

CHAPTER 3

Semiconductors

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IN THIS CHAPTER YOU WILL LEARN

1. The basic properties of semiconductors and in particular silicon, which is the material used to make most of today's electronic circuits.
2. How doping a pure silicon crystal dramatically changes its electrical conductivity, which is the fundamental idea underlying the use of semiconductors in the implementation of electronic devices.
3. The two mechanisms by which current flows in semiconductors: drift and diffusion of charge carriers.
4. The structure and operation of the *pn* junction; a basic semiconductor structure that implements the diode and plays a dominant role in transistors.

Introduction

Thus far we have dealt with electronic circuits, and notably amplifiers, as system building blocks. For instance, in Chapter 2 we learned how to use op amps to design interesting and useful circuits, taking advantage of the terminal characteristics of the op amp and without any knowledge of what is inside the op amp package. Though interesting and motivating, this approach has its limitations. Indeed, to achieve our goal of preparing the reader to become a proficient circuit designer, we have to go beyond this black-box or system-level abstraction and learn about the basic devices from which electronic circuits are assembled, namely, diodes (Chapter 4) and transistors (Chapters 5 and 6). These solid-state devices are made using semiconductor materials, predominantly silicon.

In this chapter, we briefly introduce the properties and physics of semiconductors. The objective is to provide a basis for understanding the physical operation of diodes and transistors in order to enable their effective use in the design of circuits. Although many of the concepts studied in this chapter apply to semiconductor materials in general, our treatment is heavily biased toward silicon, simply because it is the material used in the vast majority of microelectronic circuits. To complement the material presented here, Appendix A provides a description of the integrated-circuit fabrication process. As discussed in Appendix A, whether our circuit consists of a single transistor or is an **integrated circuit** containing more than 2 billion transistors, it is fabricated in a single silicon crystal, which gives rise to the name **monolithic circuit**. This chapter therefore begins with a study of the crystal structure of semiconductors and introduces the two types of charge carriers available for current conduction: electrons and holes. The most significant property of semiconductors is that their conductivity can be varied over a very wide range through the introduction of controlled amounts of impurity atoms into the semiconductor crystal in a process called **doping**. Doped semiconductors are discussed in Section 3.2. This is

followed by the study in Section 3.3 of the two mechanisms for current flow in semiconductors, namely, carrier drift and carrier diffusion.

Armed with these basic semiconductor concepts, we spend the remainder of the chapter on the study of an important semiconductor structure; the *pn* junction. In addition to being essentially a diode, the *pn* junction is the basic element of the bipolar junction transistor (BJT, Chapter 6) and plays an important role in the operation of field-effect transistors (FETs, Chapter 5).

3.1 Intrinsic Semiconductors

As their name implies, semiconductors are materials whose conductivity lies between that of conductors, such as copper, and insulators, such as glass. There are two kinds of semiconductors: single-element semiconductors, such as germanium and silicon, which are in group IV in the periodic table; and compound semiconductors, such as gallium-arsenide, which are formed by combining elements from groups III and V or groups II and VI. Compound semiconductors are useful in special electronic circuit applications as well as in applications that involve light, such as light-emitting diodes (LEDs). Of the two elemental semiconductors, germanium was used in the fabrication of very early transistors (late 1940s, early 1950s). It was quickly supplanted, however, with silicon, on which today's integrated-circuit technology is almost entirely based. For this reason, we will deal mostly with silicon devices throughout this book.¹

A silicon atom has four valence electrons, and thus it requires another four to complete its outermost shell. This is achieved by sharing one of its valence electrons with each of its four neighboring atoms. Each pair of shared electrons forms a **covalent bond**. The result is that a crystal of pure or intrinsic silicon has a regular lattice structure, where the atoms are held in their position by the covalent bonds. Figure 3.1 shows a two-dimensional representation of such a structure.

At sufficiently low temperatures, approaching absolute zero (0 K), all the covalent bonds are intact and no electrons are available to conduct electric current. Thus, at such low temperatures, the intrinsic silicon crystal behaves as an insulator.

At room temperature, sufficient thermal energy exists to break some of the covalent bonds, a process known as thermal generation. As shown in Fig. 3.2, when a covalent bond is broken, an electron is freed. The **free electron** can wander away from its parent atom, and it becomes available to conduct electric current if an electric field is applied to the crystal. As the electron leaves its parent atom, it leaves behind a net positive charge, equal to the magnitude of the electron charge. Thus, an electron from a neighboring atom may be attracted to this positive charge, and leaves its parent atom. This action fills up the “hole” that existed in the ionized atom but creates a new hole in the other atom. This process may repeat itself, with the result that we effectively have a positively charged carrier, or **hole**, moving through the silicon crystal structure and being available to conduct electric current. The charge of a hole is equal in magnitude to the charge of an electron. We can thus see that as temperature increases, more covalent bonds are broken and electron–hole pairs are generated. The increase in the numbers of free electrons and holes results in an increase in the conductivity of silicon.

¹An exception is the subject of gallium arsenide (GaAs) circuits, which though not covered in this edition of the book, is studied in some detail in material provided on the text website and on the disc accompanying the text.

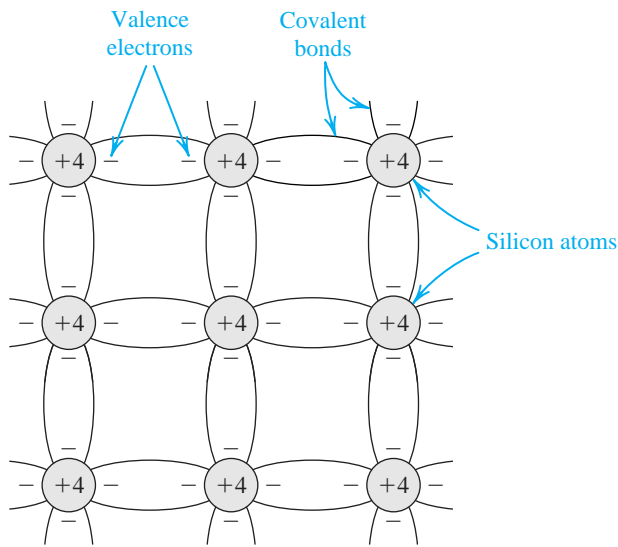


Figure 3.1 Two-dimensional representation of the silicon crystal. The circles represent the inner core of silicon atoms, with +4 indicating its positive charge of $+4q$, which is neutralized by the charge of the four valence electrons. Observe how the covalent bonds are formed by sharing of the valence electrons. At 0 K, all bonds are intact and no free electrons are available for current conduction.

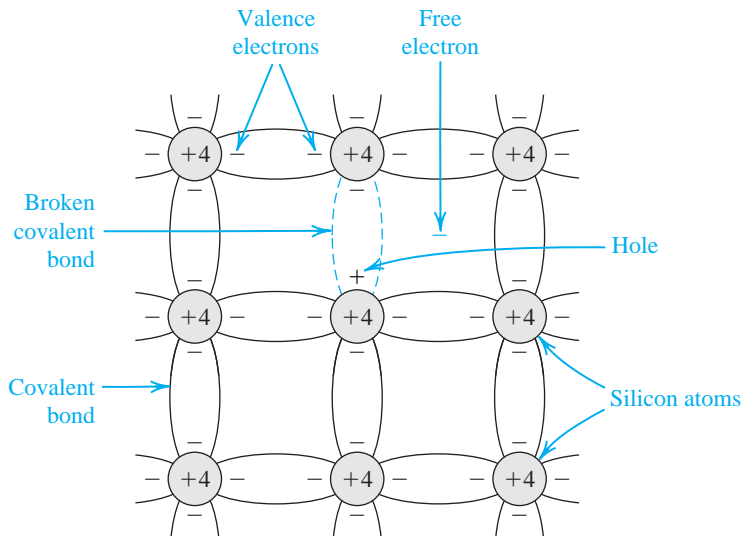


Figure 3.2 At room temperature, some of the covalent bonds are broken by thermal generation. Each broken bond gives rise to a free electron and a hole, both of which become available for current conduction.

Thermal generation results in free electrons and holes in equal numbers and hence equal concentrations, where concentration refers to the number of charge carriers per unit volume (cm^3). The free electrons and holes move randomly through the silicon crystal structure, and in the process some electrons may fill some of the holes. This process, called **recombination**, results in the disappearance of free electrons and holes. The recombination rate is

proportional to the number of free electrons and holes, which in turn is determined by the thermal **generation** rate. The latter is a strong function of temperature. In thermal equilibrium, the recombination rate is equal to the generation rate, and one can conclude that the concentration of free electrons n is equal to the concentration of holes p ,

$$n = p = n_i \quad (3.1)$$

where n_i denotes the number of free electrons and holes in a unit volume (cm^3) of intrinsic silicon at a given temperature. Results from semiconductor physics gives n_i as

$$n_i = BT^{3/2} e^{-E_g/2kT} \quad (3.2)$$

where B is a material-dependent parameter that is $7.3 \times 10^{15} \text{cm}^{-3} \text{K}^{-3/2}$ for silicon; E_g , a parameter known as the **bandgap energy**, is 1.12 electron volt (eV) for silicon²; and k is Boltzmann's constant ($8.62 \times 10^{-5} \text{ eV/K}$). It is interesting to know that the bandgap energy E_g is the minimum energy required to break a covalent bond and thus generate an electron-hole pair.

Example 3.1

Calculate the value of n_i for silicon at room temperature ($T \approx 300 \text{ K}$).

Solution

Substituting the values given above in Eq. (3.1) provides

$$\begin{aligned} n_i &= 7.3 \times 10^{15} (300)^{3/2} e^{-1.12/(2 \times 8.62 \times 10^{-5} \times 300)} \\ &= 1.5 \times 10^{10} \text{ carriers/cm}^3 \end{aligned}$$

Although this number seems large, to place it into context note that silicon has $5 \times 10^{22} \text{ atoms/cm}^3$. Thus at room temperature only one in about 5×10^{12} atoms is ionized and contributing a free electron and a hole!

Finally, it is useful for future purposes to express the product of the hole and free-electron concentration as

$$pn = n_i^2 \quad (3.3)$$

where for silicon at room temperature, $n_i \approx 1.5 \times 10^{10} / \text{cm}^3$. As will be seen shortly, this relationship extends to extrinsic or doped silicon as well.

²Note that $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$.

EXERCISE

3.1 Calculate the intrinsic carrier density n_i for silicon at $T = 50$ K and 350 K.

Ans. $9.6 \times 10^{-39} / \text{cm}^3$; $4.15 \times 10^{11} / \text{cm}^3$

3.2 Doped Semiconductors

The intrinsic silicon crystal described above has equal concentrations of free electrons and holes, generated by thermal generation. These concentrations are far too small for silicon to conduct appreciable current at room temperature. Also, the carrier concentrations and hence the conductivity are strong functions of temperature, not a desirable property in an electronic device. Fortunately, a method was developed to change the carrier concentration in a semiconductor crystal substantially and in a precisely controlled manner. This process is known as doping, and the resulting silicon is referred to as **doped silicon**.

Doping involves introducing impurity atoms into the silicon crystal in sufficient numbers to substantially increase the concentration of either free electrons or holes but with little or no change in the crystal properties of silicon. To increase the concentration of free electrons, n , silicon is doped with an element with a valence of 5, such as phosphorus. The resulting doped silicon is then said to be of **n type**. To increase the concentration of holes, p , silicon is doped with an element having a valence of 3, such as boron, and the resulting doped silicon is said to be of **p type**.

Figure 3.3 shows a silicon crystal doped with phosphorus impurity. The dopant (phosphorus) atoms replace some of the silicon atoms in the crystal structure. Since the phosphorus atom has five electrons in its outer shell, four of these electrons form covalent bonds with the

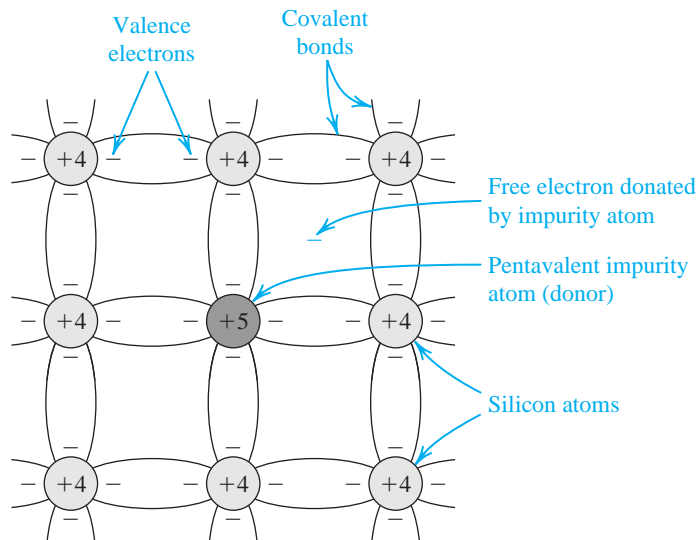


Figure 3.3 A silicon crystal doped by a pentavalent element. Each dopant atom donates a free electron and is thus called a donor. The doped semiconductor becomes n type.

neighboring atoms, and the fifth electron becomes a free electron. Thus each phosphorus atom *donates* a free electron to the silicon crystal, and the phosphorus impurity is called a **donor**. It should be clear, though, that no holes are generated by this process. The positive charge associated with the phosphorus atom is a **bound charge** that does not move through the crystal.

