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Course: ECE 3210

Subject: Lab 6, Aliasing

Date: October 18, 2024

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1 Introduction

This will be a bit of a shorter lab where we study the phenomenon of aliasing using a few techniques. We will be producing sine wave signals and then sampling them at different rates to see the affect of sampling below the Nyquist frequency.

2 Theory

A sine wave is produced with the following expression:

$$x(t) = A\sin(2\pi f_0 t + \phi)$$

Sampling that frequency at a separate sampling frequency f_s can be modeled with this expression:

$$x[k] = A\sin\left(2\pi k\left(\frac{f_0}{f_s}\right) + \phi\right)$$

With a sampling period T where $T = \frac{1}{f_s}$ we can also show $x[k]$ with this:

$$x[k] = A\sin(2\pi kT + \phi) \tag{1}$$

The other portion of this lab requires us to play a chirp signal which is a signal that has a frequency that increases linearly with time. It is shown by Eq. 2.

$$c(t) = A\cos(\pi\mu t^2 + 2\pi f_1 + \phi) \tag{2}$$

3 Results

When comparing the plots shown in Fig. 1 and Fig. 2 we can see that the stem plots appear very similar. This is due to aliasing. The frequency of the signal is so close to the frequency of the sample rate that the true waveform of the signal is lost.

The only difference between the plots is that the sampled higher frequency signal is 180 degrees out of phase from the lower frequency signal.

When sampling the chirp signal shown by at different frequencies we can hear different results. When sampling at a very high frequency — 32 kHz — the chirp signal increased linearly in frequency until the duration expired, but when we sampled at a lower frequency — 16 kHz — we heard something else. The tone got higher at the same rate as the original tone, about half way through the duration it suddenly started to get lower. Again, when sampled at 8 kHz, the tone increased until just 1/4 of the duration, then decreased until it was too low to hear. This was at just have the duration though, and the pattern repeated again in the same fashion until the entire duration expired.

The perceived tones approximately followed the pattern shown in Fig. 5.

4 Discussion and Conclusions

The aliasing phenomenon is demonstrated in Fig. 3 where a higher-frequency signal and a lower frequency signal intersect at every point where they are sampled.

This is why the signals shown in the Fig. 1 and Fig. 2 sound identical when sampled at that sample rate. Even though the frequency of one signal is dramatically higher than the other, when sampled they result in the same signal.

Fig. 5 shows that sampling at a really high frequency is necessary to prevent the high frequencies from being interpreted and transmitted as lower frequencies. In the case of telephone communication, this is impractical because a lot of data would be used to store information about really high frequencies that aren't necessary for common dialogue that occurs well below 1 kHz.

Because of this, and anti-aliasing filter, or low-pass filter, is used on the signal before being sampled. This essentially cuts off the entire right portion of the graph in Fig. 5 and doesn't disturb any of the input signal that we care about.

