COSYS - Linear Part

M1 CORO EPICO

Lab - Second Part (simulation with Matlab or Octave)

Wednesday 6 October 2021

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1 Introduction of the second part of the LAB

Several dynamical systems have been explicitly or implicitly introduced in the first part of the Lab. Let us first do a review of them and introduce some notations.

1.1 Notation for the lab

The Open loop system: The first five sections were dedicated to the analysis of the open loop system. Let us denote (Σ) the quadruple $(A, B, C, D = 0)$ or linear system

$$(\Sigma) \qquad \begin{cases} \dot{x} &= Ax + Bu \qquad\qquad x(0) = x_0 \\ y &= Cx \end{cases}$$

defined by

$$A = \begin{bmatrix} -2 & 1 \\ 0 & -3 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix} \quad D = \begin{bmatrix} 0 \end{bmatrix}$$

One can denote x_1 and x_2 the two components of its state, x_{01} and x_{02} the two components of its initial value.

The closed loop system with static state feedback: In section 6, with the calculation of the static state feedback, one implicitly introduced the closed loop system with static state feedback.

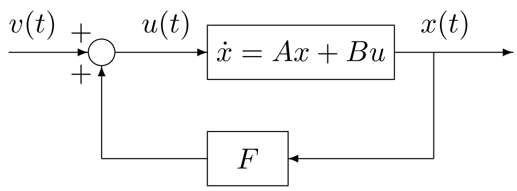

Figure 1: Closed loop with a static state feedback.

Let us denote (Σ_F) the quadruple $(A_F = A + BF, B, C, D = 0)$ or linear system

$$(\Sigma_F) \qquad \begin{cases} \dot{x} &= (A + BF)x + Bv \qquad\qquad x(0) = x_0 \\ y &= Cx \end{cases}$$

Denote x_{1_F} and x_{2_F} the two components of its state, x_{01_F} and x_{02_F} the two components of its initial value.

The closed loop system with observer and state feedback: In section 7, with the calculation of observer gain, one implicitly introduced the closed loop system with observer and state feedback (figure 2).

Let us denote (Σ_{FK}) the quadruple $(A_{FK}, B_{FK}, C_{FK}, D_{FK} = 0)$ or linear system

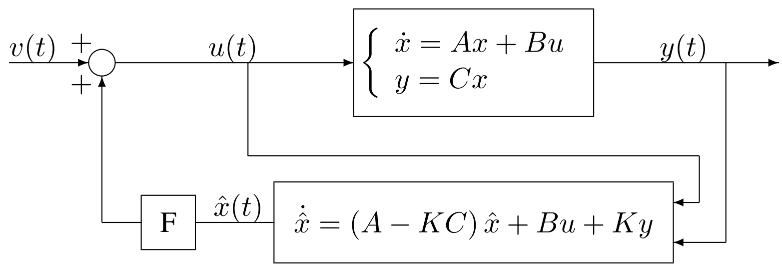

Figure 2: Closed loop with observer and state feedback.

$$(\Sigma_{FK}) \qquad \left\{ \begin{array}{l} \begin{bmatrix} \dot{x} \\ \dot{\hat{x}} \end{bmatrix} = \begin{bmatrix} A & BF \\ KC & A - KC + BF \end{bmatrix} \begin{bmatrix} x \\ \hat{x} \end{bmatrix} + \begin{bmatrix} B \\ B \end{bmatrix} v \\ y = \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} x \\ \hat{x} \end{bmatrix} \end{array} \right.$$

One can denote $x1_{FK}$, $x2_{FK}$, $\hat{x}1_{FK}$, $\hat{x}2_{FK}$ the four components of its state, and $x01_{FK}$, $x02_{FK}$, $\hat{x}01_{FK} = 0$, $\hat{x}02_{FK} = 0$ the four components of its initial value.
The following notation are used :

$$A_{FK} = \begin{bmatrix} A & BF \\ KC & A - KC + BF \end{bmatrix}, \quad B_{FK} = \begin{bmatrix} B \\ B \end{bmatrix}, \quad C_{FK} = \begin{bmatrix} C & 0 \end{bmatrix}$$

2 Open loop simulation

2.1 Checking the result of part1, subsection 5.2.1

In subsection 5.2.1 of part 1, you should have obtained that the theoretical expression of the free response is

$$x(t) = \begin{bmatrix} e^{-2t}\,(x_{01} + x_{02}) - e^{-3t} x_{02} \\ e^{-3t} x_{02} \end{bmatrix}$$

With Matlab (or Octave) the response due to initial condition x_0 can be obtained with the function "`initial`" (use help to understand how to use it).
Use this function to check that your expression is correct with $\mathbf{x_0} = [\mathbf{1}; \mathbf{10}]$.
Using the "`subplot`" function of Matlab or Octave, give a figure with two axis and the following plots

x_1, xv_1
x_2, xv_2

where x_i is the plot of the component "i" of the theoretical expression and xv_i is the corresponding simulated component.

