CHAPTER 10

Feedback

IN THIS CHAPTER YOU WILL LEARN

1. The general structure of the negative-feedback amplifier and the basic principle that underlies its operation.
2. The advantages of negative feedback, how these come about, and at what cost.
3. The appropriate feedback topology to employ with each of the four amplifier types: voltage, current, transconductance, and transresistance amplifiers.
4. An intuitive and insightful approach for the analysis of practical feedback-amplifier circuits.
5. Why and how negative-feedback amplifiers can become unstable (i.e., oscillate) and how to design the circuit to ensure stable performance.

Introduction

Most physical systems incorporate some form of feedback. It is interesting to note, though, that the theory of negative feedback has been developed by electronics engineers. In his search for methods for the design of amplifiers with stable gain for use in telephone repeaters, Harold Black, an electronics engineer with the Western Electric Company, invented the feedback amplifier in 1928. Since then the technique has been so widely used that it is almost impossible to think of electronic circuits without some form of feedback, either implicit or explicit. Furthermore, the concept of feedback and its associated theory are currently used in areas other than engineering, such as in the modeling of biological systems.

Feedback can be either **negative (degenerative)** or **positive (regenerative)**. In amplifier design, negative feedback is applied to effect one or more of the following properties:

1. *Desensitize the gain:* that is, make the value of the gain less sensitive to variations in the values of circuit components, such as might be caused by changes in temperature.
2. *Reduce nonlinear distortion:* that is, make the output proportional to the input (in other words, make the gain constant, independent of signal level).
3. *Reduce the effect of noise:* that is, minimize the contribution to the output of unwanted electric signals generated, either by the circuit components themselves, or by extraneous interference.

4. *Control the input and output resistances:* that is, raise or lower the input and output resistances by the selection of an appropriate feedback topology.
5. *Extend the bandwidth* of the amplifier.

All of the desirable properties above are obtained at the expense of a reduction in gain. It will be shown that the gain-reduction factor, called the **amount of feedback**, is the factor by which the circuit is desensitized, by which the input resistance of a voltage amplifier is increased, by which the bandwidth is extended, and so on. In short, *the basic idea of negative feedback is to trade off gain for other desirable properties.* This chapter is devoted to the study of negative-feedback amplifiers: their analysis, design, and characteristics.

Under certain conditions, the negative feedback in an amplifier can become positive and of such a magnitude as to cause oscillation. In fact, in Chapter 17 we will study the use of positive feedback in the design of oscillators and bistable circuits. Here, in this chapter, however, we are interested in the design of stable amplifiers. We shall therefore study the stability problem of negative-feedback amplifiers and their potential for oscillation.

It should not be implied, however, that positive feedback always leads to instability. In fact, positive feedback is quite useful in a number of nonregenerative applications, such as the design of active filters, which are studied in Chapter 16.

Before we begin our study of negative feedback, we wish to remind the reader that we have already encountered negative feedback in a number of applications. Almost all op-amp circuits (Chapter 2) employ negative feedback. Another popular application of negative feedback is the use of the emitter resistance R_E to stabilize the bias point of bipolar transistors and to increase the input resistance, bandwidth, and linearity of a BJT amplifier. In addition, the source follower and the emitter follower both employ a large amount of negative feedback. The question then arises about the need for a formal study of negative feedback. As will be appreciated by the end of this chapter, the formal study of feedback provides an invaluable tool for the analysis and design of electronic circuits. Also, the insight gained by thinking in terms of feedback can be extremely profitable.

10.1 The General Feedback Structure

Figure 10.1 shows the basic structure of a feedback amplifier. Rather than showing voltages and currents, Fig. 10.1 is a **signal-flow diagram**, where each of the quantities x can represent either a voltage or a current signal. The *open-loop* amplifier has a gain A; thus its output x_o is related to the input x_i by

$$x_o = Ax_i \tag{10.1}$$

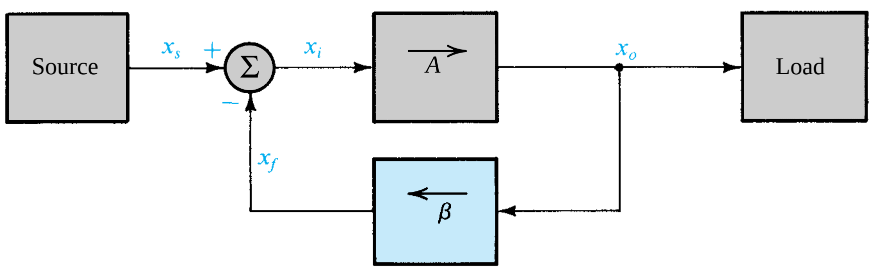


Figure 10.1 General structure of the feedback amplifier. This is a signal-flow diagram, and the quantities x represent either voltage or current signals.

The output x_o is fed to the load as well as to a feedback network, which produces a sample of the output. This sample x_f is related to x_o by the **feedback factor** β,

$$x_f = \beta x_o \tag{10.2}$$

The feedback signal x_f is *subtracted* from the source signal x_s, which is the input to the complete feedback amplifier,[1] to produce the signal x_i, which is the input to the basic amplifier,

$$x_i = x_s - x_f \tag{10.3}$$

Here we note that it is this subtraction that makes the feedback negative. In essence, negative feedback reduces the signal that appears at the input of the basic amplifier.

Implicit in the description above is that the source, the load, and the feedback network *do not* load the basic amplifier. That is, the gain A does not depend on any of these three networks. In practice this will not be the case, and we shall have to find a method for casting a real circuit into the ideal structure depicted in Fig. 10.1. Figure 10.1 also implies that the forward transmission occurs entirely through the basic amplifier and the reverse transmission occurs entirely through the feedback network.

The gain of the feedback amplifier can be obtained by combining Eqs. (10.1) through (10.3):

$$A_f \equiv \frac{x_o}{x_s} = \frac{A}{1 + A\beta} \tag{10.4}$$

The quantity $A\beta$ is called the **loop gain**, a name that follows from Fig. 10.1. For the feedback to be negative, the loop gain $A\beta$ must be positive; that is, the feedback signal x_f should have the same sign as x_s, thus resulting in a smaller difference signal x_i. Equation (10.4) indicates that for positive $A\beta$ the **gain with feedback** A_f will be smaller than the **open-loop gain** A by a factor equal to $1 + A\beta$, which is called the **amount of feedback**.

If, as is the case in many circuits, the loop gain $A\beta$ is large, $A\beta \gg 1$, then from Eq. (10.4) it follows that

$$A_f \simeq \frac{1}{\beta} \tag{10.5}$$

which is a very interesting result: *The gain of the feedback amplifier is almost entirely determined by the feedback network*. Since the feedback network usually consists of passive components, which usually can be chosen to be as accurate as one wishes, the advantage of negative feedback in obtaining accurate, predictable, and stable gain should be apparent. In other words, the overall gain will have very little dependence on the gain of the basic amplifier, A, a desirable property because the gain A is usually a function of many manufacturing and application parameters, some of which might have wide tolerances. We have seen a dramatic illustration of all of these effects in op-amp circuits in Chapter 2, where the **closed-loop gain** (which is another name for the gain-with-feedback) is almost entirely determined by the feedback elements.

Equations (10.1) through (10.3) can be combined to obtain the following expression for the feedback signal x_f:

$$x_f = \frac{A\beta}{1 + A\beta} x_s \tag{10.6}$$

[1]In earlier chapters, we used the subscript "sig" for quantities associated with the signal source (e.g., v_{sig} and R_{sig}). We did that to avoid confusion with the subscript "s," which is usually used with FETs to denote quantities associated with the source terminal of the transistor. At this point, however, it is expected that readers have become sufficiently familiar with the subject that the possibility of confusion is minimal. Therefore, we will revert to using the simpler subscript s for signal-source quantities.

Thus for $A\beta \gg 1$ we see that $x_f \simeq x_s$, which implies that the signal x_i at the input of the basic amplifier is reduced to almost zero. Thus if a large amount of negative feedback is employed, the feedback signal x_f becomes an almost identical replica of the input signal x_s. An outcome of this property is the tracking of the two input terminals of an op amp. The difference between x_s and x_f, which is x_i, is sometimes referred to as the **error signal**. Accordingly, the **input differencing circuit** is often also called a **comparison circuit**. (It is also known as a **mixer**.) An expression for x_i can be easily determined as

$$x_i = \frac{1}{1+A\beta}x_s \tag{10.7}$$

from which we can verify that for $A\beta \gg 1$, x_i becomes very small. Observe that negative feedback reduces the signal that appears at the input terminals of the basic amplifier by the amount of feedback, $(1 + A\beta)$. As will be seen later, it is this reduction of input signal that results in the increased linearity of the feedback amplifier.[2]

Example 10.1

The noninverting op-amp configuration shown in Fig. 10.2(a) provides a direct implementation of the feedback loop of Fig. 10.1.

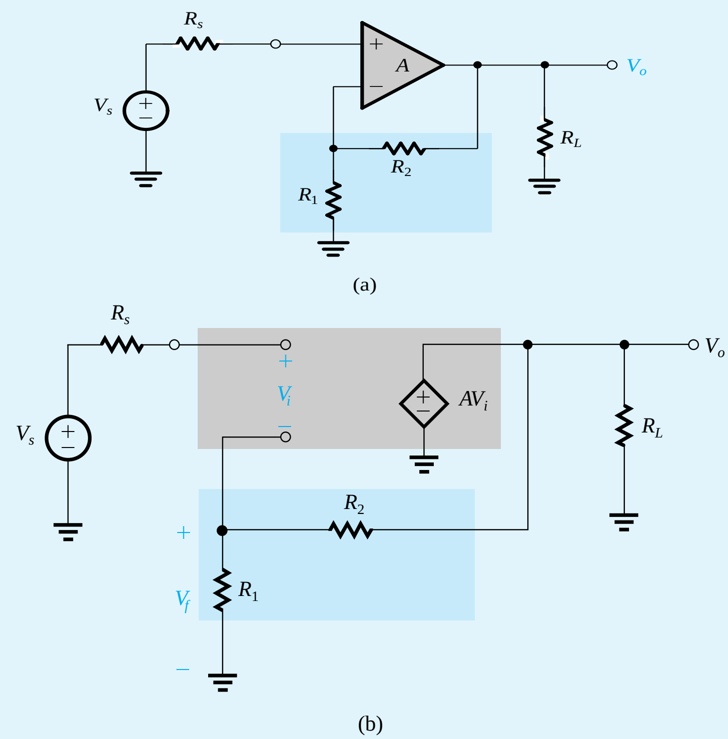


Figure 10.2 (**a**) A non-inverting op-amp circuit for Example 10.1. (**b**) The circuit in (a) with the op-amp replaced with its equivalent circuit.

[2]We have in fact already seen examples of this: adding a resistance R_e in the emitter of a CE amplifier (or a resistance R_s in the source of a CS amplifier) increases the linearity of these amplifiers because for the same input signal as before, v_{be} and v_{gs} are now smaller (by the amount of feedback).

(a) Assume that the op amp has infinite input resistance and zero output resistance. Find an expression for the feedback factor β.
(b) Find the condition under which the closed-loop gain A_f is almost entirely determined by the feedback network.
(c) If the open-loop gain $A = 10^4$ V/V, find R_2/R_1 to obtain a closed-loop gain A_f of 10 V/V.
(d) What is the amount of feedback in decibels?
(e) If $V_s = 1$ V, find V_o, V_f, and V_i.
(f) If A decreases by 20%, what is the corresponding decrease in A_f?

Solution

(a) To be able to see more clearly the direct correspondence between the circuit in Fig. 10.2(a) and the block diagram in Fig. 10.1, we replace the op amp with its equivalent-circuit model, as shown in Fig. 10.2(b). Since the op amp is assumed to have infinite input resistance and zero output resistance, its model is simply an ideal voltage-controlled voltage source of gain A. From Fig. 10.2(b) we observe that the feedback network, consisting of the voltage divider (R_1, R_2), is connected directly to the output and feeds a signal V_f to the inverting input terminal of the op amp. It is important at this point to note that the zero output resistance of the op amp causes the output voltage to be AV_i irrespective of the values of R_1 and R_2 and of R_L. That is what we meant by the statement that in the block diagram of Fig. 10.1, the feedback network and the load are assumed not to load the basic amplifier. Now we can easily determine the feedback factor β from

$$\beta \equiv \frac{V_f}{V_o} = \frac{R_1}{R_1 + R_2}$$

Let's next examine how V_f is subtracted from V_s at the input side. The subtraction is effectively performed by the differential action of the op amp; by its very nature, a differential-input amplifier takes the difference between the signals at its two input terminals. Observe also that because the input resistance of the op amp is assumed to be infinite, no current flows in R_s. Thus the value of R_s has no bearing on V_i; or the source "does not load" the amplifier input. Similarly, because of the zero input current of the op amp, V_f will depend only on the ratio R_1/R_2 and not on the absolute values of R_1 and R_2.

(b) The closed-loop gain A_f is given by

$$A_f = \frac{A}{1 + A\beta}$$

To make A_f nearly independent of A, we must ensure that the loop gain $A\beta$ is much larger than unity,

$$A\beta \gg 1$$

$$A\left(\frac{R_1}{R_1 + R_2}\right) \gg 1$$

Since under such a condition,

$$A_f \simeq \frac{1}{\beta} = \frac{R_1 + R_2}{R_1} = 1 + \frac{R_2}{R_1}$$

the condition can be stated as

$$A \gg A_f$$

(c) For $A = 10^4$ V/V and $A_f = 10$ V/V, we see that $A \gg A_f$, thus we can select R_1 and R_2 to obtain

$$\beta = \frac{1}{A_f} = 0.1$$

Example 10.1 *continued*

Thus,

$$\frac{1}{\beta} = 1 + \frac{R_2}{R_1} = A_f = 10$$

which yields

$$R_2/R_1 = 9$$

A more exact value for the ratio R_2/R_1 can be obtained from

$$A_f = \frac{A}{1 + A\beta}$$

$$10 = \frac{10^4}{1 + 10^4\beta}$$

which results in

$$\beta = 0.0999$$

and,

$$\frac{R_2}{R_1} = 9.01$$

(d) The amount of feedback is

$$1 + A\beta = \frac{A}{A_f} = \frac{10^4}{10} = 1000$$

which is 60 dB.

(e) For $V_s = 1 \text{ V}$,

$$V_o = A_f V_s = 10 \times 1 = 10 \text{ V}$$

$$V_f = \beta V_o = 0.0999 \times 10 = 0.999 \text{ V}$$

$$V_i = \frac{V_o}{A} = \frac{10}{10^4} = 0.001 \text{V}$$

Note that if we had used the approximate value of $\beta = 0.1$, we would have obtained $V_f = 1 \text{ V}$ and $V_i = 0 \text{ V}$.

(f) If A decreases by 20%, thus becoming

$$A = 0.8 \times 10^4 \text{ V/V}$$

the value of A_f becomes

$$A_f = \frac{0.8 \times 10^4}{1 + 0.8 \times 10^4 \times 0.0999} = 9.9975 \text{ V/V}$$

that is, it decreases by 0.025%, which is lower than the percentage change in A by approximately a factor $(1 + A\beta)$.

EXERCISES

10.1 Repeat Example 10.1, (c) to (f) for $A = 100$ V/V.
Ans. (c) 10.11; (d) 20 dB; (e) 10 V, 0.9 V, 0.1 V; (f) 2.44%

10.2 Repeat Example 10.1, (c) to (f) for $A_f = 10^3$ V/V. For (e) use $V_s = 0.01$ V.
Ans. (c) 1110.1; (d) 20 dB; (e) 10 V, 0.009 V, 0.001 V; (f) 2.44%

10.2 Some Properties of Negative Feedback

The properties of negative feedback were mentioned in the Introduction. In the following, we shall consider some of these properties in more detail.

10.2.1 Gain Desensitivity

The effect of negative feedback on desensitizing the closed-loop gain was demonstrated in Example 10.1, where we saw that a 20% reduction in the gain of the basic amplifier gave rise to only a 0.025% reduction in the gain of the closed-loop amplifier. This sensitivity-reduction property can be analytically established as follows.

Assume that β is constant. Taking differentials of both sides of Eq. (10.4) results in

$$dA_f = \frac{dA}{(1 + A\beta)^2} \tag{10.8}$$

Dividing Eq. (10.8) by Eq. (10.4) yields

$$\frac{dA_f}{A_f} = \frac{1}{(1 + A\beta)} \frac{dA}{A} \tag{10.9}$$

which says that the percentage change in A_f (due to variations in some circuit parameter) is smaller than the percentage change in A by a factor equal to the amount of feedback. For this reason, the amount of feedback, $1 + A\beta$, is also known as the **desensitivity factor**.

EXERCISE

10.3 An amplifier with a nominal gain $A = 1000$ V/V exhibits a gain change of 10% as the operating temperature changes from 25° C to 75°C. If it is required to constrain the change to 0.1% by applying negative feedback, what is the largest closed-loop gain possible? If three of these feedback amplifiers are placed in cascade, what overall gain and gain stability are achieved?
Ans. 10 V/V; 1000 V/V, with a maximum variability of 0.3% over the specified temperature range.

10.2.2 Bandwidth Extension

Consider an amplifier whose high-frequency response is characterized by a single pole. Its gain at mid and high frequencies can be expressed as

$$A(s) = \frac{A_M}{1 + s/\omega_H} \tag{10.10}$$

where A_M denotes the midband gain and ω_H is the upper 3-dB frequency. Application of negative feedback, with a frequency-independent factor β, around this amplifier results in a closed-loop gain $A_f(s)$ given by

$$A_f(s) = \frac{A(s)}{1 + \beta A(s)}$$

Substituting for $A(s)$ from Eq. (10.10) results, after a little manipulation, in

$$A_f(s) = \frac{A_M/(1 + A_M\beta)}{1 + s/\omega_H(1 + A_M\beta)} \tag{10.11}$$

Thus the feedback amplifier will have a midband gain of $A_M/(1 + A_M\beta)$ and an upper 3-dB frequency ω_{Hf} given by

$$\omega_{Hf} = \omega_H(1 + A_M\beta) \tag{10.12}$$

It follows that the upper 3-dB frequency is increased by a factor equal to the amount of feedback.

Similarly, it can be shown that if the open-loop gain is characterized by a dominant low-frequency pole giving rise to a lower 3-dB frequency ω_L, then the feedback amplifier will have a lower 3-dB frequency ω_{Lf},

$$\omega_{Lf} = \frac{\omega_L}{1 + A_M\beta} \tag{10.13}$$

Note that the amplifier bandwidth is increased by the same factor by which its midband gain is decreased, *maintaining the gain–bandwidth product at a constant value*. This point is further illustrated by the Bode Plot in Fig. 10.3.

Finally, note that the action of negative feedback in extending the amplifier bandwidth should not be surprising: Negative feedback works to minimize the change in gain magnitude, including its change with frequency.

EXERCISE

10.4 Consider the noninverting op-amp circuit of Example 10.1. Let the open-loop gain A have a low-frequency value of 10^4 and a uniform –6-dB/octave rolloff at high frequencies with a 3-dB frequency of 100 Hz. Find the low-frequency gain and the upper 3-dB frequency of a closed-loop amplifier with $R_1 = 1\ \text{k}\Omega$ and $R_2 = 9\ \text{k}\Omega$.

Ans. 9.99 V/V; 100.1 kHz

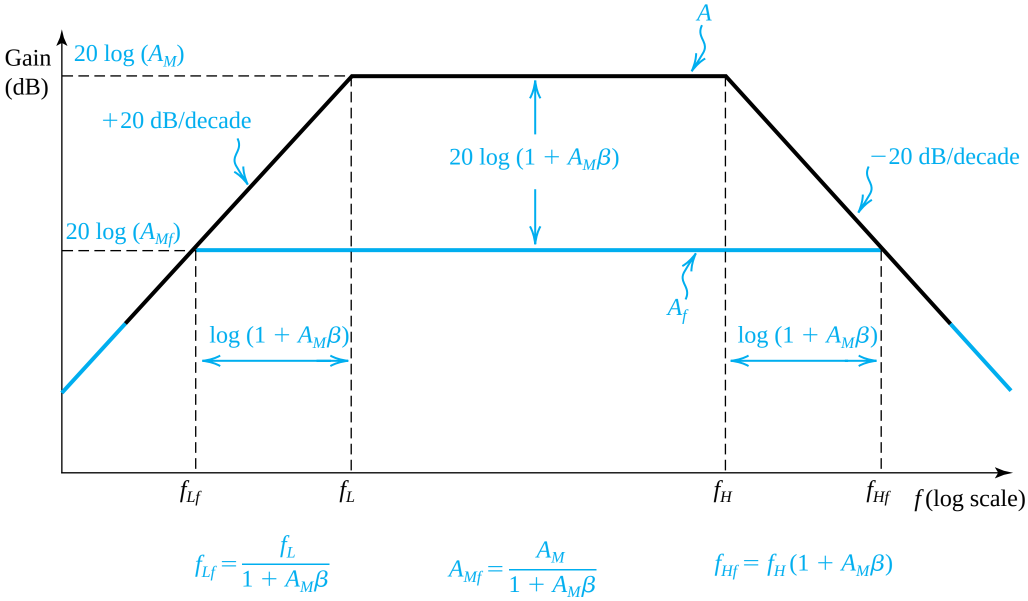


Figure 10.3 Application of negative feedback reduces the midband gain, increases f_H, and reduces f_L, all by the same factor, $(1+A_M\beta)$, which is equal to the amount of feedback.

10.2.3 Interference Reduction

Negative feedback can be employed to reduce the interference in an amplifier or, more precisely, to increase the ratio of signal to interference. However, as we shall now explain, this interference-reduction process is possible only under certain conditions. Consider the situation illustrated in Fig. 10.4. Figure 10.4(a) shows an amplifier with gain A_1, an input signal V_s, and interference, V_n. It is assumed that for some reason this amplifier suffers from interference and that the interference can be assumed to be introduced at the input of the amplifier. The **signal-to-interference ratio** for this amplifier is

$$S/I = V_s/V_n \tag{10.14}$$

Consider next the circuit in Fig. 10.4(b). Here we assume that it is possible to build another amplifier stage with gain A_2 that does not suffer from the interference problem. If this is the case, then we may precede our original amplifier A_1 by the *clean* amplifier A_2 and apply negative feedback around the overall cascade of such an amount as to keep the overall gain constant. The output voltage of the circuit in Fig. 10.4(b) can be found by superposition:

$$V_o = V_s \frac{A_1 A_2}{1 + A_1 A_2 \beta} + V_n \frac{A_1}{1 + A_1 A_2 \beta} \tag{10.15}$$

Thus the signal-to-interference ratio at the output becomes

$$\frac{S}{I} = \frac{V_s}{V_n} A_2 \tag{10.16}$$

which is A_2 times higher than in the original case.

We emphasize once more that the improvement in signal-to-interference ratio by the application of feedback is possible only if one can precede the interference-prone stage

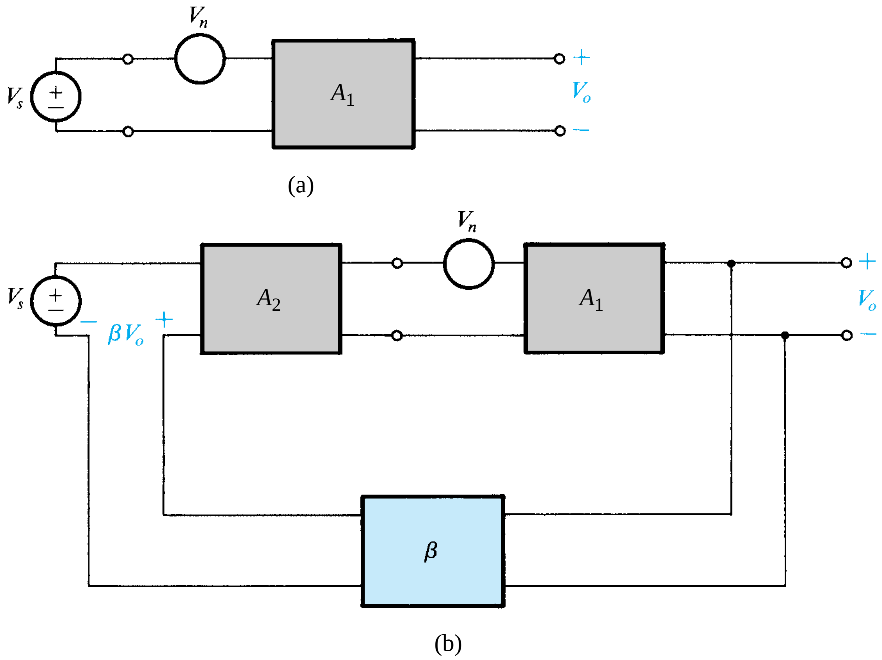


Figure 10.4 Illustrating the application of negative feedback to improve the signal-to-interference ratio in amplifiers.

by a (relatively) interference-free stage. This situation, however, is not uncommon in practice. The best example is found in the output power-amplifier stage of an audio amplifier. Such a stage usually suffers from a problem known as **power-supply hum**. The problem arises because of the large currents that this stage draws from the power supply and the difficulty of providing adequate power-supply filtering inexpensively. The power-output stage is required to provide large power gain but little or no voltage gain. We may therefore precede the power-output stage by a small-signal amplifier that provides large voltage gain, and apply a large amount of negative feedback, thus restoring the voltage gain to its original value. Since the small-signal amplifier can be fed from another, less hefty (and hence better regulated) power supply, it will not suffer from the hum problem. The hum at the output will then be reduced by the amount of the voltage gain of this added **preamplifier**.

EXERCISE

10.5 Consider a power-output stage with voltage gain $A_1 = 1$, an input signal $V_s = 1$ V, and a hum V_n of 1 V. Assume that this power stage is preceded by a small-signal stage with gain $A_2 = 100$ V/V and that overall feedback with $\beta = 1$ is applied. If V_s and V_n remain unchanged, find the signal and interference voltages at the output and hence the improvement in S/I.

Ans. $\simeq 1$ V; $\simeq 0.01$ V; 100 (40 dB)

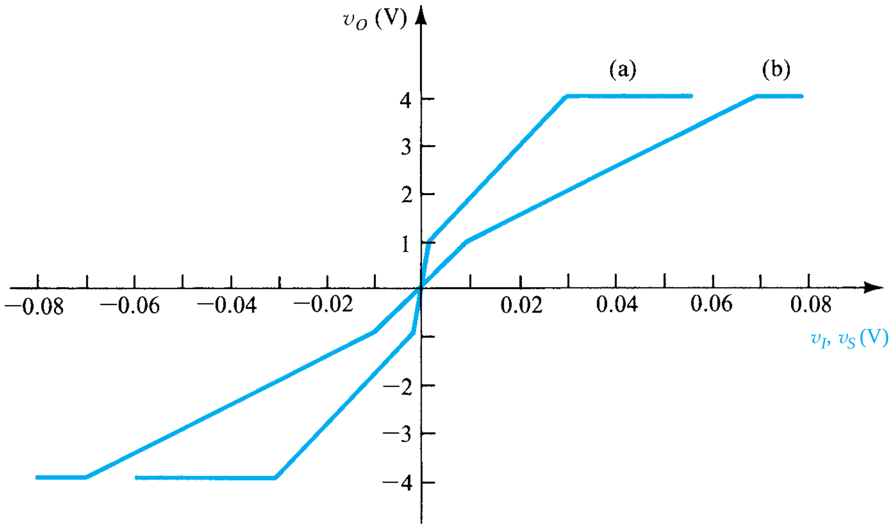


Figure 10.5 Illustrating the application of negative feedback to reduce the nonlinear distortion in amplifiers. Curve (a) shows the amplifier transfer characteristic (v_O versus v_I) without feedback. Curve (b) shows the characteristic (v_O versus v_S) with negative feedback (β = 0.01) applied.

10.2.4 Reduction in Nonlinear Distortion

Curve (a) in Fig. 10.5 shows the transfer characteristic v_O versus v_I of an amplifier. As indicated, the characteristic is piecewise linear, with the voltage gain changing from 1000 to 100 and then to 0. This nonlinear transfer characteristic will result in this amplifier generating a large amount of nonlinear distortion.

The amplifier transfer characteristic can be considerably **linearized** (i.e., made less nonlinear) through the application of negative feedback. That this is possible should not be too surprising, since we have already seen that negative feedback reduces the dependence of the overall closed-loop amplifier gain on the open-loop gain of the basic amplifier. Thus large changes in open-loop gain (1000 to 100 in this case) give rise to much smaller corresponding changes in the closed-loop gain.

To illustrate, let us apply negative feedback with $\beta = 0.01$ to the amplifier whose open-loop voltage transfer characteristic is depicted in Fig. 10.5. The resulting transfer characteristic of the closed-loop amplifier, v_O versus v_S, is shown in Fig. 10.5 as curve (b). Here the slope of the steepest segment is given by

$$A_{f1} = \frac{1000}{1 + 1000 \times 0.01} = 90.9$$

and the slope of the next segment is given by

$$A_{f2} = \frac{100}{1 + 100 \times 0.01} = 50$$

Thus the order-of-magnitude change in slope has been considerably reduced. The price paid, of course, is a reduction in voltage gain. Thus if the overall gain has to be restored, a preamplifier should be added. This preamplifier should not present a severe nonlinear-distortion problem, since it will be dealing with smaller signals.

Finally, it should be noted that negative feedback can do nothing at all about amplifier saturation, since in saturation the gain is very small (almost zero) and hence the amount of feedback is almost unity.

10.3 The Four Basic Feedback Topologies

Based on the quantity to be amplified (voltage or current) and on the desired form of output (voltage or current), amplifiers can be classified into four categories. These categories were discussed in Chapter 1. In the following, we shall review this amplifier classification and point out the feedback topology appropriate in each case.

10.3.1 Voltage Amplifiers

Voltage amplifiers are intended to amplify an input voltage signal and provide an output voltage signal. The voltage amplifier is essentially a voltage-controlled voltage source. The input resistance is required to be high, and the output resistance is required to be low. Since the signal source is essentially a voltage source, it is convenient to represent it in terms of a Thévenin equivalent circuit. In a voltage amplifier, the output quantity of interest is the output voltage. It follows that the feedback network should *sample* the output *voltage*, just as a voltmeter measures a voltage. Also, because of the Thévenin representation of the source, the feedback signal x_f should be a *voltage* that can be *mixed* with the source voltage in *series*.

The most suitable feedback topology for the voltage amplifier is the **voltage-mixing**, **voltage-sampling** one shown in Fig. 10.6. Because of the series connection at the input and the parallel or shunt connection at the output, this feedback topology is also known as **series–shunt feedback**. As will be shown, this topology not only stabilizes the voltage gain but also results in a higher input resistance (intuitively, a result of the series connection at the input) and a lower output resistance (intuitively, a result of the parallel connection at the output), which are desirable properties for a voltage amplifier.

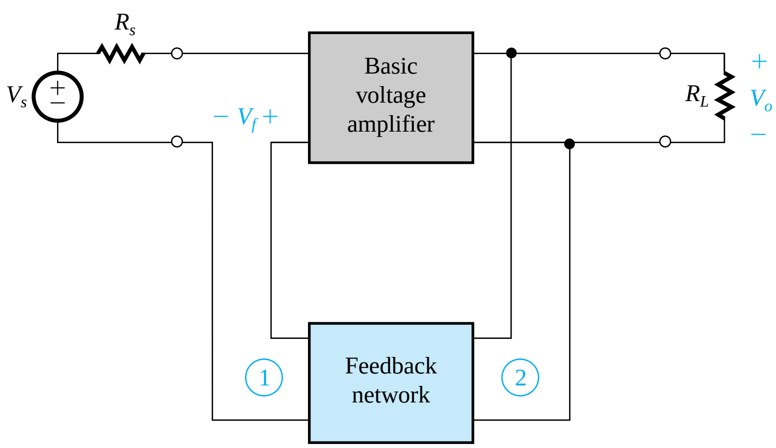


Figure 10.6 Block diagram of a feedback voltage amplifier. Here the appropriate feedback topology is series–shunt.

The increased input resistance results because V_f subtracts from V_s, resulting in a smaller signal V_i at the input of the basic amplifier. The lower V_i, in turn, causes the input current to be smaller, with the result that the resistance seen by V_s will be larger. We shall derive a formula for the input resistance of the feedback voltage amplifier in the next section.

The decreased output resistance results because the feedback works to keep V_o as constant as possible. Thus if the current drawn from the amplifier output changes by ΔI_o, the change ΔV_o in V_o will be lower than it would have been if feedback were not present. Thus the output resistance $\Delta V_o / \Delta I_o$ will be lower than that of the open-loop amplifier. In the following section we shall derive an expression for the output resistance of the feedback voltage amplifier.

Three examples of series–shunt feedback amplifiers are shown in Fig. 10.7. The amplifier in Fig. 10.7(a) is the familiar noninverting op-amp configuration. The feedback network, composed of the voltage divider (R_1, R_2), develops a voltage V_f that is applied to the negative input terminal of the op amp. The subtraction of V_f from V_s is achieved by utilizing the differencing action of the op-amp differential input. For the feedback to be negative, V_f must be of the same polarity as V_s, thus resulting in a smaller signal at the input of the basic amplifier. To ascertain that this is the case, we follow the signal around the loop, as follows: As V_s increases, V_o increases and the voltage divider causes V_f to increase. Thus the change in V_f is of the same polarity as the change in V_s, and the feedback is negative.

The second feedback voltage amplifier, shown in Fig. 10.7(b), utilizes two MOSFET amplifier stages in cascade. The output voltage V_o is sampled by the feedback network composed of the voltage divider (R_1, R_2), and the feedback signal V_f is fed to the source terminal of Q_1. The subtraction is implemented by applying V_s to the gate of Q_1 and V_f to its source, with the result that the signal at this amplifier input $V_i = V_{gs} = V_s - V_f$. To ascertain that the feedback is negative, let V_s increase. The drain voltage of Q_1 will decrease, and since this is applied to the gate of Q_2, its drain voltage V_o will increase. The feedback network will then cause V_f to increase, which is the same change in polarity initially assumed for V_s. Thus the feedback is indeed negative.

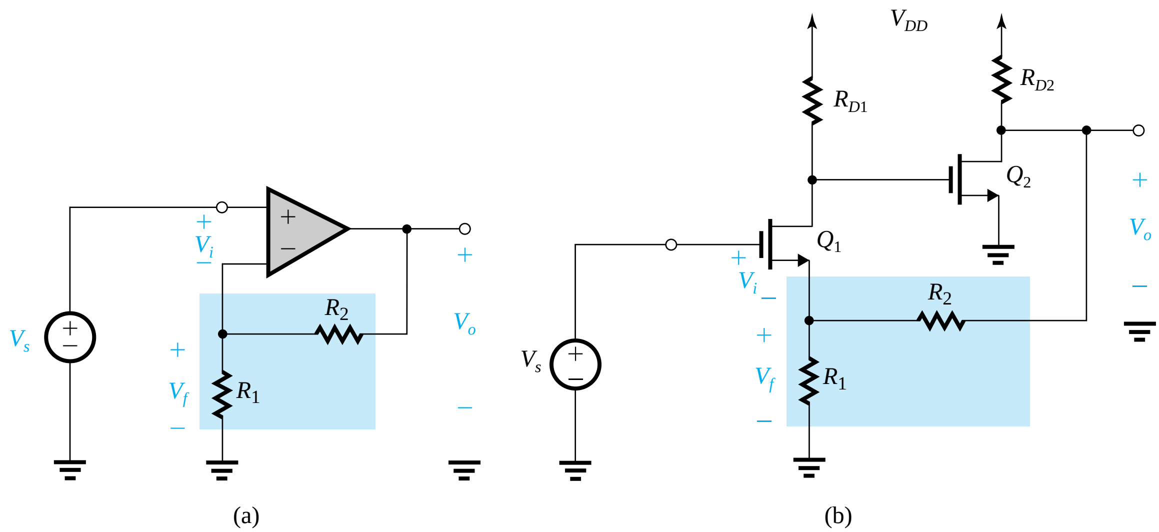


Figure 10.7 Examples of a feedback voltage amplifier. All these circuits employ series–shunt feedback. Note that the dc bias circuits are only partially shown.

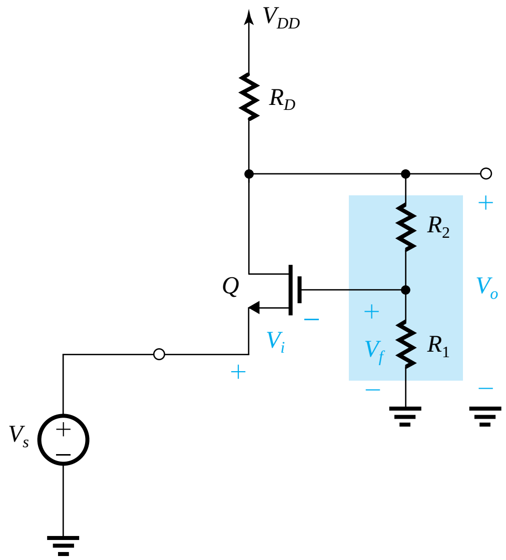


Figure 10.7 *continued*

The third example of series–shunt feedback, shown in Fig. 10.7(c), utilizes a CG transistor Q with a fraction V_f of the output voltage V_o fed back to the gate through a voltage divider (R_1, R_2). Observe that the subtraction of V_f from V_s is effected by applying V_s to the source, thus the input V_i to the CG amplifier is obtained as $V_s - V_f$. As usual, however, we must check the polarity of the feedback: If V_s increases, V_d (which is V_o) will increase and V_f will correspondingly increase. Thus V_f and V_s change in the same direction, verifying that the feedback is negative.

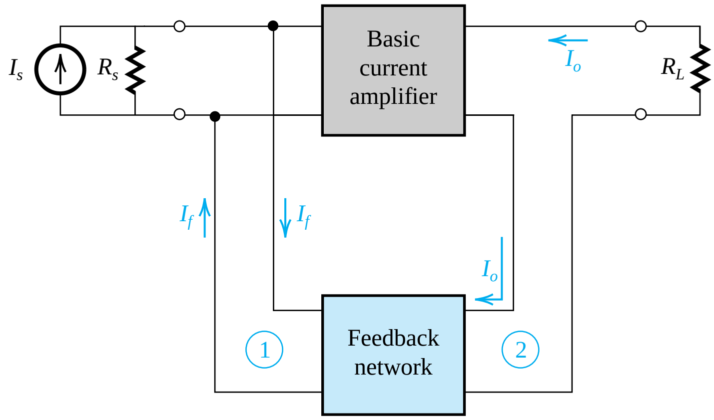
EXERCISE

10.6 For the circuit in Fig. 10.7(c) let $(R_1 + R_2) \gg R_D$. Using small-signal analysis, find expressions for the open-loop gain $A \equiv V_o/V_i$; the feedback factor $\beta \equiv V_f/V_o$; and the closed loop gain $A_f \equiv V_o/V_s$. For $A\beta \gg 1$, find an approximate expression for A_f. Neglect r_o.
Ans. $A = g_m R_D$; $\beta = R_1/(R_1 + R_2)$;

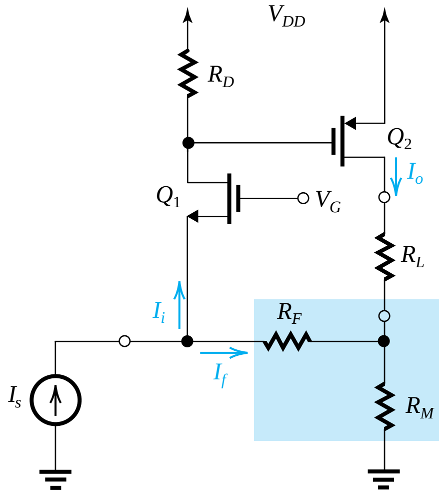
$$A_f = \frac{g_m R_D}{1 + g_m R_D R_1/(R_1 + R_2)}; \left(1 + \frac{R_2}{R_1}\right)$$

10.3.2 Current Amplifiers

The input signal in a current amplifier is essentially a current, and thus the signal source is most conveniently represented by its Norton equivalent. The output quantity of interest is current; hence the feedback network should *sample* the output *current*, just as a current meter measures a current. The feedback signal should be in *current* form so that it may be *mixed* in *shunt* with the source current. Thus the feedback topology most suitable for a current amplifier is the **current-mixing**, **current-sampling** topology, illustrated in Fig. 10.8(a). Because of the parallel (or shunt) connection at the input, and the series connection



(a)



(b)

Figure 10.8 (**a**) Block diagram of a feedback current amplifier. Here, the appropriate feedback topology is the shunt–series. (**b**) Example of a feedback current amplifier.

at the output, this feedback topology is also known as **shunt–series feedback**. As will be shown, this topology not only stabilizes the current gain but also results in a lower input resistance, and a higher output resistance, both desirable properties for a current amplifier.

The decrease in input resistance results because the feedback current I_f subtracts from the input current I_s, and thus a lower current enters the basic current amplifier. This in turn results in a lower voltage at the amplifier input, that is, across the current source I_s. It follows that the input resistance of the feedback current amplifier will be lower than that of the open-loop amplifier. We shall derive an expression for R_{if} in Section 10.5.

The increase in output resistance is simply a result of the action of negative feedback in keeping the value of I_o as constant as possible. Thus if the voltage across R_L is changed, the resulting change in I_o will be lower than it would have been without the feedback, which implies that the output resistance is increased. An expression for R_{of} will be derived in Section 10.5.

An example of a feedback current amplifier is shown in Fig. 10.8(b). It utilizes a CG stage Q_1 followed by a CS stage Q_2. The output current I_o is fed to a load resistance R_L. A sample of I_o is obtained by placing a small resistance R_M in series with R_L. The voltage developed across R_M is fed via a large resistance R_F to the source node of Q_1. The feedback current I_f that flows through R_F is subtracted from I_s at the source node, resulting in the input current $I_i = I_s - I_f$. For the feedback to be negative, I_f must have the same polarity as I_s. To ascertain that this is the case, we assume an increase in I_s and follow the change around the loop: An increase in I_s causes I_i to increase and the drain voltage of Q_1 will increase. Since this voltage is applied to the gate of the *p*-channel device Q_2, its increase will cause I_o, the drain current of Q_2, to decrease. Thus, the voltage across R_M will decrease, which will cause I_f to increase. This is the same polarity assumed for the initial change in I_s, verifying that the feedback is indeed negative.

Example 10.2

For the feedback current amplifier shown in Fig 10.8(b), find expressions for the open-loop gain $A \equiv I_o/I_i$, the feedback factor $\beta = (I_f/I_o)$, and the closed-loop gain $A_f \equiv I_o/I_s$. For simplicity, neglect the Early effect in Q_1 and Q_2.

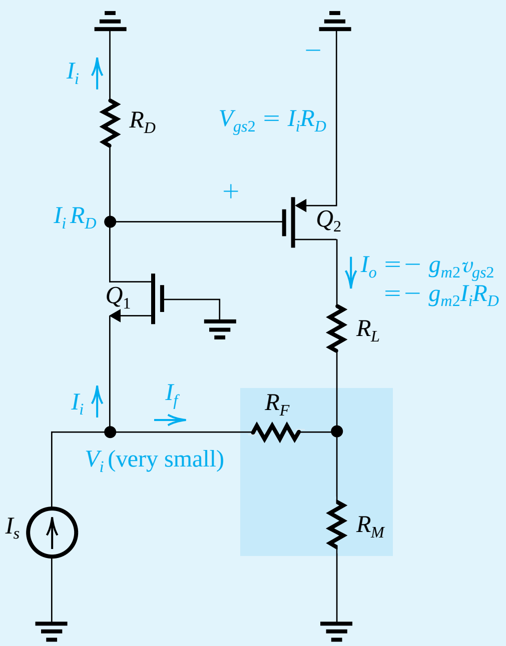


Figure 10.9 Analysis of the feedback current amplifier of Fig. 10.8(b) to obtain $A \equiv I_o/I_i$ and $\beta \equiv I_f/I_o$.

