DEVOPS RESEARCH SIMULATION PROJECT - BANKING

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♦ PROBLEM DESCRIPTION

Our bank is opening a new location and would like to conduct a study to determine how daily banking operations might operate based on 4 different scenarios. These scenarios will be modeled as a banking system using 4 different queueing models with the following properties:

System	Entities	Attributes	Activities	Events	State Variables
Banking	Customers	Chequing account, balance	Making Deposits, Opening an account	Arrival, Departure	Number of customers in the building

The following terminology will be used to describe tools used:

JMT – *Java Modelling Tools v.1.0.5*

Excel – *Microsoft Excel*

♦ THE PROBLEM MAPPING TO EACH OF THE QUEUING MODELS

The problem will map to 4 of the following queue models and will be explained below:

1. **M/M/1**
   Will be modeled with exponential arrival rate. Where each customer will be serviced at a random rate based on an exponential distribution. This model will aim to simulate customers arriving to the bank, making a deposit account balance at the single teller in the system, and departing.

2. **M/G/1**
   Will be modeled with an exponential arrival rate and a uniform service rate. Encompasses the scenario where each customer is only allotted 0,1 or 2 minutes at the teller and simulates customers arriving to the bank, making a deposit to their account at the single teller in the system, and departing.

3. **M/M/4/16**
   Will be modeled with an exponential arrival rate and an exponential service rate with 4 tellers and a system capacity of 16. Models the scenario with 4 tellers and a total bank capacity of 16 customers, the bank will refuse entry as any more customers in the building is a potential fire hazard. Customers will arrive to the bank, make a deposit to their account balance at one of the 4 tellers in the system, and departing.

4. **NETWORK OF QUEUES**
   Will demonstrate a network of queues with 3 separate M/M/1 queues. Models the scenario where a customer wants to open a bank account and must first see one of two bank tellers (with differing arrival rates) and then be redirected to a bank manager, open an account, and depart.

◆ SIMULATION GOALS AND SIMULATION PARAMETERS

Goals

The goal of this simulation will be to compare the theoretical values of our metrics to the simulated model for each of the for queues listed to observe how potential scenarios will play out in our bank. Another goal of the simulation will be to see whether observed values fall within computed confidence intervals. Finally, analysis of the data will allow us to determine whether our models are correct, need more samples, or if they must be refined.

Parameters

All of the simulations conducted in this report followed a template(See Figure 1.) with only variations to the seed. Any concerns about these parameters determined via observation will be noted in the analysis portion of the report in which they occur.

Simulation Parameters
Define simulation parameters and initial customer locations.

Simulation random seed:	5000	☐ random
Maximum duration (sec):	30	☐ infinite
Maximum simulated time:	600	☑ infinite
Maximum number of samples:	1,000,000	☐ no automatic stop
Maximum number of events:	1,000,000	☑ infinite
Animation update interval (sec):	1	☑ animation

Figure 1. *Simulation Parameters Template*

♦ METHODOLOGY

ν Tools Used to Translate the Models

To translate the queue models, I used JMT - Java Modelling Tools v.1.0.5. Simulation data was then extracted from JMT and calculation of theoretical values was done in Microsoft Excel.

ν Simulation Setups

1. **M/M/1**
   Modeled in JMT as depicted in (Figure.2). Arrival distribution in Source 1 was set up as exp(0.1) in 8 steps from 0.1 to 0.8. Service distribution was then set up as exp(1).

Figure 2. *M/M/1 Queue*

2. **M/G/1**
   Modeled in JMT as depicted in (Figure.3). Arrival distribution in Source 1 was set up as exp(0.1) in 8 steps from 0.1 to 0.8. Service distribution was then set up as Uniform(0,2) .
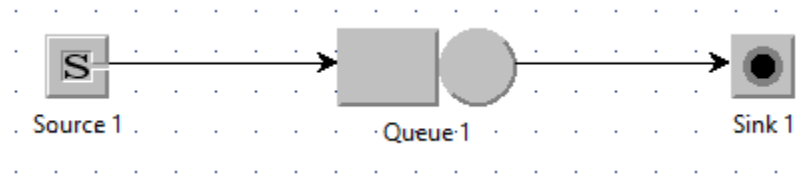
Figure 3. *M/G/1 Queue using Uniform Service distribution*

3. **M/M/4/16**
   Modeled in JMT as depicted in (Figure.4). Arrival distribution in Source 1 was set up as exp(0.4) in 8 steps from 0.4 to 3.2 (With respective utilizations of 0.1 to 0.8). Service distribution was then set up as exp(1) with 4 servers and a finite system capacity of 16.
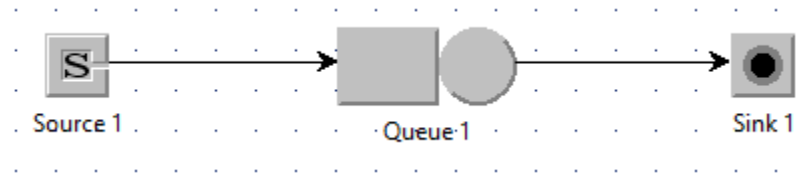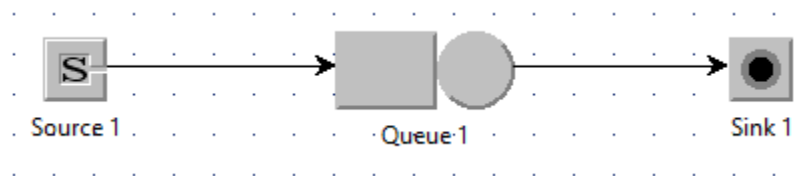
Figure 4. *M/M/4/16 Queue*

4. NETWORK OF QUEUES

Modeled in JMT as depicted in (Figure.5). Arrival distribution in Source 1 was set up as exp(0.03) in 8 steps from 100% to 800% (Class 1). Arrival distribution in Source 2 was set up as exp(0.07) in 8 steps from 100% to 800% (Class 2). Service distribution was then set up as exp(1) in each queue. Note that customers in Class 1 go through Queue 1 and Queue 3 and customers in Class 2 go through Queue 2 and Queue 3. Metrics are compared via reference to their respective Class in the analysis portion of the report.
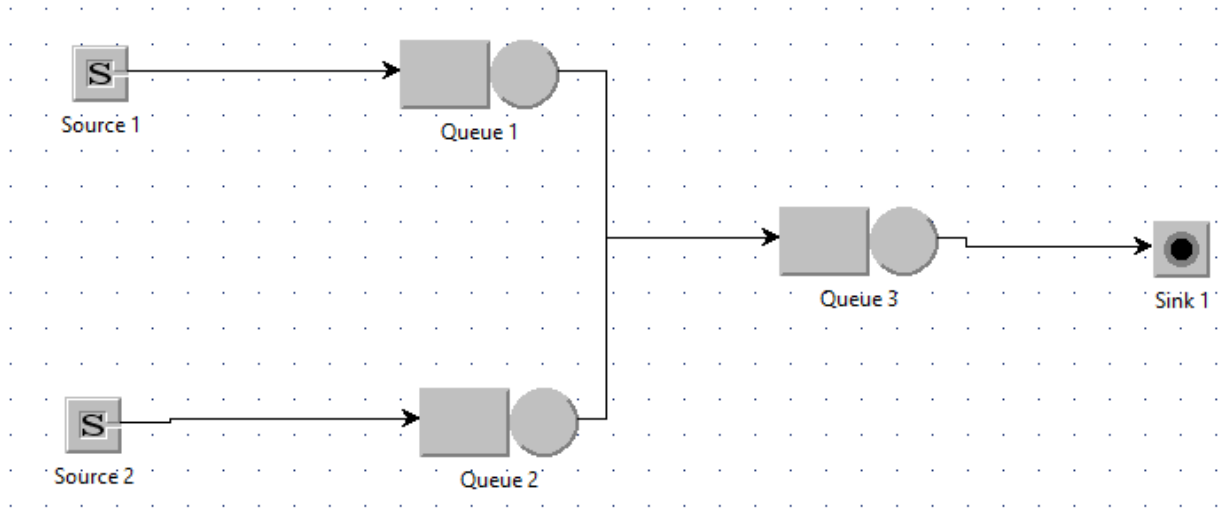
Figure 5. *Network of Queues*