If the concentration of donor atoms is N_D , where N_D is usually much greater than n_i , the concentration of free electrons in the n -type silicon will be

$$n_n \approx N_D \quad (3.4)$$

where the subscript n denotes n -type silicon. Thus n_n is determined by the doping concentration and not by temperature. This is not the case, however, for the hole concentration. All the holes in the n -type silicon are those generated by thermal ionization. Their concentration p_n can be found by noting that the relationship in Eq. (3.3) applies equally well for doped silicon, provided thermal equilibrium is achieved. Thus for n -type silicon

$$p_n n_n = n_i^2$$

Substituting for n_n from Eq. (3.4), we obtain for p_n

$$p_n \approx \frac{n_i^2}{N_D} \quad (3.5)$$

Thus p_n will have the same dependence on temperature as that of n_i^2 . Finally, we note that in n -type silicon the concentration of free electrons n_n will be much larger than that of holes. Hence electrons are said to be the **majority** charge carriers and holes the **minority** charge carriers in n -type silicon.

To obtain p -type silicon in which holes are the majority charge carriers, a trivalent impurity such as boron is used. Figure 3.4 shows a silicon crystal doped with boron. Note that the boron atoms replace some of the silicon atoms in the silicon crystal structure. Since each

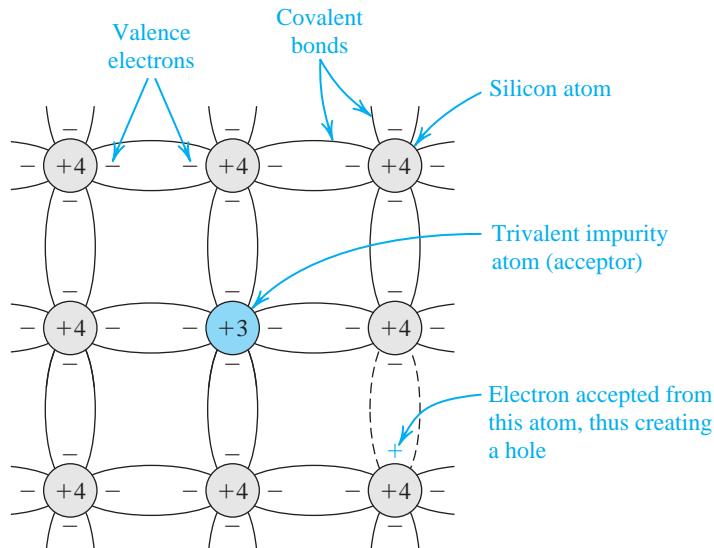


Figure 3.4 A silicon crystal doped with a trivalent impurity. Each dopant atom gives rise to a hole, and the semiconductor becomes p type.

boron atom has three electrons in its outer shell, it **accepts** an electron from a neighboring atom, thus forming covalent bonds. The result is a hole in the neighboring atom and a bound negative charge at the **acceptor** (boron) atom. It follows that each acceptor atom provides a hole. If the acceptor doping concentration is N_A , where $N_A \gg n_i$; the hole concentration becomes

$$p_p \simeq N_A \quad (3.6) \quad \text{!}$$

where the subscript p denotes p -type silicon. Thus, here the majority carriers are holes and their concentration is determined by N_A . The concentration of minority electrons can be found by using the relationship

$$p_p n_p = n_i^2 \quad \text{!}$$

and substituting for p_p from Eq. (3.6),

$$n_p \simeq \frac{n_i^2}{N_A} \quad (3.7) \quad \text{!}$$

Thus, the concentration of the minority electrons will have the same temperature dependence as that of n_i^2 .

It should be emphasized that a piece of n -type or p -type silicon is electrically neutral; the charge of the majority free carriers (electrons in the n -type and holes in the p -type silicon) are neutralized by the bound charges associated with the impurity atoms.

Example 3.2

Consider an n -type silicon for which the dopant concentration $N_D = 10^{17}/\text{cm}^3$. Find the electron and hole concentrations at $T = 300$ K.

Solution

The concentration of the majority electrons is

$$n_n \simeq N_D = 10^{17}/\text{cm}^3$$

The concentration of the minority holes is

$$p_n \simeq \frac{n_i^2}{N_D}$$

In Example 3.1 we found that at $T = 300$ K, $n_i = 1.5 \times 10^{10}/\text{cm}^3$. Thus,

$$\begin{aligned} p_n &= \frac{(1.5 \times 10^{10})^2}{10^{17}} \\ &= 2.25 \times 10^3/\text{cm}^3 \end{aligned}$$

Observe that $n_n \gg n_i$ and that n_n is vastly higher than p_n .

EXERCISES

- 3.2** For the situation in Example 3.2, find the electron and hole concentrations at 350 K. You may use the value of n_i at $T = 350$ K found in Exercise 3.1.

Ans. $n_n = 10^{17}/\text{cm}^3$, $p_n = 1.72 \times 10^6/\text{cm}^3$

- 3.3** For a silicon crystal doped with boron, what must N_A be if at $T = 300$ K the electron concentration drops below the intrinsic level by a factor of 10^6 ?

Ans. $N_A = 1.5 \times 10^{16}/\text{cm}^3$

3.3 Current Flow in Semiconductors

There are two distinctly different mechanisms for the movement of charge carriers and hence for current flow in semiconductors: drift and diffusion.

3.3.1 Drift Current

When an electrical field E is established in a semiconductor crystal, holes are accelerated in the direction of E , and free electrons are accelerated in the direction opposite to that of E . This situation is illustrated in Fig. 3.5. The holes acquire a velocity $v_{p\text{-drift}}$ given by

$$\text{①} \quad v_{p\text{-drift}} = \mu_p E \quad (3.8)$$

where μ_p is a constant called the **hole mobility**: It represents the degree of ease by which holes move through the silicon crystal in response to the electrical field E . Since velocity has the units of centimeters per second and E has the units of volts per centimeter, we see from Eq. (3.8) that the mobility μ_p must have the units of centimeters squared per volt-second ($\text{cm}^2/\text{V}\cdot\text{s}$). For intrinsic silicon $\mu_p = 480 \text{ cm}^2/\text{V}\cdot\text{s}$.

The free electrons acquire a drift velocity $v_{n\text{-drift}}$ given by

$$\text{①} \quad v_{n\text{-drift}} = -\mu_n E \quad (3.9)$$

where the result is negative because the electrons move in the direction opposite to E . Here μ_n is the **electron mobility**, which for intrinsic silicon is about $1350 \text{ cm}^2/\text{V}\cdot\text{s}$. Note that μ_n is about 2.5 times μ_p , signifying that electrons move with much greater ease through the silicon crystal than do holes.

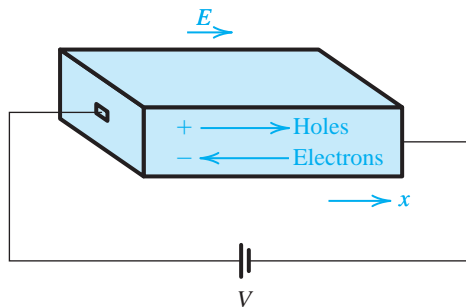


Figure 3.5 An electric field E established in a bar of silicon causes the holes to drift in the direction of E and the free electrons to drift in the opposite direction. Both the hole and electron drift currents are in the direction of E .

Let's now return to the single-crystal silicon bar shown in Fig. 3.5. Let the concentration of holes be p and that of free electrons n . We wish to calculate the current component due to the flow of holes. Consider a plane perpendicular to the x direction. In one second, the hole charge that crosses that plane will be $(Aqp v_{p\text{-drift}})$ coulombs, where A is the cross-sectional area of the silicon bar and q is the magnitude of electron charge. This then must be the hole component of the drift current flowing through the bar,

$$I_p = Aqp v_{p\text{-drift}} \quad (3.10)$$

Substituting for $v_{p\text{-drift}}$ from Eq. (3.9), we obtain

$$I_p = Aqp \mu_p E$$

We are usually interested in the current density J_p , which is the current per unit cross-sectional area,

$$J_p = \frac{I_p}{A} = qp \mu_p E \quad (3.11) \quad \text{!}$$

The current component due to the drift of free electrons can be found in a similar manner. Note, however, that electrons drifting from right to left result in a current component from left to right. This is because of the convention of taking the direction of current flow as the direction of flow of positive charge and opposite to the direction of flow of negative charge. Thus,

$$I_n = -Aqn v_{n\text{-drift}}$$

Substituting for $v_{n\text{-drift}}$ from Eq. (3.9), we obtain the current density $J_n = I_n/A$ as

$$J_n = qn \mu_n E \quad (3.12) \quad \text{!}$$

The total drift current density can now be found by summing J_p and J_n from Eqs. (3.11) and (3.12),

$$J = J_p + J_n = q(p \mu_p + n \mu_n) E \quad (3.13) \quad \text{!}$$

This relationship can be written as

$$J = \sigma E \quad (3.14)$$

or

$$J = E/\rho \quad (3.15)$$

where the **conductivity** σ is given by

$$\sigma = q(p \mu_p + n \mu_n) \quad (3.16)$$

and the **resistivity** ρ is given by

$$\rho \equiv \frac{1}{\sigma} = \frac{1}{q(p \mu_p + n \mu_n)} \quad (3.17) \quad \text{!}$$

Observe that Eq. (3.15) is a form of Ohm's law and can be written alternately as

$$\rho = \frac{E}{J} \quad (3.18)$$

Thus the units of ρ are ohm-centimeters $\left(\Omega \cdot \text{cm} = \frac{\text{V}/\text{cm}}{\text{A}/\text{cm}^2} \right)$.

Example 3.3

Find the resistivity of (a) intrinsic silicon and (b) p -type silicon with $N_A = 10^{16}/\text{cm}^3$. Use $n_i = 1.5 \times 10^{10}/\text{cm}^3$, and assume that for intrinsic silicon $\mu_n = 1350 \text{ cm}^2/\text{V}\cdot\text{s}$ and $\mu_p = 480 \text{ cm}^2/\text{V}\cdot\text{s}$, and for the doped silicon $\mu_n = 1110 \text{ cm}^2/\text{V}\cdot\text{s}$ and $\mu_p = 400 \text{ cm}^2/\text{V}\cdot\text{s}$. (Note that doping results in reduced carrier mobilities).

Solution

(a) For intrinsic silicon,

$$p = n = n_i = 1.5 \times 10^{10}/\text{cm}^3$$

Thus,

$$\begin{aligned}\rho &= \frac{1}{q(p\mu_p + n\mu_n)} \\ \rho &= \frac{1}{1.6 \times 10^{-19} (1.5 \times 10^{10} \times 480 + 1.5 \times 10^{10} \times 1350)} \\ &= 2.28 \times 10^5 \text{ } \Omega \cdot \text{cm}\end{aligned}$$

(b) For the p -type silicon

$$\begin{aligned}p_p &\approx N_A = 10^{16}/\text{cm}^3 \\ n_p &\approx \frac{n_i^2}{N_A} = \frac{(1.5 \times 10^{10})^2}{10^{16}} = 2.25 \times 10^4/\text{cm}^3\end{aligned}$$

Thus,

$$\begin{aligned}\rho &= \frac{1}{q(p\mu_p + n\mu_n)} \\ &= \frac{1}{1.6 \times 10^{-19} (10^{16} \times 400 + 2.25 \times 10^4 \times 1110)} \\ &\approx \frac{1}{1.6 \times 10^{-19} \times 10^{16} \times 400} = 1.56 \text{ } \Omega \cdot \text{cm}\end{aligned}$$

Observe that the resistivity of the p -type silicon is determined almost entirely by the doping concentration. Also observe that doping the silicon reduces its resistivity by a factor of about 10^5 , a truly remarkable change.

EXERCISE

3.4 A uniform bar of n -type silicon of 2 μm length has a voltage of 1 V applied across it. If $N_D = 10^{16}/\text{cm}^3$ and $\mu_n = 1350 \text{ cm}^2/\text{V} \cdot \text{s}$, find (a) the electron drift velocity, (b) the time it takes an electron to cross the 2- μm length, (c) the drift-current density, and (d) the drift current in the case the silicon bar has a cross sectional area of $0.25 \text{ } \mu\text{m}^2$.

Ans. $6.75 \times 10^6 \text{ cm/s}$; 30 ps; $1.08 \times 10^4 \text{ A/cm}^2$; 27 μA

3.3.2 Diffusion Current

Carrier diffusion occurs when the density of charge carriers in a piece of semiconductor is not uniform. For instance, if by some mechanism the concentration of, say, holes, is made higher in one part of a piece of silicon than in another, then holes will diffuse from the region of high concentration to the region of low concentration. Such a diffusion process is like that observed if one drops a few ink drops in a water-filled tank. The diffusion of charge carriers gives rise to a net flow of charge, or **diffusion current**.

