

# MY457/MY557: Causal Inference for Observational and Experimental Studies

## Week 1: Causal Frameworks

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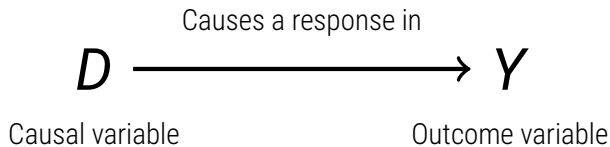
Winter Term 2025

# Lecture Map

- 1 Potential Outcomes
- 2 Causal Estimands
- 3 Identification
- 4 Graphical Causal Framework
- 5 Assignment Mechanisms
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# A (Very) Simple Causal Model



# Effects of Causes

Causes and their effects have two properties: they are **successive** and can be reasoned about in **counterfactual** terms:

*[...] We may define a **cause** to be an object **followed** by another, [...] where, **if** the first object had not been, the second never had existed.*

– Hume, 1748

*[...] would not have died **if** he had not eaten of it, people would be apt to say that eating of that dish was the **cause** of his death.*

– Mill, 1843

One important implication is that causal variables must be **manipulable**:

*No causation without **manipulation**.*

– Holland, 1986

# Good Causal Questions

Manipulability means we must think very carefully about causal questions...

- (Largely) immutable characteristics?
  - Judges' sex assigned at birth → decision making
  - Race and ethnicity → employment outcomes
  - Country of origin → political beliefs
- Major global events?
  - Russian revolution → Karl Marx's intellectual popularity
  - 9/11 → Arab Spring
- Non-successive chains?
  - Monthly expenditure → monthly savings
  - Holocaust → modern AFD election support

# Concepts: Treatment, Outcomes, and Potential Outcomes

## Treatment:

$D_i$ : Indicator of treatment intake for *unit i*

$$D_i = \begin{cases} 1 & \text{if unit } i \text{ received the treatment} \\ 0 & \text{otherwise.} \end{cases}$$

## Observed Outcome:

$Y_i$ : Observed outcome variable of interest for unit *i*

## Potential Outcome:

$Y_{0i}$  and  $Y_{1i}$ : Potential outcomes for unit *i*:

$Y_{1i}$  Outcome for unit *i* when  $D_i = 1$

$Y_{0i}$  Outcome for unit *i* when  $D_i = 0$

(Alternative notation:  $Y_i(d)$ ,  $Y_i^d$ , etc.)

# Concepts: Treatment, Outcomes, and Potential Outcomes

Under further assumptions ('SUTVA', more later), we can connect these three concepts mathematically:

$$Y_i = D_i \cdot Y_{1i} + (1 - D_i) \cdot Y_{0i}$$

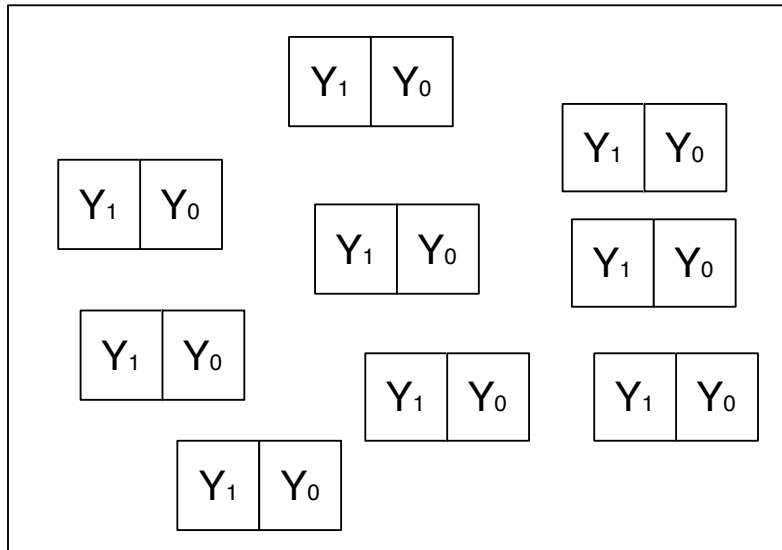
i.e.  $Y_i = \begin{cases} Y_{1i} & \text{if } D_i = 1 \\ Y_{0i} & \text{if } D_i = 0 \end{cases}$

The punchline:

