Project Report

Electrical Machines - Implementation of Modified Winding Function Theory in Synchronous Machines

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02/04/2022

EPiCo
Electric Vehicle Propulsion and Control

1) winding arrangement of phase in the stator slots based on the data

Data:-

synchronous motor, Poles - P=4, Power - 475KW, 480V, 0.8 P.F,

48 slots, $L = 273.05\,mm$, $r = 422.656\,mm$, $g = 2.54\,mm$, $N_r = 108$ T/P

$N_s = 3$ turns/coil, $R_s = 0.1592\,\Omega$, $R_r = .3632\,\Omega$, 8/12 pitch

$$\text{slots per pole per phase} = \frac{\text{Total number of slots}}{\text{No. of Poles} \times \text{Phases}} = \frac{48}{4 \times 3} = 4$$

$$\text{Pole Pitch} = \cancel{8}/12 \times 180^\circ = 60^\circ$$

$$\frac{2}{\cancel{3}} \times \cancel{180}^{30} = 60^\circ$$

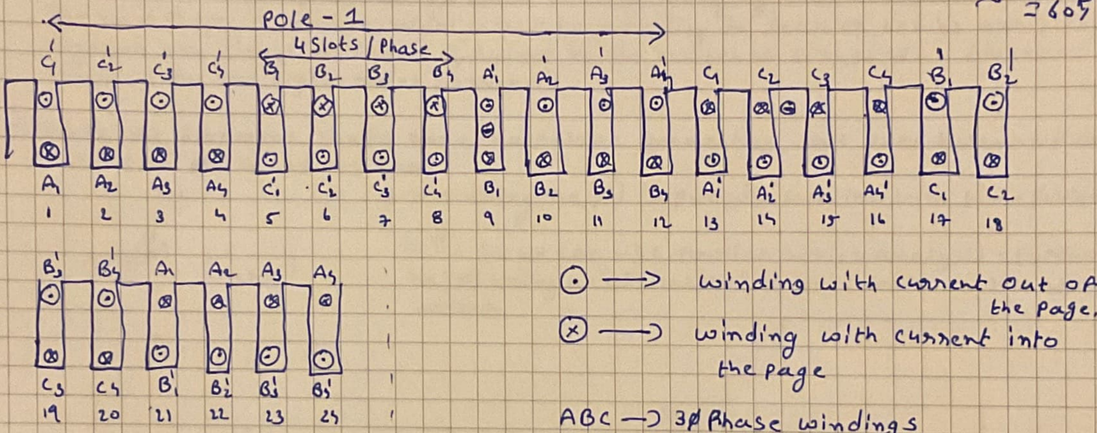


Fig:- winding distribution for 24 slots

- Here we have used "Short pitch" winding i.e., 8/12. For "48 slots" the slots per pole per phase is equal to '4'. The coil span is 1-9. That is we use 8 slots gap between the coil that goes inside the slot and comes outside.

- The "electrical angle" b/w phases is "120°". Therefore the mechanical angle will be 60°. ($\because \theta_e = \frac{P}{2}\theta_m$)

2) Fourier series of winding function ($N\phi$)

Slot Pitch $= \frac{360}{48} = 7.5^\circ_{mech} \Rightarrow 15^\circ$ electrical.

Fourier series formula

$$f(x) = \tfrac{1}{2} a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n x \pi}{T}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n x}{T}(2\pi)\right)$$

where

$$a_0 = \frac{2}{\pi}\int_{t_0}^{t_0+T} f(x)\,dx, \quad a_n = \frac{2}{\pi}\int_{t_0}^{t_0+T} f(x)\cos nx\,dx, \quad b_n = \frac{2}{\pi}\int_{t_0}^{t_0+T} f(x)\sin nx\,dx$$

$n = 1, 2, 3, 4, \ldots$

-) The airgap flux setup by the 3ϕ stator winding carrying sinusoidal currents. The waveshape is non sinusoidal in nature. According to Fourier series analysis, any non-sinusoidal flux is equivalent to the number of sinusoidal fluxes of fundamental and higher order harmonics.

-) ~~Since the flux wave shape have half wave symmentry, all "even harmonics" are absent in the Fourier series~~

-) Considering $n_a(\phi)$ is periodic function with time period $T = \pi$. So we can use Fourier series analysis. Further $n_a(\phi)$ being an "odd function" all cosine and Dc values are absent.

