

Chapter 6 - BJTs

Threshold voltage: $V_T \approx 25.9 \text{ mV}$.

Two junctions in BJT, EBJ (Emitter Base Junction) and CBJ (Collector Base Junction).

Mode	EBJ	CBJ
Cutoff	Reverse	Reverse
Active	Forward	Reverse
Saturation	Forward	Forward

NPN TRANSISTOR:

In the cutoff mode:

$$V_{BC} < 0.4 \text{ V} \mid V_{BE} < 0.5 \text{ V} \mid$$

$$I_C = 0 \mid I_B = 0$$

In the active mode:

$$V_{BC} < 0.4 \text{ V} \mid V_{BE} \approx 0.7 \text{ V} \mid V_{CE} > 0.3 \text{ V}$$

$$I_C = \beta I_B \mid I_B > 0$$

In the saturation mode:

$$V_{BC} \approx 0.5 \text{ V} \mid V_{BE} \approx 0.7 \text{ V} \mid$$

$$V_{CE_{SAT}} \approx 0.2 \text{ V}$$

$$I_C = \beta_{forced} I_B \mid I_B > 0$$

PNP TRANSISTOR:

In the cutoff mode:

$$V_{CB} < 0.4 \text{ V} \mid V_{EB} < 0.5 \text{ V} \mid$$

$$I_C = 0 \mid I_B = 0$$

In the active mode:

$$V_{CB} < 0.4 \text{ V} \mid V_{EB} \approx 0.7 \text{ V} \mid V_{EC} > 0.3 \text{ V}$$

$$I_C = \beta I_B \mid I_B > 0$$

In the saturation mode:

$$V_{CB} \approx 0.5 \text{ V} \mid V_{EB} \approx 0.7 \text{ V} \mid$$

$$V_{EC_{SAT}} \approx 0.2 \text{ V}$$

$$I_C = \beta_{forced} I_B \mid I_B > 0$$

Current relationships in a BJT transistor.

I_S is known as Saturation Current

$$i_E = i_C + i_B \mid i_E = \frac{\beta+1}{\beta} i_C \mid i_C = \alpha i_E \mid$$

$$\alpha = \frac{\beta}{\beta+1} \mid \beta = \frac{\alpha}{1-\alpha}$$

α is the **common-base current gain**.

A BJT is in the Saturation Region if - The CBJ is Forward biased by more than 0.4V

- The Ratio of i_C/i_B is lower than β

$$i_C = I_S e^{v_{BE}/V_T}$$

Considering the Early voltage

$$i_C = I_S e^{v_{BE}/V_T} \left(1 + \frac{v_{CE}}{V_A} \right)$$

$$r_o = \left[\frac{\delta i_C}{\delta v_{CE}} \Big|_{V_{BE}=\text{constant}} \right]^{-1}$$

$$r_o = \frac{V_A + V_{CE}}{I_C}$$

$$r_o = \frac{V_A}{I_C}$$

Where $I'_C = I_S e^{V_{BE}/V_T}$

$$R_{CE_{SAT}} \equiv \frac{\delta v_{CE}}{\delta i_C} \Big|_{i_B=I_B \mid i_C=I_{CSAT}}$$

Amplifier stuff

$$v_{CE} = V_{CC} - i_C R_C$$

Operating point Q occurs at (V_{BE}, V_{CE}) .

$$A_v = - \left(\frac{I_C}{V_T} \right) R_C = - \frac{V_{RC}}{V_T}$$

$$V_{RC} = V_{CC} - V_{CE}$$

$$A_{vmax} \approx \frac{V_{CC}}{V_T}$$

Small signal stuff

$$i_C = I_C + \frac{I_C}{V_T} v_{be}$$

$$i_C = I_C + i_c$$

$$i_c = \frac{I_C}{V_T} v_{be}$$

$$i_c = g_m v_{be}$$

$$g_m = \frac{I_C}{V_T}$$

Base current:

$$i_B = \frac{i_C}{\beta} = \frac{I_C}{\beta} + \frac{1}{\beta} \frac{I_C}{V_T} v_{be}$$

$$i_B = I_B + i_b$$

$$i_b = \frac{1}{\beta} \frac{I_C}{V_T} v_{be}$$

We know that $I_C/V_T = g_m$ so

$$i_b = \frac{g_m}{\beta} v_{be}$$

The small-signal input resistance looking into the base, is denoted by r_π and is defined as

$$r_\pi \equiv \frac{v_{be}}{i_b} = \frac{\beta}{g_m} = \frac{V_T}{I_B}$$

Emitter current:

$$i_E = \frac{i_C}{\alpha} = \frac{I_C}{\alpha} + \frac{i_c}{\alpha}$$

$$i_E = I_E + i_e$$

$$i_e = \frac{i_c}{\alpha} = \frac{I_C}{\alpha V_T} v_{be} = \frac{I_E}{V_T} v_{be}$$