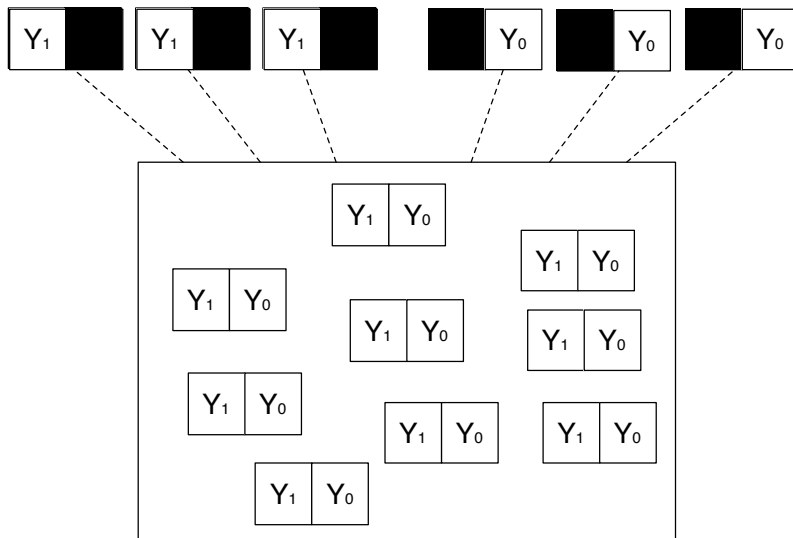
- *A priori* each potential outcome **could be observed** (manipulability!)
- After treatment assignment, one **is observed**, the other **is counterfactual**



# Neyman Urn Model



# Neyman Urn Model



# Causal Inference as a Missing Data Problem

Imagine a population with 4 units:

$i$	$D_i$	$Y_i$	$Y_{1i}$	$Y_{0i}$
1	1	3	3	0
2	1	1	1	1
3	0	0	1	0
4	0	1	1	1

We take the values of both  $Y_{1i}$  and  $Y_{0i}$  to be real and fixed for all  $i$

But we **can only observe** one of them for any  $i$  ...

This is known as the **fundamental problem of causal inference** (FPCI)

# Causal Inference as a Missing Data Problem

... because of the FPCI we see only this:

$i$	$D_i$	$Y_i$	$Y_{1i}$	$Y_{0i}$
1	1	3	3	?
2	1	1	1	?
3	0	0	?	0
4	0	1	?	1

The goal:

1. Define causal **estimands** in terms of potential outcomes (previous table)
2. **Estimate** them using observable data on this slide (present table)

Essentially: fill in the **missing counterfactuals** as best as possible!

# Stable Unit Treatment Value Assumption (SUTVA)

Recall:  $Y_i = Y_{D_i}$ , or equivalently  $Y_i = D_i Y_{1i} + (1 - D_i) Y_{0i}$

This notation implicitly makes an assumption:

SUTVA:

$$Y_{(D_1, D_2, \dots, D_N)i} = Y_{(D'_1, D'_2, \dots, D'_N)i} \quad \text{if} \quad D_i = D'_i$$

Read: For each  $i$  given a fixed value of  $D_i$ , potential outcomes under all possible randomization vectors are equal.

SUTVA comprises two sub-assumptions:

1. No **interference** between units: Potential outcomes for a unit not affected by treatment status of other units  
→ Violations: spill-over effects, contagion, dilution
2. No **different versions** of treatment (stability, consistency): Nominally identical treatments are in fact identical  
→ Violations: variable levels of treatment, technical errors

# Causal Inference Without SUTVA

Let  $\mathbf{D} = (D_1, D_2)$  be a vector of binary treatments for  $N = 2$ .

How many different values can  $\mathbf{D}$  possibly take?

$$(D_1, D_2) = (0, 0) \text{ or } (1, 0) \text{ or } (0, 1) \text{ or } (1, 1)$$

How many potential outcomes for unit 1?

$$Y_{(0,0)1}, Y_{(1,0)1}, Y_{(0,1)1}, Y_{(1,1)1}.$$

How many causal effects for unit 1?

$$\begin{aligned} Y_{(1,1)1} - Y_{(0,0)1}, & \quad Y_{(1,1)1} - Y_{(0,1)1}, \\ Y_{(1,0)1} - Y_{(0,0)1}, & \quad Y_{(1,0)1} - Y_{(0,1)1}, \\ Y_{(1,1)1} - Y_{(1,0)1}, & \quad Y_{(0,1)1} - Y_{(0,0)1}. \end{aligned}$$

How many observed outcomes for unit 1? Only one:  $Y_1 = Y_{(D_1, D_2)1}$

Without SUTVA, causal inference is exponentially more difficult as  $n \uparrow$ .

