École Centrale de Nantes

Control System - Linear Part

Lab Report

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1. Aims/Objectives

The aim of this laboratory is to enable students to develop a better understanding on linear system control. Different controllers' structures and the corresponding system responses are studied in this lab. In the meanwhile, multiple observers are also introduced to the system and the system responses are observed and analyzed. Besides, comparisons are made between different controllers and observers based on the system responses.

2. Open loop simulation

2.1 Free response simulation

Given an open loop system Σ

$$\begin{aligned} \dot{x} &= Ax + Bu \quad x(0) = x_0 \\ y &= Cx \end{aligned} \tag{2.1}$$

which is defined by

$$A = \begin{bmatrix} -2 & 1 \\ 0 & -3 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix} \quad D = [0] \tag{2.2}$$

The time domain expression of the free response is

$$x_{ic}(t) = \begin{bmatrix} e^{-2t}\left(x_{01} + x_{02}\right) - e^{-3t}x_{02} \\ e^{-3t}x_{02} \end{bmatrix} \tag{2.3}$$

where x_{01} and x_{02} the two components of its initial state value.

Let x_0 = [1;10], by using the 'initial' function in Matlab, the simulated result of the free response of system Σ can be obtained. In Figure 2.1, the simulation results and the theoretical responses expression (2.3) are plotted respectively.

It can be observed that without any noise or perturbation, a system's free response is consistent with its theoretical expression. Additionally, the free responses of state x_1 and x_2 components both converge to 0 as time tends to infinity.

3. Closed loop simulation

3.1 Computation of state feedback gain and observer gain

After verifying that the system Σ is both controllable and observable, it is feasible to design a state feedback controller and develop an observer for this system. The state feedback gain F and the observer gain K can be computed by using 'acker' function in Matlab. The 'acker' function calculates the feedback gain and observer gain corresponding to specific eigenvalues.

Place the eigenvalues of A+BF on {-5, -5}, the output of 'acker' function is F = [-9, -5]. Similarly, when the desired eigenvalues for A-KC are {-10, -10}, the result obtained in Matlab is K = [15; 49]. Therefore, the feedback gain F is [-9, -5] and the observer gain K is [15; 49].