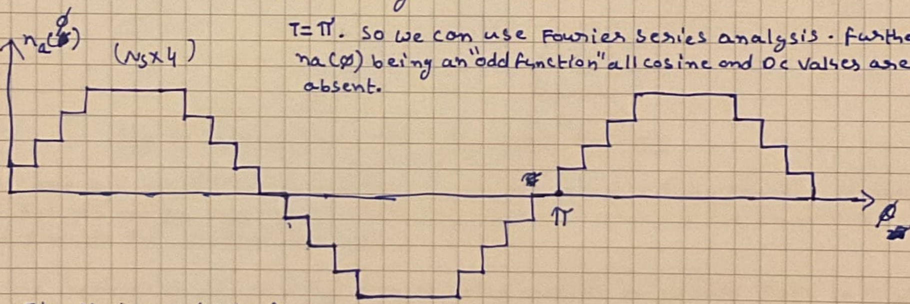


Fig - Stator $a\phi$ turns function

-) So, it takes 24 slot's to complete 1 complete mechanical rotation.

-) The Fourier series for $n_a(\phi)$ can be written as follows considering each step as 15°. i.e., $\sum_{n=1}^{\infty} b_n \sin\left(\frac{n x}{T}(2\pi)\right)$

where

$$b_n = \frac{2}{\pi}\left(\int_0^{\pi/24} 1\sin(2nt)\,dt + \int_{\pi/24}^{2\pi/24} 2\sin(2nt)\,dt + \int_{2\pi/24}^{3\pi/24} 3\sin(2nt)\,dt + \cdots\right.$$

$$\left.\cdots \int_{20\pi/24}^{21\pi/24} (-3)\sin(2nt)\,dt + \int_{21\pi/24}^{22\pi/24} (-2)\sin(2nt)\,dt + \int_{22\pi/24}^{23\pi/24} (-1)\sin(2nt)\,dt\right)$$

-) Similarly, Phase ϕb and ϕc turns function are same as ϕa. except that they are shifted by $60°$ and $120°$ mechanical respectively.

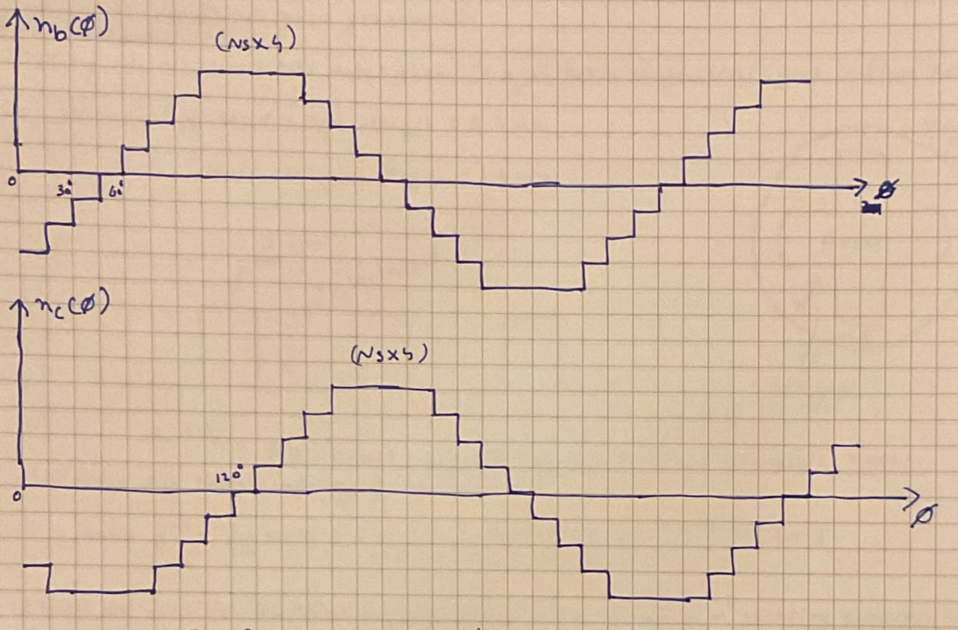


Fig-3 stator Phase ϕb & ϕc turns function

-) we can use the Fourier series formula to generate the turn function similar to for ϕb and ϕc i.e., $n_b(\phi)$ and $n_c(\phi)$, we must consider the Phase Shift and provide the limits accordingly.

-) So, we implement mathematical process using matlab to genrate plots to find the correctness of the winding function.

-) For $n_a(\phi)$, $n_b(\phi)$, $n_c(\phi)$ we use only "sin" component of Fourier series

i.e., $\sum_{n=1}^{\infty} b_n \sin\left(\frac{n \times (2\pi)}{T}\right)$

where b_n for $n_a(\phi)$ is $= \frac{2}{\pi}\left(\int_0^{\pi/24} 1 \sin(2nt)\,dt + \int_{\pi/24}^{2\pi/24} 2 \sin(2nt)\,dt + \cdots\right.$

~~b_n for $n_b(\phi)$ is~~ $= \frac{2}{\pi}\left(\int_{8\pi/24}^{9\pi/24} 1 \sin(2nt)\,dt + \int_{9\pi/24}^{10\pi/24} 2 \sin(2nt)\,dt + \cdots\right.$

b_n for $n_b(\phi)$ and $n_c(\phi)$ can be obtained by shifting $n_a(\phi)$ by $60°$ and $120°$ mech respectively.

