}(R_L \parallel r_o) \tag{1.18}$$

Substituting for v_{be} from Eq. (1.17) yields the voltage-gain expression

$$\frac{v_o}{v_s} = -\frac{r_\pi}{r_\pi + R_s} g_m (R_L \parallel r_o) \tag{1.19}$$

Observe that the gain is negative, indicating that this amplifier is inverting. For the given component values,

$$\frac{v_o}{v_s} = -\frac{2.5}{2.5+5} \times 40 \times (5 \parallel 100)$$
$$= -63.5 \text{ V/V}$$

Neglecting the effect of r_o, we obtain

$$\frac{v_o}{v_s} \simeq -\frac{2.5}{2.5+5} \times 40 \times 5$$
$$= -66.7 \text{ V/V}$$

which is quite close to the value obtained including r_o. This is not surprising, since $r_o \gg R_L$.

(b) For the model in Fig. 1.19(c) to be equivalent to that in Fig. 1.19(a),

$$\beta i_b = g_m v_{be}$$

But $i_b = v_{be}/r_\pi$; thus,

$$\beta = g_m r_\pi$$

For the values given,

$$\beta = 40 \text{ mA/V} \times 2.5 \text{ k}\Omega$$
$$= 100 \text{ A/A}$$

EXERCISES

1.18 Consider a current amplifier having the model shown in the second row of Table 1.1. Let the amplifier be fed with a signal current-source i_s having a resistance R_s, and let the output be connected to a load resistance R_L. Show that the overall current gain is given by

$$\frac{i_o}{i_s} = A_{is} \frac{R_s}{R_s + R_i} \frac{R_o}{R_o + R_L}$$

1.19 Consider the transconductance amplifier whose model is shown in the third row of Table 1.1. Let a voltage signal source v_s with a source resistance R_s be connected to the input and a load resistance R_L be connected to the output. Show that the overall voltage-gain is given by

$$\frac{v_o}{v_s} = G_m \frac{R_i}{R_i + R_s} (R_o \parallel R_L)$$

1.20 Consider a transresistance amplifier having the model shown in the fourth row of Table 1.1. Let the amplifier be fed with a signal current-source i_s having a resistance R_s, and let the output be connected to a load resistance R_L. Show that the overall gain is given by

$$\frac{v_o}{i_s} = R_m \frac{R_s}{R_s + R_i} \frac{R_L}{R_L + R_o}$$

1.21 Find the input resistance between terminals B and G in the circuit shown in Fig. E1.21. The voltage v_x is a test voltage with the input resistance R_{in} defined as $R_{in} \equiv v_x / i_x$.

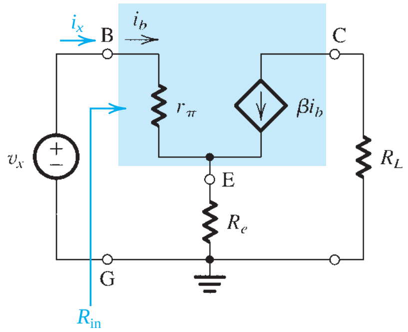


Figure E1.21

Ans. $R_{in} = r_\pi + (\beta + 1)R_e$

1.6 Frequency Response of Amplifiers[2]

From Section 1.2 we know that the input signal to an amplifier can always be expressed as the sum of sinusoidal signals. It follows that an important characterization of an amplifier is in terms of its response to input sinusoids of different frequencies. Such a characterization of amplifier performance is known as the amplifier frequency response.

1.6.1 Measuring the Amplifier Frequency Response

We shall introduce the subject of amplifier frequency response by showing how it can be measured. Figure 1.20 depicts a linear voltage amplifier fed at its input with a sine-wave signal of amplitude V_i and frequency ω. As the figure indicates, the signal measured at the amplifier output also is sinusoidal with exactly the same frequency ω. This is an important point to note: *Whenever a sine-wave signal is applied to a linear circuit, the resulting output is sinusoidal with the same frequency as the input*. In fact, the sine wave is the only signal that does not change shape as it passes through a linear circuit. Observe, however, that the output sinusoid will in general have a different amplitude and will be shifted in phase relative to the input. The ratio of the amplitude of the output sinusoid (V_o) to the amplitude of the input sinusoid (V_i) is the magnitude of the amplifier gain (or transmission) at the test frequency ω. Also, the angle ϕ is the phase of the amplifier transmission at the test frequency ω. If we denote the **amplifier transmission**, or **transfer function** as it is more commonly

[2]Except for its use in the study of the frequency response of op-amp circuits in Sections 2.5 and 2.7, the material in this section will not be needed in a substantial manner until Chapter 9.

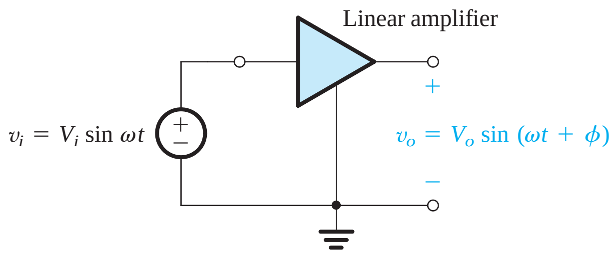


Figure 1.20 Measuring the frequency response of a linear amplifier: At the test frequency ω, the amplifier gain is characterized by its magnitude (V_o/V_i) and phase ϕ.

known, by $T(\omega)$, then

$$|T(\omega)| = \frac{V_o}{V_i}$$

$$\angle T(\omega) = \phi$$

The response of the amplifier to a sinusoid of frequency ω is completely described by $|T(\omega)|$ and $\angle T(\omega)$. Now, to obtain the complete frequency response of the amplifier we simply change the frequency of the input sinusoid and measure the new value for $|T|$ and $\angle T$. The end result will be a table and/or graph of gain magnitude $[|T(\omega)|]$ versus frequency and a table and/or graph of phase angle $[\angle T(\omega)]$ versus frequency. These two plots together constitute the frequency response of the amplifier; the first is known as the **magnitude** or **amplitude response**, and the second is the **phase response**. Finally, we should mention that it is a common practice to express the magnitude of transmission in decibels and thus plot $20 \log |T(\omega)|$ versus frequency.

1.6.2 Amplifier Bandwidth

Figure 1.21 shows the magnitude response of an amplifier. It indicates that the gain is almost constant over a wide frequency range, roughly between ω_1 and ω_2. Signals whose frequencies are below ω_1 or above ω_2 will experience lower gain, with the gain decreasing as we move farther away from ω_1 and ω_2. The band of frequencies over which the gain of the amplifier is almost constant, to within a certain number of decibels (usually 3 dB), is called the **amplifier bandwidth**. Normally the amplifier is designed so that its bandwidth coincides with the spectrum of the signals it is required to amplify. If this were not the case, the amplifier would *distort* the frequency spectrum of the input signal, with different components of the input signal being amplified by different amounts.

1.6.3 Evaluating the Frequency Response of Amplifiers

Above, we described the method used to measure the frequency response of an amplifier. We now briefly discuss the method for analytically obtaining an expression for the frequency response. What we are about to say is just a preview of this important subject, whose detailed study is in Chapter 9.

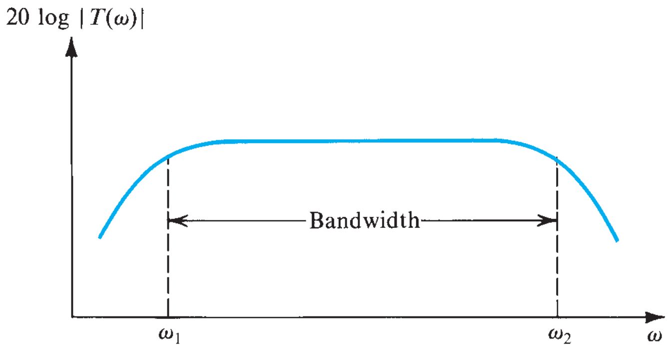


Figure 1.21 Typical magnitude response of an amplifier: $|T(\omega)|$ is the magnitude of the amplifier transfer function—that is, the ratio of the output $V_o(\omega)$ to the input $V_i(\omega)$.

To evaluate the frequency response of an amplifier, one has to analyze the amplifier equivalent circuit model, taking into account all reactive components.[3] Circuit analysis proceeds in the usual fashion but with inductances and capacitances represented by their reactances. An inductance L has a reactance or impedance $j\omega L$, and a capacitance C has a reactance or impedance $1/j\omega C$ or, equivalently, a susceptance or admittance $j\omega C$. Thus in a *frequency-domain* analysis we deal with impedances and/or admittances. The result of the analysis is the amplifier transfer function $T(\omega)$

$$T(\omega) = \frac{V_o(\omega)}{V_i(\omega)}$$

where $V_i(\omega)$ and $V_o(\omega)$ denote the input and output signals, respectively. $T(\omega)$ is generally a complex function whose magnitude $|T(\omega)|$ gives the magnitude of transmission or the magnitude response of the amplifier. The phase of $T(\omega)$ gives the phase response of the amplifier.

In the analysis of a circuit to determine its frequency response, the algebraic manipulations can be considerably simplified by using the **complex frequency variable** s. In terms of s, the impedance of an inductance L is sL and that of a capacitance C is $1/sC$. Replacing the reactive elements with their impedances and performing standard circuit analysis, we obtain the transfer function $T(s)$ as

$$T(s) \equiv \frac{V_o(s)}{V_i(s)}$$

Subsequently, we replace s by $j\omega$ to determine the transfer function for **physical frequencies**, $T(j\omega)$. Note that $T(j\omega)$ is the same function we called $T(\omega)$ above[4]; the additional j is included in order to emphasize that $T(j\omega)$ is obtained from $T(s)$ by replacing s with $j\omega$.

[3]Note that in the models considered in previous sections no reactive components were included. These were simplified models and cannot be used alone to predict the amplifier frequency response.
[4]At this stage, we are using s simply as a shorthand for $j\omega$. We shall not require detailed knowledge of s-plane concepts until Chapter 9. A brief review of s-plane analysis is presented in Appendix F.

1.6.4 Single-Time-Constant Networks

In analyzing amplifier circuits to determine their frequency response, one is greatly aided by knowledge of the frequency-response characteristics of single-time-constant (STC) networks. An STC network is one that is composed of, or can be reduced to, one reactive component (inductance or capacitance) and one resistance. Examples are shown in Fig. 1.22. An STC network formed of an inductance L and a resistance R has a time constant $\tau = L/R$. The time constant τ of an STC network composed of a capacitance C and a resistance R is given by $\tau = CR$.