Solution

Figure 10.9 shows the circuit prepared for small-signal analysis. Some of the analysis is also indicated on the diagram. Since, as indicated,

$$I_o = -g_{m2} R_D I_i$$

the open-loop gain A is given by

$$A \equiv \frac{I_o}{I_i} = -g_{m2}R_D$$

To obtain β, we observe that I_o is fed to a current divider formed by R_M and R_F. Since current mixing results in a reduced input resistance, the voltage at the source node of Q_2 will be close to zero, and R_F in effect appears in parallel with R_M, enabling us to obtain β as

$$\beta \equiv \frac{I_f}{I_o} \simeq -\frac{R_M}{R_F + R_M}$$

where the negative sign is a result of the reference directions used for I_o and I_f. Note, however, that the loop gain $A\beta$ will be positive, as should always be the case in a negative feedback amplifier. We can now combine A and β to obtain A_f as

$$A_f \equiv \frac{I_o}{I_s} = -\frac{g_{m2}R_D}{1 + g_{m2}R_D \Big/ \left(1 + \dfrac{R_F}{R_M}\right)}$$

EXERCISE

10.7 For the feedback current amplifier of Fig. 10.8(b), analyzed in Example 10.2, find an approximate expression for the closed-loop current gain under the condition that the loop gain is large. Also, state the condition precisely.

Ans. $A_f \simeq -\left(1 + \dfrac{R_F}{R_M}\right)$; $g_{m2}R_D \gg \left(1 + \dfrac{R_F}{R_M}\right)$

10.3.3 Transconductance Amplifiers

In transconductance amplifiers the input signal is a voltage and the output signal is a current. It follows that the appropriate feedback topology is the **voltage-mixing**, **current-sampling** topology, illustrated in Fig. 10.10(a). The presence of the series connection at both the input and the output gives this feedback topology the alternative name **series–series feedback**.

As in the case of the feedback voltage amplifier, the series connection at the input results in an increased input resistance. The sampling of the output current results in an increased output resistance. Thus the series–series feedback topology provides the transconductance amplifier with the desirable properties of increased input and output resistances.

Examples of feedback transconductance-amplifiers are shown in Fig. 10.10 (b) and (c). The circuit in Fig. 10.10(b) utilizes a differential amplifier A_1 followed by a CS stage Q_2. The output current I_o is fed to R_L and to a series resistance R_F, which develops a feedback voltage V_f. The latter is applied to the positive input terminal of the differential amplifier A_1. The subtraction of V_f from V_s is performed by the differencing action of the differential-amplifier input. At this point we must check that V_f and V_s have the same polarity: A positive change in V_s will result in a negative change at the gate of Q_2, which in turn causes I_o to increase. The increase in I_o results in a positive change in V_f, which is the same polarity assumed for the change in V_s, verifying that the feedback is negative.

The transconductance amplpifier in Fig.10.10(c) utilizes a CS amplifier Q_1 in cascade with another CS amplifier, Q_2. The output current I_o is fed to R_L and to a series resistance

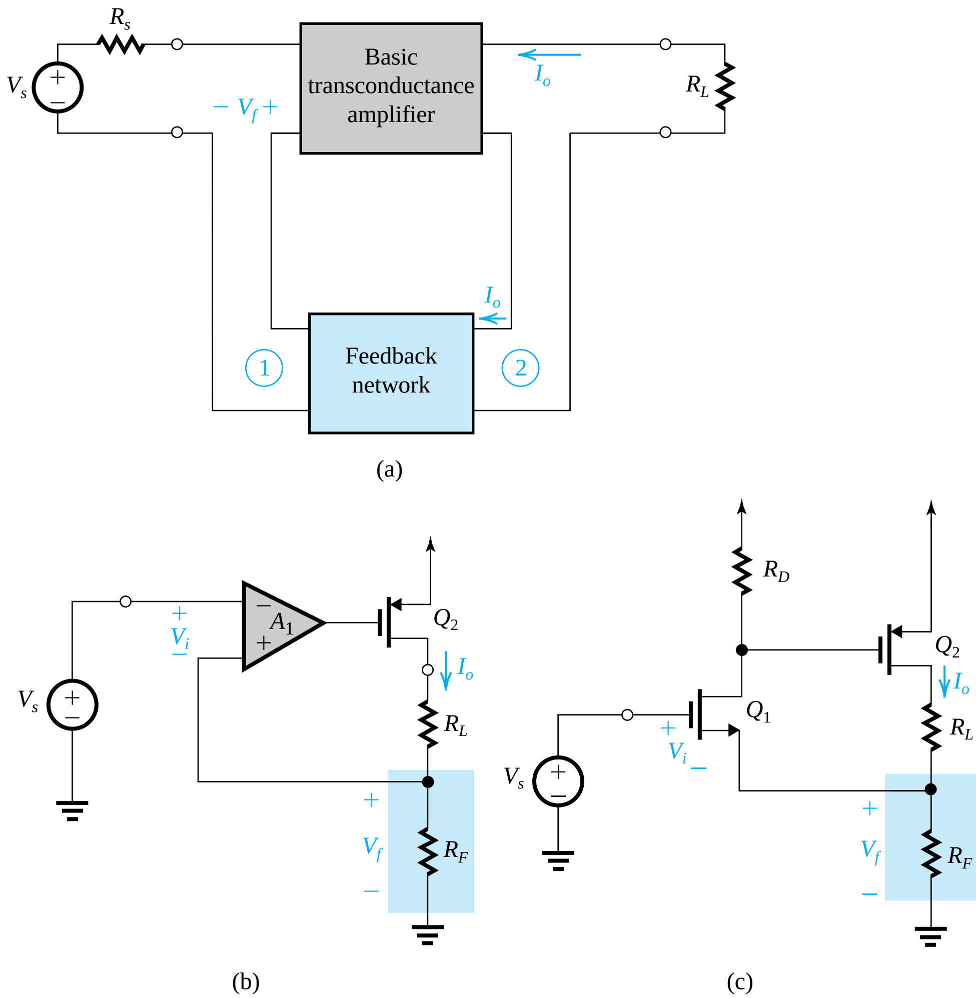


Figure 10.10 (**a**) Block diagram of a feedback transconductance amplifier. Here, the appropriate feedback topology is series–series. (**b**) Example of a feedback transconductance amplifier. (**c**) Another example.

R_F that develops a feedback voltage V_f. The latter is fed to the source of Q_1, thus utilizing the input of Q_1 to implement the subtraction; $V_i = V_s - V_f$. The reader is urged to verify that V_f has the same polarity as V_s and thus that the feedback is negative.

EXERCISE

10.8 For the circuit in Fig. 10.10(b), let the differential amplifier A_1 have an infinite input resistance. Use small-signal analysis to obtain expressions for the open-loop gain $A \equiv I_o/V_i$, the feedback factor $\beta \equiv V_f/I_o$, and the closed-loop gain $A_f \equiv I_o/V_s$. If the loop gain is much greater than unity, find an approximate expression for A_f. Neglect r_{o2}.

Ans. $A = A_1 g_{m2}$; $\beta = R_F$; $A_f = \dfrac{A_1 g_{m2}}{1 + A_1 g_{m2} R_F}$; $A_f \simeq 1/R_F$

10.3.4 Transresistance Amplifiers

In transresistance amplifiers the input signal is current and the output signal is voltage. It follows that the appropriate feedback topology is of the **current-mixing**, **voltage-sampling** type, shown in Fig. 10.11(a). The presence of the parallel (or shunt) connection at

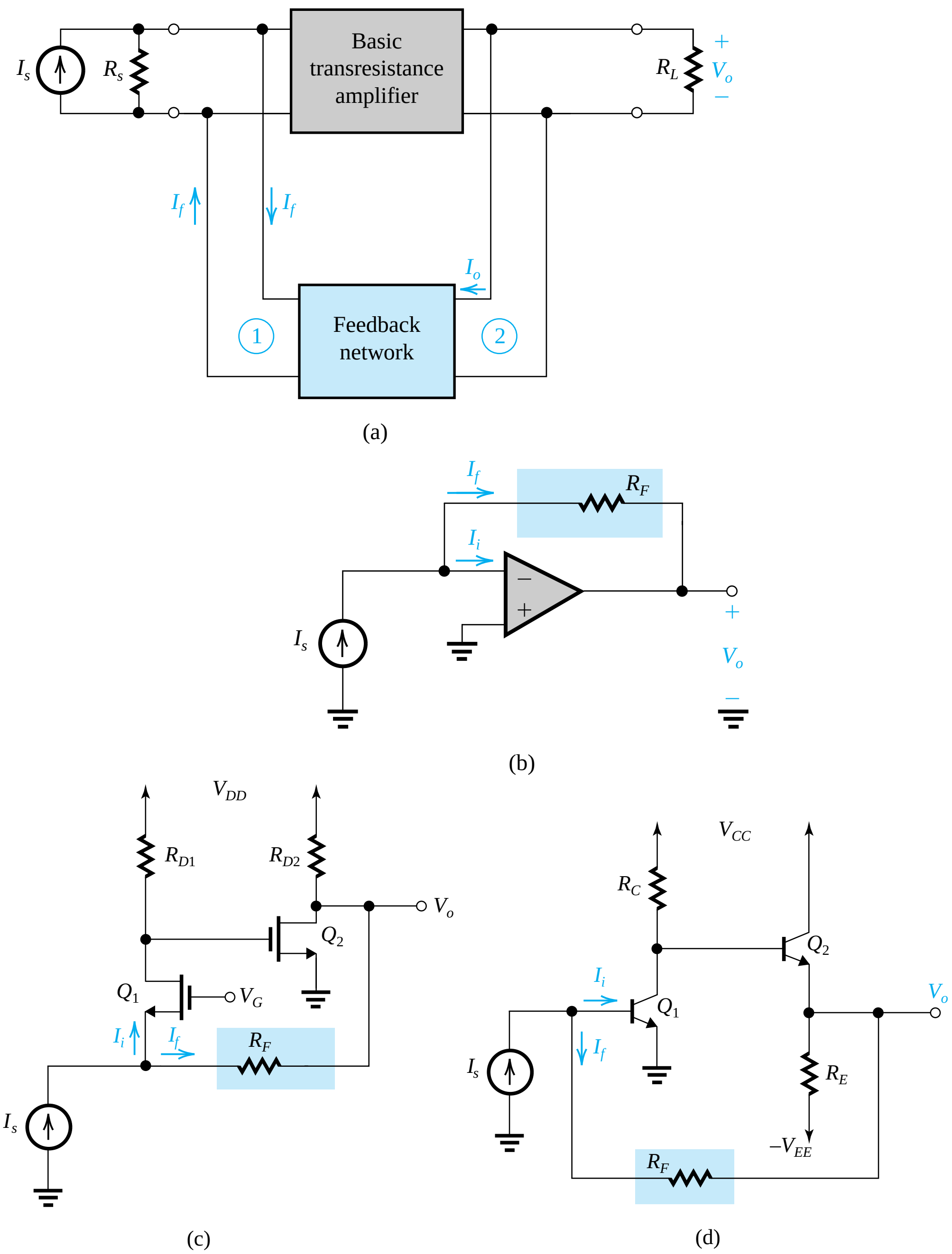


Figure 10.11 **(a)** Block diagram of a feedback transresistance amplifier. Here, the appropriate feedback topology is shunt–shunt. **(b)**, **(c)**, and **(d)** Examples of feedback transresistance amplifiers.

both the input and the output makes this feedback topology also known as **shunt–shunt** feedback.

The shunt connection at the input causes the input resistance to be reduced. The shunt connection at the output stabilizes the output voltage and thus causes the output resistance to be reduced. Thus, the shunt–shunt topology equips the transresistance amplifier with the desirable attributes of a low input and a low output resistance.

Three examples of feedback transresistance amplifiers are shown in Fig. 10.11(b), (c), and (d). The circuit in Fig. 10.11(b) utilizes an op amp with a feedback resistance R_F that senses V_o and provides a feedback current I_f that is subtracted from I_s at the input node. To see that the feedback is negative, let I_s increase. The input current I_i will increase, causing the voltage of the negative input terminal to rise. In response, the output voltage will decrease, causing an increase in I_f. Thus I_f and I_s have the same polarity, and the feedback is negative.

The circuit in Fig. 10.11(c) utilizes a CG stage Q_1 cascaded with a CS stage Q_2. A feedback resistor R_F senses V_o and feeds a current I_f to the input node, where the subtraction from I_s takes place. The reader is urged to show that I_f and I_s have the same polarity and thus the feedback is negative.

Finally, the BJT feedback transresistance amplifier in Fig. 10.11(d) utilizes a CE stage Q_1 cascaded with an emitter follower Q_2. A feedback resistor R_F senses V_o and feeds back a current I_f to the input node, where it is subtracted from I_s. The reader is urged to show that the feedback is indeed negative.

EXERCISE

10.9 For the circuit in Fig. 10.11(b), let the op amp have an open-loop gain A, a differential input resistance R_{id}, and a zero output resistance. Analyze the circuit from first principles (i.e., do not use the feedback analysis approach) to determine $A_f \equiv V_o/I_s$. Under what conditions does $A_f \simeq -R_F$?

Ans. $A_f = -R_F \Big/ \left(1 + \dfrac{1}{A} + \dfrac{R_F}{AR_{id}}\right)$; $A \gg 1$ and $AR_{id} \gg R_F$

10.3.5 A Concluding Remark

Throughout this section we introduced examples of the four different types of feedback amplifier. However, in order to use the feedback analysis approach, we had to make a variety of approximations. For instance, in Example 10.2, to find β we had to assume that the input resistance of the closed-loop amplifier was very low. Also, in Exercise 10.6 we assumed that $(R_1 + R_2) \gg R_D$, that is, that the feedback network does not load the basic amplifier. The need to make such approximations in a seemingly ad hoc manner is no doubt somewhat disconcerting to the reader. There is, however, very good news: Starting in the next section we will present a systematic approach for the analysis of feedback amplifiers that takes into account the various loading effects and thus obviates the need for ad hoc approximations.

10.4 The Feedback Voltage Amplifier (Series–Shunt)

10.4.1 The Ideal Case

As mentioned before, series–shunt is the appropriate feedback topology for a voltage amplifier. The ideal structure of the series–shunt feedback amplifier is shown in Fig. 10.12(a). It consists of a *unilateral* open-loop amplifier (the A circuit) and an ideal voltage-sampling, voltage-mixing feedback network (the β circuit). The A circuit has an input resistance R_i, an open-circuit voltage gain A, and an output resistance R_o. It is assumed that the source and load resistances have been absorbed inside the A circuit (more on this point later). Furthermore, note that the β circuit does *not* load the A circuit; that is, connecting the β circuit does not change the value of A (defined as $A \equiv V_o/V_i$).

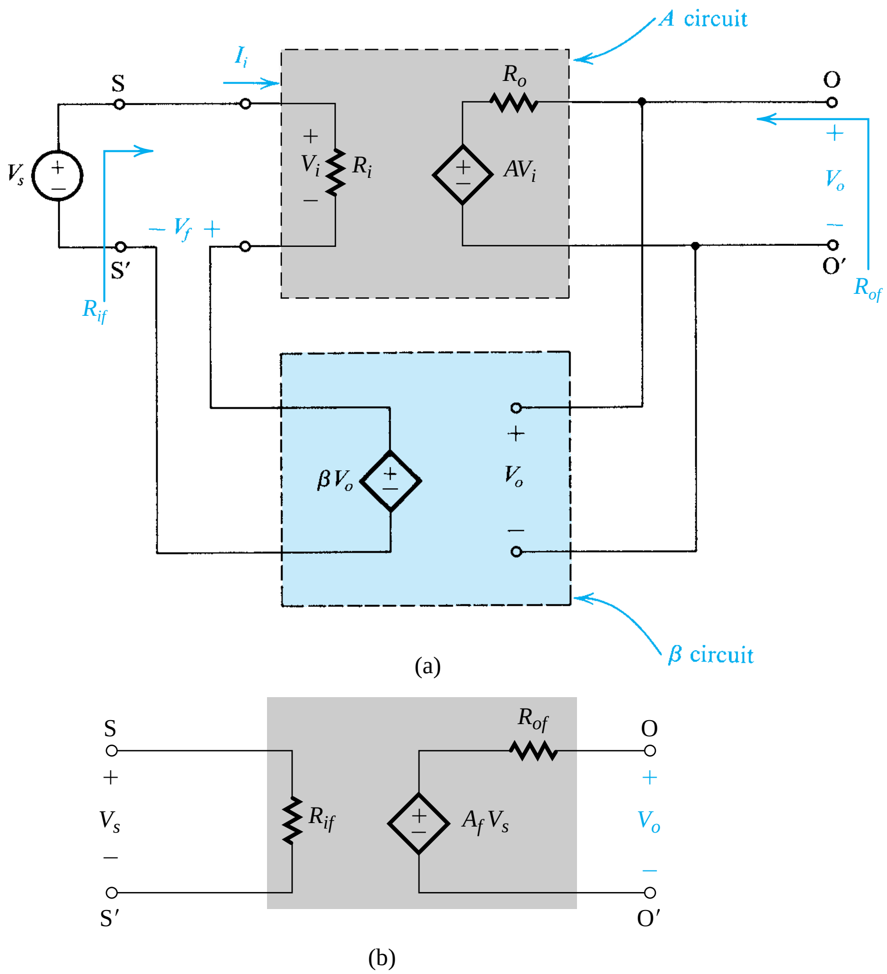


Figure 10.12 The series–shunt feedback amplifier: (**a**) ideal structure; (**b**) equivalent circuit.

The circuit of Fig. 10.12(a) exactly follows the ideal feedback model of Fig. 10.1. Therefore the closed-loop voltage gain A_f is given by

$$A_f \equiv \frac{V_o}{V_s} = \frac{A}{1 + A\beta} \tag{10.17}$$

The equivalent circuit model of the series–shunt feedback amplifier is shown in Fig. 10.12(b). Observe that A_f is the open-circuit voltage gain of the feedback amplifier, R_{if} is its input resistance, and R_{of} is its output resistance. Expressions for R_{if} and R_{of} can be derived as follows.

For R_{if}, refer to the input loop of the circuit in Fig. 10.12(a). The series mixing subtracts V_f from V_s and thus reduces V_i by a factor equal to the amount of feedback (Eq. 10.7),

$$V_i = \frac{V_s}{1 + A\beta}$$

Thus the input current I_i becomes

$$I_i = \frac{V_i}{R_i} = \frac{V_s}{(1 + A\beta)R_i} \tag{10.18}$$

Since I_i is the current drawn from V_s, the input resistance R_{if} can be expressed as

$$R_{if} \equiv \frac{V_s}{I_i}$$

and using Eq. (10.18) is found to be

$$R_{if} = (1 + A\beta)R_i \tag{10.19}$$

Thus, as expected, the series-mixing feedback results in an increase in the amplifier input resistance by a factor equal to the amount of feedback, $(1 + A\beta)$, a highly desirable property for a voltage amplifier.

It should be clear from the above derivation that the increased input resistance is a result only of the series mixing and is independent of the type of sampling. Thus, the transconductance amplifier, which is the other amplifier type in which series mixing is employed, will also exhibit an increased input resistance even though the feedback network samples its output current (series sampling).

To determine the output resistance R_{of} of the feedback amplifier in Fig. 10.12(a), we set $V_s = 0$ and apply a test voltage V_x between the output terminals, as shown in Fig. 10.13. If the current drawn from V_x is I_x, the output resistance R_{of} is

$$R_{of} \equiv \frac{V_x}{I_x} \tag{10.20}$$

An equation for the output loop yields

$$I_x = \frac{V_x - AV_i}{R_o} \tag{10.21}$$

From the input loop we see that

$$V_i = -V_f$$

Now $V_f = \beta V_o = \beta V_x$; thus,

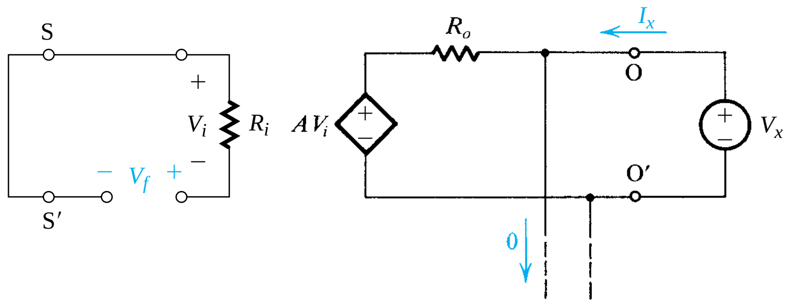


Figure 10.13 Determining the output resistance of the feedback amplifier of Fig. 10.12(a): $R_{of} = V_x/I_x$.

$$V_i = -\beta V_x$$

which when substituted in Eq. (10.21) yields

$$I_x = \frac{V_x(1 + A\beta)}{R_o}$$

Substituting this value of I_x into Eq. (10.20) provides the following expression for R_{of},

$$R_{of} = \frac{R_o}{1 + A\beta} \tag{10.22}$$

Thus, as expected, the shunt sampling (or voltage sampling) at the output results in a decrease in the amplifier output resistance by a factor equal to the amount of negative feedback, $(1 + A\beta)$, a highly desirable property for a voltage amplifier.

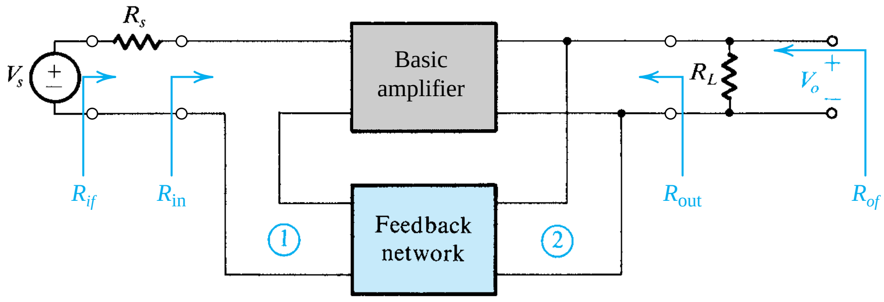
Although perhaps not entirely obvious, the reduction of the output resistance is a result only of the method of sampling the output and does not depend on the method of mixing. Thus, the transistance amplifier, which is the other amplifier type in which shunt (or voltage) sampling is employed, will also exhibit a reduced output resistance.

10.4.2 The Practical Case

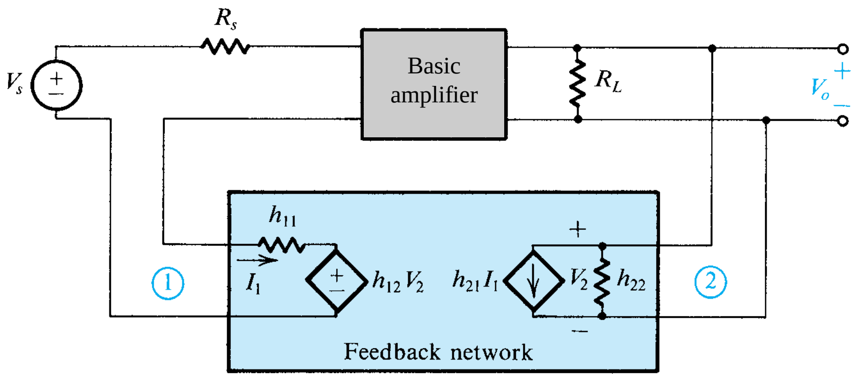
In a practical series–shunt feedback amplifier, the feedback network will not be an ideal voltage-controlled voltage source. Rather, the feedback network is usually resistive and hence will load the basic amplifier and thus affect the values of A, R_i, and R_o. In addition, the source and load resistances will affect these three parameters. Thus the problem we have is as follows: Given a series–shunt feedback amplifier represented by the block diagram of Fig. 10.14(a), find the A circuit and the β circuit.

Our problem essentially involves representing the amplifier of Fig. 10.14(a) by the ideal structure of Fig. 10.12(a). As a first step toward that end we observe that the source and load resistances should be lumped with the basic amplifier. This, together with representing the two-port feedback network in terms of its h parameters (see Appendix C), is illustrated in Fig. 10.14(b). The choice of h parameters is based on the fact that this is the only parameter set that represents the feedback network by a series network at port 1 and a parallel network at port 2. Such a representation is obviously convenient in view of the series connection at the input and the parallel connection at the output.

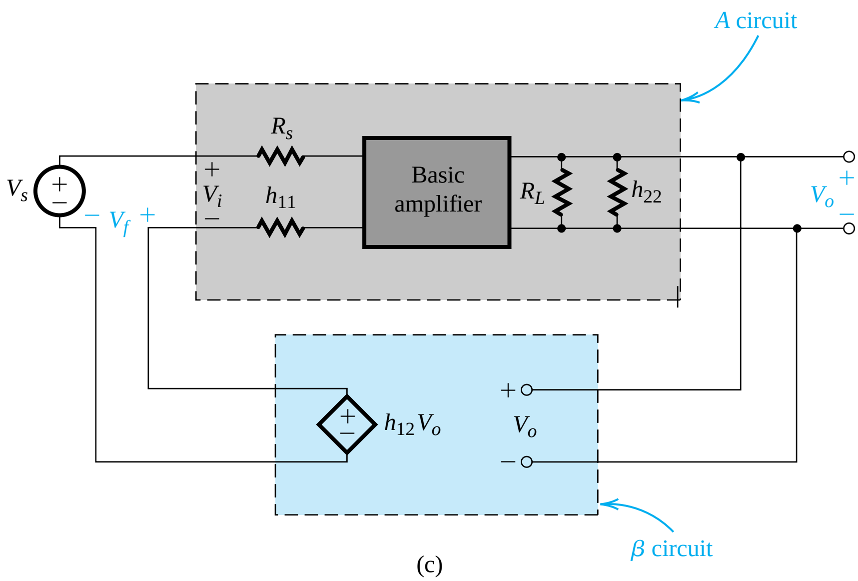
Examination of the circuit in Fig. 10.14(b) reveals that the current source $h_{21}I_1$ represents the forward transmission of the feedback network. Since the feedback network is usually



(a)



(b)



(c)

Figure 10.14 Derivation of the A circuit and β circuit for the series–shunt feedback amplifier. **(a)** Block diagram of a practical series–shunt feedback amplifier. **(b)** The circuit in **(a)** with the feedback network represented by its h parameters. **(c)** The circuit in **(b)** with h_{21} neglected.

passive, its forward transmission can be neglected in comparison to the much larger forward transmission of the basic amplifier. We will therefore assume that $|h_{21}|_{\substack{\text{feedback}\\\text{network}}} \ll |h_{21}|_{\substack{\text{basic}\\\text{amplifier}}}$ and thus omit the controlled source $h_{21}I_1$ altogether.

Compare the circuit of Fig. 10.14(b) (after eliminating the current source $h_{21}I_1$) with the ideal circuit of Fig. 10.12(a). We see that by including h_{11} and h_{22} with the basic amplifier, we obtain the circuit shown in Fig. 10.14(c), which is very similar to the ideal circuit. Now, if the basic amplifier is unilateral (or almost unilateral)—that is it does not contain internal feedback—then the circuit of Fig. 10.14(c) is equivalent to the ideal circuit. It follows then that the *A* circuit is obtained by augmenting the basic amplifier at the input with the source resistance R_s and the resistance h_{11} of the feedback network, and at the output with the load resistance R_L and the conductance h_{22} of the feedback network.

We conclude that the loading effect of the feedback network on the basic amplifier is represented by the components h_{11} and h_{22}. From the definitions of the *h* parameters in Appendix C we see that h_{11} is the resistance looking into port 1 of the feedback network with port 2 short-circuited. Since port 2 of the feedback network is connected in *shunt* with the output port of the amplifier, short-circuiting port 2 destroys the feedback. Similarly, h_{22} is the conductance looking into port 2 of the feedback network with port 1 open-circuited. Since port 1 of the feedback network is connected in *series* with the amplifier input, open-circuiting port 1 destroys the feedback.

These observations suggest a simple rule for finding the loading effects of the feedback network on the basic amplifier: The loading effect is found by looking into the appropriate port of the feedback network while the other port is open-circuited or short-circuited so as to destroy the feedback. If the connection is a shunt one, we short-circuit the port; if it is a series one, we open-circuit it. In Sections 10.5, 10.6, and 10.7 it will be seen that this simple rule applies also to the other three feedback topologies.[3]

We next consider the determination of β. From Fig. 10.14(c), we see that β is equal to h_{12} of the feedback network,

$$\beta = h_{12} \equiv \left.\frac{V_1}{V_2}\right|_{I_1=0} \tag{10.23}$$

Thus to measure β, one applies a voltage to port 2 of the feedback network and measures the voltage that appears at port 1 while the latter port is open-circuited. This result is intuitively appealing because the object of the feedback network is to sample the output voltage ($V_2 = V_o$) and provide a voltage signal ($V_1 = V_f$) that is mixed in series with the input source. The series connection at the input suggests that (as in the case of finding the loading effects of the feedback network) β should be found with port 1 open-circuited.

10.4.3 Summary

A summary of the rules for finding the *A* circuit and β for a given series–shunt feedback amplifier of the form in Fig. 10.14(a) is given in Fig. 10.15. As for using the feedback formulas in Eqs. (10.19) and (10.22) to determine the input and output resistances, it is important to note that:

1. R_i and R_o are the input and output resistances, respectively, of the *A* circuit in Fig. 10.15(a).
2. R_{if} and R_{of} are the input and output resistances, respectively, of the feedback amplifier, *including* R_s and R_L (see Fig. 10.14a).

[3] A simple rule to remember: If the connection is *sh*unt, *sh*ort it; if *se*ries, *se*ver it.

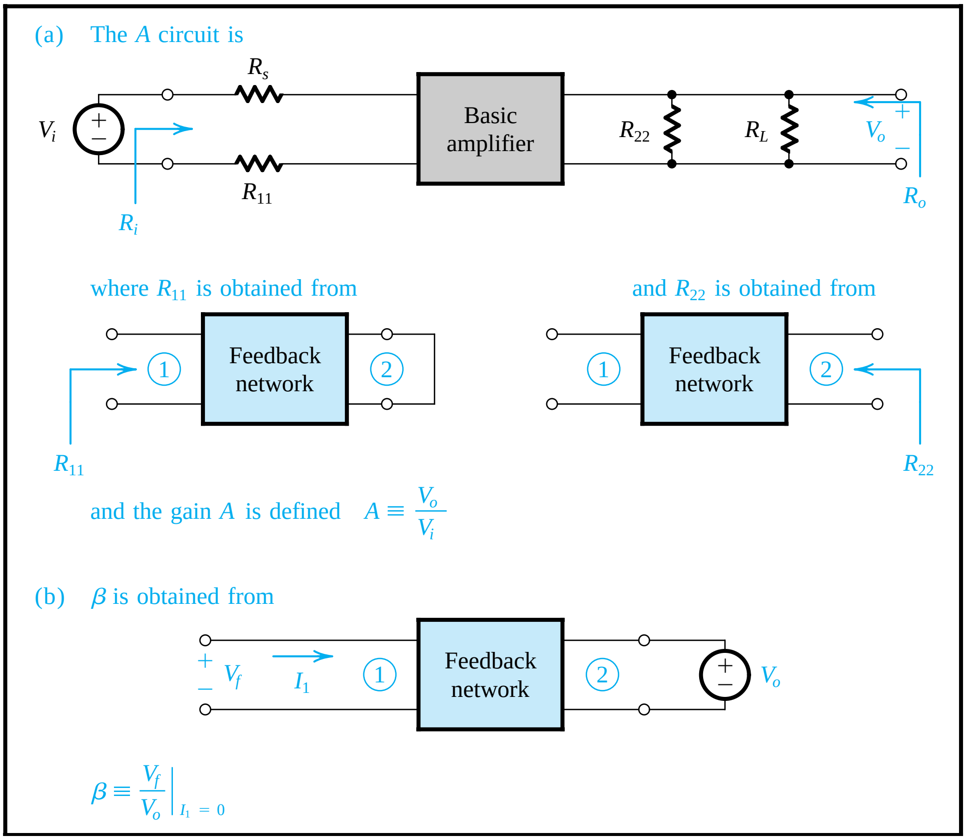


Figure 10.15 Summary of the rules for finding the A circuit and β for the series–shunt case of Fig. 10.14(a).

3. The actual input and output resistances of the feedback amplifier usually exclude R_s and R_L. These are denoted R_{in} and R_{out} in Fig. 10.14(a) and can be easily determined as

$$R_{in} = R_{if} - R_s \tag{10.24}$$

$$R_{out} = 1\Big/\left(\frac{1}{R_{of}} - \frac{1}{R_L}\right) \tag{10.25}$$

Example 10.3

Figure 10.16(a) shows an op amp connected in the noninverting configuration. The op amp has an open-loop gain μ, a differential input resistance R_{id}, and an output resistance r_o. Recall that in our analysis of op-amp circuits in Chapter 2, we neglected the effects of R_{id} (assumed it to be infinite) and of r_o (assumed it to be zero). Here we wish to use the feedback method to analyze the circuit taking both R_{id} and r_o into account. Find expressions for A, β, the closed-loop gain V_o/V_s, the input resistance R_{in} (see Fig. 10.16a), and the output resistance R_{out}. Also find numerical values, given $\mu = 10^4$, $R_{id} = 100$ kΩ, $r_o = 1$ kΩ, $R_L = 2$ kΩ, $R_1 = 1$ kΩ, $R_2 = 1$ MΩ, and $R_s = 10$ kΩ.

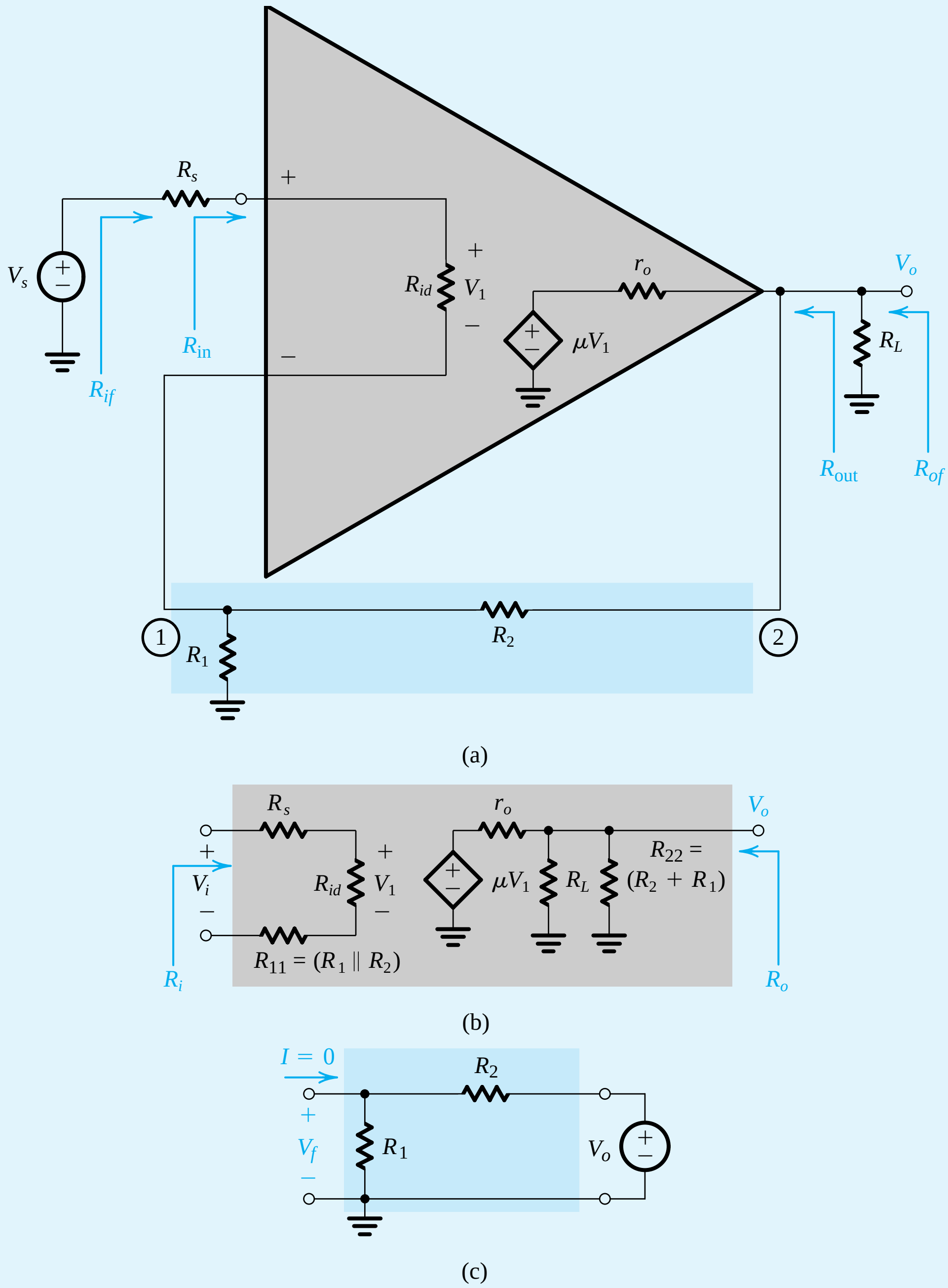


Figure 10.16 Circuits for Example 10.3.

Example 10.3 *continued*

Solution

We observe that the feedback network consists of R_2 and R_1. This network samples the output voltage V_o and provides a voltage signal (across R_1) that is mixed in series with the input source V_s.

The A circuit can be easily obtained following the rules of Fig. 10.15, and is shown in Fig. 10.16(b). Observe that the loading effect of the feedback network at the input side is obtained by short-circuiting port 2 of the feedback network (because it is connected in shunt) and looking into port 1, with the result that $R_{11} = R_1 \| R_2$. The loading effect of the feedback network at the output side is found by open-circuiting port 1 (because it is connected in series) and looking into port 2, with the result that $R_{22} = R_2 + R_1$. For the resulting A circuit in Fig. 10.16(b), we can write by inspection:

$$A \equiv \frac{V_o}{V_i} = \mu \frac{R_L \| (R_1 + R_2)}{[R_L \| (R_1 + R_2)] + r_o} \frac{R_{id}}{R_{id} + R_s + (R_1 \| R_2)}$$

For the values given, we find that $A \simeq 6000$ V/V.

The circuit for determining β is shown in Fig. 10.16(c), from which we obtain

$$\beta \equiv \frac{V_f}{V_o} = \frac{R_1}{R_1 + R_2} \simeq 10^{-3} \text{ V/V}$$

The voltage gain with feedback can now be obtained as

$$A_f \equiv \frac{V_o}{V_s} = \frac{A}{1 + A\beta} = \frac{6000}{7} = 857 \text{ V/V}$$

The input resistance R_{if} determined by the feedback equations is the resistance seen by the external source (see Fig. 10.16a), and is given by

$$R_{if} = R_i(1 + A\beta)$$

where R_i is the input resistance of the A circuit in Fig. 10.16(b):

$$R_i = R_s + R_{id} + (R_1 \| R_2)$$

For the values given, $R_i \simeq 111$ kΩ, resulting in

$$R_{if} = 111 \times 7 = 777 \text{ k}\Omega$$

This, however, is not the resistance asked for. What is required is R_{in}, indicated in Fig. 10.16(a). To obtain R_{in} we subtract R_s from R_{if}:

$$R_{\text{in}} = R_{if} - R_s$$

For the values given, $R_{\text{in}} = 739$ kΩ. The resistance R_{of} given by the feedback equations is the output resistance of the feedback amplifier, including the load resistance R_L, as indicated in Fig. 10.16(a). R_{of} is given by

$$R_{of} = \frac{R_o}{1 + A\beta}$$

where R_o is the output resistance of the A circuit. R_o can be obtained by inspection of Fig.10.16(b) as

$$R_o = r_o \| R_L \| (R_2 + R_1)$$

For the values given, $R_o \simeq 667$ Ω, and

$$R_{of} = \frac{667}{7} = 95.3 \ \Omega$$

The resistance asked for, R_{out}, is the output resistance of the feedback amplifier excluding R_L. From Fig. 10.16(a) we see that

$$R_{of} = R_{\text{out}} \parallel R_L$$

Thus

$$R_{\text{out}} \simeq 100\ \Omega$$

Example 10.4

As another example of a series–shunt feedback amplifier, consider the circuit shown in Fig. 10.7(b) which is repeated in Fig. 10.17(a). It is required to analyze this amplifier to obtain its voltage gain V_o/V_s, input resistance R_{in}, and output resistance R_{out}. Find numerical values for the case $g_{m1} = g_{m2} = 4$ mA/V, $R_{D1} = R_{D2} = 10$ kΩ, $R_1 = 1$ kΩ, and $R_2 = 9$ kΩ. For simplicity, neglect r_o of each of Q_1 and Q_2.

Solution

We identify the feedback network as the voltage divider (R_1, R_2). Its loading effect at the input is obtained by short circuiting its port 2 (because it is connected in shunt with the output). Then, looking into its port 1, we see $R_1 \parallel R_2$. The loading effect at the output is obtained by open-circuiting port 1 of the feedback network (because it is connected in series with the input). Then, looking into port 2, we see R_2 in series with R_1. The A circuit will therefore be as shown in Fig. 10.17(b). The gain A is determined as the product of the gain of Q_1 and the gain of Q_2 as follows:

$$A_1 = \frac{V_{d1}}{V_i} = -\frac{R_{D1}}{1/g_{m1} + (R_1 \parallel R_2)} = -\frac{g_{m1}R_{D1}}{1 + g_{m1}(R_1 \parallel R_2)}$$

$$A_2 = \frac{V_o}{V_{d1}} = -g_{m2}[R_{D2} \parallel (R_1 + R_2)]$$

$$A = \frac{V_o}{V_i} = A_1A_2 = \frac{g_{m1}R_{D1}g_{m2}[R_{D2} \parallel (R_1 + R_2)]}{1 + g_{m1}(R_1 \parallel R_2)}$$

For the numerical values given,

$$A = \frac{4 \times 10 \times 4[10 \parallel (1 + 9)]}{1 + 4(1 \parallel 9)} = 173.9 \quad \text{V/V}$$

The value of β is determined from the β circuit in Fig. 10.17(c),

$$\beta \equiv \frac{V_f}{V_o} = \frac{R_1}{R_1 + R_2}$$

For the numerical values given,

$$\beta = \frac{1}{1 + 9} = 0.1$$

The closed-loop gain V_o/V_s can now be found as

$$\frac{V_o}{V_s} = A_f = \frac{A}{1 + A\beta} = \frac{173.9}{1 + 173.9 \times 0.1} = 9.5 \ \text{V/V}$$

Example 10.4 *continued*

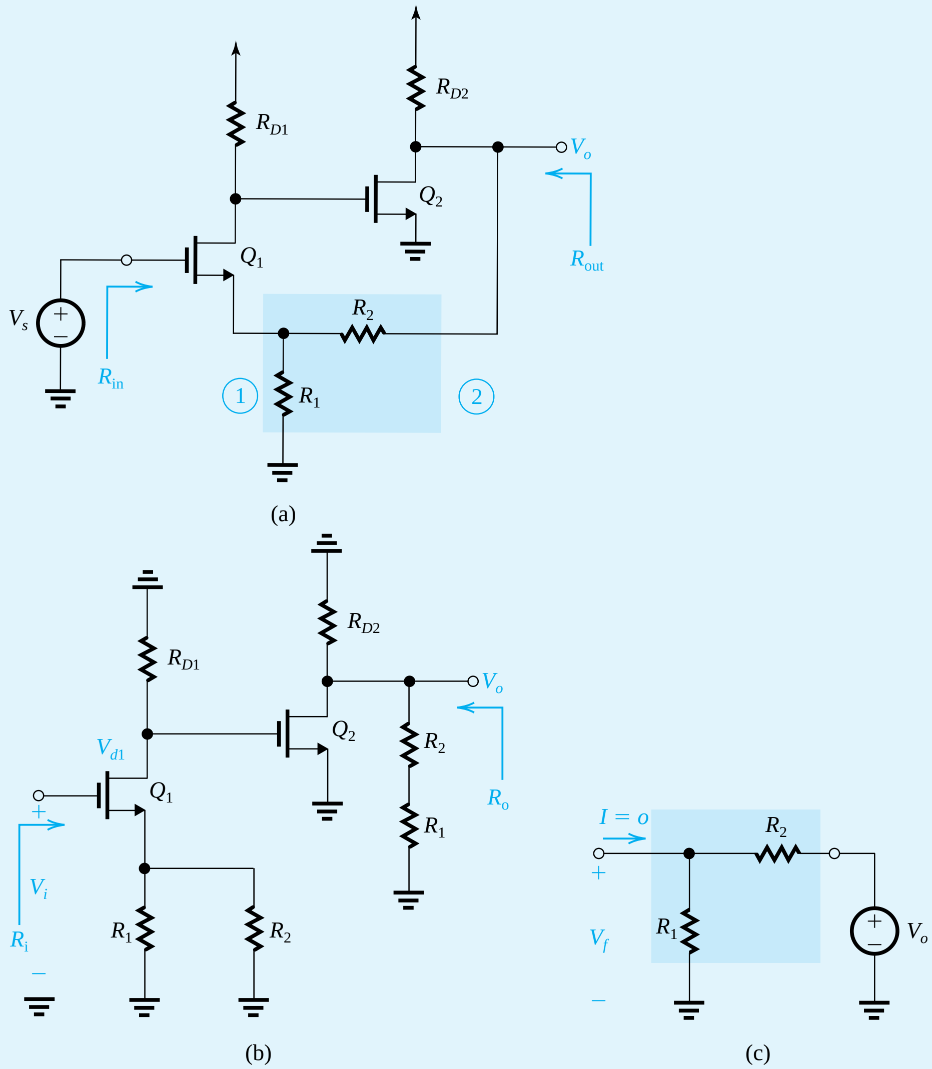


Figure 10.17 (**a**) Series–shunt feedback amplifier for Example 10.4; (**b**) The A circuit; (**c**) The β circuit.