~~b_n for $n_c(\phi)$ is~~ $= \frac{2}{\pi}\left(\int_{16\pi/24}^{17\pi/24} 1 \sin(2nt)\,dt + \int_{17\pi/24}^{18\pi/24} 2 \sin(2nt)\,dt + \cdots\right.$

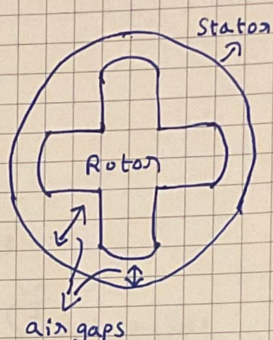
3) Generate Inverse Airgap function $g^{-1}(\phi, \theta_m)$

-) Airgap is the space b/w the stator and the rotor. where the RMF (Rotating magnatic field) can robe generated.

-) Being it's a salient pole rotor, there is a non uniform ~~magnatic~~ air gap in time period.

Considering max airgap length = 2.7mm

min airgap length = 1.4mm



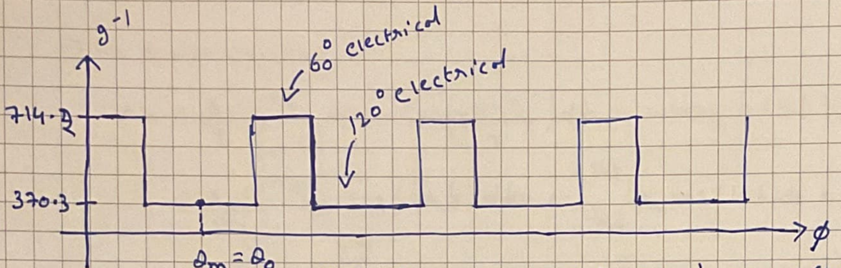


Fig:- Inverse airgap function in case of Symmetric rotor.

calculation for Inverse airgap g^{-1} =

$$\max g^{-1} = \frac{1}{1.4} \times 1000 = 714.2 \quad , \quad \min g^{-1} = \frac{1}{2.7} \times 1000 = 370.3$$

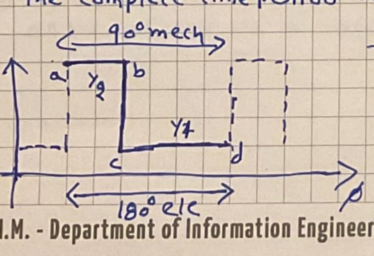
-) here we consider 'θ_m' rotor position angle as a 'θ_0' initial or starting rotor position

-) Using the Fourier series we can generate the airgap function. since it is ~~not~~ an even ~~odd~~ function ~~the odd harmonics are absent~~ ie., ~~cosine~~ sine term is zero. So we consider only fundmental component 'a_0' and cosine component 'a_n'.

$$g^{-1}(\phi) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n x}{T}(2\pi)\right)$$

-) In the inverse airgap function the square wave represent's the ~~th~~ airgap length b/w the stator and the rotor.

-) The complete time period for one pole ~~of~~ to take 90° mechanical is 180° electrical.



$T_{ab} + T_{cd} = T$ (Time period) = 90° mech = 180° elec

$T_{ab} = \frac{2\pi}{4} \times \frac{1}{3} = 30°$ mech = 60° electrical

$T_{cd} = \frac{2\pi}{4} \times \frac{2}{3} = 60°$ mech = 120° electrical

→ Fourier series for inverse air gap function. $= g^{-1}(\phi) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \operatorname{cosin}\left(\frac{2\pi n x}{T}\right)$

fundmental 'or' DC component 'a_0'

$$a_0 = \frac{2}{\pi} \times \left(\frac{2\pi}{3} \times Y_1 + \frac{\pi}{3} \times Y_2\right)$$

$\because$ Length of airgap 'Y_1' is $2\pi/3$

Length of airgap 'Y_2' is $\pi/3$

→ For cosine component

$$a_n = \frac{2}{T} \int_{t_0}^{t_0+T} f(t) \cdot \cos\left(\frac{2\pi n t}{T}\right) dt \quad \text{where } n = 0, 1, 2, 3 \ldots$$

$$= \frac{2}{\pi}\left(\int_0^{\pi/6} Y_1 \cos(2nt)\,dt + \int_{\pi/6}^{\pi/3} Y_2 \cos(2nt)\,dt + \int_{\pi/3}^{\frac{2\pi}{3}} Y_1 \cos(2nt)\,dt + \int_{\frac{2\pi}{3}}^{\frac{5\pi}{6}} Y_2 \cos(2nt)\,dt + \int_{\frac{5\pi}{6}}^{\pi} Y_1 \cos(2nt)\,dt\right)$$

$$= \frac{1}{n\pi}\left(Y_1\left(\sin\frac{2n\pi}{6} - 0\right) + \left(\sin\frac{4n\pi}{3} - \sin\frac{2n\pi}{3}\right) + \left(\sin 2n\pi - \sin\frac{5n\pi}{3}\right)\right)$$
$$+ Y_2\left[\left(\sin\frac{2n\pi}{3}\pi - \sin\frac{n\pi}{3}\right) + \left(\sin\frac{5n\pi}{3} - \sin\frac{4n\pi}{3}\right)\right]$$