ν Process of Stat Collection

Stats were collected through the "What If Analysis" feature provided in JMT (See Figure 6.) then transferred to a table in Excel for further analysis. Theoretical values were calculated using an excel sheet (See Figure 7.) set up to produce values based on input of lambda, mu and sigma2.
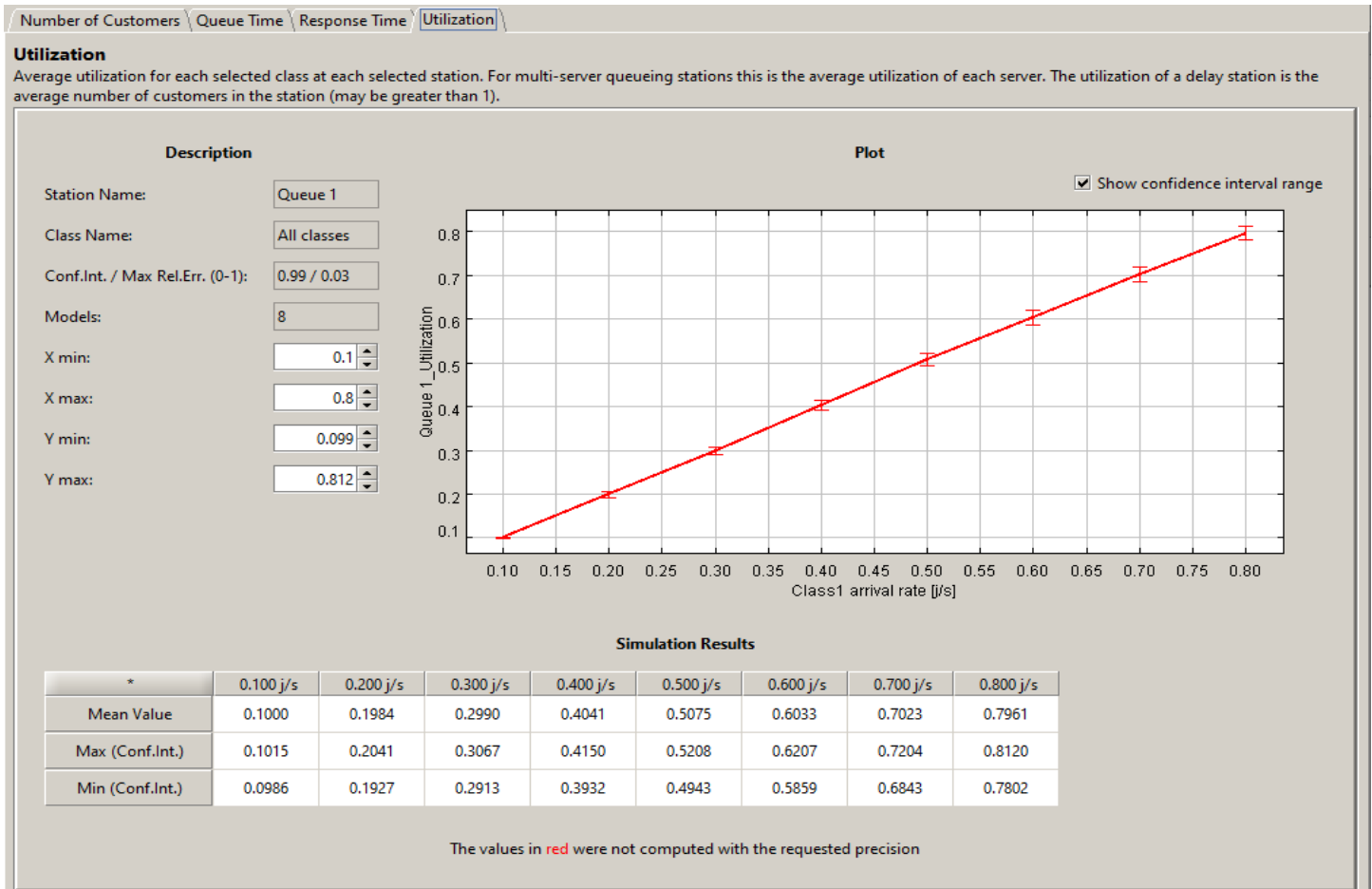
Figure 6. *Example of "What If Analysis" feature in JMT for a M/M/1 Queue showing stats for utilization*

Figure 7. *Example of Excel Sheet used for computing theoretical values for a M/G/1 Queue*

♦ ANALYSIS

Note: *Tables in this section should be read with the following in mind* - **Confidence intervals** under any header "**AVERAGE OF REPLICATIONS**" are **calculated** using the averages of the sampled stats from the prior 3 replications and are NOT the averages of the confidence intervals that JMT has reported. The best and worse case are then calculated using the theoretical mean of the stat. It should also be noted that **steps** are reported in terms of expected server utilization.

Note 2: *All tables appearing in this section can be found in the github repo as "**ProjectDataCollection.xlsx**"*

1. **M/M/1**
   - Compares observed performance metrics "p , L , w , wq" to their theoretical counterparts
   - Statistics collection for inter-arrival distribution
   - Testing of JMT's random number generator
   - Collected system state histogram compared with theory

2. **M/G/1**
   - Compares observed performance metrics "p , L , w , wq" to their theoretical counterparts

3. **M/M/4/16**
   - Compares observed performance metrics "p , L , w , wq, pn (drop rate)" to their theoretical counterparts
   - Study of Loss Behavior

4. **NETWORK OF QUEUES**
   - Compares observed performance metric "w" to it's theoretical counterpart for each class.
   - Comparison of Class 1 to Class 2

M/M/1

ν Reporting of Collected Statistics Including Confidence Intervals

M/M/1 Number of customers - "L"

Simulated Number of customers - "L"								
Replication 1 (Seed: *5000*) -								
Step:	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**
Mean Value (j)	0.111	0.2476	0.4241	0.6734	1.0141	1.4921	2.3332	3.9345
Max (j.) Confidence Interval	0.1144	0.2543	0.4328	0.6916	1.0404	1.5328	2.3993	4.1202
Min (j.) Confidence Interval	0.1077	0.2409	0.4154	0.6553	0.9879	1.4515	2.2672	3.7488
Replication 2 (Seed: *10000*) -								
Step:	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**
Mean Value (j)	0.1108	0.2491	0.4273	0.6637	1.0163	1.511	2.3357	4.0541
Max (j.) Confidence Interval	0.1141	0.2531	0.4361	0.6773	1.0411	1.5425	2.3938	4.2334
Min (j.) Confidence Interval	0.1076	0.2451	0.4184	0.6502	0.9914	1.4794	2.2776	3.8748
Replication 3 (Seed: *15000*) -								
Step:	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**
Mean Value (j)	0.1114	0.2502	0.429	0.6598	0.9827	1.4817	2.2944	4.0361
Max (j.) Confidence Interval	0.1142	0.2559	0.438	0.6748	1.0028	1.5243	2.3599	4.1993
Min (j.) Confidence Interval	0.1086	0.2446	0.42	0.6449	0.9626	1.439	2.2289	3.873
AVERAGE OF REPLICATIONS								
Step:	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**
Mean Value (j)	0.111067	0.248967	0.4268	0.665633	1.004367	1.494933	2.3211	4.008233
Point Estimator	0.111067	0.248967	0.4268	0.665633	1.004367	1.494933	2.3211	4.008233
T0.005,2	9.92	9.92	9.92	9.92	9.92	9.92	9.92	9.92
Sample Variance:	9.33E-08	1.7E-06	6.19E-06	4.9E-05	0.000353	0.000221	0.000536	0.004158
Standard Deviation	0.000306	0.001305	0.002488	0.007003	0.018796	0.014854	0.023157	0.064486
Max (j.) Confidence Interval	0.112816	0.256441	0.441049	0.705742	1.112018	1.580007	2.453725	4.377565
Min (j.) Confidence Interval	0.109317	0.241492	0.412551	0.625524	0.896715	1.409859	2.188475	3.638901
Best Case	0.001705	0.006441	0.012478	0.039076	0.103285	0.080007	0.120392	0.361099
Worst Case	0.001794	0.008508	0.016021	0.041142	0.112018	0.090141	0.144859	0.377565

Theoretical Number of customers - "L"								
Step:	**0.1**	0.2	**0.3**	**0.4**	0.5	0.6	**0.7**	0.8
Mean Value	0.111111	0.25	0.428571	0.666667	1	1.5	2.333333	4

Comparison:

Simulated values match up with theoretical values and fall within computed confidence intervals.