The input resistance is obviously infinite because of the infinite input resistance of the MOSFET. The output resistance R_{out} is obtained as follows,

$$R_{out} = R_{of} = \frac{R_o}{1 + A\beta}$$

where R_o is the output resistance of the A circuit. From Fig. 10.17(b),

$$R_o = R_{D2} \| (R_1 + R_2)$$

$$= 10 \| 10 = 5\ \text{k}\Omega$$

The amount of feedback is

$$1 + A\beta = 1 + (173.9 \times 0.1) = 18.39$$

Thus,

$$R_{out} = \frac{5000}{18.39} = 272\ \Omega$$

which is relatively low given that the open-loop amplifier has $R_o = 5000\ \Omega$.

EXERCISES

10.10 If the op amp of Example 10.3 has a uniform –6-dB/octave high-frequency rolloff with f_{3dB} = 1 kHz, find the 3-dB frequency of the closed-loop gain V_o/V_s.
Ans. 7 kHz

10.11 The circuit shown in Fig. E10.11 consists of a differential stage followed by an emitter follower, with series–shunt feedback supplied by the resistors R_1 and R_2. Assuming that the dc component of V_s is zero, and that β of the BJTs is very high, find the dc operating current of each of the three transistors and show that the dc voltage at the output is approximately zero. Then find the values of A, β, $A_f \equiv V_o/V_s$, R_{in}, and R_{out}. Assume that the transistors have $\beta = 100$.
Ans. 85.7 V/V; 0.1 V/V; 8.96 V/V; 191 kΩ; 19.1 Ω.

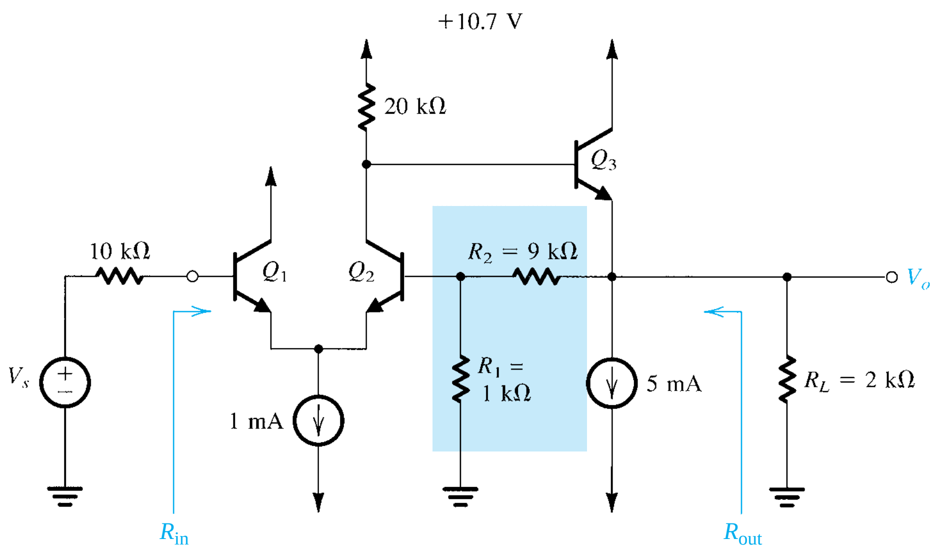


Figure E10.11

10.12 For the series–shunt amplifier in Fig. 10.7(c), find A, β, A_f, R_{in}, and R_{out}. Neglect r_o of Q.
Ans. $A = g_m[R_D \| (R_1 + R_2)]$; $\beta = R_1/(R_1 + R_2)$;

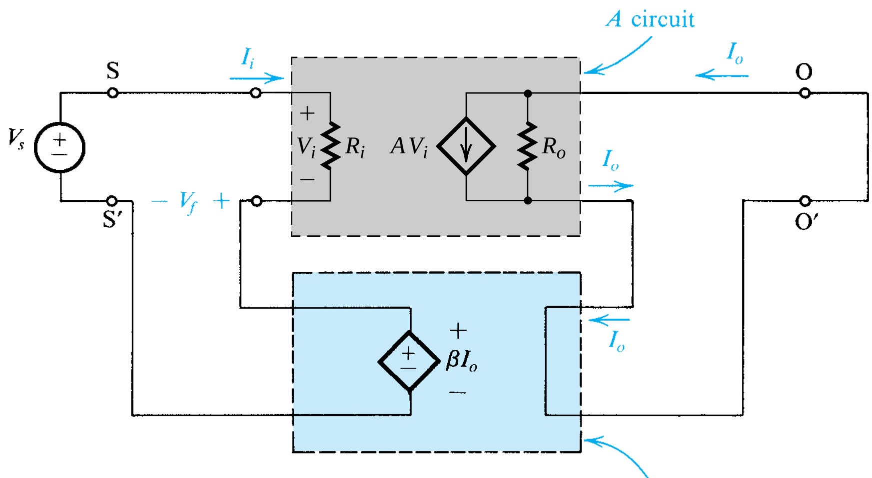
$A_f = A/(1 + A\beta)$; $R_{in} = (1/g_m)(1 + A\beta)$;

$R_{out} = [R_D \| (R_1 + R_2)]/(1 + A\beta)$

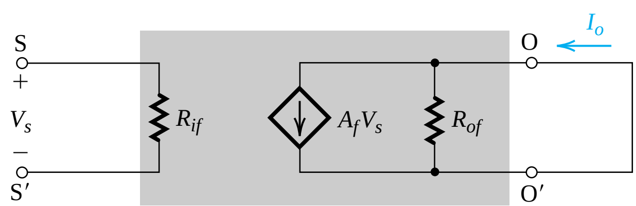
10.5 The Feedback Transconductance Amplifier (Series–Series)

10.5.1 The Ideal Case

As mentioned in Section 10.3, the series–series feedback topology stabilizes I_o/V_s and is therefore best suited for transconductance amplifiers. Figure 10.18(a) shows the ideal structure for the series–series feedback amplifier. It consists of a unilateral open-loop amplifier (the A circuit) and an ideal feedback network. The A circuit has an input resistance R_i, a short-circuit transconductance $A \equiv I_o/V_i$, and an output resistance R_o. The β circuit samples the short-circuit output current I_o and provides a feedback voltage V_f that is subtracted from V_s in the series input loop. Note that the β circuit presents zero resistance to the output loop, and thus does not load the amplifier output. Also, the feedback signal $V_f = \beta I_o$ is an ideal voltage source, thus the β circuit does not load the amplifier input. Also observe that while A is a transconductance, β is a transresistance, and thus the loop gain $A\beta$ is, as expected, a dimensionless quantity. Finally, note that the source and the load resistances have been absorbed inside the A circuit (more on this later).



(a)



(b)

Figure 10.18 The series–series feedback amplifier: (**a**) ideal structure; (**b**) equivalent circuit.

Since the structure of Fig. 10.18(a) follows the ideal feedback structure of Fig. 10.1, we can obtain the closed-loop gain A_f as

$$A_f \equiv \frac{I_o}{V_s} = \frac{A}{1+A\beta} \tag{10.26}$$

The feedback transconductance amplifier can be represented by the equivalent circuit in Fig. 10.18(b). Note that A_f is the short-circuit transconductance. Because of the series mixing, the input resistance with feedback, R_{if}, will be larger than the input resistance of the A circuit, R_i, by a factor equal to the amount of feedback,

$$R_{if} = R_i(1+A\beta) \tag{10.27}$$

Recall that the derivation we employed in the previous section to obtain R_{if} of the series–shunt feedback amplifier did not depend on the method of sampling. Thus it applies equally well to the series–series amplifier we are considering here.

Next we consider the output resistance R_{of} of the feedback transconductance amplifier. From the equivalent circuit in Fig. 10.18(b) we observe that R_{of} is the resistance seen by breaking the output loop (say at OO′) and setting V_s to zero. Thus to find the output resistance R_{of} of the series–series feedback amplifier of Fig. 10.18(a) we reduce V_s to zero and break the output circuit to apply a test current I_x, as shown in Fig. 10.19:

$$R_{of} \equiv \frac{V_x}{I_x} \tag{10.28}$$

In this case, $V_i = -V_f = -\beta I_o = -\beta I_x$. Thus for the circuit in Fig. 10.19 we obtain

$$V_x = (I_x - AV_i)R_o = (I_x + A\beta I_x)R_o$$

Hence

$$R_{of} = (1+A\beta)R_o \tag{10.29}$$

That is, in this case the negative feedback increases the output resistance. This should have been expected, since the negative feedback tries to make I_o constant in spite of changes in the output voltage, which means increased output resistance. This result also confirms our earlier observation: The relationship between R_{of} and R_o is a function only of the method of sampling.

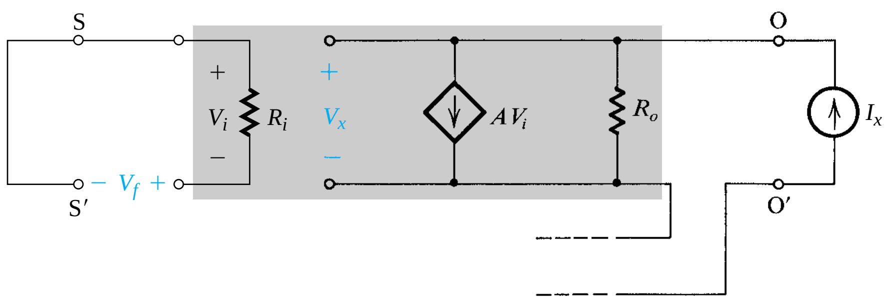


Figure 10.19 Determining the output resistance R_{of} of the series–series feedback amplifier.

While voltage (shunt) sampling reduces the output resistance, current (series) sampling increases it.

We conclude that the series–series feedback topology increases both the input and the output resistance, a highly desirable outcome for a transconductance amplifier.

10.5.2 The Practical Case

Figure 10.20(a) shows a block diagram for a practical series–series feedback amplifier. To be able to apply the feedback equations to this amplifier, we have to represent it by the ideal structure of Fig. 10.18(a). Our objective therefore is to devise a simple method for finding A and β. Observe the definition of the amplifier input resistance R_{in} and output resistance R_{out}. It is important to note that these are different from R_{if} and R_{of}, which are determined by the feedback equations, as will become clear shortly.

The series–series amplifier of Fig. 10.20(a) is redrawn in Fig. 10.20(b) with R_s and R_L shown closer to the basic amplifier, and the two-port feedback network represented by its z parameters (Appendix C). This parameter set has been chosen because it is the only one that provides a representation of the feedback network with a series circuit at the input and a series circuit at the output. This is obviously convenient in view of the series connections at input and output. The input and output resistances with feedback, R_{if} and R_{of}, are indicated on the diagram.

As we have done in the case of the series–shunt amplifier, we shall assume that the forward transmission through the feedback network is negligible in comparison to that through the basic amplifier, and thus we can dispense with the voltage source $z_{21}I_1$ in Fig. 10.20(b). Doing this, and redrawing the circuit to include z_{11} and z_{22} with the basic amplifier, results in the circuit in Fig. 10.20(c). Now if the basic amplifier is unilateral (or almost unilateral), then the circuit in Fig. 10.20(c) is equivalent to the ideal circuit of Fig. 10.18(a).

It follows that the A circuit is composed of the basic amplifier augmented at the input with R_s and z_{11} and augmented at the output with R_L and z_{22}. Since z_{11} and z_{22} are the impedances looking into ports 1 and 2, respectively, of the feedback network with the other port open-circuited, we see that finding the loading effects of the feedback network on the basic amplifier follows the rule formulated in Section 10.4. That is, we look into one port of the feedback network while the other port is open-circuited or short-circuited so as to destroy the feedback (open if series and short if shunt).

From Fig. 10.20(c) we see that β is equal to z_{12} of the feedback network,

$$\beta = z_{12} \equiv \left.\frac{V_1}{I_2}\right|_{I_1=0} \tag{10.30}$$

This result is intuitively appealing. Recall that in this case the feedback network samples the output current $[I_2 = I_o]$ and provides a voltage $[V_f = V_1]$ that is mixed in series with the input source. Again, the series connection at the input suggests that β is measured with port 1 open.

10.5.3 Summary

For future reference we present in Fig. 10.21 a summary of the rules for finding A and β for a given series–series feedback amplifier of the type shown in Fig. 10.20(a). Note that R_i is the input resistance of the A circuit, and its output resistance is R_o, which can be

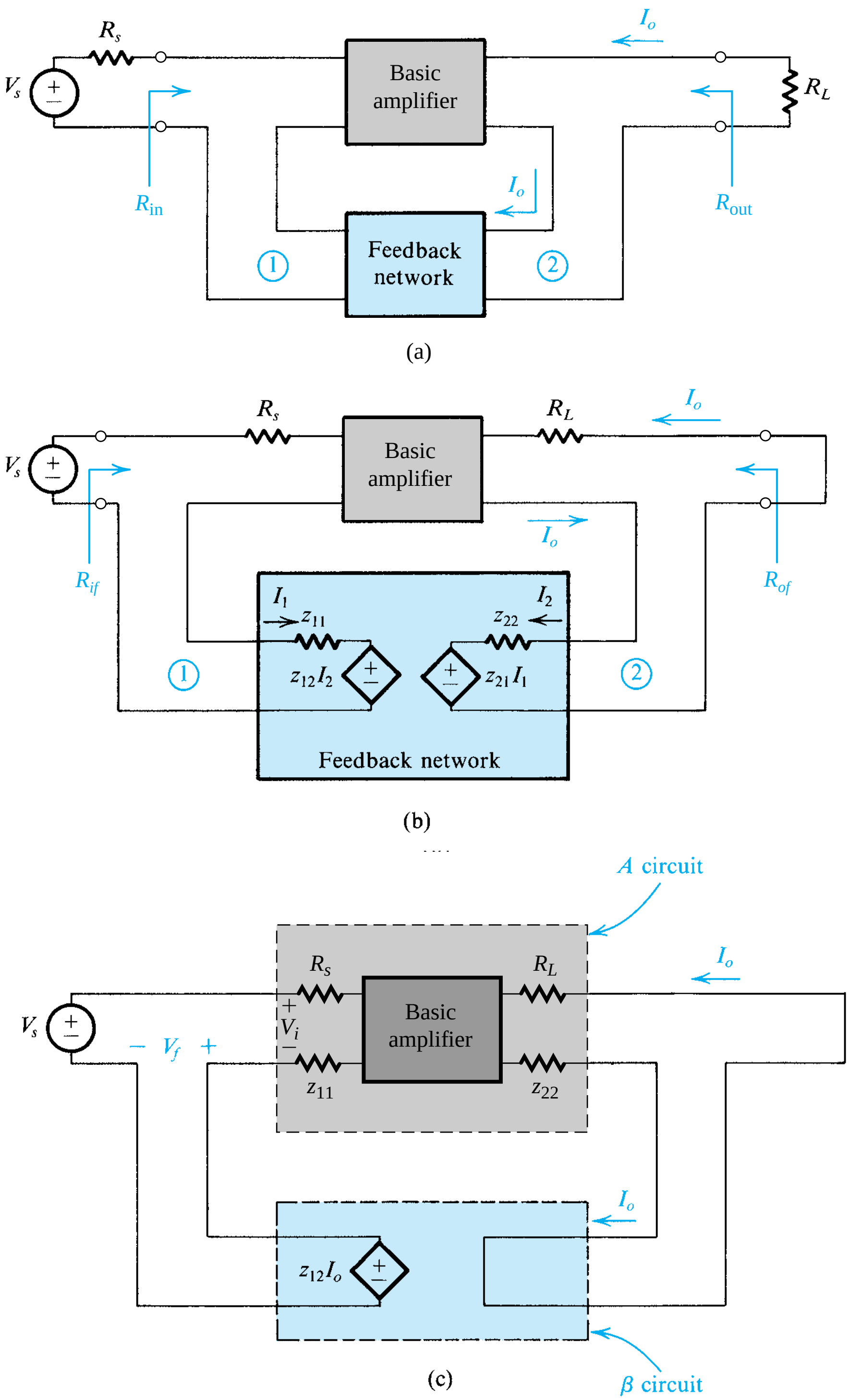


Figure 10.20 Derivation of the A circuit and the β circuit for series–series feedback amplifiers. **(a)** A series–series feedback amplifier. **(b)** The circuit of **(a)** with the feedback network represented by its z parameters. **(c)** A redrawing of the circuit in **(b)** with z_{21} neglected.

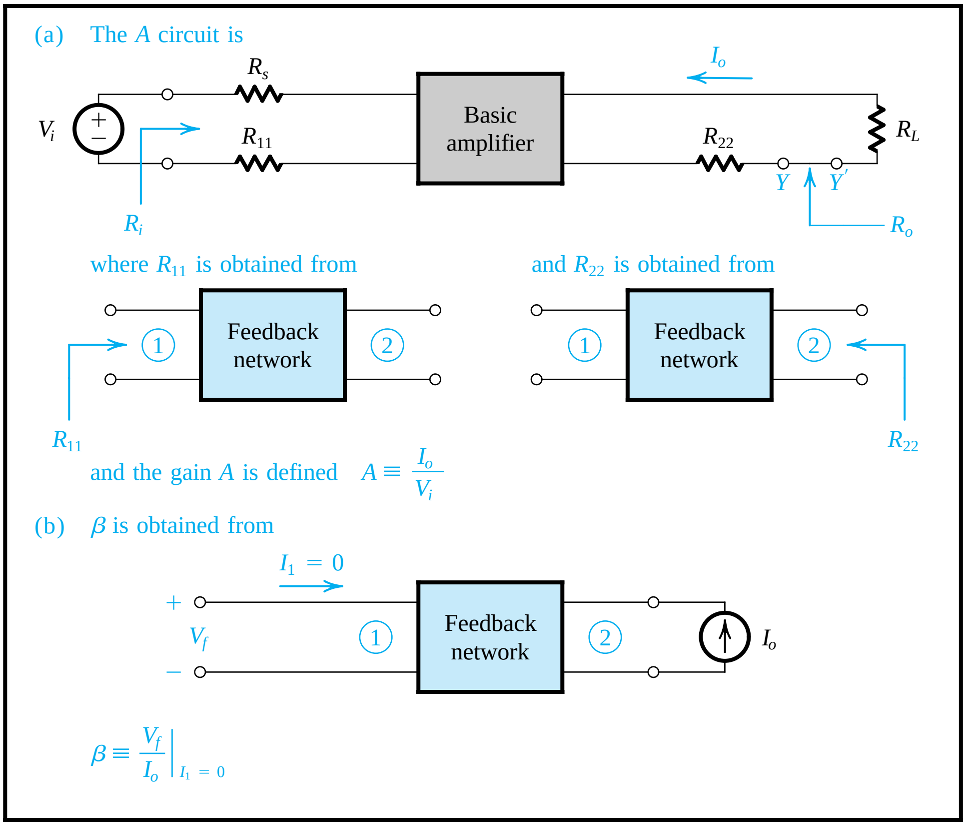


Figure 10.21 Finding the A circuit and β for the series–series feedback amplifier.

determined by breaking the output loop and looking between Y and Y' while V_i is set to zero. R_i and R_o can be used in Eqs. (10.27) and (10.29) to determine R_{if} and R_{of} (see Fig. 10.20b). The input and output resistances of the feedback amplifier can then be found by subtracting R_s from R_{if} and R_L from R_{of},

$$R_{\text{in}} = R_{if} - R_s \tag{10.31}$$

$$R_{\text{out}} = R_{of} - R_L \tag{10.32}$$

Example 10.5

As a first example of a feedback transconductance amplifier, consider the circuit shown in Fig. 10.22(a). This is the same circuit we presented in Fig. 10.10(b) and was the subject of Exercise 10.8. Here, for generality we not only assume that A_1 has finite input and output resistances but include a source resistance R_s. The objective is to analyze this circuit to determine its closed-loop gain $A_f \equiv I_o/V_s$, the input resistance of the feedback amplifier R_{in}, and the output resistance R_{out}. The latter is the resistance seen between the two terminals of R_L looking back into the output loop.

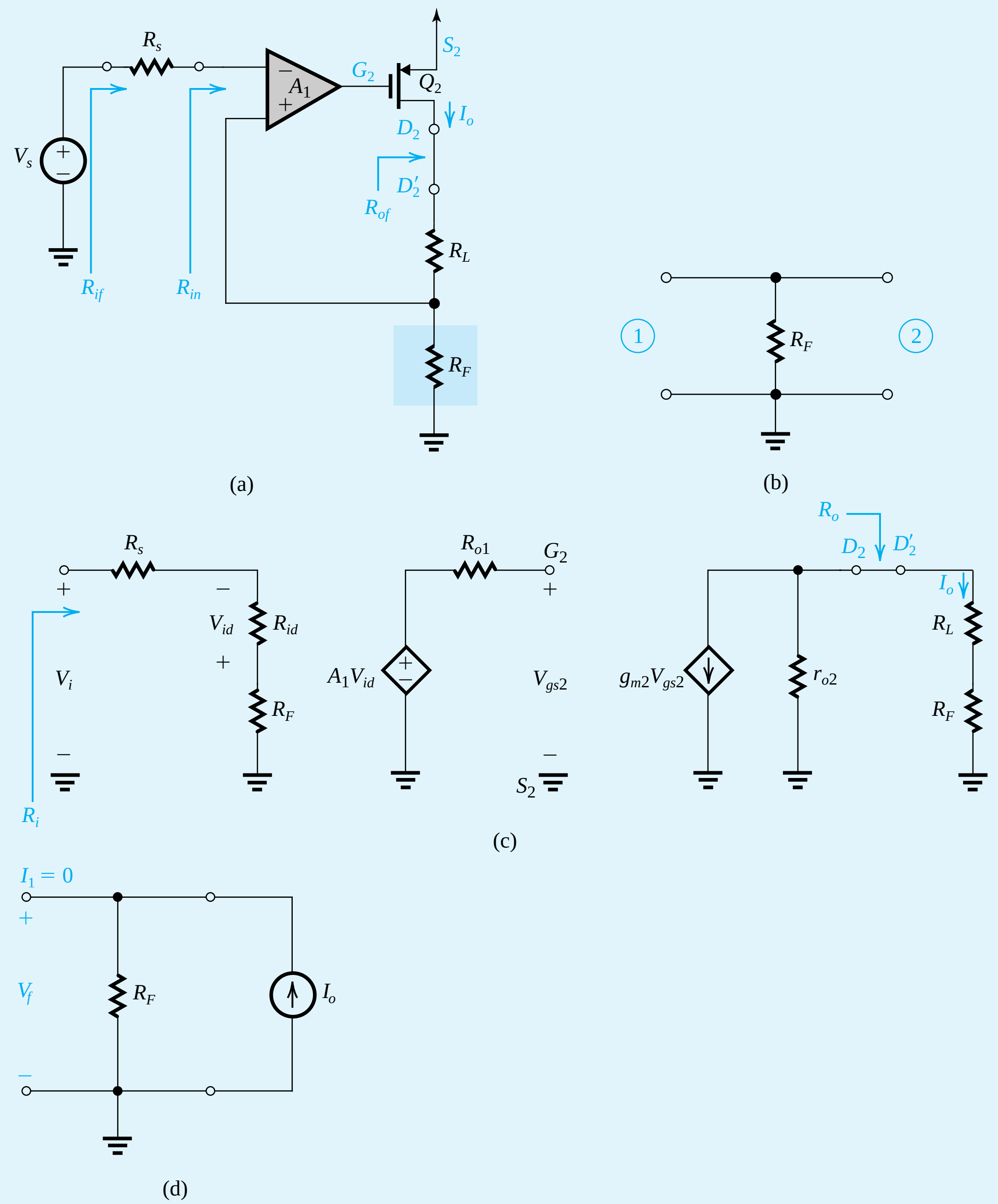


Figure 10.22 Circuits for Example 10.5.

Solution

First we identify the basic amplifier and the feedback circuit. The basic amplifier consists of the differential amplifier A_1 cascaded with the CS PMOS transistor Q_2. The output current I_o is sensed by the series

Example 10.5 *continued*

resistance R_F. The latter is the feedback network (Fig. 10.22b). It develops a voltage V_f that is mixed in series with the input loop.

The second step is to ascertain that the feedback is negative. We have already done this in Section 10.3.

Next, we determine an approximate value for $A_f \equiv I_o/V_s$ under the assumption that the loop gain $A\beta$ is much greater than unity. This value, found before any analysis is undertaken, will help us determine at the end whether our analysis is correct: If the loop gain is found to be much greater than unity, then the final A_f should be close to the value initially determined. From the circuit of Fig. 10.22(a),

$$\beta = R_F$$

and thus for large $A\beta$,

$$A_f \simeq \frac{1}{\beta} = \frac{1}{R_F}$$

Next, we determine the A circuit. Since the feedback network (Fig. 10.22b) is connected in series with both the input and output loops, we include a resistance R_F in each of these loops (which is equivalent to saying we include, at the input, the input resistance of the feedback circuit with port 2 open and, at the output, the input resistance of the feedback circuit with port 1 open). Doing this, including R_s and R_L in the A circuit, and replacing A_1 and Q_2 with their small-signal models, results in the A circuit shown in Fig. 10.22(c). Analysis of this circuit is straightforward:

$$V_{id} = -V_i \frac{R_{id}}{R_{id} + R_s + R_F} \tag{10.33}$$

$$V_{gs2} = A_1 V_{id} \tag{10.34}$$

$$I_o = -g_{m2} V_{gs2} \frac{r_{o2}}{r_{o2} + R_L + R_F} \tag{10.35}$$

Combining these three equations results in

$$A \equiv \frac{I_o}{V_i} = (A_1 g_{m2})\left(\frac{R_{id}}{R_{id} + R_s + R_F}\right)\left(\frac{r_{o2}}{r_{o2} + R_L + R_F}\right) \tag{10.36}$$

Usually $R_{id} \gg (R_s + R_F)$, $r_{o2} \gg (R_L + R_F)$, resulting in the approximate expression for A:

$$A \simeq A_1 g_{m2} \tag{10.37}$$

The input resistance R_i can be found by inspection as

$$R_i = R_s + R_{id} + R_F \tag{10.38}$$

The output resistance R_o is found by setting $V_i = 0$, and breaking the output loop at any location, say between D_2 and D_2'. Thus,

$$R_o = r_{o2} + R_L + R_F \tag{10.39}$$

Finally, β can be found from Fig. 10.22(d) as

$$\beta \equiv \frac{V_f}{I_o} = R_F$$

The loop gain $A\beta$ is thus

$$A\beta = (A_1 g_{m2} R_F)\left(\frac{R_{id}}{R_{id} + R_s + R_F}\right)\left(\frac{r_{o2}}{r_{o2} + R_L + R_F}\right) \tag{10.40}$$

$$\simeq A_1 g_{m2} R_F \tag{10.41}$$

With numerical values, one can now obtain the value of $A\beta$ and determine whether it is indeed much greater than unity. We next determine the closed-loop gain

$$A_f = \frac{A}{1 + A\beta}$$

Substituting for A from Eq. (10.37) and for $A\beta$ from Eq. (10.41), we have

$$A_f \simeq \frac{A_1 g_{m2}}{1 + A_1 g_{m2} R_F}$$

For $A_1 g_{m2} R_F \gg 1$,

$$A_f \simeq \frac{1}{R_F}$$

which is the value we found at the outset.

The series mixing raises the input resistance with feedback,

$$R_{if} = R_i(1 + A\beta)$$

Substituting for R_i from Eq. (10.38) and for $A\beta$ from the full expression in Eq. (10.40), we obtain

$$\begin{aligned} R_{if} &= (R_s + R_{id} + R_F)(1 + A\beta) \\ &= R_s + R_{id} + R_F + A_1 g_{m2} R_F R_{id} \frac{r_{o2}}{r_{o2} + R_L + R_F} \end{aligned}$$

which for $r_{o2} \gg R_L + R_F$ yields

$$R_{if} \simeq R_s + R_{id} + R_F + A_1 g_{m2} R_F R_{id}$$

To obtain R_{in}, we subtract R_s from R_{if} (see Fig. 10.22a):

$$R_{\text{in}} = R_{id} + R_F + A_1 g_{m2} R_F R_{id}$$

Usually $R_F \ll R_{id}$,

$$R_{\text{in}} \simeq R_{id}(1 + A_1 g_{m2} R_F) \tag{10.42}$$

which is an intuitively appealing result: The series mixing at the input raises the input resistance R_{id} by a factor equal to the approximate value of $(1 + A\beta)$.

To obtain R_{of}, we note that the series connection at the output raises the output resistance, thus,

$$\begin{aligned} R_{of} &= R_o(1 + A\beta) \\ &= (r_{o2} + R_L + R_F)(1 + A\beta) \\ &= r_{o2} + R_L + R_F + A\beta(r_{o2} + R_L + R_F) \end{aligned}$$

Example 10.5 *continued*

Substituting for $A\beta$ from Eq. (10.40) and making the approximation $R_{id} \gg (R_s + R_F)$, we write

$$R_{of} \simeq r_{o2} + R_L + R_F + A_1 g_{m2} R_F r_{o2}$$

To obtain R_{out}, which is the resistance seen by R_L in the circuit of Fig. 10.22(a), we subtract R_L from R_{of},

$$R_{\text{out}} = r_{o2} + R_F + A_1 g_{m2} R_F r_{o2}$$

usually $R_F \ll r_{o2}$; thus,

$$R_{\text{out}} \simeq r_{o2}(1 + A_1 g_{m2} R_F)$$

which is an intuitively appealing result: The series connection at the output raises the output resistance of Q_2 (r_{o2}) by a factor equal to the amount of feedback.

Finally, we note that we have deliberately solved this problem in great detail to illustrate the beauty

EXERCISE

D10.13 For the circuit analyzed in Example 10.5, select a value for R_F that will result in $A_f \simeq 5$ mA/V. Now, for $A_1 = 200$ V/V, $g_{m2} = 2$ mA/V, $R_{id} = 100$ kΩ, $r_{o2} = 20$ kΩ, and assuming that $R_s \ll R_{id}$ and $R_L \ll r_{o2}$, find the value of A_f realized and the input and output resistances of the feedback transconductance amplifier. If for some reason g_{m2} drops in value by 50%, what is the corresponding percentage change in A_f?

Ans. 200 Ω; 4.94 mA/V; 8.1 MΩ; 1.62 MΩ; –1.25 %

Example 10.6

Because negative feedback extends the amplifier bandwidth, it is commonly used in the design of broadband amplifiers. One such amplifier is the MC1553. Part of the circuit of the MC1553 is shown in Fig. 10.23(a). The circuit shown (called a **feedback triple**) is composed of three gain stages with series–series feedback provided by the network composed of R_{E1}, R_F, and R_{E2}.

Observe that the feedback network samples the emitter current I_o of Q_3, and thus I_o is the output quantity of the feedback amplifier. However, practically speaking, I_o is rather difficult to utilize. Thus usually the collector current of Q_3, I_c, is taken as the output. This current is of course almost equal to I_o; $I_c = \alpha I_o$. Thus, as a transconductance amplifier with I_c as the output current, the output resistance of interest is that labeled R_{out} in Fig. 10.23(a). In some applications, I_c is passed through a load resistance, such as R_{C3}, and the voltage V_o is taken as the output. Assume that the bias circuit, which is not shown, establishes $I_{C1} = 0.6$ mA, $I_{C2} = 1$ mA, and $I_{C3} = 4$ mA. Also assume that for all three transistors,[4] $h_{fe} = 100$ and $r_o = \infty$.

[4]To avoid possible confusion of the BJT current gain β and the feedback factor β, we sometimes use h_{fe} to denote the transistor β.

(a) Anticipating that the loop gain will be large, find an approximate expression and value for the closed-loop gain $A_f \equiv I_o/V_s$ and hence for I_c/V_s. Also find V_o/V_s.
(b) Use feedback analysis to find A, β, A_f, V_o/V_s, R_{in}, and R_{out}. For the calculation of R_{out}, assume that r_o of Q_3 is 25 kΩ.

Solution

(a) When $A\beta \gg 1$,

$$A_f \equiv \frac{I_o}{V_s} \simeq \frac{1}{\beta}$$

where the feedback factor β can be found from the feedback network. The feedback network is highlighted in Fig. 10.23(a), and the determination of the value of β is illustrated in Fig. 10.23(b), from which we find

$$\beta \equiv \frac{V_f}{I_o} = \frac{R_{E2}}{R_{E2}+R_F+R_{E1}} \times R_{E1}$$

$$= \frac{100}{100+640+100} \times 100 = 11.9\ \Omega$$

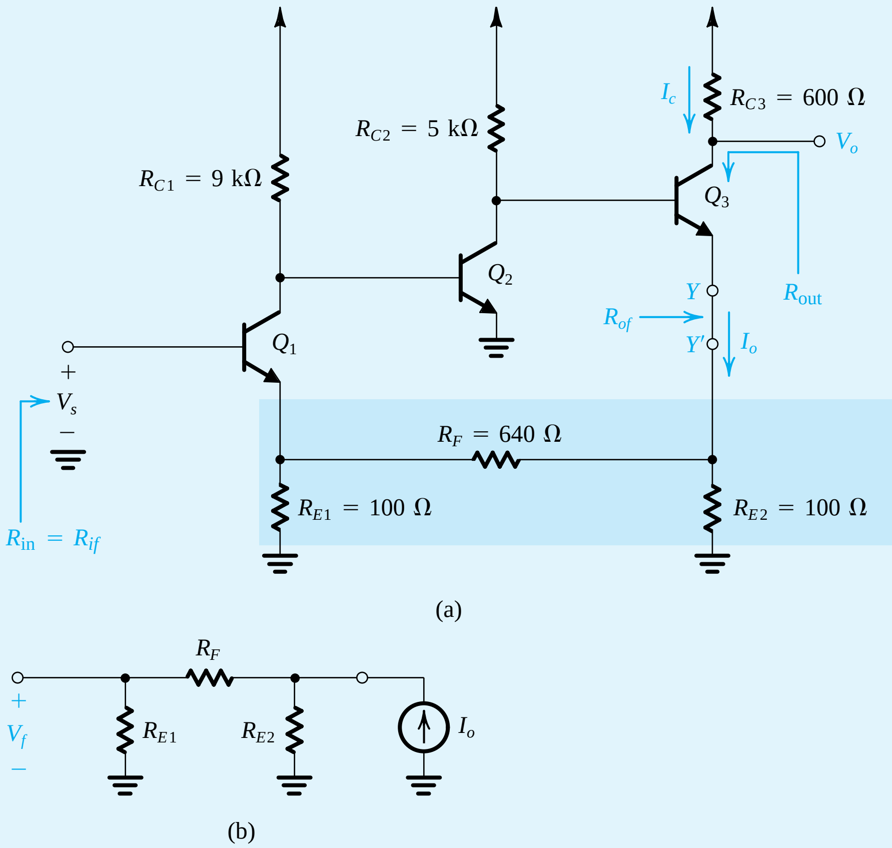


Figure 10.23 Circuits for Example 10.6.

Example 10.6 *continued*

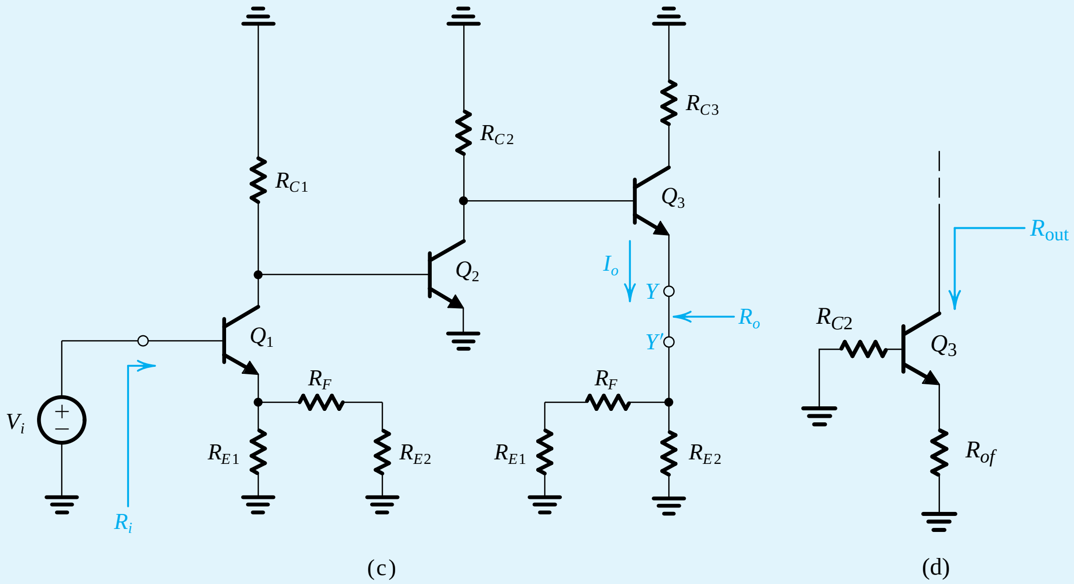


Figure 10.23 *continued*

Thus,

$$A_f \simeq \frac{1}{\beta}$$
$$= \frac{1}{R_{E2}}\left(1 + \frac{R_{E2} + R_F}{R_{E1}}\right)$$
$$= \frac{1}{11.9} = 84 \text{ mA/V}$$
$$\frac{I_c}{V_s} \simeq \frac{I_o}{V_s} = 84 \text{ mA/V}$$
$$\frac{V_o}{V_s} = \frac{-I_c R_{C3}}{V_s} = -84 \times 0.6 = -50.4 \text{ V/V}$$

(b) Employing the loading rules given in Fig. 10.21, we obtain the A circuit shown in Fig. 10.23(c). To find $A \equiv I_o/V_i$ we first determine the gain of the first stage. This can be written by inspection as

$$\frac{V_{c1}}{V_i} = \frac{-\alpha_1(R_{C1} \parallel r_{\pi 2})}{r_{e1} + [R_{E1} \parallel (R_F + R_{E2})]}$$

Since Q_1 is biased at 0.6 mA, r_{e1} = 41.7 Ω. Transistor Q_2 is biased as 1 mA; thus $r_{\pi 2} = h_{fe}/g_{m2} = 100/40 = 2.5$ kΩ. Substituting these values together with $\alpha_1 = 0.99$, $R_{C1} = 9$ kΩ, $R_{E1} = 100$ Ω, $R_F = 640$ Ω, and $R_{E2} = 100$ Ω, results in

$$\frac{V_{c1}}{V_i} = -14.92 \text{ V/V}$$

Next, we determine the gain of the second stage, which can be written by inspection (noting that $V_{b2} = V_{c1}$) as

$$\frac{V_{c2}}{V_{c1}} = -g_{m2}\{R_{C2} \parallel (h_{fe}+1)[r_{e3} + (R_{E2} \parallel (R_F + R_{E1}))]\}$$

Substituting $g_{m2} = 40$ mA/V, $R_{C2} = 5$ kΩ, $h_{fe} = 100$, $r_{e3} = 25/4 = 6.25\ \Omega$, $R_{E2} = 100\ \Omega$, $R_F = 640\ \Omega$, and $R_{E1} = 100\ \Omega$, results in

$$\frac{V_{c2}}{V_{c1}} = -131.2 \text{ V/V}$$

Finally, for the third stage we can write by inspection

$$\frac{I_o}{V_{c2}} = \frac{I_{e3}}{V_{b3}} = \frac{1}{r_{e3} + (R_{E2} \parallel (R_F + R_{E1}))}$$

$$= \frac{1}{6.25 + (100 \parallel 740)} = 10.6 \text{ mA/V}$$

Combining the gains of the three stages results in

$$A \equiv \frac{I_o}{V_i} = -14.92 \times -131.2 \times 10.6 \times 10^{-3}$$

$$= 20.7 \text{ A/V}$$

The closed-loop gain A_f can now be found from

$$A_f \equiv \frac{I_o}{V_s} = \frac{A}{1 + A\beta}$$

$$= \frac{20.7}{1 + 20.7 \times 11.9} = 83.7 \text{ mA/V}$$

which we note is very close to the approximate value found in (a) above.

The voltage gain is found from

$$\frac{V_o}{V_s} = \frac{-I_c R_{C3}}{V_s} \simeq \frac{-I_o R_{C3}}{V_s} = -A_f R_{C3}$$

$$= -83.7 \times 10^{-3} \times 600 = -50.2 \text{ V/V}$$

which is also very close to the approximate value found in (a) above.

The input resistance of the feedback amplifier is given by

$$R_{in} = R_{if} = R_i(1 + A\beta)$$

where R_i is the input resistance of the A circuit. The value of R_i can be found from the circuit in Fig. 10.23(c) as follows:

$$R_i = (h_{fe} + 1)[r_{e1} + (R_{E1} \parallel (R_F + R_{E2}))]$$

$$= 13.65 \text{ k}\Omega$$

Thus,

$$R_{if} = 13.65(1 + 20.7 \times 11.9) = 3.38 \text{ M}\Omega$$

To determine the output resistance R_{out}, which is the resistance looking into the collector of Q_3, we face a dilemma. The feedback does not sample I_c and thus we cannot employ the feedback formulas directly.[5] Nevertheless, we present a somewhat indirect solution to this problem below. Here we note parenthetically that had Q_1 been a MOSFET, this problem would not have existed, since $I_d = I_s$.

Since the feedback senses the emitter current I_o, the output resistance given by the feedback analysis will be the resistance seen in the emitter circuit, say between Y and Y',

$$R_{of} = R_o(1 + A\beta)$$

[5] This important point was first brought to the authors' attention by Gordon Roberts (see Roberts and Sedra, 1992).

Example 10.6 *continued*

where R_o can be determined from the A circuit in Fig. 10.23(c) by breaking the circuit between Y and Y'. The resistance looking between these two nodes can be found to be

$$R_o = [R_{E2} \| (R_F + R_{E1})] + r_{e3} + \frac{R_{C2}}{h_{fe} + 1}$$

which, for the values given, yields $R_o = 143.9\ \Omega$. The output resistance R_{of} of the feedback amplifier can now be found as

$$R_{of} = R_o(1 + A\beta) = 143.9(1 + 20.7 \times 11.9) = 35.6\ \text{k}\Omega$$

We can now use the value of R_{of} to obtain an approximate value for R_{out}. To do this, we assume that the effect of the feedback is to place a resistance R_{of} (35.6 kΩ) in the emitter of Q_3, and find the output resistance from the equivalent circuit shown in Fig. 10.23(d). This is the output resistance of a BJT with a resistance R_{of} in its emitter and a resistance R_{C2} in its base. The formula we have for this (Eq. 7.50) does not unfortunately account for a resistance in the base. The formula, however, can be modified (see Problem 10.48) to obtain

$$\begin{aligned} R_{\text{out}} &= r_{o3} + [R_{of} \| (r_{\pi 3} + R_{C2})] \left[1 + g_{m3} r_{o3} \frac{r_{\pi 3}}{r_{\pi 3} + R_{C2}}\right] \\ &= 25 + [35.6 \| (0.625 + 5)] \left[1 + 160 \times 25 \times \frac{0.625}{0.625 + 5}\right] \\ &= 2.19\ \text{M}\Omega \end{aligned}$$

Thus R_{out} is increased (from r_{o3}) but not by $(1 + A\beta)$.