Note : the answer to this question is given in the M-file : `TP_epico_student.m`

3 Closed loop simulation

3.1 Checking the static feedback F and observer gain K of part1

In part1 of the exercise, a static feedback F and a output injection K were calculated such that $\sigma^d_{(A+BF)} = \{-5, -5\}$ and $\sigma^d_{(A-KC)} = \{-10, -10\}$. You can check your result using the `acker` or `place` functions of Matlab (or Octave).

Note : remember that the conventions of the course imply that one should use: `F=-acker(A,B,pf)` (where "pf" is the vector which defined the desired eigenvalues for $A + BF$). And that one should use: `K=acker(A.',C.',pk).'` (where "pk" is the vector which defined the desired eigenvalues for $A - KC$).

3.2 A new feedback gain and two new Observer gains for simulations

In the first chapter of the course, it is explain that when a A matrix has multiple equal eigenvalues, the matrix A could be not diagonalizable. In order to avoid this case. The desired eigenvalues for $A+BF$ and $A-KC$ will be changed for the rest of the lab.

3.2.1 A new feedback gain

With Matlab (or Octave), compute the feedback gain F such that $\sigma^d_{(A+BF)} = \{-5.0, -5.1\}$.
Define the system (Σ_F) corresponding to the quadruple $(A_F = A + BF, B, I_2, D = 0)$.

Note: you can use the following code `AF=A+B*F;Gss_AF=ss(AF,B,eye(2),D);`
Remark that in the proposed code the output matrix is the identity (I_2, $eye(2)$ with Matlab or Octave). This will give the possibility, in the simulation to visualize $x1_F$ and $x2_F$.

3.2.2 Two new observer gains

One aim of the Lab will be to illustrate the difference in the use of a slow observer and a fast observer. It is proposed here to define two observer gains.
Compute the observer gain K_1 such that $\sigma^d_{(A-K_1C)} = \{-5.2, -5.3\}$.
Compute the feedback gain K_2 such that $\sigma^d_{(A-K_2C)} = \{-10.0, -10.1\}$.
Define the system (Σ_{FK_1}) corresponding to the quadruple $(A_{FK_1}, B_{FK_1}, I_4, D_{FK_1} = 0)$.
Define the system (Σ_{FK_2}) corresponding to the quadruple $(A_{FK_2}, B_{FK_2}, I_4, D_{FK_2} = 0)$.

Note: Remark that in the proposed code the output matrix is the identity (I_4, $eye(4)$ with Matlab or Octave). This will give the possibility, in the simulation to visualize $x1_{FK_1}$, $x2_{FK_1}$, $\hat{x}1_{FK_1}$, $\hat{x}2_{FK_1}$.

3.3 Closed loop simulation with the first observer

Simulate the free response ($v = 0$) of the closed loop of figure 2, using the system (Σ_{FK_1}) with $x0_{FK_1} = [x01_{FK_1}; x02_{FK_1}] = [1; 10]$ and $\hat{x}0_{FK_1} = [\hat{x}01_{FK_1}; \hat{x}02_{FK_1}] = [0; 0]$.
Check that $\hat{x}1_{FK_1} \to x1_{FK_1}$ and $\hat{x}2_{FK_1} \to x2_{FK_1}$ when $t \to \infty$, or equivalently that $\tilde{x}1_{FK_1} = x_1 - \hat{x}1_{FK_1} \to 0$ and $\tilde{x}2_{FK_1} = x2_{FK_1} - \hat{x}2_{FK_1} \to 0$ when $t \to \infty$.
Plot the results on a figure with 4 axis with the plots arrange in the following way,

$x1_{FK_1}$, $\hat{x}1_{FK_1}$	$x2_{FK_1}$, $\hat{x}2_{FK_1}$
$\tilde{x}1_{FK_1}$	$\tilde{x}2_{FK_1}$

For these simulations, it is proposed to use the following time vector : `t=[0:0.01:3];`
Give the values of $x1_{FK_1}$, $\hat{x}1_{FK_1}$ and $\tilde{x}1_{FK_1}$ for $t = 2.8$ and $t = 2.95$.
Give the values of $x2_{FK_1}$, $\hat{x}2_{FK_1}$ and $\tilde{x}2_{FK_1}$ for $t = 2.8$ and $t = 2.95$.