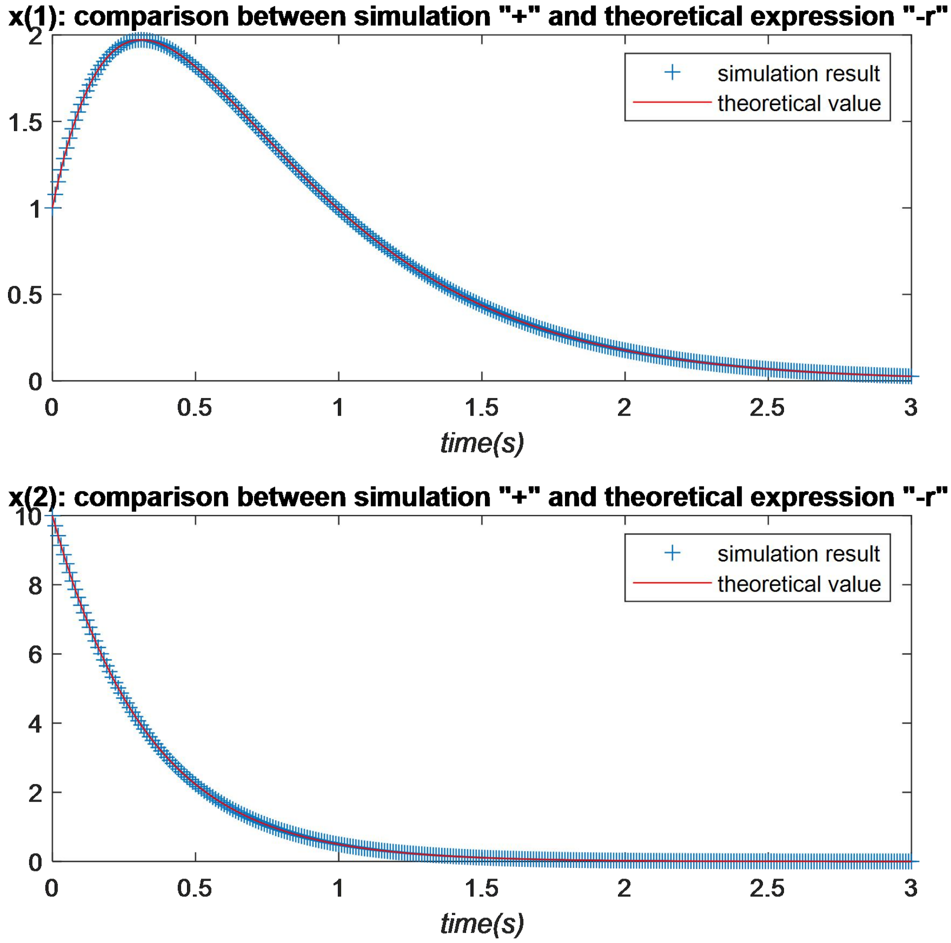

Figure 2.1: Comparison between simulation and theoretical results of an open loop system.

3.2 A new feedback gain and two new Observer gains for simulations

Considering that the matrix A is not diagonalizable when it has multiple equal eigenvalues, the desired eigenvalues for $A+BF$ and A-KC should be changed.

3.2.1 A new feedback gain

Relocate the eigenvalues of $A+BF$ on {-5.0, -5.1}, the result is F = [-9.3, -5.1]. Define the system Σ_F corresponding to the quadruple ($A_F=A+BF$, B, $C=I_2$, $D=0$)

$$\begin{cases} \dot{x} = (A+BF)x + Bv \quad x(0) = x_0 \\ y = I_2 x \end{cases} \tag{3.1}$$

which gives possibility to visualize state values directly in further simulation. The diagram of system Σ_F is shown as Figure 3.1.

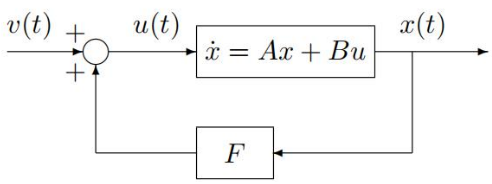

Figure 3.1: Closed loop with state feedback.

3.2.2 Two new observer gains

Since the eigenvalues of A-K_1C is located at {-5.2, -5.3} in system Σ_{FK1}: (A_{FK1}, B_{FK1}, $C=I_4$, $D_{FK1}=0$). The corresponding result obtained in Matlab is K_1 = [5.50; 5.06].

For system Σ_{FK2}: (A_{FK2}, B_{FK2}, $C=I_4$, $D_{FK2}=0$), in which eigenvalues of A-K_2C are {-10.0; -10.1}, the computed result of observer gain is K_2 = [15.1; 49.7].

Based on system Σ_F, an observer can be added to estimate the values of state, as shown in Figure 3.2.

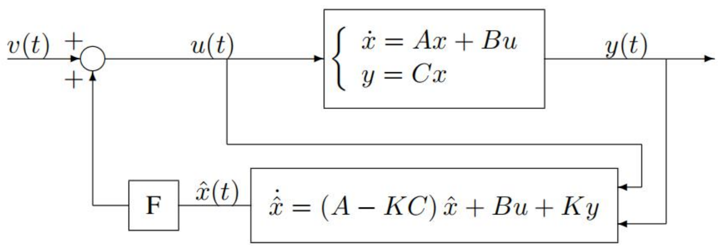

Figure 3.2: Closed loop with observer and state feedback.