Appendix E presents a study of STC networks and their responses to sinusoidal, step, and pulse inputs. Knowledge of this material will be needed at various points throughout this book, and the reader will be encouraged to refer to the appendix. At this point we need in particular the frequency response results; we will, in fact, briefly discuss this important topic, now.

Most STC networks can be classified into two categories,[5] **low pass (LP)** and **high pass (HP)**, with each of the two categories displaying distinctly different signal responses. As an example, the STC network shown in Fig. 1.22(a) is of the *low-pass* type and that in Fig. 1.22(b) is of the *high-pass* type. To see the reasoning behind this classification, observe that the transfer function of each of these two circuits can be expressed as a voltage-divider ratio, with the divider composed of a resistor and a capacitor. Now, recalling how the impedance of a capacitor varies with frequency ($Z = 1/j\omega C$), it is easy to see that the transmission of the circuit in Fig. 1.22(a) will decrease with frequency and approach zero as ω approaches ∞. Thus the circuit of Fig. 1.22(a) acts as a **low-pass filter**[6]; it passes low-frequency, sine-wave inputs with little or no attenuation (at $\omega = 0$, the transmission is unity) and attenuates high-frequency input sinusoids. The circuit of Fig. 1.22(b) does the opposite; its transmission is unity at $\omega = \infty$ and decreases as ω is reduced, reaching 0 for $\omega = 0$. The latter circuit, therefore, performs as a **high-pass filter**.

Table 1.2 provides a summary of the frequency-response results for STC networks of both types.[7] Also, sketches of the magnitude and phase responses are given in Figs. 1.23 and 1.24. These frequency-response diagrams are known as **Bode plots** and the **3-dB frequency**

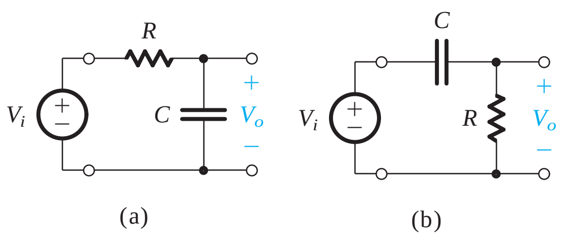


Figure 1.22 Two examples of STC networks: **(a)** a low-pass network and **(b)** a high-pass network.

[5]An important exception is the **all-pass** STC network studied in Chapter 16.
[6]A filter is a circuit that passes signals in a specified frequency band (the filter passband) and stops or severely attenuates (filters out) signals in another frequency band (the filter stopband). Filters will be studied in Chapter 16.
[7]The transfer functions in Table 1.2 are given in general form. For the circuits of Fig. 1.22, $K = 1$ and $\omega_0 = 1/CR$.

Table 1.2 Frequency Response of STC Networks

	Low-Pass (LP)	High-Pass (HP)
Transfer Function $T(s)$	$\dfrac{K}{1+(s/\omega_0)}$	$\dfrac{Ks}{s+\omega_0}$
Transfer Function (for physical frequencies) $T(j\omega)$	$\dfrac{K}{1+j(\omega/\omega_0)}$	$\dfrac{K}{1-j(\omega_0/\omega)}$
Magnitude Response $\lvert T(j\omega)\rvert$	$\dfrac{\lvert K\rvert}{\sqrt{1+(\omega/\omega_0)^2}}$	$\dfrac{\lvert K\rvert}{\sqrt{1+(\omega_0/\omega)^2}}$
Phase Response $\angle T(j\omega)$	$-\tan^{-1}(\omega/\omega_0)$	$\tan^{-1}(\omega_0/\omega)$
Transmission at $\omega = 0$ (dc)	K	0
Transmission at $\omega = \infty$	0	K
3-dB Frequency	$\omega_0 = 1/\tau$; $\tau \equiv$ time constant $\tau = CR$ or L/R	
Bode Plots	in Fig. 1.23	in Fig. 1.24

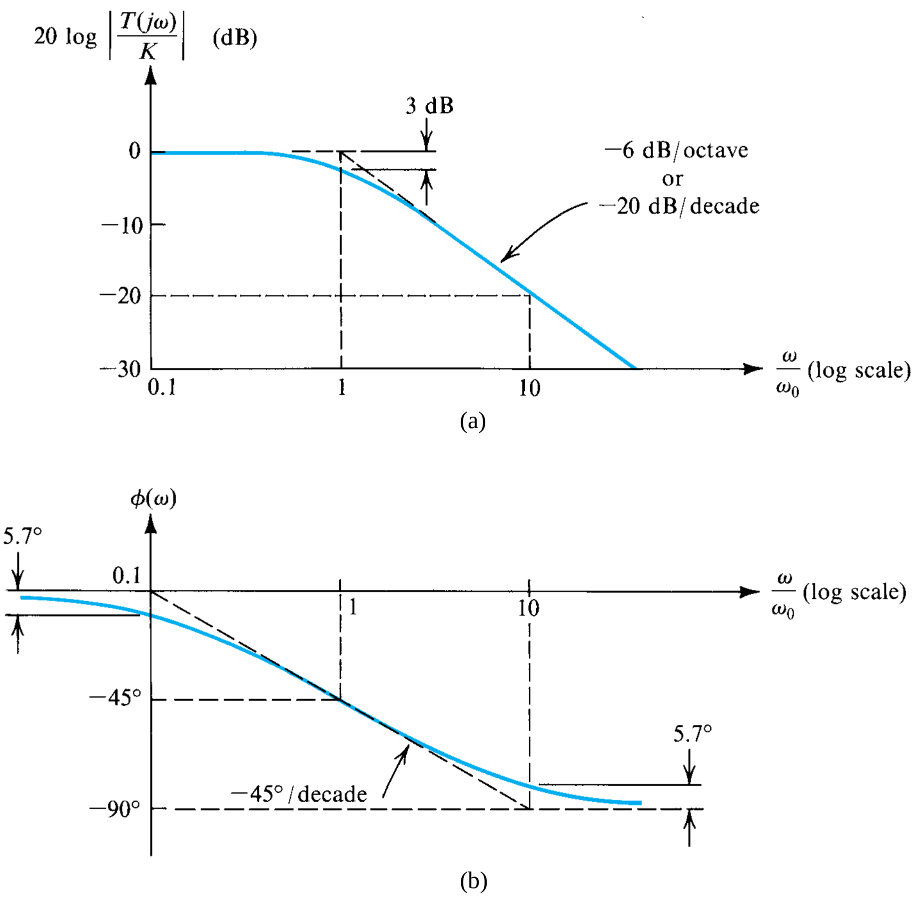


Figure 1.23 **(a)** Magnitude and **(b)** phase response of STC networks of the low-pass type.

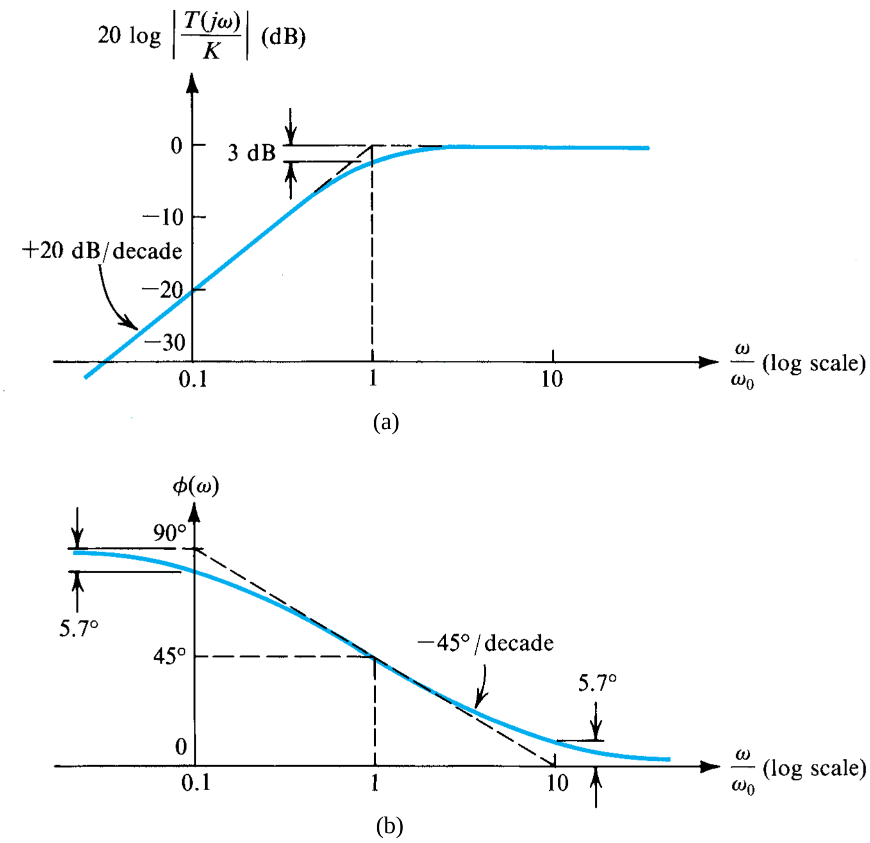


Figure 1.24 **(a)** Magnitude and **(b)** phase response of STC networks of the high-pass type.

(ω_0) is also known as the **corner frequency**, **break frequency**, or **pole frequency**. The reader is urged to become familiar with this information and to consult Appendix E if further clarifications are needed. In particular, it is important to develop a facility for the rapid determination of the time constant τ of an STC circuit. The process is very simple: Set the independent voltge or current source to zero; "grab hold" of the two terminals of the reactive element (capacitor C or inductor L); and determine the equivalent resistance R that appears between these two terminals. The time-constant is then CR or L/R.

Example 1.5

Figure 1.25 shows a voltage amplifier having an input resistance R_i, an input capacitance C_i, a gain factor μ, and an output resistance R_o. The amplifier is fed with a voltage source V_s having a source resistance R_s, and a load of resistance R_L is connected to the output.

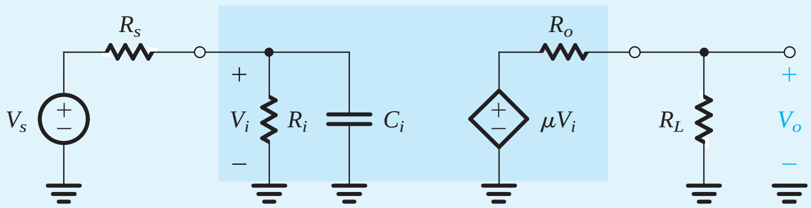


Figure 1.25 Circuit for Example 1.5.

