Solution to Problem V-2

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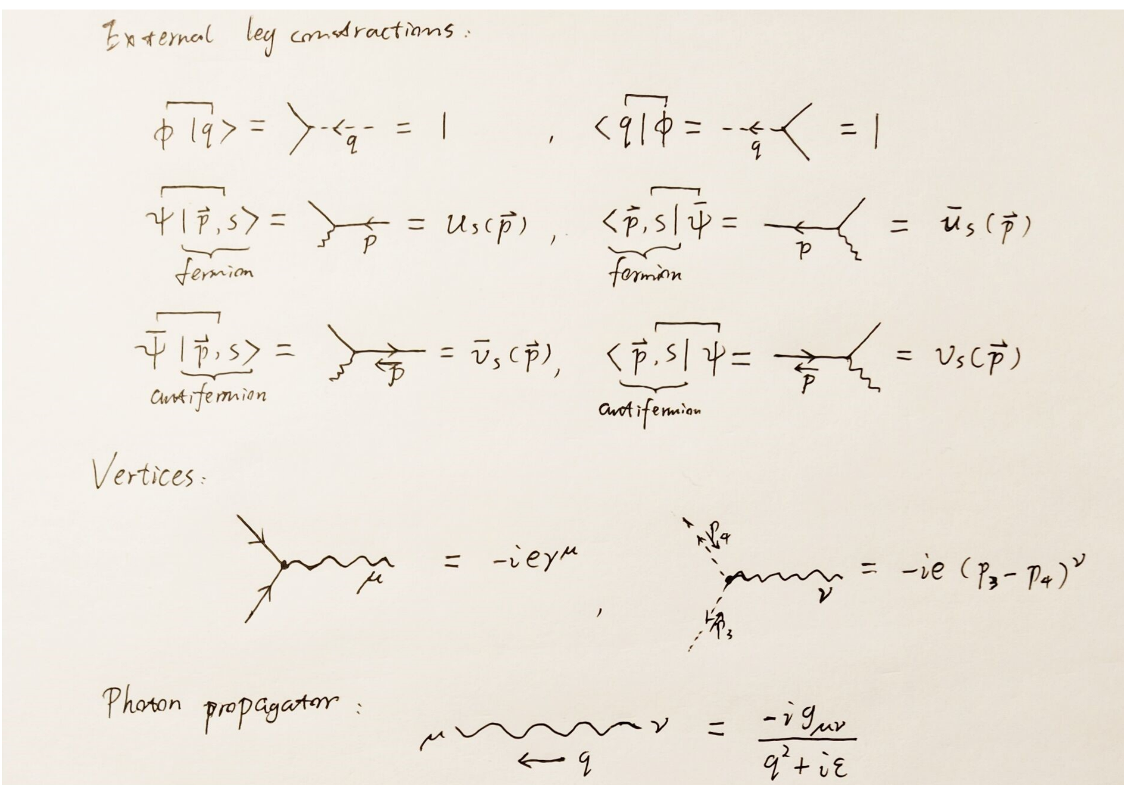
(a)

Trivial.

(b)

In this $e^- + e^+ \to \tilde{\mu}^- + \tilde{\mu}^+$, We can write down the Feynman rules:



Thus we can write the scattering amplitude as

$$
\begin{aligned}
& i\mathcal{M}\left(e_r^-\left(\vec{p}_1\right)+e_s^+\left(\vec{p}_2\right)\rightarrow\tilde{\mu}^-\left(\vec{p}_3\right)+\tilde{\mu}^+\left(\vec{p}_4\right)\right) \\
=& \bar{v}_s(\vec{p}_2)\big(-ieQ_e\gamma^\mu\big)u_r(\vec{p}_1)\frac{-ig_{\mu\nu}}{(p_1+p_2)^2+i\epsilon}\big(-ieQ_\mu(p_3-p_4)^\nu\big) \\
=& \frac{i\left(Q_\mu Q_e e^2\right)}{(p_1+p_2)^2+i\epsilon}\left(\bar{v}_s\left(\vec{p}_2\right)\gamma^\mu u_r\left(\vec{p}_1\right)\right)\left(p_3-p_4\right)_\mu .
\end{aligned}
\tag{1}
$$

(c)

In the center of mass frame ($\vec{p}_2=-\vec{p}_1$ and $\vec{p}_4=-\vec{p}_3$), the average of $|\mathcal{M}|^2$ can be written as

$$
\begin{aligned}
\frac{1}{4}\sum_{r,s}|\mathcal{M}|^2 &= \frac{1}{4}\frac{(Q_\mu Q_e e^2)^2}{s^2}[\bar{v}_s(\vec{p}_2)\gamma^\mu u_r(\vec{p}_1)][\bar{u}_r(\vec{p}_1)\gamma^\nu v_s(\vec{p}_2)](p_3-p_4)_\mu(p_3-p_4)_\nu \\
&= \frac{1}{4}\frac{(Q_\mu Q_e e^2)^2}{s^2}\,\mathrm{Tr}\left[\gamma^\mu(\not{p}_1+m_e)\gamma^\nu(\not{p}_2-m_e)\right](p_3-p_4)_\mu(p_3-p_4)_\nu .
\end{aligned}
\tag{2}
$$