Small-signal resistance looking into the emitter is

$$r_e \equiv \frac{v_{be}}{i_e}$$

$$r_e = \frac{V_T}{I_E}$$

$$r_e = \frac{\alpha}{g_m} \approx \frac{1}{g_m}$$

Relationship between r_π and r_e :

$$v_{be} = i_b r_\pi = i_e r_\pi$$

$$r_\pi = (i_e/i_b) r_e$$

$$r_\pi = (\beta + 1) r_e$$

Voltage gain of the amplifier:

$$A_v \equiv \frac{v_{ce}}{v_{be}} = -g_m R_C$$

$$A_v = - \frac{I_C R_C}{V_T}$$

Hybrid- π model includes the r_π resistor:

$$i_e = \frac{v_{be}}{r_\pi} + g_m v_{be}$$

$$\frac{v_{be}}{r_\pi} (1 + g_m r_\pi)$$

$$g_m v_{be} = g_m (i_b r_\pi)$$

T-model includes the r_e resistor:

$$g_m = I_C/V_T$$

$$r_e = \frac{V_T}{I_E} = \frac{\alpha}{g_m}$$

If we include r_o , the output voltage becomes:

$$v_o = -g_m v_{be} (R_C \parallel r_o)$$

Three different amplifier configurations:

Common-emitter:

Common-base

Common-collector (also known as

emitter follower)

But first, for all amplifier configurations:

$$R_{in} \equiv \frac{v_i}{i_i}$$

$$v_i = \frac{R_{in}}{R_{in} + R_{sig}} v_{sig}$$

$$A_{vo} \equiv \frac{v_o}{v_i} \Big|_{R_L=\infty}$$

$$R_x = \frac{v_x}{i_x}$$

$$v_o = \frac{R_L}{R_L + R_o} A_{vo} v_i$$

$$A_v \equiv \frac{v_o}{v_i} = A_{vo} \frac{R_L}{R_L + R_o}$$

$$G_v \equiv \frac{v_o}{v_{sig}}$$

$$G_v = \frac{R_{in}}{R_{in} + R_{sig}} A_v$$

$$R_{in} = r_\pi$$

$$v_o = -(g_m v_\pi)(R_C \parallel r_o)$$

$$A_{vo} = -g_m (R_C \parallel r_o)$$

Often, you can neglect r_o , so:

$$A_{vo} \approx (-g_m R_C)$$

R_o is output resistance.

$$R_o = R_C \parallel r_o$$

$$A_v = -\alpha \frac{R_C \parallel R_L \parallel r_o}{r_e}$$

$$A_v = -\alpha \frac{\text{Total resistance in collector}}{\text{Total resistance in emitter}}$$

$$G_v = -\beta \frac{R_C \parallel R_L \parallel r_o}{R_{sig} + r_\pi}$$

Common emitter with emitter resistance:

$$\begin{aligned} A_v &= A_{vo} \frac{R_L}{R_L + R_o} \\ &= -\alpha \frac{R_C}{r_e + R_e} \frac{R_L}{R_L + R_C} \\ &= -\alpha \frac{R_C \parallel R_L}{r_e + R_e} \end{aligned}$$

$$A_{vo} = -\frac{\alpha}{r_e} \frac{R_C}{1 + R_e/r_e}$$

$$A_{vo} = -\frac{g_m R_C}{1 + R_e/r_e} \approx -\frac{g_m R_C}{1 + g_m R_e}$$

$$v_o = -i_c R_C$$

$$= -\alpha i_e R_C$$

) gives

$$A_{vo} = -\alpha \frac{R_C}{r_e + R_e}$$

$$\begin{aligned} \frac{R_{in}(\text{with } R_e \text{ included})}{R_{in}(\text{without } R_e)} &= \frac{(\beta + 1)(r_e + R_e)}{(\beta + 1)r_e} \\ &= 1 + \frac{R_e}{r_e} \simeq 1 + g_m R_e \end{aligned}$$

$$A_v = g_m(R_C \parallel R_L) \qquad R_o = r_e$$

$A_{vo} = 1$ yields A_v in Eq. (6.96), thus con

Common base:

$$\begin{aligned} G_v &= \frac{r_e}{R_{sig} + r_e} g_m(R_C \parallel R_L) \\ &= \alpha \frac{R_C \parallel R_L}{R_{sig} + r_e} \end{aligned}$$

Common collector:

$$R_{in} = r_e$$

$$v_o = -\alpha i_e R_C$$

$$i_e = -\frac{v_i}{r_e}$$

$$A_{vo} \equiv \frac{v_o}{v_i} = \frac{\alpha}{r_e} R_C = g_m R_C$$

$$R_{in} = \frac{v_i}{i_b}$$

Substituting for $i_b = i_e/(\beta + 1)$ where i_e is given by

$$i_e = \frac{v_i}{r_e + R_L}$$

$$R_{in} = (\beta + 1)(r_e + R_L)$$

$$A_v \equiv \frac{v_o}{v_i} = \frac{R_L}{R_L + r_e}$$

$$A_{vo} = 1$$

$$\frac{v_i}{v_{sig}} = \frac{R_{in}}{R_{sig} + R_{in}} = \frac{r_e}{R_{sig} + r_e}$$

We now proceed to determine the ov

$$\begin{aligned} \frac{v_i}{v_{sig}} &= \frac{R_{in}}{R_{in} + R_{sig}} \\ &= \frac{(\beta + 1)(r_e + R_L)}{(\beta + 1)(r_e + R_L) + R_{sig}} \end{aligned}$$

$$G_v \equiv \frac{v_o}{v_{sig}} = \frac{v_i}{v_{sig}} \times A_v$$

Eq. (6.96), results in

$$G_v = \frac{(\beta + 1)R_L}{(\beta + 1)R_L + (\beta + 1)r_e + R_{sig}}$$

$$R_{out} = r_e + \frac{R_{sig}}{\beta + 1}$$