(a) Derive an expression for the amplifier voltage gain V_o/V_s as a function of frequency. From this find expressions for the dc gain and the 3-dB frequency.
(b) Calculate the values of the dc gain, the 3-dB frequency, and the frequency at which the gain becomes 0 dB (i.e., unity) for the case $R_s = 20\ \text{k}\Omega$, $R_i = 100\ \text{k}\Omega$, $C_i = 60$ pF, $\mu = 144$ V/V, $R_o = 200\ \Omega$, and $R_L = 1\ \text{k}\Omega$.
(c) Find $v_o(t)$ for each of the following inputs:
  (i) $v_i = 0.1 \sin 10^2 t$, V
  (ii) $v_i = 0.1 \sin 10^5 t$, V
  (iii) $v_i = 0.1 \sin 10^6 t$, V
  (iv) $v_i = 0.1 \sin 10^8 t$, V

Solution

(a) Utilizing the voltage-divider rule, we can express V_i in terms of V_s as follows

$$V_i = V_s \frac{Z_i}{Z_i + R_s}$$

where Z_i is the amplifier input impedance. Since Z_i is composed of two parallel elements, it is obviously easier to work in terms of $Y_i = 1/Z_i$. Toward that end we divide the numerator and denominator by Z_i, thus obtaining

$$V_i = V_s \frac{1}{1 + R_s Y_i}$$

$$= V_s \frac{1}{1 + R_s[(1/R_i) + sC_i]}$$

Thus,

$$\frac{V_i}{V_s} = \frac{1}{1 + (R_s/R_i) + sC_iR_s}$$

This expression can be put in the standard form for a low-pass STC network (see the top line of Table 1.2) by extracting $[1 + (R_s/R_i)]$ from the denominator; thus we have

$$\frac{V_i}{V_s} = \frac{1}{1 + (R_s/R_i)} \frac{1}{1 + sC_i[(R_sR_i)/(R_s + R_i)]} \tag{1.20}$$

At the output side of the amplifier we can use the voltage-divider rule to write

$$V_o = \mu V_i \frac{R_L}{R_L + R_o}$$

This equation can be combined with Eq. (1.20) to obtain the amplifier transfer function as

$$\frac{V_o}{V_s} = \mu \frac{1}{1 + (R_s/R_i)} \frac{1}{1 + (R_o/R_L)} \frac{1}{1 + sC_i[(R_sR_i)/(R_s + R_i)]} \tag{1.21}$$

We note that only the last factor in this expression is new (compared with the expression derived in the last section). This factor is a result of the input capacitance C_i, with the time constant being

$$\begin{aligned} \tau &= C_i \frac{R_sR_i}{R_s + R_i} \\ &= C_i(R_s \| R_i) \end{aligned} \tag{1.22}$$

We could have obtained this result by inspection: From Fig. 1.25 we see that the input circuit is an STC network and that its time constant can be found by reducing V_s to zero, with the result that the resistance seen by C_i is R_i in parallel with R_s. The transfer function in Eq. (1.21) is of the form $K/(1 + (s/\omega_0))$, which corresponds to a low-pass STC network. The dc gain is found as

$$K \equiv \frac{V_o}{V_s}(s = 0) = \mu \frac{1}{1 + (R_s/R_i)} \frac{1}{1 + (R_o/R_L)} \tag{1.23}$$

The 3-dB frequency ω_0 can be found from

$$\omega_0 = \frac{1}{\tau} = \frac{1}{C_i(R_s \| R_i)} \tag{1.24}$$

Since the frequency response of this amplifier is of the low-pass STC type, the Bode plots for the gain magnitude and phase will take the form shown in Fig. 1.23, where K is given by Eq. (1.23) and ω_0 is given by Eq. (1.24).

(b) Substituting the numerical values given into Eq. (1.23) results in

$$K = 144 \frac{1}{1 + (20/100)} \frac{1}{1 + (200/1000)} = 100 \text{ V/V}$$

Thus the amplifier has a dc gain of 40 dB. Substituting the numerical values into Eq. (1.24) gives the 3-dB frequency

$$\begin{aligned} \omega_0 &= \frac{1}{60 \text{ pF} \times (20 \text{ k}\Omega // 100 \text{ k}\Omega)} \\ &= \frac{1}{60 \times 10^{-12} \times (20 \times 100/(20 + 100)) \times 10^3} = 10^6 \text{ rad/s} \end{aligned}$$

Example 1.5 *continued*

Thus,

$$f_0 = \frac{10^6}{2\pi} = 159.2 \text{ kHz}$$

Since the gain falls off at the rate of –20 dB/decade, starting at ω_0 (see Fig. 1.23a) the gain will reach 0 dB in two decades (a factor of 100); thus we have

$$\text{Unity-gain frequency} = 100 \times \omega_0 = 10^8 \text{ rad/s or } 15.92 \text{ MHz}$$

(c) To find $v_o(t)$ we need to determine the gain magnitude and phase at 10^2, 10^5, 10^6, and 10^8 rad/s. This can be done either approximately utilizing the Bode plots of Fig. 1.23 or exactly utilizing the expression for the amplifier transfer function,

$$T(j\omega) \equiv \frac{V_o}{V_s}(j\omega) = \frac{100}{1 + j(\omega/10^6)}$$

We shall do both:

(i) For $\omega = 10^2$ rad/s, which is $(\omega_0/10^4)$, the Bode plots of Fig. 1.23 suggest that $|T| = K = 100$ and $\phi = 0°$. The transfer function expression gives $|T| \simeq 100$ and $\phi = -\tan^{-1} 10^{-4} \simeq 0°$. Thus,

$$v_o(t) = 10 \sin 10^2 t, \text{ V}$$

(ii) For $\omega = 10^5$ rad/s, which is $(\omega_0/10)$, the Bode plots of Fig. 1.23 suggest that $|T| \simeq K = 100$ and $\phi = -5.7°$. The transfer function expression gives $|T| = 99.5$ and $\phi = -\tan^{-1} 0.1 = -5.7°$. Thus,

$$v_o(t) = 9.95 \sin(10^5 t - 5.7°), \text{ V}$$

(iii) For $\omega = 10^6$ rad/s $= \omega_0$, $|T| = 100/\sqrt{2} = 70.7$ V/V or 37 dB and $\phi = -45°$. Thus,

$$v_o(t) = 7.07 \sin(10^6 t - 45°), \text{ V}$$

(iv) For $\omega = 10^8$ rad/s, which is $(100\omega_0)$, the Bode plots suggest that $|T| = 1$ and $\phi = -90°$. The transfer function expression gives

$$|T| \simeq 1 \quad \text{and} \quad \phi = -\tan^{-1} 100 = -89.4°$$

Thus,

$$v_o(t) = 0.1 \sin(10^8 t - 89.4°), \text{ V}$$

1.6.5 Classification of Amplifiers Based on Frequency Response

Amplifiers can be classified based on the shape of their magnitude-response curve. Figure 1.26 shows typical frequency-response curves for various amplifier types. In Fig. 1.26(a) the gain remains constant over a wide frequency range, but falls off at low and high frequencies. This type of frequency response is common in audio amplifiers.

As will be shown in later chapters, **internal capacitances** in the device (a transistor) cause the falloff of gain at high frequencies, just as C_i did in the circuit of Example 1.5. On the other hand, the falloff of gain at low frequencies is usually caused by **coupling capacitors** used to connect one amplifier stage to another, as indicated in Fig. 1.27. This practice is usually adopted to simplify the design process of the different stages. The coupling capacitors

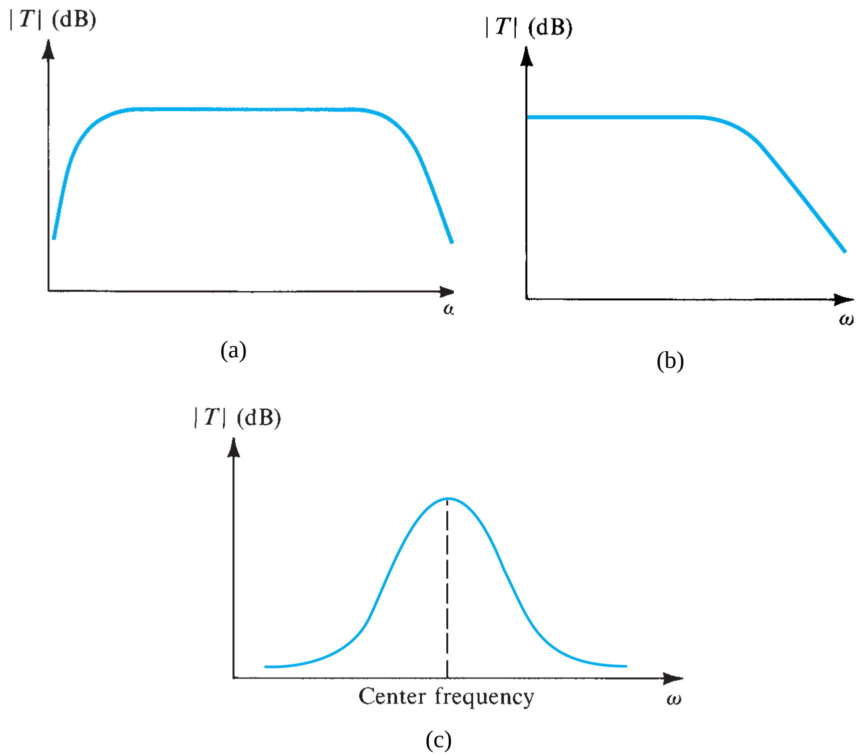


Figure 1.26 Frequency response for **(a)** a capacitively coupled amplifier, **(b)** a direct-coupled amplifier, and **(c)** a tuned or bandpass amplifier.