EXERCISE

D10.14 For the feedback triple in Fig. 10.23(a), analyzed in Example 10.6, modify the value of R_F to obtain a closed-loop transconductance I_o/V_s of approximately 100 mA/V. Assume that the loop gain remains large. What is the new value of R_F? For this value, what is the approximate value of the voltage gain if the output voltage is taken at the collector of Q_3?

Ans. 800 Ω; –60 V/V

10.6 The Feedback Transresistance Amplifier (Shunt–Shunt)

10.6.1 The Ideal Case

As mentioned in Section 10.3, the shunt–shunt feedback topology stabilizes V_o/I_s and is thus best suited for transresistance amplifiers. Figure 10.24(a) shows the ideal structure for the shunt–shunt feedback amplifier. It consists of a unilateral open-loop amplifier (the A circuit) and an ideal feedback network. The A circuit has an input resistance R_i, an open-circuit transresistance $A \equiv V_o/I_i$, and an output resistance R_o. The β circuit samples the open-circuit

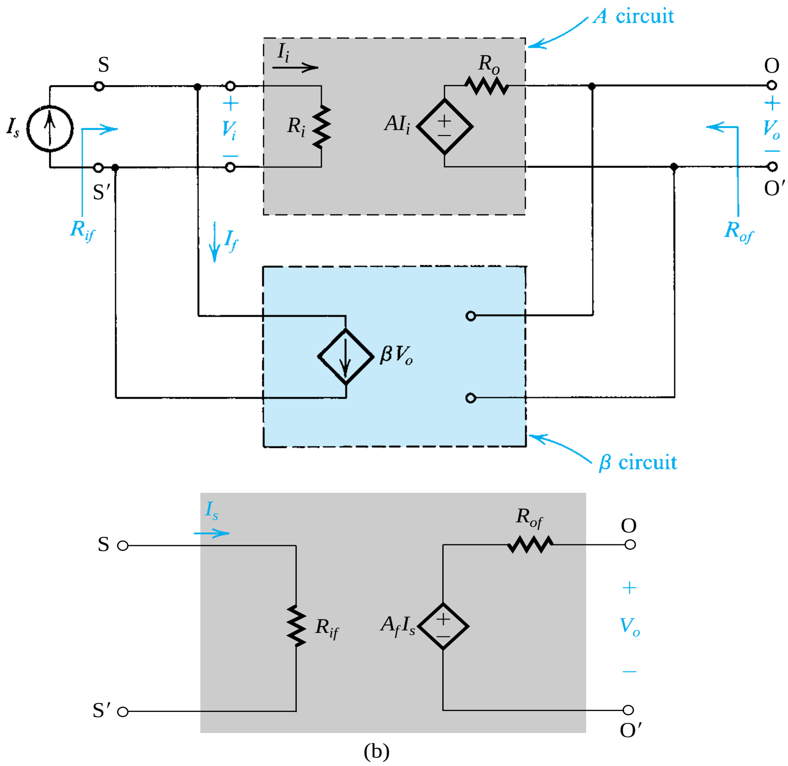


Figure 10.24 (**a**) Ideal structure for the shunt–shunt feedback amplifier. (**b**) Equivalent circuit of the amplifier in (a).

output voltage V_o and provides a feedback current I_f that is subtracted from the signal-source current I_s at the input nodes. Note that the β circuit presents an infinite impedance to the amplifier output and thus does not load the amplifier output. Also, the feedback signal $I_f = \beta V_o$ is provided as an ideal current source, and thus the β circuit does not load the amplifier input. Also observe that while A is a transresistance, β is a transconductance and thus the loop gain $A\beta$ is, as expected, a dimensionless quantity. Finally, note that the source and load resistances have been absorbed inside the A circuit (more on this later).

Since the structure of Fig. 10.24(a) follows the ideal feedback structure of Fig. 10.1, we can obtain the closed-loop gain A_f as

$$A_f \equiv \frac{V_o}{I_s} = \frac{A}{1 + A\beta} \tag{10.43}$$

The feedback transresistance amplifier can be represented by the equivalent circuit in Fig. 10.24(b). Note that A_f is the open-circuit transresistance. To obtain the input resistance R_{if}, refer to the input side of the block diagram in Fig. 10.24(a). The shunt connection at the input causes the feedback current to subtract from I_s resulting in a reduced current I_i into the A circuit,

$$I_i = I_s - I_f$$

Substituting $I_f = \beta V_o = \beta A I_i$ and rearranging, results in

$$I_i = \frac{I_s}{1 + A\beta}$$

which indicates that the shunt mixing reduces the input current by the amount of feedback. This is, of course, a direct application of Eq. (10.7), where in the case of shunt mixing, $x_s = I_s$ and $x_i = I_i$. The input resistance with feedback, R_{if}, can now be obtained from

$$R_{if} \equiv \frac{V_i}{I_s} = \frac{V_i}{(1 + A\beta)I_i}$$

Substituting for $V_i/I_i = R_i$, which is the input resistance of the A circuit, results in

$$R_{if} = \frac{R_i}{1 + A\beta} \tag{10.44}$$

Thus, as expected, the shunt connection at the input lowers the input resistance by a factor equal to the amount of feedback. The lowered input resistance is a welcome result for the transresistance amplifier; the lower the input resistance, the easier it is for the signal current source that feeds the amplifier input.

Turning our attention next to the output resistance, we can follow an approach identical to that used in the case of the series–shunt amplifier (Section 10.4) to show that the shunt connection at the output lowers the output resistance by a factor equal to the amount of feedback,

$$R_{of} = \frac{R_o}{1 + A\beta} \tag{10.45}$$

This also is a welcome result for the transresistance amplifier as it makes its voltage-output circuit more ideal; the output voltage will change less as we draw current from the amplifier output. Finally, note that *the shunt feedback connection, whether at the input or at the output, always reduces the corresponding resistance*.

10.6.2 The Practical Case

Figure 10.25 shows a block diagram for a practical shunt–shunt feedback amplifier. To be able to apply the feedback equations to this amplifier, we have to represent it by the ideal structure of Fig. 10.24(a). Our objective therefore is to devise a simple method for finding the A circuit and β. Building on the insight we have gained from our study of the series–shunt and series–series topologies, we present the method for the shunt–shunt case, without

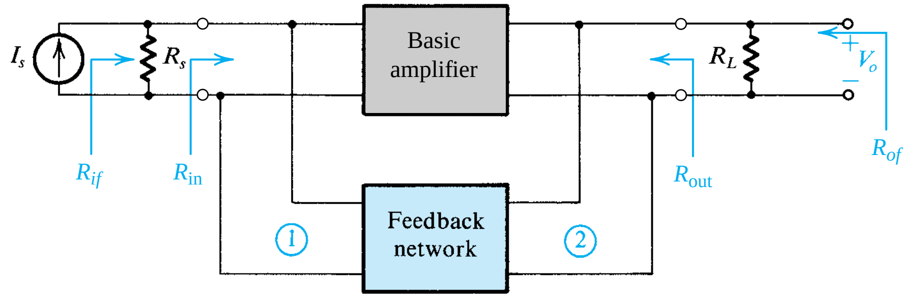


Figure 10.25 Block diagram for a practical shunt–shunt feedback amplifier.

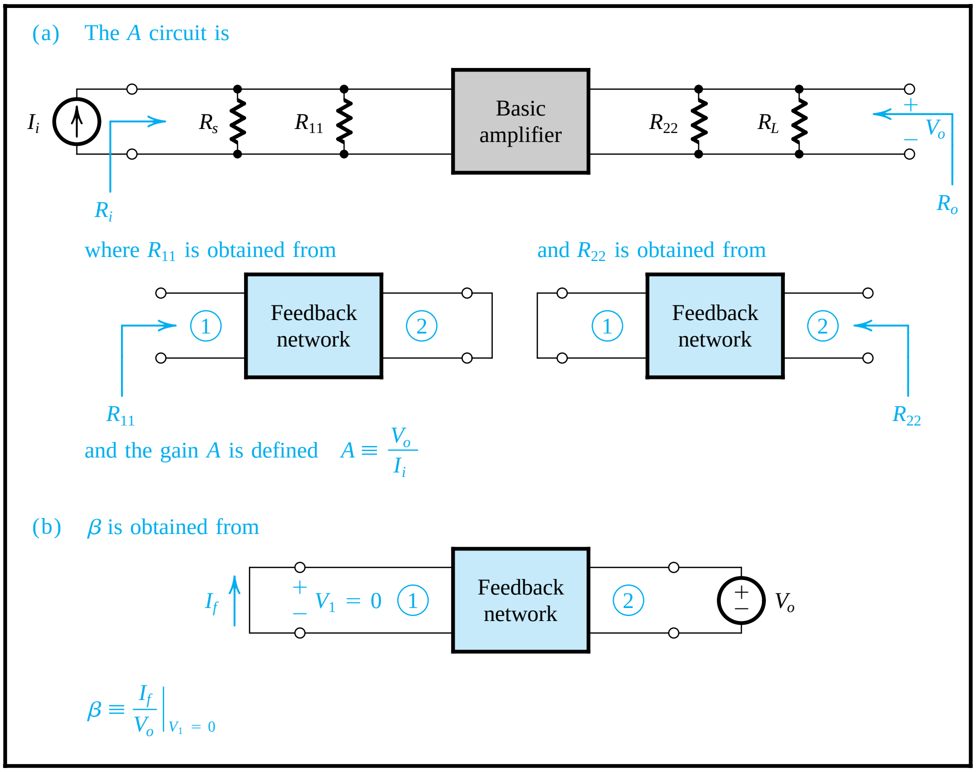


Figure 10.26 Finding the A circuit and β for the shunt–shunt feedback amplifier in Fig. 10.25.

derivation, in Fig. 10.26. As in previous cases, the method of Fig. 10.26 assumes that the basic amplifier is unilateral (or almost so) and that the feedforward transmission through the feedback network is negligibly small.

As indicated in Fig. 10.26, the A circuit is obtained by including R_s across the input terminals of the amplifier and R_L across its output terminals. The loading effect of the feedback network on the amplifier input is represented by the resistance R_{11} and its loading effect at the output is represented by the resistance R_{22}. The value of R_{11} is obtained by looking into port 1 of the feedback network while port 2 is shorted (because it is connected in shunt). Similarly, R_{22} is found by looking into port 2 while port 1 is shorted (because it is connected in shunt). Finally, observe that since the feedback network senses V_o, it is fed by a voltage V_o; and since it delivers a current I_f that is mixed in shunt at the input, its port 1 is short-circuited and β is found as I_f/V_o, where I_f is the current that flows through the short circuit.

The open-loop resistances R_i and R_o are determined from the A circuit and are used in Eqs. (10.44) and (10.45) to determine R_{if} and R_{of}. Finally, the resistances R_{in} and R_{out} that characterize the feedback amplifier are obtained from R_{in} and R_{of} by reference to Fig. 10.25 as follows:

$$R_{\text{in}} = 1 \Big/ \left(\frac{1}{R_{if}} - \frac{1}{R_s}\right) \tag{10.46}$$

$$R_{\text{out}} = 1 \Big/ \left(\frac{1}{R_{of}} - \frac{1}{R_L}\right) \tag{10.47}$$

Example 10.7

Figure 10.27(a) shows a feedback transresistance amplifier. It is formed by connecting a resistance R_F in the negative-feedback path of a voltage amplifier with gain μ, an input resistance R_{id}, and an output resistance r_o. The amplifier μ can be implemented with an op amp, a simple differential amplifier, a single-ended inverting amplifier, or, in the limit, a single-transistor CE or CS amplifier. The latter case will be considered in Exercise 10.15. Of course, the higher the gain μ, the more ideal the characteristics of the feedback transresistance amplifier will be, simply because of the concomitant increase in loop gain.

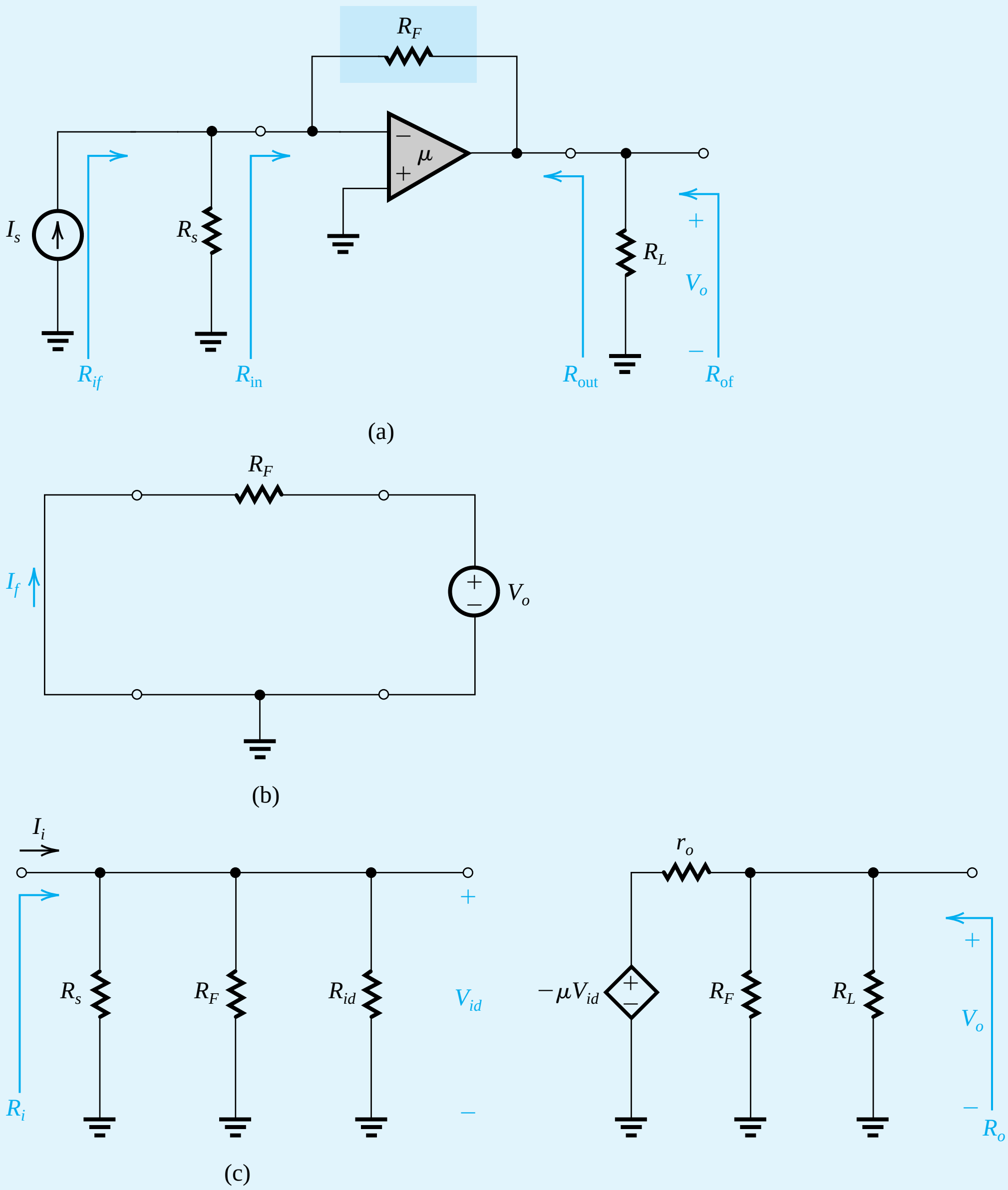


Figure 10.27 **(a)** A feedback transresistance amplifier; **(b)** the β circuit; **(c)** the A circuit.

(a) If the loop gain is large, find an approximate expression for the closed-loop open-circuit transresistance of the feedback amplifier.
(b) Find the A circuit and expressions for A, R_i, and R_o.
(c) Find expressions for the loop gain, A_f, R_{if}, R_{in}, R_{of}, and R_{out}.
(d) Find the values of R_i, R_o, A, β, A_f, R_{if}, R_{in}, R_{of}, and R_{out} for the case $\mu = 10^4$ V/V, $R_{id} = \infty$, $r_o = 100\ \Omega$, $R_F = 10\ \text{k}\Omega$, and $R_s = R_L = 1\ \text{k}\Omega$.
(e) If instead of a current source I_s having a source resistance $R_s = 1\ \text{k}\Omega$, the amplifier is fed from a voltage source V_s having a source resistance $R_s = 1\ \text{k}\Omega$, find an expression for and the value of the voltage gain V_o/V_s.

Solution

(a) If the loop gain $A\beta$ is large,

$$A_f \equiv \frac{V_o}{I_s} \simeq \frac{1}{\beta}$$

where β can be found from the β circuit in Fig. 10.27(b) as

$$\beta \equiv \frac{I_f}{V_o} = -\frac{1}{R_F} \tag{10.48}$$

Thus,

$$\frac{V_o}{I_s} \simeq -R_F$$

Note that in this case the voltage at the input node (the inverting input terminal of μ) will be very close to ground and thus very little, if any, current flows into the input terminal of the amplifier. Nearly all of I_s will flow through R_F, resulting in $V_o \simeq 0 - I_s R_F = -I_s R_F$. This should be reminiscent of the inverting op-amp configuration studied in Section 2.2.

(b) Since the feedback network consists of R_F, the loading effect at the amplifier input and output will simply be R_F. This is indicated in the A circuit shown in Fig. 10.27(c). The open-loop transresistance A can be obtained as follows:

$$V_{id} = I_i R_i \tag{10.49}$$

where

$$R_i = R_{id} \parallel R_F \parallel R_s \tag{10.50}$$

$$V_o = -\mu V_{id} \frac{(R_F \parallel R_L)}{r_o + (R_F \parallel R_L)} \tag{10.51}$$

Combining Eqs. (10.49) and (10.51) gives

$$A \equiv \frac{V_o}{I_i} = -\mu R_i \frac{(R_F \parallel R_L)}{r_o + (R_F \parallel R_L)} \tag{10.52}$$

The open-loop output resistance can be obtained by inspection of the A circuit with I_i set to 0. We see that $V_{id} = 0$, and

$$R_o = r_o \parallel R_F \parallel R_L \tag{10.53}$$

(c) The loop gain $A\beta$ can be obtained by combining Eqs. (10.48) and (10.52),

$$A\beta = \mu\left(\frac{R_i}{R_F}\right) \frac{(R_F \parallel R_L)}{r_o + (R_F \parallel R_L)}$$

Example 10.7 *continued*

Observe that although both A and β are negative, $A\beta$ is positive, a comforting fact confirming that the feedback is negative. Also note that $A\beta$ is dimensionless, as it must always be.

The closed-loop gain A_f can now be found as

$$A_f \equiv \frac{V_o}{I_s} = \frac{A}{1 + A\beta}$$

Thus

$$A_f = \frac{-\mu R_i \dfrac{(R_F \parallel R_L)}{r_o + (R_F \parallel R_L)}}{1 + \mu \dfrac{R_i}{R_F} \dfrac{(R_F \parallel R_L)}{r_o + (R_F \parallel R_L)}} \tag{10.54}$$

Note that the condition of $A\beta \gg 1$ which results in $A_f \simeq -R_F$ corresponds to

$$\mu\left(\frac{R_i}{R_F}\right)\frac{(R_F \parallel R_L)}{r_o + (R_F \parallel R_L)} \gg 1 \tag{10.55}$$

The input resistance with feedback, R_{if}, is obtained by dividing R_i by $(1 + A\beta)$ with the result

$$R_{if} = \frac{R_i}{1 + A\beta}$$

or

$$\frac{1}{R_{if}} = \frac{1}{R_i} + \frac{A\beta}{R_i} = \frac{1}{R_i} + \frac{\mu}{R_F}\,\frac{(R_F \parallel R_L)}{r_o + (R_F \parallel R_L)}$$

Substituting for R_i from Eq. (10.50) and replacing $\mu(R_F \parallel R_L)/[r_o + (R_F \parallel R_L)]$ by μ', where μ' is lower than but usually close to the value of μ, results in

$$R_{if} = R_{id} \parallel R_F \parallel R_s \parallel (R_F / \mu')$$

The two terms containing R_F can be combined,

$$R_{if} = R_s \parallel R_{id} \parallel [R_F/(\mu' + 1)] \tag{10.56}$$

Since $R_{if} = R_s \parallel R_{\text{in}}$, we see that

$$R_{\text{in}} = R_{id} \parallel [R_F \parallel (\mu' + 1)]$$

Usually R_{id} is large and thus

$$R_{\text{in}} \simeq \frac{R_F}{\mu' + 1} \simeq \frac{R_F}{\mu'} \tag{10.57}$$

from which we observe that for large amplifier gain μ, the input resistance will be low.

The output resistance with feedback R_{of} can be found by dividing R_o by $(1 + A\beta)$:

$$R_{of} = \frac{R_o}{1 + A\beta}$$

Thus,

$$\begin{aligned}\frac{1}{R_{of}} &= \frac{1}{R_o} + \frac{A\beta}{R_o} \\ &= \frac{1}{R_o} + \mu\frac{R_i}{R_F}\,\frac{(R_F \parallel R_L)}{r_o + (R_F \parallel R_L)}\frac{1}{R_o}\end{aligned}$$

Substituting for R_o from Eq. (10.53),

$$\frac{1}{R_{of}} = \frac{1}{R_L} + \frac{1}{R_F} + \frac{1}{r_o} + \mu\frac{R_i}{R_F}\frac{1}{r_o}$$
$$= \frac{1}{R_L} + \frac{1}{R_F} + \frac{1}{r_o}\left(1 + \mu\frac{R_i}{R_F}\right)$$

Thus,

$$R_{of} = R_L \parallel R_F \parallel \frac{r_o}{1 + \mu\dfrac{R_i}{R_F}}$$

Since, moreover,

$$R_{of} = R_L \parallel R_{\text{out}}$$

we obtain for R_{out}

$$R_{\text{out}} = R_F \parallel \frac{r_o}{1 + \mu\dfrac{R_i}{R_F}}$$

Usually $R_F \gg r_o/[(1 + \mu(R_i/R_F)]$; thus,

$$R_{\text{out}} \simeq \frac{r_o}{1 + \mu\dfrac{R_i}{R_F}} \simeq \left(\frac{R_F}{R_i}\right)\left(\frac{r_o}{\mu}\right)$$

from which we see that for large μ, the output resistance will be considerably reduced.

(d) For the numerical values given:

$$R_i = R_{id} \parallel R_F \parallel R_s$$
$$= \infty \parallel 10 \parallel 1 = 0.91 \text{ k}\Omega$$
$$R_o = r_o \parallel R_F \parallel R_s$$
$$= 0.1 \parallel 10 \parallel 1 = 90 \ \Omega$$
$$A = -\mu R_i \frac{(R_F \parallel R_L)}{r_o + (R_F \parallel R_L)}$$
$$= -10^4 \times 0.91 \times \frac{(10 \parallel 1)}{0.1 + (10 \parallel 1)} = -8198 \text{ k}\Omega$$
$$\beta = -\frac{1}{R_F} = -\frac{1}{10} = -0.1 \text{ mA/V}$$
$$A\beta = 819.8$$
$$1 + A\beta = 820.8$$
$$A_f = \frac{A}{1 + A\beta} = -\frac{8198}{820.8} = -9.99 \text{ k}\Omega$$

which is very close to the ideal value of $-R_F = -10$ kΩ.

$$R_{if} = \frac{R_i}{1 + A\beta} = \frac{910}{820.8} = 1.11 \ \Omega$$

$$R_{\text{in}} = \frac{1}{\dfrac{1}{R_{if}} - \dfrac{1}{R_i}} - \frac{1}{\dfrac{1}{1.11} - \dfrac{1}{1000}} \simeq 1.11 \ \Omega$$

Example 10.7 *continued*

which is very low, a highly desirable property. We also have

$$R_{of} = \frac{R_o}{1 + A\beta} = \frac{90}{820.8} = 0.11\ \Omega$$

$$R_{out} = \frac{1}{\dfrac{1}{R_{of}} - \dfrac{1}{R_L}} = \frac{1}{\dfrac{1}{0.11} - \dfrac{1}{1000}} \simeq 0.11\ \Omega$$

which as well is very low, another highly desirable property.

(e) If the amplifier is fed with a voltage source V_s having a resistance $R_s = 1\ \text{k}\Omega$, the output voltage can be found from

$$V_o = A_f\ I_s = A_f\ \frac{V_s}{R_s}$$

Thus,

$$\frac{V_o}{V_s} = \frac{A_f}{R_s} = -\frac{9.99\ \text{k}\Omega}{1\ \text{k}\Omega} = -9.99\ \text{V/V}$$

EXERCISE

10.15 For the transresistance amplifier in Fig. E10.15, replace the MOSFET with its equivalent-circuit model and use feedback analysis to show the following:

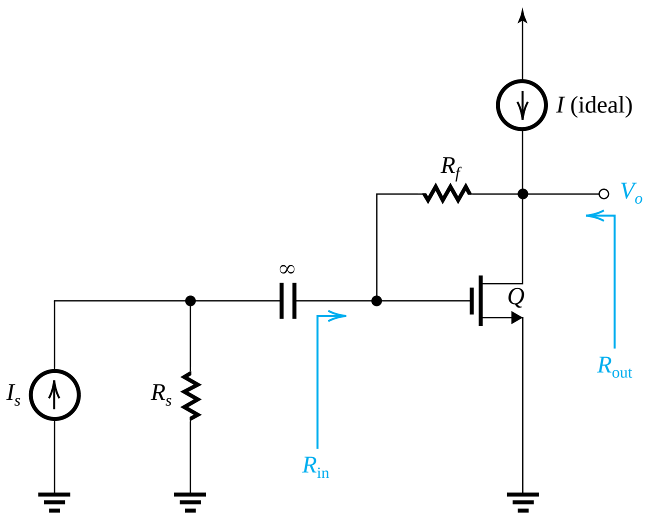


Figure E10.15

(a) For large loop gain (which cannot be achieved here), $A_f \equiv V_o/I_s \simeq -R_f$.

(b) $$A_f = \frac{-(R_s \| R_f)g_m(r_o \| R_f)}{1 + (R_s \| R_f)g_m(r_o \| R_f)/R_f}$$

(c) $$R_{in} = \frac{R_f}{[1 + g_m(r_o \| R_f)]}$$

(d) $R_{\text{out}} = r_o \| \dfrac{R_f}{1 + g_m(R_s \| R_f)}$

(e) For $g_m = 5$ mA/V, $r_o = 20$ kΩ, $R_f = 10$ kΩ, and $R_s = 1$ kΩ, find A, β, $A\beta$, A_f R_i, R_o, R_{if}, R_{in}, R_{of}, and R_{out}.

Ans. (e) –30.3 kΩ; –0.1 mA/V; 3.03; –7.52 kΩ (compare to the ideal value of –10 kΩ); 909 Ω; 6.67 kΩ; 226 Ω; 291 Ω; 1.66 kΩ; 1.66 kΩ

10.6.3 An Important Note

The feedback analysis method is predicated on the assumption that all (or most) of the feedforward transmission occurs in the basic amplifier and all (or most) of the feedback transmission occurs in the feedback network. The circuit considered in Exercise 10.15 above is simple and can be analyzed directly (i.e., without invoking the feedback approach) to determine A_f. In this way we can check the validity of our assumptions. This point is illustrated in Problem 10.58, where we find that for the circuit in Fig. E10.15, all of the feedback transmission occurs in the feedback circuit. Also, as long as g_m is much greater than $1/R_f$, the assumption that most of the feedforward transmission occurs in the basic amplifiers is valid, and thus the feedback analysis is reasonably accurate.

10.7 The Feedback Current Amplifier (Shunt–Series)

10.7.1 The Ideal Case

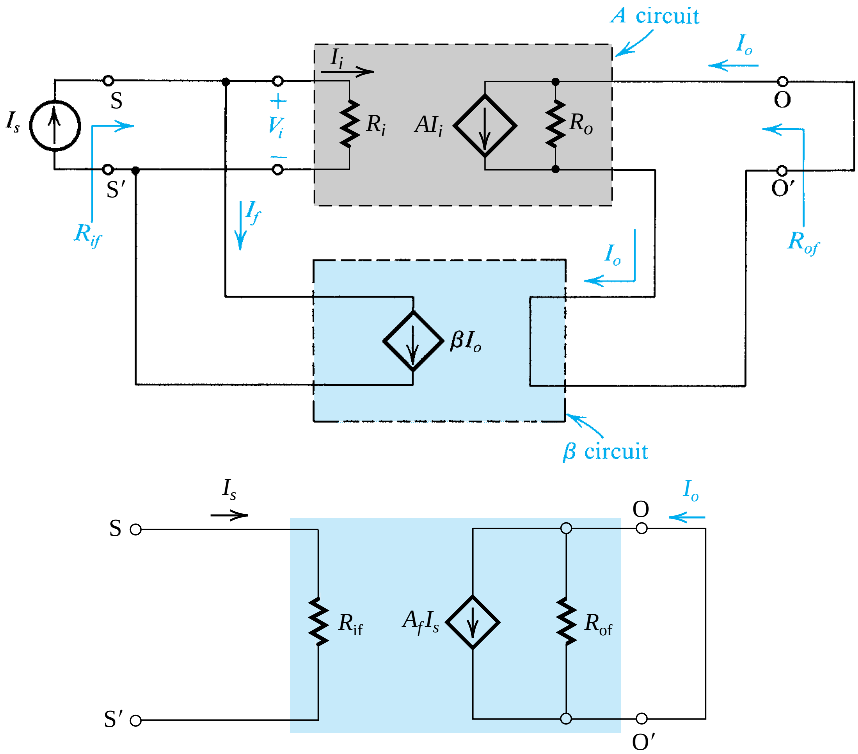
As mentioned in Section 10.3, the shunt–series feedback topology is best suited for current amplifiers: The shunt connection at the input reduces the input resistance, making it easier to feed the amplifier with a current signal; the sampling of output current stabilizes I_o, which is the output signal in a current amplifier, and the series connection at the output increases the output resistance, making the output current value less susceptible to changes in load resistance.

Figure 10.28(a) shows the ideal structure for the shunt–series feedback amplifier. It consists of a unilateral open-loop amplifier (the A circuit) and an ideal feedback network. The A circuit has an input resistance R_i, a short-circuit current gain $A \equiv I_o/I_i$, and an output resistance R_o. The β circuit samples the short-circuit output current I_o and provides a feedback current I_f that is subtracted from the signal-source current I_s at the input node. Note that the β circuit presents a zero resistance to the output loop and thus does not load the amplifier output. Also, the feedback signal $I_f = \beta I_o$ is provided as an ideal current source, and thus the β circuit does not load the amplifier input. Also observe that both A and β are current gains and $A\beta$ is a dimensionless quantity. Finally, note that the source and load resistances have been absorbed inside the A circuit (more on this later).

Since the structure of Fig. 10.28(a) follows the ideal feedback structure of Fig. 10.1, we can obtain the closed-loop current gain A_f as

$$A_f \equiv \frac{I_o}{I_s} = \frac{A}{1 + A\beta} \tag{10.59}$$

The feedback current amplifier can be represented by the equivalent circuit in Fig. 10.28(b).



(b)

Figure 10.28 (**a**) Ideal structure for the shunt–series feedback amplifier. (**b**) Equivalent circuit of the amplifier in (**a**).

Note that A_f is the short-circuit current gain. The input resistance R_{if} is found by dividing R_i by $(1 + A\beta)$, which is a result of the shunt connection at the input. Thus,

$$R_{if} = \frac{R_i}{1 + A\beta} \tag{10.60}$$

The output resistance R_{of} is the resistance obtained by setting $I_s = 0$, breaking the short-circuit output loop, at say OO', and measuring the resistance between the two terminals thus created. Since the series feedback connection always raises resistance, we can obtain R_{of} by multiplying R_o by $(1 + A\beta)$,

$$R_{of} = (1 + A\beta)R_o \tag{10.61}$$

10.7.2 The Practical Case

Figure 10.29 shows a block diagram for a practical shunt–series feedback amplifier. To be able to apply the feedback equations to this amplifier, we have to represent it by the ideal structure of Fig. 10.28(a). Our objective therefore is to devise a simple method for finding the A and β circuits. Building on the insight we have gained from the study of the three other topologies, we present the method for the shunt–series case without derivation, in Fig. 10.30. As in previous cases, the method of Fig. 10.30 assumes that the basic amplifier is unilateral (or almost so) and that the feedforward transmission in the feedback network is negligibly small.

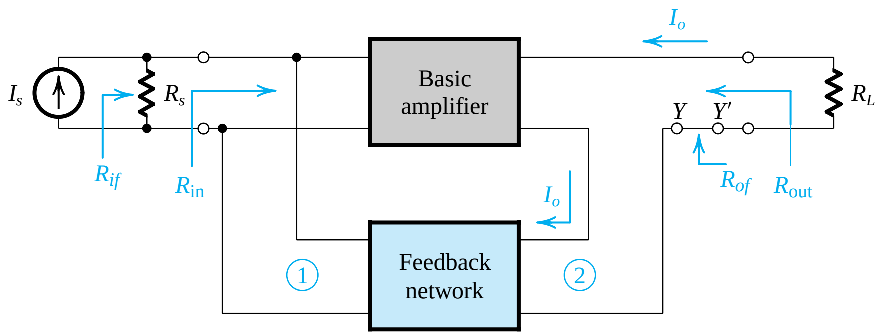


Figure 10.29 Block diagram for a practical shunt–series feedback amplifier.

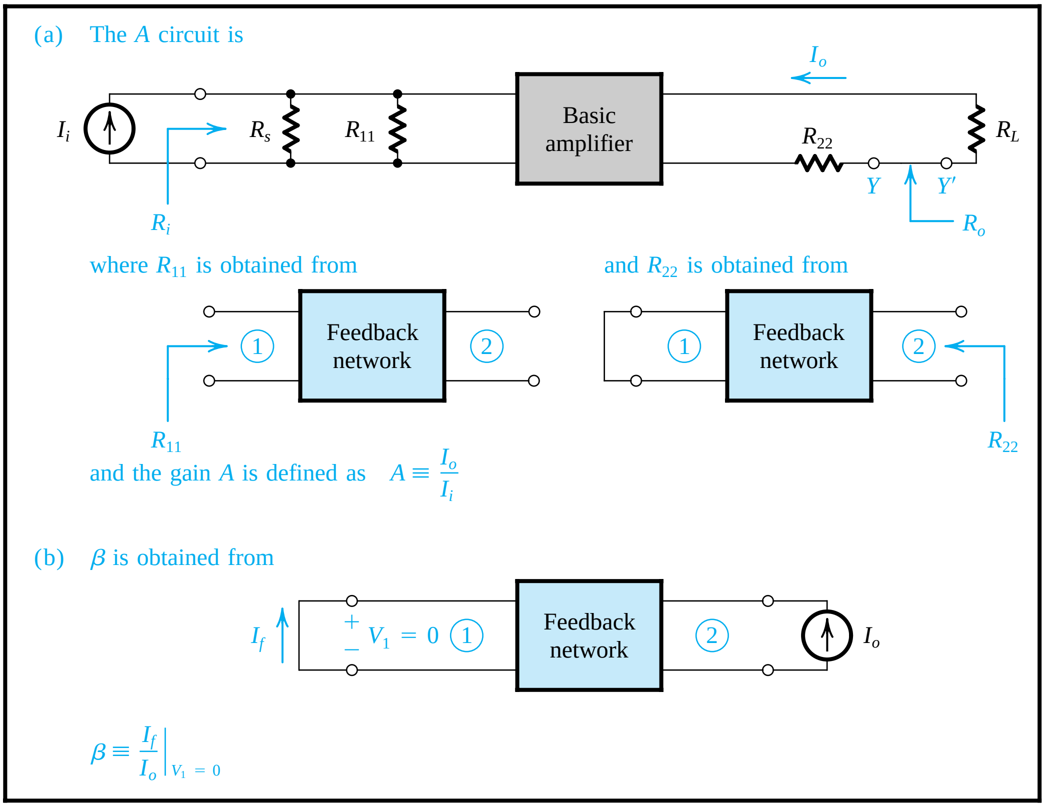


Figure 10.30 Finding the A circuit and β for the current-mixing current-sampling (shunt–series) feedback amplifier of Fig. 10.29.

As indicated in Fig. 10.30, the A circuit is obtained by including R_s across the input terminals of the amplifier and R_L in series with its output loop. The loading effect of the feedback network on the amplifier input is represented by the resistance R_{11}, and its loading effect at the amplifier output is represented by resistance R_{22}. The value of R_{11} is obtained by looking into port 1 of the feedback network while its port 2 is open-circuited (because it is connected in series). The value of R_{22} is obtained by looking into port 2 of the feedback network while its port 1 is short-circuited (because it is connected in shunt). Finally, observe that since the feedback network senses I_o, it is fed by a current I_o; and since it delivers a current I_f that is mixed in shunt at the input, its port 1 is short-circuited and β is found as I_f/I_o, where I_f is the current that flows through the short circuit.

The open-loop resistances R_i and R_o are determined from the A circuit as indicated. Observe that R_o is found by breaking the output loop at say YY' and measuring the resistance between Y and Y'. Resistances R_i and R_o are then used in Eqs. (10.60) and (10.61), respectively, to determine R_{if} and R_{of}. Finally, the resistances R_{in} and R_{out} that characterized the feedback amplifier are obtained from R_{if} and R_{of} by reference to Fig. 10.29, as follows:

$$R_{\text{in}} = 1\Big/\left(\frac{1}{R_{if}} - \frac{1}{R_s}\right) \tag{10.62}$$

$$R_{\text{out}} = R_{of} - R_L \tag{10.63}$$

Example 10.8

Figure 10.31 shows a feedback current amplifier formed by cascading an inverting voltage amplifier μ with a MOSFET Q. The output current I_o is the drain current of Q. The feedback network, consisting of resistors R_1 and R_2, senses an exactly equal current, namely, the source current of Q, and provides a feedback current signal that is mixed with I_s at the input node. Note that the bias arrangement is *not* shown.

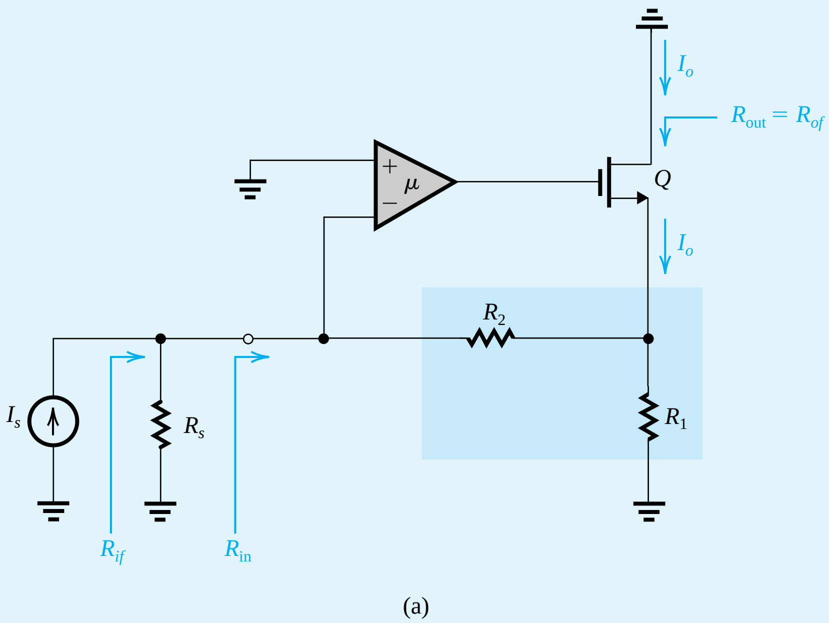


Figure 10.31 Circuit for Example 10.8.

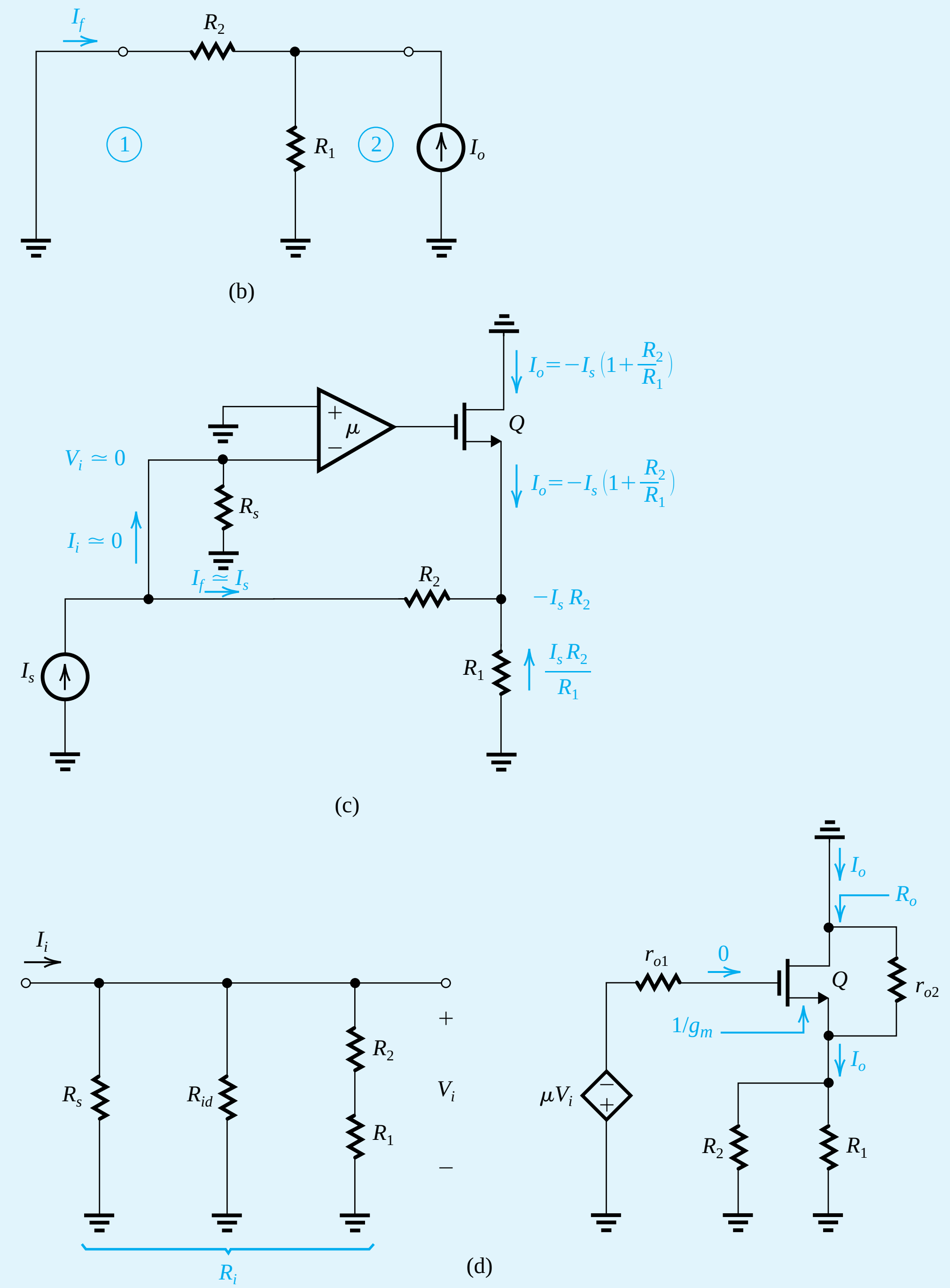


Figure 10.31 *continued*

Example 10.8 *continued*

The amplifier μ can be implemented in a variety of ways, including by means of an op amp, a differential amplifier, or a single-ended inverting amplifier. The simplest approach is to implement μ with a CS MOSFET amplifier. However, in such a case the loop gain will be very limited. Assume that the amplifier μ has an input resistance R_{id}, an open-circuit voltage gain μ, and an output resistance r_{o1}.

