Advanced Control Laboratory Report

Ex2: Numerical Analysis, Simulation and Control

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Abstract

The goal of this exercise is to learn the usage of the software package MATLAB/SIMULINK and its application in control engineering systems.

1. Results of Exercise 1.8

Table 1 shows different combinations utilised in our simulation.

Table 1: Parameter combinations for the measurement noise signal covariance, process covariance matrix and measurement covariance matrix.

$Idx.$	σ^2	σ_Q^2	σ_R^2
1	10^{-3}	10^{-5}	10^{-3}
2	10^{-3}	10^{-5}	10^{-2}
3	10^{-3}	10^{-5}	10^{-4}
4	10^{-3}	10^{-3}	10^{-3}
5	10^{-2}	10^{-5}	10^{-3}
6	10^{-4}	10^{-5}	10^{-3}

Since the obtained graphs are similar, here we only show the results of one combination, as shown in Figure 1. The selected parameters correspond to index 1 in Table 1.

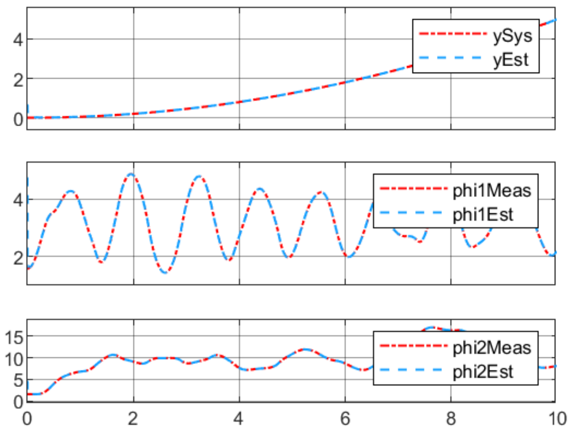

Figure 1: Extended Kalman filter result plot.

To facilitate the visualization of the convergence of the estimated states, we extend the simulation time to 30s and plot the estimation errors in Figure 2.

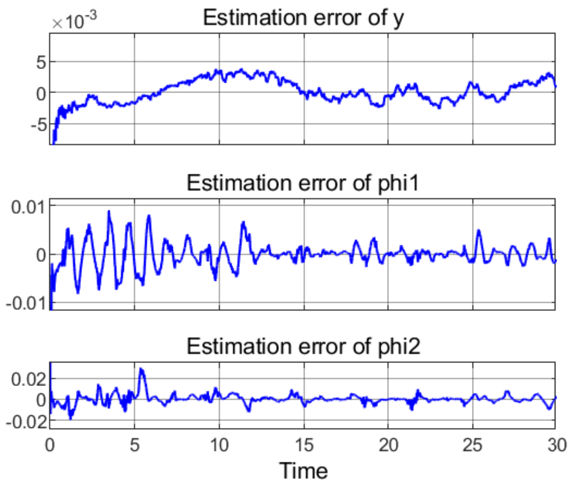

Figure 2: Estimation errors of extended Kalman filter ($Idx.1$).

2. Analysis

According to the results in Figure 1 and Figure 2, it can be seen that the extended Kalman filter estimates the states with a high accuracy. Estimation errors are initially large but gradually converge to 0. Plus, the magnitude of the estimation error of EKF on y is the smallest, while the one on φ_2 is the largest.

In order to evaluate the performance of the Kalman filter in various combinations, here we introduce an index, mean squared error (MSE):

$$\mathrm{MSE} = \frac{1}{n}\sum_{i=1}^{n}\left(Y_i - \hat{Y}_i\right)^2. \tag{1}$$

where Y_i is the ith actual output and $\hat{Y}_i$ is the estimate of Y_i.

In this way, the covariances in the 6 combinations are respectively substituted into the Simulink model, and the corresponding output parameter curves and further MSE values are obtained, as shown in Table 2.

Table 2: MSE results for 6 combinations.

$Idx.$	MSE_y	$MSE_{\varphi 1}$	$MSE_{\varphi 2}$
1	1.01×10^{-5}	4.93×10^{-7}	2.50×10^{-5}
2	3.17×10^{-7}	1.42×10^{-4}	1.48×10^{-5}
3	1.21×10^{-5}	1.95×10^{-7}	2.66×10^{-5}
4	1.00×10^{-5}	5.74×10^{-8}	2.63×10^{-5}
5	9.71×10^{-5}	6.04×10^{-6}	2.38×10^{-4}
6	1.16×10^{-6}	7.43×10^{-8}	2.41×10^{-6}

Moreover, the plots of estimation errors in the cases from index 2 to index 6 are attached in appendix.

Make a comparison among Figure 2, Figure A.3 and Figure A.4, it can be observed that an increased σ_R^2 will cause the estimation error of y to decrease, and vice versa. It is worth noting that the changes have little effect on the estimation of φ_1 and φ_2, which can also be corroborated by Table 2.

According to Figure 2 and Figure A.5, one can infer that the estimation error of y gets larger while σ_Q^2 increases.

Observe the last group of Figures A.6 and A.7, the estimation error of y decreases with a decreasing σ^2, and vice versa.

3. Conclusion

- The applied EKF can achieve convergent estimation of the states.
- The magnitude of the estimation error of y is the smallest, while the one of φ_2 is the largest.
- The covariance of measurement noise signal significantly affects the accuracy of the Kalman filter. To be more specific: the larger the covariance, the worse the precision, and vice versa.
- The covariances of the 2 matrices Q and R have little effect on the estimation of φ_1 and φ_2.
- An increase in σ_Q^2 will make the estimate of y worse but an increase in σ_R^2 will make it more precise.

