A Fuzzy LQR PID Control for a Two-Legged Wheel Robot with Uncertainties and Variant Height

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***Abstract*—This paper proposes a fuzzy LQR PID control for a two-legged wheeled balancing robot for keeping stability against uncertainties and variant heights. The proposed control includes the fuzzy supervisor, LQR, PID, and two calibrations. The fuzzy LQR is conducted to control the stability and motion of the robot while its posture changes with respect to time. The fuzzy supervisor is used to adjust the LQR control according to the robotic height. It consists of one input and one output. The input and output have three membership functions, respectively, to three postures of the robot. The PID control is used to control the posture of the robot. The first calibration is used to compensate for the bias value of the tilting angle when the robot changes its posture. The second calibration is applied to compute the robotic height according to the hip angle. In order to verify the effectiveness of the proposed control, a practical robot with the variant height is constructed, and the proposed control is embedded in the control board. Finally, two experiments are also conducted to verify the balancing and moving ability of the robot with the variant posture.**

***Keywords—Fuzzy LQR Control; Two-Legged Wheeled Balancing Robot; PID Control; Fuzzy Logic System.***

**I. INTRODUCTION**

In recent years, autonomous robots have been invented rapidly to share manpower requirements in factories, restaurants [1], airports [2], and delivery [3][4]. Unlike conventional robots and human workers operating as separate workspaces, human-robot cooperation is used to complete complex tasks. As a result, the human-robot interaction must be considered in the robot design, which means that the robots and humans can share the workspace together [5]. Two-wheel inverted pendulum (TWIP) robots [6], an extended system of inverted pendulums, and mobile robots, own advantages such as compactness, mobility, and human-like functions. This kind of robot has more applications in logistics transportation, commuting, and navigation, self-balancing capability. Because the TWIP mobile platform [7] is classified as an underactuated system that implements the 3-DOF motion of pitch, yaw, and straight movement with only two actuator inputs, high-performance motion control for this robot is a highly challenging task for the control community and recently numerous results have been reported as well-classified.

When the TWIP robots work at a small pitch angle around the balancing point, some conventional linear control techniques such as PID control [8-11] and linear quadratic regulator (LQR) [8][9][12-16] have been employed. However, when the robot operates in nonlinear regions with large pitch angles because of external disturbances, modeling errors, or internal maneuvers, the control performance of the linear control approaches will be degraded. In order to alleviate these problems and enhance the control performance, many nonlinear control methods have been investigated, such as feedback linearization control [17][18], sliding mode control [19-21], backstepping control [22-24], and model predictive control [25][26]. These approaches usually require a mathematical model for the design procedure. In practice, it is hard to determine exactly the mathematical model, parameters usually change with respect to time due to the aging and affections of the external environment. In order to handle these challenges, adaptive control [19][27-29], neural network control [19][30-32], and fuzzy control [33-36] have been investigated for the TWIP robots. In [19], a neural network was used to estimate the unknown model parameters and a robust adaptive control was applied to compensate for the estimator errors and uncertainties in a two-wheeled self-balancing robot system. In [27], an adaptive backstepping control was constructed for a wheeled inverted pendulum system under the presence of the model parameter uncertainties. In [30], an adaptive neural network was used to compensate for the unknown terms in the output dynamics of a self-balancing robot. In [33], a fuzzy logic control and a pole placement state-feedback controller were both designed for a two-wheeled self-balancing robot against the disturbance force. The pole placement state-feedback controller was used to keep the balance of the robot, and the fuzzy logic control was used to control the position of the robot. In [37], a fuzzy and PD control was applied for a two-wheeled self-balancing robot with structured and unstructured uncertainties. The PD control was used to control the balancing of the robot and the fuzzy PD control was utilized to control the position of the robot.