M/M/1 Utilization - "p"

Simulated Utilization - "p"								
Replication 1 (Seed: *5000*) -								
Step:	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**
Mean Value (j)	0.1	0.1984	0.299	0.4041	0.5075	0.6033	0.7023	0.7961
Max (j.) Confidence Interval	0.1015	0.2041	0.3067	0.415	0.5208	0.6207	0.7204	0.812
Min (j.) Confidence Interval	0.0986	0.1927	0.2913	0.3932	0.4943	0.5859	0.6843	0.7802
Replication 2 (Seed: *10000*) -								
Step:	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**
Mean Value (j)	0.1001	0.1986	0.3001	0.3999	0.4925	0.5975	0.6922	0.7886
Max (j.) Confidence Interval	0.1027	0.2039	0.3074	0.4101	0.5062	0.6096	0.706	0.8123
Min (j.) Confidence Interval	0.0975	0.1934	0.2927	0.3898	0.4787	0.5855	0.6784	0.765
Replication 3 (Seed: *15000*) -								
Step:	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**
Mean Value (j)	0.1001	0.1996	0.2997	0.4016	0.5006	0.6034	0.7025	0.804
Max (j.) Confidence Interval	0.1023	0.2046	0.307	0.4111	0.5107	0.6194	0.717	0.8188
Min (j.) Confidence Interval	0.098	0.1945	0.2924	0.392	0.4905	0.5874	0.6881	0.7893
AVERAGE OF REPLICATIONS								
Step:	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**
Mean Value (j)	0.100067	0.198867	0.2996	0.401867	0.5002	0.6014	0.699	0.796233
Point Estimator	0.100067	0.198867	0.2996	0.401867	0.5002	0.6014	0.699	0.796233
$T_{0.005,2}$	9.92	9.92	9.92	9.92	9.92	9.92	9.92	9.92
Sample Variance:	3.33E-09	4.13E-07	3.1E-07	4.46E-06	5.64E-05	1.14E-05	3.47E-05	5.93E-05
Standard Deviation	5.77E-05	0.000643	0.000557	0.002113	0.007508	0.003378	0.00589	0.007701
Max (j.) Confidence Interval	0.100397	0.202549	0.302789	0.413967	0.543201	0.620746	0.732733	0.840339
Min (j.) Confidence Interval	0.099736	0.195185	0.296411	0.389767	0.457199	0.582054	0.665267	0.752128
Best Case	0.000264	0.002549	0.002789	0.010233	0.042801	0.017946	0.032733	0.040339
Worst Case	0.000397	0.004815	0.003589	0.013967	0.043201	0.020746	0.034733	0.047872

Theoretical Utilization - "p"								
Step:	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**
Mean Value	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8

Comparison:

Simulated values match up with theoretical values and fall within computed confidence intervals**.**

M/M/1 Response Time - "w"

Simulated Response Time - "w"								
Replication 1 (Seed: 5000) -								
Step:	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**
Mean Value (j)	1.1112	1.2453	1.4186	1.6959	2.021	2.4838	3.3199	4.9752
Max (j.) Confidence Interval	1.1376	1.2801	1.4599	1.7451	2.0689	2.5449	3.3988	5.114
Min (j.) Confidence Interval	1.0847	1.2105	1.3772	1.6466	1.9731	2.4228	3.2409	4.8364
Replication 2 (Seed: 10000) -								
Step:	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**
Mean Value (j)	1.1168	1.2471	1.4244	1.6599	2.0172	2.5113	3.3456	5.0485
Max (j.) Confidence Interval	1.1474	1.2747	1.457	1.6868	2.059	2.5751	3.4166	5.1719
Min (j.) Confidence Interval	1.0862	1.2195	1.3918	1.6331	1.9754	2.4475	3.2747	4.925
Replication 3 (Seed: 15000) -								
Step:	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**
Mean Value (j)	1.1194	1.2564	1.4328	1.6455	1.9724	2.4705	3.2902	5.0133
Max (j.) Confidence Interval	1.1397	1.2875	1.4665	1.6879	2.0098	2.5422	3.3733	5.1572
Min (j.) Confidence Interval	1.0991	1.2252	1.3991	1.603	1.9349	2.3988	3.2072	4.8693
AVERAGE OF REPLICATIONS								
Step:	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**
Mean Value (j)	1.1158	1.2496	1.425267	1.6671	2.003533	2.488533	3.318567	5.012333
Point Estimator	1.1158	1.2496	1.425267	1.6671	2.003533	2.488533	3.318567	5.012333
T0.005,2	9.92	9.92	9.92	9.92	9.92	9.92	9.92	9.92
Sample Variance:	1.76E-05	3.55E-05	5.1E-05	0.000674	0.000731	0.000433	0.000769	0.001344
Standard Deviation	0.00419	0.005957	0.00714	0.02596	0.027029	0.020808	0.027724	0.03666
Max (j.) Confidence Interval	1.1398	1.28372	1.466157	1.815781	2.158338	2.607706	3.477351	5.222294
Min (j.) Confidence Interval	1.0918	1.21548	1.384376	1.518419	1.848729	2.369361	3.159782	4.802372
Best Case	0.019311	0.03372	0.037586	0.148248	0.151271	0.107706	0.144018	0.197628
Worst Case	0.028689	0.03452	0.044195	0.149114	0.158338	0.130639	0.173551	0.222294

Theoretical Response Time - "w"								
Step:	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
Mean Value	1.111111	1.25	1.428571	1.666667	2	2.5	3.333333	5

Comparison:

Simulated values match up with theoretical values and fall within computed confidence intervals.

M/M/1 Queue Time - "wq"

Simulated Queue Time - "wq"								
Replication 1 (Seed: 5000) -								
Step:	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**
Mean Value (j)	0.1122	0.2524	0.4346	0.6739	1.0078	1.5083	2.3291	3.9931
Max (j.) Confidence Interval	0.1151	0.2587	0.446	0.6927	1.0279	1.5418	2.3876	4.1402
Min (j.) Confidence Interval	0.1093	0.246	0.4233	0.6551	0.9878	1.4749	2.2706	3.846
Replication 2 (Seed: 10000) -								
Step:	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**
Mean Value (j)	0.1114	0.2495	0.4277	0.6643	0.9949	1.5072	2.3444	4.0483
Max (j.) Confidence Interval	0.1137	0.2555	0.4403	0.6798	1.0218	1.5477	2.4124	4.1694
Min (j.) Confidence Interval	0.1092	0.2434	0.4151	0.6489	0.9679	1.4667	2.2765	3.9272
Replication 3 (Seed: *15000*) -								
Step:	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**
Mean Value (j)	0.1106	0.2459	0.4206	0.661	0.9866	1.4778	2.3174	4.0168
Max (j.) Confidence Interval	0.1129	0.2517	0.4291	0.6748	1.0082	1.511	2.3695	4.1365
Min (j.) Confidence Interval	0.1083	0.2402	0.412	0.6472	0.965	1.4447	2.2652	3.897
AVERAGE OF REPLICATIONS								
Step:	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**
Mean Value (j)	0.1114	0.249267	0.427633	0.6664	0.996433	1.497767	2.3303	4.0194
Point Estimator	0.1114	0.249267	0.427633	0.6664	0.996433	1.497767	2.3303	4.0194
T0.005,2	9.92	9.92	9.92	9.92	9.92	9.92	9.92	9.92
Sample Variance:	6.4E-07	1.06E-05	4.9E-05	4.49E-05	0.000114	0.000299	0.000183	0.000767
Standard Deviation	0.0008	0.003256	0.007	0.006701	0.010683	0.0173	0.01354	0.027692
Max (j.) Confidence Interval	0.115982	0.267916	0.467726	0.704782	1.057617	1.596851	2.407848	4.177999
Min (j.) Confidence Interval	0.106818	0.230617	0.387541	0.628018	0.935249	1.398682	2.252752	3.860801
Best Case	0.004293	0.017916	0.039154	0.038115	0.057617	0.096851	0.074514	0.139199
Worst Case	0.004871	0.019383	0.041031	0.038648	0.064751	0.101318	0.080581	0.177999

Theoretical Queue Time - "wq"								
Step:	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**
Mean Value	0.111111	0.25	0.428571	0.666667	1	1.5	2.333333	4

Comparison:

Simulated values match up with theoretical values and fall within computed confidence intervals.

