CORO

AV (first part) & STATES: sample exam 2

1 Probability and estimation

Reminder: $\sum_{k=1}^{n} k = \frac{n\,(n+1)}{2}$, $\sum_{k=1}^{n} k^2 = \frac{n\,(n+1)\,(2\,n+1)}{6}$
A discrete valued random variable y is uniformly distributed over $\{1, \ldots, \mathsf{x}\}$ where x is integer valued.

a) Write the PMF of y given x.

b) Write the expectation of y given x.

c) Write the variance of y given x.

d) y is measured. Write the maximum likelihood estimator of x.

e) $\vec{\mathsf{y}} = (\mathsf{y}_1, \ldots, \mathsf{y}_n)$ is a set of n random variables independent given x, and driven by the distribution above. $\vec{\mathsf{y}}$ is measured. Write the maximum likelihood estimator of x.

2 Kalman filter

n is the minute index. An autonomous robot leaves its base position at minute $n = 1$. It moves along a straight line. $\mathsf{x}[n]$ is the actual position at minute n (so $\mathsf{x}[1]$ is the base position).
The base position is perfectly known (and its value is 0).
The odometer gives a measure $\mathsf{u}[n]$ of the distance between , but the actual distance browsed between minute n and minute $n+1$, but the actual distance is $\mathsf{u}[n] + \mathsf{v}[n]$. $(\mathsf{v}[n])_{n\geq 1}$ is a zero-mean white noise with variance q.
Every day, the embedded GPS navigator gives a measure $\mathsf{y}[n]$ of position with an error $\mathsf{w}[n]$. $(\mathsf{w}[n])_{n\geq 1}$ is a zero mean white noise with variance r.
The objective is to estimate $\mathsf{x}[n]$ given $\mathsf{y}[1], \ldots, \mathsf{y}[n]$ by means of a Kalman filter.

a) Propose a value for the initial prediction $\mathsf{x}^{|0}[1]$ and its variance $P^{|0}[1]$.

b) Give the update equations of the Kalman filter ($\mathsf{x}^{|n}[n]$ and $P^{|n}[n]$ in function of $\mathsf{x}^{|n-1}[n]$, $P^{|n-1}[n]$, r and the new observation $\mathsf{y}[n]$.

c) Give the prediction equations of the Kalman filter ($\mathsf{x}^{|n}[n+1]$ and $P^{|n}[n+1]$ in function of $\mathsf{x}^{|n}[n]$, $P^{|n}[n]$, D and q).

d) The GPS is out of order ($r = +\infty$). Give $\mathsf{x}^{|n}[n]$ and $P^{|n}[n]$ in function of n, $(\mathsf{u}[k])_{k\leq n}$ and q.

e) The odometer is out of order ($q = +\infty$). Give $\mathsf{x}^{|n}[n]$ and $P^{|n}[n]$ in function of $\mathsf{y}[n]$ and r.