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# Esti-what?

Estimand:

↪ Unobserved population parameter or function.

Estimator:

↪ A function that can be applied to observed data.

Estimate:

↪ A specific output of said function.



# Unit-Level Causal Estimands

## Individual Treatment Effect:

$$\tau_i = Y_{1i} - Y_{0i}$$

Read: Effect of the causal variable on outcome for unit  $i$ , defined by the comparison of two unit-level potential outcomes.

This **cannot be observed**, and is also very hard to estimate:

- We cannot observe both potential outcomes  $Y_{1i}$  and  $Y_{0i}$  for the same unit  $i$ .
- Hard to reliably fill in the missing potential outcome for any one unit  $i$ .

# Group-Level Causal Estimands

Consider a fixed group (**population\***) of units  $i = 1, \dots, N$

Values of the potential outcomes for this population can be represented as two **only partially observed** vectors:

$$\mathbf{Y}_1 = (Y_{11}, Y_{12}, \dots, Y_{1N})$$

$$\mathbf{Y}_0 = (Y_{01}, Y_{02}, \dots, Y_{0N})$$

A group-level causal estimand is a comparison of  $\mathbf{Y}_1$  and  $\mathbf{Y}_0$

A common choice is a difference of their **expected values**.

\*Note: The use of population here is possibly not what you are used to! We actually mean the particular realized sample. We can talk more about this if you want, but the idea is that there are two distinct sampling processes going on here – sampling from a super-population that generates the study sample, and sampling from potential outcomes of the realized sample. We are primarily interested in the latter, not the former.

# Causal Estimand: The ATE

## Average Treatment Effect:

$$\tau_{ATE} = \frac{1}{N} \sum_{i=1}^N (Y_{1i} - Y_{0i})$$

or equivalently

$$\tau_{ATE} = \mathbb{E}[Y_{1i} - Y_{0i}]$$

Read: Effect of the causal variable on outcome on average for the entire group, defined by the comparison of two group-level averages of potential outcomes.

Note: We represent the **estimand** as a greek letter (in this case  $\tau$ , but could be anything). We typically represent an **estimator** for that estimand as a greek letter with something on top (e.g.  $\tilde{\tau}$  or  $\hat{\tau}$ ). An **estimate** will be a realised number (interval, etc.).

# Causal Estimand: The ATT

## Average Treatment Effect on the Treated:

$$\tau_{ATT} = \frac{1}{N_1} \sum_{i=1}^N D_i (Y_{1i} - Y_{0i}) \quad \text{where} \quad N_1 = \sum_{i=1}^N D_i$$

or equivalently

$$\tau_{ATT} = \mathbb{E}[Y_{1i} - Y_{0i} | D_i = 1]$$

(Note: The mathematical symbol  $|$  means “conditional on”.)

Read: Effect of the causal variable on outcome on average for those units in the treated group, defined by the comparison of two group-level averages of potential outcomes.

What's going on here?

- When would  $\tau_{ATT} \neq \tau_{ATE}$ ? When  $D_i$  and  $Y_{di}$  are associated.
- Exercise: Define  $\tau_{ATU}$ , ATE on the untreated (control) units, also called the Average Treatment Effect on the Untreated:  $\tau_{ATU} = \mathbb{E}[Y_{1i} - Y_{0i} | D_i = 0]$

# Causal Estimand: The CATE

## Conditional Average Treatment Effect:

$$\tau_{CATE}(\mathbf{x}) = \mathbb{E}[Y_{1i} - Y_{0i} | X_i = \mathbf{x}]$$

where  $X_i$  is a **pre-treatment covariate** for unit  $i$  (more soon)

Read: Effect of the causal variable on outcome on average for those units where  $X_i = \mathbf{x}$ , defined by the comparison of two subgroup averages of potential outcomes.

This estimand sometimes goes by other names (e.g. local average treatment effect or LATE, which we will see in a few weeks).

Generally, this is an increasingly important area for causal inference (e.g. optimal policy targeting, tailored medical therapies, algorithmic advertising).