This new system's expression is

$$\begin{cases} \begin{bmatrix} \dot{x} \\ \dot{\hat{x}} \end{bmatrix} = \begin{bmatrix} A & BF \\ KC & A-KC+BF \end{bmatrix} \begin{bmatrix} x \\ \hat{x} \end{bmatrix} + \begin{bmatrix} B \\ B \end{bmatrix} v \\ y = \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} x \\ \hat{x} \end{bmatrix} \end{cases} \tag{3.2}$$

To simplify the expression, the following notations are used:

$$A_{FK} = \begin{bmatrix} A & BF \\ KC & A-KC+BF \end{bmatrix}, \quad B_{FK} = \begin{bmatrix} B \\ B \end{bmatrix}, \quad C_{FK} = \begin{bmatrix} C & 0 \end{bmatrix} \tag{3.3}$$

Therefore, the system Σ_{FK1} is defined as

$$A_{FK1} = \begin{bmatrix} -2 & 1 & 0 & 0 \\ 0 & -3 & -9.3 & -5.1 \\ 5.5 & 0 & -7.5 & 1 \\ 5.1 & 0 & -14.4 & -8.1 \end{bmatrix}, B_{FK1} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}, C_{FK1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.4}$$

The system Σ_{FK2} is defined as

$$A_{FK2} = \begin{bmatrix} -2 & 1 & 0 & 0 \\ 0 & -3 & -9.3 & -5.1 \\ 15.1 & 0 & -17.1 & 1 \\ 49.7 & 0 & -59 & -8.1 \end{bmatrix}, B_{FK2} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}, C_{FK2} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.5}$$

3.3 Closed loop simulation with the first observer

Set the initial state values of system Σ_{FK1} as [x_{0FK1}; $\hat{x}_{0FK1}$] = [1; 10; 0; 0]. The simulated free response of it is shown as Figure 3.3.

According to Figure 3.3, it can be seen that the estimation errors converge to 0. Therefore, the estimated states in observer tend to have the same values as the true state when simulation time approaches to infinity.

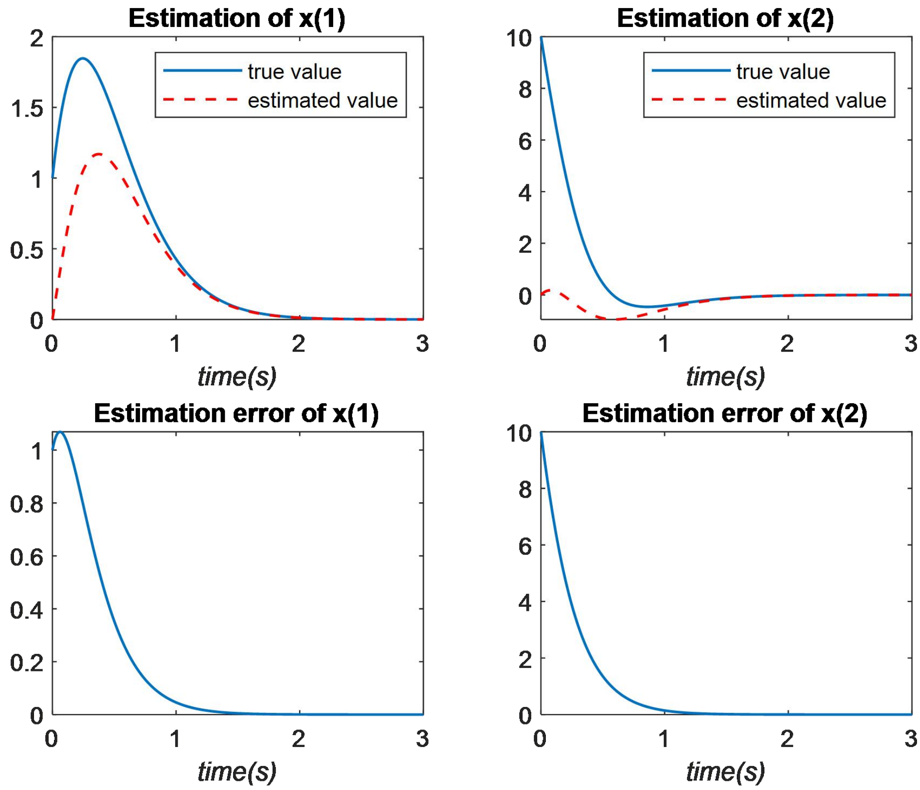

Figure 3.3: Observation of state values in system Σ_{FK1}.

