1 System analysis with Matlab

M.Sc. Laboratory Advanced Control (WS 22/23)

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Additional tasks

1.1 Double pendulum (continued)

The double pendulum of Exercise 1.8 (see Figure 1.2) is revisited.

Exercise 1.1.

(i) *Transfer the second order equations of motion into a first-order ODE system. Set up the state vector as* $\boldsymbol{x} = \begin{bmatrix}\varphi_1 & \dot{\varphi}_1 & \varphi_2 & \dot{\varphi}_2\end{bmatrix}^T$.

(ii) *Extend the ODE system by the carriage position* y *and the carriage velocity* $\dot{y}$ *such that the state vector reads* $\boldsymbol{x} = \begin{bmatrix}y & \dot{y} & \varphi_1 & \dot{\varphi}_1 & \varphi_2 & \dot{\varphi}_2\end{bmatrix}^T$. *The available measurements are the carriage position* y *and the angles* φ_1, φ_2.

(iii) *Identify the equilibrium positions of the system without any calculations. Verify your model by testing whether these states are actually equilibrium positions.*

(iv) *Linearize the system around the equilibrium positions.*
Self-check: For the equilibrium position $\boldsymbol{x}_R = \boldsymbol{0}$ *the system and input matrix read*

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 46.0600 & -0.0739 & -11.9414 & 0.0301 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -38.2860 & 0.0904 & 35.1713 & -0.0539 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \\ 0 \\ -3.4779 \\ 0 \\ 0.3175 \end{bmatrix}.$$

The eigenvalues of A and B are thereby given as

$$\lambda_1 = -7.9758\, s^{-1}, \quad \lambda_2 = -4.3126\, s^{-1}, \quad \lambda_3 = 0\, s^{-1},$$
$$\lambda_4 = 0\, s^{-1}, \quad \lambda_5 = 4.3017\, s^{-1}, \quad \lambda_6 = 7.8588\, s^{-1}.$$

Additionally for the equilibrium position $\boldsymbol{x}_R = \begin{bmatrix}0 & 0 & 0 & 0 & \pi & 0\end{bmatrix}^T$ *the system and input matrix read*

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 46.0600 & -0.0466 & -11.9414 & 0.0028 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 38.2860 & -0.0099 & -35.1713 & -0.0266 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \\ 0 \\ -3.4779 \\ 0 \\ -0.3175 \end{bmatrix},$$

with the eigenvalues

$$\lambda_1 = -6.3001\, s^{-1}, \quad \lambda_2 = -(0.0140 + i5.3933)\, s^{-1}, \quad \lambda_3 = -(0.0140 - i5.3933)\, s^{-1},$$
$$\lambda_4 = 0\, s^{-1}, \quad \lambda_5 = 0\, s^{-1}, \quad \lambda_6 = 6.3453\, s^{-1}.$$

(v) *Create callable Matlab-files (*.m) with your numeric model, one for the linearized and one for the non-linear model. To achieve this, review the documentation of the* `odeFunction` *of Matlab. First, read the help-page on Matlabs-Website and afterwards implement it yourself.*

(vi) *Create a new Matlab-Script (*.m). Use your generated numerical model-function-file to simulate your system. Plot the results and make a comparison.*

(vii) *Implement your own integration scheme. Start with the explicit Euler and continue with the Runge-Kutta scheme of order four.*

(viii) *Compare the simulation result based on the chosen step-width and derivation from the equilibrium.*

References

[1] T. Meurer. „Control Systems - Lecture notes“. In: `http://www.control.tf.uni-kiel.de/en/teaching/winter-term` (2019).

[2] T. Meurer. „Rigid Body Dynamics and Robotics - Lecture notes“. In: `http://www.control.tf.uni-kiel.de/en/teaching/winter-term` (2019).