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Course: ECE 3210

Subject: Lab 02, Impulse Response

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1 Introduction

In this lab we analyze a simple circuit (the same as in Lab 01) to find its impulse response. After brief analysis we will build the circuit and measure the input response to compare.

2 Theory

The circuit shown in Fig. 1 is the circuit we analyze. Kirchoff's Current Law suggests that the current through the resistor is equal to the sum of the currents through the capacitor and inductor. Written out this can be shown with

$$i_R = i_C + i_L$$

Substituting each item in that equation with equivalent expressions that allow us to solve the circuit results in Eq. 1.

$$\frac{f(t) - y(t)}{\mathrm{R}} = \mathrm{C}y'(t) + \frac{1}{\mathrm{L}}\int y(t)dt \tag{1}$$

After a bit of algebra and moving the equation around we reach this equation:

$$\mathrm{C}y'(t) + \frac{1}{\mathrm{L}}\int y(t)dt + \frac{y(t)}{\mathrm{R}} = \frac{f(t)}{\mathrm{R}}$$

Now, after differentiating and putting the equation into the notation shown in the textbook, we arrive at Eq. 2

$$(\mathrm{D}^2 + \frac{1}{\mathrm{RC}}\mathrm{D} + \frac{1}{\mathrm{LC}})y(t) = (\frac{1}{\mathrm{RC}}\mathrm{D})f(t) \tag{2}$$

When solving for the equation $y_n(t)$, we find that the solutions to the characteristic equation $\mathrm{Q}(\lambda)$ are complex conjugates. This means that with $\lambda_1 = \alpha + j\beta$ and $\lambda_2 = *\lambda_1$, the solution of $y_n(t)$ will follow the general form

$$y_n(t) = e^{\alpha t}\big(\mathrm{C}_1\cos(\beta t) + \mathrm{C}_2\sin(\beta t)\big)$$

Finally, solving the equation for the impulse reponse by putting it in the form $h(t) = b_n\delta(t) + \mathrm{P(D)}y(t)u(t)$ and using the initial conditions $y_n(t) = 0$ and $y_n'(t) = 1$, we get the final equation for the impulse response in Eq. 3

$$h(t) = -2647.5e^{-15151t}\sin(173417t) + 30303e^{-15151t}\cos(173417t) \tag{3}$$

Notably, the math in this lab looked a lot cleaner before we put the literal values into the equation, but life doesn't give us nice numbers so I guess it's just good practice for the real world.

3 Results

The blue line shown in Fig. 2 is the analytical solution shown in Eq. 3, and the orange line shows the measured values captured from the oscilloscope. We measured over the interval $t \in [-50, 450]$ µs as opposed to the recommended $t \in [0, 300]$ µs because it showed the response better, and matched the scale of the oscilloscope more appropriately.

4 Discussion and Conclusions

The real part of the solution to the characteristic equation Q(λ) is less than 0, so the system can be called BIBO stable, which is to say, any bounded input will result in a bounded output. You can see this in Fig. 2, when the input function was given an impulse, the output of the circuit fell to a stable position after some oscillation.

The measured response fits very close to the analytical solution that was derived by hand. The frequency of the oscillation of the response was just a bit higher than the derived function, but it can be attributed to tolerances of the components in the system.

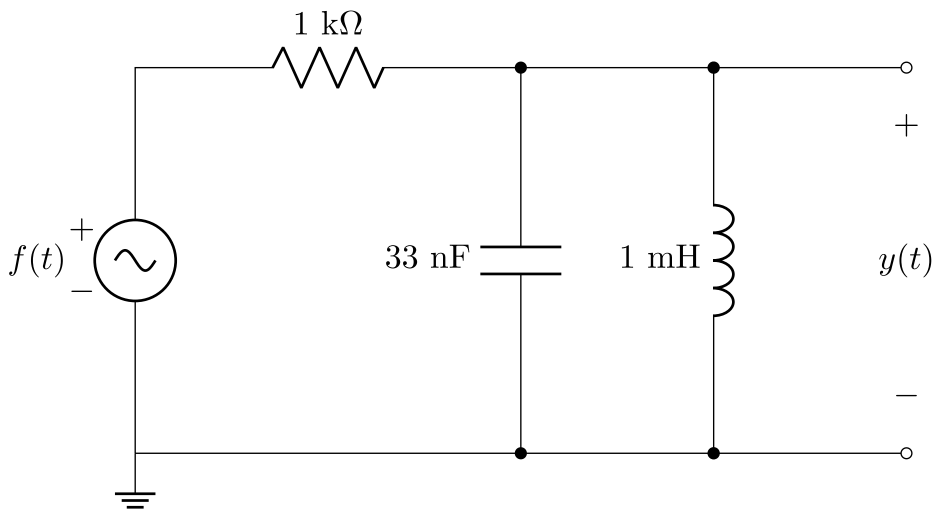

Figure 1: Circuit for analysis in the lab.

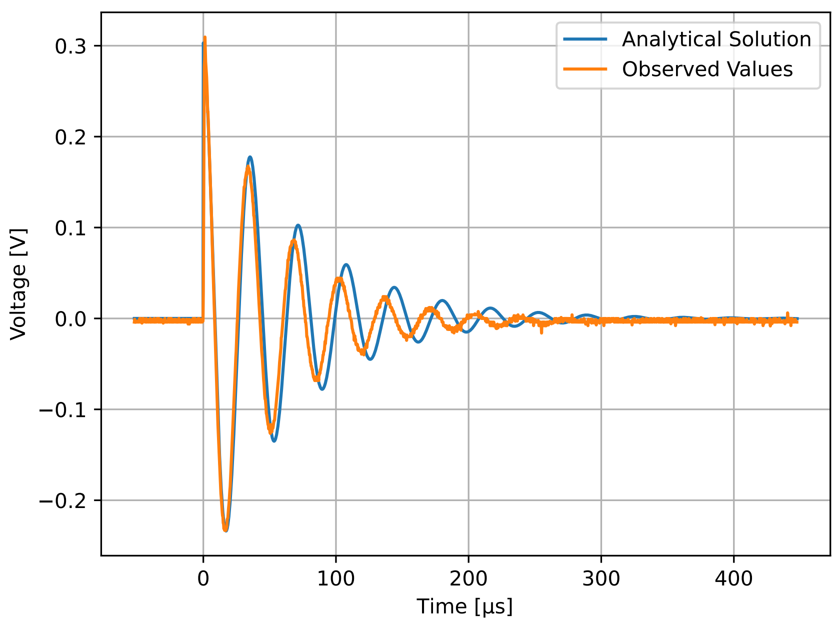

Figure 2: Graph of results comparing analytical solution and measured values.