The conventional TWIP robots are well known moving fast and stably on a flat road. However, when they move in uneven terrain, such as gullies and slopes, their limitations appear. They cannot overcome an obstacle when the radius of the wheel is less than the height of the obstacle or the contact point is above the center of the wheel [38]. In order to manage this challenge, some advanced self-balancing robots [38-41] are constructed to work corporately with people. The study in [38] designed a terrain-adaptive two-legged wheeled robot with leg mechanisms which can jump over obstacles. Klemm et al. [39] described the fundamental design of Ascento, atwo-legged wheeled jumping robot, that moved on uneven terrain and also climbed the stairs by jumping. Zhou et al. [40] proposed a centroidal adjustment control to let the robot have higher robustness in moving. Some conventional control approaches [39][42-48] have been investigated to manage the balance and height control problems in these types of robot. In [39], a linear quadratic regulator (LQR) and PID controller were applied for the Ascento robot to control the stabilization, driving and jumping. For the stabilization and driving, the LQR controller was designed from linearization state space models linearized around ten different leg heights. In [42], a cascade PID controller was proposed for a bipedal leg-wheeled robot to guarantee the stabilization and driving. In [49], LQR controller and fuzzy PD controller were investigated on a new type of wheel legged robot with parallel four-bar mechanism for stable movement and jumping over obstacles.

Based on the above analysis, this paper presents a fuzzy LQR PID control for a two-legged wheeled robot (TLWR) for keeping stability against uncertainties and variant heights. The proposed control is designed based on the fuzzy supervisor, LQR, and PID. As a result, this approach does not require the rigor mathematical model. The fuzzy LQR, including a LQR control and a fuzzy supervisor, is utilized to control the stability and motion of the robot with the variant posture. According to the robotic height, the fuzzy supervisor will estimate the gains of the LQR control. The supervisor consists of one input and one output which have three membership functions, respectively, to three postures of the robot. The PID control is used to control the posture of the robot. In order to verify the effectiveness of the proposed control, the practical robot was constructed and the proposed control was embedded in the control board. Additionally, two experiments were also conducted to verify the balancing and moving ability of the robot. Because the sensor displacement plane is tilted according to the posture of the robot, some computations are implemented to compensate this tilting angle. The main contributions of this paper are summarized as follows:

1. A proposed control is constructed based on the fuzzy supervisor and three LQR controllers which are respectively designed according to three postures of the TLWR, low, medium and high postures. As a result, the complexity in the TLWR is reduced in the control design.

2. The effectiveness of the proposed control is verified on a practical testbench and the challenges, measuring the robotic height and compensating angles for the pitch angle in the real testbench, are also discussed in this paper.

This paper is constructed as follows: In section 2, the problem formulation of the two-legged wheeled balancing robot with variant height, including equivalent centroid calculation, TLWR Modeling, and linear state space model, is discussed. The proposed controller consisting of PID control and Fuzzy LQR control are designed in Section 3. In Section 4, some experiments are conducted in practical robot and the results of the proposed control are compared to another control. Finally, some conclusions and future works are mentioned in Section 5.

**II. DESCRIPTION AND MODELING OF THE TWO-LEGGED WHEELED ROBOT**

图示

描述已自动生成The structure of the self-balancing two-legged wheeled robot is presented in Fig. 1. The robotic system is equipped with a control board, an inertial measurement unit (IMU), a Zigbee module, three DC motors including encoders, and one 12V rechargeable lead-acid battery. The control board is designed as a primary controller, with the IMU being used to calculate the rate and angle of platform inclination. Additionally, the control board can drive the robotic platform’s yaw control. Two motors, including encoders are installed at the feet of the robot to drive the robotic motion. Another is added in the hip to adjust the height of the robot. In order to save energy of the hip motor, torsion springs are installed in inner joints at the knees of the robot. Dead-reckoning computations are manipulated based on the information from two optical encoders mounted in the drive motors.

Fig. 1. The CAD model of the robot

**Remark 1:** In this paper, we limit the application of the upper body to sagittal motion. The yaw control of the robot is realized by the differential motion of the two wheels, and the pitch angle of the torso is controlled by the hip joint. The roll angle of the TLWR can be controlled by adjusting the height of the two legs, but in this paper, both legs perform the same motion, so the roll angle is always kept at zero. Additionally, the symbols of the TLWR utilized in this paper are summarized in Table I.