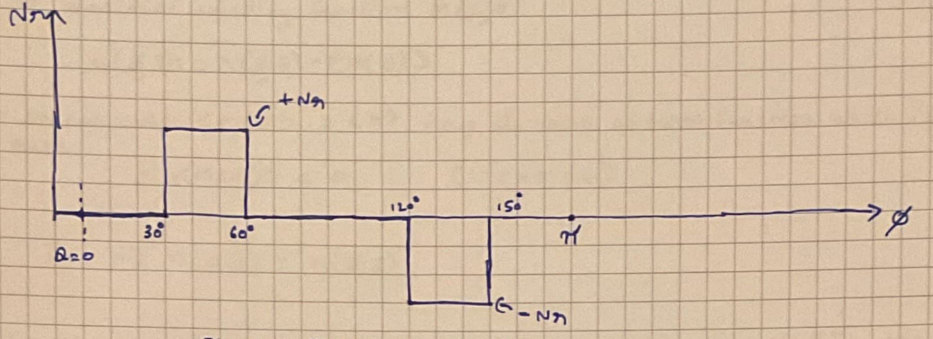
→ The plots for the $g^{-1}(\phi)$ is generated by using the matlab. The values of a_0 & a_n are used for the math.

4) Rotor Turns function (N_r)

→ The rotor turns function can be found simiar to that of stators turn function

→ Rotor turns function 'N_r' is a periodic function with time period 'π'

→ So, we can use the Fourier series analysis. Further 'N_r' being an "odd function" all the "cos" and the "DC" values are ignored.



Figure!- Rotor turns function

→ so, The Fourier series used for the "N_r" can be written as follows

$$f(\phi) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{2\pi n}{T}\right)$$

$$b_n = \frac{2}{T}\int_{t_0}^{t_0+T} f(t)\cdot\sin\left(\frac{2\pi n t}{T}\right) dt \quad \text{where } n = 1, 2, 3, \ldots$$

$$T = \pi$$

$$\therefore\ b_n = \frac{2}{\pi}\left[\int_{\pi/6}^{\pi/3}(N_r)\sin(2n\phi) + \int_{\frac{2\pi}{3}}^{\frac{5\pi}{6}}(-N_r)\sin(2n\phi)\right]$$

$$= \frac{N_r}{n\pi}\left[\cos\left(\frac{n\pi}{3}\right) - \cos\left(\frac{2n\pi}{3}\right) + \cos\left(\frac{5n\pi}{3}\right) - \cos\left(\frac{4n\pi}{3}\right)\right]$$

calculations of self-inductance and mutual inductance

→ In order to get the self-inductance and mutual we require the information of stator turns function, inverse airgap function and modified winding function.

→ we already have the stator turns function and air gap function from the above information. we now need to get the modified winding function.

→ In the paper it is been mentioned that for the air gap with only "even harmonics", the modified winding function can be represented as

$$\langle M(\theta)\rangle = \langle n\rangle$$

where $\langle n\rangle$ is the 'dc' value of the turns function.

→ So modified winding function $M(\phi, \theta)$

$$M(\phi\,\theta) = n(\phi\,\theta) - \langle M(\theta) \rangle$$

In our case $\langle M(\theta) \rangle = \langle n \rangle$ i.e., dc value of turns function which is equal to zero

$$\therefore \langle M(\theta) \rangle = 0 \qquad (\because \langle n \rangle = 0)$$

$$\rightarrow M(\phi\,\theta) = n(\phi, \theta)$$

→ So, as now we have the information of "stator turns function", "modified winding function" & "inverse airgap function". we can get the

<u>self inductance</u> by using

$$L_{AA} = \mu_0 r L \int_0^{2\pi} n_A(\phi, \theta)\, M_A(\phi, \theta)\, g^{-1}(\phi, \theta)\, d\phi$$

<u>mutual inductance</u> by using

$$L_{BA} = \frac{\lambda_{BA}}{i_A} = \mu_0 r L \int_0^{2\pi} n_B(\phi\,\theta)\, M_A(\phi\,\theta)\, g^{-1}(\phi\,\theta)\, d\phi$$

Similarly we can get the info of L_{BB}, L_{CC} & L_{CA}, L_{BC}.