## Illustration: Average Treatment Effect

Let's return to our population of 4 units:

$i$	$D_i$	$Y_i$
1	1	3
2	1	1
3	0	0
4	0	1
$\mathbb{E}[Y_i \mid D_i = 1]$		2
$\mathbb{E}[Y_i \mid D_i = 0]$		0.5
$\mathbb{E}[Y_i \mid D_i = 1] - \mathbb{E}[Y_i \mid D_i = 0]$		1.5

What is  $\tau_{ATE} = \mathbb{E}[Y_{1i} - Y_{0i}]$ ?

Naïve estimator:

$$\begin{aligned}\tilde{\tau}_{ATE} &= \mathbb{E}[Y_i \mid D_i = 1] - \mathbb{E}[Y_i \mid D_i = 0] \quad (\text{observed difference in means}) \\ &= \frac{3+1}{2} - \frac{0+1}{2} = 1.5 \quad \text{Could this be wrong?}\end{aligned}$$

## Illustration: Average Treatment Effect

Let's return to our population of 4 units:

$i$	$D_i$	$Y_i$	$Y_{1i}$	$Y_{0i}$	$\tau_i$
1	1	3	3	0	3
2	1	1	1	1	0
3	0	0	1	0	1
4	0	1	1	1	0
$\mathbb{E}[Y_{1i}]$			1.5		
$\mathbb{E}[Y_{0i}]$				0.5	
$\mathbb{E}[Y_{1i} - Y_{0i}]$					1

$$\tau_{ATE} = \mathbb{E}[Y_{1i} - Y_{0i}] = \mathbb{E}[\tau_i] = \frac{3 + 0 + 1 + 0}{4} = 1.$$

But recall that  $\tilde{\tau}_{ATE} = 1.5$

Why does  $\tau_{ATE} \neq \tilde{\tau}$ ? When would they be equal?

# Illustration: Average Treatment Effect on the Treated

Again suppose we observe a population of 4 units:

$i$	$D_i$	$Y_i$	$Y_{1i}$	$Y_{0i}$	$\tau_i$
1	1	3	3	0	3
2	1	1	1	1	0
3	0	0	1	0	1
4	0	1	1	1	0
$\mathbb{E}[Y_{1i} \mid D_i = 1]$			2		
$\mathbb{E}[Y_{0i} \mid D_i = 1]$			0.5		
$\mathbb{E}[Y_{1i} - Y_{0i} \mid D_i = 1]$			1.5		

$$\tau_{ATT} = \mathbb{E}[Y_{1i} - Y_{0i} \mid D_i = 1] = \mathbb{E}[\tau_i \mid D_i = 1] = \frac{3 + 0}{2} = 1.5.$$



# Average Treatment Effect on the Treated

In this example, why does  $\tau_{ATT} \neq \tau_{ATE}$ ?

Because  $\mathbb{E}[Y_{1i}] \neq \mathbb{E}[Y_{1i}|D_i = 1]$  (and likewise for  $\mathbb{E}[Y_{0i}]$ )

That is,  $D_i$  and  $Y_{di}$  are associated

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# The Identification Problem for Causal Inference

## Identification:

In statistics, an **estimand** (parameter) is **identified** if its value can, asymptotically, be uniquely (**mapped to** observed data and unidentified otherwise).

In the language of research design, if an estimand is not identified, there are **alternative explanations** (mappings) connecting the observed data (the estimate) and the estimand of interest. (We will dive into this intuition in a moment).

Recall that in causal inference, estimands are typically population causal effects but the **FPCI** tells us that at least half of the potential outcomes are always missing.

## Identification Strategy:

A combination of **data** and **assumptions** which allows us to **identify** a causal estimand by estimating (“filling in”) the missing potential outcomes (usually at a group level) in expectation.

# Selection Bias

Let's see this idea in practice.

Consider again the naïve difference of observed means in the treatment groups:

$$\underbrace{E(Y_i|D_i = 1) - E(Y_i|D_i = 0)}_{\text{Observed difference in average outcome measures}} = E(Y_{1i}|D_i = 1) - E(Y_{0i}|D_i = 0)$$
$$= \underbrace{E(Y_{1i}|D_i = 1) - E(Y_{0i}|D_i = 1)}_{\text{ATT}} + \underbrace{E(Y_{0i}|D_i = 1) - E(Y_{0i}|D_i = 0)}_{\text{Selection bias}}$$

If we would want to use ATE for estimand we would need to include both unobserved terms ( $Y_1 + D = 0$ )  
But we would need to adjust the selection bias to include both  $Y_0 + D_1$  and  $Y_1 + D_0$

Read: The same observed mean difference could be due to **different combinations** of the ATT (estimand!) and selection bias terms.