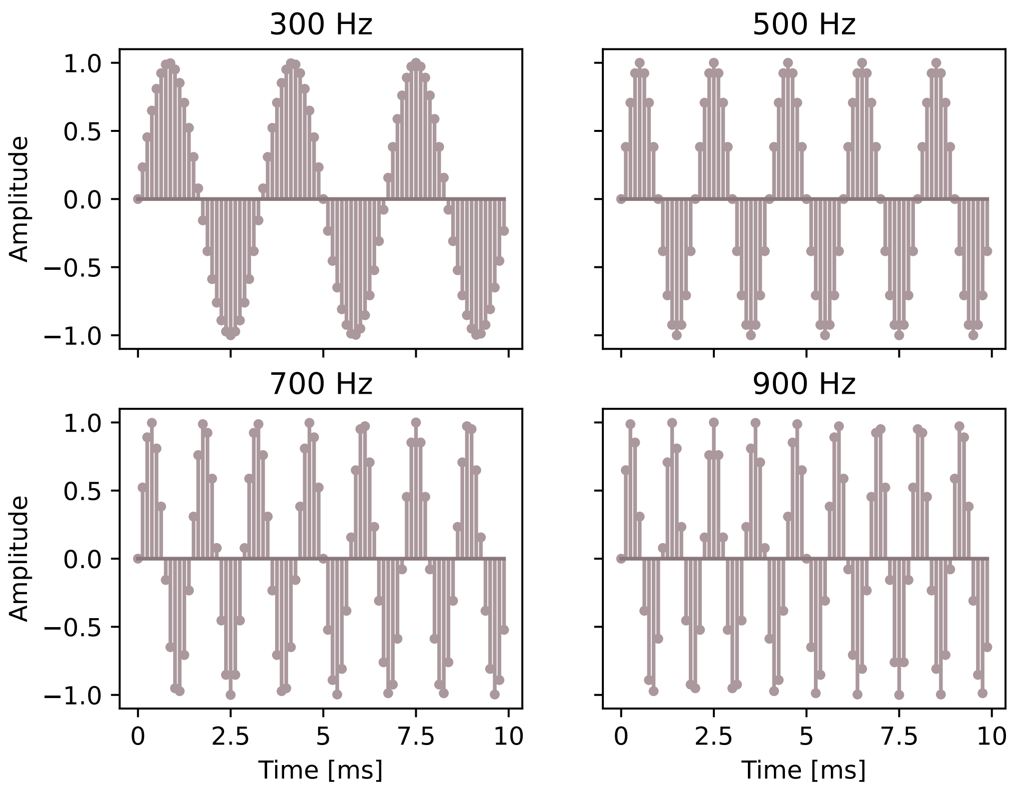

Figure 1: Low Frequency Signals

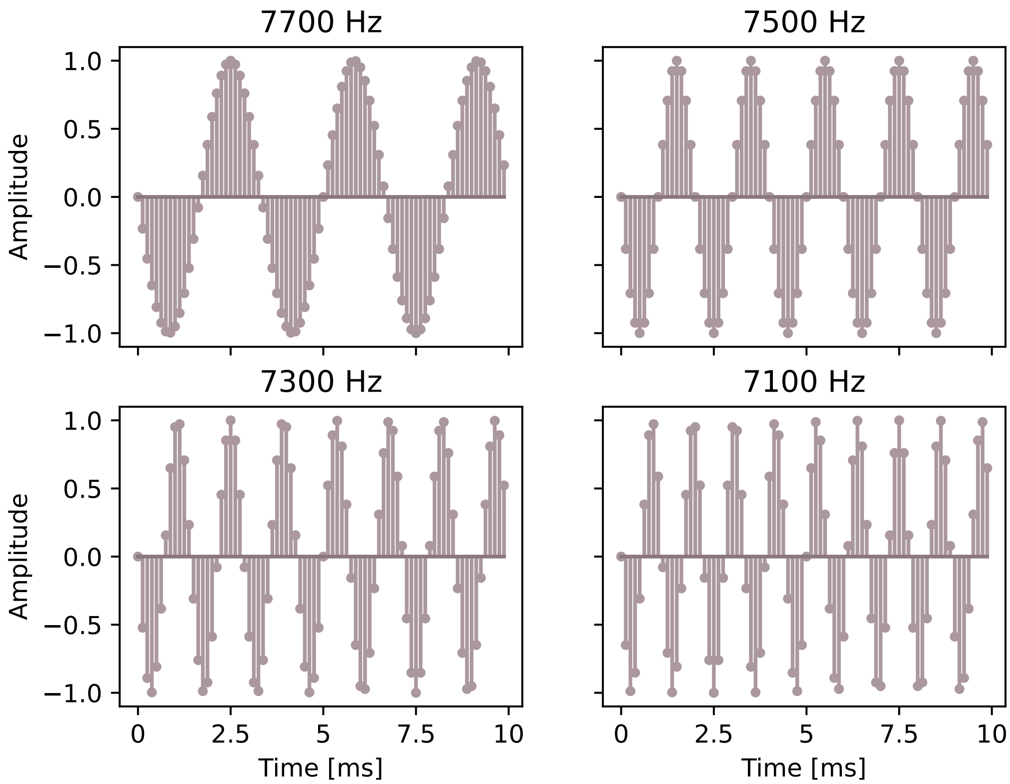

Figure 2: High Frequency Signals

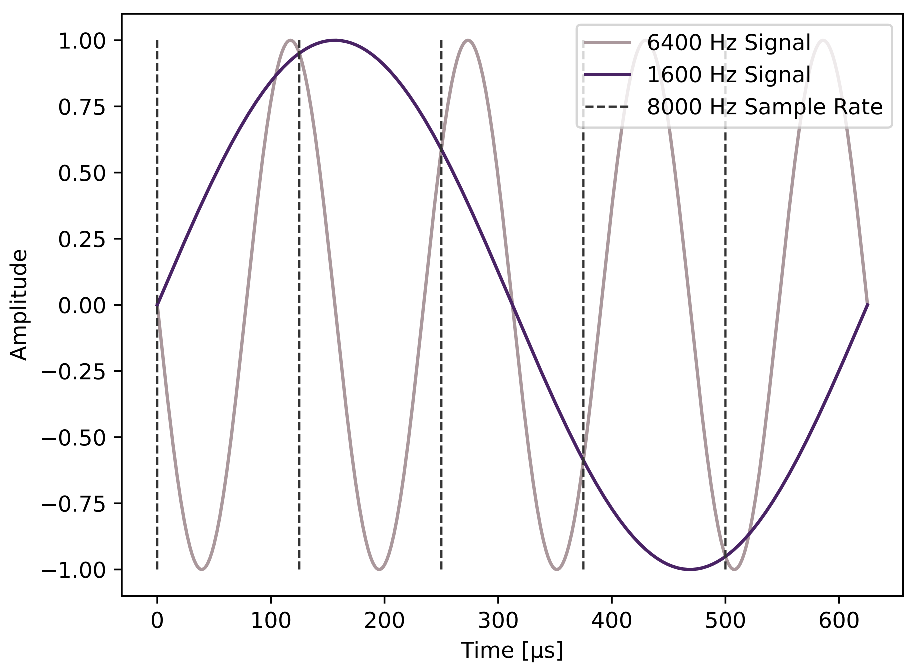