As an example, consider the bar of silicon shown in Fig. 3.6(a): By some unspecified process, we have arranged to inject holes into its left side. This continuous hole injection gives rise to and maintains a hole **concentration profile** such as that shown in Fig. 3.6(b). This profile in turn causes holes to diffuse from left to right along the silicon bar, resulting in a hole current in the x direction. The magnitude of the current at any point is proportional to the slope of the concentration profile, or the **concentration gradient**, at that point,

$$J_p = -qD_p \frac{dp(x)}{dx} \quad (3.19)$$

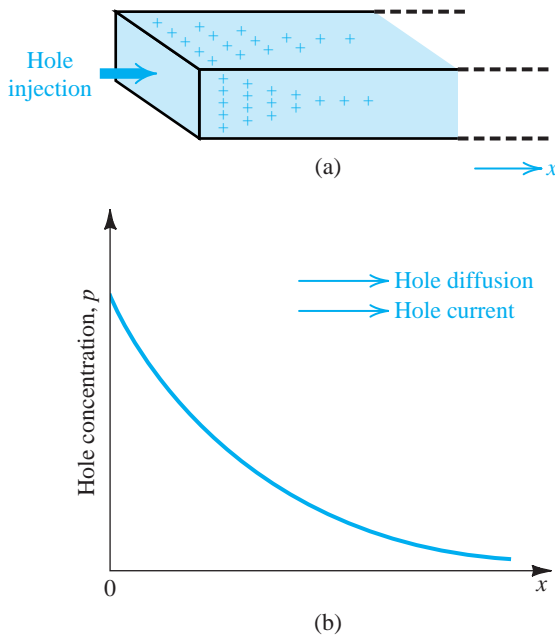


Figure 3.6 A bar of silicon (a) into which holes are injected, thus creating the hole concentration profile along the x axis, shown in (b). The holes diffuse in the positive direction of x and give rise to a hole-diffusion current in the same direction. Note that we are not showing the circuit to which the silicon bar is connected.

where J_p is the hole-current density (A/cm^2), q is the magnitude of electron charge, D_p is a constant called the **diffusion constant** or **diffusivity** of holes; and $p(x)$ is the hole concentration at point x . Note that the gradient (dp/dx) is negative, resulting in a positive current in the x direction, as should be expected.

In the case of electron diffusion resulting from an electron concentration gradient (see Fig. 3.7), a similar relationship applies, giving the electron-current density,

$$J_n = qD_n \frac{dn(x)}{dx} \quad (3.20)$$

where D_n is the diffusion constant or diffusivity of electrons. Observe that a negative (dn/dx) gives rise to a negative current, a result of the convention that the positive direction of current is taken to be that of the flow of positive charge (and opposite to that of the flow of negative charge). For holes and electrons diffusing in intrinsic silicon, typical values for the diffusion constants are $D_p = 12 \text{ cm}^2/\text{s}$ and $D_n = 35 \text{ cm}^2/\text{s}$.

At this point the reader is probably wondering where the diffusion current in the silicon bar in Fig. 3.6(a) goes. A good question as we are not showing how the right-side end of the bar is connected to the rest of the circuit. We will address this and related questions in detail in our discussion of the pn junction in later sections.

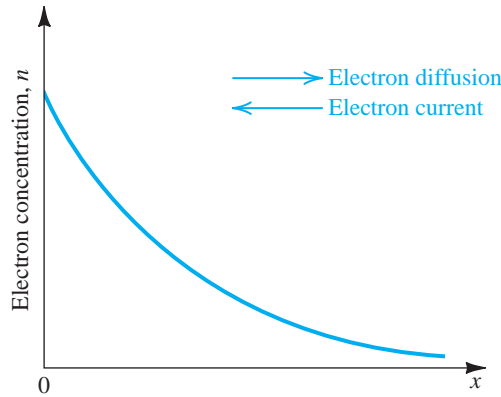


Figure 3.7 If the electron-concentration profile shown is established in a bar of silicon, electrons diffuse in the x direction, giving rise to an electron-diffusion current in the negative $-x$ direction.

Example 3.4

Consider a bar of silicon in which a hole concentration profile described by

$$p(x) = p_0 e^{-x/L_p}$$

is established. Find the hole-current density at $x = 0$. Let $p_0 = 10^{16}/\text{cm}^3$ and $L_p = 1 \text{ } \mu\text{m}$. If the cross-sectional area of the bar is $100 \text{ } \mu\text{m}^2$, find the current I_p .

Solution

$$\begin{aligned}
 J_p &= -qD_p \frac{dp(x)}{dx} \\
 &= -qD_p \frac{d}{dx}[p_0 e^{-x/L_p}]
 \end{aligned}$$

Thus,

$$\begin{aligned}
 J_p(0) &= q \frac{D_p}{L_p} p_0 \\
 &= 1.6 \times 10^{-19} \times \frac{12}{1 \times 10^{-4}} \times 10^{16} \\
 &= 192 \text{ A/cm}^2
 \end{aligned}$$

The current I_p can be found from

$$\begin{aligned}
 I_p &= J_p \times A \\
 &= 192 \times 100 \times 10^{-8} \\
 &= 192 \text{ } \mu\text{A}
 \end{aligned}$$

EXERCISE

- 3.5** The linear electron-concentration profile shown in Fig. E3.5 has been established in a piece of silicon. If $n_0 = 10^{17}/\text{cm}^3$ and $W = 1 \text{ } \mu\text{m}$, find the electron-current density in micro amperes per micron squared ($\mu\text{A}/\mu\text{m}^2$). If a diffusion current of 1 mA is required what must the cross-sectional area (in a direction perpendicular to the page) be?

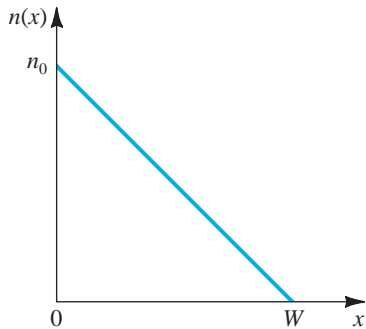


Figure E3.5

Ans. $56 \mu\text{A}/\mu\text{m}^2$; $18 \mu\text{m}^2$

3.3.3 Relationship between D and μ

A simple but powerful relationship ties the diffusion constant with the mobility,

$$\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = V_T \quad (3.21)$$

where $V_T = kT/q$. The parameter V_T is known as the **thermal voltage**. At room temperature, $T \approx 300$ K and $V_T = 25.9$ mV. We will encounter V_T repeatedly throughout this book. The relationship in Eq. (3.21) is known as the **Einstein relationship**.

EXERCISE

3.6 Use the Einstein relationship to find D_n and D_p for intrinsic silicon using $\mu_n = 1350$ cm²/V·s and $\mu_p = 480$ cm²/V·s.

Ans. 35 cm²/s; 12.4 cm²/s

3.4 The pn Junction with Open-Circuit Terminals (Equilibrium)

Having learned important semiconductor concepts, we are now ready to consider our first practical semiconductor structure—the pn junction. As mentioned previously, the pn junction implements the diode (Chapter 4) and plays the dominant role in the structure and operation of the bipolar junction transistor (BJT). As well, understanding pn junctions is very important to the study of the MOSFET operation (Chapter 5).

3.4.1 Physical Structure

Figure 3.8 shows a simplified physical structure of the pn junction. It consists of p -type semiconductor (e.g., silicon) brought into close contact with an n -type semiconductor material (also silicon). In actual practice, both the p and n regions are part of the same silicon crystal; that is, the pn junction is formed within a single silicon crystal by creating regions of different dopings (p and n regions). Appendix A provides a description of the fabrication process of integrated circuits including pn junctions. As indicated in Fig. 3.8, external wire connections are made to the p and n regions through metal (aluminum) contacts. If the pn junction is used as a diode, these constitute the diode terminals and are therefore labeled “anode” and “cathode” in keeping with diode terminology.³

³This terminology in fact is a carryover from that used with vacuum-tube technology, which was the technology for making diodes and other electronic devices until the invention of the transistor in 1947. This event ushered in the era of solid-state electronics, which changed not only electronics, communications, and computers but indeed the world!

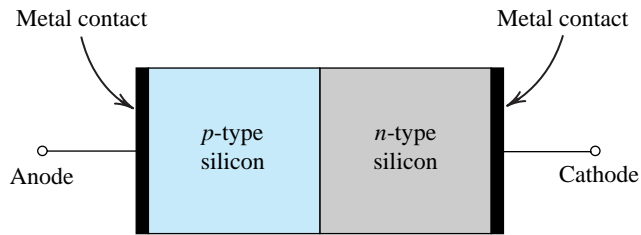


Figure 3.8 Simplified physical structure of the pn junction. (Actual geometries are given in Appendix A.) As the pn junction implements the junction diode, its terminals are labeled anode and cathode.

3.4.2 Operation with Open-Circuit Terminals

Figure 3.9 shows a pn junction under open-circuit conditions—that is, the external terminals are left open. The “+” signs in the p -type material denote the majority holes. The charge of these holes is neutralized by an equal amount of bound negative charge associated with the acceptor atoms. For simplicity, these bound charges are not shown in the diagram. Also not shown are the minority electrons generated in the p -type material by thermal ionization.

In the n -type material the majority electrons are indicated by “−” signs. Here also, the bound positive charge, which neutralizes the charge of the majority electrons, is not shown in order to keep the diagram simple. The n -type material also contains minority holes generated by thermal ionization but not shown in the diagram.

The Diffusion Current I_D Because the concentration of holes is high in the p region and low in the n region, holes diffuse across the junction from the p side to the n side; similarly, electrons diffuse across the junction from the n side to the p side. These two current components add together to form the diffusion current I_D , whose direction is from the p side to the n side, as indicated in Fig. 3.9.

The Depletion Region The holes that diffuse across the junction into the n region quickly recombine with some of the majority electrons present there and thus disappear from the scene. This recombination process results also in the disappearance of some free electrons from the n -type material. Thus some of the bound positive charge will no longer be neutralized by free electrons, and this charge is said to have been **uncovered**. Since recombination takes place close to the junction, there will be a region close to the junction that is *depleted of free electrons* and contains uncovered bound positive charge, as indicated in Fig. 3.9.

The electrons that diffuse across the junction into the p region quickly recombine with some of the majority holes there, and thus disappear from the scene. This results also in the disappearance of some majority holes, causing some of the bound negative charge to be uncovered (i.e., no longer neutralized by holes). Thus, in the p material close to the junction, there will be a region *depleted of holes* and containing uncovered bound negative charge, as indicated in Fig. 3.9.

From the above it follows that a **carrier-depletion region** will exist on both sides of the junction, with the n side of this region positively charged and the p side negatively charged. This carrier-depletion region—or, simply, **depletion region**—is also called the **space-charge region**. The charges on both sides of the depletion region cause an electric field E to be

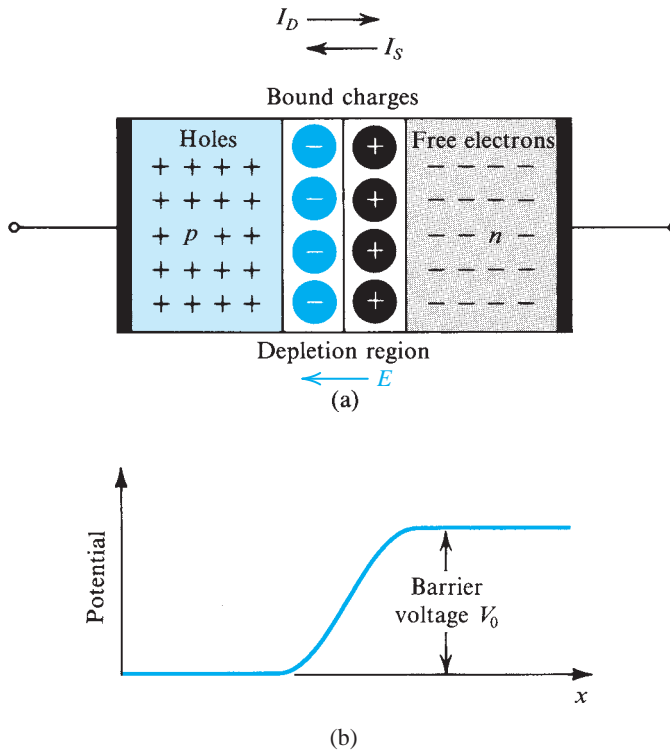


Figure 3.9 (a) The pn junction with no applied voltage (open-circuited terminals). (b) The potential distribution along an axis perpendicular to the junction.

established across the region in the direction indicated in Fig. 3.9. Hence a potential difference results across the depletion region, with the n side at a positive voltage relative to the p side, as shown in Fig. 3.9(b). Thus the resulting electric field opposes the diffusion of holes into the n region and electrons into the p region. In fact, the voltage drop across the depletion region acts as a **barrier** that has to be overcome for holes to diffuse into the n region and electrons to diffuse into the p region. The larger the barrier voltage, the smaller the number of carriers that will be able to overcome the barrier and hence the lower the magnitude of diffusion current. Thus it is the appearance of the barrier voltage V_0 that limits the carrier diffusion process. It follows that the diffusion current I_D depends strongly on the voltage drop V_0 across the depletion region.

