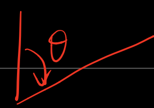
1\. a) $\tan\theta[n] = -\dfrac{x_H - X[n]}{z_H} = Y[n]$



θ is negative

$\therefore\ X[n] = x_H + z_H Y[n]$

b) $Y[n] = Y^{*}[n] + w[n]$ $\quad \cancel{r - \frac{q}{z_H}}$

d) $X[n+1] = X[n] + D + v[n]$; $X[1] = 0$ with $Var(v[n]) = q$

c) $Y[n] = \dfrac{X[n] - X_H}{z_H} + w[n]$ with $Var(w[n]) = r$

e) $\hat{x}^{1|0}[1] = 0$ see algo. 3.1 in the book.

$P^{1|0}[1] = 0$

f) Loop $n \geq 1$

$$\left[\begin{array}{l} \hat{y}|^{n-1}[n] = \dfrac{\hat{x}|^{n-1}[n] - x_H}{z_H} \\ C_{x,y}[n] = P|^{n-1}[n] / z_H \\ C_{y,y}[n] = P|^{n-1}[n] / z_H^2 + r \end{array}\right.$$

$y[n]$ is observed (new information) $\quad$ Kalman gain

$$\left[\begin{array}{l} x|^{n}[n] = x|^{n-1}[n] + \dfrac{C_{x,y}[n]}{C_{y,y}[n]}\left(y[n] - \hat{y}|^{n-1}[n]\right) \\ p|^{n}[n] = \dfrac{P|^{n-1}[n] \cdot z_H^2 r}{P|^{n-1}[n] + z_H^2 r} \end{array}\right.$$

g)

$$\left[\begin{array}{l} \hat{x}|^{n}[n+1] = \hat{x}|^{n}[n] + D \\ P|^{n}[n+1] = P|^{n}[n] + q \end{array}\right.$$

h) $r = +\infty$ $\quad \hat{x}|^{n}[n] = (n-1)\cdot D$ $\quad$ Kalman gain $= 0$

$P|^{n}[n] = (n-1)\cdot q$ $\quad P|^{n}[n] = P|^{n-1}[n]$

i) $q = +\infty$ $\quad \hat{x}|^{n}[n] = x_H + z_H \cdot y[n]$

$P|^{n}[n] = z_H^2 r$ $\quad$ see question 1

b) $x[n] = X_H + z_H \cdot Y_{true}[n]$

$\hat{x}[n] = X_H + z_H\left(Y_{true}[n] + w[n]\right)$

error $\quad \hat{x}[n] - x[n] = z_H w[n]$ $\quad \boxed{Var(Ax+B) = A\,Var(x)\,A^{T}}$

error variance $\quad Var\left(\hat{x}[n] - x[n]\right) = z_H^2\ Var(w[n])$

$= z_H^2 r$

Kalman recursion

...

input $\hat{X}^{|n-1}[n]$ with $P^{|n-1}[n]$

inside $\hat{X}^{|n}[n]$ with $P^{|n}[n]$

output $\hat{X}^{|n}[n+1]$ with $P^{|n}[n+1]$

$$P - \frac{P/Z_H}{P/Z_H^2 + r} \cdot \frac{P}{Z_H}$$

$$= P - \frac{P \cancel{Z_H}^{2}}{P + Z_H^2 r} \cdot \frac{\cancel{P}}{\cancel{Z_H}}$$

$$= \frac{\cancel{P^2} + Z_H^2 r P - \cancel{P^2}}{P + Z_H^2 r}$$

$$P - \frac{P/Z_H}{P/Z_H^2 + r} \cdot \frac{P}{Z_H}$$

$$\downarrow$$

$$\frac{P^2 \cdot \cancel{Z_H}}{P + r \cdot Z_H^2} \cdot \frac{\cancel{P}}{\cancel{Z_H}}$$

$$\frac{\cancel{P^2} + P \cdot r \cdot Z_H^2 - \cancel{P^2}}{P + r \cdot Z_H^2}$$