Homework 02

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(1)

The electric quadrupole moment of this deformed nucleus can be calculated with the integral:

$$
\begin{aligned}
Q_d &= \frac{1}{e}\int\limits_V \rho(3z^2 - r^2)dv \\
&= \frac{\rho}{e}\int\limits_V (2z^2 - x^2 - y^2)dv \\
&= \frac{Z}{V}\Big(\frac{2}{5}\frac{R^2}{1.2}V - \frac{2}{5}R^2V\Big)
\end{aligned}
$$

with $R = 1.2A^{\frac{1}{3}}\text{fm}$, $A = 200$ and $Z = 80$, the quadrupole moment is

$$
Q_d = -\frac{Z}{15}R^2 = -262.65\text{fm}^2
$$

(2)

The expression of spherical harmonics:

$$
\begin{aligned}
Y_{20} &= \frac{1}{4}\sqrt{\frac{5}{\pi}}(3cos^2\theta - 1) \\
Y_{2-2} &= \frac{1}{\sqrt{2\pi}}\exp(-2i\varphi)\frac{\sqrt{15}}{4}sin^2\theta \\
Y_{22} &= \frac{1}{\sqrt{2\pi}}\exp(2i\varphi)\frac{\sqrt{15}}{4}sin^2\theta
\end{aligned}
$$

With $L = 2$ and $M_L = 2$, the angular part

$$\begin{aligned}
\int\limits_0^2 \pi \int_0^\pi Y^*_{LM_L} Y_{20} Y_{LM_L} sin\theta d\theta d\varphi &= \int_0^\pi \frac{15}{64}\sqrt{\frac{5}{\pi}} sin^5\theta(3cos^2\theta - 1)d\theta \\
&= -\frac{15}{64}\sqrt{\frac{5}{\pi}} \int_0^\pi sin^4\theta(3cos^2\theta - 1)dcos\theta \\
&= -\frac{15}{64}\sqrt{\frac{5}{\pi}} \int_0^\pi (1 - cos^2\theta)^2(3cos^2\theta - 1)dcos\theta \\
&= -\frac{15}{64}\sqrt{\frac{5}{\pi}} \int_1^{-1} (1 - x^2)^2(3x^2 - 1)dx \\
&= -\frac{1}{7}\sqrt{\frac{5}{\pi}}
\end{aligned}$$