The Drift Current I_S and Equilibrium In addition to the current component I_D due to majority-carrier diffusion, a component due to minority carrier drift exists across the junction. Specifically, some of the thermally generated holes in the n material move toward the junction and reach the edge of the depletion region. There, they experience the electric field in the depletion region, which sweeps them across that region into the p side. Similarly, some of the minority thermally generated electrons in the p material move to the edge of the depletion region and get swept by the electric field in the depletion region across that region into the n side. These two current components—electrons moved by drift from p to n and holes moved by drift from n to p —add together to form the drift current I_S , whose direction is from the n side to the p side of the junction, as indicated in Fig. 3.9. Since the current I_S is carried

by thermally generated minority carriers, its value is strongly dependent on temperature; however, it is independent of the value of the depletion-layer voltage V_0 . This is due to the fact that the drift current is determined by the number of minority carriers that make it to the edge of the depletion region; any minority carriers that manage to get to the edge of the depletion region will be swept across by E irrespective of the value of E or, correspondingly, of V_0 .

Under open-circuit conditions (Fig. 3.9) no external current exists; thus the two opposite currents across the junction must be equal in magnitude:

$$I_D = I_S$$

This equilibrium condition⁴ is maintained by the barrier voltage V_0 . Thus, if for some reason I_D exceeds I_S , then more bound charge will be uncovered on both sides of the junction, the depletion layer will widen, and the voltage across it (V_0) will increase. This in turn causes I_D to decrease until equilibrium is achieved with $I_D = I_S$. On the other hand, if I_S exceeds I_D , then the amount of uncovered charge will decrease, the depletion layer will narrow, and the voltage across it (V_0) will decrease. This causes I_D to increase until equilibrium is achieved with $I_D = I_S$.

The Junction Built-In Voltage With no external voltage applied, the barrier voltage V_0 across the pn junction can be shown to be given by⁵

$$V_0 = V_T \ln \left(\frac{N_A N_D}{n_i^2} \right) \quad (3.22) \quad \text{①}$$

where N_A and N_D are the doping concentrations of the p side and n side of the junction, respectively. Thus V_0 depends both on doping concentrations and on temperature. It is known as the **junction built-in voltage**. Typically, for silicon at room temperature, V_0 is in the range of 0.6 V to 0.9 V.

When the pn junction terminals are left open-circuited, the voltage measured between them will be zero. That is, the voltage V_0 across the depletion region *does not* appear between the junction terminals. This is because of the contact voltages existing at the metal-semiconductor junctions at the terminals, which counter and exactly balance the barrier voltage. If this were not the case, we would have been able to draw energy from the isolated pn junction, which would clearly violate the principle of conservation of energy.

Width of and Charge Stored in the Depletion Region Figure 3.10 provides further illustration of the situation that obtains in the pn junction when the junction is in equilibrium. In Fig. 3.10(a) we show a junction in which $N_A > N_D$, a typical situation in practice. This is borne out by the carrier concentration on both sides of the junction, as shown in Fig 3.10(b). Note that we have denoted the minority carrier concentrations in both sides by n_{p0} and p_{n0} , with the additional subscript “0” signifying equilibrium (i.e., before external voltages are applied as will be seen in the next section). Observe that the depletion region extends in both the p and n materials and that equal amounts of charge exist on both sides (Q_+ and Q_- in Fig. 3.10c). However, since usually unequal dopings N_A and N_D are used, as in the case illustrated in Fig. 3.10, the width of the depletion layer will not be the same on the two sides. Rather, to uncover the same amount of charge, the depletion layer will extend deeper into the more lightly doped material. Specifically, if we denote the width of the

⁴In fact, in equilibrium the equality of drift and diffusion currents applies not just to the total currents but also to their individual components. That is, the hole drift current must equal the hole diffusion current and, similarly, the electron drift current must equal the electron diffusion current.

⁵The derivation of this formula and of a number of others in this chapter can be found in textbooks dealing with devices, such as that by Streetman and Bannerjee (see the reading list in Appendix G).

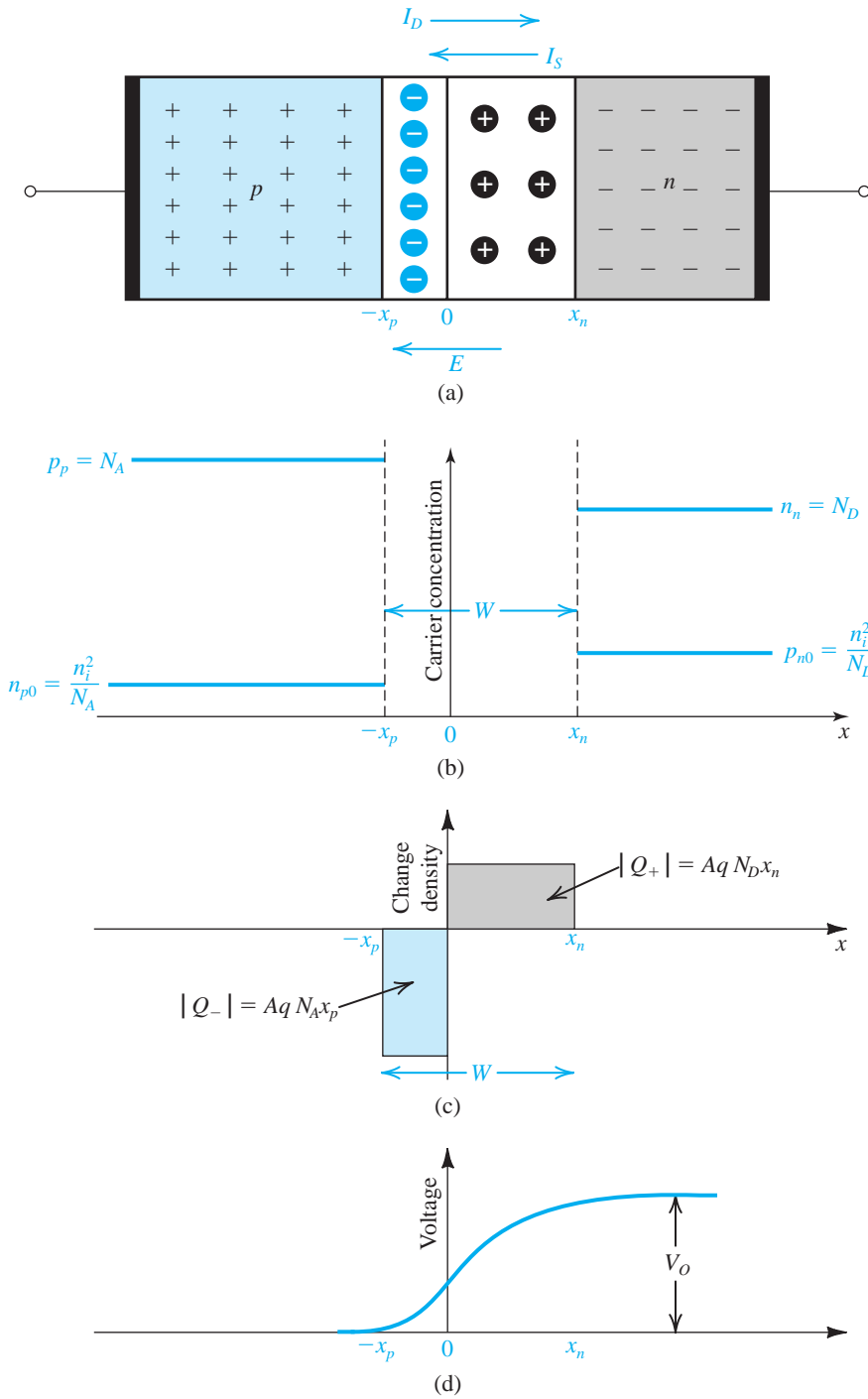


Figure 3.10 (a) A pn junction with the terminals open circuited. (b) Carrier concentrations; note that $N_A \gg N_D$. (c) The charge stored in both sides of the depletion region; $Q_J = |Q_+| = |Q_-|$. (d) The built-in voltage V_0 .

depletion region in the p side by x_p and in the n side by x_n , we can express the magnitude of the charge on the n side of the junction as

$$|Q_+| = qAx_nN_D \quad (3.23)$$

and that on the p side of the junction as

$$|Q_-| = qAx_pN_A \quad (3.24)$$

where A is the cross-sectional area of the junction in the plane perpendicular to the page. The charge equality condition can now be written as

$$qAx_nN_D = qAx_pN_A$$

which can be rearranged to yield

$$\frac{x_n}{x_p} = \frac{N_A}{N_D} \quad (3.25) \quad \text{!}$$

In actual practice, it is usual for one side of the junction to be much more heavily doped than the other, with the result that the depletion region exists almost entirely on one side (the lightly doped side).

The width W of the depletion layer can be shown to be given by

$$W = x_n + x_p = \sqrt{\frac{2\epsilon_s}{q} \left(\frac{1}{N_A} + \frac{1}{N_D} \right) V_0} \quad (3.26) \quad \text{!}$$

where ϵ_s is the electrical permittivity of silicon $= 11.7\epsilon_0 = 11.7 \times 8.85 \times 10^{-14} \text{ F/cm} = 1.04 \times 10^{-12} \text{ F/cm}$. Typically W is in the range $0.1 \mu\text{m}$ to $1 \mu\text{m}$. Eqs. (3.25) and (3.26) can be used to obtain x_n and x_p in terms of W as

$$x_n = W \frac{N_A}{N_A + N_D} \quad (3.27) \quad \text{!}$$

$$x_p = W \frac{N_D}{N_A + N_D} \quad (3.28) \quad \text{!}$$

The charge stored on either side of the depletion region can be expressed in terms of W by utilizing Eqs. (3.23) and (3.27) to obtain

$$\begin{aligned} Q_J &= |Q_+| = |Q_-| \\ Q_J &= Aq \left(\frac{N_A N_D}{N_A + N_D} \right) W \end{aligned} \quad (3.29) \quad \text{!}$$

Finally, we can substitute for W from Eq. (3.26) to obtain

$$Q_J = A \sqrt{2\epsilon_s q \left(\frac{N_A N_D}{N_A + N_D} \right) V_0} \quad (3.30) \quad \text{!}$$

These expressions for Q_J will prove useful in subsequent sections.

Example 3.5

Consider a pn junction in equilibrium at room temperature ($T = 300$ K) for which the doping concentrations are $N_A = 10^{18}/\text{cm}^3$ and $N_D = 10^{16}/\text{cm}^3$ and the cross-sectional area $A = 10^{-4} \text{ cm}^2$. Calculate p_p , n_{p0} , n_n , p_{n0} , V_0 , W , x_n , x_p , and Q_J . Use $n_i = 1.5 \times 10^{10}/\text{cm}^3$.

Solution

$$\begin{aligned} p_p &\approx N_A = 10^{18} \text{ cm}^{-3} \\ n_{p0} &= \frac{n_i^2}{p_p} \approx \frac{n_i^2}{N_A} = \frac{(1.5 \times 10^{10})^2}{10^{18}} = 2.25 \times 10^2 \text{ cm}^{-3} \\ n_n &\approx N_D = 10^{16} \text{ cm}^{-3} \\ p_{n0} &= \frac{n_i^2}{n_n} \approx \frac{n_i^2}{N_D} = \frac{(1.5 \times 10^{10})^2}{10^{16}} = 2.25 \times 10^4 \text{ cm}^{-3} \end{aligned}$$

To find V_0 we use Eq. (3.22),

$$V_0 = V_T \ln \left(\frac{N_A N_D}{n_i^2} \right)$$

where

$$\begin{aligned} V_T &= \frac{kT}{q} = \frac{8.62 \times 10^{-5} \times 300 \text{ (eV)}}{q \text{ (e)}} \\ &= 25.9 \times 10^{-3} \text{ V} \end{aligned}$$

Thus,

$$\begin{aligned} V_0 &= 25.9 \times 10^{-3} \ln \left(\frac{10^{18} \times 10^{16}}{2.25 \times 10^{20}} \right) \\ &= 0.814 \text{ V} \end{aligned}$$

To determine W we use Eq. (3.26):

$$\begin{aligned} W &= \sqrt{\frac{2 \times 1.04 \times 10^{-12}}{1.6 \times 10^{-19}} \left(\frac{1}{10^{18}} + \frac{1}{10^{16}} \right) \times 0.814} \\ &= 3.27 \times 10^{-5} \text{ cm} = 0.327 \text{ } \mu\text{m} \end{aligned}$$

To determine x_n and x_p we use Eq. (3.27) and (3.28), respectively:

$$\begin{aligned} x_n &= W \frac{N_A}{N_A + N_D} \\ &= 0.327 \frac{10^{18}}{10^{18} + 10^{16}} = 0.324 \text{ } \mu\text{m} \\ x_p &= W \frac{N_D}{N_A + N_D} \\ &= 0.327 \frac{10^{16}}{10^{18} + 10^{16}} = 0.003 \text{ } \mu\text{m} \end{aligned}$$

Finally, to determine the charge stored on either side of the depletion region, we use Eq. (3.29)

$$\begin{aligned} Q_J &= 10^{-4} \times 1.6 \times 10^{-19} \left(\frac{10^{18} \times 10^{16}}{10^{18} + 10^{16}} \right) \times 0.327 \times 10^{-4} \\ &= 5.18 \times 10^{-12} \text{ C} = 5.18 \text{ pC} \end{aligned}$$

EXERCISES

3.7 Show that

$$V_0 = \frac{1}{2} \left(\frac{q}{\epsilon_s} \right) \left(\frac{N_A N_D}{N_A + N_D} \right) W^2$$

3.8 Show that for a pn junction in which the p side is much more heavily doped than the n side, (i.e. $N_A \gg N_D$), referred to as a p^+n diode, Eqs. (3.26), (3.27), (3.28), (3.29), and (3.30) can be simplified as follows:

$$W \approx \sqrt{\frac{2\epsilon_s}{qN_D} V_0} \quad (3.26')$$

$$x_n \approx W \quad (3.27')$$

$$x_p \approx (W/(N_A/N_D)) \quad (3.28')$$

$$Q_J \approx AqN_D W \quad (3.29')$$

$$Q_J \approx A\sqrt{2\epsilon_s q N_D V_0} \quad (3.30')$$

3.9 If in the fabrication of the pn junction in Example 3.5, it is required to increase the minority carrier concentration in the n region by a factor of 2, what must be done?