TABLE I. THE PARAMETERS OF THE TLWR

|  |  |
| --- | --- |
| **Symbol** | **Definitions** |
| 𝜃𝑤 (rad) | Rotation angle of the wheel |
| 𝜃𝑏 (rad) | Tilt angle of the body of the equivalent WIP model |
| *r* | Radius of the driving wheel |
| 𝐼𝑏 | Total moments of inertia of above the driving wheel in the TLWR model |
| 𝑙𝑐 | Distance between the center of mass (CoM) position and wheel axis in the TLWR model |
| 𝑑 | Distance between two wheels |
| 𝑚1 | Mass of the shank |
| 𝑚2 | Mass of the lower thigh |
| 𝑚3 | Mass of the upper thigh |
| 𝑚4 | Mass of the body |
| 𝑙1 | Length of the shank |
| 𝑙2 | Length of the lower thigh |
| 𝑙3 | Length of the upper thigh |
| 𝑙4 | Distance between the coordinate system 4th and 5th |
| 𝑙5 | Distance between the coordinate system 5th and 6th |
| 𝜏𝐿 | Torque of the left wheel |
| 𝜏𝑅 | Torque of the right wheel |
| 𝑙𝑐1 | Position of CoM of the shank |
| 𝑙𝑐2 | Position of CoM of the lower thigh |
| 𝑙𝑐3 | Position of CoM of the upper thigh |
| 𝑙𝑐4 | Position of CoM of the body |
| 𝜃1 | The joint angle between the coordinate system 0th and 1th |
| 𝜃2 | The joint angle between the coordinate system 1th and 2th |
| 𝜃3 | The joint angle between the coordinate system 2th and 3th |
| 𝜃4 | The joint angle between the coordinate system 2th and 4th |
| 𝜃5 | The joint angle between the coordinate system 4th and 5th |
| 𝜃6 | The joint angle between the coordinate system 5th and 6th |
| 𝑎 | Distance between the coordinate system 2th and 3th |

*A. Equivalent Centroid Calculation*

The coordinate systems involved in this paper are exhibited with 𝑂𝐺 𝑥𝐺 𝑦𝐺 𝑧𝐺of universal frame and 𝑂𝑤𝑥𝑤𝑦𝑤𝑧𝑤 of wheel-axle frame. In the decoupling process, the five-link multi-rigid body system is equivalent to a lumped mass point as presented in Fig. 2. The position of the equivalent centroid is weighted by the masses of the individual links and their centroid positions. To establish the relationship between this center of mass and the axle coordinate system, the Denavit– Hartenberg (D-H) convention was used to establish the kinematic model. By setting up the coordinates as presented in Fig. 3, the D-H parameters of the TLWR are shown in Table II.

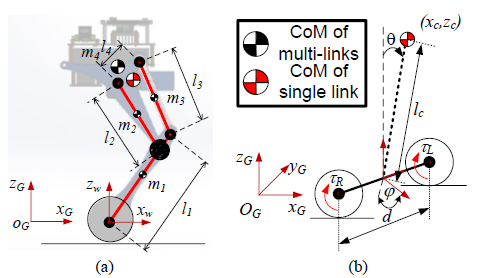


Fig. 2. Schematics of differential types of dynamic model of the robot (a) the two- legged wheeled robot, (b) The decoupled equivalent WIP model

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Fig. 3. Kinematic parameters of the TLWR

TABLE II. DH-PARAMETERS OF THE TLWR

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Coordinate** | 𝒂𝒊−𝟏 | 𝜶𝒊−𝟏 | 𝒅𝒊 | 𝜽𝒊 |
| 1 | 0 | 0 | 0 | 𝜃1 |
| 2 | 𝑙1 − 𝑎 | 0 | 0 | 𝜃2 |
| 3 | 𝑙2 | 0 | 0 | 𝜃4 |
| 4 | 𝑙4 | 0 | 0 | 𝜃5 |
| 5 | 𝑙5 | 0 | 0 | 𝜃6 |

The homogeneous transformation matrix from the ith coordinate to the i-1-th coordinate is given as (1).

The homogeneous transformation matrix between the coordinate system i and the axle coordinate system is as (2).