(a) If the loop gain is large, find an approximate expression for the closed-loop gain $A_f \equiv I_o/I_s$.
(b) Find the A circuit and derive expressions for A, R_i, and R_o.
(c) Give expressions for $A\beta$, A_f, R_{if}, R_{in}, R_{of}, and R_{out}.
(d) Find numerical values for A, β, $A\beta$, A_f, R_i, R_{if}, R_{in}, R_o, R_{of}, and R_{out} for the following case: $\mu = 1000$ V/V, $R_s = \infty$, $R_{id} = \infty$, $r_{o1} = 1\ \text{k}\Omega$, $R_1 = 10\ \text{k}\Omega$, $R_2 = 90\ \text{k}\Omega$, and for Q: $g_m = 5$ mA/V and $r_o = 20\ \text{k}\Omega$.

Solution

(a) When the loop gain $A\beta \gg 1$, $A_f \simeq 1/\beta$. To determine β refer to Fig. 10.31(b),

$$\beta \equiv \frac{I_f}{I_o} = -\frac{R_1}{R_1+R_2} \tag{10.64}$$

Thus,

$$A_f \simeq \frac{1}{\beta} = -\left(1+\frac{R_2}{R_1}\right) \tag{10.65}$$

To see what happens in this case more clearly, refer to Fig. 10.31(c). Here we have assumed the loop gain to be large, so that $I_i \simeq 0$ and thus $I_f \simeq I_s$. Also note that because $I_i \simeq 0$, V_i will be close to zero. Thus, we can easily determine the voltage at the source of Q as $-I_f R_2 \simeq -I_s R_2$. The current through R_1 will then be $I_s R_2/R_1$. The source current of Q will be $-(I_s + I_s R_2/R_1)$, which means that the output current I_o will be

$$I_o = -I_s\left(1+\frac{R_2}{R_1}\right)$$

which confirms the expression for A_f obtained above (Eq. 10.65).

(b) To obtain the A circuit we load the input side of the basic amplifier with R_s and R_{11}. The latter in this case is simply $R_1 + R_2$ (because port 2 of the feedback network is opened). We also load the output of the basic amplifier with R_{22}, which in this case is $R_1 \| R_2$ (because port 1 of the feedback network is shorted). The resulting A circuit is shown in Fig., 10.31(d), where we have replaced the amplifier μ with its equivalent circuit. Analysis of the A circuit is straightforward and proceeds as follows:

$$R_i = R_s \| R_{id} \| (R_1+R_2) \tag{10.66}$$

$$V_i = I_i R_i \tag{10.67}$$

$$I_o = -\mu V_i \frac{1}{1/g_m + (R_1 \| R_2 \| r_{o2})} \frac{r_{o2}}{r_{o2} + (R_1 \| R_2)} \tag{10.68}$$

Combining Eqs. (10.67) and (10.68) results in A:

$$A \equiv \frac{I_o}{I_i} = -\mu \frac{R_i}{1/g_m + (R_1 \| R_2 \| r_{o2})} \frac{r_{o2}}{r_{o2} + (R_1 \| R_2)} \tag{10.69}$$

For the case $1/g_m \ll (R_1 \| R_2 \| r_{o2})$,

$$A \simeq -\mu \frac{R_i}{R_1 \| R_2 \| r_{o2}} \frac{r_{o2}}{r_{o2} + (R_1 \| R_2)}$$

Which reduces to

$$A = -\mu \frac{R_i}{R_1 \| R_2} \tag{10.70}$$

Noting that R_o is the output resistance of Q, which has a resistance $(R_1 \| R_2)$ in its source lead, we can write

$$R_o = r_{o2} + (R_1 \| R_2) + (g_m r_{o2})(R_1 \| R_2)$$

$$\simeq g_m r_{o2}(R_1 \| R_2) \tag{10.71}$$

(c) The loop gain is obtained by combining Eqs. (10.64) and (10.69),

$$A\beta = \mu \frac{R_i}{\frac{1}{g_m} + (R_1 \| R_2 \| r_{o2})} \frac{r_{o2}}{r_{o2} + (R_1 \| R_2)} \frac{R_1}{R_1 + R_2} \tag{10.72}$$

For the case $1/g_m \ll (R_1 \| R_2 \| r_{o2})$,

$$A\beta \simeq \mu \frac{R_i}{R_1 \| R_2} \frac{R_1}{R_1 + R_2} = \mu \frac{R_i}{R_2} \tag{10.73}$$

The input resistance R_{if} is found as

$$R_{if} = R_i/(1 + A\beta)$$

$$\frac{1}{R_{if}} = \frac{1}{R_i} + \frac{A\beta}{R_i}$$

We can substitute for $A\beta$ from the full expression in Eq. (10.72). For the approximate case, we use $A\beta$ from Eq. (10.73):

$$\frac{1}{R_{if}} = \frac{1}{R_i} + \frac{\mu}{R_2}$$

That is,

$$R_{if} = R_i \| \frac{R_2}{\mu}$$

Substituting for R_i from Eq. (10.66), we write

$$R_{if} = R_s \| R_{id} \| (R_1 + R_2) \| \frac{R_2}{\mu}$$

Since by definition,

$$R_{if} = R_s \| R_{\text{in}}$$

we can easily find R_{in} as

$$R_{\text{in}} = R_{id} \| (R_1 + R_2) \| \frac{R_2}{\mu} \tag{10.74}$$

Example 10.8 *continued*

Usually the third component on the right-hand side is the smallest; thus,

$$R_{\text{in}} \simeq \frac{R_2}{\mu} \tag{10.75}$$

For the output resistance, we have

$$R_{of} = R_o(1 + A\beta) \simeq A\beta R_o$$

Substituting for R_o for Eq. (10.71) and for $A\beta$ from the approximate expression in Eq. (10.73), we have

$$R_{of} \simeq \mu\left(\frac{R_i}{R_2}\right)(g_m r_{o2})(R_1 \parallel R_2)$$

$$R_{of} = \mu\frac{R_i}{R_1 + R_2}(g_m r_{o2})R_1 \tag{10.76}$$

Finally, we note that

$$R_{\text{out}} = R_{of} = \mu\frac{R_i}{R_1 + R_2}g_m r_{o2}R_1 \tag{10.77}$$

(d) For the numerical values given,

$$R_i = \infty \parallel \infty \parallel (10 + 90) = 100\ \text{k}\Omega$$

Since $1/g_m = 0.2\ \text{k}\Omega \ll (10 \parallel 90 \parallel 20)$,

$$A \simeq -\mu\frac{R_i}{R_1 \parallel R_2}$$

$$= -1000\frac{100}{10 \parallel 90} = -11.11 \times 10^3\ \text{A/A}$$

$$\beta = -\frac{R_1}{R_1 + R_2} = -\frac{10}{10 + 90} = -0.1\ \text{A/A}$$

$$A\beta = 1111$$

$$A_f = -\frac{11.11 \times 10^3}{1 + 1111} = -9.99\ \text{A/A}$$

which is very close to the ideal value of

$$A_f \simeq -\left(1 + \frac{R_2}{R_1}\right) = -\left(1 + \frac{90}{10}\right) = -10\ \text{A/A}$$

$$R_{\text{in}} = \frac{R_2}{\mu} = \frac{90\ \text{k}\Omega}{1000} = 90\ \Omega$$

$$R_o = g_m r_{o2}(R_1 \parallel R_2)$$

$$= 5 \times 20(10 \parallel 90) = 900\ \text{k}\Omega$$

$$R_{\text{out}} = (1 + A\beta)R_o = 1112 \times 900 = 1000\ \text{M}\Omega$$

EXERCISES

10.16 For the amplifier in Example 10.8, find the values of A_f, R_{in}, and R_{out} when the value of μ is 10 times lower, that is when $\mu = 100$.
Ans. –9.91 A/A; 900 Ω; 100 MΩ

10.17 If in the circuit in Fig. 10.31(a), R_2 is short-circuited, find the ideal value of A_f. For the case $R_s = R_{id} = \infty$, give expressions for R_i, R_o, A, β, A_f, R_{in}, and R_{out}
Ans. $A_f = 1$ A/A; $R_i = R_1$; $R_o = r_{o2}$, $A = -\mu g_m R_1$; $\beta = -1$; $A_f = \mu g_m R_1/(1+\mu g_m R_1)$; $R_{in} = 1/\mu g_m$; $R_{out} \simeq \mu(g_m r_{o2})R_1$.

10.8 Summary of the Feedback Analysis Method

Table 10.1 provides a summary of the rules and relationships employed in the analysis and design of the four types of feedback amplifier. In addition to the wealth of information in Table 10.1, we offer the following important analysis tips.

1. Always begin the analysis by determining an approximate value for the closed-loop gain A_f, assuming that the loop gain $A\beta$ is large and thus

$$A_f \simeq 1/\beta$$

   This value should serve as a check on the final value you find for A_f. How close the actual A_f is to the approximate value will depend on how large $A\beta$ is compared to unity.
2. The shunt connection at input or output always results in reducing the corresponding resistance (input or output). The series connection at input or output always results in increasing the corresponding resistance (input or output).
3. In utilizing negative feedback to improve the properties of an amplifier under design, the starting point in the design is the selection of the feedback topology appropriate for the application at hand. Then the required amount of negative feedback $(1 + A\beta)$ can be ascertained utilizing the fact that it is this quantity that determines the magnitude of improvement in the various amplifier parameters. Also, the feedback factor β can be determined from

$$\beta \simeq 1/A_f$$

10.9 Determining the Loop Gain

We have already seen that the loop gain $A\beta$ is a very important quantity that characterizes a feedback loop. Furthermore, in the following sections it will be shown that $A\beta$ determines whether the feedback amplifier is stable (as opposed to oscillatory). In this section, we shall describe an alternative approach to the determination of loop gain.

Table 10.1 Summary of Relationships for the Four Feedback-Amplifier Topologies

Feedback Amplifier	Feedback Topology	x_i	x_o	x_f	x_s	A	β	A_f	Source Form	Loading of Feedback Network is Obtained: At Input	Loading of Feedback Network is Obtained: At Output	To Find β, Apply to Port 2 of Feedback Network	R_{if}	R_{of}	Refer to Figs.
Voltage	Series–shunt	V_i	V_o	V_f	V_s	$\frac{V_o}{V_i}$	$\frac{V_f}{V_o}$	$\frac{V_o}{V_s}$	Thévenin	By short-circuiting port 2 of feedback network	By open-circuiting port 1 of feedback network	a voltage, and find the open-circuit voltage at port 1	$R_i(1+A\beta)$	$\frac{R_o}{1+A\beta}$	10.6 10.12 10.14 10.15
Current	Shunt–series	I_i	I_o	I_f	I_s	$\frac{I_o}{I_i}$	$\frac{I_f}{I_o}$	$\frac{I_o}{I_s}$	Norton	By open-circuiting port 2 of feedback network	By short-circuiting port 1 of feedback network	a current, and find the short-circuit current at port 1	$\frac{R_i}{1+A\beta}$	$R_o(1+A\beta)$	10.8(a) 10.28 10.29 10.30
Transconductance	Series–series	V_i	I_o	V_f	V_s	$\frac{I_o}{V_i}$	$\frac{V_f}{I_o}$	$\frac{I_o}{V_s}$	Thévenin	By open-circuiting port 2 of feedback network	By open-circuiting port 1 of feedback network	a current, and find the open-circuit voltage at port 1	$R_i(1+A\beta)$	$R_o(1+A\beta)$	10.10(a) 10.18 10.20 10.21
Transresistance	Shunt–shunt	I_i	V_o	I_f	I_s	$\frac{V_o}{I_i}$	$\frac{I_f}{V_o}$	$\frac{V_o}{I_s}$	Norton	By short-circuiting port 2 of feedback network	By short-circuiting port 1 of feedback network	a voltage, and find the short-circuit current at port 1	$\frac{R_i}{1+A\beta}$	$\frac{R_o}{1+A\beta}$	10.11(a) 10.24 10.25 10.26

10.9.1 An Alternative Approach for Finding Aβ

First, consider again the general feedback amplifier shown in Fig. 10.1. Let the external source x_s be set to zero. Open the feedback loop by breaking the connection of x_o to the feedback network and apply a test signal x_t. We see that the signal at the output of the feedback network is $x_f = \beta x_t$; that at the input of the basic amplifier is $x_i = -\beta x_t$; and the signal at the output of the amplifier, where the loop was broken, will be $x_o = -A\beta x_t$. It follows that the loop gain $A\beta$ is given by the negative of the ratio of the *returned* signal to the applied test signal; that is, $A\beta = -x_o/x_t$. It should also be obvious that this applies regardless of where the loop is broken.

However, in breaking the feedback loop of a practical amplifier circuit, we must ensure that the conditions that existed prior to breaking the loop do not change. This is achieved by terminating the loop where it is opened with an impedance equal to that seen before the loop was broken. To be specific, consider the conceptual feedback loop shown in Fig. 10.32(a). If we break the loop at XX′, and apply a test voltage V_t to the terminals thus created to the left of XX′, the terminals at the right of XX′ should be loaded with an impedance Z_t as shown in Fig. 10.32(b). The impedance Z_t is equal to that previously seen looking to the left of XX′. The loop gain $A\beta$ is then determined from

$$A\beta = -\frac{V_r}{V_t} \tag{10.78}$$

Finally, it should be noted that in some cases it may be convenient to determine $A\beta$ by applying a test current I_t and finding the returned current signal I_r. In this case, $A\beta = -I_r/I_t$.

An alternative equivalent method for determining $A\beta$ (see Rosenstark, 1986) that is usually convenient to employ especially in SPICE simulations is as follows: As before, the loop is broken at a convenient point. Then the open-circuit voltage transfer function T_{oc} is determined as indicated in Fig. 10.32(c), and the short-circuit current transfer function T_{sc} is determined as shown in Fig. 10.32(d). These two transfer functions are then combined to obtain the loop gain $A\beta$,

$$A\beta = -1\bigg/\left(\frac{1}{T_{oc}} + \frac{1}{T_{sc}}\right) \tag{10.79}$$

This method is particularly useful when it is not easy to determine the termination impedance Z_t.

To illustrate the process of determining loop gain, we consider the feedback loop shown in Fig. 10.33(a). This feedback loop represents both the inverting and the noninverting op-amp configurations. Using a simple equivalent-circuit model for the op amp, we obtain the circuit of Fig. 10.33(b). Examination of this circuit reveals that a convenient place to break the loop is at the input terminals of the op amp. The loop, broken in this manner, is shown in Fig. 10.33(c) with a test signal V_t applied to the right-hand-side terminals and a resistance R_{id} terminating the left-hand-side terminals. The returned voltage V_r is found by inspection as

$$V_r = -\mu V_1 \frac{\{R_L \parallel [R_2 + R_1 \parallel (R_{id}+R)]\}}{\{R_L \parallel [R_2 + R_1 \parallel (R_{id}+R)]\} + r_o} \frac{[R_1 \parallel (R_{id}+R)]}{[R_1 \parallel (R_{id}+R)] + R_2} \frac{R_{id}}{R_{id}+R} \tag{10.80}$$

This equation can be used directly to find the loop gain $L = A\beta = -V_r/V_t = -V_r/V_1$.

Since the loop gain L is generally a function of frequency, it is usual to call it **loop transmission** and to denote it by $L(s)$ or $L(j\omega)$.

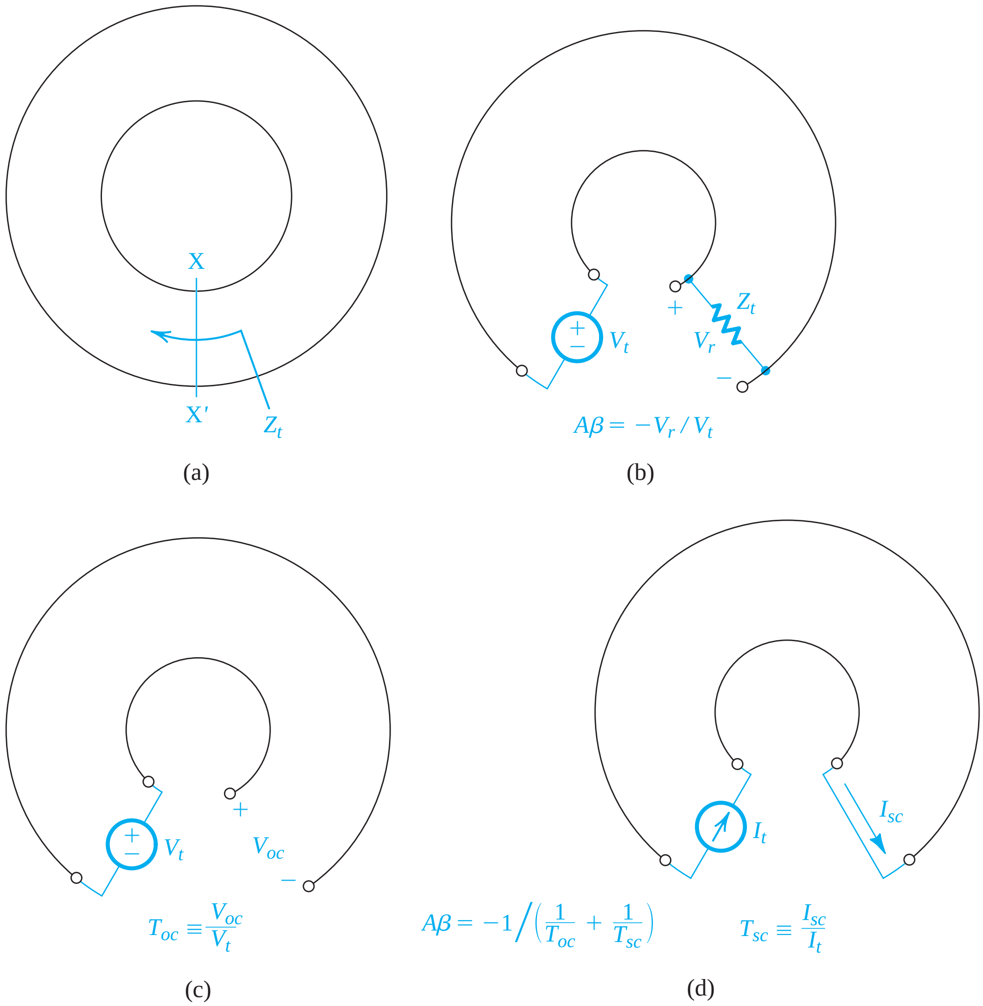


Figure 10.32 A conceptual feedback loop is broken at XX′ and a test voltage V_t is applied. The impedance Z_t is equal to that previously seen looking to the left of XX′. The loop gain $A\beta = -V_r/V_t$, where V_r is the *returned* voltage. As an alternative, $A\beta$ can be determined by finding the open-circuit transfer function T_{oc}, as in **(c)**, and the short-circuit transfer function T_{sc}, as in **(d)**, and combining them as indicated.

Finally, we note that the value of the loop gain determined using the method discussed here may differ somewhat from the value determined by the approach studied in the previous sections. The difference stems from the approximations made in the feedback analysis method utilized in the previous sections. However, as the reader will find by solving the end-of-chapter problems, the difference is usually limited to a few percent.

10.9.2 Equivalence of Circuits from a Feedback-Loop Point of View

From the study of circuit theory we know that the poles of a circuit are independent of the external excitation. In fact the poles, or the natural modes (which is a more appropriate

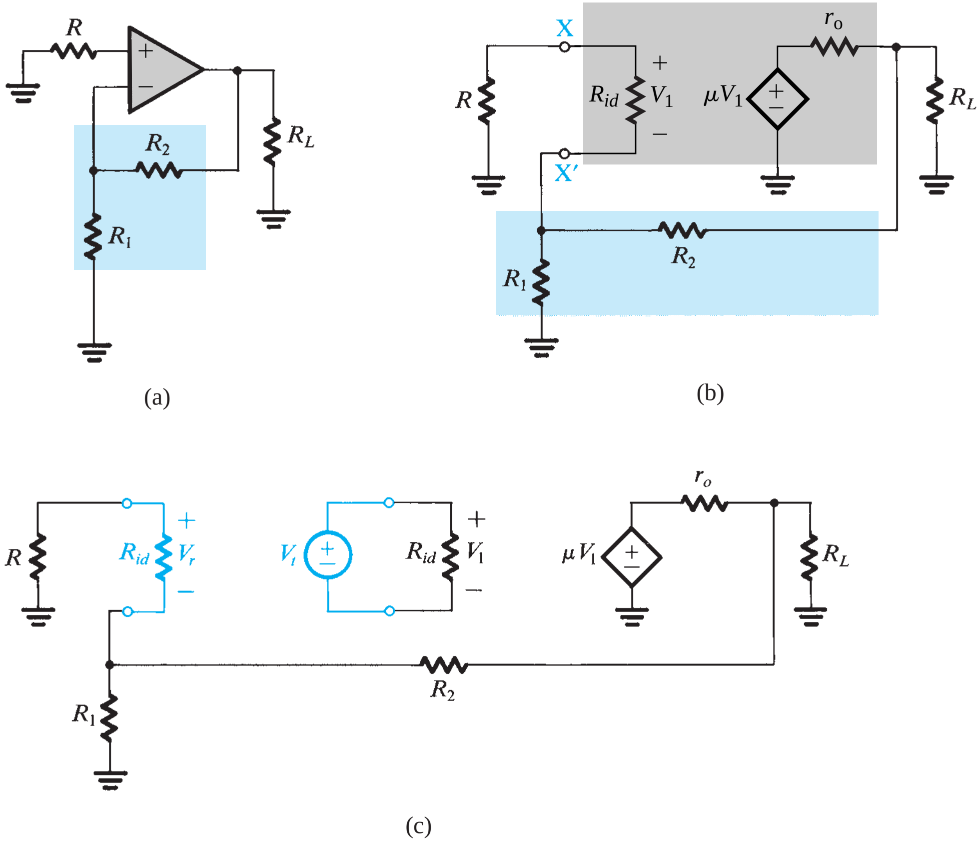


Figure 10.33 (**a**) A feedback loop that represents both the inverting and the noninverting op-amp configurations; (**b**) equivalent circuit; (**c**) determination of the loop gain.

name), can be determined by setting the external excitation to zero. It follows that the poles of a feedback amplifier depend only on the feedback loop. This will be confirmed in a later section, where we show that the **characteristic equation** (whose roots are the poles) is completely determined by the loop gain. Thus, a given feedback loop may be used to generate a number of circuits having the same poles but different **transmission zeros**. The closed-loop gain and the transmission zeros depend on how and where the input signal is injected into the loop.

As an example, return to the feedback loop of Fig. 10.33(a). This loop can be used to generate the noninverting op-amp circuit by feeding the input voltage signal to the terminal of R that is connected to ground; that is, we lift this terminal off ground and connect it to V_s. The same feedback loop can be used to generate the inverting op-amp circuit by feeding the input voltage signal to the terminal of R_1 that is connected to ground.

Recognition of the fact that two or more circuits are equivalent from a feedback-loop point of view is very useful because (as will be shown in Section 10.10) stability is a function of the loop. Thus one needs to perform the stability analysis only once for a given loop.

In Chapter 16 we shall employ the concept of **loop equivalence** in the synthesis of active filters.

EXERCISES

10.18 Find the loop gain $A\beta$ for the feedback amplifier in Fig. 10.17 (Example 10.4). Set $V_s = 0$, break the loop at the gate of Q_2, apply a voltage V_t to the gate of Q_2, and determine the returned voltage V_r at the drain of Q_1. Evaluate the expression for $A\beta$ for the values given in Example 10.4 and compare to the value obtained in Example 10.4. Neglect r_{o1} and r_{o2}.

Ans. $A\beta = \dfrac{g_{m2}R_{D2}}{R_{D2} + R_2 + \left(R_1 \| \dfrac{1}{g_{m2}}\right)} \dfrac{R_1 R_{D1}}{R_1 + \dfrac{1}{g_{m1}}}$; 16.67 (compared to 17.39 obtained in Example 10.4)

10.19 Find the loop gain $A\beta$ for the feedback amplifier in Fig. E10.15 (Exercise 10.15). Set $I_s = 0$, break the loop at the gate of Q, apply a voltage V_t to the gate of Q, and determine the voltage V_r that appears across R_s. Find the value of $A\beta$ using the component values given in Exercise 10.15, and compare to the value given in the answer to Exercise 10.15.

Ans. $A\beta = \dfrac{g_m r_o R_s}{r_o + R_f + R_s}$; 3.22 (compared to 3.03 obtained in Exercise 10.15)

10.10 The Stability Problem

10.10.1 Transfer Function of the Feedback Amplifier

In a feedback amplifier such as that represented by the general structure of Fig. 10.1, the open-loop gain A is generally a function of frequency, and it should therefore be more accurately called the **open-loop transfer function**, $A(s)$. Also, we have been assuming for the most part that the feedback network is resistive and hence that the feedback factor β is constant, but this need not be always the case. We shall therefore assume that in the general case the **feedback transfer function** is $\beta(s)$. It follows that the **closed-loop transfer function** $A_f(s)$ is given by

$$A_f(s) = \frac{A(s)}{1 + A(s)\beta(s)} \tag{10.81}$$

To focus attention on the points central to our discussion in this section, we shall assume that the amplifier is direct coupled with constant dc gain A_0 and with poles and zeros occurring in the high-frequency band. Also, for the time being let us assume that at low frequencies $\beta(s)$ reduces to a constant value. Thus at low frequencies the loop gain $A(s)\beta(s)$ becomes a constant, which should be a positive number; otherwise the feedback would not be negative. The question then is: What happens at higher frequencies?

For physical frequencies $s = j\omega$, Eq. (10.81) becomes

$$A_f(j\omega) = \frac{A(j\omega)}{1 + A(j\omega)\beta(j\omega)} \tag{10.82}$$

Thus the loop gain $A(j\omega)\beta(j\omega)$ is a complex number that can be represented by its magnitude and phase,

$$\begin{aligned} L(j\omega) &\equiv A(j\omega)\beta(j\omega) \\ &= |A(j\omega)\beta(j\omega)|e^{j\phi(\omega)} \end{aligned} \tag{10.83}$$

It is the manner in which the loop gain varies with frequency that determines the stability or instability of the feedback amplifier. To appreciate this fact, consider the frequency at which the phase angle $\phi(\omega)$ becomes 180°. At this frequency, ω_{180}, the loop gain $A(j\omega)\beta(j\omega)$ will be a real number with a negative sign. Thus at this frequency the feedback will become positive. If at $\omega = \omega_{180}$ the magnitude of the loop gain is less than unity, then from Eq. (10.82) we see that the closed-loop gain $A_f(j\omega)$ will be greater than the open-loop gain $A(j\omega)$, since the denominator of Eq. (10.82) will be smaller than unity. Nevertheless, the feedback amplifier will be stable.

On the other hand, if at the frequency ω_{180} the magnitude of the loop gain is equal to unity, it follows from Eq. (10.82) that $A_f(j\omega)$ will be infinite. This means that the amplifier will have an output for zero input; this is by definition an **oscillator**. To visualize how this feedback loop may oscillate, consider the general loop of Fig. 10.1 with the external input x_s set to zero. Any disturbance in the circuit, such as the closure of the power-supply switch, will generate a signal $x_i(t)$ at the input to the amplifier. Such a noise signal usually contains a wide range of frequencies, and we shall now concentrate on the component with frequency $\omega = \omega_{180}$, that is, the signal $X_i \sin(\omega_{180} t)$. This input signal will result in a feedback signal given by

$$X_f = A(j\omega_{180})\beta(j\omega_{180})X_i = -X_i$$

Since X_f is further multiplied by –1 in the summer block at the input, we see that the feedback causes the signal X_i at the amplifier input to be *sustained*. That is, from this point on, there will be sinusoidal signals at the amplifier input and output of frequency ω_{180}. Thus the amplifier is said to oscillate at the frequency ω_{180}.

The question now is: What happens if at ω_{180} the magnitude of the loop gain is greater than unity? We shall answer this question, not in general, but for the restricted yet very important class of circuits in which we are interested here. The answer, which is not obvious from Eq. (10.82), is that the circuit will oscillate, and the oscillations will grow in amplitude until some nonlinearity (which is always present in some form) reduces the magnitude of the loop gain to exactly unity, at which point sustained oscillations will be obtained. This mechanism for starting oscillations by using positive feedback with a loop gain greater than unity, and then using a nonlinearity to reduce the loop gain to unity at the desired amplitude, will be exploited in the design of sinusoidal oscillators in Chapter 17. Our objective here is just the opposite: Now that we know how oscillations could occur in a negative-feedback amplifier, we wish to find methods to prevent their occurrence.

10.10.2 The Nyquist Plot

The Nyquist plot is a formalized approach for testing for stability based on the discussion above. It is simply a polar plot of loop gain with frequency used as a parameter. Figure 10.34 shows such a plot. Note that the radial distance is $|A\beta|$ and the angle is the phase angle ϕ. The solid-line plot is for positive frequencies. Since the loop gain—and for that matter any gain function of a physical network—has a magnitude that is an even function of frequency and a phase that is an odd function of frequency, the $A\beta$ plot for negative frequencies (shown in Fig. 10.34 as a broken line) can be drawn as a mirror image through the Re axis.

The Nyquist plot intersects the negative real axis at the frequency ω_{180}. Thus, if this intersection occurs to the left of the point (–1, 0), we know that the magnitude of loop gain at this frequency is greater than unity and the amplifier will be unstable. On the other hand, if the intersection occurs to the right of the point (–1, 0) the amplifier will be stable. It follows that if the Nyquist plot *encircles* the point (–1, 0) then the amplifier will be

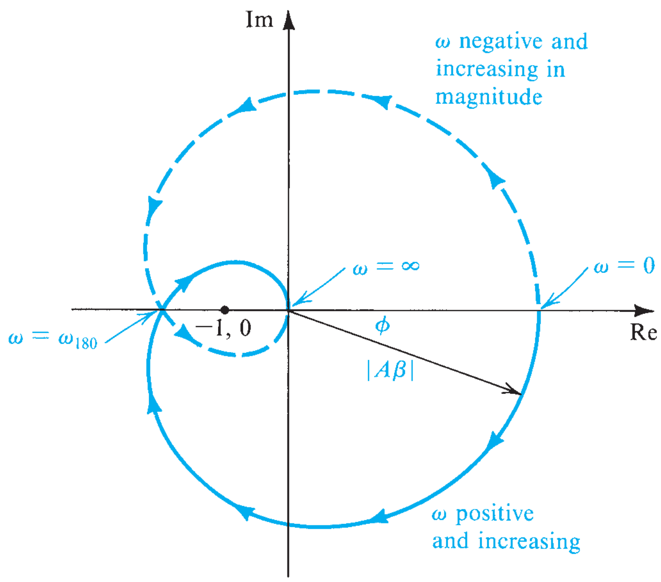


Figure 10.34 The Nyquist plot of an unstable amplifier.

unstable. It should be mentioned, however, that this statement is a simplified version of the **Nyquist criterion**; nevertheless, it applies to all the circuits in which we are interested. For the full theory behind the Nyquist method and for details of its application, consult Haykin (1970).

EXERCISE

10.20 Consider a feedback amplifier for which the open-loop transfer function $A(s)$ is given by

$$A(s) = \left(\frac{10}{1 + s/10^4}\right)^3$$

Let the feedback factor β be a constant independent of frequency. Find the frequency ω_{180} at which the phase shift is 180°. Then, show that the feedback amplifier will be stable if the feedback factor β is less than a critical value β_{cr} and unstable if $\beta \geq \beta_{cr}$, and find the value of β_{cr}.

Ans. $\omega_{180} = \sqrt{3} \times 10^4$ rad/s; $\beta_{cr} = 0.008$

10.11 Effect of Feedback on the Amplifier Poles

The amplifier frequency response and stability are determined directly by its poles. Therefore we shall investigate the effect of feedback on the poles of the amplifier.[6]

[6] For a brief review of poles and zeros and related concepts, refer to Appendix F.

10.11.1 Stability and Pole Location

We shall begin by considering the relationship between stability and pole location. For an amplifier or any other system to be stable, its poles should lie in the left half of the s plane. A pair of complex-conjugate poles on the $j\omega$ axis gives rise to sustained sinusoidal oscillations. Poles in the right half of the s plane give rise to growing oscillations.

To verify the statement above, consider an amplifier with a pole pair at $s = \sigma_0 \pm j\omega_n$. If this amplifier is subjected to a disturbance, such as that caused by closure of the power-supply switch, its transient response will contain terms of the form

$$v(t) = e^{\sigma_0 t}[e^{+j\omega_n t} + e^{-j\omega_n t}] = 2e^{\sigma_0 t}\cos(\omega_n t) \tag{10.84}$$

This is a sinusoidal signal with an envelope $e^{\sigma_0 t}$. Now if the poles are in the left half of the s plane, then σ_0 will be negative and the oscillations will decay exponentially toward zero, as shown in Fig. 10.35(a), indicating that the system is stable. If, on the other hand, the poles are in

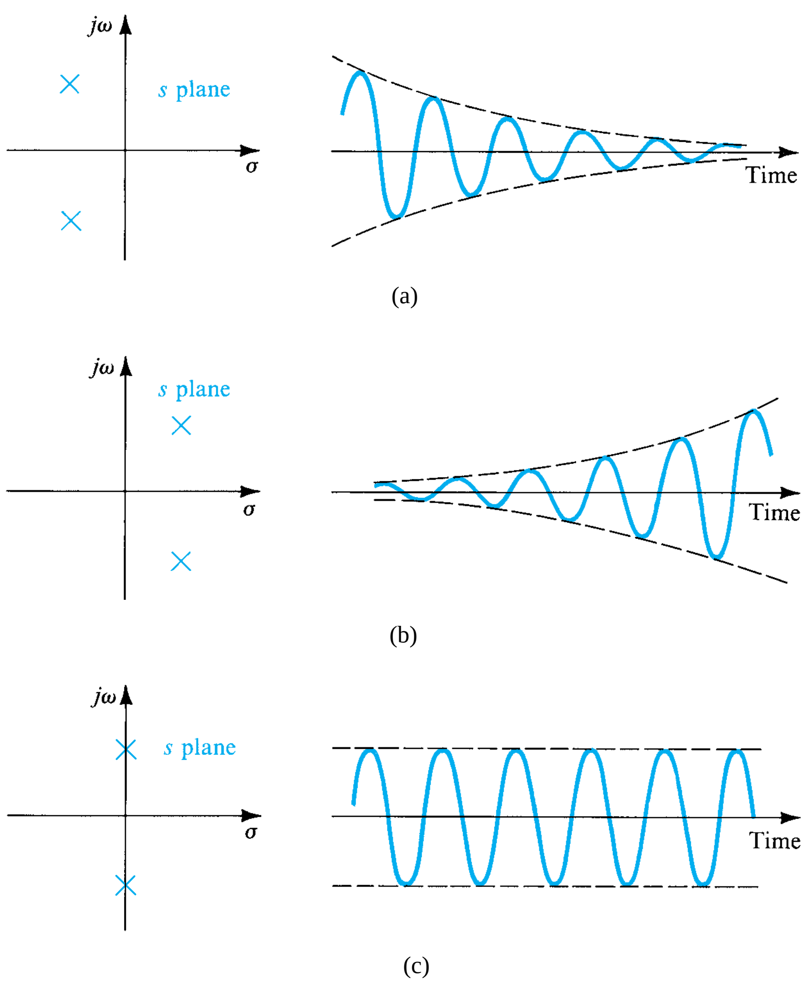


Figure 10.35 Relationship between pole location and transient response.

the right half-plane, then σ_0 will be positive, and the oscillations will grow exponentially (until some nonlinearity limits their growth), as shown in Fig. 10.35(b). Finally, if the poles are on the $j\omega$ axis, then σ_0 will be zero and the oscillations will be sustained, as shown in Fig. 10.35(c).

Although the discussion above is in terms of complex-conjugate poles, it can be shown that the existence of any right-half-plane poles results in instability.

10.11.2 Poles of the Feedback Amplifier

From the closed-loop transfer function in Eq. (10.81), we see that the poles of the feedback amplifier are the zeros of $1 + A(s)\beta(s)$. That is, the feedback-amplifier poles are obtained by solving the equation

$$1 + A(s)\beta(s) = 0 \tag{10.85}$$

which is called the **characteristic equation** of the feedback loop. It should therefore be apparent that applying feedback to an amplifier changes its poles.

In the following, we shall consider how feedback affects the amplifier poles. For this purpose we shall assume that the open-loop amplifier has real poles and no finite zeros (i.e., all the zeros are at $s = \infty$). This will simplify the analysis and enable us to focus our attention on the fundamental concepts involved. We shall also assume that the feedback factor β is independent of frequency.

10.11.3 Amplifier with a Single-Pole Response

Consider first the case of an amplifier whose open-loop transfer function is characterized by a single pole:

$$A(s) = \frac{A_0}{1 + s/\omega_P} \tag{10.86}$$

The closed-loop transfer function is given by

$$A_f(s) = \frac{A_0/(1 + A_0\beta)}{1 + s/\omega_P(1 + A_0\beta)} \tag{10.87}$$

Thus the feedback moves the pole along the negative real axis to a frequency ω_{Pf},

$$\omega_{Pf} = \omega_P(1 + A_0\beta) \tag{10.88}$$

This process is illustrated in Fig. 10.36(a). Figure 10.36(b) shows Bode plots for $|A|$ and $|A_f|$. Note that while at low frequencies the difference between the two plots is $20\log(1 + A_0\beta)$, the two curves coincide at high frequencies. One can show that this indeed is the case by approximating Eq. (10.87) for frequencies $\omega \gg \omega_P(1 + A_0\beta)$:

$$A_f(s) \simeq \frac{A_0\omega_P}{s} \simeq A(s) \tag{10.89}$$

Physically speaking, at such high frequencies the loop gain is much smaller than unity and the feedback is ineffective.

Figure 10.36(b) clearly illustrates the fact that applying negative feedback to an amplifier results in extending its bandwidth at the expense of a reduction in gain. Since the pole of the closed-loop amplifier never enters the right half of the s plane, the single-pole amplifier is stable for any value of β. Thus this amplifier is said to be **unconditionally stable**. This

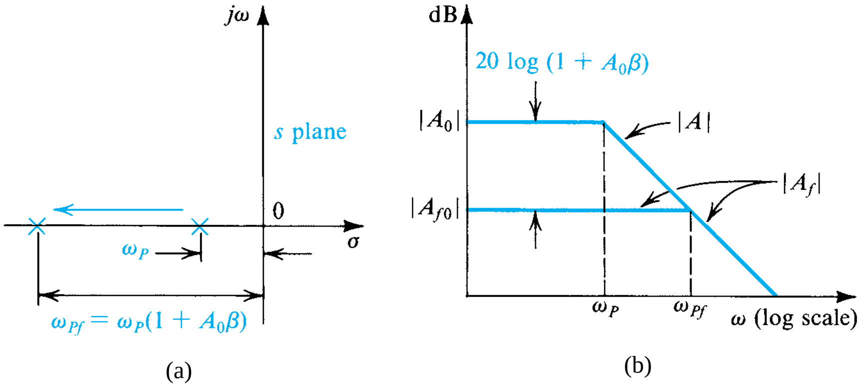


Figure 10.36 Effect of feedback on **(a)** the pole location and **(b)** the frequency response of an amplifier having a single-pole, open-loop response.

result, however, is hardly surprising, since the phase lag associated with a single-pole response can never be greater than 90°. Thus the loop gain never achieves the 180° phase shift required for the feedback to become positive.

EXERCISE

10.21 An op amp having a single-pole rolloff at 100 Hz and a low-frequency gain of 10^5 is operated in a feedback loop with $\beta = 0.01$. What is the factor by which feedback shifts the pole? To what frequency? If β is changed to a value that results in a closed-loop gain of +1, to what frequency does the pole shift?
Ans. 1001; 100.1 kHz; 10 MHz

10.11.4 Amplifier with Two-Pole Response

Consider next an amplifier whose open-loop transfer function is characterized by two real-axis poles:

$$A(s) = \frac{A_0}{(1 + s/\omega_{P1})(1 + s/\omega_{P2})} \tag{10.90}$$

In this case, the closed-loop poles are obtained from $1 + A(s)\beta = 0$, which leads to

$$s^2 + s(\omega_{P1} + \omega_{P2}) + (1 + A_0\beta)\omega_{P1}\omega_{P2} = 0 \tag{10.91}$$

Thus the closed-loop poles are given by

$$s = -\frac{1}{2}(\omega_{P1} + \omega_{P2}) \pm \frac{1}{2}\sqrt{(\omega_{P1} + \omega_{P2})^2 - 4(1 + A_0\beta)\omega_{P1}\omega_{P2}} \tag{10.92}$$

From Eq. (10.92) we see that as the loop gain $A_0\beta$ is increased from zero, the poles are brought closer together. Then a value of loop gain is reached at which the poles become coincident. If the loop gain is further increased, the poles become complex conjugate and move along a vertical line. Figure 10.37 shows the locus of the poles for increasing loop gain. This plot is called a **root-locus diagram**, where "root" refers to the fact that the poles are the roots of the characteristic equation.

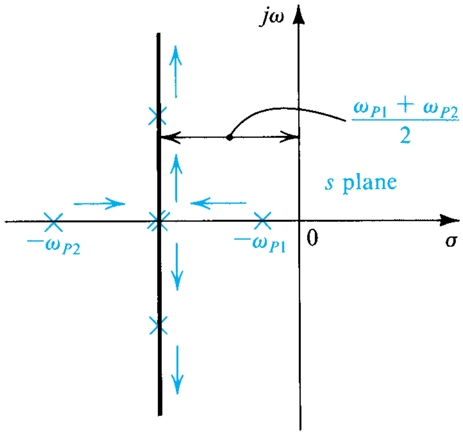


Figure 10.37 Root-locus diagram for a feedback amplifier whose open-loop transfer function has two real poles.

From the root-locus diagram of Fig. 10.37 we see that this feedback amplifier also is unconditionally stable. Again, this result should come as no surprise; the maximum phase shift of $A(s)$ in this case is 180° (90° per pole), but this value is reached at $\omega = \infty$. Thus there is no finite frequency at which the phase shift reaches 180°.

Another observation to make on the root-locus diagram of Fig. 10.37 is that the open-loop amplifier might have a dominant pole, but this is not necessarily the case for the closed-loop amplifier. The response of the closed-loop amplifier can, of course, always be plotted once the poles have been found from Eq. (10.92). As is the case with second-order responses generally, the closed-loop response can show a peak (see Chapter 16). To be more specific, the characteristic equation of a second-order network can be written in the standard form

$$s^2 + s\frac{\omega_0}{Q} + \omega_0^2 = 0 \tag{10.93}$$

where ω_0 is called the **pole frequency** and Q is called **pole *Q* factor**. The poles are complex if Q is greater than 0.5. A geometric interpretation for ω_0 and Q of a pair of complex-conjugate poles is given in Fig. 10.38, from which we note that ω_0 is the radial distance of the poles from the origin and that Q indicates the distance of the poles from the $j\omega$ axis. Poles on the $j\omega$ axis have $Q = \infty$.

By comparing Eqs. (10.91) and (10.93) we obtain the Q factor for the poles of the feedback amplifier as

$$Q = \frac{\sqrt{(1 + A_0\beta)\,\omega_{P1}\omega_{P2}}}{\omega_{P1} + \omega_{P2}} \tag{10.94}$$

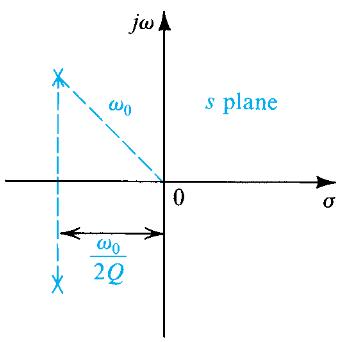


Figure 10.38 Definition of ω_0 and Q of a pair of complex-conjugate poles.