Ans. Lower N_D by a factor of 2.

3.5 The pn Junction with an Applied Voltage

Having studied the open-circuited pn junction in detail, we are now ready to apply a dc voltage between its two terminals to find its electrical conduction properties. If the voltage is applied so that the p side is made more positive than the n side, it is referred to as a forward-bias⁶ voltage. Conversely, if our applied dc voltage is such that it makes the n side more positive than the p side, it is said to be a reverse-bias voltage. As will be seen, the pn junction exhibits vastly different conduction properties in its forward and reverse directions.

Our plan is as follows. We begin by a simple qualitative description in Section 3.5.1 and then consider an analytical description of the $i-v$ characteristic of the junction in Section 3.5.2.

3.5.1 Qualitative Description of Junction Operation

Figure 3.11 shows the pn junction under three different conditions: (a) the open-circuit or equilibrium condition studied in the previous section; (b) the reverse-bias condition, where a dc voltage V_R is applied; and (c) the forward-bias condition where a dc voltage V_F is applied. Observe that in the open-circuit case, a barrier voltage V_0 develops, making n more positive than p , and limiting the diffusion current I_D to a value exactly equal to the drift current I_S ,

⁶For the time being, we take the term *bias* to refer simply to the application of a dc voltage. We will see in later chapters that it has a deeper meaning in the design of electronic circuits.

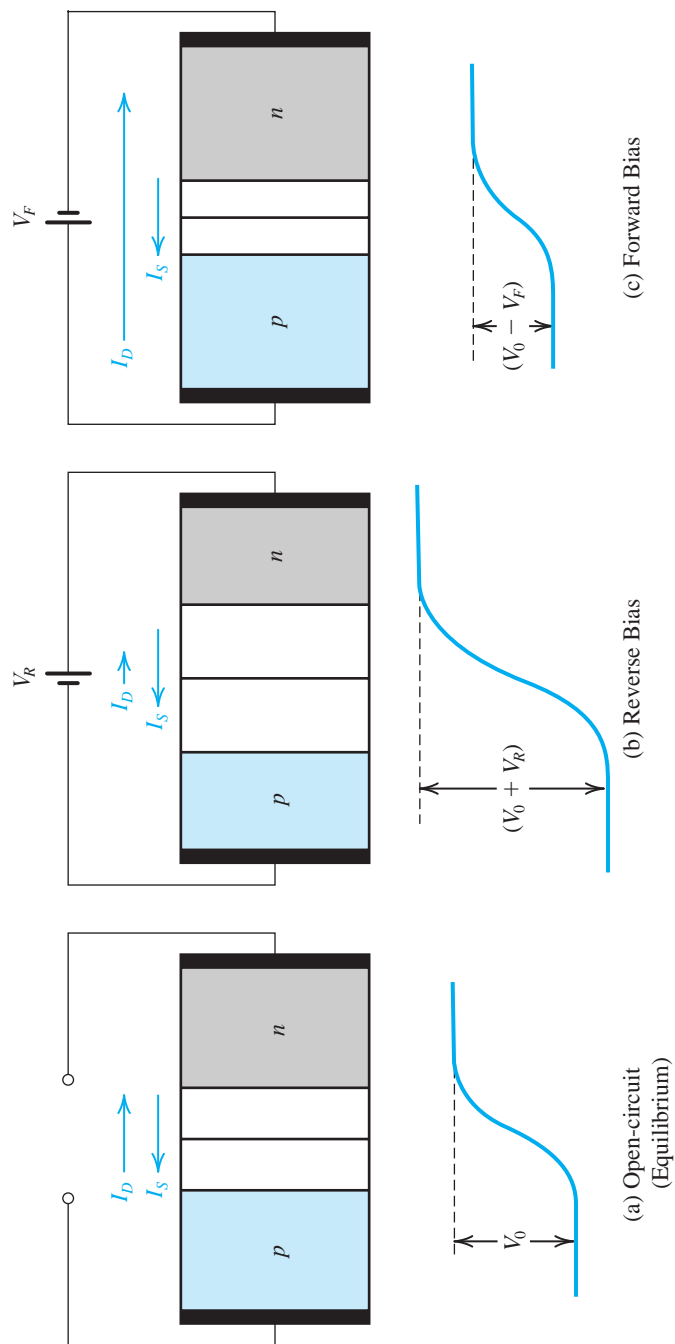


Figure 3.11 The *pn* junction in: (a) equilibrium; (b) reverse bias; (c) forward bias.

thus resulting in a zero current at the junction terminals, as should be the case since the terminals are open circuited. Also, as mentioned previously, the barrier voltage V_0 , though it establishes the current equilibrium across the junction, does *not* in fact appear between the junction terminals.

Consider now the reverse-bias case in (b). The externally applied reverse-bias voltage V_R is in the direction to add to the barrier voltage, and it does, thus increasing the effective barrier voltage to $(V_0 + V_R)$ as shown. This reduces the number of holes that diffuse into the n region and the number of electrons that diffuse into the p region. The end result is that the diffusion current I_D is dramatically reduced. As will be seen shortly, a reverse-bias voltage of a volt or so is sufficient to cause $I_D \approx 0$, and the current across the junction and through the external circuit will be equal to I_S . Recalling that I_S is the current due to the drift across the depletion region of the thermally generated minority carriers, we expect I_S to be very small and to be strongly dependent on temperature. We will show this to be the case very shortly. We thus conclude that in the reverse direction, the pn junction conducts a very small and almost-constant current equal to I_S .

Before leaving the reverse-bias case, observe that the increase in barrier voltage will be accompanied by a corresponding increase in the stored uncovered charge on both sides of the depletion region. This in turn means a wider depletion region, needed to uncover the additional charge required to support the larger barrier voltage $(V_0 + V_R)$. Analytically, these results can be obtained easily by a simple extension of the results of the equilibrium case. Thus the width of the depletion region can be obtained by replacing V_0 in Eq. (3.26) by $(V_0 + V_R)$,

$$W = x_n + x_p = \sqrt{\frac{2\epsilon_s}{q} \left(\frac{1}{N_A} + \frac{1}{N_D} \right) (V_0 + V_R)} \quad (3.31) \quad \text{!}$$

and the magnitude of the charge stored on either side of the depletion region can be determined by replacing V_0 in Eq. (3.30) by $(V_0 + V_R)$,

$$Q_J = A \sqrt{2\epsilon_s q \left(\frac{N_A N_D}{N_A + N_D} \right) (V_0 + V_R)} \quad (3.32) \quad \text{!}$$

We next consider the forward-bias case shown in Fig. 3.11(c). Here the applied voltage V_F is in the direction that subtracts from the built-in voltage V_0 , resulting in a reduced barrier voltage $(V_0 - V_F)$ across the depletion region. This reduced barrier voltage will be accompanied by reduced depletion-region charge and correspondingly narrower depletion-region width W . Most importantly, the lowering of the barrier voltage will enable more holes to diffuse from p to n and more electrons to diffuse from n to p . Thus the diffusion current I_D increases substantially and, as will be seen shortly, can become many orders of magnitude larger than the drift current I_S . The current I in the external circuit is of course the difference between I_D and I_S ,

$$I = I_D - I_S$$

and it flows in the forward direction of the junction, from p to n . We thus conclude that the pn junction can conduct a substantial current in the forward-bias region and that current is mostly a diffusion current whose value is determined by the forward-bias voltage V_F .

3.5.2 The Current–Voltage Relationship of the Junction

We are now ready to find an analytical expression that describes the current–voltage relationship of the pn junction. In the following we consider a junction operating with a

forward applied voltage V and derive an expression for the current I that flows in the forward direction (from p to n). However, our derivation is general and will be seen to yield the reverse current when the applied voltage V is made negative.

From the qualitative description above we know that a forward-bias voltage V subtracts from the built-in voltage V_0 , thus resulting in a lower barrier voltage ($V_0 - V$). The lowered barrier in turn makes it possible for a greater number of holes to overcome the barrier and diffuse into the n region. A similar statement can be made about electrons from the n region diffusing into the p region.

Let us now consider the holes injected into the n region. The concentration of holes in the n region at the edge of the depletion region will increase considerably. In fact, an important result from device physics shows that the steady-state concentration at the edge of the depletion region will be

$$p_n(x_n) = p_{n0} e^{V/V_T} \quad (3.33)$$

That is, the concentration of the minority holes increases from the equilibrium value of p_{n0} (see Fig. 3.10) to the much larger value determined by the value of V , given by Eq. (3.33).

We describe this situation as follows: The forward-bias voltage V results in an **excess concentration** of minority holes at $x = x_n$, given by

$$\begin{aligned} \text{Excess concentration} &= p_{n0} e^{V/V_T} - p_{n0} \\ &= p_{n0} (e^{V/V_T} - 1) \end{aligned} \quad (3.34)$$

The increase in minority carrier concentration in Eqs. (3.33) and (3.34) occurs at the edge of the depletion region ($x = x_n$). As the injected holes diffuse into the n material, some will recombine with the majority electrons and disappear. Thus, the excess hole concentration will decay exponentially with distance. As a result, in the total hole concentration in the n material will be given by

$$p_n(x) = p_{n0} + (\text{Excess concentration}) e^{-(x-x_n)/L_p}$$

Substituting for the “Excess concentration” from Eq. (3.34) gives

$$p_n(x) = p_{n0} + p_{n0} (e^{V/V_T} - 1) e^{-(x-x_n)/L_p} \quad (3.35)$$

The exponential decay is characterized by the constant L_p , which is called the **diffusion length** of holes in the n material. The smaller the value of L_p , the faster the injected holes will recombine with the majority electrons, resulting in a steeper decay of minority carrier concentration.

Figure 3.12 shows the steady-state minority carrier concentration profiles on both sides of a pn junction in which $N_A \gg N_D$. Let’s stay a little longer with the diffusion of holes into the n region. Note that the shaded region under the exponential represents the excess minority carriers (holes). From our study of diffusion in Section 3.3, we know that the establishment of a carrier concentration profile such as that in Fig. 3.12 is essential to support a steady-state diffusion current. In fact, we can now find the value of the hole–diffusion current density by applying Eq. (3.19),

$$J_p(x) = -qD_p \frac{dp_n(x)}{dx}$$

Substituting for $p_n(x)$ from Eq. (3.35) gives

$$J_p(x) = q \left(\frac{D_p}{L_p} \right) p_{n0} (e^{V/V_T} - 1) e^{-(x-x_n)/L_p} \quad (3.36)$$

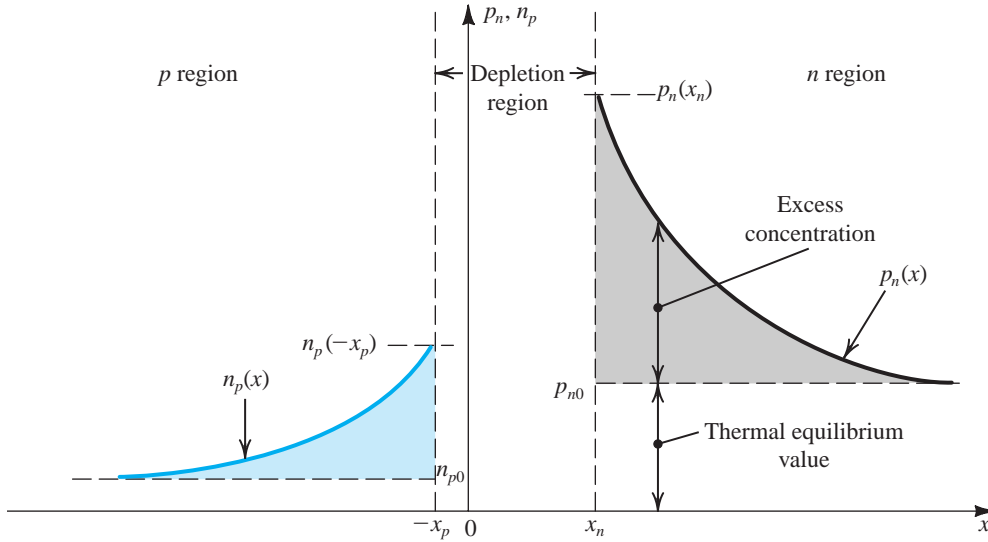


Figure 3.12 Minority-carrier distribution in a forward-biased pn junction. It is assumed that the p region is more heavily doped than the n region; $N_A \gg N_D$.