3.4 Closed loop simulation with the second observer

Simulate the free response ($v = 0$) of the closed loop of figure 2, using the system (Σ_{FK_2}) with $x0_{FK_2} = [x01_{FK_2}; x02_{FK_2}] = [1; 10]$ and $\hat{x}0_{FK_2} = [\hat{x}01_{FK_2}; \hat{x}02_{FK_2}] = [0; 0]$.
Check that $\hat{x}1_{FK_2} \to x1_{FK_2}$ and $\hat{x}2_{FK_2} \to x2_{FK_2}$ when $t \to \infty$, or equivalently that $\tilde{x}1_{FK_2} = x_1 - \hat{x}1_{FK_2} \to 0$ and $\tilde{x}2_{FK_2} = x2_{FK_2} - \hat{x}2_{FK_2} \to 0$ when $t \to \infty$.
Plot the results on a figure with 4 axis with the plots arrange in the following way,

$x1_{FK_2}$, $\hat{x}1_{FK_2}$	$x2_{FK_2}$, $\hat{x}2_{FK_2}$
$\tilde{x}1_{FK_2}$	$\tilde{x}2_{FK_2}$

For these simulations, it is proposed to use the following time vector : `t=[0:0.01:3];`
Give the values of $x1_{FK_2}$, $\hat{x}1_{FK_2}$ and $\tilde{x}1_{FK_2}$ for $t = 2.8$ and $t = 2.95$.
Give the values of $x2_{FK_2}$, $\hat{x}2_{FK_2}$ and $\tilde{x}2_{FK_2}$ for $t = 2.8$ and $t = 2.95$.

3.5 Which observer is the faster observer

Plot the previous results on a figure with 4 axis where the plots are arranged in the following way, (use adequate colors and the possibility to use dotted plot)

$x1_{FK_1}$, $\hat{x}1_{FK_1}$, $x1_{FK_2}$, $\hat{x}1_{FK_2}$	$x2_{FK_1}$, $\hat{x}2_{FK_1}$, $x2_{FK_2}$, $\hat{x}2_{FK_2}$
$\tilde{x}1_{FK_1}$, $\tilde{x}1_{FK_2}$	$\tilde{x}2_{FK_1}$, $\tilde{x}2_{FK_2}$

Which observer is the faster one ?
Clearly detail your answer (not just the first or the second one, use a zoom, indicators like the previously obtained values ...).

3.6 Comparison of the free response of (Σ), (Σ_F), (Σ_{FK_1}) and (Σ_{FK_2})

From the values of the eigenvalues of the modes of the different systems could you predict a classification of the convergence to zero of the four free responses?

3.6.1 Simulation with initial condition $\mathbf{x_0} = [\mathbf{1}; \mathbf{10}]$

Compare, with $x_0 = x_{0_F} = x_{0_{FK_i}}=[1; 10]$ and $\hat{x}_{0_{FK_i}} = [0; 0]$, the free responses of the states components of (Σ), (Σ_F), (Σ_{FK_1}) and (Σ_{FK_2}), on a figure with two axis like below.

x_1, $x1_F$, $x1_{FK_1}$, $x1_{FK_2}$
x_2, $x2_F$, $x2_{FK_1}$, $x2_{FK_2}$

Which response converge the faster to zero ? does this correspond to your prediction?
Once again, clearly detail your answer (not just the first or the second one).

3.6.2 Simulation with initial condition $\mathbf{x_0} = [\mathbf{2}; \mathbf{4}]$

Do the same comparison but with $x_0 = x_{0_F} = x_{0_{FK_i}}=[2; 4]$ and $\hat{x}_{0_{FK_i}} = [0; 0]$.

x_1, $x1_F$, $x1_{FK_1}$, $x1_{FK_2}$
x_2, $x2_F$, $x2_{FK_1}$, $x2_{FK_2}$

Is it the same classification result ? Give an explanation.