In Table 3.1, the system's freedom response is listed with values corresponding to two specific time.

Table 3.1: Response of system Σ_{FK1} at $t = 2.8$s and $t = 2.95$s.

	x_{1FK1}	$\hat{x}_{1FK1}$	$\tilde{x}_{1FK1}$	x_{2FK1}	$\hat{x}_{2FK1}$	$\tilde{x}_{2FK1}$
$t = 2.8$s	4.957×10^{-4}	4.863×10^{-4}	0.904×10^{-4}	-0.011	-0.011	0
$t = 2.95$s	2.632×10^{-4}	2.587×10^{-4}	0.045×10^{-4}	-5.899×10^{-4}	-6.015×10^{-4}	0.116×10^{-4}

3.4 Closed loop simulation with the second observer

Similar to Section 3.3, the initial state values of system Σ_{FK2} are $[x_{0FK2};\ \hat{x}_{0FK2}] = [1; 10; 0; 0]$. The corresponding simulation result is shown in Figure 3.4 and Table 3.2.

A similar result is obtained in system Σ_{FK2}. Compared with the simulation result in Section 3.3, the estimated values can converge to true values more quickly, which proves that the second observer is faster.

Table 3.2: Response of system Σ_{FK2} at $t = 2.8$s and $t = 2.95$s.

	x_{1FK2}	$\hat{x}_{1FK2}$	$\tilde{x}_{1FK2}$	x_{2FK2}	$\hat{x}_{2FK2}$	$\tilde{x}_{2FK2}$
$t = 2.8$s	5.926×10^{-5}	5.926×10^{-5}	0	-1.585×10^{-4}	-1.585×10^{-4}	0
$t = 2.95$s	2.934×10^{-5}	2.934×10^{-5}	0	-7.907×10^{-5}	-7.907×10^{-5}	0