As expected, $J_p(x)$ is highest at $x = x_n$,

$$J_p(x_n) = q \left(\frac{D_p}{L_p} \right) p_{n0} (e^{V/V_T} - 1) \quad (3.37)$$

and decays exponentially for $x > x_n$, as the minority holes recombine with the majority electrons. This recombination, however, means that the majority electrons will have to be replenished by a current that injects electrons from the external circuit into the n region of the junction. This latter current component has the same direction as the hole current (because electrons moving from right to left give rise to current in the direction from left to right). It follows that as $J_p(x)$ decreases, the electron current component increases by exactly the same amount, making the total current in the n material constant at the value given by Eq. (3.37).

An exactly parallel development can be applied to the electrons that are injected from the n to the p region, resulting in an electron diffusion current given by a simple adaptation of Eq. (3.37),

$$J_n(-x_p) = q \left(\frac{D_n}{L_n} \right) n_{p0} (e^{V/V_T} - 1) \quad (3.38)$$

Now, although the currents in Eqs. (3.37) and (3.38) are found at the two edges of the depletion region, their values do not change in the depletion region. Thus we can drop the location descriptors (x_n), ($-x_p$), add the two current densities, and multiply by the junction area A to obtain the total current I as

$$\begin{aligned} I &= A(J_p + J_n) \\ I &= Aq \left(\frac{D_p}{L_p} p_{n0} + \frac{D_n}{L_n} n_{p0} \right) (e^{V/V_T} - 1) \end{aligned}$$

Substituting for $p_{n0} = n_i^2/N_D$ and for $n_{p0} = n_i^2/N_A$ gives

$$I = Aqn_i^2 \left(\frac{D_p}{L_p N_D} + \frac{D_n}{L_n N_A} \right) (e^{V/V_T} - 1) \quad (3.39)$$

From this equation we note that for a negative V (reverse bias) with a magnitude of a few times V_T (25.9 mV), the exponential term becomes essentially zero, and the current across the junction becomes negative and constant. From our qualitative description in Section 3.5.1, we know that this current must be I_S . Thus,

$$I = I_S(e^{V/V_T} - 1) \quad (3.40)$$

where

$$I_S = Aqn_i^2 \left(\frac{D_p}{L_p N_D} + \frac{D_n}{L_n N_A} \right) \quad (3.41)$$

Figure (3.13) shows the I - V characteristic of the pn junction (Eq. 3.40). Observe that in the reverse direction the current saturates at a value equal to $-I_S$. For this reason, I_S is given the name **saturation current**. From Eq. (3.41) we see that I_S is directly proportional to the cross-sectional area A of the junction. Thus, another name for I_S , one we prefer to use in this book, is the junction **scale current**. Typical values for I_S , for junctions of various areas, range from 10^{-18} to 10^{-12} A.

Besides being proportional to the junction area A , the expression for I_S in Eq. (3.41) indicates that I_S is proportional to n_i^2 which is a very strong function of temperature (see Eq. 3.2).

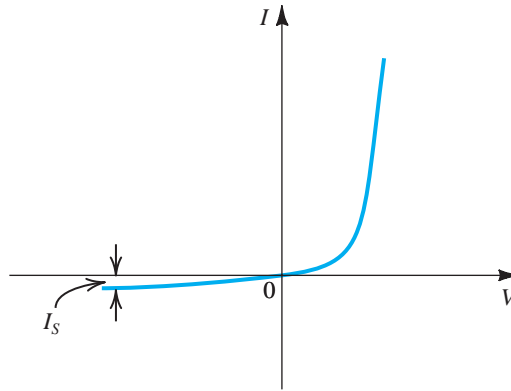


Figure 3.13 The pn junction I - V characteristic.

Example 3.6

For the pn junction considered in Example 3.5 for which $N_A = 10^{18}/\text{cm}^3$, $N_D = 10^{16}/\text{cm}^3$, $A = 10^{-4}\text{cm}^2$, $n_i = 1.5 \times 10^{10}/\text{cm}^3$, let $L_p = 5\text{ }\mu\text{m}$, $L_n = 10\text{ }\mu\text{m}$, D_p (in the n region) $= 10\text{ cm}^2/\text{V}\cdot\text{s}$, and D_n (in the p region) $= 18\text{ cm}^2/\text{V}\cdot\text{s}$. The pn junction is forward biased and conducting a current $I = 0.1\text{ mA}$. Calculate: (a) I_S ; (b) the forward-bias voltage V ; and (c) the component of the current I due to hole injection and that due to electron injection across the junction.

Solution

(a) Using Eq. (3.41), we find I_S as

$$\begin{aligned} I_S &= 10^{-4} \times 1.6 \times 10^{-19} \times (1.5 \times 10^{10})^2 \times \\ &\quad \left(\frac{10}{5 \times 10^{-4} \times 10^{16}} + \frac{18}{10 \times 10^{-4} \times 10^{18}} \right) \\ &= 7.3 \times 10^{-15}\text{ A} \end{aligned}$$

(b) In the forward direction,

$$\begin{aligned} I &= I_S(e^{V/V_T} - 1) \\ &\approx I_S e^{V/V_T} \end{aligned}$$

Thus,

$$V = V_T \ln\left(\frac{I}{I_S}\right)$$

For $I = 0.1\text{ mA}$,

$$\begin{aligned} V &= 25.9 \times 10^{-3} \ln\left(\frac{0.1 \times 10^{-3}}{7.3 \times 10^{-15}}\right) \\ &= 0.605\text{ V} \end{aligned}$$

(c) The hole-injection component of I can be found using Eq. (3.37)

$$\begin{aligned} I_p &= Aq \frac{D_p}{L_p} p_{n0}(e^{V/V_T} - 1) \\ &= Aq \frac{D_p}{L_p} \frac{n_i^2}{N_D}(e^{V/V_T} - 1) \end{aligned}$$

Similarly I_n can be found using Eq. (3.39),

$$I_n = Aq \frac{D_n}{L_n} \frac{n_i^2}{N_A}(e^{V/V_T} - 1)$$

Thus,

$$\frac{I_p}{I_n} = \left(\frac{D_p}{D_n}\right)\left(\frac{L_n}{L_p}\right)\left(\frac{N_A}{N_D}\right)$$

For our case,

$$\frac{I_p}{I_n} = \frac{10}{18} \times \frac{10}{5} \times \frac{10^{18}}{10^{16}} = 1.11 \times 10^2 = 111$$

Example 3.6 *continued*

Thus most of the current is conducted by holes injected into the n region.

Specifically,

$$I_p = \frac{111}{112} \times 0.1 = 0.0991 \text{ mA}$$

$$I_n = \frac{1}{112} \times 0.1 = 0.0009 \text{ mA}$$

This stands to reason, since the p material has a doping concentration 100 times that of the n material.

EXERCISES

3.10 Show that if $N_A \gg N_D$,

$$I_S \approx Aqn_i^2 \frac{D_p}{L_p N_D}$$

3.11 For the pn junction in Example 3.6, find the value of I_S and that of the current I at $V = 0.605 \text{ V}$ (same voltage found in Example 3.6 at a current $I = 0.1 \text{ mA}$) if N_D is reduced by a factor of 2.

Ans. $1.46 \times 10^{-14} \text{ A}$; 0.2 mA

3.12 For the pn junction considered in Examples 3.5 and 3.6, find the width of the depletion region W corresponding to the forward-bias voltage found in Example 3.6. (*Hint*: Use the formula in Eq. (3.31) with V_R replaced with $-V_F$.)

Ans. $0.166 \text{ } \mu\text{m}$

3.13 For the pn junction considered in Examples 3.5 and 3.6, find the width of the depletion region W and the charge stored in the depletion region Q_J when a 2-V reverse bias is applied. Also find the value of the reverse current I .

Ans. $0.608 \text{ } \mu\text{m}$; 9.63 pC ; $7.3 \times 10^{-15} \text{ A}$

3.5.3 Reverse Breakdown

The description of the operation of the pn junction in the reverse direction, and the $I - V$ relationship of the junction in Eq. (3.40), indicate that at a reverse-bias voltage $-V$, with $V \gg V_T$, the reverse current that flows across the junction is approximately equal to I_S and thus is very small. However, as the magnitude of the reverse-bias voltage V is increased, a value is reached at which a very large reverse current flows as shown in Fig. 3.14. Observe that as V reaches the value V_Z , the dramatic increase in reverse current is accompanied by a very small increase in the reverse voltage; that is, the reverse voltage across the junction remains very close to the value V_Z . The phenomenon that occurs at $V = V_Z$ is known as **junction breakdown**. It is not a destructive phenomenon. That is, the pn junction can be repeatedly operated in the breakdown region without a permanent effect on its characteristics. This, however, is predicated on the assumption that the magnitude of the reverse-breakdown current is

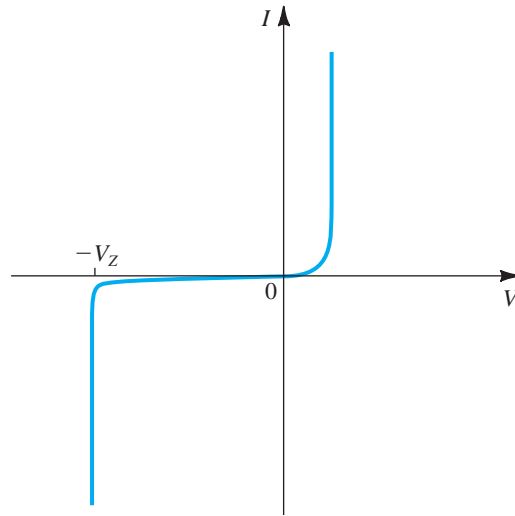


Figure 3.14 The I - V characteristic of the pn junction showing the rapid increase in reverse current in the breakdown region.

limited by the external circuit to a “safe” value. The “safe” value is one that results in the limitation of the power dissipated in the junction to a safe, allowable level.

There are two possible mechanisms for pn junction breakdown: the **zener effect**⁷ and the **avalanche effect**. If a pn junction breaks down with a breakdown voltage $V_Z < 5$ V, the breakdown mechanism is usually the zener effect. Avalanche breakdown occurs when V_Z is greater than approximately 7 V. For junctions that break down between 5 V and 7 V, the breakdown mechanism can be either the zener or the avalanche effect or a combination of the two.

Zener breakdown occurs when the electric field in the depletion layer increases to the point of breaking covalent bonds and generating electron-hole pairs. The electrons generated in this way will be swept by the electric field into the n side and the holes into the p side. Thus these electrons and holes constitute a reverse current across the junction. Once the zener effect starts, a large number of carriers can be generated, with a negligible increase in the junction voltage. Thus the reverse current in the breakdown region will be large and its value must be determined by the external circuit, while the reverse voltage appearing between the diode terminals will remain close to the specified breakdown voltage V_Z .

The other breakdown mechanism, avalanche breakdown, which occurs when the minority carriers that cross the depletion region under the influence of the electric field gain sufficient kinetic energy to be able to break covalent bonds in atoms with which they collide. The carriers liberated by this process may have sufficiently high energy to be able to cause other carriers to be liberated in another ionizing collision. This process keeps repeating in the fashion of an avalanche, with the result that many carriers are created that are able to support any value of reverse current, as determined by the external circuit, with a negligible change in the voltage drop across the junction.

⁷ Named after an early worker in the area. Note that the subscript Z in V_Z denotes *zener*. We will use V_Z to denote the breakdown voltage whether the breakdown mechanism is the zener effect or the avalanche effect.

As will be seen in Chapter 4, some pn junction diodes are fabricated to operate specifically in the breakdown region, where use is made of the nearly constant voltage V_Z .

3.6 Capacitive Effects in the pn Junction

There are two charge storage mechanisms in the pn junction. One is associated with the charge stored in the depletion region, and the other associated with the minority carrier charge stored in the n and p materials as a result of the concentration profiles established by carrier injection. While the first is easier to see when the pn junction is reverse biased, the second is in effect only when the junction is forward biased.

3.6.1 Depletion or Junction Capacitance

When a pn junction is reverse biased with a voltage V_R , the charge stored on either side of the depletion region is given by Eq. (3.32),

$$Q_J = A \sqrt{2\epsilon_s q \frac{N_A N_D}{N_A + N_D} (V_0 + V_R)}$$

Thus, for a given pn junction,

$$Q_J = \alpha \sqrt{V_0 + V_R} \quad (3.42)$$

where α is given by

$$\alpha = A \sqrt{2\epsilon_s q \frac{N_A N_D}{N_A + N_D}} \quad (3.43)$$

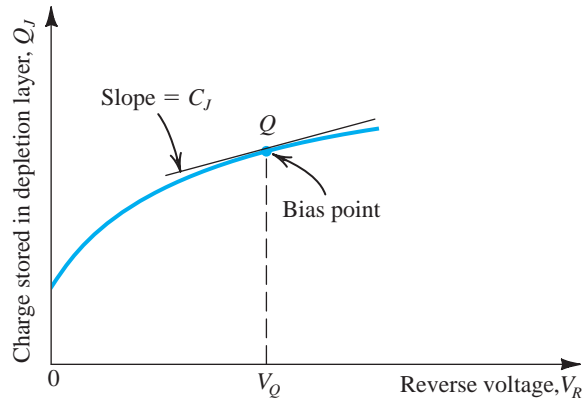


Figure 3.15 The charge stored on either side of the depletion layer as a function of the reverse voltage V_R .