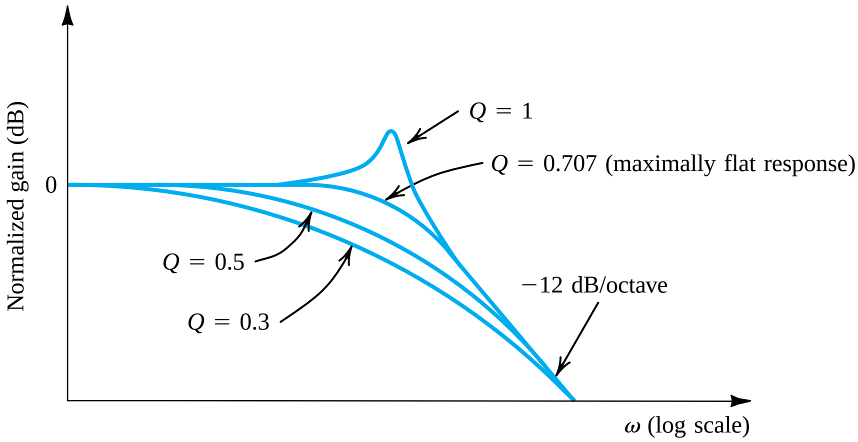


Figure 10.39 Normalized gain of a two-pole feedback amplifier for various values of Q. Note that Q is determined by the loop gain according to Eq. (10.94).

From the study of second-order network responses in Chapter 16, it will be seen that the response of the feedback amplifier under consideration shows no peaking for $Q \le 0.707$. The boundary case corresponding to $Q = 0.707$ (poles at 45° angles) results in the **maximally flat** response. Figure 10.39 shows a number of possible responses obtained for various values of Q (or, correspondingly, various values of $A_0\beta$).

EXERCISE

10.22 An amplifier with a low-frequency gain of 100 and poles at 10^4 rad/s and 10^6 rad/s is incorporated in a negative-feedback loop with feedback factor β. For what value of β do the poles of the closed-loop amplifier coincide? What is the corresponding Q of the resulting second-order system? For what value of β is a maximally flat response achieved? What is the low-frequency closed-loop gain in the maximally flat case?

Ans. 0.245; 0.5; 0.5; 1.96 V/V

Example 10.9

As an illustration of some of the ideas just discussed, we consider the positive-feedback circuit shown in Fig. 10.40(a). Find the loop transmission $L(s)$ and the characteristic equation. Sketch a root-locus diagram for varying K, and find the value of K that results in a maximally flat response and the value of K that makes the circuit oscillate. Assume that the amplifier has frequency-idependent gain, infinite input impedance, and zero output impedance.

Solution

To obtain the loop transmission, we short-circuit the signal source and break the loop at the amplifier input. We then apply a test voltage V_t and find the returned voltage V_r, as indicated in Fig. 10.40(b). The

Example 10.9 *continued*

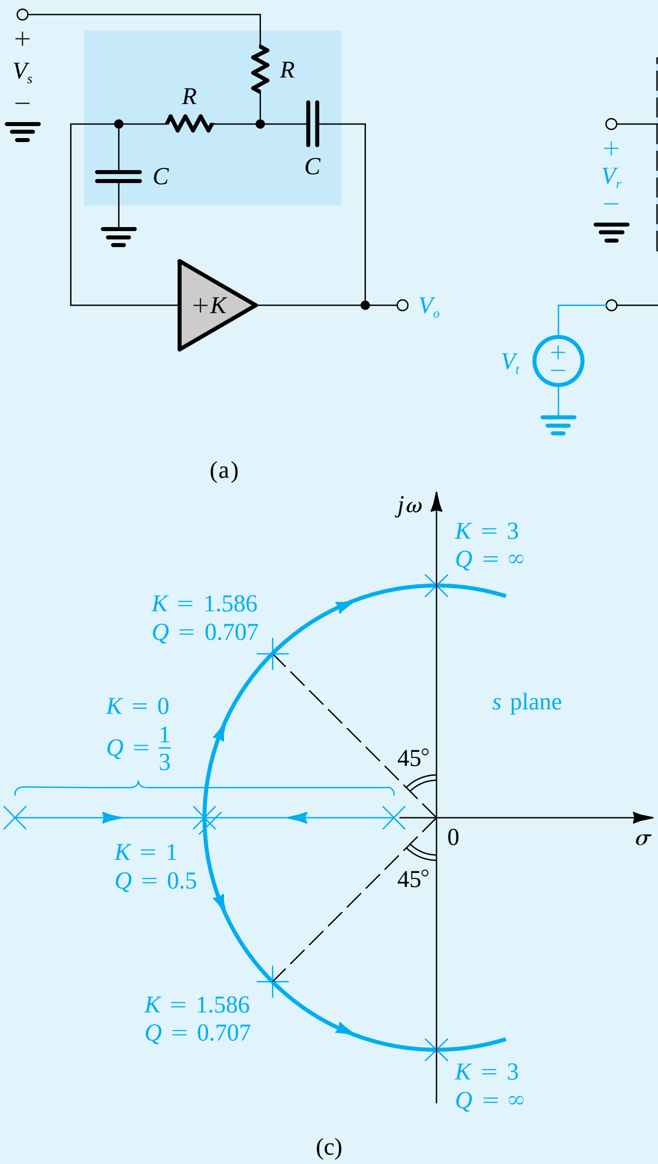


Figure 10.40 Circuits and plot for Example 10.9.

loop transmission $L(s) \equiv A(s)\beta(s)$ is given by

$$L(s) = -\frac{V_r}{V_t} = -KT(s) \tag{10.95}$$

where $T(s)$ is the transfer function of the two-port RC network shown inside the broken-line box in Fig. 10.40(b):

$$T(s) \equiv \frac{V_r}{V_1} = \frac{s(1/CR)}{s^2 + s(3/CR) + (1/CR)^2} \tag{10.96}$$

Thus,

$$L(s) = \frac{-s(K/CR)}{s^2 + s(3/CR) + (1/CR)^2} \tag{10.97}$$

The characteristic equation is

$$1 + L(s) = 0 \tag{10.98}$$

that is,

$$s^2 + s\frac{3}{CR} + \left(\frac{1}{CR}\right)^2 - s\frac{K}{CR} = 0$$

$$s^2 + s\frac{3-K}{CR} + \left(\frac{1}{CR}\right)^2 = 0 \tag{10.99}$$

By comparing this equation to the standard form of the second-order characteristic equation (Eq. 10.93) we see that the pole frequency ω_0 is given by

$$\omega_0 = \frac{1}{CR} \tag{10.100}$$

and the Q factor is

$$Q = \frac{1}{3-K} \tag{10.101}$$

Thus for $K = 0$, the poles have $Q = \frac{1}{3}$ and are therefore located on the negative real axis. As K is increased, the poles are brought closer together and eventually coincide ($Q = 0.5$, $K = 1$). Further increasing K results in the poles becoming complex and conjugate. The root locus is then a circle because the radial distance ω_0 remains constant (Eq. 10.100) independent of the value of K.

The maximally flat response is obtained when $Q = 0.707$, which results when $K = 1.586$. In this case the poles are at 45° angles, as indicated in Fig. 10.40(c). The poles cross the $j\omega$ axis into the right half of the s plane at the value of K that results in $Q = \infty$, that is, $K = 3$. Thus for $K \geq 3$ this circuit becomes unstable. This might appear to contradict our earlier conclusion that the feedback amplifier with a second-order response is unconditionally stable. Note, however, that the circuit in this example is quite different from the negative-feedback amplifier that we have been studying. Here we have an amplifier with a positive gain K and a feedback network whose transfer function $T(s)$ is frequency dependent. This feedback is in fact *positive,* and the circuit will oscillate at the frequency for which the phase of $T(j\omega)$ is zero.

Example 10.9 illustrates the use of feedback (positive feedback in this case) to move the poles of an RC network from their negative real-axis locations to complex-conjugate locations. One can accomplish the same task using negative feedback, as the root-locus diagram of Fig. 10.37 demonstrates. The process of pole control is the essence of *active-filter design,* as will be discussed in Chapter 16.

10.11.5 Amplifiers with Three or More Poles

Figure 10.41 shows the root-locus diagram for a feedback amplifier whose open-loop response is characterized by three poles. As indicated, increasing the loop gain from zero moves the highest-frequency pole outward while the two other poles are brought closer together. As $A_0\beta$ is increased further, the two poles become coincident and then become complex and conjugate. A value of $A_0\beta$ exists at which this pair of complex-conjugate poles enters the right half of the s plane, thus causing the amplifier to become unstable.

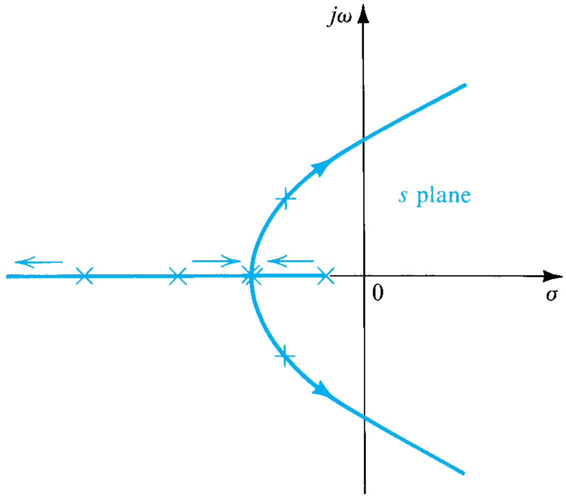


Figure 10.41 Root-locus diagram for an amplifier with three poles. The arrows indicate the pole movement as $A_0\beta$ is increased.

This result is not entirely unexpected, since an amplifier with three poles has a phase shift that reaches –270° as ω approaches ∞. Thus there exists a finite frequency, ω_{180}, at which the loop gain has 180° phase shift.

From the root-locus diagram of Fig. 10.41, we observe that one can always maintain amplifier stability by keeping the loop gain $A_0\beta$ smaller than the value corresponding to the poles entering the right half-plane. In terms of the Nyquist diagram, the critical value of $A_0\beta$ is that for which the diagram passes through the (–1, 0) point. Reducing $A_0\beta$ below this value causes the Nyquist plot to shrink and thus intersect the negative real axis to the right of the (–1, 0) point, indicating stable amplifier performance. On the other hand, increasing $A_0\beta$ above the critical value causes the Nyquist plot to expand, thus encircling the (–1, 0) point and indicating unstable performance.

For a given open-loop gain A_0 the conclusions above can be stated in terms of the feedback factor β. That is, there exists a *maximum value* for β above which the feedback amplifier becomes unstable. Alternatively, we can state that there exists a *minimum value* for the closed-loop gain A_{f0} below which the amplifier becomes unstable. To obtain lower values of closed-loop gain one needs therefore to alter the loop transfer function $L(s)$. This is the process known as *frequency compensation*. We shall study the theory and techniques of frequency compensation in Section 10.13.

Before leaving this section we point out that construction of the root-locus diagram for amplifiers having three or more poles as well as finite zeros is an involved process for which a systematic procedure exists. However, such a procedure will not be presented here, and the interested reader should consult Haykin (1970). Although the root-locus diagram provides the amplifier designer with considerable insight, other, simpler techniques based on Bode plots can be effectively employed, as will be explained in Section 10.12.

EXERCISE

10.23 Consider a feedback amplifier for which the open-loop transfer function $A(s)$ is given by

$$A(s) = \left(\frac{10}{1 + s/10^4}\right)^3$$

Let the feedback factor β be frequency independent. Find the closed-loop poles as functions of β, and show that the root locus is that of Fig. E10.23. Also find the value of β at which the amplifier becomes unstable. (*Note:* This is the same amplifier that was considered in Exercise 10.20.)

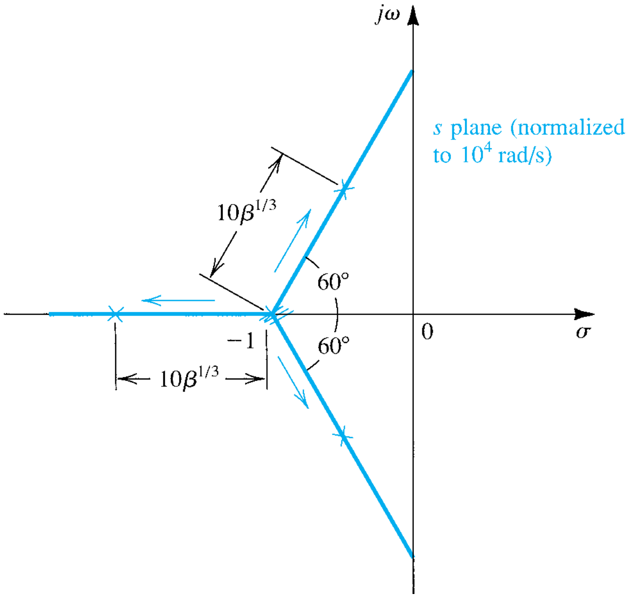


Figure 10.E23

Ans. See Fig. E10.23; $\beta_{critical} = 0.008$

10.12 Stability Study Using Bode Plots

10.12.1 Gain and Phase Margins

From Sections 10.10 and 10.11 we know that whether a feedback amplifier is or is not stable can be determined by examining its loop gain $A\beta$ as a function of frequency. One of the simplest and most effective means for doing this is through the use of a Bode plot for $A\beta$, such as the one shown in Fig. 10.42. (Note that because the phase approaches –360°, the network examined is a fourth-order one.) The feedback amplifier whose loop gain is plotted in Fig. 10.42 will be stable, since at the frequency of 180° phase shift, ω_{180}, the magnitude of the loop gain is less than unity (negative dB). The difference between the value of $|A\beta|$ at ω_{180} and unity, called the **gain margin**, is usually expressed in decibels. The gain margin represents the amount by which the loop gain can be increased while stability is maintained. Feedback amplifiers are usually designed to have sufficient gain margin to allow for the inevitable changes in loop gain with temperature, time, and so on.

Another way to investigate the stability and to express its degree is to examine the Bode plot at the frequency for which $|A\beta| = 1$, which is the point at which the magnitude plot crosses the 0-dB line. If at this frequency the phase angle is less (in magnitude) than 180°, then the amplifier is stable. This is the situation illustrated in Fig. 10.42. The difference between the phase angle at this frequency and 180° is termed the **phase margin**. On the other hand, if at the frequency of unity loop-gain magnitude, the phase lag is in excess of 180°, the amplifier will be unstable.

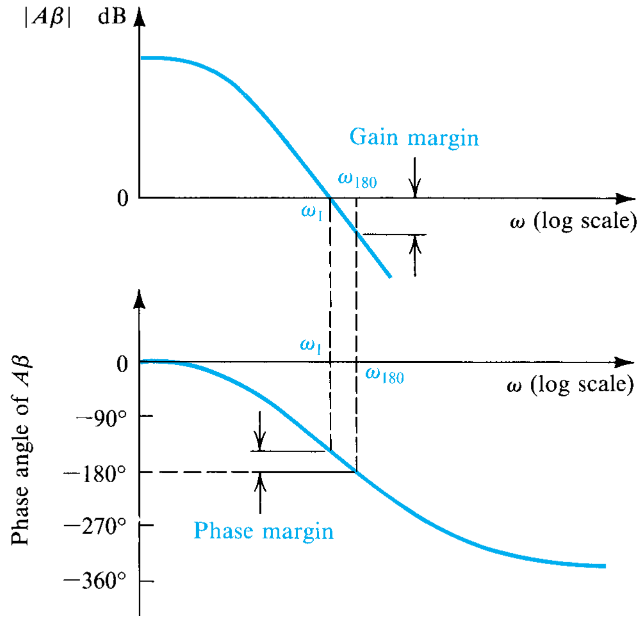


Figure 10.42 Bode plot for the loop gain $A\beta$ illustrating the definitions of the gain and phase margins.

EXERCISE

10.24 Consider an op amp having a single-pole, open-loop response with $A_0 = 10^5$ and $f_P = 10$ Hz. Let the op amp be ideal otherwise (infinite input impedance, zero output impedance, etc.). If this amplifier is connected in the noninverting configuration with a nominal low-frequency, closed-loop gain of 100, find the frequency at which $|A\beta| = 1$. Also, find the phase margin.
Ans. 10^4 Hz; 90°

10.12.2 Effect of Phase Margin on Closed-Loop Response

Feedback amplifiers are normally designed with a phase margin of at least 45°. The amount of phase margin has a profound effect on the shape of the closed-loop gain response. To see this relationship, consider a feedback amplifier with a large low-frequency loop gain, $A_0\beta \gg 1$. It follows that the closed-loop gain at low frequencies is approximately $1/\beta$. Denoting the frequency at which the magnitude of loop gain is unity by ω_1, we have (refer to Fig. 10.42)

$$A(j\omega_1)\beta = 1 \times e^{-j\theta} \tag{10.102a}$$

where

$$\theta = 180° - \text{phase margin} \tag{10.102b}$$

At ω_1 the closed-loop gain is

$$A_f(j\omega_1) = \frac{A(j\omega_1)}{1 + A(j\omega_1)\beta} \tag{10.103}$$

Substituting from Eq. (10.102a) gives

$$A_f(j\omega_1) = \frac{(1/\beta)e^{-j\theta}}{1 + e^{-j\theta}} \tag{10.104}$$

Thus the magnitude of the gain at ω_1 is

$$|A_f(j\omega_1)| = \frac{1/\beta}{|1 + e^{-j\theta}|} \tag{10.105}$$

For a phase margin of 45°, $\theta = 135°$; and we obtain

$$|A_f(j\omega_1)| = 1.3\frac{1}{\beta} \tag{10.106}$$

That is, the gain peaks by a factor of 1.3 above the low-frequency value of $1/\beta$. This peaking increases as the phase margin is reduced, eventually reaching ∞ when the phase margin is zero. Zero phase margin, of course, implies that the amplifier can sustain oscillations [poles on the $j\omega$ axis; Nyquist plot passing through (–1, 0)].

EXERCISE

10.25 Find the closed-loop gain at ω_1 relative to the low-frequency gain when the phase margin is 30°, 60°, and 90°.
Ans. 1.93; 1; 0.707

10.12.3 An Alternative Approach for Investigating Stability

Investigating stability by constructing Bode plots for the loop gain $A\beta$ can be a tedious and time-consuming process, especially if we have to investigate the stability of a given amplifier for a variety of feedback networks. An alternative approach, which is much simpler, is to construct a Bode plot for the open-loop gain $A(j\omega)$ only. Assuming for the time being that β is independent of frequency, we can plot $20 \log(1/\beta)$ as a horizontal straight line on the same plane used for $20 \log|A|$. The difference between the two curves will be

$$20 \log|A(j\omega)| - 20 \log\frac{1}{\beta} = 20 \log|A\beta| \tag{10.107}$$

which is the loop gain (in dB). We may therefore study stability by examining the difference between the two plots. If we wish to evaluate stability for a different feedback factor, we simply draw another horizontal straight line at the level $20 \log(1/\beta)$.

To illustrate, consider an amplifier whose open-loop transfer function is characterized by three poles. For simplicity let the three poles be widely separated—say, at 0.1 MHz, 1 MHz, and 10 MHz, as shown in Fig. 10.43. Note that because the poles are widely separated, the

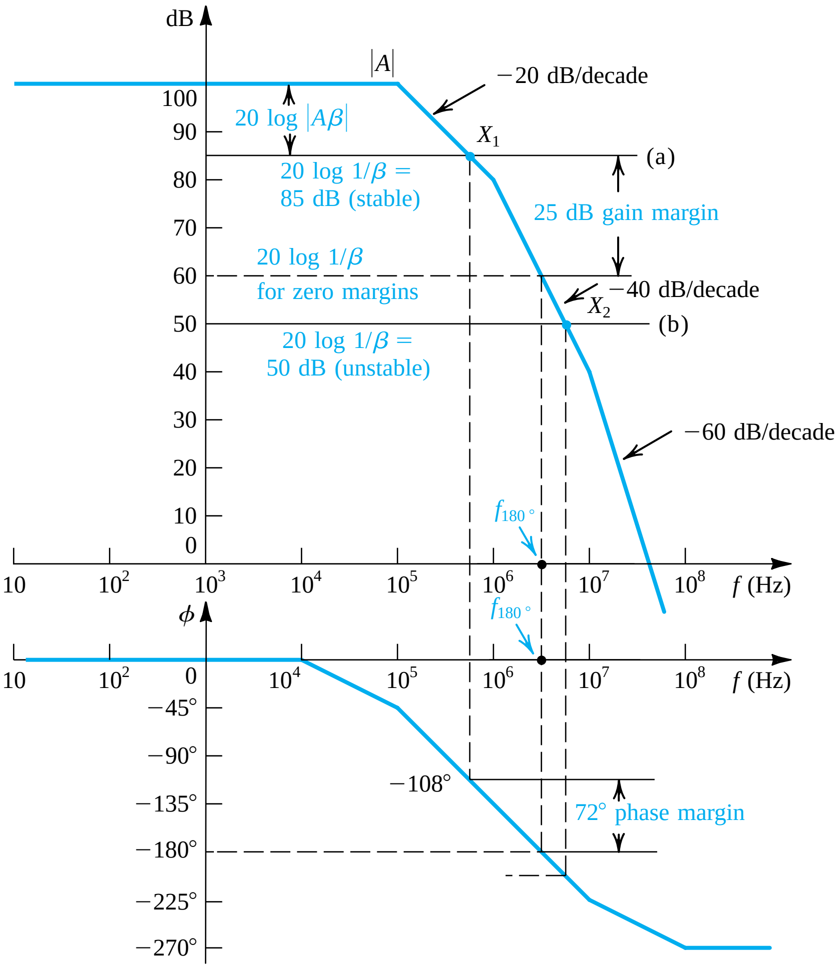


Figure 10.43 Stability analysis using Bode plot of $|A|$.

phase is approximately –45° at the first pole frequency, –135° at the second, and –225° at the third. The frequency at which the phase of $A(j\omega)$ is –180° lies on the –40-dB/decade segment, as indicated in Fig. 10.43.

The open-loop gain of this amplifier can be expressed as

$$A = \frac{10^5}{(1+jf/10^5)(1+jf/10^6)(1+jf/10^7)} \tag{10.108}$$

from which $|A|$ can be easily determined for any frequency f (in Hz), and the phase can be obtained as

$$\phi = -[\tan^{-1}(f/10^5) + \tan^{-1}(f/10^6) + \tan^{-1}(f/10^7)] \tag{10.109}$$

The magnitude and phase graphs shown in Fig. 10.43 are obtained using the method for constructing Bode plots (Appendix F). These graphs provide approximate values for important amplifier parameters, with more exact values obtainable from Eqs. (10.108) and (10.109). For example, the frequency f_{180} at which the phase angle is 180° can be found from Fig. 10.43 to be approximately 3.2×10^6 Hz. Using this value as a starting point, a more exact value can be found by trial and error using Eq. (10.109). The result is $f_{180} = 3.34 \times 10^6$ Hz. At this frequency, Eq. (10.108) gives a gain magnitude of 58.2 dB, which is reasonably close to the approximate value of 60 dB given by Fig. 10.43.

Consider next the straight line labeled (a) in Fig. 10.43. This line represents a feedback factor for which $20 \log(1/\beta) = 85$ dB, which corresponds to $\beta = 5.623 \times 10^{-5}$ and a closed-loop gain of 83.6 dB. Since the loop gain is the difference between the $|A|$ curve and the $1/\beta$ line, the point of intersection X_1 corresponds to the frequency at which $|A\beta| = 1$. Using the graphs of Fig. 10.43, this frequency can be found to be approximately 5.6×10^5 Hz. A more exact value of 4.936×10^5 can be obtained using the transfer-function equations. At this frequency the phase angle is approximately −108°. Thus the closed-loop amplifier, for which $20 \log(1/\beta) = 85$ dB, will be stable with a phase margin of 72°. The gain margin can be easily obtained from Fig. 10.43; it is 25 dB.

Next, suppose that we wish to use this amplifier to obtain a closed-loop gain of 50-dB nominal value. Since $A_0 = 100$ dB, we see that $A_0\beta \gg 1$ and $20 \log(A_0\beta) \simeq 50$ dB, resulting in $20 \log(1/\beta) \simeq 50$ dB. To see whether this closed-loop amplifier is or is not stable, we draw line (b) in Fig. 10.43 with a height of 50 dB. This line intersects the open-loop gain curve at point X_2, where the corresponding phase is greater than 180°. Thus the closed-loop amplifier with 50-dB gain will be unstable.

In fact, it can easily be seen from Fig. 10.43 that the *minimum* value of $20 \log(1/\beta)$ that can be used, with the resulting amplifier being stable, is 60 dB. In other words, the minimum value of stable closed-loop gain obtained with this amplifier is approximately 60 dB. At this value of gain, however, a manufactured version of this amplifier may still oscillate, since no margin is left to allow for possible changes in gain.

Since the 180°-phase point always occurs on the –40-dB/decade segment of the Bode plot for $|A|$, a rule of thumb to guarantee stability is as follows: *The closed-loop amplifier will be stable if the* $20 \log(1/\beta)$ *line intersects the* $20 \log|A|$ *curve at a point on the* –20-dB/decade *segment*. Following this rule ensures that a phase margin of at least 45° is obtained. For the example of Fig. 10.43, the rule implies that the maximum value of β is 10^{-4}, which corresponds to a closed-loop gain of approximately 80 dB.

The rule of thumb above can be generalized for the case in which β is a function of frequency. The general rule states that *at the intersection of* $20 \log[1/|\beta(j\omega)|]$ *and* $20 \log|A(j\omega)|$ *the difference of slopes* (called the **rate of closure**) *should not exceed* 20 dB/decade.

EXERCISE

10.26 Consider an op amp whose open-loop gain is identical to that of Fig. 10.43. Assume that the op amp is ideal otherwise. Let the op amp be connected as a differentiator. Use the rule of thumb above to show that for stable performance the differentiator time constant should be greater than 159 ms. [*Hint:* Recall that for a differentiator, the Bode plot for $1/|\beta(j\omega)|$ has a slope of +20 dB/decade and intersects the 0-dB line at $1/\tau$, where τ is the differentiator time constant.]

10.13 Frequency Compensation

In this section, we shall discuss methods for modifying the open-loop transfer function $A(s)$ of an amplifier having three or more poles so that the closed-loop amplifier is stable for a given desired value of closed-loop gain.

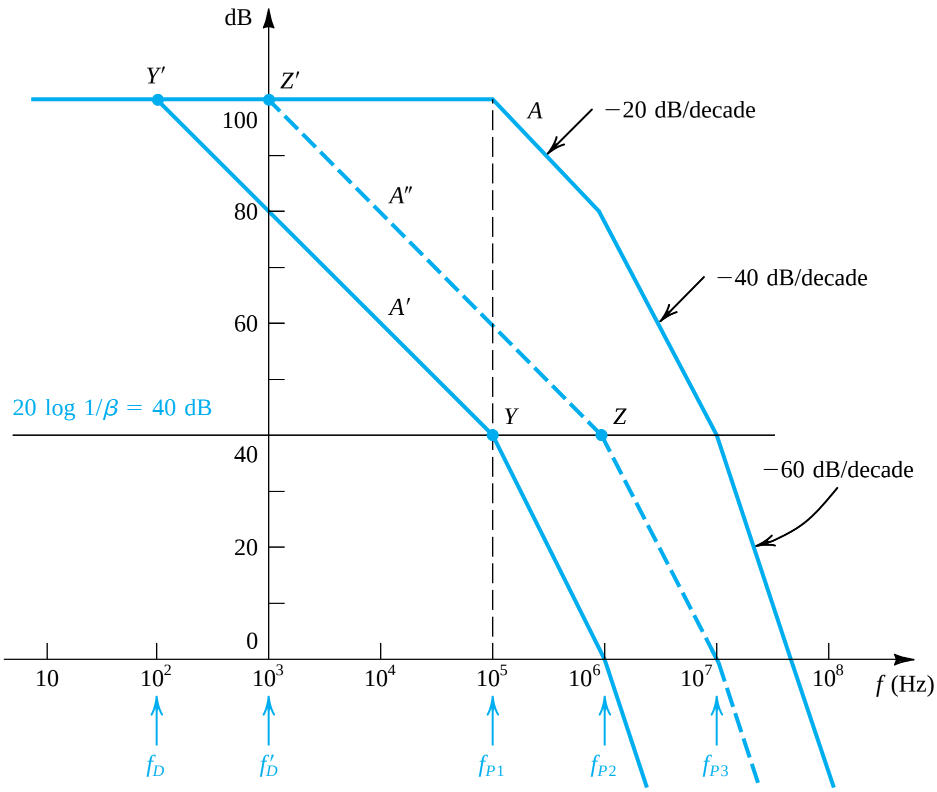


Figure 10.44 Frequency compensation for $\beta = 10^{-2}$. The response labeled A' is obtained by introducing an additional pole at f_D. The A'' response is obtained by moving the original low-frequency pole to f_D'.

10.13.1 Theory

The simplest method of frequency compensation consists of introducing a new pole in the function $A(s)$ at a sufficiently low frequency, f_D, such that the modified open-loop gain, $A'(s)$, intersects the $20 \log(1/|\beta|)$ curve with a slope difference of 20 dB/decade. As an example, let it be required to compensate the amplifier whose $A(s)$ is shown in Fig. 10.44 such that closed-loop amplifiers with β as high as 10^{-2} (i.e., closed-loop gains as low as approximately 40 dB) will be stable. First, we draw a horizontal straight line at the 40-dB level to represent $20 \log(1/\beta)$, as shown in Fig. 10.44. We then locate point Y on this line at the frequency of the first pole, f_{P1}. From Y we draw a line with –20-dB/decade slope and determine the point at which this line intersects the dc gain line, point Y'. This latter point gives the frequency f_D of the new pole that has to be introduced in the open-loop transfer function.

The compensated open-loop response $A'(s)$ is indicated in Fig. 10.44. It has four poles: at f_D, f_{P1}, f_{P2}, and f_{P3}. Thus $|A'|$ begins to roll off with a slope of −20 dB/decade at f_D. At f_{P1} the slope changes to −40 dB/decade, at f_{P2} it changes to −60 dB/decade, and so on. Since the 20 log(1/β) line intersects the $20\log|A'|$ curve at point Y on the −20-dB/decade segment, the closed-loop amplifier with this β value (or lower values) will be stable.

A serious disadvantage of this compensation method is that at most frequencies the open-loop gain has been drastically reduced. This means that at most frequencies the amount of feedback available will be small. Since all the advantages of negative feedback are directly proportional to the amount of feedback, the performance of the compensated amplifier will be impaired.

Careful examination of Fig. 10.44 shows that the gain $A'(s)$ is low because of the pole at f_{P1}. If we can somehow eliminate this pole, then—rather than locating point Y, drawing YY', and so on—we can start from point Z (at the frequency of the second pole) and draw the line ZZ'. This would result in the open-loop curve $A''(s)$, which shows considerably higher gain than $A'(s)$.

Although it is not possible to eliminate the pole at f_{P1}, it is usually possible to shift that pole from $f = f_{P1}$ to $f = f_D'$. This makes the pole dominant and eliminates the need for introducing an additional lower-frequency pole, as will be explained next.

10.13.2 Implementation

We shall now address the question of implementing the frequency-compensation scheme discussed above. The amplifier circuit normally consists of a number of cascaded gain stages, with each stage responsible for one or more of the transfer-function poles. Through manual and/or computer analysis of the circuit, one identifies which stage introduces each of the important poles f_{P1}, f_{P2}, and so on. For the purpose of our discussion, assume that the first pole f_{P1} is introduced at the interface between the two cascaded differential stages shown in Fig. 10.45(a). In Fig. 10.45(b) we show a simple small-signal model of the circuit at this interface. Current source I_x represents the output-signal current of the Q_1–Q_2 stage. Resistance R_x and capacitance C_x represent the total resistance and capacitance between the two nodes B and B′. It follows that the pole f_{P1} is given by

$$f_{P1} = \frac{1}{2\pi C_x R_x} \tag{10.110}$$

Let us now connect the compensating capacitor C_C between nodes B and B′. This will result in the modified equivalent circuit shown in Fig. 10.45(c) from which we see that the pole introduced will no longer be at f_{P1}; rather, the pole can be at any desired lower frequency f_D':

$$f_D' = \frac{1}{2\pi(C_x + C_C)R_x} \tag{10.111}$$

We thus conclude that one can select an appropriate value for C_C to shift the pole frequency from f_{P1} to the value f_D' determined by point Z' in Fig. 10.44.

At this juncture it should be pointed out that adding the capacitor C_C will usually result in changes in the location of the other poles (those at f_{P2} and f_{P3}). One might therefore need to calculate the new location of f_{P2} and perform a few iterations to arrive at the required value for C_C.

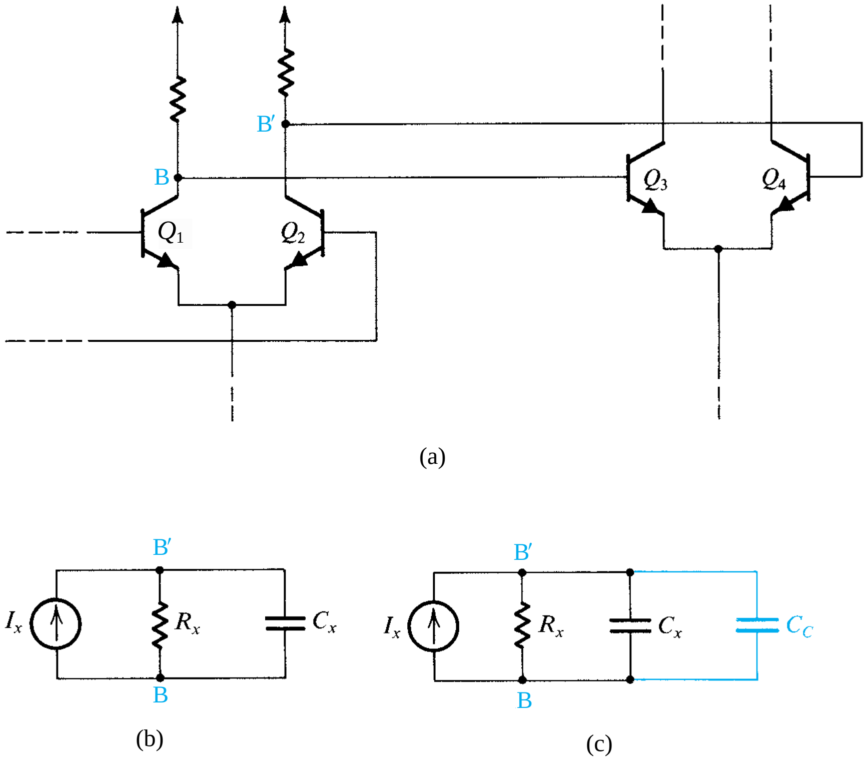


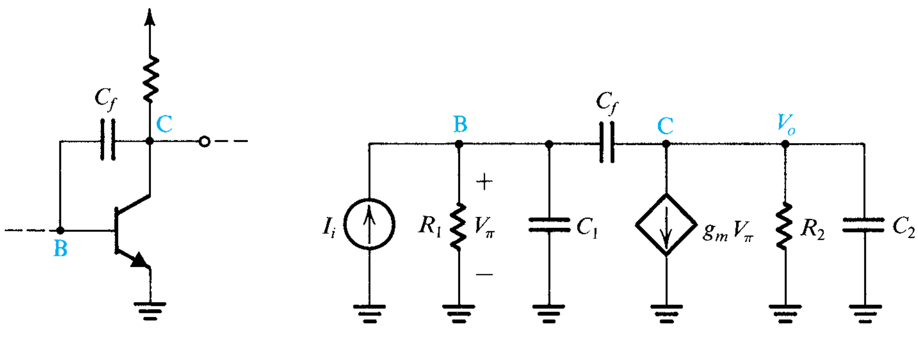
Figure 10.45 **(a)** Two cascaded gain stages of a multistage amplifier. **(b)** Equivalent circuit for the interface between the two stages in (**a**). **(c)** Same circuit as in (**b**), but with a compensating capacitor C_C added. Note that the analysis here applies equally well to MOS amplifiers.

A disadvantage of this implementation method is that the required value of C_C is usually quite large. Thus if the amplifier to be compensated is an IC op amp, it will be difficult, and probably impossible, to include this compensating capacitor on the IC chip. (As pointed out in Chapter 7 and in Appendix A, the maximum practical size of a monolithic capacitor is about 100 pF.) An elegant solution to this problem is to connect the compensating capacitor in the feedback path of an amplifier stage. Because of the Miller effect, the compensating capacitance will be multiplied by the stage gain, resulting in a much larger effective capacitance. Furthermore, as explained later, another unexpected benefit accrues.

10.13.3 Miller Compensation and Pole Splitting

Figure 10.46(a) shows one gain stage in a multistage amplifier. For simplicity, the stage is shown as a common-emitter amplifier, but in practice it can be a more elaborate circuit. In the feedback path of this common-emitter stage we have placed a compensating capacitor C_f.

Figure 10.46(b) shows a simplified equivalent circuit of the gain stage of Fig. 10.46(a). Here R_1 and C_1 represent the total resistance and total capacitance between node B and ground. Similarly, R_2 and C_2 represent the total resistance and total capacitance between node C and ground. Furthermore, it is assumed that C_1 includes the Miller component due to capacitance C_μ, and C_2 includes the input capacitance of the succeeding amplifier stage. Finally, I_i represents the output signal current of the preceding stage.



(a) (b)

Figure 10.46 **(a)** A gain stage in a multistage amplifier with a compensating capacitor connected in the feedback path, and **(b)** an equivalent circuit. Note that although a BJT is shown, the analysis applies equally well to the MOSFET case.

In the absence of the compensating capacitor C_f, we can see from Fig. 10.46(b) that there are two poles—one at the input and one at the output. Let us assume that these two poles are f_{P1} and f_{P2} of Fig. 10.44; thus,

$$f_{P1} = \frac{1}{2\pi C_1 R_1} \qquad f_{P2} = \frac{1}{2\pi C_2 R_2} \tag{10.112}$$

With C_f present, analysis of the circuit yields the transfer function

$$\frac{V_o}{I_i} = \frac{(sC_f - g_m)R_1R_2}{1 + s[C_1R_1 + C_2R_2 + C_f(g_mR_1R_2 + R_1 + R_2)] + s^2[C_1C_2 + C_f(C_1 + C_2)]R_1R_2} \tag{10.113}$$

The zero is usually at a much higher frequency than the dominant pole, and we shall neglect its effect. The denominator polynomial $D(s)$ can be written in the form

$$D(s) = \left(1 + \frac{s}{\omega'_{P1}}\right)\left(1 + \frac{s}{\omega'_{P2}}\right) = 1 + s\left(\frac{1}{\omega'_{P1}} + \frac{1}{\omega'_{P2}}\right) + \frac{s^2}{\omega'_{P1}\omega'_{P2}} \tag{10.114}$$

where ω'_{P1} and ω'_{P2} are the new frequencies of the two poles. Normally one of the poles will be dominant; $\omega'_{P1} \ll \omega'_{P2}$. Thus,

$$D(s) \simeq 1 + \frac{s}{\omega'_{P1}} + \frac{s^2}{\omega'_{P1}\omega'_{P2}} \tag{10.115}$$

Equating the coefficients of s in the denominator of Eq. (10.113) and in Eq. (10.115) results in

$$\omega'_{P1} = \frac{1}{C_1R_1 + C_2R_2 + C_f(g_mR_1R_2 + R_1 + R_2)}$$

which can be approximated by

$$\omega'_{P1} \simeq \frac{1}{g_mR_2C_fR_1} \tag{10.116}$$

To obtain ω'_{P2} we equate the coefficients of s^2 in the denominator of Eq. (10.113) and in Eq. (10.115) and use Eq. (10.116):

$$\omega'_{P2} \simeq \frac{g_m C_f}{C_1 C_2 + C_f (C_1 + C_2)} \tag{10.117}$$

From Eqs. (10.116) and (10.117), we see that as C_f is increased, ω'_{P1} is reduced and ω'_{P2} is increased. This action is referred to as **pole splitting**. Note that the increase in ω'_{P2} is highly beneficial; it allows us to move point Z (see Fig. 10.44) further to the right, thus resulting in higher compensated open-loop gain. Finally, note from Eq. (10.116) that C_f is multiplied by the Miller-effect factor $g_m R_2$, thus resulting in a much larger effective capacitance, $g_m R_2 C_f$. In other words, the required value of C_f will be much smaller than that of C_C in Fig. 10.45.

Example 10.10

Consider an op amp whose open-loop transfer function is identical to that shown in Fig. 10.43. We wish to compensate this op amp so that the closed-loop amplifier with resistive feedback is stable for any gain (i.e., for β up to unity). Assume that the op-amp circuit includes a stage such as that of Fig. 10.46 with C_1 = 100 pF, C_2 = 5 pF, and g_m = 40 mA/V, that the pole at f_{P1} is caused by the input circuit of that stage, and that the pole at f_{P2} is introduced by the output circuit. Find the value of the compensating capacitor for two cases: either if it is connected between the input node B and ground, or in the feedback path of the transistor.

Solution

First we determine R_1 and R_2 from

$$f_{P1} = 0.1 \text{ MHz} = \frac{1}{2\pi C_1 R_1}$$

Thus,

$$R_1 = \frac{10^5}{2\pi}\ \Omega$$

$$f_{P2} = 1 \text{ MHz} = \frac{1}{2\pi C_2 R_2}$$

Thus,

$$R_2 = \frac{10^5}{\pi}\ \Omega$$

If a compensating capacitor C_C is connected across the input terminals of the transistor stage, then the frequency of the first pole changes from f_{P1} to f'_D:

$$f'_D = \frac{1}{2\pi(C_1 + C_C)R_1}$$

The second pole remains unchanged at 1-MHz. The required value for f'_D is determined by drawing a –20-dB/decade line from the 1-MHz frequency point on the 20 log(1/β) = 20 log 1 = 0 dB line. This line will intersect the 100-dB dc gain line at 10 Hz. Thus,

$$f'_D = 10 \text{ Hz} = \frac{1}{2\pi(C_1 + C_C)R_1}$$

which results in $C_C \simeq 1\ \mu F$, which is quite large and certainly cannot be included on the IC chip.

Next, if a compensating capacitor C_f is connected in the feedback path of the transistor, then both poles change location to the values given by Eqs. (10.116) and (10.117):

$$f'_{P1} \simeq \frac{1}{2\pi g_m R_2 C_f R_1} \qquad f'_{P2} \simeq \frac{g_m C_f}{2\pi[C_1 C_2 + C_f(C_1 + C_2)]} \tag{10.118}$$

To determine where we should locate the first pole, we need to know the value of f'_{P2}. As an approximation, let us assume that $C_f \gg C_2$, which enables us to obtain

$$f'_{P2} \simeq \frac{g_m}{2\pi(C_1 + C_2)} = 60.6 \text{ MHz}$$

Thus it appears that this pole will move to a frequency higher than f_{P3} (which is 10 MHz). Let us therefore assume that the second pole will be at f_{P3}. This requires that the first pole be located at

$$f'_{P1} = \frac{f_{P3}}{A_0} = \frac{10^7 \text{ Hz}}{10^5} = 100 \text{ Hz}$$

Thus,

$$f'_{P1} = 100 \text{ Hz} = \frac{1}{2\pi g_m R_2 C_f R_1}$$

which results in $C_f = 78.5$ pF. Although this value is indeed much greater than C_2, we can determine the location of the pole f'_{P2} from Eq. (10.118), which yields $f'_{P2} = 57.2$ MHz, confirming that this pole has indeed been moved past f_{P3}.

We conclude that using Miller compensation not only results in a much smaller compensating capacitor but, owing to pole splitting, also enables us to place the dominant pole a decade higher in frequency. This results in a wider bandwidth for the compensated op amp.