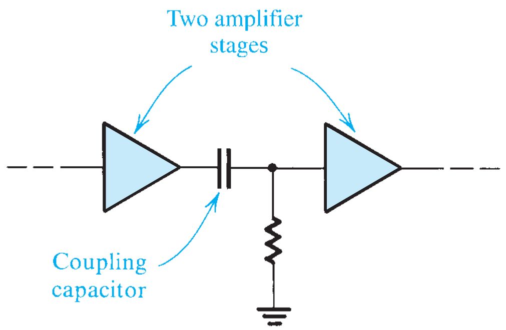


Figure 1.27 Use of a capacitor to couple amplifier stages.

are usually chosen quite large (a fraction of a microfarad to a few tens of microfarads) so that their reactance (impedance) is small at the frequencies of interest. Nevertheless, at sufficiently low frequencies the reactance of a coupling capacitor will become large enough to cause part of the signal being coupled to appear as a voltage drop across the coupling capacitor, thus not reaching the subsequent stage. Coupling capacitors will thus cause loss of gain at low frequencies and cause the gain to be zero at dc. This is not at all surprising, since from

Fig. 1.27 we observe that the coupling capacitor, acting together with the input resistance of the subsequent stage, forms a high-pass STC circuit. It is the frequency response of this high-pass circuit that accounts for the shape of the amplifier frequency response in Fig. 1.26(a) at the low-frequency end.

There are many applications in which it is important that the amplifier maintain its gain at low frequencies down to dc. Furthermore, monolithic integrated-circuit (IC) technology does not allow the fabrication of large coupling capacitors. Thus IC amplifiers are usually designed as **directly coupled** or **dc amplifiers** (as opposed to **capacitively coupled**, or **ac amplifiers**). Figure 1.26(b) shows the frequency response of a dc amplifier. Such a frequency response characterizes what is referred to as a **low-pass amplifier**.

In a number of applications, such as in the design of radio and TV receivers, the need arises for an amplifier whose frequency response peaks around a certain frequency (called the **center frequency**) and falls off on both sides of this frequency, as shown in Fig. 1.26(c). Amplifiers with such a response are called **tuned amplifiers**, **bandpass amplifiers**, or **bandpass filters**. A tuned amplifier forms the heart of the front-end or tuner of a communication receiver; by adjusting its center frequency to coincide with the frequency of a desired communications channel (e.g., a radio station), the signal of this particular channel can be received while those of other channels are attenuated or filtered out.

EXERCISES

1.22 Consider a voltage amplifier having a frequency response of the low-pass STC type with a dc gain of 60 dB and a 3-dB frequency of 1000 Hz. Find the gain in dB at $f = 10$ Hz, 10 kHz, 100 kHz, and 1 MHz.
Ans. 60 dB; 40 dB; 20 dB; 0 dB

D1.23 Consider a transconductance amplifier having the model shown in Table 1.1 with $R_i = 5$ kΩ, $R_o = 50$ kΩ, and $G_m = 10$ mA/V. If the amplifier load consists of a resistance R_L in parallel with a capacitance C_L, convince yourself that the voltage transfer function realized, V_o/V_i, is of the low-pass STC type. What is the lowest value that R_L can have while a dc gain of at least 40 dB is obtained? With this value of R_L connected, find the highest value that C_L can have while a 3-dB bandwidth of at least 100 kHz is obtained.
Ans. 12.5 kΩ; 159.2 pF

D1.24 Consider the situation illustrated in Fig. 1.27. Let the output resistance of the first voltage amplifier be 1 kΩ and the input resistance of the second voltage amplifier (including the resistor shown) be 9 kΩ. The resulting equivalent circuit is shown in Fig. E1.24 where V_s and R_s are the output voltage and output resistance of the first amplifier, C is a coupling capacitor, and R_i is the input resistance of the second amplifier. Convince yourself that V_2/V_s is a high-pass STC function. What is the smallest value for C that will ensure that the 3-dB frequency is not higher than 100 Hz?
Ans. 0.16 μF

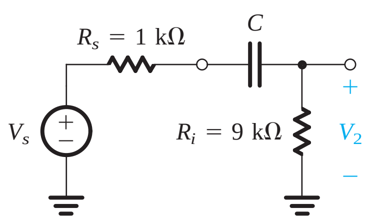


Figure E1.24

Summary

- An electrical signal source can be represented in either the Thévenin form (a voltage source v_s in series with a source resistance R_s) or the Norton form (a current source i_s in parallel with a source resistance R_s). The Thévenin voltage v_s is the open-circuit voltage between the source terminals; the Norton current i_s is equal to the short-circuit current between the source terminals. For the two representations to be equivalent, v_s and $R_s i_s$ must be equal.

- A signal can be represented either by its waveform versus time or as the sum of sinusoids. The latter representation is known as the frequency spectrum of the signal.

- The sine-wave signal is completely characterized by its peak value (or rms value which is the peak $/\sqrt{2}$), its frequency (ω in rad/s or f in Hz; $\omega = 2\pi f$ and $f = 1/T$, where T is the period in seconds), and its phase with respect to an arbitrary reference time.

- Analog signals have magnitudes that can assume any value. Electronic circuits that process analog signals are called analog circuits. Sampling the magnitude of an analog signal at discrete instants of time and representing each signal sample by a number results in a digital signal. Digital signals are processed by digital circuits.

- The simplest digital signals are obtained when the binary system is used. An individual digital signal then assumes one of only two possible values: low and high (say, 0 V and +5 V), corresponding to logic 0 and logic 1, respectively.

- An analog-to-digital converter (ADC) provides at its output the digits of the binary number representing the analog signal sample applied to its input. The output digital signal can then be processed using digital circuits. Refer to Fig. 1.10 and Eq. (1.3).

- The transfer characteristic, v_O versus v_I, of a linear amplifier is a straight line with a slope equal to the voltage gain. Refer to Fig. 1.12.

- Amplifiers increase the signal power and thus require dc power supplies for their operation.

- The amplifier voltage gain can be expressed as a ratio A_v in V/V or in decibels, $20 \log|A_v|$, dB. Similarly, for current gain: A_i A/A or $20 \log|A_i|$, dB. For power gain: A_p W/W or $10 \log A_p$, dB.

- Depending on the signal to be amplified (voltage or current) and on the desired form of output signal (voltage or current), there are four basic amplifier types: voltage, current, transconductance, and transresistance amplifiers. For the circuit models and ideal characteristics of these four amplifier types, refer to Table 1.1. A given amplifier can be modeled by any one of the four models, in which case their parameters are related by the formulas in Eqs. (1.14) to (1.16).

- A sinusoid is the only signal whose waveform is unchanged through a linear circuit. Sinusoidal signals are used to measure the frequency response of amplifiers.

- The transfer function $T(s) \equiv V_o(s)/V_i(s)$ of a voltage amplifier can be determined from circuit analysis. Substituting $s = j\omega$ gives $T(j\omega)$, whose magnitude $|T(j\omega)|$ is the magnitude response, and whose phase $\phi(\omega)$ is the phase response, of the amplifier.

- Amplifiers are classified according to the shape of their frequency response, $|T(j\omega)|$. Refer to Fig. 1.26.

- Single-time-constant (STC) networks are those networks that are composed of, or can be reduced to, one reactive component (L or C) and one resistance (R). The time constant τ is either L/R or CR.

- STC networks can be classified into two categories: low-pass (LP) and high-pass (HP). LP networks pass dc and low frequencies and attenuate high frequencies. The opposite is true for HP networks.

- The gain of an LP (HP) STC circuit drops by 3 dB below the zero-frequency (infinite-frequency) value at a frequency $\omega_0 = 1/\tau$. At high frequencies (low frequencies) the gain falls off at the rate of 6 dB/octave or 20 dB/decade. Refer to Table 1.2 on page 34 and Figs. 1.23 and 1.24. Further details are given in Appendix E.

Computer Simulation Problems

Problems involving design are marked with D throughout the text. As well, problems are marked with asterisks to describe their degree of difficulty. Difficult problems are marked with an asterisk (*); more difficult problems with two asterisks (**); and very challenging and/or time-consuming problems with three asterisks (***).

Circuit Basics

As a review of the basics of circuit analysis and in order for the readers to gauge their preparedness for the study of electronic circuits, this section presents a number of relevant circuit analysis problems. For a summary of Thévenin's and Norton's theorems, refer to Appendix D. The problems are grouped in appropriate categories.

Resistors and Ohm's Law

1.1 Ohm's law relates V, I, and R for a resistor. For each of the situations following, find the missing item:

(a) $R = 1$ kΩ, $V = 10$ V
(b) $V = 10$ V, $I = 1$ mA
(c) $R = 10$ kΩ, $I = 10$ mA
(d) $R = 100$ Ω, $V = 10$ V

1.2 Measurements taken on various resistors are shown below. For each, calculate the power dissipated in the resistor and the power rating necessary for safe operation using standard components with power ratings of 1/8 W, 1/4 W, 1/2 W, 1 W, or 2 W:

(a) 1 kΩ conducting 30 mA
(b) 1 kΩ conducting 40 mA
(c) 10 kΩ conducting 3 mA
(d) 10 kΩ conducting 4 mA
(e) 1 kΩ dropping 20 V
(f) 1 kΩ dropping 11 V

1.3 Ohm's law and the power law for a resistor relate V, I, R, and P, making only two variables independent. For each pair identified below, find the other two:

(a) $R = 1$ kΩ, $I = 10$ mA
(b) $V = 10$ V, $I = 1$ mA
(c) $V = 10$ V, $P = 1$ W
(d) $I = 10$ mA, $P = 0.1$ W
(e) $R = 1$ kΩ, $P = 1$ W

Combining Resistors

1.4 You are given three resistors whose values are 10 kΩ, 20 kΩ, and 40 kΩ. How many different resistances can you create using series and parallel combinations of these three? List them in value order, lowest first. Be thorough and organized. (*Hint:* In your search, first consider all parallel combinations, then consider series combinations, and then consider series-parallel combinations, of which there are two kinds).

1.5 In the analysis and test of electronic circuits, it is often useful to connect one resistor in parallel with another to obtain a nonstandard value, one which is smaller than the smaller of the two resistors. Often, particularly during circuit testing, one resistor is already installed, in which case the second, when connected in parallel, is said to "shunt" the first. If the original resistor is 10 kΩ, what is the value of the shunting resistor needed to reduce the combined value by 1%, 5%, 10%, and 50%? What is the result of shunting a 10-kΩ resistor by 1 MΩ? By 100 kΩ? By 10 kΩ?

Voltage Dividers

1.6 Figure P1.6(a) shows a two-resistor voltage divider. Its function is to generate a voltage V_O (smaller than the power-supply voltage V_{DD}) at its output node X. The circuit looking back at node X is equivalent to that shown in Fig. P1.6(b). Observe that this is the Thévenin equivalent of the voltage divider circuit. Find expressions for V_O and R_O.

