Reference book: Bodson Sostry.

Consider a system : $\dot{x} = \theta x^2 + u$, θ is a function of t.

① if one can know $\theta(t)$ perfectly, control law can be designed as:

$u = -\theta x^2 - kx$ $\quad k>0$

$\Downarrow$

$\dot{x} = -kx$

② if one just know the nominal value of $\theta(t)$,

$u = -\theta_0 x^2 - kx$

$\Downarrow$

$\dot{x} = \underline{(\theta-\theta_0)} x^2 - kx$

the system might be stable, but it's introduced a disturbance.

③ in practice, one can get the estimation of $\theta(t)$.

$u = -\hat{\theta} x^2 - kx$

$\Downarrow$

$\dot{x} = (\theta - \hat{\theta}) x^2 - kx$

☆ $\frac{d}{dt}(\theta - \hat{\theta}) = \dot{\theta} - \dot{\hat{\theta}} = -\dot{\hat{\theta}}$ → need to be designed later.

↑ here we suppose $\dot{\theta} = 0$ bcs θ changes slowly.

$V = \frac{1}{2}x^2 + \frac{(\theta-\hat{\theta})^2}{2}$ $\quad e = \theta - \hat{\theta}$ should tends to zero.

$\dot{V} = x(\theta x^2 + u) - (\theta - \hat{\theta})\dot{\hat{\theta}}$

$\quad = (\theta - \hat{\theta})x^3 - kx^2 - (\theta - \hat{\theta})\dot{\hat{\theta}}$

one can design $\dot{\hat{\theta}} = x^3$

$\Downarrow$

$\dot{V} = -kx^2 \le 0$

↓ Lyapunov theory.

$x(t)$ is bounded.

Moreover, one can use Le Salle's method.

$E = \left\{ \begin{pmatrix} x \\ \theta-\hat{\theta} \end{pmatrix} \middle| \dot{V} = 0 \right\} = \left\{ \begin{pmatrix} x \\ \theta-\hat{\theta} \end{pmatrix} \middle| x = 0 \right\}$

invariance $x \equiv 0 \Rightarrow \varepsilon^+ = E$

$\dot{x} = 0$

Conclusion: $\hat{\theta}$ doesn't have to identify θ.

Just adaping θ is enough to obtain $x(t) \xrightarrow{t \to \infty} 0$