ν Statistics Collection for Inter-Arrival Distribution
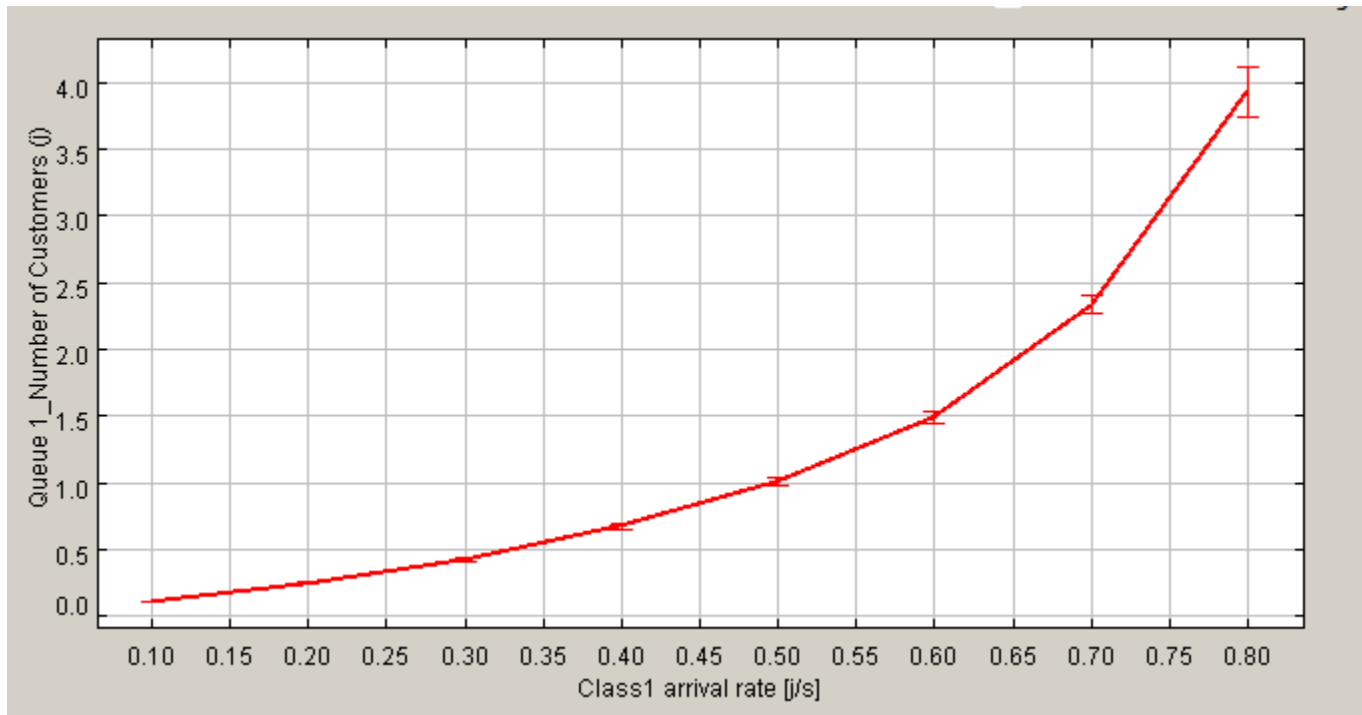
Figure 8. *Graph of collected inter-arrival distribution*

Conclusion:

Inter-arrival curve shape (See Figure 8.) matches exponential distribution.

ν Testing of Random Number Generator Used

Note 1: Testing of JMT random number generator was done by generating numbers within JMT and testing them against both Kolmogogrov-Smirnov test and Chi-Squared Test to verify acceptable amount of randomization.

*Note 2: Code for testing can be found on repo as "**randNumGenCSVInput.java**"*
    *JMT generated number data can be found on repo as "**Logger1RandomData.csv**"*

Example Program Output:

```
Getting data from CSV file...
Conducting Kolmogorov-Smirnov Test with 10 JMT-Random Numbers...
 -Level of Significance -> 0.01:
    -h0 is not rejected... d0.01: 0.49>d:0.12156940309796482
 -Level of Significance -> 0.05:
    -h0 is not rejected... d0.05: 0.41>d:0.12156940309796482
---------------------*TEST COMPLETE*-----------------------------


Conducting Chi-square Test with 100 JMT-Random Numbers...
```

```
 -Level of Significance -> 0.01:
    -h0 is not rejected... d0.01: 21.7>d:11.2
 -Level of Significance -> 0.05:
    -h0 is not rejected... d0.05: 16.9>d:11.2
---------------------*TEST COMPLETE*---------------------------
Conducting Kolmogorov-Smirnov Test with 10000 JMT-Random Numbers...
 -Level of Significance -> 0.01:
    -h0 is not rejected... d0.01: 0.49>d:4.2767846025526524E-5
 -Level of Significance -> 0.05:
    -h0 is not rejected... d0.05: 0.41>d:4.2767846025526524E-5
---------------------*TEST COMPLETE*---------------------------
Conducting Chi-square Test with 10000 JMT-Random Numbers...
 -Level of Significance -> 0.01:
    -h0 is not rejected... d0.01: 21.7>d:11.084
 -Level of Significance -> 0.05:
    -h0 is not rejected... d0.05: 16.9>d:11.084
---------------------*TEST COMPLETE*---------------------------
Program exiting...
```

Conclusions:

The random generator in JMT did not fail the uniformity test using Kolmogorov-Smirnov or Chi-Squared in any of my trials with a significance of 0.01. In a further study of 30 trials, neither test failed.

ν Collection of System State Histogram Compared with Theory

System State Histogram of 1000 Samples and CDF Curve
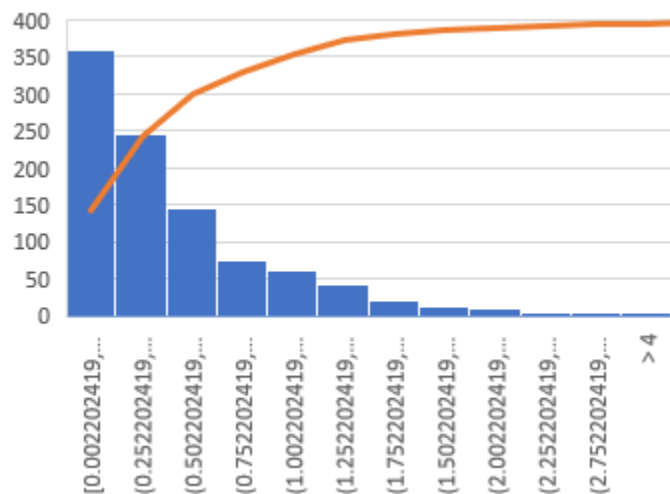
Figure 9. System State Histogram of 1000 Samples and CDF Curve
Note: Y-Axis Values are scaled by 1000 and should instead be 0, 0.1,0.2, … to 1 to represent respective probabilities in terms of the CDF. Otherwise, Y-Axis denotes number of occurrences of a value falling in the range below each respective bar – increments are ~0.25 in range.

M/M/1 Solution for P_k
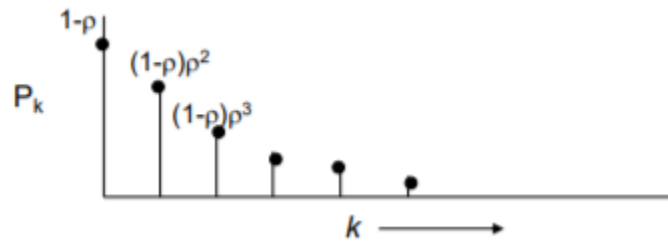
Figure 10. Solution for P_k for a M/M/1 Queue
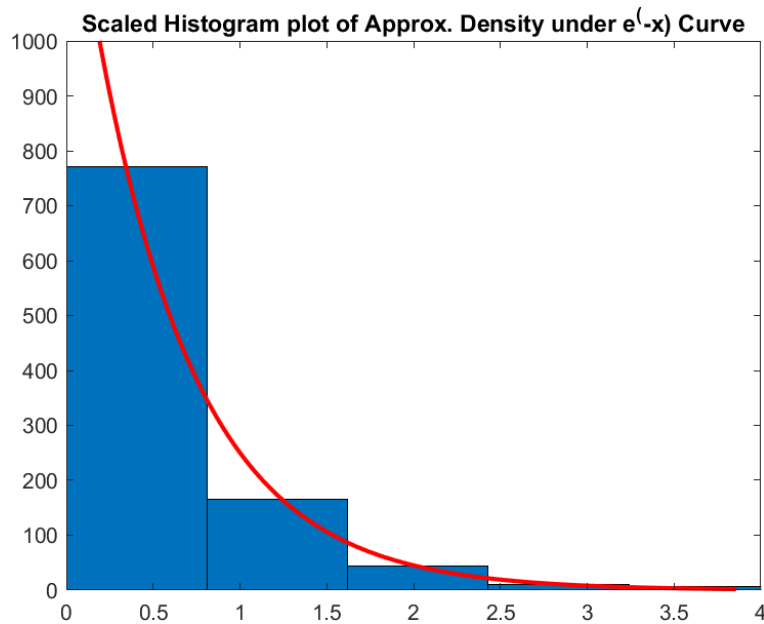
Figure 11. Normalized Histogram plot of 1000 samples under respective theoretical e(-x) density curve
Note: Y-Axis Values are scaled by 1000 and should instead be 0, 0.1,0.2, … to 1 to represent respective probabilities
Note 2: Program used to generate data included within the bars can be found in the repo as "exprv.xlsx"

Conclusion:

The shape of our collected system state for the M/M/1 queue (See Figure 9.) matches its theoretical counterpart (See Figure 10.). Finally, the first 1000 samples of our inter-arrival times were normalized and compared to theoretical density. This plot (Figure 11.) revealed that our observed density was close to our theoretical density.