The position of the CoM of the upper body relative to the world coordinate system can be presented as (3).

where 𝑚𝑖 is the mass of the ith link, 𝑞𝑏 = [𝑞𝑏1, . . . , 𝑞𝑏4]𝑇is the actual angles, 𝑃𝑤𝐶 = [𝑥𝑐 , 𝑧𝑐 ]𝑇 is the position coordinate of the equivalent CoM relative to the axle coordinate system, 𝑃𝑤𝐶𝑖 is the position coordinate of the CoM of the ith link in the wheel axis coordinate system, and 𝑃 𝑖 𝐶𝑖 is the position of the CoM of the ith link in the local coordinate system. Based on the coordinate of CoM, the pendulum length 𝑙𝑐 and inclination angle 𝜃of the inverted pendulum can be obtained as in equation (4) and (5).

*B. TLWR Modelling*

**Assumption 2:** The driving wheels are subject to rolling constraints and there is no slippage between the wheel and the ground.

In this study, the pendulum length in (4) is used to analyze the dynamics of the inverted pendulum. The state variables of the wheel inverted pendulum are selected as 𝑞𝑤 = [𝑥 𝜃 𝜑]𝑇 . By using Euler Lagrange approach, the dynamic equations of the TLWR can be expressed as (6).

where 𝑀(𝑞𝑤) ∈ 𝑅3×3, 𝑉(𝑞𝑤, 𝑞̇𝑤) ∈ 𝑅3, and 𝐺(𝑞𝑤) ∈ 𝑅3 are respectively the inertia matrix, the Coriolis and Centripetal forces vector, the Gravitational force vector; 𝐵 is the input matrix, 𝜏𝑒𝑥𝑡 is the lumped uncertainties including the modeling error and external torque, 𝜏𝑤 = [𝜏𝐿 𝜏𝑅 ]𝑇 is the wheel driving torques. In the differential-drive mobile robot,the input matrix is defined as

*C. Linear State Space Model*

In this study, three postures of the TLWR presented in Fig. 4 are considered in the control design. Its linear state space models are given as (7 ).

where 𝑥 = [𝑥 𝜃 𝜑 𝑥̇ 𝜃̇ 𝜑̇]𝑇 ∈ 𝑅6×1 is the state space, 𝑢 = 𝜏𝑤 ∈ 𝑅2×1 is the control input. 𝐴𝑖 , 𝐵𝑖 , 𝐶𝑖 with 𝑖 = 1,2,3 are respectively the matrices in three postures of the TLWR, which are computed as [50].

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Fig. 4. Three postures of the TLWR

III. CONTROL DESIGN

As presented in Fig. 5, the structure of the proposed control includes the Fuzzy LQR control and Posture control. The posture control is designed from the PID control to adjust the hip angle of the robot. The height of the robot is calculated by the calibration 1 which presents the relationship between the hip angle and the height of the robot. The Fuzzy LQR control combines a fuzzy supervisor and a LQR control. The fuzzy supervisor includes one input and six outputs which are the control gains of the LQR control. The input fuzzy has three membership functions respect to the low, medium and high postures of the robot. With different postures, the center of mass of the robots are different so the dynamics of the robot is also changed. Because the control gains of LQR are dependent on the robotic dynamics, they are also different at each posture. The Fuzzy supervisor is designed to calculate the control gain of the LQR with respect to the height of the robot. When the posture of the robot changes, the inclination angle of IMU sensor also alters. As a result, inclination angles, calculated from IMU sensors, should be adjusted by the calibration 2. Based on the kinematic of the robot, the auxiliary angles are calculated with respect to the height of the robot. The input transformation matrix is used to convert the control inputs computed from the proposed control into the input voltage at each wheel motor.

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*A. PID Control*

In order to control the posture of the TLWR, a PID control is used to drive the hip motor based on the difference of the posture profile and the real posture of the TLWR. Based on the geometric structure of the TLWR in Fig. 3, the pendulum length can be calculated following the hip angle. The calibration 1 is utilized to compute the robotic height

according to hip angle. The PID control law is presented as (8).

where 𝐾𝑝 is a proportional gain, 𝐾𝑖 is an integral gain and 𝐾𝑑 is a differential gain.

*B. Linear Quaratic Regulator(LQR)*

In the LQR controller, the optimal control gains, K, are computed based on the cost function (𝐽), which optimize states, 𝑥(𝑡) and control signal, 𝑢(𝑡) of the systems (9). The control signal, 𝑢(𝑡), and the cost function, 𝐽, are selected as in (9) and (10).