Appendix A. Additional plots

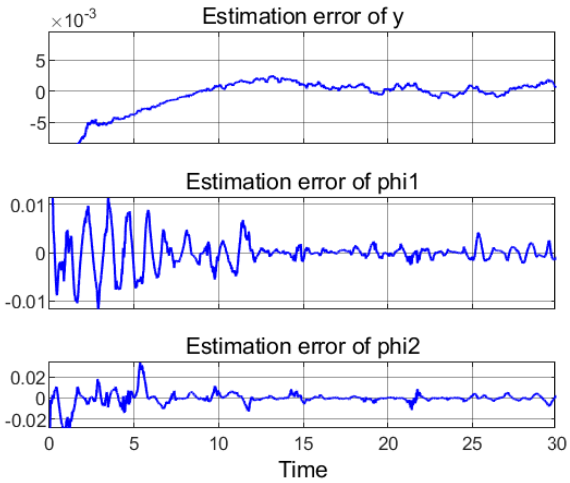

Figure A.3: Estimation errors of extended Kalman filter ($Idx.2$).

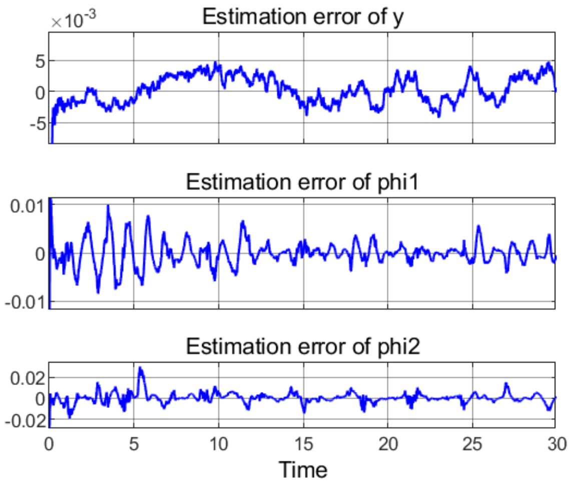

Figure A.4: Estimation errors of extended Kalman filter ($Idx.3$).

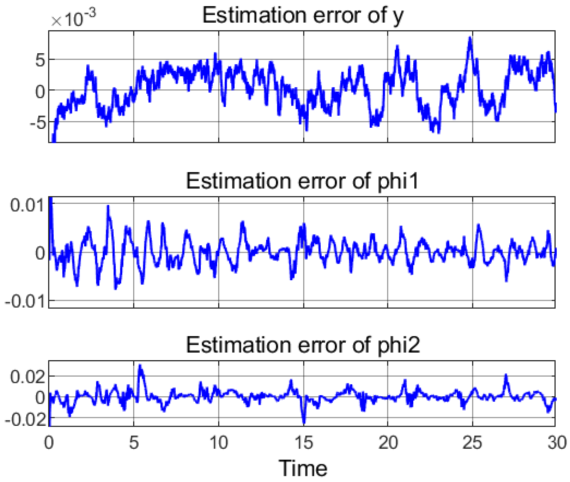

Figure A.5: Estimation errors of extended Kalman filter ($Idx.4$).

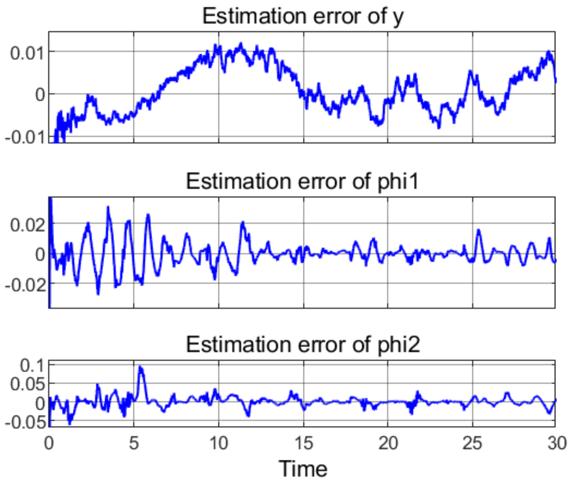

Figure A.6: Estimation errors of extended Kalman filter ($Idx.5$).

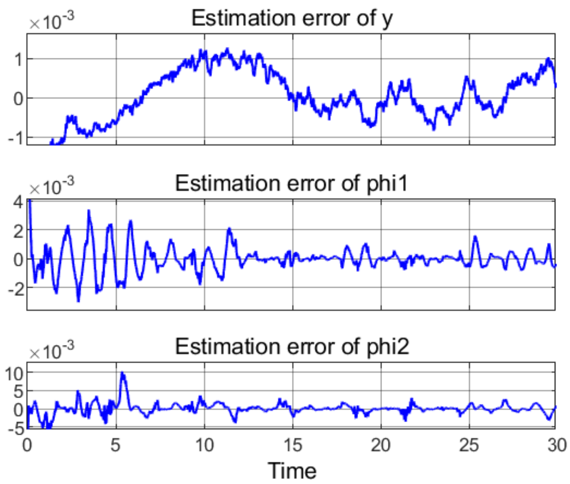

Figure A.7: Estimation errors of extended Kalman filter ($Idx.6$).