Matlab Plots

Figure 1- Stator Turns Function for Phase A, Phase B and Phase C
The Highest Harmonic Order **N = 5.**

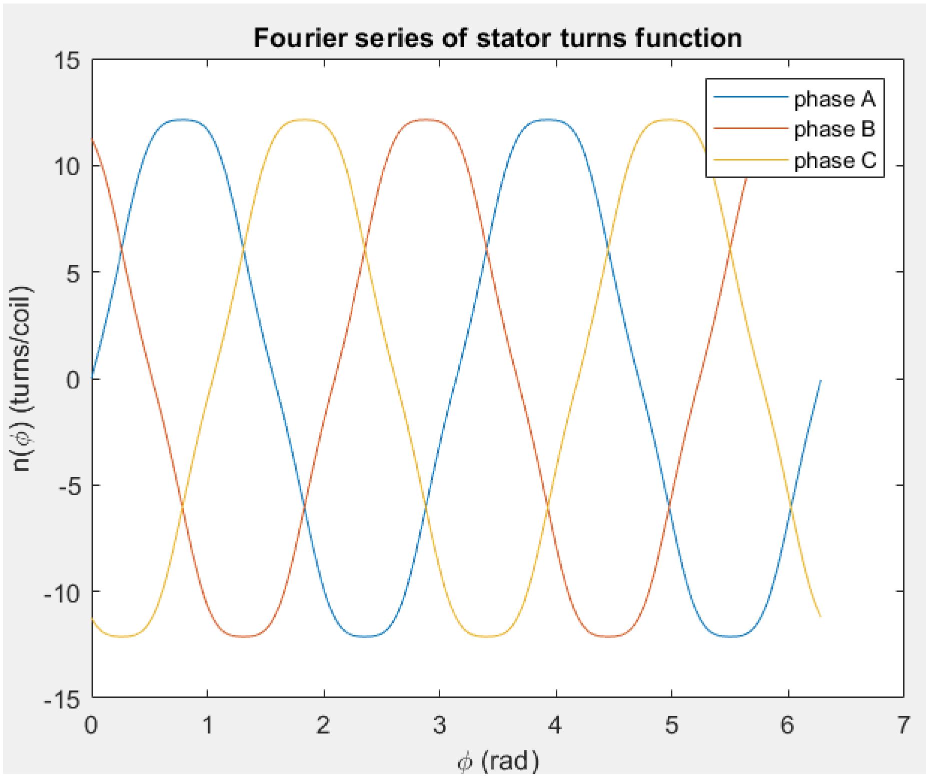


Figure 2- Inverse Air Gap Function
The Highest Harmonic Order **N = 2.**

