a) $\text{Mean}(w) = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$

$\text{Var}(w) = \begin{bmatrix} \sigma^2 & & 0 \\ & \ddots & \\ 0 & & \sigma^2 \end{bmatrix}$

b) $Y[n] = u[n]x + w[n]$

$$\underbrace{Y \to \begin{bmatrix} Y(1) \\ \vdots \\ Y[n] \end{bmatrix} = u \to \begin{bmatrix} u[1] \\ \vdots \\ u[N] \end{bmatrix} X + w \to \begin{bmatrix} w[1] \\ \vdots \\ w[n] \end{bmatrix}}$$

c) $(H^T C_{ww}^{-1} H)^{-1} H^T C_{ww}^{-1} (Y - m_w)$

$H \to u$, $ww \to w$

$$\underbrace{(u^T C_{ww}^{-1} u)}_{G_1}{}^{-1} \underbrace{u^T C_{ww}^{-1}}_{G_2} (Y - m_w), \quad m_w \to 0$$

$u^T = u(1) \cdots u[n]$

$$\underbrace{\underset{1\times N}{[u(1) \cdots u[N]]} \underset{N\times N}{\begin{bmatrix} \sigma^{-2} & & 0 \\ & \ddots & \\ 0 & & \sigma^{-2} \end{bmatrix}}}\underset{N\times 1}{\begin{bmatrix} u(1) \\ \vdots \\ u(N) \end{bmatrix}}$$

$C_{ww} \begin{bmatrix} \sigma^2 & & 0 \\ & \ddots & \\ 0 & & \sigma^2 \end{bmatrix}$

$C_{ww}^{-1} = \begin{bmatrix} \frac{1}{\sigma^2} & & 0 \\ & \ddots & \\ 0 & & \frac{1}{\sigma^2} \end{bmatrix}$

$$= \sigma^{-2} [u(1) \cdots u(N)] \begin{bmatrix} u(1) \\ \vdots \\ u(N) \end{bmatrix} = \frac{1}{\sigma^2} \sum_{i=1}^{N} u(i)^2 = G_1$$

$$u^T C_{ww}^{-1} = \frac{1}{\sigma^2} [u(1) \cdots u(N)] = G_2$$

$m_w = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$

$$G_1^{-1} G_2 y = \frac{1}{\sum_{i=1}^{N} u(i)^2} [u(1) \cdots u(N)] \begin{bmatrix} y[1] \\ \vdots \\ y[N] \end{bmatrix} \Rightarrow \hat{x} = \frac{\sum_{i=1}^{N} u(i) y(i)}{\sum_{i=1}^{N} u(i)^2}$$

d) Bias $= 0$

$$\text{Errvar}(x) = (H^T C_{ww}^{-1} H)^{-1} = (U^T C_{ww}^{-1} U)^{-1} = \frac{\sigma^2}{\sum_{i=1}^{N} u(i)^2}$$

e)