M/G/1

ν Reporting of Collected Statistics Including Confidence Intervals

M/G/1 Number of customers - "L"

Simulated Number of customers - "L"								
Replication 1 (Seed: *6000*) -								
Step:	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
Mean Value (j)	0.1065	0.2307	0.3894	0.5837	0.8376	1.203	1.8173	2.9725
Max (j.) Confidence Interval	0.1094	0.2369	0.3989	0.5988	0.8577	1.2301	1.8647	3.0555
Min (j.) Confidence Interval	0.1036	0.2245	0.3799	0.5685	0.8174	1.1759	1.7699	2.8895
Replication 2 (Seed: *12000*) -								
Step:	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
Mean Value (j)	0.1077	0.2335	0.3811	0.571	0.8251	1.178	1.7603	2.9476
Max (j.) Confidence Interval	0.1101	0.2395	0.3904	0.5822	0.8463	1.2122	1.8097	3.0426
Min (j.) Confidence Interval	0.1054	0.2276	0.3718	0.5599	0.8039	1.1437	1.7109	2.8526
Replication 3 (Seed: *18000*) -								
Step:	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
Mean Value (j)	0.1077	0.2321	0.3816	0.5768	0.8406	1.1989	1.7907	2.9735
Max (j.) Confidence Interval	0.11	0.2386	0.3922	0.5927	0.862	1.2246	1.8345	3.0888
Min (j.) Confidence Interval	0.1055	0.2255	0.371	0.561	0.8192	1.1731	1.7468	2.8582
AVERAGE OF REPLICATIONS								
Step:	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
Mean Value (j)	0.1073	0.2321	0.384033	0.577167	0.834433	1.1933	1.789433	2.964533
Point Estimator	0.1073	0.2321	0.384033	0.577167	0.834433	1.1933	1.789433	2.964533
T0.005,2	9.92	9.92	9.92	9.92	9.92	9.92	9.92	9.92
Sample Variance:	4.8E-07	1.96E-06	2.17E-05	4.04E-05	6.76E-05	0.00018	0.000813	0.000215
Standard Deviation	0.000693	0.0014	0.004654	0.006358	0.008221	0.013408	0.028521	0.014673
Max (j.) Confidence Interval	0.111268	0.240118	0.41069	0.613581	0.881517	1.270091	1.952783	3.048571
Min (j.) Confidence Interval	0.103332	0.224082	0.357376	0.540753	0.78735	1.116509	1.626084	2.880495
Best Case	0.002224	0.000918	0.00691	0.007419	0.03735	0.066509	0.109417	0.480495
Worst Case	0.005712	0.015118	0.046405	0.080247	0.131517	0.220091	0.436116	0.648571

Theoretical Number of customers - "L"									
Step:		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
Mean Value		0.105556	0.225	0.364286	0.533333	0.75	1.05	1.516667	2.4

Comparison:

Simulated values are larger than theoretical values and do not fall within theoretically computed confidence intervals after step 0.3.

M/G/1 Utilization - "p"

Simulated Utilization - "p"								
Replication 1 (Seed: *6000*) -								
Step:	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**
Mean Value (j)	0.0988	0.1984	0.2985	0.3961	0.4977	0.5995	0.6963	0.7982
Max (j.) Confidence Interval	0.1012	0.2024	0.3045	0.4048	0.5075	0.611	0.7115	0.8118
Min (j.) Confidence Interval	0.0964	0.1944	0.2926	0.3873	0.488	0.588	0.681	0.7846
Replication 2 (Seed: *12000*) -								
Step:	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**
Mean Value (j)	0.1008	0.1994	0.299	0.4006	0.4961	0.597	0.6949	0.8059
Max (j.) Confidence Interval	0.1038	0.2036	0.3056	0.4115	0.5047	0.6064	0.707	0.828
Min (j.) Confidence Interval	0.0978	0.1952	0.2923	0.3897	0.4874	0.5876	0.6828	0.7837
Replication 3 (Seed: *18000*) -								
Step:	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**
Mean Value (j)	0.0994	0.198	0.2959	0.3955	0.4949	0.6009	0.695	0.7986
Max (j.) Confidence Interval	0.1018	0.2029	0.3034	0.4037	0.5065	0.6103	0.7099	0.8158
Min (j.) Confidence Interval	0.0969	0.193	0.2885	0.3873	0.4833	0.5914	0.6801	0.7813
AVERAGE OF REPLICATIONS								
Step:	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**
Mean Value (j)	0.099667	0.1986	0.2978	0.3974	0.496233	0.599133	0.6954	0.8009
Point Estimator	0.099667	0.1986	0.2978	0.3974	0.496233	0.599133	0.6954	0.8009
T0.005,2	9.92	9.92	9.92	9.92	9.92	9.92	9.92	9.92
Sample Variance:	1.05E-06	5.2E-07	2.77E-06	7.77E-06	1.97E-06	3.9E-06	6.1E-07	1.88E-05
Standard Deviation	0.001026	0.000721	0.001664	0.002787	0.001405	0.001976	0.000781	0.004335
Max (j.) Confidence Interval	0.105545	0.20273	0.307332	0.413365	0.504279	0.610449	0.699873	0.825726
Min (j.) Confidence Interval	0.093789	0.19447	0.288268	0.381435	0.488188	0.587818	0.690927	0.776074
Best Case	0.005545	0.00273	0.007332	0.013365	0.004279	0.010449	0.000127	0.023926
Worst Case	0.006211	0.00553	0.011732	0.018565	0.011812	0.012182	0.009073	0.025726

Theoretical Utilization - "p"								
Step:	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**
Mean Value	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8

Comparison:

Simulated values match up with theoretical values and fall within computed confidence intervals**.**

M/G/1 Response Time - "w"

Simulated Response Time - "w"								
Replication 1 (Seed: 6000) -								
Step:	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**
Mean Value (j)	1.0738	1.1637	1.2811	1.4456	1.6616	1.993	2.5703	3.6991
Max (j.) Confidence Interval	1.0901	1.186	1.3069	1.4769	1.7055	2.0434	2.6451	3.806
Min (j.) Confidence Interval	1.0575	1.1414	1.2554	1.4144	1.6178	1.9427	2.4955	3.5921
Replication 2 (Seed: 12000) -								
Step:	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**
Mean Value (j)	1.0749	1.1682	1.2793	1.4344	1.6581	1.9901	2.5394	3.7122
Max (j.) Confidence Interval	1.0889	1.2001	1.2988	1.46	1.6977	2.0372	2.5927	3.7999
Min (j.) Confidence Interval	1.061	1.1362	1.2598	1.4088	1.6185	1.943	2.4861	3.6244
Replication 3 (Seed: 18000) -								
Step:	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**
Mean Value (j)	1.0739	1.1628	1.2805	1.4284	1.6538	2.0038	2.5767	3.6967
Max (j.) Confidence Interval	1.0931	1.1837	1.3062	1.4707	1.6999	2.0356	2.6511	3.7853
Min (j.) Confidence Interval	1.0547	1.1419	1.2547	1.386	1.6077	1.972	2.5024	3.6082
AVERAGE OF REPLICATIONS								
Step:	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**
Mean Value (j)	1.0742	1.1649	1.2803	1.436133	1.657833	1.995633	2.562133	3.702667
Point Estimator	1.0742	1.1649	1.2803	1.436133	1.657833	1.995633	2.562133	3.702667
T0.005,2	9.92	9.92	9.92	9.92	9.92	9.92	9.92	9.92
Sample Variance:	3.7E-07	8.37E-06	8.4E-07	7.62E-05	1.53E-05	5.21E-05	0.000398	6.96E-05
Standard Deviation	0.000608	0.002893	0.000917	0.00873	0.003907	0.00722	0.019946	0.008343
Max (j.) Confidence Interval	1.077684	1.18147	1.285549	1.486133	1.680209	2.036983	2.67637	3.750449
Min (j.) Confidence Interval	1.070716	1.14833	1.275051	1.386134	1.635458	1.954284	2.447896	3.654884
Best Case	0.015161	0.02333	0.060765	0.0528	0.135458	0.204284	0.28123	0.654884
Worst Case	0.022128	0.05647	0.071263	0.1528	0.180209	0.286983	0.509704	0.750449

Theoretical Response Time - "w"								
Step:		**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**
Mean Value		1.055556	1.125	1.214286	1.333333	1.5	1.75	2.166667	3

Comparison:

Simulated values are larger than theoretical values and do not fall within theoretically computed confidence intervals after step 0.1.