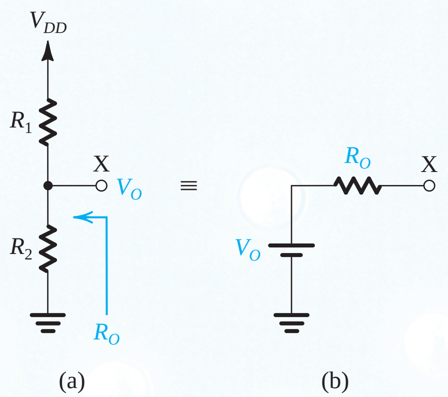


Figure P1.6

1.7 A two-resistor voltage divider employing a 3.3-kΩ and a 6.8-kΩ resistor is connected to a 9-V ground-referenced power supply to provide a relatively low voltage (close to 3V). Sketch the circuit. Assuming exact-valued resistors, what output voltage (measured to ground) and equivalent output resistance result? If the resistors used are not ideal but have a ±5% manufacturing tolerance, what are the extreme output voltages and resistances that can result?

1.8 You are given three resistors, each of 10 kΩ, and a 9-V battery whose negative terminal is connected to ground. With a voltage divider using some or all of your resistors, how many positive-voltage sources of magnitude less than 9 V can you design? List them in order, smallest first. What is the output resistance (i.e., the Thévenin resistance) of each?

D *1.9 Two resistors, with nominal values of 4.7 kΩ and 10 kΩ, are used in a voltage divider with a +15-V supply to create a nominal +10-V output. Assuming the resistor values to be exact, what is the actual output voltage produced? Which resistor must be shunted (paralleled) by what third resistor to create a voltage-divider output of 10.00 V? If an output resistance of exactly 3.33 kΩ is also required, what do you suggest? What should be done if the original 4.7-kΩ and 10-kΩ resistors are used but the requirement is 10.00 V and 3.00 kΩ?

Current Dividers

1.10 Current dividers play an important role in circuit design. Therefore it is important to develop a facility for dealing with current dividers in circuit analysis. Figure P1.10 shows a two-resistor current divider fed with an ideal current source I. Show that

$$I_1 = \frac{R_2}{R_1 + R_2} I$$

$$I_2 = \frac{R_1}{R_1 + R_2} I$$

and find the voltage V that develops across the current divider.

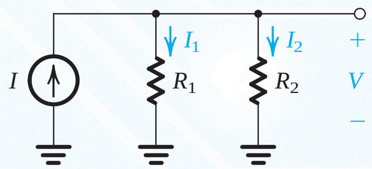


Figure P1.10

D 1.11 Design a simple current divider that will reduce the current provided to a 1-kΩ load to 20% of that available from the source.

D 1.12 A designer searches for a simple circuit to provide one-third of a signal current I to a load resistance R. Suggest a solution using one resistor. What must its value be? What is the input resistance of the resulting current divider? For a particular value R, the designer discovers that the otherwise-best-available resistor is 10% too high. Suggest two circuit topologies using one additional resistor that will solve this problem. What is the value of the resistor required? What is the input resistance of the current divider in each case?

D 1.13 A particular electronic signal source generates currents in the range 0 mA to 1 mA under the condition that its load voltage not exceed 1 V. For loads causing more than 1 V to appear across the generator, the output current is no longer assured but will be reduced by some unknown amount. This circuit limitation, occurring, for example, at the peak of a sine-wave signal, will lead to undesirable signal distortion that must be avoided. If a 10-kΩ load is to be connected, what must be done? What is the name of the circuit you must use? How many resistors are needed? What is (are) the(ir) value(s)?

Thévenin Equivalent Circuits

1.14 For the circuit in Fig. P1.14, find the Thévenin equivalent circuit between terminals (a) 1 and 2, (b) 2 and 3, and (c) 1 and 3.

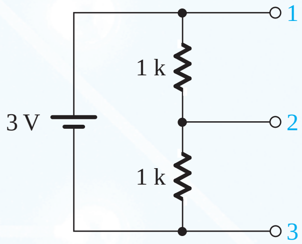


Figure P1.14

1.15 Through repeated application of Thévenin's theorem, find the Thévenin equivalent of the circuit in Fig. P1.15 between node 4 and ground, and hence find the current that flows through a load resistance of 1.5 kΩ connected between node 4 and ground.

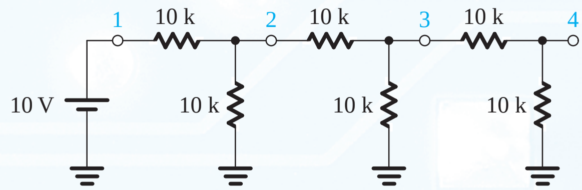


Figure P1.15

Circuit Analysis

1.16 For the circuit shown in Fig. P1.16, find the current in all resistors and the voltage (with respect to ground) at their common node using two methods:

(a) Current: Define branch currents I_1 and I_2 in R_1 and R_2, respectively; identify two equations; and solve them.
(b) Voltage: Define the node voltage V at the common node; identify a single equation; and solve it.

Which method do you prefer? Why?

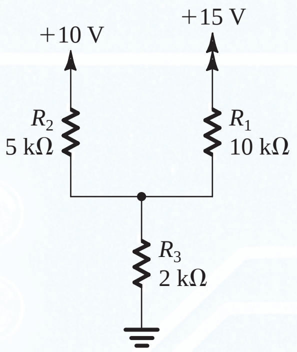


Figure P1.16

1.17 The circuit shown in Fig. P1.17 represents the equivalent circuit of an unbalanced bridge. It is required to calculate the current in the detector branch (R_5) and the voltage across it. Although this can be done by using loop and node equations, a much easier approach is possible: Find the Thévenin equivalent of the circuit to the left of node 1 and the Thévenin equivalent of the circuit to the right of node 2. Then solve the resulting simplified circuit.

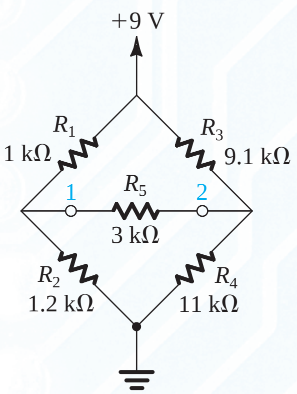


Figure P1.17

1.18 For the circuit in Fig. P1.18, find the equivalent resistance to ground, R_{eq}. To do this, apply a voltage V_x between terminal X and ground and find the current drawn from V_x. Note that you can use particular special properties of the circuit to get the result directly! Now, if R_4 is raised to 1.2 kΩ, what does R_{eq} become?

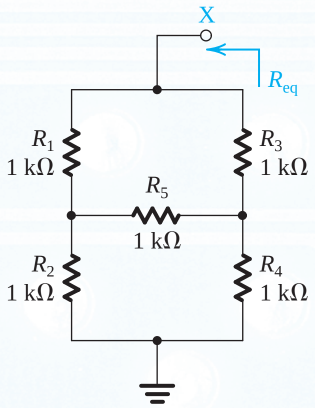


Figure P1.18

AC Circuits

1.19 The periodicity of recurrent waveforms, such as sine waves or square waves, can be completely specified using only one of three possible parameters: radian frequency, ω, in radians per second (rad/s); (conventional) frequency, f, in hertz (Hz); or period T, in seconds (s). As well, each of the parameters can be specified numerically in one of several ways: using letter prefixes associated with the basic units, using scientific notation, or using some combination of both. Thus, for example, a particular period may be specified as 100 ns, 0.1 µs, 10^{-1} µs, 10^{5} ps, or 1×10^{-7} s. (For the definition of the various prefixes used in electronics, see Appendix H) For each of the measures listed below, express the trio of terms in scientific notation associated with the basic unit (e.g., 10^{-7} s rather than 10^{-1} µs).

(a) $T = 10^{-4}$ ms
(b) $f = 1$ GHz
(c) $\omega = 6.28 \times 10^{2}$ rad/s
(d) $T = 10$ s
(e) $f = 60$ Hz
(f) $\omega = 1$ krad/s
(g) $f = 1900$ MHz

1.20 Find the complex impedance, Z, of each of the following basic circuit elements at 60 Hz, 100 kHz, and 1 GHz:

(a) $R = 1$ kΩ
(b) $C = 10$ nF
(c) $C = 2$ pF
(d) $L = 10$ mH
(e) $L = 1$ nH

1.21 Find the complex impedance at 10 kHz of the following networks:

(a) 1 kΩ in series with 10 nF
(b) 1 kΩ in parallel with 0.01 µF

(c) 100 kΩ in parallel with 100 pF
(d) 100 Ω in series with 10 mH

Section 1.1: Signals

1.22 Any given signal source provides an open-circuit voltage, v_{oc}, and a short-circuit current i_{sc}. For the following sources, calculate the internal resistance, R_s; the Norton current, i_s; and the Thévenin voltage, v_s:

(a) $v_{oc} = 10$ V, $i_{sc} = 100$ μA
(b) $v_{oc} = 0.1$ V, $i_{sc} = 10$ μA

1.23 A particular signal source produces an output of 30 mV when loaded by a 100-kΩ resistor and 10 mV when loaded by a 10-kΩ resistor. Calculate the Thévenin voltage, Norton current, and source resistance.

1.24 A temperature sensor is specified to provide 2 mV/°C. When connected to a load resistance of 10 kΩ, the output voltage was measured to change by 10 mV, corresponding to a change in temperature of 10°C. What is the source resistance of the sensor?

1.25 Refer to the Thévenin and Norton representations of the signal source (Fig. 1.1). If the current supplied by the source is denoted i_o and the voltage appearing between the source output terminals is denoted v_o, sketch and clearly label v_o versus i_o for $0 \le i_o \le i_s$.

1.26 The connection of a signal source to an associated signal processor or amplifier generally involves some degree of signal loss as measured at the processor or amplifier input. Considering the two signal-source representations shown in Fig. 1.1, provide two sketches showing each signal-source representation connected to the input terminals (and corresponding input resistance) of a signal processor. What signal-processor input resistance will result in 90% of the open-circuit voltage being delivered to the processor? What input resistance will result in 90% of the short-circuit signal current entering the processor?