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where 𝑄 and 𝑅 are positive semi-defined matric. In order to minimize the cost function, 𝐽, the control gain, 𝐾, is given as (11).

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where 𝑃 is solution of the differential equation of Riccati as (12).

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**Remark 2:** The LQR controller performance is dependent on the selection of weight matrices and linearization matrices, A and B. As a result, different postures will give different optimal control gains, 𝐾1, 𝐾2 and 𝐾3 in the low, medium and high postures, respectively.

**Remark 3:** When the height of the TLWR changes respect to the posture, the sensor plane will be titled at an angle correspondingly. The calibration 2 is used to compensate this titling angle of the sensor. The equation in the calibration 2 is calculated by applying the algebraic solution technique. The transformation matrix between the 6th coordination system and the origin coordination can be calculated from equation (13).

@@

By implementing some manipulations, transformation matrix, 𝑇 6 0 , will only depend on the hip angle. Because the height of two legs is adjusted simultaneously, the orientation matrix of the transformation matrix, 𝑇 6 0 , is a rotation operation around the z-axis. As a result, this matrix can be presented as (14).

@@

The compensation angle for the sensor is calculated as (15).

@@

The output of the calibration is presented as (16).

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*C. Fuzzy Supervisor*

According to the robotic height estimated from the hip angles, the fuzzy supervisor will compute the control gains in the LQR, respectively. Fig. 6 presents the structure of the fuzzy supervisor, which includes an input and 8 outputs, control gains of the LQR. Three membership functions in the input are presented as Low, Medium and High in Fig. 7.

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Fig. 6. Structure of the fuzzy supervisor

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Fig. 7. Input membership function of the fuzzy supervisor

* If ℎ is *Low* then 𝑲 = 𝑲1,
* If ℎ is *Medium* then 𝑲 = 𝑲2,
* If ℎ is *High* then 𝑲 = 𝑲3*.*

Where ℎ is the height of the robot; 𝑲𝑖(𝑖=1,2,3) are control gains of the low, medium and high postures, respectively.

The MAX – PROD aggregation method and “centroid” defuzzification method are utilized. The control gains can be computed in (17).

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IV. EXPERIMENT DISCUSSION

*A. Test Bench Description*

The practical TLWR robot shown in Fig. 8 includes a board Control, two-wheel motors with encoders, a hip motor, a hip encoder, and an IMU sensor. The control board developed form ATmega2560 is used to control the stability, posture and motion of the robot. Two-wheel motors are two DC motors, JGB37-520 with the speed of 333 revolution per minute (RPM), which are attached to incremental encoder with 11 pulses per revolution (PPR). Based on the control signals which are generated from the control board, they will keep the robot stable or drive the motion of the robot. In order to adjust the posture of the robot, a high torque 5840-31ZT motor and an encoder AMT332D-V are mounted in the hip. The IMU sensor, MPU9250, is utilized to compute the titling of the robot. The power of the whole system is supplied by a 12 V lithium battery. After practical measurements, the weight of the robot is 3.15 kg, and the battery life is about 2.5 h.

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Fig. 8. Two- legged wheeled robot can change height in practice

The hardware connection diagram of the test bench is shown in Fig. 9. A graphic user interface, built by Visual studio C# on a laptop, acquires the robotic position, pitch, hip and yaw angles and control signals, and send the control gains to the robot through wireless communication in real-time through radio frequency (RF) transceiver, Zigbee C2530. The control board is constructed to conduct the proposed control from the information acquiring from IMU sensor and encoders. After the control signals are computed by the control board, they will be provided to the drivers of the hip motor and leg motor.

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Fig. 9. Hardware connection diagram

**Remark 4:** The parameters of the TLWR, such as lengths, masses, moments inertia, and radius of wheels, in Table III are specified and calculated by using the mass properties in the SOLIDWORK.