M/G/1 Queue Time - "wq"

Simulated Queue Time - "wq"								
Replication 1 (Seed: 6000) -								
Step:	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**
Mean Value (j)	0.0742	0.1701	0.2917	0.4534	0.6712	1.0073	1.571	2.7257
Max (j.) Confidence Interval	0.076	0.175	0.2993	0.4663	0.6899	1.0302	1.6124	2.8062
Min (j.) Confidence Interval	0.0725	0.1651	0.284	0.4406	0.6525	0.9843	1.5297	2.6452
Replication 2 (Seed: 12000) -								
Step:	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**
Mean Value (j)	0.0746	0.166	0.2841	0.4442	0.6674	0.9903	1.5726	2.6927
Max (j.) Confidence Interval	0.0765	0.1699	0.2907	0.4531	0.6871	1.0138	1.6049	2.7662
Min (j.) Confidence Interval	0.0726	0.1622	0.2774	0.4354	0.6477	0.9667	1.5402	2.6192
Replication 3 (Seed: *18000*) -								
Step:	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**
Mean Value (j)	0.074	0.1656	0.2836	0.4427	0.6673	1.0006	1.5559	2.6998
Max (j.) Confidence Interval	0.0756	0.1701	0.2906	0.4539	0.6841	1.0292	1.5913	2.7864
Min (j.) Confidence Interval	0.0723	0.1612	0.2766	0.4315	0.6506	0.972	1.5204	2.6132
AVERAGE OF REPLICATIONS								
Step:	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**
Mean Value (j)	0.074267	0.167233	0.286467	0.446767	0.668633	0.9994	1.5665	2.706067
Point Estimator	0.074267	0.167233	0.286467	0.446767	0.668633	0.9994	1.5665	2.706067
T0.005,2	9.92	9.92	9.92	9.92	9.92	9.92	9.92	9.92
Sample Variance:	9.33E-08	6.2E-06	2.06E-05	3.36E-05	4.94E-06	7.33E-05	8.49E-05	0.000302
Standard Deviation	0.000306	0.002491	0.004539	0.005793	0.002223	0.008563	0.009215	0.01737
Max (j.) Confidence Interval	0.076016	0.181498	0.312463	0.479947	0.681367	1.048445	1.619275	2.805548
Min (j.) Confidence Interval	0.072517	0.152969	0.26047	0.413586	0.655899	0.950355	1.513725	2.606585
Best Case	0.016961	0.027969	0.046184	0.080253	0.155899	0.200355	0.347058	0.606585
Worst Case	0.020461	0.056498	0.098178	0.146614	0.181367	0.298445	0.452609	0.805548

Theoretical Queue Time - "wq"								
Step:	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**
Mean Value	0.055556	0.125	0.214286	0.333333	0.5	0.75	1.166667	2

Comparison:

Simulated values are larger and do not match up with theoretical values nor fall within computed confidence intervals. Best and worst cases show that more replications and longer simulation times are required at higher utilizations.

M/M/4/16

ν Reporting of collected statistics including confidence intervals

M/M/4/16 Number of customers - "L"

Simulated Number of customers - "L"								
Replication 1 (Seed: 7000) -								
Step:	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**
Mean Value (j)	0.3982	0.789	1.1938	1.6466	2.1694	2.8198	3.6697	5.0996
Max (j.) Confidence Interval	0.4054	0.8123	1.2282	1.6904	2.2217	2.8816	3.7677	5.2487
Min (j.) Confidence Interval	0.3911	0.7657	1.1595	1.6028	2.1171	2.758	3.5718	4.9506
Replication 2 (Seed: 14000) -								
Step:	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**
Mean Value (j)	0.4066	0.8097	1.2273	1.6722	2.1955	2.8069	3.71	5.028
Max (j.) Confidence Interval	0.4184	0.8251	1.2582	1.7026	2.2563	2.8701	3.7915	5.1498
Min (j.) Confidence Interval	0.3948	0.7943	1.1964	1.6417	2.1346	2.7436	3.6285	4.9062
Replication 3 (Seed: 21000) -								
Step:	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**
Mean Value (j)	0.4029	0.8084	1.2178	1.6526	2.1996	2.8002	3.7834	4.9823
Max (j.) Confidence Interval	0.414	0.8325	1.2484	1.6944	2.2583	2.876	3.8619	5.1072
Min (j.) Confidence Interval	0.3918	0.7842	1.1871	1.6107	2.1409	2.7245	3.7049	4.8575
AVERAGE OF REPLICATIONS								
Step:	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**
Mean Value (j)	0.402567	0.802367	1.212967	1.657133	2.188167	2.808967	3.721033	5.036633
Point Estimator	0.402567	0.802367	1.212967	1.657133	2.188167	2.808967	3.721033	5.036633
T0.005,2	9.92	9.92	9.92	9.92	9.92	9.92	9.92	9.92
Sample Variance:	1.77E-05	0.000134	0.000298	0.000179	0.000268	9.92E-05	0.003323	0.003496
Standard Deviation	0.00421	0.011594	0.017265	0.013389	0.016381	0.009962	0.057647	0.059125
Max (j.) Confidence Interval	0.426678	0.86877	1.311849	1.733814	2.281987	2.866023	4.051198	5.375259
Min (j.) Confidence Interval	0.378455	0.735964	1.114084	1.580453	2.094346	2.751911	3.390869	4.698008
Best Case	0.021633	0.066375	0.095971	0.073357	0.079231	0.041335	0.315786	0.312085
Worst Case	0.02659	0.066432	0.101794	0.080004	0.10841	0.072778	0.344544	0.365166
Theoretical Number of customers - "L"								
Step:	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**
Mean Value	0.400088	0.802395	1.215878	1.660457	2.173577	2.824688	3.735412	5.063174

Comparison:

Simulated values match up with theoretical values and fall within computed confidence intervals.

M/M/4/16 Utilization - "p"

Simulated Utilization - "p"								
Replication 1 (Seed: *7000*) -								
Step:	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**
Mean Value (j)	0.0995	0.1971	0.299	0.393	0.4984	0.6051	0.6986	0.7905
Max (j.) Confidence Interval	0.1014	0.2022	0.3062	0.4048	0.5103	0.6209	0.7173	0.814
Min (j.) Confidence Interval	0.0977	0.192	0.2918	0.3813	0.4864	0.5894	0.6799	0.7669
Replication 2 (Seed: *14000*) -								
Step:	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**
Mean Value (j)	0.1016	0.2018	0.3026	0.4047	0.5045	0.5965	0.7091	0.7979
Max (j.) Confidence Interval	0.1044	0.2061	0.3085	0.4146	0.5183	0.6123	0.729	0.816
Min (j.) Confidence Interval	0.0988	0.1974	0.2966	0.3947	0.4908	0.5807	0.6892	0.7797
Replication 3 (Seed: 21*000*) -								
Step:	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**
Mean Value (j)	0.1007	0.2016	0.3007	0.3984	0.5037	0.5932	0.6987	0.7892
Max (j.) Confidence Interval	0.1035	0.2066	0.3095	0.4079	0.5173	0.6097	0.7157	0.8028
Min (j.) Confidence Interval	0.0979	0.1966	0.2918	0.3889	0.49	0.5766	0.6818	0.7756
AVERAGE OF REPLICATIONS								
Step:	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**
Mean Value (j)	0.1006	0.200167	0.300767	0.3987	0.5022	0.598267	0.702133	0.792533
Point Estimator	0.1006	0.200167	0.300767	0.3987	0.5022	0.598267	0.702133	0.792533
T0.005,2	9.92	9.92	9.92	9.92	9.92	9.92	9.92	9.92
Sample Variance:	1.11E-06	7.06E-06	3.24E-06	3.43E-05	1.1E-05	3.77E-05	3.64E-05	2.2E-05
Standard Deviation	0.001054	0.002658	0.001801	0.005856	0.003315	0.006144	0.006034	0.004693
Max (j.) Confidence Interval	0.106634	0.215388	0.311081	0.432238	0.521187	0.633453	0.736689	0.819411
Min (j.) Confidence Interval	0.094566	0.184945	0.290452	0.365162	0.483213	0.563081	0.667577	0.765656
Best Case	0.005434	0.015055	0.009548	0.032238	0.016776	0.033603	0.031171	0.026191
Worst Case	0.006634	0.015388	0.011081	0.034837	0.021197	0.036769	0.03794	0.027564

Theoretical Utilization - "p"								
Step:	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**
Mean Value	0.1	0.2	0.3	0.4	0.499989	0.59985	0.698749	0.79322

Comparison:

Simulated values match up with theoretical values and fall within computed confidence intervals.