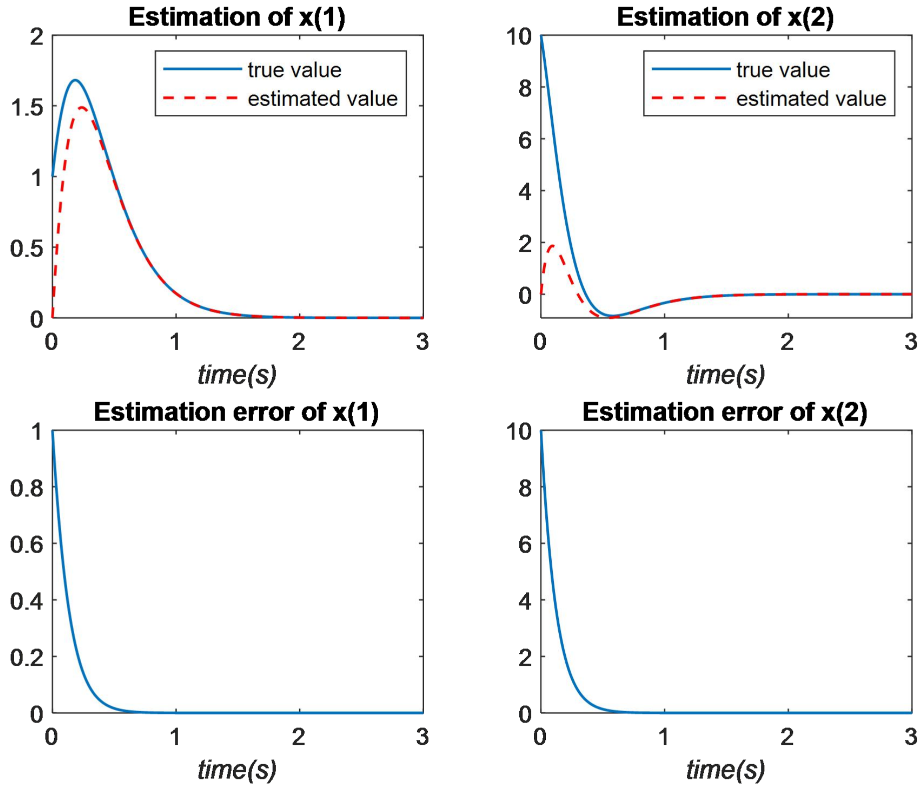

Figure 3.4: Observation of state values in system Σ_{FK2}.

3.5 Which observer is the faster observer

Plot the observations of system Σ_{FK1} and Σ_{FK2} in the same figure, as shown below:

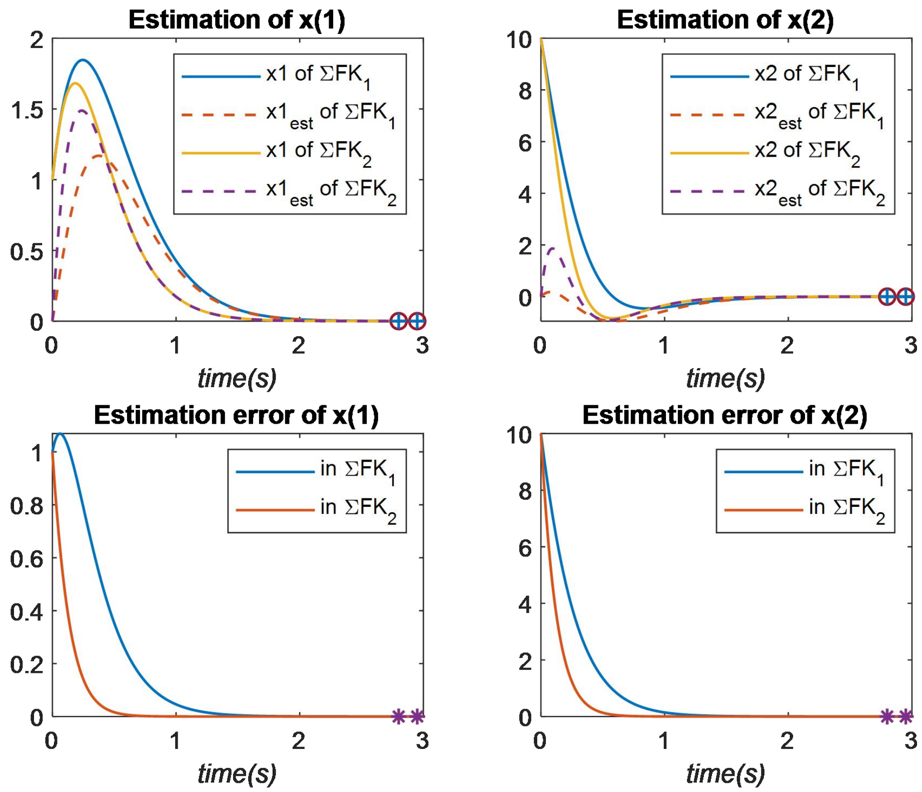

Figure 3.5: Observations of state values in system Σ_{FK1} and Σ_{FK2}.

In order to see more information, zoom the region where time is from 2.75s to 3s. In this region the results in two specific time are also included, as shown in Figure 3.6.