Section 1.2: Frequency Spectrum of Signals

1.27 To familiarize yourself with typical values of angular frequency ω, conventional frequency f, and period T, complete the entries in the following table:

Case	ω (rad/s)	f (Hz)	T (s)
a		1×10^9	
b	1×10^9		
c			1×10^{-10}
d		60	
e	6.28×10^3		
f			1×10^{-6}

1.28 For the following peak or rms values of some important sine waves, calculate the corresponding other value:

(a) 117 V rms, a household-power voltage in North America
(b) 33.9 V peak, a somewhat common peak voltage in rectifier circuits
(c) 220 V rms, a household-power voltage in parts of Europe
(d) 220 kV rms, a high-voltage transmission-line voltage in North America

1.29 Give expressions for the sine-wave voltage signals having:

(a) 10-V peak amplitude and 10-kHz frequency
(b) 120-V rms and 60-Hz frequency
(c) 0.2-V peak-to-peak and 1000-rad/s frequency
(d) 100-mV peak and 1-ms period

1.30 Using the information provided by Eq. (1.2) in association with Fig. 1.5, characterize the signal represented by $v(t) = 1/2 + 2/\pi\,(\sin 2000\pi t + \frac{1}{3}\sin 6000\pi t + \frac{1}{5}\sin 10{,}000\pi t + \cdots)$. Sketch the waveform. What is its average value? Its peak-to-peak value? Its lowest value? Its highest value? Its frequency? Its period?

1.31 Measurements taken of a square-wave signal using a frequency-selective voltmeter (called a spectrum analyzer) show its spectrum to contain adjacent components (spectral lines) at 98 kHz and 126 kHz of amplitudes 63 mV and 49 mV, respectively. For this signal, what would direct measurement of the fundamental show its frequency and amplitude to be? What is the rms value of the fundamental? What are the peak-to-peak amplitude and period of the originating square wave?

1.32 What is the fundamental frequency of the highest-frequency square wave for which the fifth harmonic is barely audible by a relatively young listener? What is the fundamental frequency of the lowest-frequency square wave for which the fifth and some of the higher harmonics are directly heard? (Note that the psychoacoustic properties of human hearing allow a listener to sense the lower harmonics as well.)

1.33 Find the amplitude of a symmetrical square wave of period T that provides the same power as a sine wave of peak amplitude $\hat{V}$ and the same frequency. Does this result depend on equality of the frequencies of the two waveforms?

Section 1.3: Analog and Digital Signals

1.34 Give the binary representation of the following decimal numbers: 0, 5, 8, 25, and 57.

1.35 Consider a 4-bit digital word $b_3b_2b_1b_0$ in a format called signed-magnitude, in which the most significant bit, b_3, is interpreted as a sign bit—0 for positive and 1 for negative values. List the values that can be represented by this scheme. What is peculiar about the representation of zero? For a particular analog-to-digital converter (ADC), each change in b_0 corresponds to a 0.5-V change in the analog input. What is the full range of the analog signal that can be represented? What signed-magnitude digital code results for an input of +2.5 V? For –3.0 V? For +2.7 V? For –2.8 V?

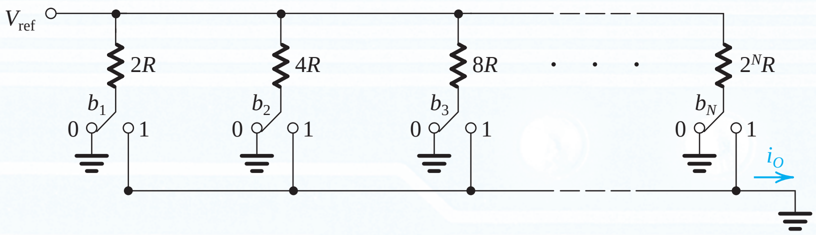


Figure P1.37

1.36 Consider an N-bit ADC whose analog input varies between 0 and V_{FS} (where the subscript FS denotes "full scale").

(a) Show that the least significant bit (LSB) corresponds to a change in the analog signal of $V_{FS}/(2^N - 1)$. This is the resolution of the converter.
(b) Convince yourself that the maximum error in the conversion (called the quantization error) is half the resolution; that is, the quantization error $= V_{FS}/2(2^N - 1)$.
(c) For $V_{FS} = 10$ V, how many bits are required to obtain a resolution of 5 mV or better? What is the actual resolution obtained? What is the resulting quantization error?

1.37 Figure P1.37 shows the circuit of an N-bit digital-to-analog converter (DAC). Each of the N bits of the digital word to be converted controls one of the switches. When the bit is 0, the switch is in the position labeled 0; when the bit is 1, the switch is in the position labeled 1. The analog output is the current i_O. V_{ref} is a constant reference voltage.

(a) Show that

$$i_O = \frac{V_{ref}}{R}\left(\frac{b_1}{2^1} + \frac{b_2}{2^2} + \cdots + \frac{b_N}{2^N}\right)$$

(b) Which bit is the LSB? Which is the MSB?
(c) For $V_{ref} = 10$ V, $R = 5$ kΩ, and $N = 6$, find the maximum value of i_O obtained. What is the change in i_O resulting from the LSB changing from 0 to 1?

1.38 In compact-disc (CD) audio technology, the audio signal is sampled at 44.1 kHz. Each sample is represented by 16 bits. What is the speed of this system in bits per second?

Section 1.4: Amplifiers

1.39 Various amplifier and load combinations are measured as listed below using rms values. For each, find the voltage, current, and power gains (A_v, A_i, and A_p, respectively) both as ratios and in dB:

(a) $v_I = 100$ mV, $i_I = 100$ μA, $v_O = 10$ V, $R_L = 100$ Ω
(b) $v = 10$ μV, $i_I = 100$ nA, $v_O = 2$ V, $R_L = 10$ kΩ
(c) $v_I = 1$ V, $i_I = 1$ mA, $v_O = 10$ V, $R_L = 10$ Ω

1.40 An amplifier operating from ±3-V supplies provides a 2.2-V peak sine wave across a 100-Ω load when provided with a 0.2-V peak input from which 1.0 mA peak is drawn. The average current in each supply is measured to be 20 mA. Find the voltage gain, current gain, and power gain expressed as ratios and in decibels as well as the supply power, amplifier dissipation, and amplifier efficiency.

1.41 An amplifier using balanced power supplies is known to saturate for signals extending within 1.2 V of either supply. For linear operation, its gain is 500 V/V. What is the rms value of the largest undistorted sine-wave output available, and input needed, with ±5-V supplies? With ±10-V supplies? With ±15-V supplies?

1.42 Symmetrically saturating amplifiers, operating in the so-called clipping mode, can be used to convert sine waves to pseudo-square waves. For an amplifier with a small-signal gain of 1000 and clipping levels of ±9 V, what peak value of input sinusoid is needed to produce an output whose extremes are just at the edge of clipping? Clipped 90% of the time? Clipped 99% of the time?

Section 1.5: Circuit Models for Amplifiers

1.43 Consider the voltage-amplifier circuit model shown in Fig. 1.16(b), in which $A_{vo} = 10$ V/V under the following conditions:

(a) $R_i = 10R_s$, $R_L = 10R_o$
(b) $R_i = R_s$, $R_L = R_o$
(c) $R_i = R_s/10$, $R_L = R_o/10$

Calculate the overall voltage gain v_o/v_s in each case, expressed both directly and in decibels.

1.44 An amplifier with 40 dB of small-signal, open-circuit voltage gain, an input resistance of 1 MΩ, and an output resistance of 10 Ω, drives a load of 100 Ω. What voltage and power gains (expressed in dB) would you expect with the load connected? If the amplifier has a peak output-current limitation of 100 mA, what is the rms value of the largest sine-wave input for which an undistorted output is possible? What is the corresponding output power available?

1.45 A 10-mV signal source having an internal resistance of 100 kΩ is connected to an amplifier for which the input resistance is 10 kΩ, the open-circuit voltage gain is 1000 V/V, and the output resistance is 1 kΩ. The amplifier is connected in turn to a 100-Ω load. What overall voltage gain results as measured from the source internal voltage to the load? Where did all the gain go? What would the gain be if the source was connected directly to the load? What is the ratio of these two gains? This ratio is a useful measure of the benefit the amplifier brings.

1.46 A buffer amplifier with a gain of 1 V/V has an input resistance of 1 MΩ and an output resistance of 10 Ω. It is connected between a 1-V, 100-kΩ source and a 100-Ω load. What load voltage results? What are the corresponding voltage, current, and power gains (in dB)?

1.47 Consider the cascade amplifier of Example 1.3. Find the overall voltage gain v_o/v_s obtained when the first and second stages are interchanged. Compare this value with the result in Example 1.3, and comment.

1.48 You are given two amplifiers, A and B, to connect in cascade between a 10-mV, 100-kΩ source and a 100-Ω load. The amplifiers have voltage gain, input resistance, and output resistance as follows: for A, 100 V/V, 10 kΩ, 10 kΩ, respectively; for B, 1 V/V, 100 kΩ, 100 Ω, respectively. Your problem is to decide how the amplifiers should be connected. To proceed, evaluate the two possible connections between source S and load L, namely, SABL and SBAL. Find the voltage gain for each both as a ratio and in decibels. Which amplifier arrangement is best?

D *1.49 A designer has available voltage amplifiers with an input resistance of 10 kΩ, an output resistance of 1 kΩ, and an open-circuit voltage gain of 10. The signal source has a 10-kΩ resistance and provides a 10-mV rms signal, and it is required to provide a signal of at least 2 V rms to a 1-kΩ load. How many amplifier stages are required? What is the output voltage actually obtained?

D *1.50 Design an amplifier that provides 0.5 W of signal power to a 100-Ω load resistance. The signal source provides a 30-mV rms signal and has a resistance of 0.5 MΩ. Three types of voltage-amplifier stages are available:

(a) A high-input-resistance type with $R_i = 1$ MΩ, $A_{vo} = 10$, and $R_o = 10$ kΩ
(b) A high-gain type with $R_i = 10$ kΩ, $A_{vo} = 100$, and $R_o = 1$ kΩ
(c) A low-output-resistance type with $R_i = 10$ kΩ, $A_{vo} = 1$, and $R_o = 20$ Ω

Design a suitable amplifier using a combination of these stages. Your design should utilize the minimum number of stages and should ensure that the signal level is not reduced below 10 mV at any point in the amplifier chain. Find the load voltage and power output realized.

D *1.51 It is required to design a voltage amplifier to be driven from a signal source having a 10-mV peak amplitude and a source resistance of 10 kΩ to supply a peak output of 3 V across a 1-kΩ load.