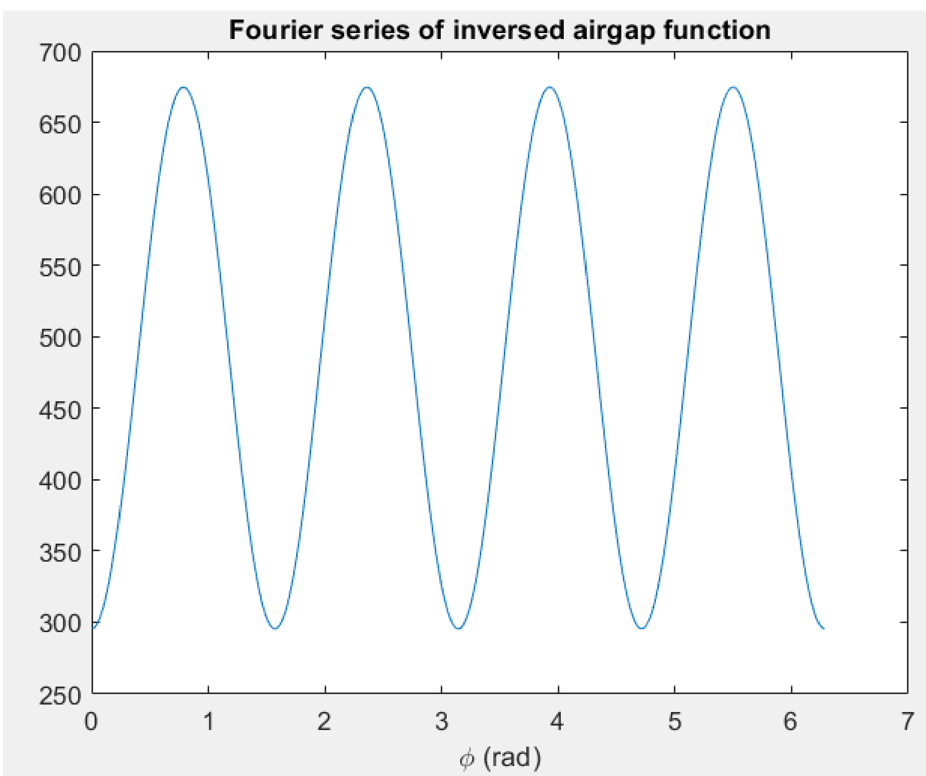


Figure 3- Rotor Turns Function
The Highest Harmonic Order **N = 4.**

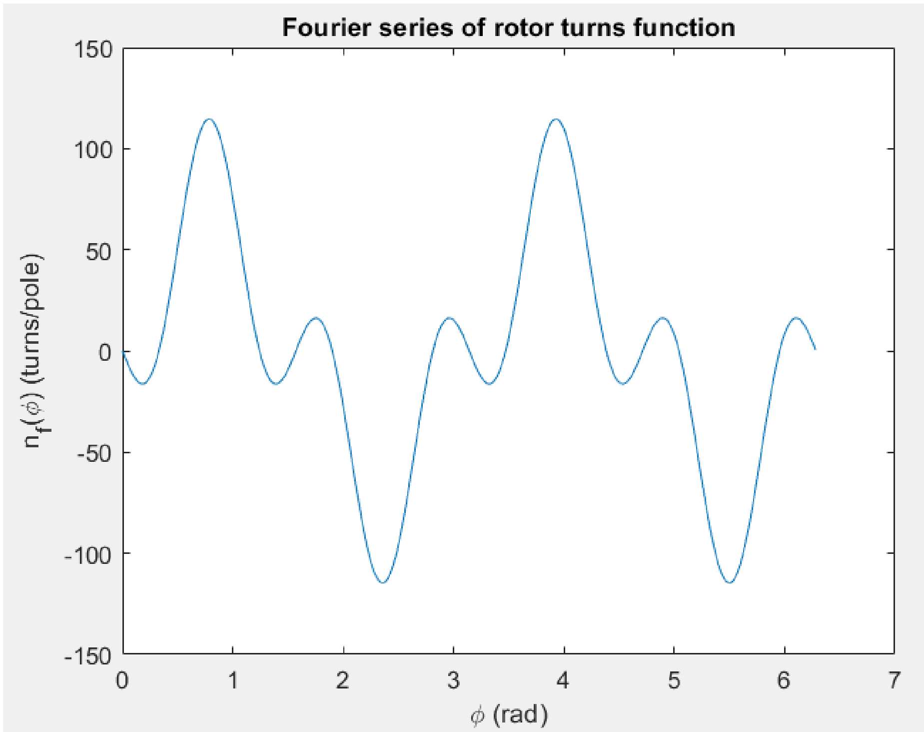


Figure 4- Self -Inductance

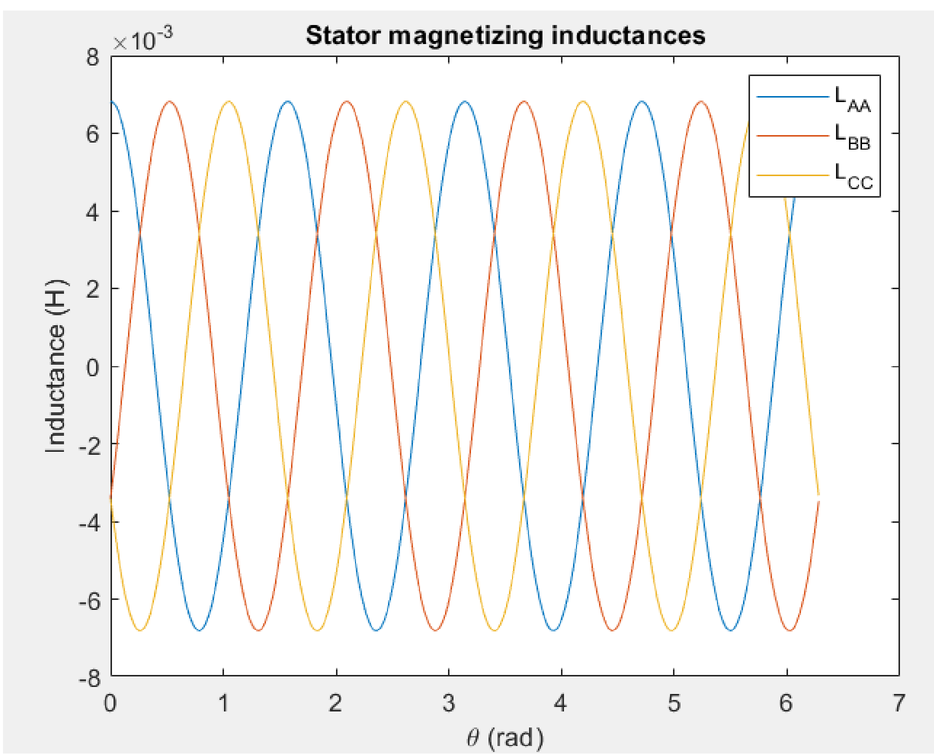
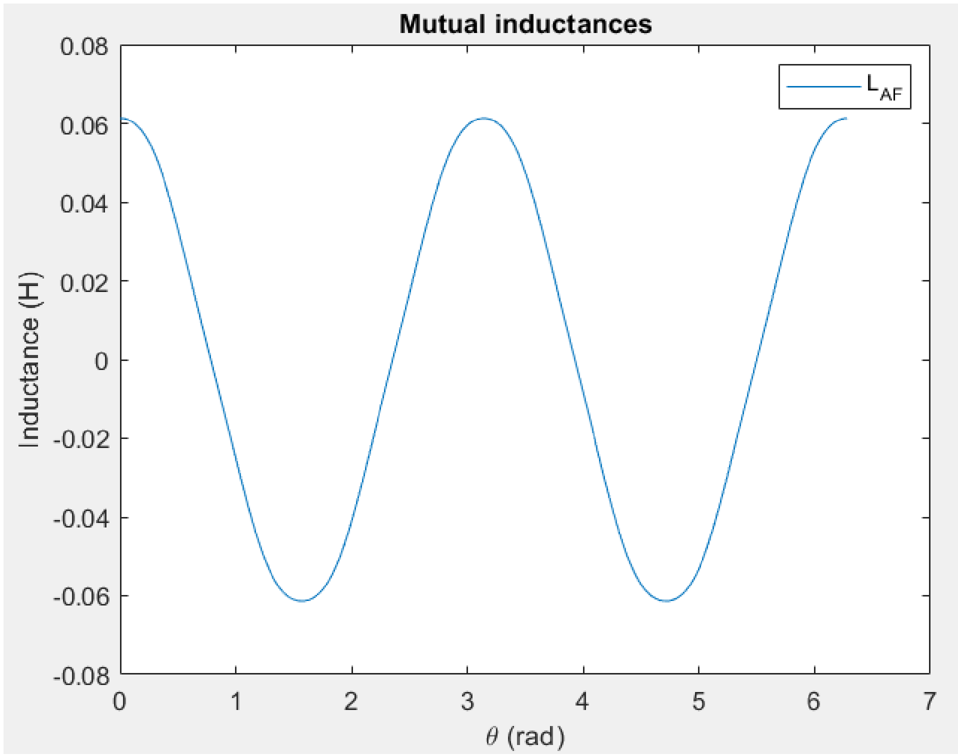