EXERCISE

10.27 A multipole amplifier having a first pole at 1 MHz and an open-loop gain of 100 dB is to be compensated for closed-loop gains as low as 20 dB by the introduction of a new dominant pole. At what frequency must the new pole be placed?
Ans. 100 Hz

10.28 For the amplifier described in Exercise 10.27, rather than introducing a new dominant pole, we can use additional capacitance at the circuit node at which the first pole is formed to reduce the frequency of the first pole. If the frequency of the second pole is 10 MHz and if it remains unchanged while additional capacitance is introduced as mentioned, find the frequency to which the first pole must be lowered so that the resulting amplifier is stable for closed-loop gains as low as 20 dB. By what factor must the capacitance at the controlling node be increased?
Ans. 1000 Hz; 1000

Summary

- Negative feedback is employed to make the amplifier gain less sensitive to component variations; to control input and output impedances; to extend bandwidth; to reduce nonlinear distortion; and to enhance signal-to-interference ratio.
- The advantages above are obtained at the expense of a reduction in gain and at the risk of the amplifier becoming unstable (that is, oscillating). The latter problem is solved by careful design.
- For each of the four basic types of amplifier, there is an appropriate feedback topology. The four topologies, together with their analysis procedure and their effects on input and output impedances, are summarized in **Table 10.1** in Section 10.8.
- The key feedback parameters are the loop gain ($A\beta$), which for negative feedback must be a positive dimensionless number, and the amount of feedback $(1 + A\beta)$. The latter directly determines gain reduction, gain desensitivity, bandwidth extension, and changes in R_i and R_o.
- Since A and β are in general frequency dependent, the poles of the feedback amplifier are obtained by solving the characteristic equation $1 + A(s)\beta(s) = 0$.
- For the feedback amplifier to be stable, its poles must all be in the left half of the s plane.
- Stability is guaranteed if at the frequency for which the phase angle of $A\beta$ is 180° (i.e., ω_{180}), $|A\beta|$ is less than unity; the amount by which it is less than unity, expressed in decibels, is the gain margin. Alternatively, the amplifier is stable if, at the frequency at which $|A\beta| = 1$, the phase angle is less than 180°; the difference is the phase margin.
- The stability of a feedback amplifier can be analyzed by constructing a Bode plot for $|A|$ and superimposing on it a plot for $1/|\beta|$. Stability is guaranteed if the two plots intersect with a difference in slope no greater than 6 dB/octave.
- To make a given amplifier stable for a given feedback factor β, the open-loop frequency response is suitably modified by a process known as frequency compensation.
- A popular method for frequency compensation involves connecting a feedback capacitor across an inverting stage in the amplifier. This causes the pole formed at the input of the amplifier stage to shift to a lower frequency and thus become dominant, while the pole formed at the output of the amplifier stage is moved to a very high frequency and thus becomes unimportant. This process is known as pole splitting.

PROBLEMS

Computer Simulation Problems

SIM Problems identified by this icon are intended to demonstrate the value of using SPICE simulation to verify hand analysis and design, and to investigate important issues such as allowable signal swing and amplifier nonlinear distortion. Instructions to assist in setting up PSpice and Multisim simulations for all the indicated problems can be found in the corresponding files on the disc. Note that if a particular parameter value is not specified in the problem statement, you are to make a reasonable assumption. * difficult problem; ** more difficult; *** very challenging and/or time-consuming; D: design problem.

Section 10.1: The General Feedback Structure

10.1 A negative-feedback amplifier has a closed-loop gain $A_f = 100$ and an open-loop gain $A = 10^4$. What is the feedback factor β? If a manufacturing error results in a reduction of A to 10^3, what closed-loop gain results? What is the percentage change in A_f corresponding to this factor of 10 reduction in A?

10.2 Consider the op-amp circuit shown in Fig. P10.2, where the op amp has infinite input resistance and zero output resistance but finite open-loop gain A.

(a) Convince yourself that $\beta = R_1/(R_1 + R_2)$
(b) If $R_1 = 10\ \text{k}\Omega$, find R_2 that results in $A_f = 10$ V/V for the following three cases: (i) $A = 1000$ V/V; (ii) $A = 100$ V/V; (iii) $A = 12$ V/V.

(c) For each of the three cases in (b), find the percentage change in A_f that results when A decreases by 20%. Comment on the results.

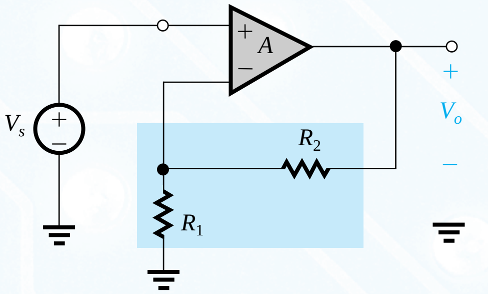


Figure P10.2

10.3 The noninverting buffer op-amp configuration shown in Fig. P10.3 provides a direct implementation of the feedback loop of Fig. 10.1. Assuming that the op amp has infinite input resistance and zero output resistance, what is β? If $A = 1000$, what is the closed-loop voltage gain? What is the amount of feedback (in dB)? For $V_s = 1$ V, find V_o and V_i. If A decreases by 10%, what is the corresponding percentage decrease in A_f?

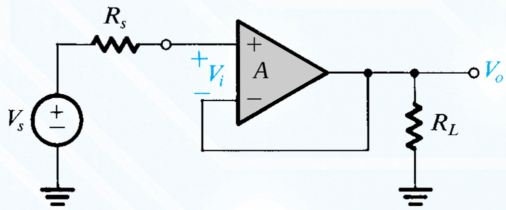


Figure P10.3

10.4 In a particular circuit represented by the block diagram of Fig. 10.1, a signal of 1 V from the source results in a difference signal of 10 mV being provided to the amplifying element A, and 10 V applied to the load. For this arrangement, identify the values of A and β that apply.

10.5 Find the loop gain and the amount of feedback of a voltage amplifier for which A_f and $1/\beta$ differ by (a) 1%, (b) 5%, (c) 10%, (d) 50%.

10.6 In a particular amplifier design, the β network consists of a linear potentiometer for which β is 0.00 at one end, 1.00 at the other end, and 0.50 in the middle. As the potentiometer is adjusted, find the three values of closed-loop gain that result when the amplifier open-loop gain is (a) 1 V/V, (b) 10 V/V, (c) 100 V/V, (d) 10,000 V/V.

10.7 A newly constructed feedback amplifier undergoes a performance test with the following results: With the feedback connection removed, a source signal of 5 mV is required to provide a 10-V output to the load; with the feedback connected, a 10-V output requires a 200-mV source signal. For this amplifier, identify values of A, β, $A\beta$, the closed-loop gain, and the amount of feedback (in dB).

Section 10.2: Some Properties of Negative Feedback

10.8 For the negative-feedback loop of Fig. 10.1, find the loop gain $A\beta$ for which the sensitivity of closed-loop gain to open-loop gain [i.e., $(dA_f/A_f)/(dA/A)$] is –40 dB. For what value of $A\beta$ does the sensitivity become 1/2?

D 10.9 A designer is considering two possible designs of a feedback amplifier. The ultimate goal is $A_f = 20$ V/V. One design employs an amplifier for which $A = 500$ V/V and the other uses $A = 250$ V/V. Find β and the desensitivity factor in both cases. If the $A = 500$ amplifier units have a gain uncertainty of ±10%, what is the gain uncertainty for the closed-loop amplifiers utilizing this amplifier type? If the same result is to be achieved with the $A = 250$ amplifier, what is the maximum allowable uncertainty in its gain?

D 10.10 A designer is required to achieve a closed-loop gain of $25 \pm 1\%$ V/V using a basic amplifier whose gain variation is $\pm 10\%$. What nominal value of A and β (assumed constant) are required?

D 10.11 A circuit designer requires a gain of $25 \pm 1\%$ V/V using an amplifier whose gain varies by a factor of 10 over temperature and time. What is the lowest gain required? The nominal gain? The value of β?

D 10.12 A power amplifier employs an output stage whose gain varies from 2 to 12 for various reasons. What is the gain of an ideal (non varying) amplifier connected to drive it so that an overall gain with feedback of $100 \pm 5\%$ V/V can be achieved? What is the value of β to be used? What are the requirements if A_f must be held within $\pm 0.5\%$? For each of these situations, what preamplifier gain and feedback factor β are required if A_f is to be 10 V/V (with the two possible tolerances)?

D 10.13 It is required to design an amplifier with a gain of 100 that is accurate to within ±1%. You have available amplifier stages with a gain of 1000 that is accurate to within ±30%. Provide a design that uses a number of these gain stages in cascade, with each stage employing negative feedback of an appropriate amount. Obviously, your design should use the lowest possible number of stages while meeting specification.

10.14 Consider an amplifier having a midband gain A_M and a low-frequency response characterized by a pole at $s = -\omega_L$ and a zero at $s = 0$. Let the amplifier be connected in a negative-feedback loop with a feedback factor β. Find an expression for the midband gain and the lower 3-dB frequency of the closed-loop amplifier. By what factor have both changed?

D *10.15 It is required to design an amplifier to have a nominal closed-loop gain of 10 V/V using a battery-operated amplifier whose gain reduces to half its normal full-battery value over the life of the battery. If only 2% drop in closed-loop gain is desired, what nominal open-loop amplifier gain must be used in the design? (Note that since the change in A is large, it is inaccurate to use differentials.) What value of β should be chosen? If component-value variation in the β network may produce as much as a ±1% variation in β, to what value must A be raised to ensure the required minimum gain?

10.16 A capacitively coupled amplifier has a midband gain of 1000 V/V, a single high-frequency pole at 10 kHz, and a single low-frequency pole at 100 Hz. Negative feedback is employed so that the midband gain is reduced to 10. What are the upper and lower 3-dB frequencies of the closed-loop gain?

D 10.17 Low-cost audio power amplifiers often avoid direct coupling of the loudspeaker to the output stage because any resulting dc bias current in the speaker can use up (and thereby waste) its limited mechanical dynamic range. Unfortunately, the coupling capacitor needed can be large! But feedback helps. For example, for an 8-Ω loudspeaker and f_L = 100 Hz, what size capacitor is needed? Now, if feedback is arranged around the amplifier and the speaker so that a closed-loop gain A_f = 10 V/V is obtained from an amplifier whose open-loop gain is 1000 V/V, what value of f_{Lf} results? If the ultimate product-design specification requires a 50-Hz cutoff, what capacitor can be used?

D **10.18 It is required to design a dc amplifier with a low-frequency gain of 1000 and a 3-dB frequency of 0.5 MHz. You have available gain stages with a gain of 1000 but with a dominant high-frequency pole at 10 kHz. Provide a design that employs a number of such stages in cascade, each with negative feedback of an appropriate amount. Use identical stages. [*Hint:* When negative feedback of an amount $(1 + A\beta)$ is employed around a gain stage, its x-dB frequency is increased by the factor $(1 + A\beta)$.]

D 10.19 Design a supply-ripple-reduced power amplifier for which the output stage can be modeled by the block diagram of Fig. 10.4, where A_1 = 0.9 V/V, and the power-supply ripple V_N = +1 V. A closed-loop gain of 10 V/V is desired. What is the gain of the low-ripple preamplifier needed to reduce the output ripple to ±100 mV? To ±10 mV? To ±1 mV? For each case, specify the value required for the feedback factor β.

D 10.20 Design a feedback amplifier that has a closed-loop gain of 100 V/V and is relatively insensitive to change in basic-amplifier gain. In particular, it should provide a reduction in A_f to 99 V/V for a reduction in A to one-tenth its nominal value. What is the required loop gain? What nominal value of A is required? What value of β should be used? What would the closed-loop gain become if A were increased tenfold? If A were made infinite?

D 10.21 A feedback amplifier is to be designed using a feedback loop connected around a two-stage amplifier. The first stage is a direct-coupled, small-signal amplifier with a high upper 3-dB frequency. The second stage is a power-output stage with a midband gain of 10 V/V and upper and lower 3-dB frequencies of 8 kHz and 80 Hz, respectively. The feedback amplifier should have a midband gain of 100 V/V and an upper 3-dB frequency of 40 kHz. What is the required gain of the small-signal amplifier? What value of β should be used? What does the lower 3-dB frequency of the overall amplifier become?

***10.22** The complementary BJT follower shown in Fig. P10.22(a) has the approximate transfer characteristic

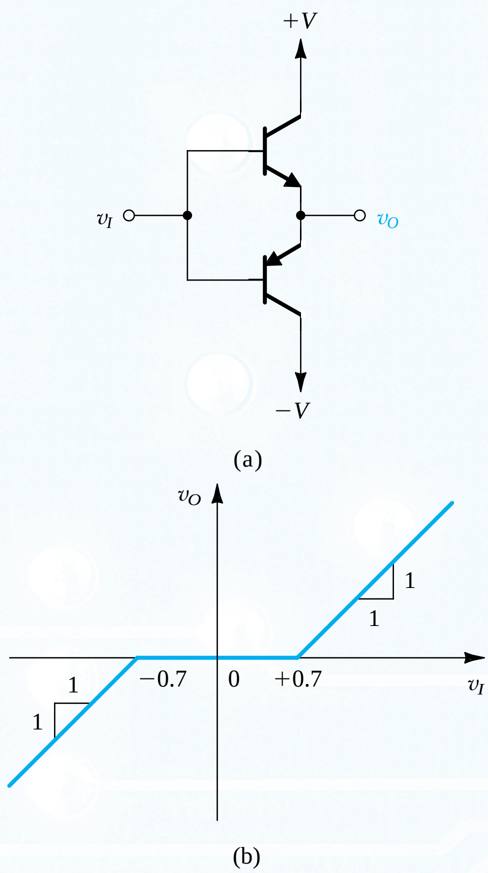


Figure P10.22

shown in Fig. P10.22(b). Observe that for –0.7 V ≤ v_I ≤ +0.7 V, the output is zero. This "dead band" leads to crossover distortion (see Section 11.3). Consider this follower to be driven by the output of a differential amplifier of gain 100 whose positive-input terminal is connected to the input signal source v_S and whose negative-input terminal is connected to the emitters of the follower. Sketch the transfer characteristic v_O versus v_S of the resulting feedback amplifier. What are the limits of the dead band, and what are the gains outside the dead band?

D 10.23 A particular amplifier has a nonlinear transfer characteristic that can be approximated as follows:

(a) For small input signals, $|v_I| \le 10$ mV, $v_O/v_I = 10^3$
(b) For intermediate input signals, 10 mV $\le |v_I| \le 60$ mV, $\Delta v_O/\Delta v_I = 10^2$
(c) For large input signals, $|v_I| \ge 60$ mV, the output saturates

If the amplifier is connected in a negative-feedback loop, find the feedback factor β that reduces the factor-of-10 change in gain (occurring at $|v_I| = 10$ mV) to only a 10% change. What is the transfer characteristic v_O versus v_S of the amplifier with feedback?

Section 10.3: The Four Basic Feedback Topologies

D 10.24 For the feedback voltage amplifier of Fig. 10.7(a) let the op amp have an infinite input resistance, a zero output resistance, and a finite open-loop gain $A = 10^4$ V/V. If $R_1 = 1$ kΩ, find the value of R_2 that results in a closed-loop gain of 100 V/V. What does the gain become if R_1 is removed?

10.25 Consider the feedback voltage amplifier of Fig. 10.7(c). Neglect r_o and assume that $(R_1 + R_2) \gg R_D$.

(a) Find expressions for A and β and hence the amount of feedback.
(b) Noting that the feedback can be eliminated by removing R_1 and R_2 and connecting the gate of Q to a constant dc voltage (signal ground) give the input resistance R_i and the output resistance R_o of the open-loop amplifier.
(c) Using standard circuit analysis (i.e, without invoking the feedback approach), find the input resistance R_{if} and the output resistance R_{of} of the circuit in Fig. 10.7(b). How does R_{if} relate to R_i, and R_{of} to R_o?

10.26 The feedback current amplifier in Fig. P10.26 utilizes an op amp with an input differential resistance R_{id}, an open-loop gain μ, and an output resistance r_o. The output current I_o that is delivered to the load resistance R_L is sensed by the feedback network composed of the two resistances R_M and R_F, and a fraction I_f is fed back to the amplifier input node. Find expressions for $A \equiv I_o/I_i$, $\beta \equiv I_f/I_o$, and $A_f \equiv I_o/I_s$, assuming that the feedback causes the voltage at the input node to be near ground. If the loop gain is large, what does the closed-loop current gain become? State precisely the condition under which this is obtained. For $\mu = 10^4$ V/V, $R_{id} = 1$ MΩ, $r_o = 100$ Ω, $R_L = 10$ kΩ, $R_M = 100$ Ω, and $R_F = 10$ kΩ, find A, β, and A_f.

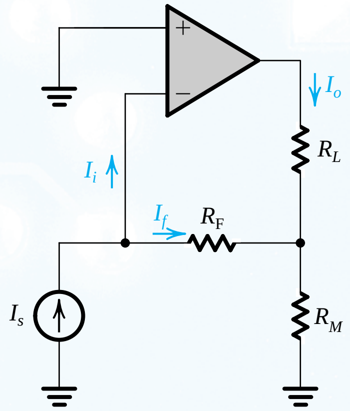


Figure P10.26

10.27 Figure P10.27 shows a feedback transconductance amplifier utilizing an op amp with open-loop gain μ, very large input resistance, and a very small output resistance, and an NMOS transistor Q. The amplifier delivers its output current to R_L. The feedback network, composed of resistor R, senses the equal current in the source terminal of Q and delivers a proportional voltage V_f to the negative input terminal of the op amp.

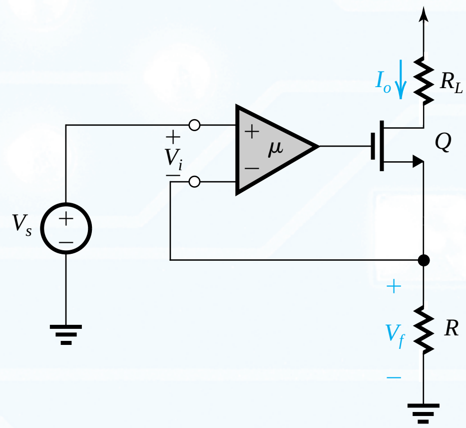


Figure P10.27

(a) Show that the feedback is negative.
(b) Open the feedback loop by breaking the connection of R to the negative input of the op amp and grounding the negative input terminal. Find an expression for $A \equiv I_o/V_i$.

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(c) Find an expression for $\beta \equiv V_f/I_o$.
(d) Find an expression for $A_f \equiv I_o/V_s$.
(e) What is the condition to obtain $I_o \simeq V_s/R$?

10.28 Figure P10.28 shows a feedback transconductance amplifier implemented using an op amp with open-loop gain μ, a very large input resistance, and an output resistance r_o. The output current I_o that is delivered to the load resistance R_L is sensed by the feedback network composed of the three resistances R_M, R_1, and R_2, and a proportional voltage V_f is fed back to the negative-input terminal of the op amp. Find expressions for $A \equiv I_o/V$, $\beta \equiv V_f/I_o$, and $A_f \equiv I_o/V_s$. If the loop gain is large, find an approximate expression for A_f and state precisely the condition for which this applies.

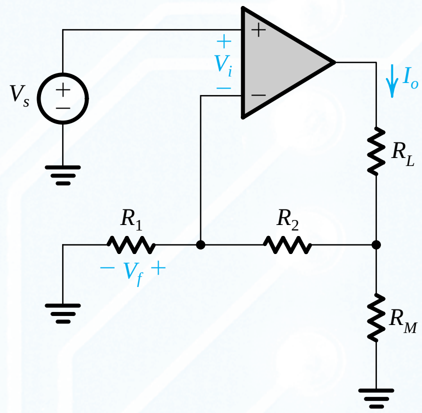

Figure P10.28

10.29 For the feedback transresistance amplifier in Fig. 10.11(d), use small-signal analysis to find the open-loop gain $A \equiv V_o/I_i$, the feedback factor $\beta \equiv I_f/V_o$, and the closed-loop gain $A_f \equiv V_o/I_s$. Neglect r_o of each of Q_1 and Q_2 and assume that $R_C \ll \beta_2 R_E$ and $R_E \ll R_F$, and that the feedback causes the signal voltage at the input node to be nearly zero. Evaluate V_o/I_s for the following component values: $\beta_1 = \beta_2 = 100$, $R_C = R_E = 10\ \text{k}\Omega$, and $R_F = 100\ \text{k}\Omega$.

10.30 For the feedback transresistance amplifier in Fig. P10.30, let $R_F \gg R_C$ and $r_o \gg R_C$, and assume that the feedback causes the signal voltage at the input node to be nearly zero. Derive expressions for $A \equiv V_o/I_i$, $\beta \equiv I_f/V_o$, and $A_f \equiv V_o/I_s$. Find the value of A_f for the case of $R_C = 10\ \text{k}\Omega$, $R_F = 100\ \text{k}\Omega$, and the transistor current gain $\beta = 100$.

Section 10.4: The Feedback Voltage Amplifier (Series–Shunt)

10.31 A series–shunt feedback amplifier employs a basic amplifier with input and output resistances each of 2 kΩ and

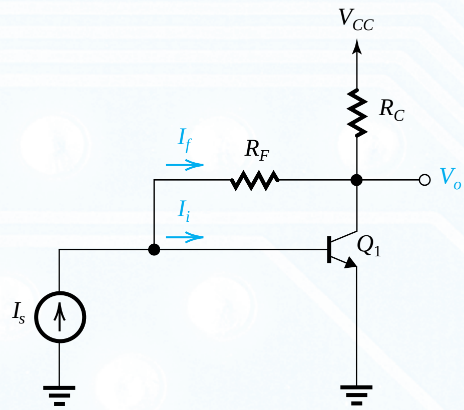

Figure P10.30

gain $A = 1000$ V/V. The feedback factor $\beta = 0.1$ V/V. Find the gain A_f, the input resistance R_{if}, and the output resistance R_{of} of the closed-loop amplifier.

10.32 For a particular amplifier connected in a feedback loop in which the output voltage is sampled, measurement of the output resistance before and after the loop is connected shows a change by a factor of 100. Is the resistance with feedback higher or lower? What is the value of the loop gain $A\beta$? If R_{of} is 100 Ω, what is R_o without feedback?

10.33 The formulas for R_{if} and R_{of} in Eqs. (10.19) and (10.22), respectively, also apply for the case in which A is a function of frequency. In this case, the resulting impedances Z_{if} and Z_{of} will be functions of frequency. Consider the case of a series–shunt amplifier that has an input resistance R_i, an output resistance R_o, and open-loop gain $A = A_0/(1+(s/\omega_H))$, and a feedback factor β that is independent of frequency. Find Z_{if} and Z_{of} and give an equivalent circuit for each, together with the values of all the elements in the equivalent circuits.

10.34 A series–shunt feedback amplifier utilizes the feedback circuit shown in Fig. P10.34.

(a) Find expressions for the h parameters of the feedback circuit (see Fig. 10.14b).
(b) If $R_1 = 1$ kΩ and $\beta = 0.01$, what are the values of all four h parameters? Give the units of each parameter.
(c) For the case $R_s = 1$ kΩ and $R_L = 1$ kΩ, sketch and label an equivalent circuit following the model in Fig. 10.14(c).

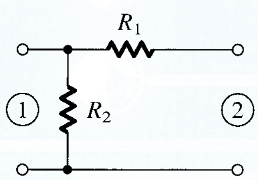

Figure P10.34

10.35 A feedback amplifier utilizing voltage sampling and employing a basic voltage amplifier with a gain of 1000 V/V and an input resistance of 1000 Ω has a closed-loop input resistance of 10 kΩ. What is the closed-loop gain? If the basic amplifier is used to implement a unity-gain voltage buffer, what input resistance do you expect?

***10.36** In the series–shunt feedback amplifier shown in Fig. P10.36, the transistors are biased with ideal current-sources $I_1 = 0.1$ mA and $I_2 = 1$ mA, the devices operate with $V_{BE} = 0.7$ V and have $\beta_1 = \beta_2 = 100$. The input signal V_s has a zero dc component. Resistances $R_s = 100\ \Omega$, $R_1 = 1\,\text{k}\Omega$, $R_2 = 10\,\text{k}\Omega$, and $R_L = 1\,\text{k}\Omega$.

(a) If the loop gain is large, what do you expect the closed-loop gain V_o/V_s to be? Give both an expression and its approximate value.
(b) Find the dc emitter current in each of Q_1 and Q_2. Also find the dc voltage at the emitter of Q_2.
(c) Sketch the A circuit without the dc sources. Derive expressions for A, R_i, and R_o, and find their values.
(d) Give an expression for β and find its value.
(e) Find the closed-loop gain V_o/V_s, the input resistance R_{in}, and the output resistance R_{out}. By what percentage does the value of A_f differ from the approximate value found in (a)?

SIM **D *10.37** Figure P10.37 shows a series–shunt amplifier with a feedback factor $\beta = 1$. The amplifier is designed so that $v_O = 0$ for $v_S = 0$, with small deviations in v_O from 0 V dc being minimized by the negative-feedback action. The technology utilized has $k_n' = 2k_p' = 120\ \mu\text{A/V}^2$, $|V_t| = 0.7$ V, and $|V_A'| = 24$ V/µm.
(a) Show that the feedback is negative.
(b) With the feedback loop opened at the gate of Q_2, and the gate terminals of Q_1 and Q_2 grounded, find the dc current and the overdrive voltage at which each of Q_1 to Q_5 is operating. Ignore the Early effect. Also find the dc voltage at the output.
(c) Find g_m and r_o of each of the five transistors.
(d) Find the expressions and values of A and R_o. Assume that the bias current sources are ideal.
(e) Find the gain with feedback, A_f, and the output resistance R_{out}.
(f) How would you modify the circuit to realize a closed loop voltage gain of 5 V/V? What is the value of output resistance obtained?

D *10.38 Figure P10.38 shows a series–shunt amplifier in which the three MOSFETs are sized to operate at $|V_{OV}| = 0.2$ V. Let $|V_t| = 0.5$ V and $|V_A| = 10$ V. The current sources utilize single transistors and thus have output resistances equal to r_o.

(a) Show that the feedback is negative.
(b) Assuming the loop gain to be large, what do you expect the closed-loop voltage gain V_o/V_s to be approximately?
(c) If V_s has a zero dc component, find the dc voltages at nodes S_1, G_2, S_3, and G_3. Verify that each of the current sources has the minimum required dc voltage across it for proper operation.
(d) Find the A circuit. Calculate the gain of each of the three stages and the overall voltage gain, A.
[*Hint*: A CS amplifier with a resistance R_s in the source lead has an effective transconductance $g_m/(1 + g_m R_s)$ and an output resistance $r_o(1 + g_m R_s)$.]
(e) Find β.
(f) Find $A_f = V_o/V_s$. By what percentage does this value differ from the approximate value obtained in (b)?
(g) Find the output resistance R_{out}.

D *10.39 The active-loaded differential amplifier in Fig. P10.39 has a feedback network consisting of the voltage divider (R_1, R_2), with $R_1 + R_2 = 1\ \text{M}\Omega$. The devices are sized to operate at $|V_{OV}| = 0.2$ V. For all devices, $|V_A| = 10$ V. The input signal source has a zero dc component.

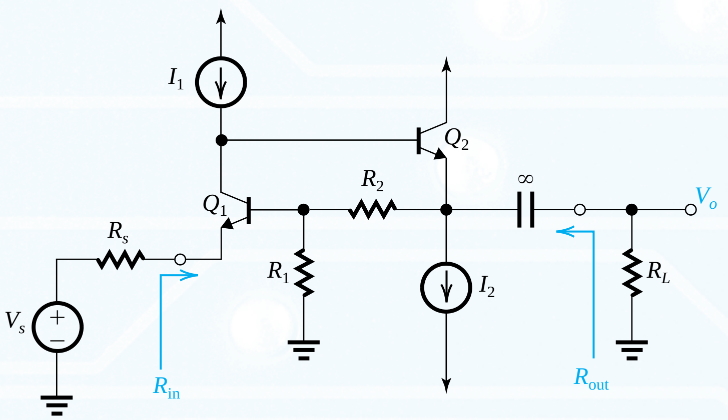

Figure P10.36

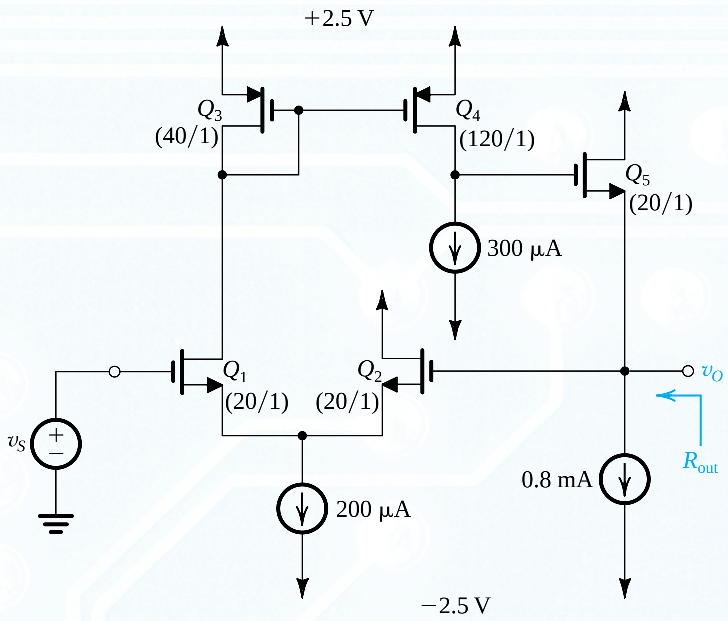

Figure P10.37

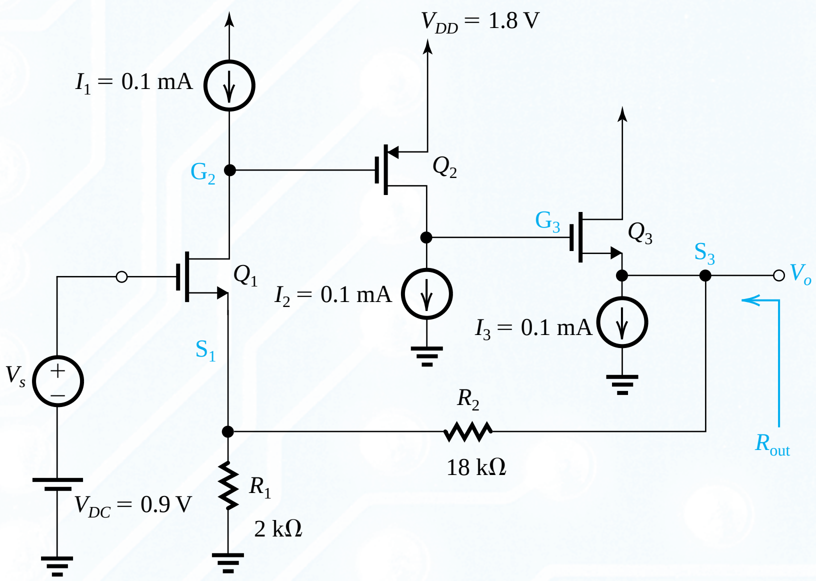

Figure P10.38

(a) Show that the feedback is negative.
(b) What do you expect the dc voltage at the gate of Q_2 to be? At the output? (Neglect the Early effect.)
(c) Find the A circuit. Derive an expression for A and find its value.
(d) Select values for R_1 and R_2 to obtain a closed-loop voltage gain $V_o/V_s = 5$ V/V.
(e) Find the value of R_{out}.
(f) Utilizing the open-circuit, closed-loop gain (5 V/V) and the value of R_{out} found in (e), find the value of gain obtained when a resistance $R_L = 10$ kΩ is connected to the output.
(g) As an alternative approach to (f) above, redo the analysis of the A circuit including R_L. Then utilize the values of R_1 and R_2 found in (d) to determine β and A_f. Compare the value of A_f to that found in (f).

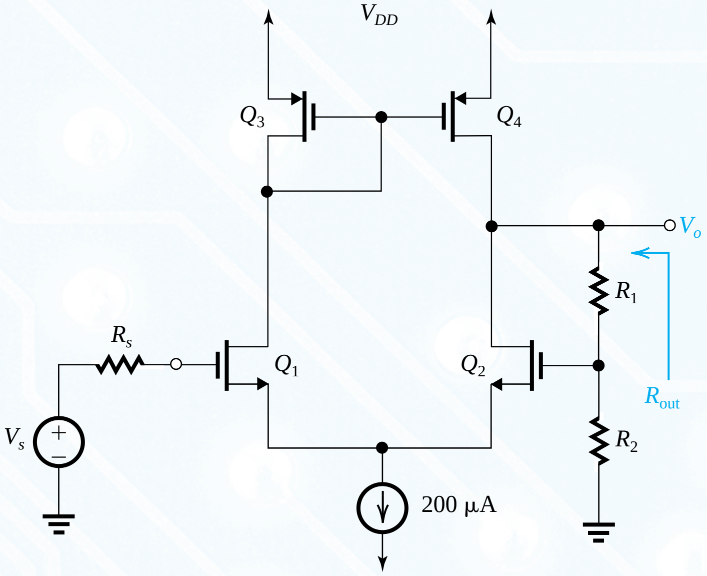

Figure P10.39

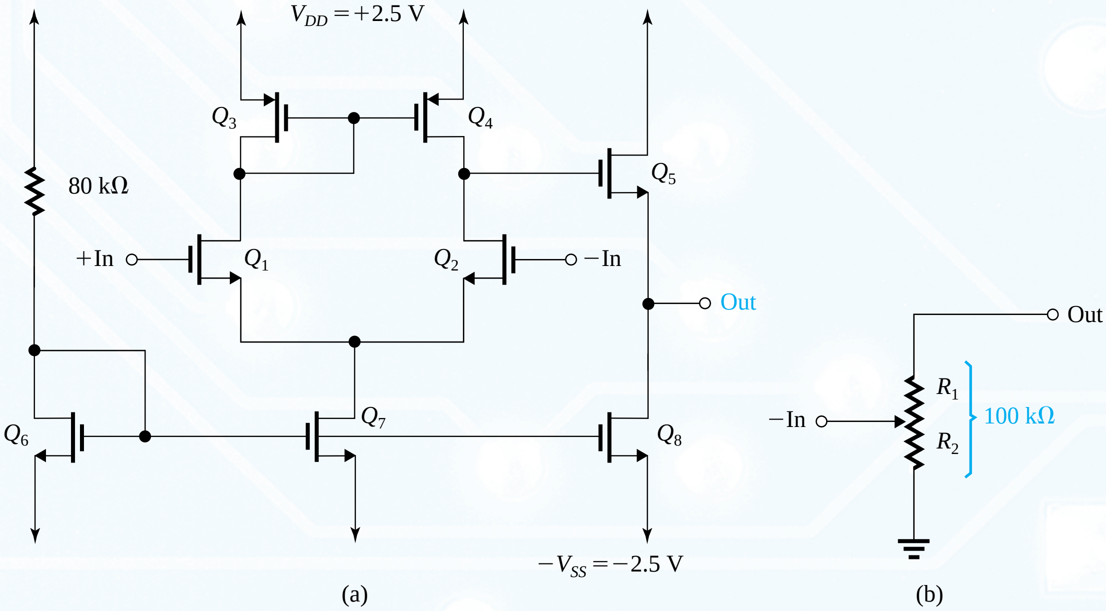

Figure P10.40

D **10.40 The CMOS op amp in Fig. P10.40(a) is fabricated in a 1-µm technology for which $V_{tn} = -V_{tp} = 0.75$ V, $\mu_n C_{ox} = 2\mu_p C_{ox} = 100$ µA/V^2, and $V_A' = 10$ V/µm. All transistors in the circuit have $L = 1$ µm.

(a) It is required to perform a dc bias design of the circuit. For this purpose, let the two input terminals be at zero volts dc and neglect channel-length modulation (i.e, let $V_A = \infty$). Design to obtain $I_{D1} = I_{D2} = 50$ µA, $I_{D5} = 250$ µA, and $V_O = 0$, and operate all transistors except for the source follower Q_5 at $V_{OV} = 0.25$ V. Assume that Q_1 and Q_2 are perfectly matched, and similarly for Q_3 and Q_4. For each transistor, find I_D and W/L.
(b) What is the allowable range of input common-mode voltage?
(c) Find g_m for each of Q_1, Q_2, and Q_5.
(d) For each transistor, calculate r_o.

(e) The 100-kΩ potentiometer shown in Fig. 10.40(b) is connected between the output terminal (Out) and the inverting input terminal (–In) to provide negative feedback whose amount is controlled by the setting of the wiper. A voltage signal V_s is applied between the noninverting input (+In) and ground. A load resistance R_L = 100 kΩ is connected between the output terminal and ground. The potentiometer is adjusted to obtain a closed-loop gain $A_f \equiv V_o/V_s \simeq 10$ V/V.

Specify the required setting of the potentiometer by giving the values of R_1 and R_2. Toward this end, find the A circuit (supply a circuit diagram), the value of A, the β circuit (supply a circuit diagram), and the value of β.

(f) What is the output resistance of the feedback amplifier, excluding R_L?

D *10.41 Figure P10.41 shows a series–shunt feedback amplifier without details of the bias circuit.

(a) Sketch the A circuit and the circuit for determining β.

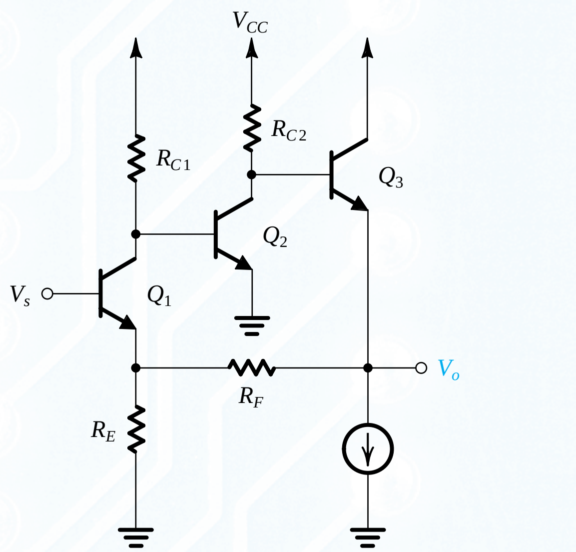

Figure P10.41

(b) Show that if $A\beta$ is large then the closed-loop voltage gain is given approximately by

$$A_f \equiv \frac{V_o}{V_s} \simeq \frac{R_F + R_E}{R_E}$$

(c) If R_E is selected equal to 50 Ω, find R_F that will result in a closed-loop gain of approximately 25 V/V.

(d) If Q_1 is biased at 1 mA, Q_2 at 2 mA, and Q_3 at 5 mA, and assuming that the transistors have h_{fe} = 100, find approximate values for R_{C1} and R_{C2} to obtain gains from the stages of the A circuit as follows: a voltage gain of Q_1 of about –10 and a voltage gain of Q_2 of about –50.

(e) For your design, what is the closed-loop voltage gain realized?

(f) Calculate the input and output resistances of the closed-loop amplifier designed.

***10.42** Figure P10.42 shows a three-stage feedback amplifier:

A_1 has an 82-kΩ differential input resistance, a 20-V/V open-circuit differential voltage gain, and a 3.2-kΩ output resistance.
A_2 has a 5-kΩ input resistance, a 20-mA/V short-circuit transconductance, and a 20-kΩ output resistance.
A_3 has a 20-kΩ input resistance, unity open-circuit voltage gain, and a 1-kΩ output resistance.
The feedback amplifier feeds a 1-kΩ load resistance and is fed by a signal source with a 9-kΩ resistance. The feedback network has R_1= 10 kΩ and R_2= 90 kΩ.

(a) Show that the feedback is negative.
(b) Supply the small-signal equivalent circuit.
(c) Sketch the A circuit and determine A.
(d) Find β and the amount of feedback.
(e) Find the closed-loop gain $A_f \equiv V_o/V_s$.
(f) Find the feedback amplifier's input resistance R_{in}.
(g) Find the feedback amplifier's output resistance R_{out}.
(h) If the high-frequency response of the open-loop gain A is dominated by a pole at 100 Hz, what is the upper 3-dB frequency of the closed-loop gain?

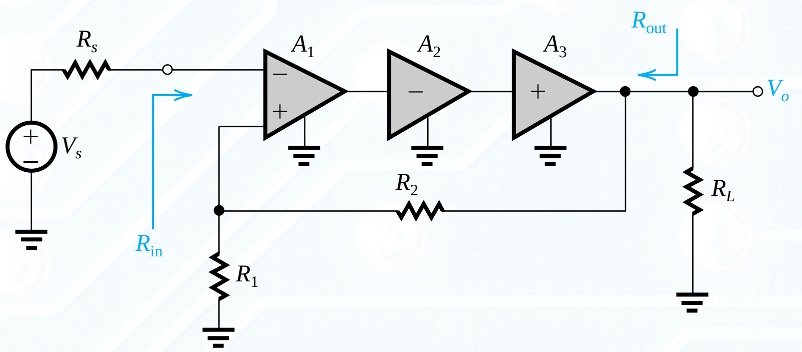

Figure P10.42

(i) If for some reason A_1 drops to half its nominal value, what is the percentage change to A_f?

Section 10.5: The Feedback Transconductance Amplifier (Series–Series)

10.43 A series–series feedback amplifier employs a transconductance amplifier having a short-circuit transconductance G_m of 0.5 A/V, input resistance of 10 kΩ, and output resistance of 100 kΩ. The feedback network has β = 100 Ω, an input resistance (with port 1 open-circuited) of 100 Ω, and an input resistance (with port 2 open-circuited) of 10 kΩ. The amplifier operates with a signal source having a resistance of 10 kΩ and with a load resistance of 10 kΩ. Find A_f, R_{in}, and R_{out}.

10.44 Reconsider the circuit in Fig. 10.23(a), analyzed in Example 10.6, this time with the output voltage taken at the emitter of Q_3. In this case the feedback can be considered to be of the series–shunt type. Note that R_{E2} should now be considered part of the basic amplifier and not of the feedback network.

(a) Determine β.
(b) Find an approximate value for $A_f \equiv V_{e3}/V_s$ assuming that the loop gain remains large (a safe assumption, since the loop in fact does not change).
[*Note*: If you continue with the feedback analysis, you'll find that $A\beta$ in fact changes somewhat; this is a result of the different approximations made in the feedback analysis approach.]

D *10.45 Figure P10.45 shows a feedback triple utilizing MOSFETs. All three MOSFETs are biased and sized to operate at g_m = 4 mA/V. You may neglect their r_o's (except for the calculation of R_{out1} as indicated below).

(a) Considering the feedback amplifier as a transconductance amplifier with output current I_o, find the value of R_F that results in a closed-loop transconductance of approximately 100 mA/V.
(b) Sketch the A circuit and find the value of $A \equiv I_o/V_i$.
(c) Find $1 + A\beta$ and $A_f \equiv V_o/I_s$. Compare to the value of A_f you designed for. What is the percentage difference? What resistance can you change to make A_f exactly 100 mA/V, and in which direction (increase or decrease)?
(d) Assuming that r_{o3} = 20 kΩ, find the output resistance R_{out1}. Since the current sampled by the feedback network is exactly equal to the output current, you can use the feedback formula.
(e) If the voltage V_o is taken as the output, in which case the amplifier becomes series–shunt feedback, what is the value of the closed-loop voltage gain V_o/V_s? Assume that R_F has the original value you selected in (a). Note that in this case R_{S2} should be considered part of the amplifier and not the feedback network. The feedback analysis will reveal that $A\beta$ changes somewhat, which may be puzzling given that the feedback loop did not change. The change is due to the different approximation used.
(f) What is the closed-loop output resistance R_{out2} of the voltage amplifier in (e) above?

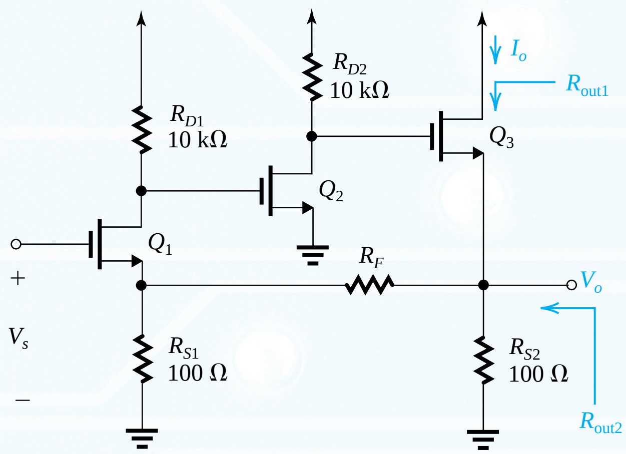

Figure P10.45

10.46 Consider the circuit in Fig. P10.46 as a transconductance amplifier with input V_s and output I_o. The transistor is specified in terms of its g_m and r_o.