(a) What is the required voltage gain from the source to the load?

(b) If the peak current available from the source is 0.1 μA, what is the smallest input resistance allowed? For the design with this value of R_i, find the overall current gain and power gain.

(c) If the amplifier power supply limits the peak value of the output open-circuit voltage to 5 V, what is the largest output resistance allowed?

(d) For the design with R_i as in (b) and R_o as in (c), what is the required value of open-circuit voltage gain $\left(\text{i.e., } \left.\frac{v_o}{v_i}\right|_{R_L=\infty}\right)$ of the amplifier?

(e) If, as a possible design option, you are able to increase R_i to the nearest value of the form 1×10^n Ω and to decrease R_o to the nearest value of the form 1×10^m Ω, find (i) the input resistance achievable; (ii) the output resistance achievable; and (iii) the open-circuit voltage gain now required to meet the specifications.

D 1.52 A voltage amplifier with an input resistance of 10 kΩ, an output resistance of 200 Ω, and a gain of 1000 V/V is connected between a 100-kΩ source with an open-circuit voltage of 10 mV and a 100-Ω load. For this situation:

(a) What output voltage results?
(b) What is the voltage gain from source to load?
(c) What is the voltage gain from the amplifier input to the load?
(d) If the output voltage across the load is twice that needed and there are signs of internal amplifier overload, suggest the location and value of a single resistor that would produce the desired output. Choose an arrangement that would cause minimum disruption to an operating circuit. (*Hint:* Use parallel rather than series connections.)

1.53 A current amplifier for which $R_i = 1$ kΩ, $R_o = 10$ kΩ, and $A_{is} = 100$ A/A is to be connected between a 100-mV source with a resistance of 100 kΩ and a load of 1 kΩ. What are the values of current gain i_o/i_i, of voltage gain v_o/v_s, and of power gain expressed directly and in decibels?

1.54 A transconductance amplifier with $R_i = 2$ kΩ, $G_m = 40$ mA/V, and $R_o = 20$ kΩ is fed with a voltage source having a source resistance of 2 kΩ and is loaded with a 1-kΩ resistance. Find the voltage gain realized.

D **1.55 A designer is required to provide, across a 10-kΩ load, the weighted sum, $v_O = 10v_1 + 20v_2$, of input signals v_1 and v_2, each having a source resistance of 10 kΩ. She has a number of transconductance amplifiers for which the input and output resistances are both 10 kΩ and $G_m = 20$ mA/V, together with a selection of suitable resistors. Sketch an appropriate amplifier topology with additional resistors selected to provide the desired result. (*Hint*: In your design, arrange to add currents.)

1.56 Figure P1.56 shows a transconductance amplifier whose output is *fed back* to its input. Find the input resistance

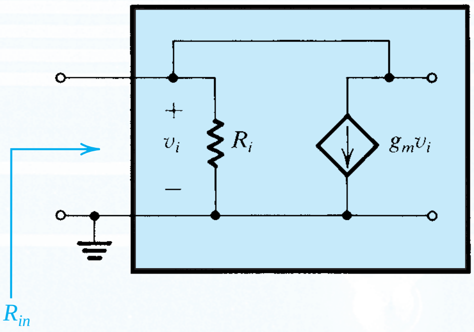


Figure P1.56

R_{in} of the resulting one-port network. (Hint: Apply a test voltage v_x between the two input terminals, and find the current i_x drawn from the source. Then, $R_{\text{in}} \equiv v_x / i_x$.)

D 1.57 It is required to design an amplifier to sense the open-circuit output voltage of a transducer and to provide a proportional voltage across a load resistor. The equivalent source resistance of the transducer is specified to vary in the range of 1 kΩ to 10 kΩ. Also, the load resistance varies in the range of 1 kΩ to 10 kΩ. The change in load voltage corresponding to the specified change in R_s should be 10% at most. Similarly, the change in load voltage corresponding to the specified change in R_L should be limited to 10%. Also, corresponding to a 10-mV transducer open-circuit output voltage, the amplifier should provide a minimum of 1 V across the load. What type of amplifier is required? Sketch its circuit model, and specify the values of its parameters. Specify appropriate values for R_i and R_o of the form 1×10^m Ω.

D 1.58 It is required to design an amplifier to sense the short-circuit output current of a transducer and to provide a proportional current through a load resistor. The equivalent source resistance of the transducer is specified to vary in the range of 1 kΩ to 10 kΩ. Similarly, the load resistance is known to vary over the range of 1 kΩ to 10 kΩ. The change in load current corresponding to the specified change in R_s is required to be limited to 10%. Similarly, the change in load current corresponding to the specified change in R_L should be 10% at most. Also, for a nominal short-circuit output current of the transducer of 10 μA, the amplifier is required to provide a minimum of 1 mA through the load. What type of amplifier is required? Sketch the circuit model of the amplifier, and specify values for its parameters. Select appropriate values for R_i and R_o in the form 1×10^m Ω.

D 1.59 It is required to design an amplifier to sense the open-circuit output voltage of a transducer and to provide a proportional current through a load resistor. The equivalent source resistance of the transducer is specified to vary in the range of 1 kΩ to 10 kΩ. Also, the load resistance is known to vary in the range of 1 kΩ to 10 kΩ. The change in the current supplied to the load corresponding to the specified change in R_s is to be 10% at most. Similarly, the change in load current corresponding to the specified change in R_L is to be 10% at most. Also, for a nominal transducer open-circuit output voltage of 10 mV, the amplifier is required to provide a minimum of 1 mA current through the load. What type of amplifier is required? Sketch the amplifier circuit model, and specify values for its parameters. For R_i and R_o, specify values in the form 1×10^m Ω.

D 1.60 It is required to design an amplifier to sense the short-circuit output current of a transducer and to provide a proportional voltage across a load resistor. The equivalent source resistance of the transducer is specified to vary in the range of 1 kΩ to 10 kΩ. Similarly, the load resistance is known to vary in the range of 1 kΩ to 10 kΩ. The change in load voltage corresponding to the specified change in R_s should be 10% at most. Similarly, the change in load voltage corresponding to the specified change in R_L is to be limited to 10%. Also, for a nominal transducer short-circuit output current of 10 μA, the amplifier is required to provide a minimum voltage across the load of 1 V. What type of amplifier is required? Sketch its circuit model, and specify the values of the model parameters. For R_i and R_o, specify appropriate values in the form 1×10^m Ω.

1.61 For the circuit in Fig. P1.61, show that

$$\frac{v_c}{v_b} = \frac{-\beta R_L}{r_\pi + (\beta + 1)R_E}$$

and

$$\frac{v_e}{v_b} = \frac{R_E}{R_E + [r_\pi / (\beta + 1)]}$$

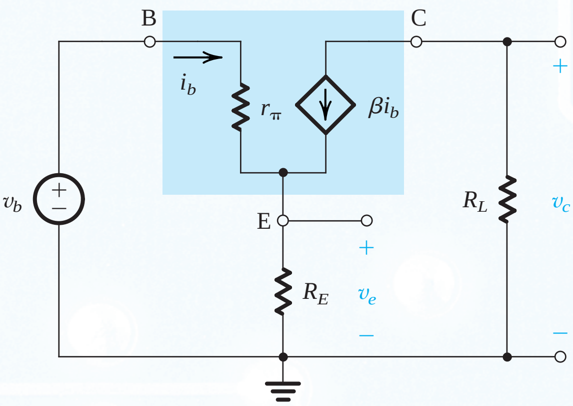


Figure P1.61

1.62 An amplifier with an input resistance of 10 kΩ, when driven by a current source of 1 μA and a source resistance of 100 kΩ, has a short-circuit output current of 10 mA and an open-circuit output voltage of 10 V. The device is driving a 4-kΩ load. Give the values of the voltage gain, current gain, and power gain expressed as ratios and in decibels?

1.63 Figure P1.63(a) shows two transconductance amplifiers connected in a special configuration. Find v_o in terms of v_1 and v_2. Let $g_m = 100$ mA/V and $R = 5$ kΩ. If $v_1 = v_2 = 1$ V, find the value of v_o. Also, find v_o for the case $v_1 = 1.01$ V and $v_2 = 0.99$ V. (Note: This circuit is called a **differential amplifier** and is given the symbol shown in Fig. P1.63(b). A particular type of differential amplifier known as an **operational amplifier** will be studied in Chapter 2.)

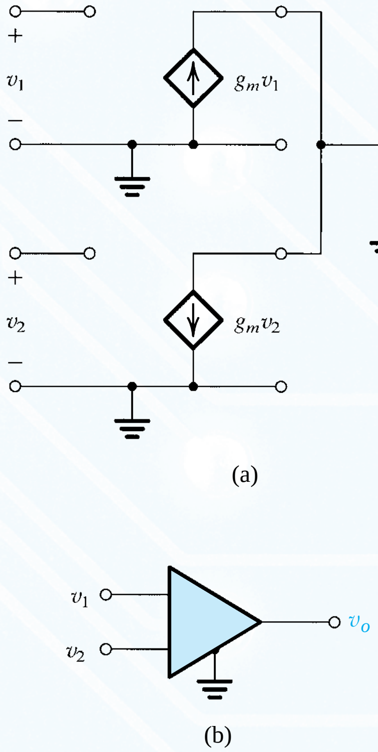


Figure P1.63

1.64 Any linear two-port network including linear amplifiers can be represented by one of four possible parameter sets, given in Appendix C. For the voltage amplifier, the most convenient representation is in terms of the g parameters. If the amplifier input port is labeled as port 1 and the output port as port 2, its g-parameter representation is described by the two equations:

$$I_1 = g_{11}V_1 + g_{12}I_2$$

$$V_2 = g_{21}V_1 + g_{22}I_2$$

Figure P1.64 shows an equivalent circuit representation of these two equations. By comparing this equivalent circuit to that of the voltage amplifier in Fig. 1.16(a), identify corresponding currents and voltages as well as the correspondence between the parameters of the amplifier equivalent circuit and the g parameters. Hence give the g parameter that corresponds to each of R_i, A_{vo} and R_o. Notice that there is an additional g parameter with no correspondence in the amplifier equivalent circuit. Which one? What does it signify? What assumption did we make about the amplifier that resulted in the absence of this particular g parameter from the equivalent circuit in Fig. 1.16(a)?