Figure 4- Mutual -Inductance LAF



Appendix - Matlab Scripts

Code - Stator Turns Function

```
% Fourier series of Stator turns function %
S_slots = 48;
intv = 2*pi/S_slots; % mechanical angle of each slot

Ns = 3; % unit: turns per coil
T = pi; % here we consider the time period of n_A is pi.
N = 5; % the highest harmonic  order

for i=1:N
    % coefficient b_n of sin components %
    bi = intg_sin2nfa(0,intv,i)+ 2*intg_sin2nfa(intv,2*intv,i)+ 3*intg_sin2nfa(2*intv,3*intv,i)+...
        4*intg_sin2nfa(3*intv,8*intv,i)+...
        3*intg_sin2nfa(8*intv,9*intv,i)+ 2*intg_sin2nfa(9*intv,10*intv,i)+ intg_sin2nfa(10*intv,11*intv,i)+...
        (-1)*intg_sin2nfa(12*intv,13*intv,i)+ (-2)*intg_sin2nfa(13*intv,14*intv,i)+
(-3)*intg_sin2nfa(14*intv,15*intv,i)+...
        (-4)*intg_sin2nfa(15*intv,20*intv,i)+...
        (-3)*intg_sin2nfa(20*intv,21*intv,i)+ (-2)*intg_sin2nfa(21*intv,22*intv,i)+
```

```
(-1)*intg_sin2nfa(22*intv,23*intv,i);
    nA_b(i) = Ns*2*bi/T;
    % elimination of calculation error %
%     if (abs(nA_b(i))-1e-10 < 0)
%         nA_b(i) = 0;
%     end
end

fa=0:0.01:2*pi; % mechanical angle in stationary frame (stator)
fs_na = zeros(size(fa)); % request memory
fs_nb = zeros(size(fa));
fs_nc = zeros(size(fa));

syms phi;
F_fsna = 0;
F_fsnb = 0;
F_fsnc = 0;

for i=1:N
    fs_na = fs_na + nA_b(i)*sin(2*pi*i*fa/T); % phase A
    fs_nb = fs_nb + nA_b(i)*sin(2*pi*i*(fa-2*pi/3)/T); % phase B
    fs_nc = fs_nc + nA_b(i)*sin(2*pi*i*(fa+2*pi/3)/T); % phase C
    F_fsna = F_fsna+nA_b(i)*sin(2*pi*i*phi/T); % symbolic function of this Fourier series
    F_fsnb = F_fsnb+nA_b(i)*sin(2*pi*i*(phi-2*pi/3)/T);
    F_fsnc = F_fsnc+nA_b(i)*sin(2*pi*i*(phi+2*pi/3)/T);
end

figure()
plot(fa,fs_na,fa,fs_nb,fa,fs_nc)
xlabel('\phi (rad)')
ylabel('n(\phi) (turns/coil)')
title('Fourier series of stator turns function')
legend('phase A','phase B','phase C')
```