M/M/4/16 Response Time - "w"

Simulated Response Time - "w"								
Replication 1 (Seed: 7000) -								
Step:	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**
Mean Value (j)	0.9985	1.002	1.0079	1.0368	1.0798	1.1743	1.3342	1.6126
Max (j.) Confidence Interval	1.0157	1.0195	1.0265	1.0575	1.1047	1.209	1.3723	1.6474
Min (j.) Confidence Interval	0.9814	0.9846	0.9893	1.0162	1.0549	1.1395	1.2961	1.5777
Replication 2 (Seed: 14000) -								
Step:	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**
Mean Value (j)	0.9982	0.9994	1.0101	1.0387	1.0896	1.1609	1.338	1.5968
Max (j.) Confidence Interval	1.0182	1.026	1.0262	1.0696	1.112	1.1928	1.3708	1.6351
Min (j.) Confidence Interval	0.9782	0.9727	0.9939	1.0078	1.0672	1.1289	1.3052	1.5584
Replication 3 (Seed: 21000) -								
Step:	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**
Mean Value (j)	1.0101	1.0146	1.0242	1.0452	1.1058	1.1694	1.3443	1.572
Max (j.) Confidence Interval	1.0297	1.0323	1.043	1.0613	1.1315	1.2043	1.3695	1.6074
Min (j.) Confidence Interval	0.9905	0.9969	1.0055	1.0291	1.0801	1.1345	1.3191	1.5366
AVERAGE OF REPLICATIONS								
Step:	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**
Mean Value (j)	1.002267	1.005333	1.014067	1.040233	1.091733	1.1682	1.338833	1.5938
Point Estimator	1.002267	1.005333	1.014067	1.040233	1.091733	1.1682	1.338833	1.5938
T0.005,2	9.92	9.92	9.92	9.92	9.92	9.92	9.92	9.92
Sample Variance:	4.6E-05	6.61E-05	7.82E-05	1.94E-05	0.000172	4.6E-05	2.6E-05	0.000419
Standard Deviation	0.006786	0.00813	0.008844	0.004405	0.013131	0.00678	0.005101	0.020466
Max (j.) Confidence Interval	1.041129	1.051895	1.064721	1.065462	1.166937	1.207032	1.36805	1.711013
Min (j.) Confidence Interval	0.963404	0.958772	0.963412	1.015005	1.01653	1.129368	1.309617	1.476587
Best Case	0.036817	0.044222	0.04982	0.022781	0.070281	0.029784	0.026848	0.115247
Worst Case	0.040909	0.048901	0.051489	0.027675	0.080125	0.04788	0.031586	0.119179

Theoretical Response Time - "w"								
Step:	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**
Mean Value	1.000221	1.002994	1.013232	1.037786	1.086812	1.177248	1.336465	1.595766

Comparison:

Simulated values match up with theoretical values and fall within computed confidence intervals.

M/M/4/16 Queue Time - "wq"

Simulated Queue Time - "wq"								
Replication 1 (Seed: 7000) -								
Step:	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**
Mean Value (j)	1.50E-04	2.96E-03	0.0131	0.0376	0.0879	0.1781	0.334	0.5988
Max (j.) Confidence Interval	1.18E-04	3.17E-03	0.0137	0.0392	0.0914	0.1835	0.3457	0.6164
Min (j.) Confidence Interval	0	2.74E-03	0.0126	0.0361	0.0844	0.1726	0.3224	0.5811
Replication 2 (Seed: 14000) -								
Step:	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**
Mean Value (j)	5.45E-04	2.93E-03	0.0132	0.038	0.0862	0.18	0.3327	0.5979
Max (j.) Confidence Interval	3.66E-04	3.16E-03	0.0137	0.0393	0.0892	0.1866	0.3444	0.6139
Min (j.) Confidence Interval	0	2.70E-03	0.0126	0.0368	0.0833	0.1734	0.3211	0.5819
Replication 3 (Seed: 21000) -								
Step:	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**
Mean Value (j)	1.58E-04	3.07E-03	0.0136	0.0378	0.0885	0.1771	0.3408	0.5945
Max (j.) Confidence Interval	2.41E-04	3.27E-03	0.0143	0.0391	0.0916	0.1837	0.3534	0.6079
Min (j.) Confidence Interval	0	2.87E-03	0.0129	0.0365	0.0855	0.1705	0.3282	0.5812
AVERAGE OF REPLICATIONS								
Step:	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**
Mean Value (j)	0.000284	0.002987	0.0133	0.0378	0.087533	0.1784	0.335833	0.597067
Point Estimator	0.000284	0.002987	0.0133	0.0378	0.087533	0.1784	0.335833	0.597067
T0.005,2	9.92	9.92	9.92	9.92	9.92	9.92	9.92	9.92
Sample Variance:	5.1E-08	5.43E-09	7E-08	4E-08	1.42E-06	2.17E-06	1.89E-05	5.14E-06
Standard Deviation	0.000226	7.37E-05	0.000265	0.0002	0.001193	0.001473	0.00435	0.002268
Max (j.) Confidence Interval	0.001577	0.003409	0.014815	0.038945	0.094366	0.186837	0.360748	0.610056
Min (j.) Confidence Interval	-0.00101	0.002564	0.011785	0.036655	0.0807	0.169963	0.310919	0.584078
Best Case	0.001229	0.000415	0.001447	0.001132	0.006111	0.007285	0.024283	0.011689
Worst Case	0.001357	0.00043	0.001583	0.001159	0.007555	0.009589	0.025546	0.014289
Theoretical Queue Time - "wq"								
Step:	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**
Mean Value	0.000221	0.002994	0.013232	0.037786	0.086812	0.177248	0.336465	0.595766

Comparison:

Simulated values match up with theoretical values and fall within computed confidence intervals. Should be noted that steps 0.2 through 0.7 could not be computed within JMTs requested precision. In a further study not depicted here, increasing simulation parameter of max duration to 20 minutes (instead) of 30 seconds did not change this result. Perhaps conducting much longer simulations would improve this result.

M/M/4/16 Drop Rate - "Pn"

Simulated Drop Rate - "Pn"								
Replication 1 (Seed: _7000_) -								
Step:	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**
Mean Value (j)	0	0	0	0	6.18E-05	6.39E-04	5.22E-03	0.027
Max (j.) Confidence Interval	0	0	0	0	-	-	6.85E-03	0.0286
Min (j.) Confidence Interval	0	0	0	0	-	-	4.22E-03	0.0255
Replication 2 (Seed: _14000_) -								
Step:	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**
Mean Value (j)	0	0	0	0	5.46E-05	5.91E-04	5.09E-03	0.0272
Max (j.) Confidence Interval	0	0	0	0	-	-	6.21E-03	0.0289
Min (j.) Confidence Interval	0	0	0	0	-	-	4.32E-03	0.0256
Replication 3 (Seed: 21_000_) -								
Step:	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**
Mean Value (j)	0	0	0	0	2.30E-04	6.41E-04	4.92E-03	0.0273
Max (j.) Confidence Interval	0	0	0	0	-	-	5.79E-03	0.0289
Min (j.) Confidence Interval	0	0	0	0	-	-	4.28E-03	0.0258
AVERAGE OF REPLICATIONS								
Step:	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**
Mean Value (j)	0	0	0	0	0.000115	0.000624	0.005077	0.027167
Point Estimator	0	0	0	0	0.000115	0.000624	0.005077	0.027167
T0.005,2	9.92	9.92	9.92	9.92	9.92	9.92	9.92	9.92
Sample Variance:	0	0	0	0	9.85E-09	8.01E-10	2.26E-08	2.33E-08
Standard Deviation	0	0	0	0	9.93E-05	2.83E-05	0.00015	0.000153
Max (j.) Confidence Interval	0	0	0	0	0.000684	0.000786	0.005938	0.028042
Min (j.) Confidence Interval	0	0	0	0	-0.00045	0.000462	0.004215	0.026292
Best Case	7.15E-16	3.14E-11	1.38E-08	9.13E-07	0.000474	0.000212	0.002428	0.017817
Worst Case	7.15E-16	3.14E-11	1.38E-08	9.13E-07	0.000663	0.000536	0.004151	0.019566

Theoretical Drop Rate - "Pn"								
Step:	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**
Mean Value	7.15E-16	3.14E-11	1.38E-08	9.13E-07	2.12E-05	0.00025	0.001787	0.008475

Comparison:

Simulated values do not match up with theoretical values nor fall within computed confidence intervals. In a further study not depicted here, increasing max simulation time to 20 minutes (instead) of 30 seconds slightly improved this result in steps 0.7 and 0.8. Perhaps conducting much longer simulations would improve this result.