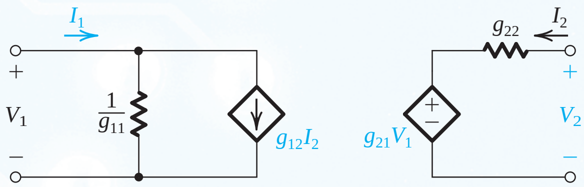


Figure P1.64

Section 1.6: Frequency Response of Amplifiers

1.65 Use the voltage-divider rule to derive the transfer functions $T(s) \equiv V_o(s)/V_i(s)$ of the circuits shown in Fig. 1.22, and show that the transfer functions are of the form given at the top of Table 1.2.

1.66 Figure P1.66 shows a signal source connected to the input of an amplifier. Here R_s is the source resistance, and R_i and C_i are the input resistance and input capacitance, respectively, of the amplifier. Derive an expression for $V_i(s)/V_s(s)$, and show that it is of the low-pass STC type. Find the 3-dB frequency for the case $R_s = 20$ kΩ, $R_i = 80$ kΩ, and $C_i = 5$ pF.

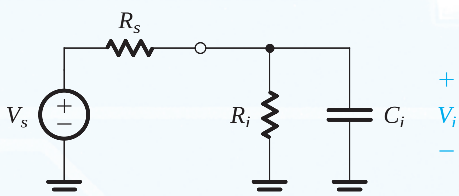


Figure P1.66

1.67 For the circuit shown in Fig. P1.67, find the transfer function $T(s) = V_o(s)/V_i(s)$, and arrange it in the appropriate standard form from Table 1.2. Is this a high-pass or a low-pass network? What is its transmission at very high frequencies? [Estimate this directly, as well as by letting $s \to \infty$ in your expression for $T(s)$.] What is the corner frequency ω_0? For $R_1 = 10$ kΩ, $R_2 = 40$ kΩ, and $C = 0.1$ μF, find f_0. What is the value of $|T(j\omega_0)|$?

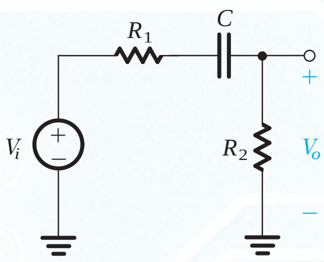


Figure P1.67

D 1.68 It is required to couple a voltage source V_s with a resistance R_s to a load R_L via a capacitor C. Derive an expression for the transfer function from source to load (i.e., V_L/V_s), and show that it is of the high-pass STC type. For $R_s = 5$ kΩ and $R_L = 20$ kΩ, find the smallest coupling capacitor that will result in a 3-dB frequency no greater than 10 Hz.

1.69 Measurement of the frequency response of an amplifier yields the data in the following table:

f (Hz)	|*T*| (dB)	∠*T* (°)
0	40	0
100	40	0
1000		
10^4	37	–45
10^5	20	
	0	

Provide plausible approximate values for the missing entries. Also, sketch and clearly label the magnitude frequency response (i.e., provide a Bode plot) for this amplifier.

1.70 Measurement of the frequency response of an amplifier yields the data in the following table:

f (Hz)		10	10^2	10^3	10^4	10^5	10^6	10^7	
|*T*| (dB)	0	20	37	40			37	20	0

Provide approximate plausible values for the missing table entries. Also, sketch and clearly label the magnitude frequency response (Bode plot) of this amplifier.

1.71 The unity-gain voltage amplifiers in the circuit of Fig. P1.71 have infinite input resistances and zero output resistances and thus function as perfect buffers. Convince yourself that the overall gain V_o/V_i will drop by 3 dB below the value at dc at the frequency for which the gain of each RC circuit is 1.0 dB down. What is that frequency in terms of CR?

1.72 A manufacturing error causes an internal node of a high-frequency amplifier whose Thévenin-equivalent node resistance is 100 kΩ to be accidentally shunted to ground by a capacitor (i.e., the node is connected to ground through a capacitor). If the measured 3-dB bandwidth of the amplifier is reduced from the expected 6 MHz to 120 kHz, estimate the value of the shunting capacitor. If the original cutoff frequency can be attributed to a small parasitic capacitor at the same internal node (i.e., between the node and ground), what would you estimate it to be?

D *1.73 A designer wishing to lower the overall upper 3-dB frequency of a three-stage amplifier to 10 kHz considers shunting one of two nodes: Node A, between the output of the first stage and the input of the second stage, and Node B, between the output of the second stage and the input of the third stage, to ground with a small capacitor. While measuring the overall frequency response of the amplifier, she connects a capacitor of 1 nF, first to node A and then to node B, lowering the 3-dB frequency from 2 MHz to 150 kHz and 15 kHz, respectively. If she knows that each amplifier stage has an input resistance of 100 kΩ, what output resistance must the driving stage have at node A? At node B? What capacitor value should she connect to which node to solve her design problem most economically?

D 1.74 An amplifier with an input resistance of 100 kΩ and an output resistance of 1 kΩ is to be capacitor-coupled to a 10-kΩ source and a 1-kΩ load. Available capacitors have values only of the form 1×10^{-n} F. What are the values of the

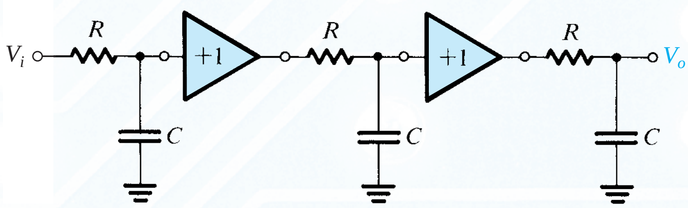


Figure P1.71

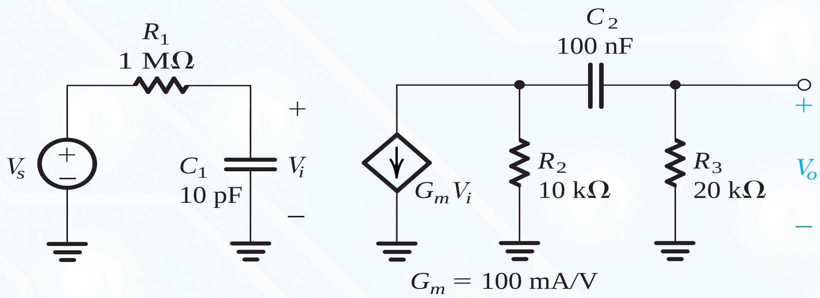


Figure P1.76

smallest capacitors needed to ensure that the corner frequency associated with each is less than 100 Hz? What actual corner frequencies result? For the situation in which the basic amplifier has an open-circuit voltage gain (A_{vo}) of 100 V/V, find an expression for $T(s) = V_o(s)/V_s(s)$.

***1.75** A voltage amplifier has the transfer function

$$A_v = \frac{100}{\left(1 + j\frac{f}{10^4}\right)\left(1 + \frac{10^2}{jf}\right)}$$

Using the Bode plots for low-pass and high-pass STC networks (Figs. 1.23 and 1.24), sketch a Bode plot for $|A_v|$. Give approximate values for the gain magnitude at $f = 10$ Hz, 10^2 Hz, 10^3 Hz, 10^4 Hz, 10^5 Hz, 10^6 Hz, and 10^7 Hz. Find the bandwidth of the amplifier (defined as the frequency range over which the gain remains within 3 dB of the maximum value).

***1.76** For the circuit shown in Fig. P1.76 first, evaluate $T_i(s) = V_i(s)/V_s(s)$ and the corresponding cutoff (corner) frequency. Second, evaluate $T_o(s) = V_o(s)/V_i(s)$ and the corresponding cutoff frequency. Put each of the transfer functions in the standard form (see Table 1.2), and combine them to form the overall transfer function, $T(s) = T_i(s) \times T_o(s)$. Provide a Bode magnitude plot for $|T(j\omega)|$. What is the bandwidth between 3-dB cutoff points?

D **1.77 A transconductance amplifier having the equivalent circuit shown in Table 1.1 is fed with a voltage source V_s having a source resistance R_s, and its output is connected to a load consisting of a resistance R_L in parallel with a capacitance C_L. For given values of R_s, R_L, and C_L, it is required to specify the values of the amplifier parameters R_i, G_m, and R_o to meet the following design constraints:

(a) At most, x% of the input signal is lost in coupling the signal source to the amplifier (i.e., $V_i \geq [1 - (x/100)]V_s$).
(b) The 3-dB frequency of the amplifier is equal to or greater than a specified value $f_{3\text{dB}}$.
(c) The dc gain V_o/V_s is equal to or greater than a specified value A_0.

Show that these constraints can be met by selecting

$$R_i \geq \left(\frac{100}{x} - 1\right)R_s$$

$$R_o \leq \frac{1}{2\pi f_{3\text{dB}} C_L - (1/R_L)}$$

$$G_m \geq \frac{A_0/[1 - (x/100)]}{(R_L \parallel R_o)}$$

Find R_i, R_o, and G_m for $R_s = 10$ kΩ, $x = 20\%$, $A_o = 80$, $R_L = 10$ kΩ, $C_L = 10$ pF, and $f_{3\text{dB}} = 3$ MHz.

***1.78** Use the voltage-divider rule to find the transfer function $V_o(s)/V_i(s)$ of the circuit in Fig. P1.78. Show that the transfer function can be made independent of frequency if the condition $C_1R_1 = C_2R_2$ applies. Under this condition the circuit is called a **compensated attenuator** and is frequently employed in the design of oscilloscope probes. Find the transmission of the compensated attenuator in terms of R_1 and R_2.

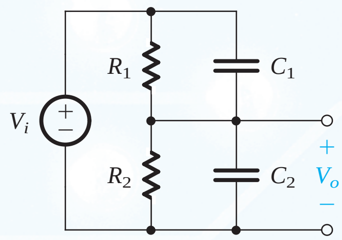


Figure P1.78

***1.79** An amplifier with a frequency response of the type shown in Fig. 1.21 is specified to have a phase shift of magnitude no greater than 11.4° over the amplifier bandwidth, which extends from 100 Hz to 1 kHz. It has been found that the gain falloff at the low-frequency end is determined by the response of a high-pass STC circuit and that at the high-frequency end it is determined by a low-pass STC circuit. What do you expect the corner frequencies of these two circuits to be? What is the drop in gain in decibels (relative to the maximum gain) at the two frequencies that define the amplifier bandwidth? What are the frequencies at which the drop in gain is 3 dB?