Probability Distributions: A Quick Reference

Statistics Department

Key Probability Distributions

Discrete Distributions

Bernoulli Distribution $X \sim \text{Bernoulli}(p)$

- PMF: $P(X = x) = p^x(1-p)^{1-x}, \quad x \in \{0, 1\}$
- Mean: $\mu = p$
- Variance: $\sigma^2 = p(1-p)$
- Application: Models binary outcomes (success/failure)

Binomial Distribution $X \sim \text{Bin}(n, p)$

- PMF: $P(X = k) = \binom{n}{k}p^k(1-p)^{n-k}, \quad k \in \{0, 1, ..., n\}$
- Mean: $\mu = np$
- Variance: $\sigma^2 = np(1-p)$
- Application: Number of successes in n independent Bernoulli trials

Poisson Distribution $X \sim \text{Poisson}(\lambda)$

- PMF: $P(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}, \quad k \in \{0, 1, 2, ...\}$
- Mean: $\mu = \lambda$
- Variance: $\sigma^2 = \lambda$
- Application: Number of events in a fixed time interval

Geometric Distribution $X \sim \text{Geom}(p)$

- PMF: $P(X = k) = (1-p)^{k-1}p, \quad k \in \{1, 2, ...\}$
- Mean: $\mu = \frac{1}{p}$
- Variance: $\sigma^2 = \frac{1-p}{p^2}$
- Application: Number of trials until first success

Continuous Distributions

Uniform Distribution $X \sim \text{Uniform}(a, b)$

- PDF: $f(x) = \frac{1}{b-a}, \quad a \leq x \leq b$
- Mean: $\mu = \frac{a+b}{2}$
- Variance: $\sigma^2 = \frac{(b-a)^2}{12}$
- Application: Models equal probability across interval

Normal Distribution $X \sim \mathcal{N}(\mu, \sigma^2)$

- PDF: $f(x) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \quad x \in \mathbb{R}$
- Mean: μ
- Variance: σ^2
- Application: Central Limit Theorem, modeling natural phenomena

Exponential Distribution $X \sim \text{Exp}(\lambda)$

- PDF: $f(x) = \lambda e^{-\lambda x}, \quad x \geq 0$
- Mean: $\mu = \frac{1}{\lambda}$
- Variance: $\sigma^2 = \frac{1}{\lambda^2}$
- Application: Time between Poisson events, memoryless property

Important Relationships

- Sum of n independent Bernoulli(p) $\Rightarrow$ Bin(n, p)
- Bin(n, p) with $n \to \infty$, $p \to 0$, $np = \lambda$ fixed $\Rightarrow$ Poisson(λ)
- Sum of n independent $\mathcal{N}(\mu_i, \sigma_i^2) \Rightarrow \mathcal{N}(\sum \mu_i, \sum \sigma_i^2)$
- Minimum of independent Exp(λ_i) $\Rightarrow$ Exp($\sum \lambda_i$)

Central Limit Theorem

For a sequence of i.i.d. random variables $X_1, X_2, ..., X_n$ with mean μ and variance σ^2, as $n \to \infty$:

$$\frac{\sum_{i=1}^{n} X_i - n\mu}{\sigma\sqrt{n}} \xrightarrow{d} \mathcal{N}(0, 1)$$