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Course: ECE 3210

Subject: Lab 5, LTI System Response to Periodic Signal

Date: October 11, 2024

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1 Introduction

In this lab we are analyzing the LTI response of a system to a periodic input. Similar to Lab 2 where we analyzed the reponse of a system to a certain input, here the input isn't a step or ramp, but rather a periodic input where the system doesn't have complete time to settle in between variations of the input.

The circuit that we are using is identical to the circuit in Lab 2, and is shown in Fig. 1. It is a simple RLC circuit.

2 Theory

The circuit from Fig. 1 is a simple RLC circuit and the differential equation derived for this circuit is shown in Eq. 1.

$$\left(\mathrm{D}^2 + \frac{1}{\mathrm{RC}}\mathrm{D} + \frac{1}{\mathrm{LC}}\right)y(t) = \left(\frac{1}{\mathrm{RC}}\mathrm{D}\right)f(t) \tag{1}$$

From this equation we can determine the system response $H(s)$, shown in Eq. 2

$$H(s) = \frac{F(s)}{Y(s)} = \frac{\frac{1}{\mathrm{RC}}s}{s^2 + \frac{1}{\mathrm{RC}}s + \frac{1}{\mathrm{LC}}} \tag{2}$$

This equation for the system response in the s domain is used to calculate $y(t)$. But first we need to find the complex exponential Fourier Series representation of the input. It follows the general form:

$$f(t) \approx \sum_{n=-m}^{m} D_n e^{jn\omega_0 t} \tag{3}$$

Where the D_n coefficients can be found using this equation:

$$D_n = \frac{1}{T_0}\int_{T_0} f_{T_0}(t)e^{-jn\omega_0 t}dt$$

For this lab we are using a frequency of $f_0 = 20$ kHz, which leads to a period $T_0 = 0.05$ ms and fundamental frequency $\omega_0 = 40000\pi$ Hz.

Using these values, the expression for D_n can be found in Eq. 4.

$$D_n = \frac{80000}{n\omega_0}\left(4\sin(n\omega_0 T_0/4)\right) \tag{4}$$

Now, all of these equations can be combined to find the system response $y(t)$ shown in Eq. 5.

$$y(t) = \sum_{-\infty}^{\infty} H(jn\omega_0) D_n e^{jn\omega_0 t} \tag{5}$$

3 Results

Fig. 2 shows the analytical solution to the system and the measured response. The two equations that are shown in the analytical portion are from Eq. 3 and Eq. 5. Clearly, you can see that the computed results line up very well with the measurements that we found experimentally in the lab. It seems that the amplitide of the measurements is just slightly lower than the computed solution, but that the values are remarkably consistent.

4 Discussion and Conclusions

In this lab we managed to analyze the resopnse of an LTI system to a periodic input. It seemed similar to the analysis of the circuit from Lab 2, but we only analyzed the impulse response of the circuit at that time. The impulse response decayed slowly when compared to the width of an impulse function. Naturally, as any input function can be seen as the the impulse function stacked alongside itself many times at varying amplitudes, the response will then fall upon itself many times and never find a chance to settle.

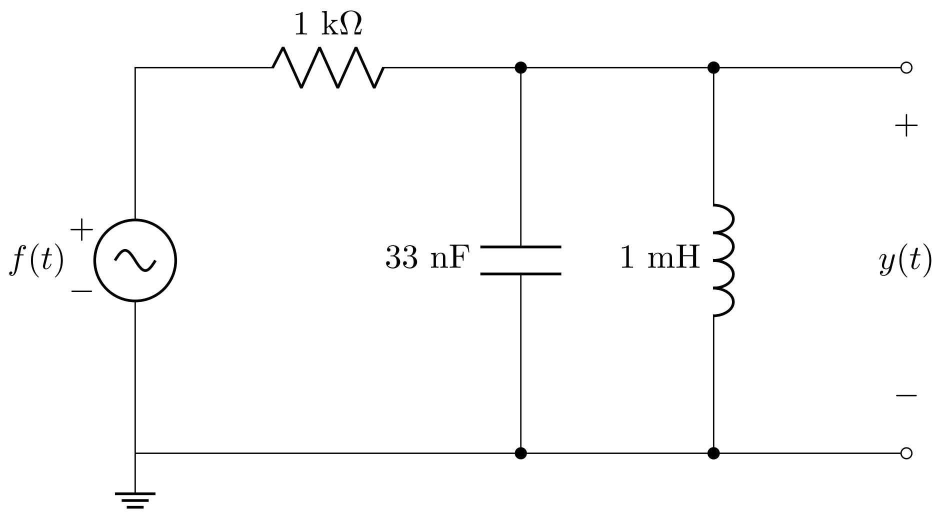

Figure 1: The Circuit Used in the Lab

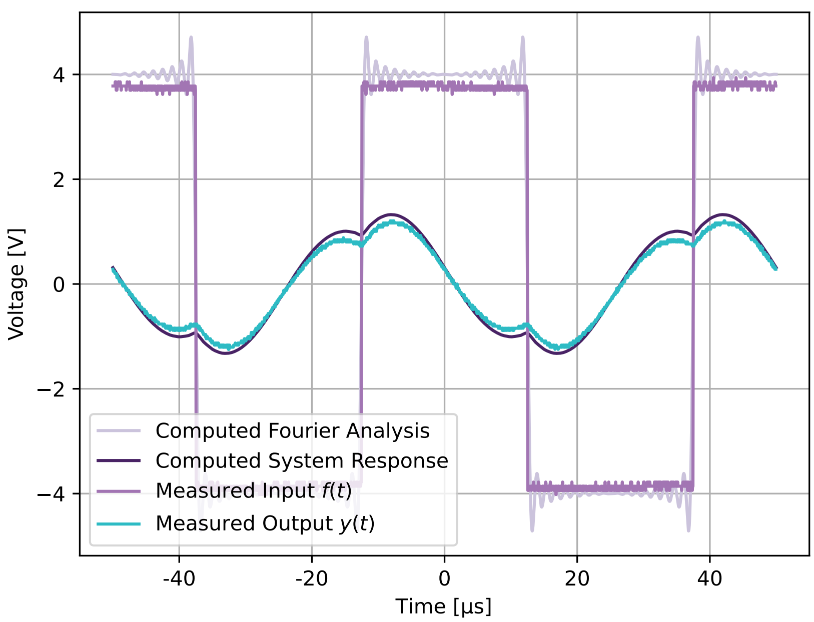

Figure 2: Analytical and Measured Results