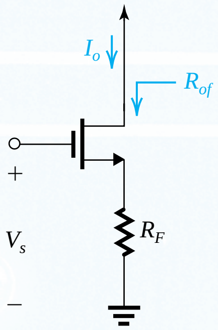

Figure P10.46

(a) Sketch the small-signal equivalent circuit and convince yourself that the feedback circuit is composed of resistor R_F.
(b) Find the A circuit and the β circuit.
(c) Derive expressions for A, β, $(1+A\beta)$, A_f, R_o, and R_{of}.

D 10.47 The transconductance amplifier in Fig. P10.47 utilizes a differential amplifier with gain μ and a very high input resistance. The differential amplifier drives a transistor Q characterized by its g_m and r_o. A resistor R_F senses the output current I_o.

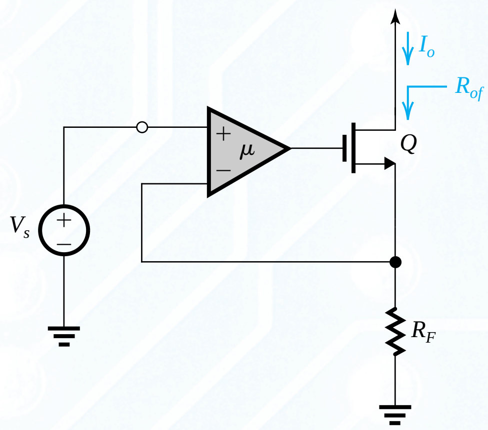

Figure P10.47

(a) For $A\beta \gg 1$, find an approximate expression for the closed-loop transconductance $A_f \equiv I_o/V_s$. Hence, select a value for R_F that results in $A_f \simeq 10$ mA/V.
(b) Find the A circuit and derive an expression for A. Evaluate A for the case $\mu = 1000$ V/V, $g_m = 2$ mA/V, $r_o = 20$ kΩ, and the value of R_F you selected in (a).
(c) Give an expression for $A\beta$ and evaluate its value and that of $1 + A\beta$.
(d) Find the closed-loop gain A_f and compare to the value you anticipated in (a) above.
(e) Find expressions and values for R_o and R_{of}.

***10.48** It is required to show that the output resistance of the BJT circuit in Fig. P10.48 is given by

$$R_o = r_o + [R_e \| (r_\pi + R_b)] \left(1 + g_m r_o \frac{r_\pi}{r_\pi + R_b}\right)$$

To derive this expression, set $V_s = 0$, replace the BJT with its small-signal, hybrid-π model, apply a test voltage V_x to the collector, and find the current I_x drawn from V_x and hence R_o as V_x/I_x. Note that the bias arrangement is not shown. For the case of $R_b = 0$, find the maximum possible value for R_o. Note that this theoretical maximum is obtained when R_e is so large that the signal current in the emitter is nearly zero. In this case, with V_x applied and $V_s = 0$, what is the current in the base, in the $g_m V_\pi$ generator, and in r_o, all in terms of I_x? Show these currents on a sketch of the equivalent circuit with R_e set to ∞.

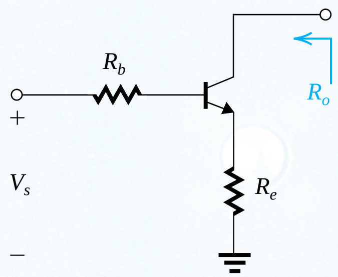

Figure P10.48

10.49 As we found out in Example 10.6, whenever the feedback network senses the emitter current of the BJT, the feedback output resistance formula cannot predict the output resistance looking into the collector. To understand this issue more clearly, consider the feedback transconductance amplifier shown in Fig. P10.49(a). To determine the output resistance, we set $V_s = 0$ and apply a test voltage V_x to the collector, as shown in Fig. P10.49(b). Now, let μ be increased to the point where the feedback signal across R_F equals the input to the positive terminal of the differential amplifier, now zero. Thus the signal current through R_F will be zero. By replacing the BJT with its hybrid-π model, show that

$$R_{\text{out}} = r_\pi + (h_{fe} + 1) r_o \simeq h_{fe} r_o$$

where h_{fe} is the transistor β. Thus for large amounts of feedback, R_{out} is limited to a maximum of $h_{fe} r_o$ independent of the amount of feedback. This should be expected, since no current flows through the feedback network R_F!

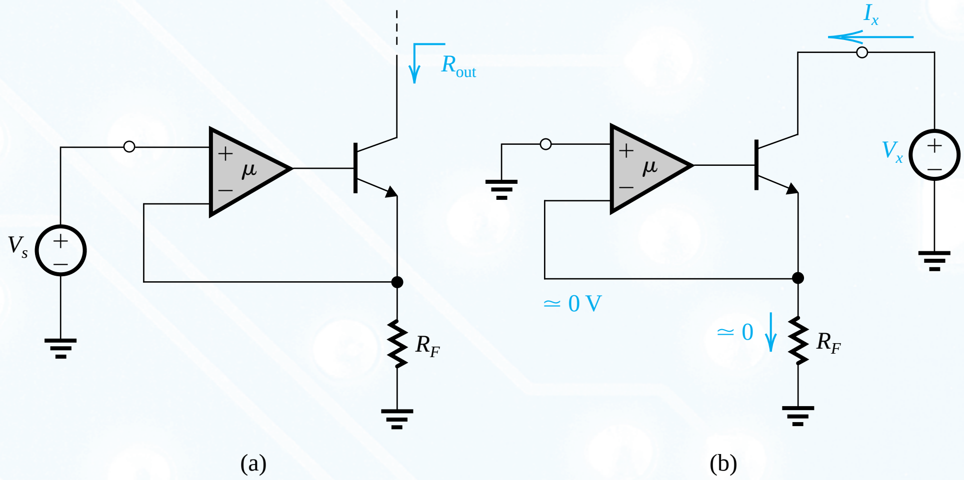

Figure P10.49

This phenomenon does *not* occur in the MOSFET version of this circuit.

10.50 For the feedback transconductance amplifier of Fig. 10.10(c) derive expressions for A, β, $A\beta$, A_f, R_o, and R_{of}. Evaluate A_f and R_{of} for the case of $g_{m1} = g_{m2} = 5$ mA/V, $R_D = 10$ kΩ, $r_{o2} = 20$ kΩ, $R_F = 100$ Ω, and $R_L = 1$ kΩ. For simplicity, neglect r_{o1} and take r_{o2} into account only when calculating output resistances.

D 10.51 For the feedback transconductance amplifier in Fig. P10.51, derive an approximate expression for the closed-loop transconductance $A_f \equiv I_o/V_s$ for the case of $A\beta \gg 1$. Hence select a value for R_2 to obtain $A_f = 100$ mA/V. If Q is biased to obtain $g_m = 1$ mA/V, specify the value of the gain μ of the differential amplifier to obtain an amount of feedback of 60 dB. If Q has $r_o = 50$ kΩ, find the output resistance R_{out}.

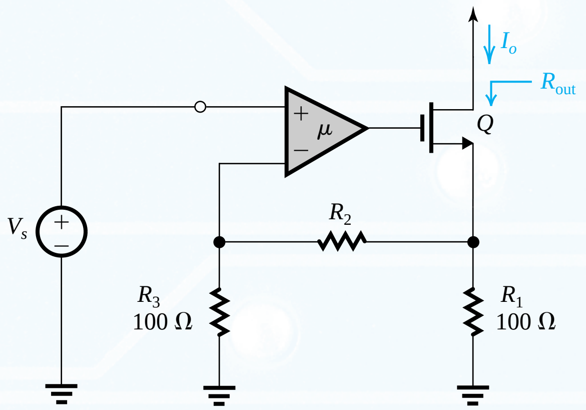

Figure P10.51

SIM 10.52 All the MOS transistors in the feedback transconductance amplifier (series–series) of Fig. P10.52 are sized to operate at $|V_{OV}| = 0.2$ V. For all transistors, $|V_t| = 0.4$ V and $|V_A| = 20$ V.

(a) If V_s has a zero dc component, find the dc voltage at the output, at the drain of Q_1, and at the drain of Q_2.
(b) Find an approximate expression and value for $A_f \equiv I_o/V_s$ for the case $A\beta \gg 1$.
(c) Use feedback analysis to obtain a more precise value for A_f.
(d) Find the value of R_{out}.
(e) If the voltage at the source of Q_5 is taken as the output, find the voltage gain using the value of I_o/V_s obtained in (c). Also find the output resistance of this series–shunt voltage amplifier.

Section 10.6: The Feedback Transresistance Amplifier (Shunt–Shunt)

10.53 For the transresistance amplifier analyzed in Example 10.7, use the formulas derived there to evaluate A_f, R_{in}, and R_{out} when μ is one-tenth the value used in the example. That is, evaluate for $\mu = 10^3$ V/V, $R_{id} = \infty$, $r_o = 100$ Ω, $R_F = 10$ kΩ, and $R_s = R_L = 1$ kΩ. Compare to the corresponding values obtained in Example 10.7.

10.54 Use the formulas derived in Example 10.7 to solve the problem in Exercise 10.15.

10.55 The CE BJT amplifier in Fig. P10.55 employs shunt–shunt feedback: Feedback resistor R_F senses the output voltage V_o and provides a feedback current to the base node.

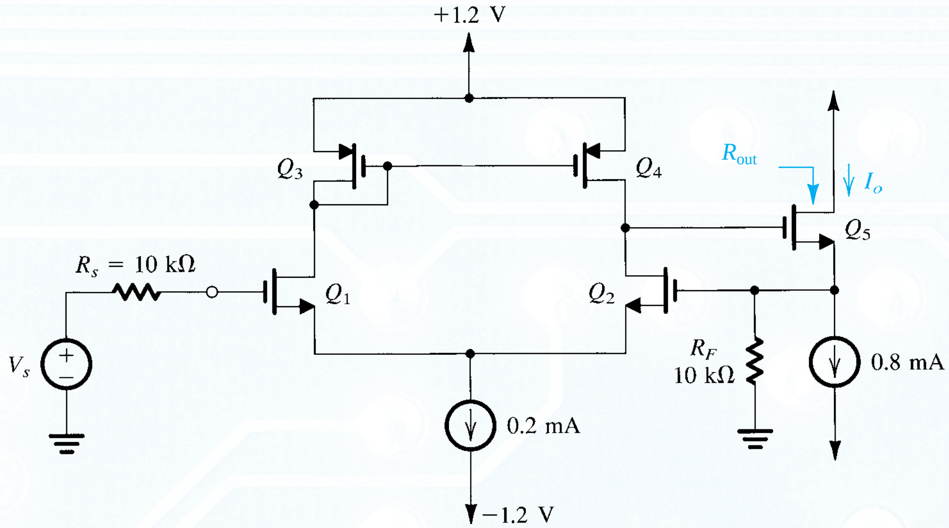

Figure P10.52

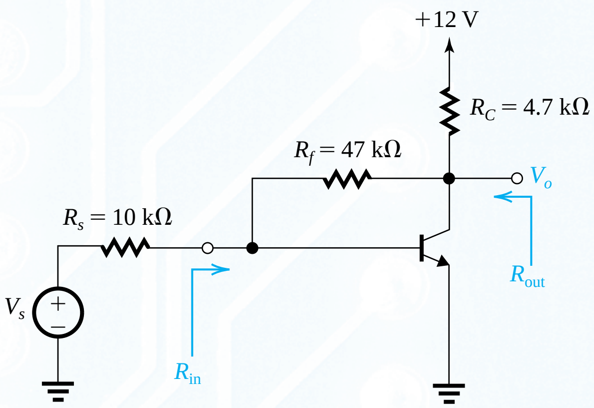

Figure P10.55

(a) If V_s has a zero dc component, find the dc collector current of the BJT. Assume the transistor $\beta = 100$.
(b) Find the small-signal equivalent circuit of the amplifier with the signal source represented by its Norton equivalent (as we usually do when the feedback connection at the input is shunt).
(c) Find the A circuit and determine the value of A, R_i and R_o.
(d) Find β and hence $A\beta$ and $1 + A\beta$.
(e) Find A_f, R_{if}, and R_{of} and hence R_{in} and R_{out}.
(f) What voltage gain V_o/V_s is realized? How does this value compare to the ideal value obtained if the loop gain is very large and thus the signal voltage at the base becomes almost zero (like what happens in an inverting op-amp circuit). Note that this single-transistor poor-man's op amp is not that bad!

D 10.56 The circuit in Fig. P10.56 utilizes a voltage amplifier with gain μ in a shunt–shunt feedback topology with the feedback network composed of resistor R_F. In order to be able to use the feedback equations, you should first convert the signal source to its Norton representation. You will then see that all the formulas derived in Example 10.7 apply here as well.

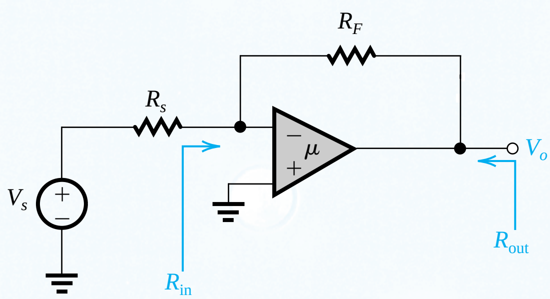

Figure P10.56

(a) If the loop gain is very large, what approximate closed-loop voltage gain V_o/V_s is realized? If $R_s = 1\text{ k}\Omega$, give the value of R_F that will result in $V_o/V_s \simeq -10$ V/V.
(b) If the amplifier μ has a dc gain of 10^3 V/V, an input resistance $R_{id} = 100\text{ k}\Omega$, and an output resistance $r_o = 1\text{ k}\Omega$, find the actual V_o/V_s realized. Also find R_{in} and R_{out} (indicated on the circuit diagram). You may use formulas derived in Example 10.7.
(c) If the amplifier μ has an upper 3-dB frequency of 1 kHz and a uniform −20 -dB/decade gain rolloff, what is the 3-dB frequency of the gain $|V_o/V_s|$?

10.57 The feedback transresistance amplifier in Fig. P10.57 utilizes two identical MOSFETs biased by ideal current sources $I = 0.5$ mA. The MOSFETs are sized to operate at $V_{OV} = 0.2$ V and have $V_t = 0.5$ V and $V_A = 10$ V. The feedback resistance $R_F = 10$ kΩ.

(a) If I_s has a zero dc component, find the dc voltage at the input, at the drain of Q_1, and at the output.
(b) Find g_m and r_o of Q_1 and Q_2.
(c) Provide the A circuit and derive an expression for A in terms of g_{m1}, r_{o1}, g_{m2}, r_{o2}, and R_F.
(d) What is β? Give an expression for the loop gain $A\beta$ and the amount of feedback $(1 + A\beta)$.
(e) Derive an expression for A_f.
(f) Derive expressions for R_i, R_{in}, R_o, and R_{out}.
(g) Evaluate A, β, $A\beta$, A_f, R_i, R_o, R_{in}, and R_{out} for the component values given.

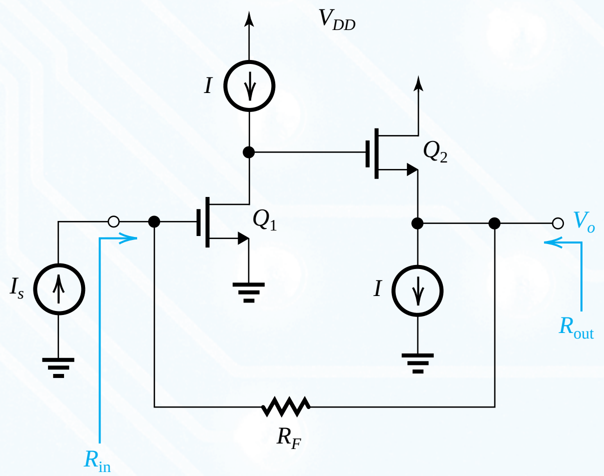

Figure P10.57

10.58 Analyze the circuit in Fig. E10.15 from first principles (i.e., do not use the feedback approach) and hence show that

$$A_f \equiv \frac{V_o}{I_s} = -\frac{(R_s \| R_f)\left(g_m - \frac{1}{R_f}\right)(r_o \| R_f)}{1 + (R_s \| R_f)\left(g_m - \frac{1}{R_f}\right)(r_o \| R_f)/R_f}$$

Comparing this expression to the one given in Exercise 10.15, part (b), you will note that the only difference is that g_m has been replaced by $(g_m - 1/R_f)$. Note that $-1/R_f$ represents the forward transmission in the feedback network, which the feedback-analysis method neglects. What is the condition then for the feedback-analysis method to be reasonably accurate for this circuit?

10.59 For the shunt–shunt feedback amplifier of Fig. 10.11(c), derive expressions for A, β, $A\beta$, A_f, R_i, R_{if}, R_o, and R_{of} in terms of g_{m1}, g_{m2}, R_{D1}, R_{D2}, and R_F. Neglect r_{o1} and r_{o2}. Present your expressions in a format that makes them easy to interpret (e.g., like those derived in Example 10.7 or those asked for in Exercise 10.15).

10.60 For the feedback transresistance amplifier in Fig. 10.11(d) let $V_{CC} = -V_{EE} = 5$ V, $R_C = R_E = R_F = 10$ kΩ. The transistors have $V_{BE} = 0.7$ V and $\beta = 100$.

(a) If I_s has a zero dc component, show that Q_1 and Q_2 are operating at dc collector currents of approximately 0.35 mA and 0.58 mA, respectively. What is the dc voltage at the output?
(b) Find the A circuit and the value of A, R_i, and R_o.
(c) Find the value of β, the loop gain, and the amount of feedback.
(d) Find $A_f \equiv V_o/I_s$, the input resistance R_{if}, and the output resistance R_{of}.

D **10.61 (a) Show that for the circuit in Fig. P10.61(a), if the loop gain is large, the voltage gain V_o/V_s is given approximately by

$$\frac{V_o}{V_s} \simeq -\frac{R_f}{R_s}$$

(b) Using three cascaded stages of the type shown in Fig. P10.61(b) to implement the amplifier μ, design a feedback amplifier with a voltage gain of approximately −100 V/V. The amplifier is to operate between a source resistance $R_s = 10$ kΩ and a load resistance $R_L = 1$ kΩ. Calculate the actual value of V_o/V_s realized, the input resistance (excluding R_s), and the output resistance (excluding R_L). Assume that the BJTs have h_{fe} of 100. [*Note:* In practice, the three amplifier stages are not made identical, for stability reasons.]

D 10.62 Negative feedback is to be used to modify the characteristics of a particular amplifier for various purposes. Identify the feedback topology to be used if:

(a) Input resistance is to be lowered and output resistance raised.
(b) Both input and output resistances are to be raised.
(c) Both input and output resistances are to be lowered.

Section 10.7: The Feedback Current Amplifier (Shunt–Series)

10.63 For the feedback current amplifier in Fig. 10.8(b):

(a) Provide the A circuit and derive expressions for R_i and A. Neglect r_o of both transistors.
(b) Provide the β circuit and an expression for β.
(c) Find an expression for $A\beta$.

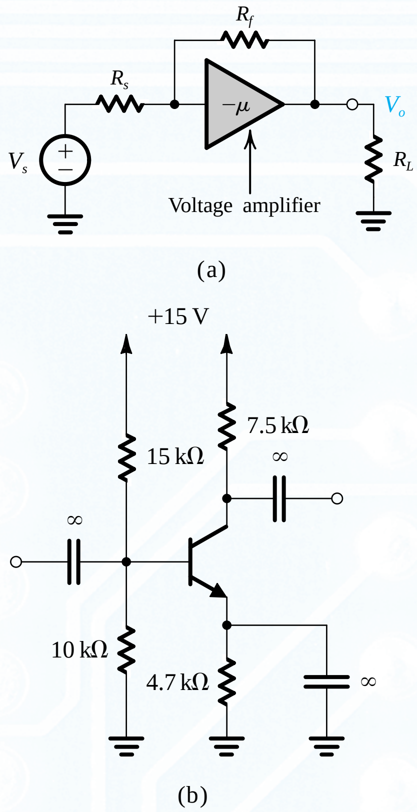

Figure P10.61

(d) For $g_{m1} = g_{m2} = 5$ mA/V, $R_D = 20$ kΩ, $R_M = 10$ kΩ, and $R_F = 90$ kΩ, find the values of A, β, $A\beta$, A_f, R_i, and R_{if}.
(e) If $r_{o2} = 20$ kΩ and $R_L = 1$ kΩ, find the output resistance as seen by R_L.

D 10.64 Design the feedback current amplifier of Fig. 10.31(a) to meet the following specifications:

(i) $A_f \equiv I_o/I_s = -100$ A/A
(ii) amount of feedback $\simeq$ 40 dB
(iii) $R_{in} \simeq 1$ kΩ

Specify the values of R_1, R_2 and μ. Assume that the amplifier μ has infinite input resistance and that $R_s = \infty$. First obtain an approximate value of μ utilizing the approximate formulas derived in Example 10.8. Then with the knowledge that for the MOSFET, $g_m = 5$ mA/V and $r_o = 20$ kΩ, modify the value of μ to meet the design specifications. What R_{out} is obtained?

10.65 The feedback current amplifier in Fig. P10.65 utilizes two identical NMOS transistors sized so that at $I_D = 0.2$ mA they operate at $V_{OV} = 0.2$ V. Both devices have $V_t = 0.5$ V and $V_A = 10$ V.

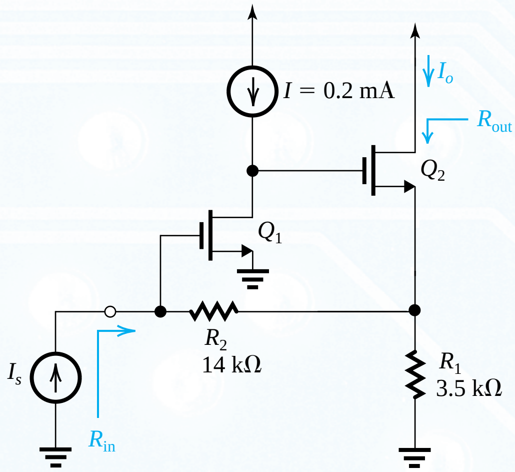

Figure P10.65

(a) If I_s has zero dc component, show that both Q_1 and Q_2 are operating at I_D= 0.2 mA. What is the dc voltage at the input?
(b) Find g_m and r_o for each of Q_1 and Q_2.
(c) Find the A circuit and the value of R_i, A, and R_o.
(d) Find the value of β.
(e) Find $A\beta$ and A_f.
(f) Find R_{in} and R_{out}.

***10.66** The feedback current amplifier in Fig. P10.66(a) can be thought of as a "super" CG transistor. Note that rather than connecting the gate of Q_2 to signal ground, an amplifier is placed between source and gate.

(a) If μ is very large, what is the signal voltage at the input terminal? What is the input resistance? What is the current gain I_o/I_s?
(b) For finite μ but assuming that the input resistance of the amplifier μ is very large, find the A circuit and derive expressions for A, R_i, and R_o.
(c) What is the value of β?
(d) Find $A\beta$ and A_f. If μ is large, what is the value of A_f?
(e) Find R_{in} and R_{out} assuming the loop gain is large.
(f) The "super" CG transistor can be utilized in the cascode configuration shown in Fig. P10.66(b), where V_G is a dc bias voltage. Replacing Q_1 by its small-signal model, use the analogy of the resulting circuit to that in Fig. P10.66(a) to find I_o and R_{out}.

***10.67** Figure P10.67 shows an interesting and very useful application of feedback to improve the performance of the current mirror formed by Q_1 and Q_2. Rather than connecting the drain of Q_1 to the gate, as is the case in simple current mirrors, an amplifier of gain $+\mu$ is connected between the drain and the gate. Note that the feedback loop does not include transistor Q_2. The feedback loop ensures that the

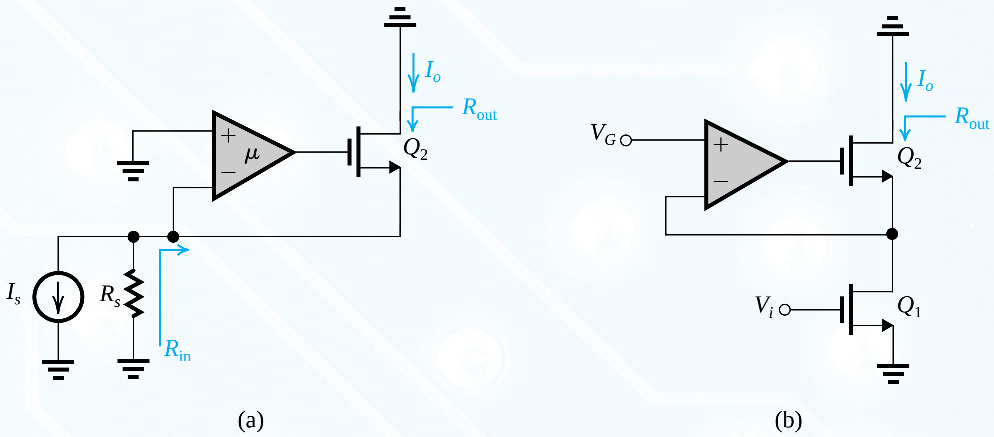

Figure P10.66

value of the gate-to-source voltage of Q_1 is such that I_{o1} equals I_s. This regulated V_{gs} is also applied to Q_2. Thus, if W/L of Q_2 is n times W/L of Q_1, $I_{o2} = nI_{o1} = nI_s$. This current tracking, however, is *not* regulated by the feedback loop.

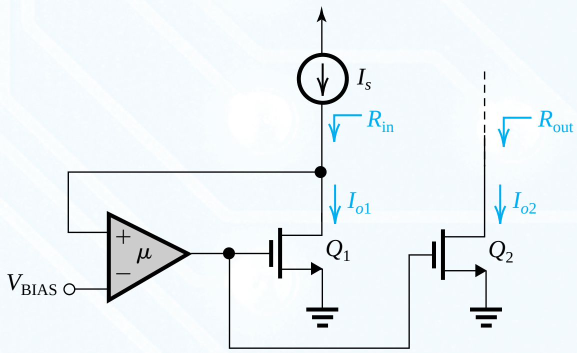

Figure P10.67

(a) Show that the feedback is negative.
(b) If μ is very large and the input resistance of the amplifier μ is infinite, what dc voltage appears at the drain of Q_1? If Q_1 is to operate at an overdrive voltage of 0.2 V, what is the minimum value that V_{BIAS} must have?
(c) Replacing Q_1 by its small-signal model, find an expression for the small-signal input resistance R_{in} assuming finite gain but infinite input resistance for the amplifier μ. Note that here it is much easier to do the analysis directly than to use the feedback-analysis approach.
(d) What is the output resistance R_{out}?

***10.68** The circuit in Fig. P10.68 is an implementation of a particular circuit building block known as **second-generation current convoyer** (CCII). It has three terminals besides ground: x, y, and z. The heart of the circuit is the feedback amplifier consisting of the differential amplifier μ and the complementary source follower (Q_N, Q_P). (Note that this feedback circuit is one we have encountered a number of times in this chapter, albeit with only one source follower transistor.) In the following, assume that the differential amplifier has a very large gain μ and infinite differential input resistance. Also, let the two current mirrors have unity current-transfer ratios.

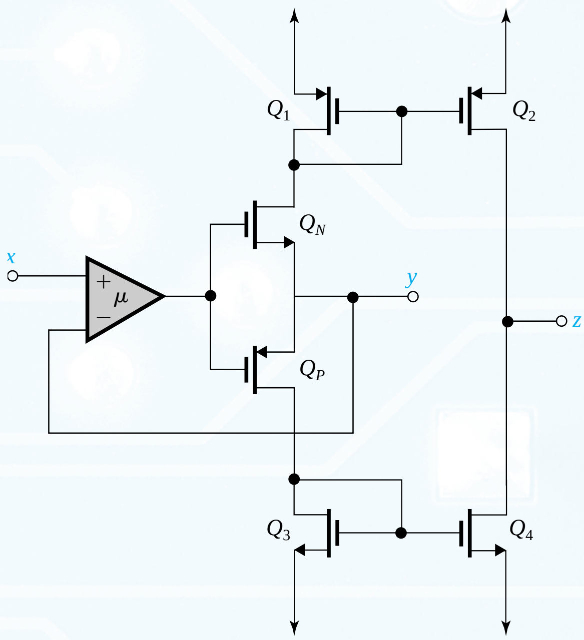

Figure P10.68

(a) If a resistance R is connected between y and ground, a voltage signal V_x is connected between x and ground, and z is short-circuited to ground. Find the current I_z through the short circuit. Show how this current is developed and its path for V_x positive and for V_x negative.

(b) If x is connected to ground, a current source I_y is connected to input terminal y, and z is connected to ground, what voltage appears at y and what is the input resistance seen by I_y? What is the current I_z that flows through the output short circuit? Also, explain the current flow through the circuit for I_y positive and for I_y negative.

(c) What is the output resistance at z?

SIM ***10.69** For the amplifier circuit in Fig. P10.69, assuming that V_s has a zero dc component, find the dc voltages at all nodes and the dc emitter currents of Q_1 and Q_2. Let the BJTs have $\beta = 100$. Use feedback analysis to find V_o/V_s and R_{in}. Let $V_{BE} = 0.7$V.

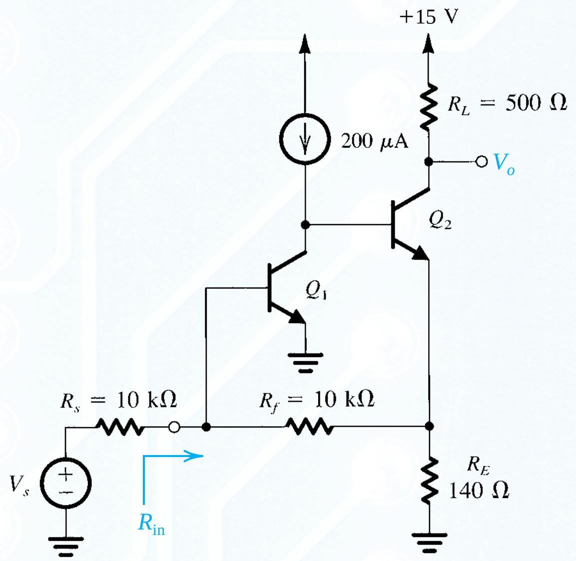

Figure P10.69

10.70 The feedback amplifier of Fig. P10.70 consists of a common-gate amplifier formed by Q_1 and R_D, and a feedback circuit formed by the capacitive divider (C_1, C_2) and the common-source transistor Q_f. Note that the bias circuit for Q_f is not shown. It is required to derive expressions for $A_f \equiv V_o/I_s$, R_{in}, and R_{out}. Assume that C_1 and C_2 are sufficiently small that their loading effect on the basic amplifier can be neglected. Also neglect r_o. Find the values of A_f, R_{in}, and R_{out} for the case in which $g_{m1} = 5$ mA/V, $R_D = 10$ kΩ, $C_1 = 0.9$ pF, $C_2 = 0.1$ pF, and $g_{mf} = 1$ mA/V.

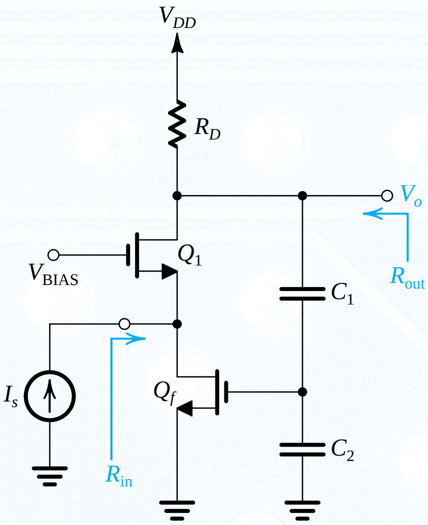

Figure P10.70

****10.71** Figure P10.71 shows a feedback amplifier utilizing the shunt–series topology. All transistors have $\beta = 100$ and $V_{BE} = 0.7$ V. Neglect r_o except in (f).

(a) Perform a dc analysis to find the dc emitter currents in Q_1 and Q_2 and hence determine their small-signal parameters.

(b) Replacing the BJTs with their hybrid-π models, give the equivalent circuit of the feedback amplifier.

(c) Give the A circuit and determine A, R_i, and R_o. Note that R_o is the resistance determined by breaking the emitter loop of Q_2 and measuring the resistance between the terminals thus created.

(d) Find the β circuit and determine the value of β.

(e) Find $A\beta$, $1 + A\beta$, A_f, R_{if}, and R_{of}. Note that R_{of} represents the resistance that in effect appears in the emitter of Q_2 as a result of the feedback.

(f) Determine I_{out}/I_{in}, R_{in}, and R_{out}. To determine R_{out}, use $V_{A2} = 75$ V and recall that the maximum possible output resistance looking into the collector of a BJT is approximately βr_o, where β is the BJT's β (see Problem 10.49).

Section 10.9: Determining the Loop Gain

10.72 Derive an expression for the loop gain $A\beta$ of the feedback amplifier in Fig. 10.22 (a) (Example 10.5). Set $V_s = 0$, break the loop at the gate of Q_2, apply a test voltage V_t to the gate of Q_2, and determine the voltage V_r that appears at the output of amplifier A_1. Put your expression in the form in Eq. (10.36) and indicate the difference.

10.73 It is required to determine the loop gain of the amplifier circuit shown in Fig. P10.41. The most convenient place to break the loop is at the base of Q_2. Thus, connect a resistance equal to $r_{\pi 2}$ between the collector of Q_1 and ground, apply a test voltage V_t to the base of Q_2, and determine the returned voltage at the collector of Q_1 (with V_s set to zero, of course). Show that

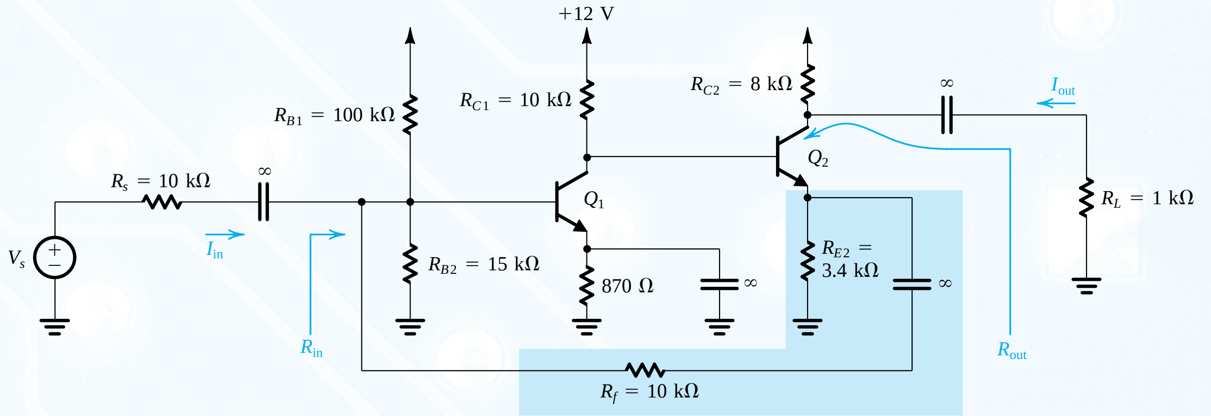

Figure P10.71

$$A\beta = \frac{g_{m2}R_{C2}(h_{fe3}+1)}{R_{C2}+(h_{fe3}+1)[r_{e3}+R_F+(R_E \parallel r_{e1})]}$$

$$\times \frac{\alpha_1 R_E}{R_E + r_{e1}}(R_{C1} \parallel r_{\pi 2})$$

10.74 Show that the loop gain of the amplifier circuit in Fig. P10.52 is

$$A\beta = g_{m1,2}(r_{o2} \parallel r_{o4})\frac{R_F \parallel r_{o5}}{(R_F \parallel r_{o5}) + 1/g_{m5}}$$

where $g_{m1,2}$ is the g_m of each of Q_1 and Q_2.

10.75 Derive an expression for the loop gain of the feedback circuit shown in Fig. P10.26. Assume that the op amp is modeled by an input resistance R_{id}, an open-circuit voltage gain μ, and an output resistance r_o.

***10.76** Find the loop gain of the feedback amplifier shown in Fig. P10.37 by breaking the loop at the gate of Q_2 (and, of course, setting $v_S = 0$). Use the values given in the statement of Problem 10.37. Determine the value of R_{out}.

10.77 Derive an expression for the loop gain of the feedback amplifier shown in Fig. 10.27(a) (Example 10.7). Evaluate $A\beta$ for the component values given in Example 10.7 and compare to the value determined there.

10.78 Derive an expression for the loop gain of the feedback amplifier in Fig. 10.31(a) (Example 10.8). Evaluate $A\beta$ for the component values given in Example 10.8 and compare to the result found there.

10.79 For the feedback amplifier in Fig. P10.70, set $I_s = 0$ and derive an expression for the loop gain by breaking the loop at the gate terminal of transistor Q_f. Refer to Problem 10.70 for more details.

Section 10.10: The Stability Problem

10.80 An op amp designed to have a low-frequency gain of 10^5 and a high-frequency response dominated by a single pole at 100 rad/s, acquires, through a manufacturing error, a pair of additional poles at 10,000 rad/s. At what frequency does the total phase shift reach 180°? At this frequency, for what value of β, assumed to be frequency independent, does the loop gain reach a value of unity? What is the corresponding value of closed-loop gain at low frequencies?

***10.81** For the situation described in Problem 10.80, sketch Nyquist plots for $\beta = 1.0$ and 10^{-3}. (Plot for $\omega = 0$ rad/s, 100 rad/s, 10^3 rad/s, 10^4 rad/s, and ∞ rad/s.)

10.82 An op amp having a low-frequency gain of 10^3 and a single-pole rolloff at 10^4 rad/s is connected in a negative-feedback loop via a feedback network having a transmission k and a two-pole rolloff at 10^4 rad/s. Find the value of k above which the closed-loop amplifier becomes unstable.

10.83 Consider a feedback amplifier for which the open-loop gain $A(s)$ is given by

$$A(s) = \frac{1000}{(1+s/10^4)(1+s/10^5)^2}$$

If the feedback factor β is independent of frequency, find the frequency at which the phase shift is 180°, and find the critical value of β at which oscillation will commence.

Section 10.11: Effect of Feedback on the Amplifier Poles

10.84 A dc amplifier having a single-pole response with pole frequency 10 Hz and unity-gain frequency of 1 MHz is operated in a loop whose frequency-independent feedback factor is 0.01. Find the low-frequency gain, the 3-dB frequency, and the unity-gain frequency of the closed-loop amplifier. By what factor does the pole shift?

***10.85** An amplifier having a low-frequency gain of 10^3 and poles at 10^4 Hz and 10^5 Hz is operated in a closed negative-feedback loop with a frequency-independent β.

(a) For what value of β do the closed-loop poles become coincident? At what frequency?
(b) What is the low-frequency gain corresponding to the situation in (a)? What is the value of the closed-loop gain at the frequency of the coincident poles?
(c) What is the value of Q corresponding to the situation in (a)?
(d) If β is increased by a factor of 10, what are the new pole locations? What is the corresponding pole Q?

D 10.86 A dc amplifier has an open-loop gain of 1000 and two poles, a dominant one at 1 kHz and a high-frequency one whose location can be controlled. It is required to connect this amplifier in a negative-feedback loop that provides a dc closed-loop gain of 10 and a maximally flat response. Find the required value of β and the frequency at which the second pole should be placed.

10.87 Reconsider Example 10.9 with the circuit in Fig. 10.40 modified to incorporate a so-called tapered network, in which the components immediately adjacent to the amplifier input are raised in impedance to $C/10$ and $10R$. Find expressions for the resulting pole frequency ω_0 and Q factor. For what value of K do the poles coincide? For what value of K does the response become maximally flat? For what value of K does the circuit oscillate?

10.88 Three identical inverting amplifier stages each characterized by a low-frequency gain K and a single-pole response with f_{3dB} = 100 kHz are connected in a feedback loop with $\beta = 1$. What is the minimum value of K at which the circuit oscillates? What would the frequency of oscillation be?

Section 10.12: Stability Study Using Bode Plots

10.89 Reconsider Exercise 10.24 for the case of the op amp wired as a unity-gain buffer. At what frequency is $|A\beta| = 1$? What is the corresponding phase margin?

10.90 Reconsider Exercise 10.24 for the case of a manufacturing error introducing a second pole at 10^4 Hz. What is now the frequency for which $|A\beta| = 1$? What is the corresponding phase margin? For what values of β is the phase margin 45° or more?

10.91 For what phase margin does the gain peaking have a value of 5%? Of 10%? Of 0.1 dB? Of 1 dB? [*Hint:* Use the result in Eq. 10.105.]

10.92 An amplifier has a dc gain of 10^5 and poles at 10^5 Hz, 3.16×10^5 Hz, and 10^6 Hz. Find the value of β, and the corresponding closed-loop gain, for which a phase margin of 45° is obtained.

10.93 A two-pole amplifier for which $A_0 = 10^3$ and having poles at 1 MHz and 10 MHz is to be connected as a differentiator. On the basis of the rate-of-closure rule, what is the smallest differentiator time constant for which operation is stable? What are the corresponding gain and phase margins?

10.94 For the amplifier described by Fig. 10.43 and with frequency-independent feedback, what is the minimum closed-loop voltage gain that can be obtained for phase margins of 90° and 45°?

Section 10.13: Frequency Compensation

D 10.95 A multipole amplifier having a first pole at 3 MHz and a dc open-loop gain of 60 dB is to be compensated for closed-loop gains as low as unity by the introduction of a new dominant pole. At what frequency must the new pole be placed?

D 10.96 For the amplifier described in Problem 10.95, rather than introducing a new dominant pole we can use additional capacitance at the circuit node at which the pole is formed to reduce the frequency of the first pole. If the frequency of the second pole is 15 MHz and if it remains unchanged while additional capacitance is introduced as mentioned, find the frequency to which the first pole must be lowered so that the resulting amplifier is stable for closed-loop gains as low as unity. By what factor is the capacitance at the controlling node increased?

10.97 Contemplate the effects of pole splitting by considering Eqs. (10.112), (10.116), and (10.117) under the conditions that $R_1 \simeq R_2 = R$, $C_2 \simeq C_1/10 = C$, $C_f \gg C$, and $g_m = 100/R$, by calculating ω_{P1}, ω_{P2}, and ω'_{P1}, ω'_{P2}.

D 10.98 An op amp with open-loop voltage gain of 10^4 and poles at 10^6 Hz, 10^7 Hz, and 10^8 Hz is to be compensated by the addition of a fourth dominant pole to operate stably with unity feedback ($\beta = 1$). What is the frequency of the required dominant pole? The compensation network is to consist of an RC low-pass network placed in the negative-feedback path of the op amp. The dc bias conditions are such that a 1-MΩ resistor can be tolerated in series with each of the negative and positive input terminals. What capacitor is required between the negative input and ground to implement the required fourth pole?

D *10.99 An op amp with an open-loop voltage gain of 80 dB and poles at 10^5 Hz, 10^6 Hz, and 2×10^6 Hz is to be compensated to be stable for unity β. Assume that the op amp incorporates an amplifier equivalent to that in Fig. 10.46, with $C_1 = 150$ pF, $C_2 = 5$ pF, and $g_m = 40$ mA/V, and that f_{P1} is caused by the input circuit and f_{P2} by the output circuit of this amplifier. Find the required value of the compensating Miller capacitance and the new frequency of the output pole.

SIM **10.100 The op amp in the circuit of Fig. P10.100 has an open-loop gain of 10^5 and a single-pole rolloff with $\omega_{3dB} = 10$ rad/s.

(a) Sketch a Bode plot for the loop gain.
(b) Find the frequency at which $|A\beta| = 1$, and find the corresponding phase margin.
(c) Find the closed-loop transfer function, including its zero and poles. Sketch a pole-zero plot. Sketch the magnitude of the transfer function versus frequency, and label the important parameters on your sketch.

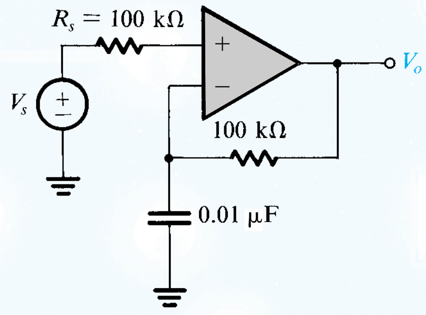

Figure P10.100