Figure 3: Aliasing Demonstration

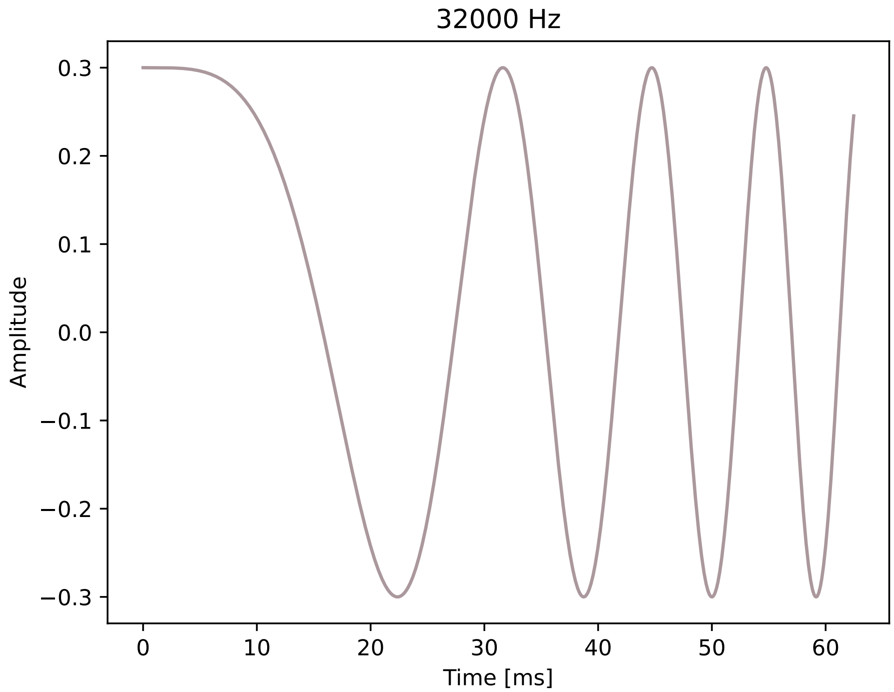

Figure 4: Chirp Signal

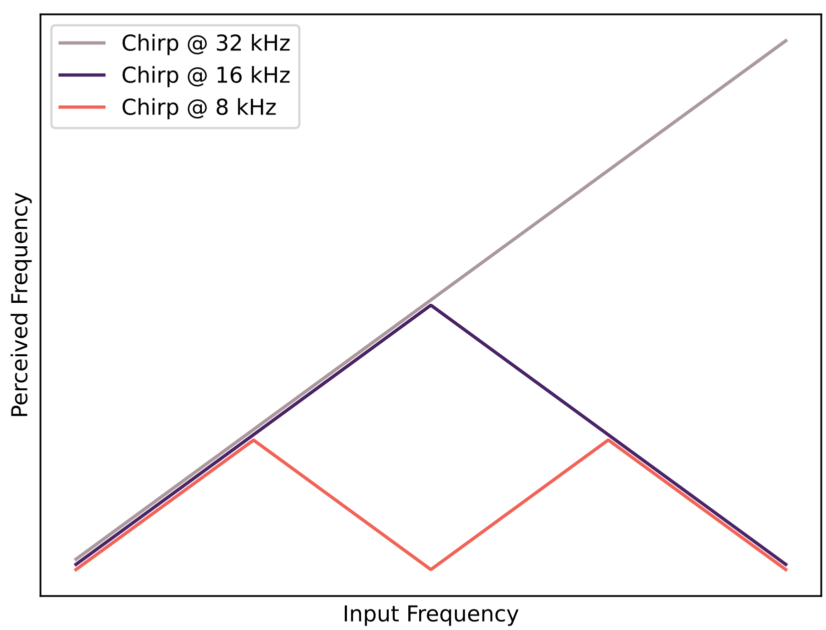

Figure 5: Chirp Samples