1. TABLE III. PARAMETERS OF THE TWO-LEGGED WHEELED ROBOT

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Symbol** | **Value** | **Unit** | **Symbol** | **Value** | **Unit** |
| 𝑎 | 0.035 | m | 𝑙2 | 0.145 | m |
| *r* | 0.0425 | m | 𝑙3 | 0.145 | m |
| 𝑑 | 0.276 | m | 𝑙4 | 0.055 | m |
| 𝑚1 | 0.56 | kg | 𝑙5 | 0.09275 | m |
| 𝑚2 | 0.44 | kg | 𝑙𝑐1 | 0.09636 | m |
| 𝑚3 | 0.19 | kg | 𝑙𝑐2 | 0.07251 | m |
| 𝑚4 | 1.96 | kg | 𝑙𝑐3 | 0.07251 | m |
| 𝑙1 | 0.19 | m | 𝑙𝑐4 | 0.028 | m |

**Remark 5:** In this study, the proposed control is carried out on the practical two-legged wheeled robot. The steps taken in this study are summarized as follows: Step 1 selects devices from the requirements; Step 2 designs a two-legged wheeled robot; Step 3 constructs a real model; Step 4 evaluates the performances of the actuators, sensors and mechanical structure; Step 5 conducts the LQR controllers with different postures on the practical mode; Finally, step 6 implements the proposed control on the robot.

*B. Experimental Description*

In order to verify the effectiveness of the proposed control, it is compared with a Fuzzy LQR control which is designed with two membership functions in the fuzzy supervisor named as fuzzy LQR control with two membership functions (FLQR with TMF). This control is designed based on the LQR controllers in the high and low postures. The control parameters of the PID and LQR control in the postures are shown in Table IV.

1. TABLE IV. CONTROL PARAMETERS

|  |  |
| --- | --- |
| **Controller** | **Parameters** |
| PID | 𝐾𝑃 = 40, 𝐾𝐼 = 3, 𝐾𝐷 = 0.02 |
| LQR Low | 𝑲1 = [12.6 18.3 680 276 24 47 12.6 18.3 680 276 −24 −47] |
| LQR Medium | 𝑲2 = [13.5 19.5 700 286 26 50 13.5 19.5 700 286 −26 −50] |
| LQR High | 𝑲3 = [14.12 22 756 296 28 53 14.12 22 756 296 −28 −53] |

**Remark 6**: Because the balancing of the robot cannot be remained during the posture of the robot change, when only the LQR in low, medium, high postures are applied independently. The fuzzy LQR control with two membership functions in fuzzy supervisor is used for comparison.

To evaluate the balancing and moving performances of the proposed controller with robot, two experiments are carried out with different scenarios to evaluate the superiority of the proposed control. In the first experiment, the robot keeps balancing in place while its height changes from high to low and from low to high during 60 seconds. In the second scenario, the robot keeps balancing, moving forward and backward, and changing its posture simultaneously during 60 seconds. The posture of the robot changes respect to time, which is illustrated in Fig. 10.

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*C. Experiment Results*

In the first scenario, the experiment is conducted to verify of the proposed control in keeping the balancing of the robot with variant postures. The robot will stay in the high posture in the initially. Its posture will change to the low posture from 15th second to 25th second. Then this status will be kept in 15 seconds after it changes to the high posture again from 40th to 50th second. Finally, the robot will stay in this posture in the last time. During the posture of the robot changes, the robot is kept balancing in place. Fig. 11 shows the output responses of the robot, which are the robot position, pitch angle, rotation angle and the height of the robot with the black lines of the reference, blue lines of the FLQR with TMF and the red lines of the proposed control. The results show that two controllers keep the robot balancing well when the posture changes with respect to time. Additionally, the proposed control with fuzzy supervisor which is designed from the controllers of the LQR controllers in three postures gives better performance than the controller with the fuzzy supervisor designed from two LQR controllers in the high and low postures Fig. 12 presents error performances of the robot, which are the robot position, pitch angle, rotation angle and the height of the robot with the blue lines of the FLQR with TMF and the red lines of the proposed control. Fig. 13 presents the control signals of the controllers in the left, right and hip motor with the blue lines of the FLQR with TMF and the red lines of the proposed control. The chattering effect in the motors is significant.

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Fig. 11. Output response of the two-legged wheeled robot with two controllers in (a) robot position; (b) pitch angle; (c) rotation angle; (d) height of robot

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Fig. 12. Error performance of two controllers in (a) robot position; (b) pitch angle; (c) rotation angle; (d) height of robot

In the second scenario, the experiment is implemented to evaluate the effectiveness of the proposed control in keeping the balancing of the robot with variant postures when the robot moves forward and backward. The robot will stay in the high posture in the initial time, then it will begin decreasing its height to the low posture from the 10th to the 20th second. It will stay the low posture in 15 seconds and increase the height to high posture from the 35th to the 45th second.