Study of Loss Behavior

The probability of loss was higher in that of the simulation compared to theoretical values. It seems that until step 0.5, probability of losing a potential customer from the system was negligible enough to be considered 0 in JMT. Perhaps due to other inefficiencies in the system, drop rate is higher than theoretical values.

NETWORK OF QUEUES

Note: *Steps such as 0.1,0.2, …, 0.8 can instead be represented as percentages of 100%, 200%, …, 800% in this section*

ν Reporting of collected statistics including confidence intervals

NETWORK OF QUEUES - CLASS 1 Response Time - "w"

CLASS 1 - Simulated Response Time - "w"								
Replication 1 (Seed: 8000) -								
Step:	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**
Mean Value (j)	2.1278	2.3147	2.4991	2.8101	3.1627	3.7535	4.6023	6.3448
Max (j.) Confidence Interval	2.1635	2.3597	2.5683	2.8659	3.2388	3.8522	4.7175	6.4866
Min (j.) Confidence Interval	2.0921	2.2696	2.4299	2.7542	3.0865	3.6547	4.4871	6.203
Replication 2 (Seed: 16000) -								
Step:	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**
Mean Value (j)	2.1148	2.2904	2.4766	2.7987	3.1645	3.691	4.5356	6.2785
Max (j.) Confidence Interval	2.1741	2.3526	2.5348	2.8423	3.2376	3.7814	4.6473	6.4619
Min (j.) Confidence Interval	2.0555	2.2282	2.4184	2.7551	3.0915	3.6006	4.424	6.0951
Replication 3 (Seed: 24000) -								
Step:	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**
Mean Value (j)	2.1545	2.308	2.5275	2.8176	3.1801	3.7398	4.5499	6.2549
Max (j.) Confidence Interval	2.1909	2.3768	2.5675	2.8774	3.2202	3.8275	4.6442	6.4161
Min (j.) Confidence Interval	2.1182	2.2393	2.4875	2.7577	3.1401	3.6521	4.4555	6.0938
AVERAGE OF REPLICATIONS								
Step:	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**
Mean Value (j)	2.132367	2.304367	2.501067	2.8088	3.1691	3.7281	4.5626	6.292733
Point Estimator	2.132367	2.304367	2.501067	2.8088	3.1691	3.7281	4.5626	6.292733
T0.005,2	9.92	9.92	9.92	9.92	9.92	9.92	9.92	9.92
Sample Variance:	0.00041	0.000158	0.000651	9.06E-05	9.16E-05	0.001079	0.001233	0.002172
Standard Deviation	0.02024	0.012551	0.025507	0.009517	0.009569	0.032852	0.035117	0.046609
Max (j.) Confidence Interval	2.248288	2.376249	2.647153	2.863306	3.223903	3.916252	4.763725	6.55968
Min (j.) Confidence Interval	2.016445	2.232484	2.35498	2.754294	3.114297	3.539948	4.361475	6.025786
Best Case	0.106249	0.062419	0.11968	0.048736	0.047432	0.179564	0.164569	0.243891
Worst Case	0.125594	0.081346	0.172492	0.060276	0.062174	0.196739	0.237681	0.290003

CLASS 1 - Theoretical Response Time - "w"								
Step:	**100%**	**200%**	**300%**	**400%**	**500%**	**600%**	**700%**	**800%**
Mean Value	2.142039	2.31383	2.527473	2.80303	3.176471	3.719512	4.599156	6.315789

Comparison:

Simulated values match up with (and are negligibly smaller than) theoretical values and fall within computed confidence intervals.

NETWORK OF QUEUES - CLASS 2 Response Time - "w"

CLASS 2 - Simulated Response Time - "w"								
Replication 1 (Seed: *8000*) -								
Step:	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**
Mean Value (j)	2.182	2.4141	2.7061	3.0716	3.5487	4.2117	5.2471	7.3011
Max (j.) Confidence Interval	2.2448	2.4683	2.7851	3.1452	3.6383	4.3317	5.3791	7.4579
Min (j.) Confidence Interval	2.1192	2.36	2.627	2.9981	3.459	4.0917	5.115	7.1443
Replication 2 (Seed: *16000*) -								
Step:	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**
Mean Value (j)	2.1587	2.3794	2.65	3.0202	3.5548	4.077	5.2234	7.2861
Max (j.) Confidence Interval	2.1923	2.4246	2.7018	3.0885	3.6378	4.194	5.3799	7.4714
Min (j.) Confidence Interval	2.1251	2.3342	2.5982	2.9519	3.4718	3.9601	5.0668	7.1009
Replication 3 (Seed: *24000*) -								
Step:	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**
Mean Value (j)	2.1811	2.4048	2.6946	3.0424	3.4678	4.2769	5.2639	7.2292
Max (j.) Confidence Interval	2.2175	2.4566	2.7506	3.1268	3.5588	4.3751	5.3827	7.4303
Min (j.) Confidence Interval	2.1447	2.353	2.6386	2.9579	3.3767	4.1787	5.1451	7.0281
AVERAGE OF REPLICATIONS								
Step:	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**
Mean Value (j)	2.173933	2.399433	2.683567	3.044733	3.523767	4.188533	5.2448	7.272133
Point Estimator	2.173933	2.399433	2.683567	3.044733	3.523767	4.188533	5.2448	7.272133
T0.005,2	9.92	9.92	9.92	9.92	9.92	9.92	9.92	9.92
Sample Variance:	0.000174	0.000323	0.000878	0.000665	0.002359	0.010393	0.000414	0.001439
Standard Deviation	0.0132	0.017962	0.029633	0.025779	0.048564	0.101944	0.020348	0.03793
Max (j.) Confidence Interval	2.249535	2.502306	2.853283	3.19238	3.80191	4.772397	5.361338	7.489372
Min (j.) Confidence Interval	2.098332	2.296561	2.51385	2.897087	3.245623	3.60467	5.128262	7.054895
Best Case	0.063155	0.089515	0.158889	0.136824	0.263449	0.548259	0.06722	0.216644
Worst Case	0.088048	0.11623	0.180544	0.158468	0.292839	0.619468	0.165855	0.217832
CLASS 2 - Theoretical Response Time - "w"								
Step:	**100%**	**200%**	**300%**	**400%**	**500%**	**600%**	**700%**	**800%**
Mean Value	2.18638	2.412791	2.694394	3.055556	3.538462	4.224138	5.294118	7.272727

Comparison:

Simulated values match up with theoretical values and fall within computed confidence intervals.

COMPARISON OF CLASS 1 AND CLASS 2

Both Class 1 and Class 2's simulated values match up with theoretical values and fall within computed confidence intervals. Interestingly, both models had negligibly smaller response times than their theoretical counterparts Arrival rate being changed from a factor of 0.03 per step (Queue 1) to a factor of 0.07 (Queue 2) did not result in response time being increased by the same factor. The response time between these two classes did not see too much of difference despite Class 2 having an arrival rate of roughly double that of Class 1. For example, at a utilization of 0.8 (or 800%), Class 2 customers were on average, only in the system 1 second longer than Class 1 customers.

♦ CONCLUSION

ν Summary of Results with Noted Discrepancies Between Simulated Results and Theory

In general, each queueing model except M/G/1 had little discrepancy between simulated and theory. However, some discrepancies did occur, and notable behavior of these models were observed as follows:

- Throughout simulations, the stats observed in M/M/1 queue and Network of Queues model most aligned to theoretical counterparts within JMT requested precision and confidence intervals. We accept these models.
- The M/G/1 queuing model saw the most discrepancy within all performance metrics except p. On average, simulation results were higher than theoretical values and did not fall within computed confidence intervals. *Further refinement of this model is necessary.*
- Longer simulation times are recommended for M/M/4/16 model to gather better results for the queue time and drop rate performance metrics.
- Results indicate that perhaps more complex systems and queuing models require more replications and simulation time to gather accurate results whereas less replications and lower simulation time should be sufficient for simpler models.
- Most of the discrepancy seen between observed and theoretical values was seen at lower utilizations and higher utilization (first and last couple steps). Perhaps this is due to the smaller values requiring higher precision and larger utilizations having more computations/complexity going on.
- In most cases simulated results were close to their theoretical values. For some utilizations it was noted that more replications and longer simulation time may be required. In cases where this action does not improve results it may be noted that making changes to the model might be necessary to see further improvement.