Thus ATT is **unidentified** from the naïve observed mean difference: it is not uniquely mapped from the observed data. We need more **assumptions**.

Correlation [association, observed difference] is not necessarily causation.

# Selection Bias

$$\begin{aligned} & E(Y_i|D_i = 1) - E(Y_i|D_i = 0) \\ &= \underbrace{E(Y_{1i}|D_i = 1) - E(Y_{0i}|D_i = 1)}_{\text{ATT}} + \underbrace{E(Y_{0i}|D_i = 1) - E(Y_{0i}|D_i = 0)}_{\text{Selection bias}} \end{aligned}$$

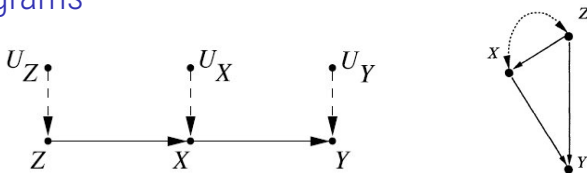
$E(Y_{0i}|D_i = 1) - E(Y_{0i}|D_i = 0)$  is called **selection bias** because, if it is not zero, treatment and control groups are systematically different in  $Y_{0i}$ . If non-zero, we might say the causal effect of  $D$  on  $Y$  is **confounded**.

Canonical example: Job training program

- Participants are **self-selected** from those in difficult labor situations
- Perhaps better resourced or more motivated individuals decide to take part
- Even in the absence of the program, post-training earnings for those people who participated would then have been higher than those for those who did not opt in:  $E[Y_0|D = 1] - E[Y_0|D = 0] > 0$

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# Causal Diagrams



So far we have reasoned about causal effects using potential outcomes. An alternative (but intimately connected) framework is the **graphical** approach.

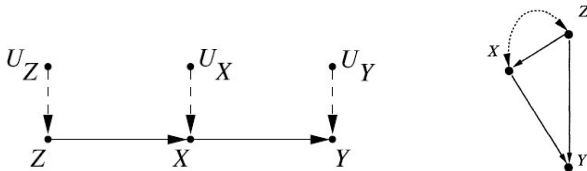
This uses **causal diagrams**, tools that allow us to:

1. Specify the variables (observed and unobserved) we care about
2. Specify how those variables are connected
3. See **what we can learn** about causal effects, and with **what assumptions**.

This can help us to:

1. Study how conditioning affects our research designs
2. Create new research designs and methodologies.

# Causal Diagrams as Directed Acyclic Graphs



Components of a causal diagram as a **Directed Acyclic Graphs (DAG)**:

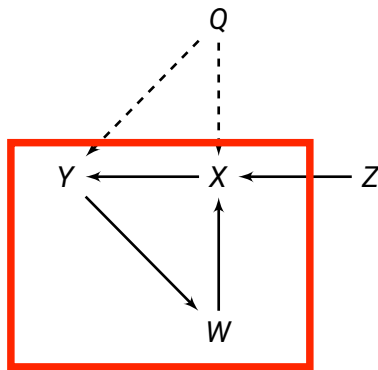
- **Nodes**: Representing “variables” (also called vertices)
- **Directed Edges**: Encoding one-way (causal) **relationships**  
Statement of belief - we believe that Z has effect on X  
→ This implies nodes are **ordered** (each pair: head and tail)  
→ These connections can be **observed** (solid) or **unobserved** (dashed)

Features of a DAG:

- Acyclic: No **directed cycles** (e.g. A does not terminate A)  
IF we have a directed cycle that is an example of underspecified theory
- Non-connections: The **absence** of relationships between variables  
This is also important because we say that a variable does not have an effect on what we measure



# Directed Acyclic Graphs: Example

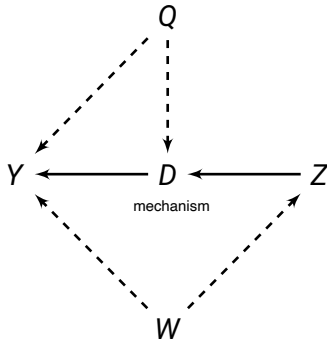


Cycle - AKA no DAG



"That's no DAG, it's a space station"

# Directed Acyclic Graphs: Example



W confounds the relationship on both Z and Y. In terms of selection bias:

it (W) sets your treatment group and it affects your potential outcome (Y) or This produces a covariance that is continuously set by W

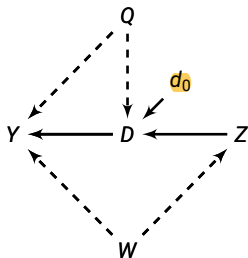
That's better! What can we learn from this DAG?