Code - Inverse Air Gap Function

```
% Fourier series of inversed air gap function g^-1
y1 = 1000/2.7; % minimum value
y2 = 1000/1.4; % maximum value
alpha0 = 1/2*(4/3*y1+2/3*y2); % DC value in Fourier series

N = 2; % the highest harmonic  order

for i=1:N
    % coefficient a_n of cos components %
    ivg_a(i) = 1/i/pi*(y1*(sin(2*i*pi/6)+sin(4*i*pi/3)-sin(2*i*pi/3)-sin(5*i*pi/3))...
               +y2*(sin(2*i*pi/3)-sin(i*pi/3)+sin(5*i*pi/3)-sin(4*i*pi/3)));
end

fa = 0:0.01:2*pi; % mechanical angle in stationary frame (stator)
fs_ivg = alpha0*ones(size(fa)); % request memory and initialization

F_fsivg = 0;
syms theta; % rotor position
```

```
for i=1:N
    fs_ivg = fs_ivg + ivg_a(i)*cos(2*i*fa);
    F_fsivg = F_fsivg + ivg_a(i)*cos(2*i*(phi-theta));
end

figure()
plot(fa,fs_ivg)
xlabel('\phi (rad)')
title('Fourier series of inverse air gap function')
)
```

Code - Rotor Turns Function

```
% Calculate the Fourier series of Turns function n_f
Nr = 108; % unit: turns/pole

N = 4; % the highest harmonic  order

for i=1:N
    % coefficient b_n of sin components %
    nf_b(i) = Nr/pi/i*(cos(i*pi/3)-cos(2*i*pi/3)+cos(5*i*pi/3)-cos(4*i*pi/3));
end

fa = 0:0.01:2*pi; % mechanical angle in stationary frame (stator)
fs_nf = zeros(size(fa)); % request memory

F_fsnf = 0;
syms theta; % rotor position

for i=1:N
    fs_nf = fs_nf + nf_b(i)*sin(2*i*fa);
    F_fsnf = F_fsnf + nf_b(i)*sin(2*i*(phi-theta));
end

figure()
plot(fa,fs_nf)
xlabel('\phi (rad)')
ylabel('n_f(\phi) (turns/pole)')
title('Fourier series of rotor turns function')
```

Code - Self and Mutual Inductances

```
mu0 = 4*pi*(1e-7); % permeability of vacuum
r = 422.656*(1e-3);
L = 273.05*(1e-3);

fcna = F_fsna*F_fsna*F_fsivg; % n_A*M_A*g^-1
fcnb = F_fsnb*F_fsnb*F_fsivg;
fcnc = F_fsnc*F_fsnc*F_fsivg;
```

```
fcnaf = F_fsnf*F_fsna*F_fsivg; % n_f*M_A*g^-1
fcnbf = F_fsnf*F_fsnb*F_fsivg;
fcncf = F_fsnf*F_fsnc*F_fsivg;

syms LAA(theta) LBB(theta) LCC(theta)
syms LAF(theta) LBF(theta) LCF(theta)

LAA(theta) = mu0*r*L*(int(fcna,phi,[0 2*pi])); % [lower bound, upper bound]
LBB(theta) = mu0*r*L*(int(fcnb,phi,[0 2*pi]));
LCC(theta) = mu0*r*L*(int(fcnc,phi,[0 2*pi]));

LAF(theta) = mu0*r*L*(int(fcnaf,phi,[0 2*pi]));
LBF(theta) = mu0*r*L*(int(fcnbf,phi,[0 2*pi]));
LCF(theta) = mu0*r*L*(int(fcncf,phi,[0 2*pi]));

% numerically approximate the integral by using vpa
LAA(theta) = vpa(LAA(theta));
LBB(theta) = vpa(LBB(theta));
LCC(theta) = vpa(LCC(theta));

 LAF(theta) = vpa(LAF(theta));
 LBF(theta) = vpa(LBF(theta));
 LCF(theta) = vpa(LCF(theta));

figure()
plot(fa,LAA(fa),fa,LBB(fa),fa,LCC(fa))
xlabel('\theta (rad)')
ylabel('Inductance (H)')
title('Stator magnetizing inductances')
legend('L_A_A','L_B_B','L_C_C')

figure()
plot(fa,LAF(fa),fa,LBF(fa),fa,LCF(fa))
xlabel('\theta (rad)')
ylabel('Inductance (H)')
title('Mutual inductances')
legend('L_A_F','L_B_F','L_C_F')

)
```