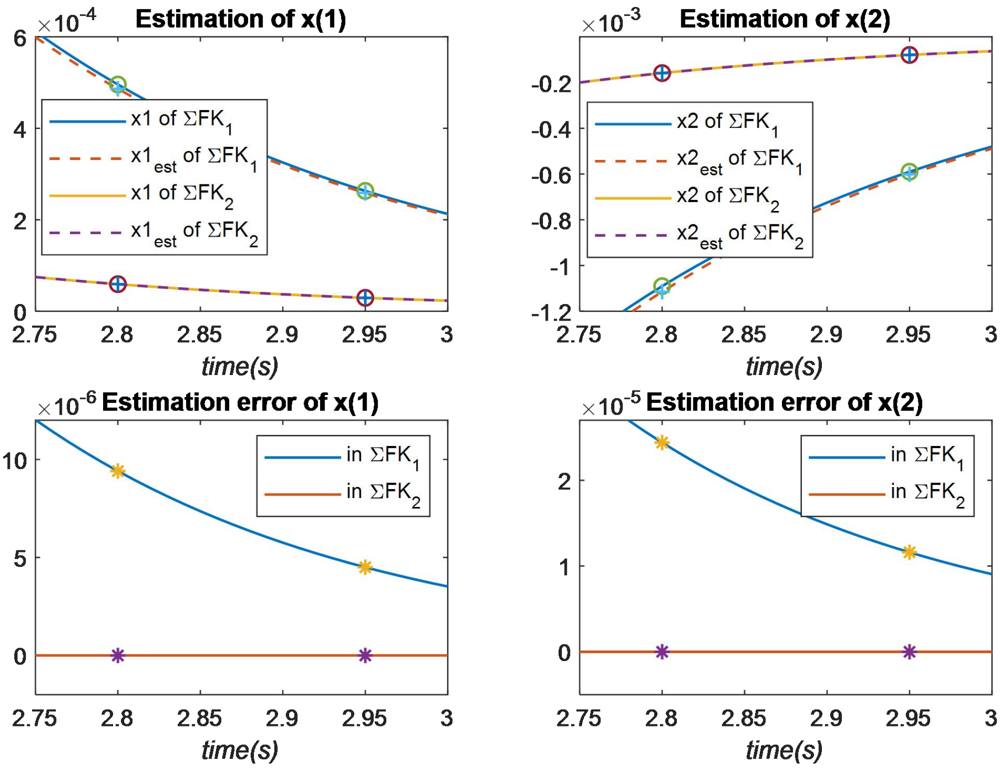

Figure 3.6: Observations of state values from 2.75s to 3s.

It can be clearly seen that in this time period the estimated values have converged to the real values in Σ_{FK2}, while there are still non-zero estimation errors in Σ_{FK1}.

This comparison strengthens the conclusion in Section 3.4, that the second observer is much faster, because it has poles which are farther away from original point in *s* domain.

3.6 Comparison of the free response of Σ, Σ_F, Σ_{FK1}, Σ_{FK2}

3.6.1 Simulation with initial condition $x_0 = [1; 10]$

The simulation results are shown in Figure 3.7, state values in Σ_F converge to zero at the fastest speed compared with other systems. The second fast system is Σ_{FK2}, which is easily understandable according to the conclusion in Section 3.5.

To explain why the free response of Σ_F is the fastest, the role of observer in a system should be mentioned. Observer gathers information of a system's input and output to estimate the values of state and send the estimated states to feedback controller. In normal case, it takes time for estimated values converge to the true values, which means observer-feedback control law behaves slower than feedback control law based on true state values.

3.6.2 Simulation with initial condition $x_0 = [2; 4]$

Change the initial condition of true state values, the simulation results are indicated in Figure 3.8 and Figure 3.9.

