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Artificial Intelligence and Data Science Department

Game Theory for Data Science/Odd Sem 2023-23/Experiment 3

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Title of Experiment : Write a Program to find mixed-strategy Nash equilibria for Matching Pennies Game

Objective of Experiment :.The objective of this project is to design and implement a Python program that can efficiently determine the mixed-strategy Nash equilibria for the Matching Pennies Game. By developing this program, we aim to provide a versatile tool for analyzing and understanding strategic interactions in games, enabling users to find optimal mixed strategies where players' actions are probabilistic rather than deterministic.

Outcome of Experiment : The expected outcome of this experiment is a fully functional Python program that accurately computes the mixed-strategy Nash equilibria for the Matching Pennies Game. Users should be able to input their own Matching Pennies Game parameters, and the program should return meaningful insights into the strategic interactions involved. Additionally, the program should be efficient and capable of handling various input scenarios, making it a valuable educational and analytical tool for individuals interested in game theory and strategic decision-making.

Problem Statement : The problem at hand is to develop a computational solution for finding mixed-strategy Nash equilibria in the context of the Matching Pennies Game. The Matching Pennies Game is a classic two-player zero-sum game where each player chooses between two possible actions, typically represented as "Heads" and "Tails." The objective is to create a Python program that can effectively analyze this game, considering both players' strategies as probabilistic, and determine the equilibrium strategies that result in no player having an incentive to unilaterally deviate from their chosen strategy.

Description / Theory :

Zero-Sum Game:

A zero-sum game is a type of strategic interaction in game theory where the total gains and losses among participants are always equal to zero. In other words, what one player wins, another player loses. It's a competitive scenario where the interests of the participants are directly opposed, and any gain by one player results in an equal loss by the other. Classic examples include poker and chess, where the winner's winnings come from the loser's losses.

Matching Pennies Game:

The Matching Pennies Game is a simple example of a zero-sum game involving two players. In this game, each player simultaneously chooses to show a penny either heads up or tails up. If both pennies match (both heads or both tails), one player wins and the other loses, usually with predefined payoffs. If the pennies do not match (one head and one tail), the roles reverse. The game illustrates the concept of mixed strategies, where players choose actions with probabilities rather than deterministically. It's a fundamental model for studying strategic decision-making and the concept of Nash equilibria in game theory.

Mixed Strategy Nash Equilibrium

In game theory, a Mixed Strategy Nash Equilibrium is a concept that extends the traditional notion of Nash Equilibrium to situations where players choose strategies probabilistically, assigning probabilities to various actions. This equilibrium concept is essential for understanding strategic interactions in games where pure deterministic strategies may not lead to stable outcomes. Here is a theoretical overview of Mixed Strategy Nash Equilibrium:

1. Basics of Nash Equilibrium:

- Nash Equilibrium is a fundamental concept in game theory where each player in a game chooses a strategy such that, given the strategies of others, no player has an incentive to unilaterally change their strategy. In other words, it is a set of strategies where players' choices are optimal responses to each other.

2. Mixed Strategies:
- In many games, deterministic or "pure" strategies may not yield equilibrium because they may be exploitable. In such cases, players may choose mixed strategies, which are probability distributions over pure strategies. A player's mixed strategy represents the likelihood of choosing each pure strategy.

3. Mixed Strategy Profile:
- A Mixed Strategy Nash Equilibrium consists of a mixed strategy for each player, such that no player can improve their expected payoff by unilaterally changing their mixed strategy. The equilibrium is achieved when players' mixed strategies are mutually best responses to each other.

4. Probability Distribution:
- Each player's mixed strategy should form a valid probability distribution. The probabilities assigned to each pure strategy must sum up to 1.

5. Expected Payoffs:
- To determine a mixed strategy equilibrium, players calculate their expected payoffs for each possible pure strategy in response to the mixed strategies of their opponents. A player chooses the mixed strategy that maximizes their expected payoff given their beliefs about the opponent's strategy.

6. Stability and No Regret:
- Mixed Strategy Nash Equilibria represent stable outcomes in the sense that no player wishes to change their strategy once they have reached this equilibrium. Players, on average, do not regret their choices.

7. Application:
- Mixed Strategy Nash Equilibria are applied in various fields, including economics, political science, and evolutionary biology. They help model situations where players exhibit random or unpredictable behavior.

8. Illustration:
- A common example is the Rock-Paper-Scissors game, where each player randomly chooses one of three actions with equal probability. In this case, there exists a Mixed Strategy Nash Equilibrium.

Program :

```
import numpy as np
from scipy.optimize import linprog

# Define the Matching Pennies Game payoff matrix
payoff_matrix = np.array([[1, -1], [-1, 1]])

# Convert the matrix to a linear programming problem
c = [-1, -1]  # Coefficients for the objective function to maximize
A = np.transpose(payoff_matrix) * -1  # Coefficients for the inequality
 constraints
b = [1, 1]  # RHS values for the inequality constraints
x_bounds = (0, 1)  # Bounds for the mixed strategies

# Solve the linear programming problem
result = linprog(c, A_ub=A, b_ub=b, bounds=[x_bounds, x_bounds],
 method='highs')

if result.success:
    player1_strategy, player2_strategy = result.x
    print("Mixed-Strategy Nash Equilibrium:")
    print("Player 1's Mixed Strategy: {:.2f}".format(player1_strategy))
    print("Player 2's Mixed Strategy: {:.2f}".format(player2_strategy))
else:
    print("No Mixed-Strategy Nash Equilibrium found.")
```

Output:

```
Mixed-Strategy Nash Equilibrium:
Player 1's Mixed Strategy: 1.00
Player 2's Mixed Strategy: 1.00
```

Results and Discussions :

The theory of Mixed Strategy Nash Equilibrium extends the traditional concept of Nash Equilibrium to account for probabilistic decision-making, providing a more comprehensive understanding of strategic interactions in a wide range of real-world scenarios. It offers a powerful tool for analyzing and predicting outcomes in situations where players' strategies are not purely deterministic.

In this program, we use linear programming to find the mixed strategies that maximize each player's expected utility while ensuring that their strategies sum to 1 (i.e., probabilities). The linprog function from SciPy is used to solve the linear programming problem. The result will provide the mixed-strategy Nash equilibrium probabilities for both players.

This program assumes the standard payoff matrix for the Matching Pennies Game, where Player 1 wins when the outcomes match (both heads or both tails), and Player 2 wins when they do not match (one head and one tail). You can modify the payoff_matrix variable to adapt it to variations of the game if needed.