- $Z \rightarrow Y$  is confounded by  $W$
- $D \rightarrow Y$  is confounded by  $Q$
- $Z \rightarrow D$  is identified

But only if our DAG is correct!

ONLY IF - All arrows and absence of arrows are correct

# Representing Interventions

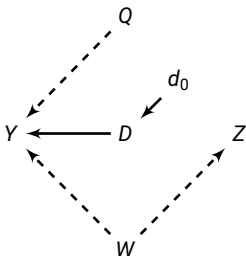


Treatments (interventions) are represented by the **`do()`** operator.

For example, **`do( $d_0$ )`** holds  $D = d_0$  exogenously.

Control

# Identification



ATE of  $D$  on  $Y$  defined as the average difference in  $Y$  between two interventions:

$$\mathbb{E}[Y \mid \text{do}(d_1)] - \mathbb{E}[Y \mid \text{do}(d_0)]$$

NO potential outcome in this equation only do values

But we cant observe both - thats the fundamental problem of casual inference  
**Problem:** Can this be estimated without an explicit intervention (identification)?

**Insight:** If the DAG is equivalent with and without  $\text{do}()$ , yes.

**Generally:** We can identify the effect of  $D$  on  $Y$  if all back-door paths are blocked.

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# Assignment Mechanism

## Assignment Mechanism:

The procedure that determines the treatment status of each unit.

Most causal inference methods achieve identification by **restricting** the (assumed) **assignment mechanism**

For example, if we are willing to assume that treatment assignment is independent of potential outcomes under no treatment, then:

$$E(Y_{0i}|D_i = 1) - E(Y_{0i}|D_i = 0) = 0$$

i.e. selection bias is zero and the observed mean difference is (in expectation) equal to ATT (and also ATE in that case)

# Different Assignment Mechanisms

Imbens and Rubin (2015, Ch. 3) present **three assumptions** about assignment mechanisms (for each unit) that provide the grounds for identification:

1. **Individualistic**: Assignment does not depend on the covariates or potential outcomes for other units.
2. **Probabilistic**: There is a nonzero probability of each treatment value, for every unit.
3. **Unconfounded**: Assignment does not depend on potential outcomes.

Assuming the above, we can distinguish:

- **(Randomized) Experiments**: The assignment mechanism is both known and controlled by the researcher (usually with some flavour of randomization).
- **Observational Studies**: The assignment mechanism is not known to, or not under the control of, the researcher.



# Our Key Assignment Mechanisms

**Randomised Experiments:** These come in many flavours, many of which we won't even discuss, for example...

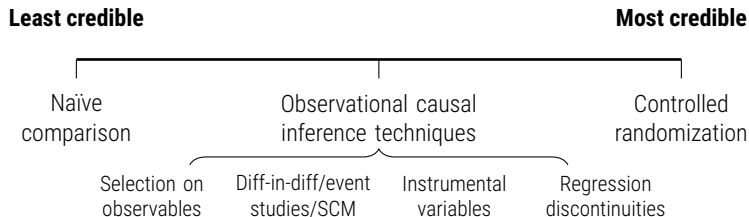
- Within designs, between designs
- Unit-randomized, cluster-randomized, dynamic randomization
- Crossover designs, stepped-wedge designs, etc. etc. etc.

**Observational Studies:**

- Selection on observables using regression, matching, etc.
- Instrumental variables, shift-share designs, etc.
- Sharp and fuzzy regression discontinuity designs, etc.
- Temporal: Diff-in-diff, event studies, synthetic control methods

Note: The first three observational designs are sometimes referred to as “design-based” methods. We'll talk about this distinction later.

# The Continuum of Credibility™



Key point: The art (and science) of applied causal inference is making defensible assumptions. There is no 'magic' solution to the fundamental problem of causal inference, only assumptions all the way down!

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# Key ideas from this week

- Learned to think about causal effects in terms of **potential outcomes**, not realized (observed) outcomes
- Observed association is **neither necessary nor sufficient** for causality – focused on one big problem, selection bias
- Introduced an alternative framework for thinking about causal models – the **graphical** approach
- Learning about causal effects should start from **understanding the assignment mechanism** for treatment
- Evaluate the **plausibility of your assumptions** to understand the credibility of your conclusions