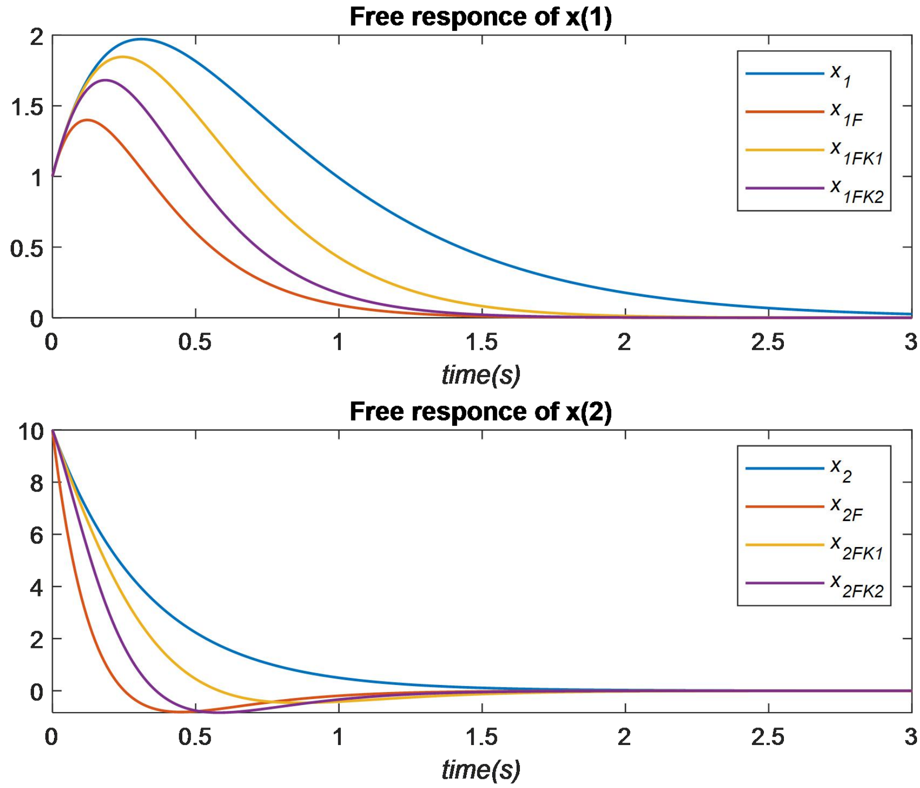

Figure 3.7: Simulation with initial condition x_0 = [1; 10].

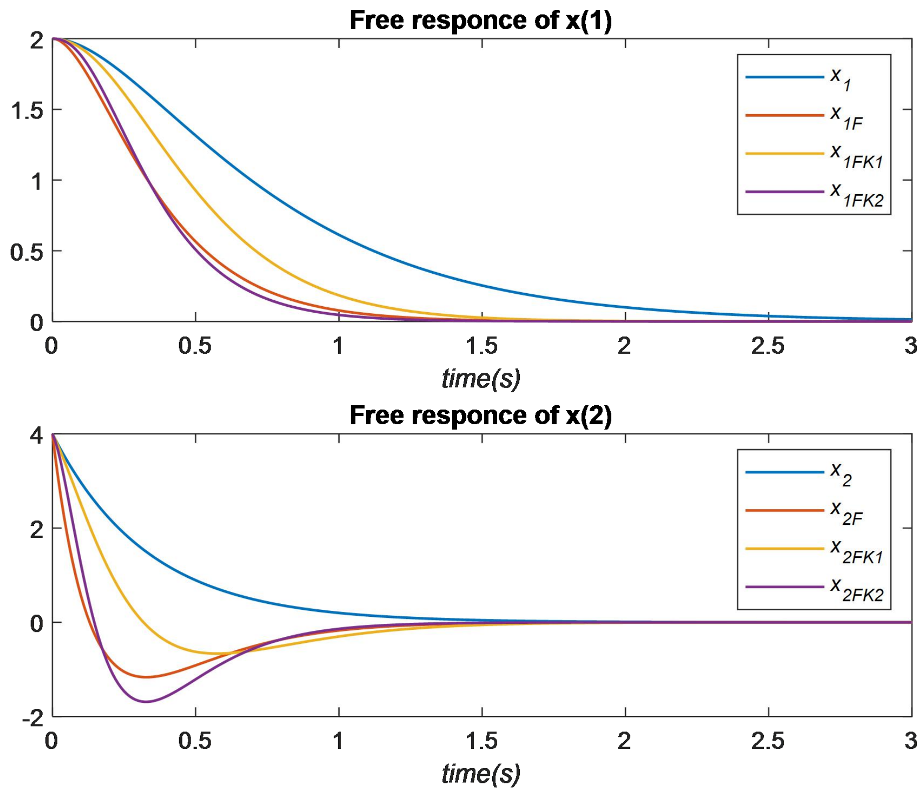

Figure 3.8: Simulation with initial condition x_0 = [2; 4].