Thus Q_J is nonlinearly related to V_R , as shown in Fig. (3.15). This nonlinear relationship makes it difficult to define a capacitance that accounts for the need to change Q_J whenever V_R is changed. We can, however, assume that the junction is operating at a point such as Q , as indicated in Fig. 3.15, and define a capacitance C_J that relates the change in the charge Q_J to a change in the voltage V_R ,

$$C_J = \left. \frac{dQ_J}{dV_R} \right|_{V_R=V_Q} \quad (3.44)$$

This incremental-capacitance approach turns out to be quite useful in electronic circuit design, as we shall see throughout this book.

Using Eq. (3.44) together with Eq. (3.42) yields

$$C_J = \frac{\alpha}{2\sqrt{V_0 + V_R}} \quad (3.45)$$

The value of C_J at zero reverse-bias can be obtained from Eq. (3.45) as

$$C_{J0} = \frac{\alpha}{2\sqrt{V_0}} \quad (3.46)$$

which enables us to express C_J as

$$C_J = \frac{C_{J0}}{\sqrt{1 + \frac{V_R}{V_0}}} \quad (3.47) \quad \text{①}$$

where C_{J0} is given by Eq. (3.46) or alternatively if we substitute for α from Eq. (3.43) by

$$C_{J0} = A \sqrt{\left(\frac{\epsilon_s q}{2}\right) \left(\frac{N_A N_D}{N_A + N_D}\right) \left(\frac{1}{V_0}\right)} \quad (3.48) \quad \text{①}$$

Before leaving the subject of depletion-region or junction capacitance we point out that in the pn junction we have been studying, the doping concentration is made to change abruptly at the junction boundary. Such a junction is known as an **abrupt junction**. There is another type of pn junction in which the carrier concentration is made to change gradually from one side of the junction to the other. To allow for such a **graded junction**, the formula for the junction capacitance (Eq. 3.47) can be written in the more general form

$$C_J = \frac{C_{J0}}{\left(1 + \frac{V_R}{V_0}\right)^m} \quad (3.49) \quad \text{①}$$

where m is a constant called the **grading coefficient**, whose value ranges from 1/3 to 1/2 depending on the manner in which the concentration changes from the p to the n side.

EXERCISE

3.12 For the pn junction considered in Examples 3.5 and 3.6, find C_{J0} and C_J at $V_R = 2$ V. Recall that $V_0 = 0.814$ V, $N_A = 10^{18}/\text{cm}^3$, $N_D = 10^{16}/\text{cm}^3$ and $A = 10^{-4} \text{ cm}^2$.

Ans. 3.2 pF; 1.7 pF

3.6.2 Diffusion Capacitance

Consider a forward-biased pn junction. In steady-state, minority carrier distributions in the p and n materials are established, as shown in Fig. 3.12. Thus a certain amount of excess minority carrier charge is stored in each of the p and n bulk regions (outside the depletion region). If the terminal voltage V changes, this charge will have to change before a new steady state is achieved. This charge-storage phenomenon gives rise to another capacitive effect, distinctly different from that due to charge storage in the depletion region.

To calculate the excess minority carrier charge, refer to Fig. 3.12. The excess hole charge stored in the n region can be found from the shaded area under the exponential as follows:⁸

$$\begin{aligned} Q_p &= Aq \times \text{shaded area under the } p_n(x) \text{ curve} \\ &= Aq[p_n(x_n) - p_{n0}]L_p \end{aligned}$$

substituting for $p_n(x_n)$ from Eq. (3.33) and using Eq. (3.37) enables us to express Q_p as

$$Q_p = \frac{L_p^2}{D_p} I_p \quad (3.50)$$

The factor (L_p^2/D_p) that relates Q_p to I_p is a useful device parameter that has the dimension of time (s) and is denoted τ_p

$$\tau_p = \frac{L_p^2}{D_p} \quad (3.51)$$

Thus,

$$Q_p = \tau_p I_p \quad (3.52)$$

The time constant τ_p is known as the excess **minority carrier (hole) lifetime**. *It is the average time it takes for a hole injected into the n region to recombine with a majority electron.* This definition of τ_p implies that the entire charge Q_p disappears and has to be replenished every τ_p seconds. The current that accomplishes the replenishing is $I_p = Q_p/\tau_p$. This is an alternate derivation for Eq. (3.52).

A relationship similar to that in Eq. (3.52) can be developed for the electron charge stored in the p region,

$$Q_n = \tau_n I_n \quad (3.53)$$

where τ_n is the electron lifetime in the p region. The total excess minority carrier charge can be obtained by adding together Q_p and Q_n ,

$$Q = \tau_p I_p + \tau_n I_n \quad (3.54)$$

This charge can be expressed in terms of the diode current $I = I_p + I_n$ as

$$Q = \tau_T I \quad (3.55)$$

where τ_T is called the **mean transit time** of the junction. Obviously, τ_T is related to τ_p and τ_n . Furthermore, for most practical devices, one side of the junction is much more heavily doped than the other. For instance, if $N_A \gg N_D$, one can show that $I_p \gg I_n$, $I \approx I_p$, $Q_p \gg Q_n$, $Q \approx Q_p$, and thus $\tau_T \approx \tau_p$.

⁸Recall that the area under an exponential curve $Ae^{-x/B}$ is equal to AB .

For small changes around a bias point, we can define an **incremental diffusion capacitance** C_d as

$$C_d = \frac{dQ}{dV} \quad (3.56)$$

and can show that

$$C_d = \left(\frac{\tau_T}{V_T} \right) I \quad (3.57) \quad \text{!}$$

where I is the forward-bias current. Note that C_d is directly proportional to the forward current I and thus is negligibly small when the diode is reverse biased. Also note that to keep C_d small, the transit time τ_T must be made small, an important requirement for a pn junction intended for high-speed or high-frequency operation.

EXERCISES

3.15 Use the definition of C_d in Eq. (3.56) to derive the expression in Eq. (3.57) by means of Eqs. (3.55) and (3.40).

3.16 For the pn junction considered in Examples 3.5 and 3.6 for which $D_p = 10 \text{ cm}^2/\text{V}\cdot\text{s}$, and $L_p = 5 \text{ }\mu\text{m}$, find τ_p and C_d at a forward-bias current of 0.1 mA. Recall that for this junction, $I_p \approx I$.

Ans. 25 ns; 96.5 pF

Summary

- Today's microelectronics technology is almost entirely based on the semiconductor material silicon. If a circuit is to be fabricated as a monolithic integrated circuit (IC) it is made using a single silicon crystal, no matter how large the circuit is [a recent chip (2009) contains 2.3 billion transistors].
- In a crystal of intrinsic or pure silicon, the atoms are held in position by covalent bonds. At very low temperatures, all the bonds are intact, and no charge carriers are available to conduct electrical current. Thus, at such low temperatures, silicon behaves as an insulator.
- At room temperature, thermal energy causes some of the covalent bonds to break, thus generating free electrons and holes that become available for current conduction.
- Current in semiconductors is carried by free electrons and holes. Their numbers are equal and relatively small in intrinsic silicon.
- The conductivity of silicon can be increased dramatically by introducing small amounts of appropriate impurity materials into the silicon crystal in a process called doping.
- There are two kinds of doped semiconductor: n -type, in which electrons are abundant, and p -type, in which holes are abundant.
- There are two mechanisms for the transport of charge carriers in semiconductor: drift and diffusion.
- Carrier drift results when an electric field E is applied across a piece of silicon. The electric field accelerates the holes in the direction of E and the electrons in the direction opposite to E . These two current components add together to produce a drift current in the direction of E .
- Carrier diffusion occurs when the concentration of charge carriers is made higher in one part of the silicon crystal than in other parts. To establish a steady-state diffusion current, a carrier concentration gradient must be maintained in the silicon crystal.
- A basic semiconductor structure is the pn junction. It is fabricated in a silicon crystal by creating a p region in close proximity to an n region. The pn junction is a diode and plays a dominant role in the structure and operation of transistors.
- When the terminals of the pn junction are left open, no current flows externally. However, two equal and opposite currents, I_D and I_S , flow across the junction, and equilibrium is maintained by a built-in voltage V_0 that develops across the junction, with the n side positive relative to the p side. Note, however, that the voltage across an open junction is 0 V, since V_0 is cancelled by potentials appearing at the metal-to-semiconductor connection interfaces.
- The voltage V_0 appears across the depletion region, which extends on both sides of the junction.
- The diffusion current I_D is carried by holes diffusing from p to n and electrons diffusing from n to p . I_D flows from p to n , which is the forward direction of the junction. Its value depends on V_0 .
- The drift current I_S is carried by thermally generated minority electrons in the p material that are swept across the depletion layer into the n side, and by thermally generated minority holes in the n side that are swept across the depletion region into the p side. I_S flows from n to p , in the reverse direction of the junction, and its value is a strong function of temperature but independent of V_0 .
- Forward biasing the pn junction, that is, applying an external voltage V that makes p more positive than n , reduces the barrier voltage to $V_0 - V$ and results in an exponential increase in I_D while I_S remains unchanged. The net result is a substantial current $I = I_D - I_S$ that flows across the junction and through the external circuit.
- Applying a negative V reverse-biases the junction and increases the barrier voltage, with the result that I_D is reduced to almost zero and the net current across the junction becomes the very small reverse current I_S .
- If the reverse voltage is increased in magnitude to a value V_Z specific to the particular junction, the junction breaks down, and a large reverse current flows. The value of the reverse current must be limited by the external circuit.
- Whenever the voltage across a pn junction is changed, some time has to pass before steady state is reached. This is due to the charge-storage effects in the junction, which are modeled by two capacitances: the junction capacitance C_j and the diffusion capacitance C_d .
- For future reference, we present in Table 3.1 a summary of pertinent relationships and the values of physical constants.

Table 3.1 Summary of Important Equations

Quantity	Relationship	Values of Constants and Parameters (for Intrinsic Si at $T = 300$ K)
Carrier concentration in intrinsic silicon (cm^{-3})	$n_i = BT^{3/2}e^{-E_g/2kT}$	$B = 7.3 \times 10^{15} \text{cm}^{-3} \text{K}^{-3/2}$ $E_g = 1.12 \text{ eV}$ $k = 8.62 \times 10^{-5} \text{ eV/K}$ $n_i = 1.5 \times 10^{10} / \text{cm}^3$
Diffusion current density (A/cm^2)	$J_p = -qD_p \frac{dp}{dx}$ $J_n = qD_n \frac{dn}{dx}$	$q = 1.60 \times 10^{-19} \text{ coulomb}$ $D_p = 12 \text{ cm}^2/\text{s}$ $D_n = 34 \text{ cm}^2/\text{s}$
Drift current density (A/cm^2)	$J_{\text{drift}} = q(p\mu_p + n\mu_n)E$	$\mu_p = 480 \text{ cm}^2/\text{V}\cdot\text{s}$ $\mu_n = 1350 \text{ cm}^2/\text{V}\cdot\text{s}$
Resistivity ($\Omega \cdot \text{cm}$)	$\rho = 1/[q(p\mu_p + n\mu_n)]$	μ_p and μ_n decrease with the increase in doping concentration
Relationship between mobility and diffusivity	$\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = V_T$	$V_T = kT/q \approx 25.8 \text{ mV}$
Carrier concentration in n -type silicon (cm^{-3})	$n_{n0} \approx N_D$ $p_{n0} = n_i^2/N_D$	
Carrier concentration in p -type silicon (cm^{-3})	$p_{p0} \approx N_A$ $n_{p0} = n_i^2/N_A$	
Junction built-in voltage (V)	$V_0 = V_T \ln \left(\frac{N_A N_D}{n_i^2} \right)$	
Width of depletion region (cm)	$\frac{x_n}{x_p} = \frac{N_A}{N_D}$ $W = x_n + x_p$ $= \sqrt{\frac{2\epsilon_s}{q} \left(\frac{1}{N_A} + \frac{1}{N_D} \right) (V_0 + V_R)}$	$\epsilon_s = 11.7\epsilon_0$ $\epsilon_0 = 8.854 \times 10^{-14} \text{ F/cm}$
Charge stored in depletion layer (coulomb)	$Q_J = q \frac{N_A N_D}{N_A + N_D} A W$	
Forward current (A)	$I = I_p + I_n$ $I_p = Aq n_i^2 \frac{D_p}{L_p N_D} (e^{V/V_T} - 1)$ $I_n = Aq n_i^2 \frac{D_n}{L_n N_A} (e^{V/V_T} - 1)$	
Saturation current (A)	$I_S = Aq n_i^2 \left(\frac{D_p}{L_p N_D} + \frac{D_n}{L_n N_A} \right)$	
I - V Relationship	$I = I_S (e^{V/V_T} - 1)$	

Table 3.1 continued		
Quantity	Relationship	Values of Constants and Parameters (for Intrinsic Si at $T = 300$ K)
Minority-carrier lifetime (s)	$\tau_p = L_p^2/D_p \quad \tau_n = L_n^2/D_n$	$L_p, L_n = 1 \text{ } \mu\text{m to } 100 \text{ } \mu\text{m}$ $\tau_p, \tau_n = 1 \text{ ns to } 10^4 \text{ ns}$
Minority-carrier charge storage (coulomb)	$Q_p = \tau_p I_p \quad Q_n = \tau_n I_n$ $Q = Q_p + Q_n = \tau_T I$	
Depletion capacitance (F)	$C_{j0} = A \sqrt{\left(\frac{\epsilon_s q}{2}\right) \left(\frac{N_A N_D}{N_A + N_D}\right) \frac{1}{V_0}}$ $C_j = C_{j0} \left(1 + \frac{V_R}{V_0}\right)^m$	$m = \frac{1}{3} \text{ to } \frac{1}{2}$
Diffusion capacitance (F)	$C_d = \left(\frac{\tau_T}{V_T}\right) I$	

PROBLEMS

Problems are marked with asterisks to describe their degree of difficulty. Difficult problems are marked with an asterisk (*); more difficult problems with two asterisks (**); and very challenging and/or time-consuming problems with three asterisks (***). Also, if in the following problems the need arises for the values of particular parameters or physical constants that are not stated, please consult Table 3.1.