Notice that in the center of mass frame,

$$
(p_3-p_4)_\mu=\begin{cases}0 & for\ \mu=0\\ 2p_{3i} & for\ \mu=i\ (i=1,2,3)\end{cases},
$$

the equation above can be written as

$$
\begin{aligned}
\frac{1}{4}\sum_{r,s}|\mathcal{M}|^2 &= 4\frac{(Q_\mu Q_e e^2)^2}{s^2}\Big[-m_e^2\delta^{ij}-(p_1\cdot p_2)\delta^{ij}+{p_1}^i{p_2}^j+{p_1}^j{p_2}^i\Big]p_{3i}p_{3j} \\
&= 4\frac{(Q_\mu Q_e e^2)^2}{s^2}\left\{\Big[-m_e^2\delta^{ij}-\Big(\frac{s}{2}-m_e^2\Big)\delta^{ij}\Big]p_{3i}p_{3j}-2\big(\vec{p}_1\cdot\vec{p}_3\big)^2\right\} \\
&= \frac{(Q_\mu Q_e e^2)^2}{s^2}\left[4m_e^2|\vec{p}_3|^2-8\big(\vec{p}_1\cdot\vec{p}_3\big)^2+4\Big(\frac{s}{2}-m_e^2\Big)|\vec{p}_3|^2\right]
\end{aligned}
\tag{3}
$$

(d)

In the center of mass frame, $s=(p_1+p_2)^2=4E_{\vec{p}_1}^2$. When the energy of the e^-+e^+ collision is just high enough for a $\tilde{\mu}^-+\tilde{\mu}^+$ pair creation ($|\vec{p}_1|\simeq E_{\vec{p}_1}\simeq M_\mu$), the average of

$|\mathcal{M}|^2$ can be written as

$$
\begin{aligned}
\frac{1}{4}\sum_{r,s}|\mathcal{M}|^2 &= \frac{(Q_\mu Q_e e^2)^2}{16E_{\vec{p}_1}^4}\Big[4m_e^2|\vec{p}_3|^2 - 8\big(\vec{p}_1\cdot\vec{p}_3\big)^2 + 4|\vec{p}_3|^2\big(2E_{\vec{p}_1}^2 - m_e^2\big)\Big] \\
&= \frac{(Q_\mu Q_e e^2)^2}{16E_{\vec{p}_1}^4}\Big[8E_{\vec{p}_1}^2|\vec{p}_3|^2 - 8\big(\vec{p}_1\cdot\vec{p}_3\big)^2\Big] \\
&\simeq \frac{(Q_\mu Q_e e^2)^2}{2}\left[\frac{|\vec{p}_3|^2}{M_\mu^2}\left(1 - \frac{(\vec{p}_1\cdot\vec{p}_3)^2}{M_\mu^2|\vec{p}_3|}\right)\right] \\
&\simeq \frac{(Q_\mu Q_e e^2)^2}{2}\left[\frac{|\vec{p}_3|^2}{M_\mu^2}\left(1 - \frac{(\vec{p}_1\cdot\vec{p}_3)^2}{|\vec{p}_1|^2|\vec{p}_3|^2}\right)\right] \\
&= \frac{(Q_\mu Q_e e^2)^2}{2}\beta_\mu^2\sin^2\theta
\end{aligned} \tag{4}
$$

where $\beta_\mu := |\vec{p}_3|/M_\mu$ is the velocity of the produced $\tilde{\mu}^\pm$ and θ is the scttering angle in the center of mass frame.

(e)

In the high energy limit, $|\vec{p}_1| \simeq E_{\vec{p}_1} \simeq |\vec{p}_3|$.

$$
\begin{aligned}
\frac{1}{4}\sum_{r,s}|\mathcal{M}|^2 &= \frac{(Q_\mu Q_e e^2)^2}{16E_{\vec{p}_1}^4}\Big[8E_{\vec{p}_1}^2|\vec{p}_3|^2 - 8\big(\vec{p}_1\cdot\vec{p}_3\big)^2\Big] \\
&= \frac{(Q_\mu Q_e e^2)^2}{2E_{\vec{p}_1}^4}\Big[E_{\vec{p}_1}^2|\vec{p}_3|^2 - |\vec{p}_1|^2|\vec{p}_3|^2\cos^2\theta\Big] \\
&\simeq \frac{(Q_\mu Q_e e^2)^2}{2}\big[1 - \cos^2\theta\big] \\
&= \frac{(Q_\mu Q_e e^2)^2}{2}\sin^2\theta
\end{aligned} \tag{5}
$$