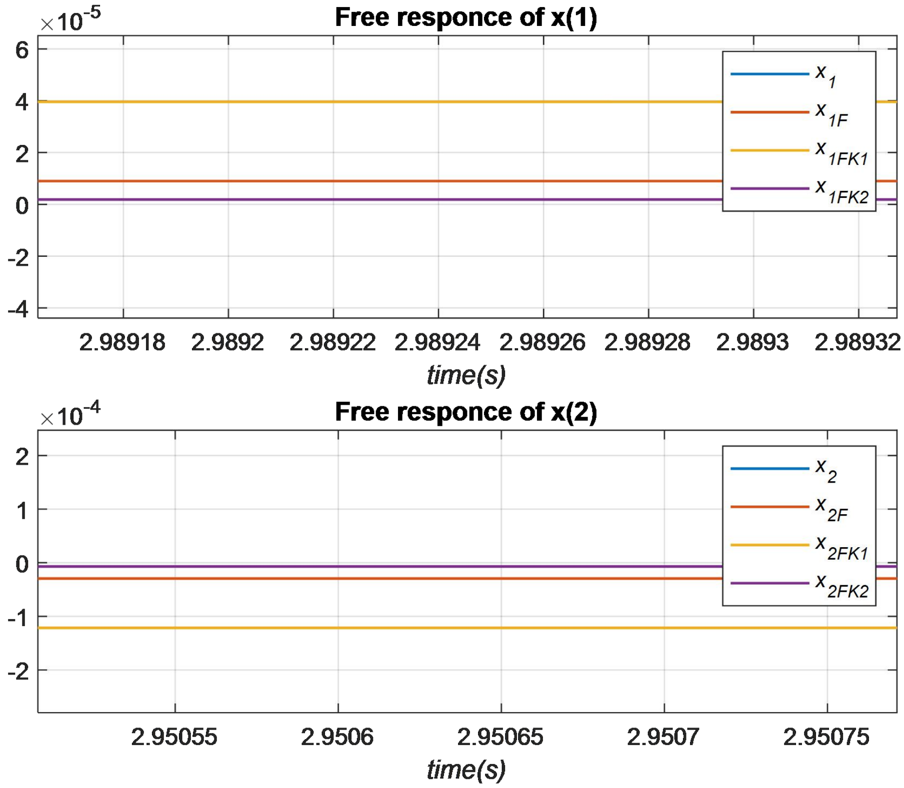

Figure 3.9: Zoom the overlapping region in Figure 3.8.

According to the figures above, the free response of Σ_{FK2} becomes the fastest in this case, which conflicts with the previous statement. It should be noticed that the deviations between initial state values and their initial estimated values are smaller than the one in Section 3.6.1. This limits the estimator influence on free response.

To explain why the output response of observer-feedback control can be faster than a feedback control based on true state values, one can suppose that a feedback control is an observer-feedback control whose initial state in observer is exactly the same as the initial state in plant. In this way, the input of feedback controller is always equivalent to the true state, which behaves the same as a feedback control law without observer. Therefore, Σ_F and Σ_{FK2} have the same state space model. The only difference between the two systems is the initial state: one is [2; 4; 2; 4], another is [2; 4; 0; 0]. Thus, it is possible for the second one to get a faster response.

4. Conclusion

In this lab, it can be concluded that under the same initial state condition, the observer-feedback control system response faster, with the poles of observer farther away from original point in s-domain. Besides, it is possible to design an observer-feedback control system to have a better response than a feedback control system, especially when the initial values in observer are closed to true values.