Section 3.1: Intrinsic Semiconductors

3.1 Find values of the intrinsic carrier concentration n_i for silicon at -70°C , 0°C , 20°C , 100°C , and 125°C . At each temperature, what fraction of the atoms is ionized? Recall that a silicon crystal has approximately 5×10^{22} atoms/cm³.

3.2 Calculate the value of n_i for gallium arsenide (GaAs) at $T = 300$ K. The constant $B = 3.56 \times 10^{14}$ (cm⁻³K^{-3/2}) and the bandgap voltage $E_g = 1.42$ eV.

Section 3.2: Doped Semiconductors

3.3 For a p -type silicon in which the dopant concentration $N_A = 10^{18}/\text{cm}^3$, find the hole and electron concentrations at $T = 300$ K.

3.4 For a silicon crystal doped with phosphorus, what must N_D be if at $T = 300$ K the hole concentration drops below the intrinsic level by a factor of 10^7 ?

3.5 In a phosphorus-doped silicon layer with impurity concentration of $10^{16}/\text{cm}^3$, find the hole and electron concentrations at 27°C and 125°C .

Section 3.3: Current Flow in Semiconductors

3.6 A young designer, aiming to develop intuition concerning conducting paths within an integrated circuit, examines the end-to-end resistance of a connecting bar $10 \text{ } \mu\text{m}$ long, $3 \text{ } \mu\text{m}$ wide, and $1 \text{ } \mu\text{m}$ thick, made of various materials. The designer considers:

- (a) intrinsic silicon
 (b) n -doped silicon with $N_D = 10^{16}/\text{cm}^3$
 (c) n -doped silicon with $N_D = 10^{18}/\text{cm}^3$
 (d) p -doped silicon with $N_A = 10^{16}/\text{cm}^3$
 (e) aluminum with resistivity of $2.8 \mu\Omega \cdot \text{cm}$

Find the resistance in each case. For intrinsic silicon, use the data in Table 3.1. For doped silicon, assume $\mu_n = 2.5\mu_p = 1200 \text{ cm}^2/\text{V}\cdot\text{s}$. (Recall that $R = \rho L/A$)

3.7 Contrast the electron and hole drift velocities through a $10\text{-}\mu\text{m}$ layer of intrinsic silicon across which a voltage of 5 V is imposed. Let $\mu_n = 1350 \text{ cm}^2/\text{V}\cdot\text{s}$ and $\mu_p = 480 \text{ cm}^2/\text{V}\cdot\text{s}$.

3.8 Find the current that flows in a silicon bar of $10\text{-}\mu\text{m}$ length having a $5\text{-}\mu\text{m} \times 4\text{-}\mu\text{m}$ cross section and having free electron and hole densities of $10^{15}/\text{cm}^3$ and $10^{15}/\text{cm}^3$, respectively, when a 1 V is applied end-to-end. Use $\mu_n = 1200 \text{ cm}^2/\text{V}\cdot\text{s}$ and $\mu_p = 500 \text{ cm}^2/\text{V}\cdot\text{s}$.

3.9 In a $10\text{-}\mu\text{m}$ long bar of donor-doped silicon, what donor concentration is needed to realize a current density of $1 \text{ mA}/\mu\text{m}^2$ in response to an applied voltage of 1 V . (Note: Although the carrier mobilities change with doping concentration, as a first approximation you may assume μ_n to be constant and use the value for intrinsic silicon, $1350 \text{ cm}^2/\text{V}\cdot\text{s}$).

3.10 Holes are being steadily injected into a region of n -type silicon (connected to other devices, the details of which are not important for this question). In the steady state, the excess-hole concentration profile shown in Fig. P3.10 is established in the n -type silicon region. Here “excess” means over and above the thermal-equilibrium concentration (in the absence of hole injection), denoted p_{n0} . If $N_D = 10^{16}/\text{cm}^3$, $n_i = 1.5 \times 10^{10}/\text{cm}^3$, $D_n = 12 \text{ cm}^2/\text{s}$, and $W = 0.1 \mu\text{m}$, find the density of the current that will flow in the x direction.

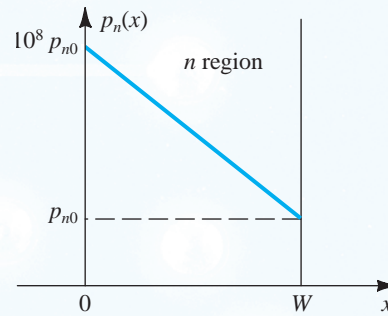


Figure P3.10

3.11 Both the carrier mobility and diffusivity decrease as the doping concentration of silicon is increased. The table below provides a few data points for μ_n and μ_p versus doping concentration. Use the Einstein relationship to obtain the corresponding values for D_n and D_p .

Section 3.4: The pn Junction with Open-Circuit Terminals (Equilibrium)

3.12 Calculate the built-in voltage of a junction in which the p and n regions are doped equally with 10^{16} atoms/ cm^3 . Assume $n_i = 1.5 \times 10^{10}/\text{cm}^3$. With the terminals left open, what is the width of the depletion region, and how far does it extend into the p and n regions? If the cross-sectional area of the junction is $100 \mu\text{m}^2$, find the magnitude of the charge stored on either side of the junction.

3.13 If, for a particular junction, the acceptor concentration is $10^{16}/\text{cm}^3$ and the donor concentration is $10^{15}/\text{cm}^3$, find the junction built-in voltage. Assume $n_i = 1.5 \times 10^{10}/\text{cm}^3$. Also, find the width of the depletion region (W) and its extent in each of the p and n regions when the junction terminals are left open. Calculate the magnitude of the charge stored on either side of the junction. Assume that the junction area is $400 \mu\text{m}^2$.

Doping Concentration (carriers/ cm^3)	μ_n ($\text{cm}^2/\text{V}\cdot\text{s}$)	μ_p ($\text{cm}^2/\text{V}\cdot\text{s}$)	D_n (cm^2/s)	D_p (cm^2/s)
Intrinsic	1350	480		
10^{16}	1100	400		
10^{17}	700	260		
10^{18}	360	150		

Table P3.11

3.14 Estimate the total charge stored in a $0.1\text{-}\mu\text{m}$ depletion layer on one side of a $10\text{-}\mu\text{m} \times 10\text{-}\mu\text{m}$ junction. The doping concentration on that side of the junction is $10^{16}/\text{cm}^3$.

3.15 In a pn junction for which $N_A \gg N_D$, and the depletion layer exists mostly on the shallowly doped side with $W = 0.3\text{ }\mu\text{m}$, find V_0 if $N_D = 10^{16}/\text{cm}^3$. Also calculate Q_J .

3.16 By how much does V_0 change if N_A or N_D is increased by a factor of 10?

Section 3.5: The pn Junction with an Applied Voltage

3.17 If a 5-V reverse-bias voltage is applied across the junction specified in Problem 3.13, find W and Q_J .

3.18 Show that for a pn junction reverse-biased with a voltage V_R , the depletion-layer width W and the charge stored on either side of the junction, Q_J , can be expressed as

$$W = W_0 \sqrt{1 + \frac{V_R}{V_0}}$$

$$Q_J = Q_{J0} \sqrt{1 + \frac{V_R}{V_0}}$$

where W_0 and Q_{J0} are the values in equilibrium.

3.19 In a forward-biased pn junction show that the ratio of the current component due to hole injection across the junction to the component due to electron injection is given by

$$\frac{I_p}{I_n} = \frac{D_p}{D_n} \frac{L_n}{L_p} \frac{N_A}{N_D}$$

Evaluate this ratio for the case $N_A = 10^{18}/\text{cm}^3$, $N_D = 10^{16}/\text{cm}^3$, $L_p = 5\text{ }\mu\text{m}$, $L_n = 10\text{ }\mu\text{m}$, $D_p = 10\text{ cm}^2/\text{s}$, and $D_n = 20\text{ cm}^2/\text{s}$, and hence find I_p and I_n for the case in which the pn junction is conducting a forward current $I = 1\text{ mA}$.

3.20 Calculate I_S and the current I for $V = 700\text{ mV}$ for a pn junction for which $N_A = 10^{17}/\text{cm}^3$, $N_D = 10^{16}/\text{cm}^3$, $A = 200\text{ }\mu\text{m}^2$, $n_i = 1.5 \times 10^{10}/\text{cm}^3$, $L_p = 5\text{ }\mu\text{m}$, $L_n = 10\text{ }\mu\text{m}$, $D_p = 10\text{ cm}^2/\text{s}$, and $D_n = 18\text{ cm}^2/\text{s}$.

3.21 Assuming that the temperature dependence of I_S arises mostly because I_S is proportional to n_i^2 , use the expression for n_i in Eq. (3.2) to determine the factor by which n_i^2 changes as T changes from 300 K to 305 K. This

will be approximately the same factor by which I_S changes for a 5°C rise in temperature. What is the factor?

3.22 A p^+n junction is one in which the doping concentration in the p region is much greater than that in the n region. In such a junction, the forward current is mostly due to hole injection across the junction. Show that

$$I \simeq I_p = Aqn_i^2 \frac{D_p}{L_p N_D} (e^{V/V_T} - 1)$$

For the specific case in which $N_D = 10^{16}/\text{cm}^3$, $D_p = 10\text{ cm}^2/\text{s}$, $L_p = 10\text{ }\mu\text{m}$, and $A = 10^4\text{ }\mu\text{m}^2$, find I_S and the voltage V obtained when $I = 0.5\text{ mA}$. Assume operation at 300 K where $n_i = 1.5 \times 10^{10}/\text{cm}^3$.

3.23 A pn junction for which the breakdown voltage is 12 V has a rated (i.e., maximum allowable) power dissipation of 0.25 W. What continuous current in the breakdown region will raise the dissipation to half the rated value? If breakdown occurs for only 10 ms in every 20 ms, what average breakdown current is allowed?

Section 3.6: Capacitive Effects in the pn Junction

3.24 For the pn junction specified in Problem 3.13, find C_{J0} and C_J at $V_R = 5\text{ V}$.

3.25 For a particular junction for which $C_{J0} = 0.6\text{ pF}$, $V_0 = 0.75\text{ V}$, and $m = 1/3$, find C_J at reverse-bias voltages of 1 V and 10 V.

3.26 The junction capacitance C_J can be thought of as that of a parallel-plate capacitor and thus given by

$$C_J = \frac{\epsilon A}{W}$$

Show that this approach leads to a formula identical to that obtained by combining Eqs. (3.43) and (3.45) [or equivalently, by combining Eqs. (3.47) and (3.48)].

3.27 A pn junction operating in the forward-bias region with a current I of 1 mA is found to have a diffusion capacitance of 10 pF. What diffusion capacitance do you expect this junction to have at $I = 0.1\text{ mA}$? What is the mean transit time for this junction?

3.28 For the p^+n junction specified in Problem 3.22, find τ_p and calculate the excess minority carrier charge and the value of the diffusion capacitance at $I = 0.2\text{ mA}$.

3.29 A **short-base diode** is one where the widths of the p and n regions are much smaller than L_n and L_p , respectively. As a result, the excess minority carrier distribution in each region is a straight line rather than the exponentials shown in Fig. 3.12.

(a) For the short-base diode, sketch a figure corresponding to Fig. 3.12 and assume as in Fig. 3.12 that $N_A \gg N_D$.

(b) Following a derivation similar to that given in Section 3.5.2, show that if the widths of the p and n regions are denoted W_p and W_n then

$$I = Aqn_i^2 \left[\frac{D_p}{(W_n - x_n)N_D} + \frac{D_n}{(W_p - x_p)N_A} \right] (e^{V/V_T} - 1)$$

and

$$Q_p = \frac{1}{2} \frac{(W_n - x_n)^2}{D_p} I_p$$

$$\simeq \frac{1}{2} \frac{W_n^2}{D_p} I_p, \text{ for } W_n \gg x_n$$

(c) Also, assuming $Q \simeq Q_p$, $I \simeq I_p$, show that

$$C_d = \frac{\tau_T}{V_T} I$$

where

$$\tau_T = \frac{1}{2} \frac{W_n^2}{D_p}$$

(d) If a designer wishes to limit C_d to 8 pF at $I = 1$ mA, what should W_n be? Assume $D_p = 10$ cm²/s.