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Fig. 13. Control signals response of the two-legged wheeled robot with two controllers in (a) left motor; (b) right motor; (c) hip motor

Finally, it will keep this status in the last time. Besides the posture change with respect to time, the robot also moves forward to the setpoint of 1.6 meter from the origin position from the the 10th to the 20th second. Then, it stays in this place in 15 seconds before moving backward to the origin position from the 35th to the 45th second.

Finally, it will stay at the origin position in the last time. Fig. 14 shows the output responses of the robot, which are the robot position, pitch angle, rotation angle and the height of the robot with the black lines of the reference, blue lines of the FLQR with TMF and the red lines of the proposed control. The results show that two controllers keep the robot balancing well when the robot moves forward and backward with variant postures.

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Fig. 14. Output response of the two-legged wheeled robot with two controllers in (a) robot position; (b) pitch angle; (c) rotation angle; (d) height of robot

Additionally, the proposed control with fuzzy supervisor gives better performance than the FLQR with TMF. Fig. 15 presents error performances of the robot, which are the robot position, pitch angle, rotation angle and the height of the robot with the blue lines of the FLQR with TMF and the red lines of the proposed control Fig. 16 presents the control signals of the controllers in the left, right and hip motor with the blue lines of the FLQR with TMF and the red lines of the proposed control. The chattering effect in the motors is also significant in this case study.

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Fig. 15. Error performance of two controllers in (a) robot position; (b) pitch angle; (c) rotation angle; (d) height of robot

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Fig. 16. Control signals response of the two-legged wheeled robot with two controllers (a) left motor; (b) right motor; and (c) hip motor

V. CONCLUSION AND FUTURE WORK

This paper presented a fuzzy LQR PID control for a two-legged wheeled balancing robot for keeping its stability against the uncertainties and variant heights. The proposed control includes the fuzzy supervisor, LQR, and PID. The fuzzy LQR is conducted to control the stability and motion of the robot with the variant postures. The fuzzy supervisor is used to adjust the LQR control according to the robotic height. It consists of one input and one output. The input and output have three membership functions respectively to three postures of the robot. The PID control is used to control the posture of the robot. In order to verify the effectiveness of the proposed control, the practical robot was constructed and the proposed control were embedded in the control board. Two experiments were also conducted to verify the balancing and moving ability of the robot. In practice, when the posture of robot changes, the sensor mounting plane is tilting an angle with respect to the height of the robot. So, a calibration was carried out to compensate with the pitch angle which is computed from the IMU sensor.

Future works in this study will focus on; 1) Suppressing chattering effect; 2) improving the robotic test bench to control the length of two legs separately; 3) applying type-2 fuzzy system to manage the uncertainties in the system; 4) Investigating some advance navigation methods by using Lidar and CCD images.

ACKNOWLEDGEMENT

The research topic was supported by The Youth Incubator for Science and Technology Programme, managed by Youth Promotion Science and Technology Center - Ho Chi Minh Communist Youth Union and Department of Science and Technology of Ho Chi Minh City, the contract number is " 22/2022/ HĐ-KHCNT-VU" signed on 30th, December, 2022.

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一种具有不确定性和可变高度的双腿车轮机器人的模糊LQR PID控制

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**摘要：本文提出了一种双腿轮式平衡机器人的模糊LQR PID控制方法，以保持其对不确定性和不同高度的稳定性。建议的控制包括模糊监督器、LQR、PID和两个校准。采用模糊 LQR来控制机器人的稳定性和运动，同时其姿态随时间的变化使用模糊监控器来根据机器人的高度来调整LQR控制。它由一个输入和一个输出组成。输入和输出分别有三个隶属函数.PID控制件用于控制机器人的姿态。第一校准用于补偿当机器人改变其姿态时的倾斜角度的偏置值。第二次校准是根据髋关节角度计算机器人的高度。为了验证该控制器的有效性，构造了一个实用的变高度机器人，并将该控制器嵌入到控制板中。最后，通过两个实验验证了机器人的平衡和移动能力。**

***关键词-模糊LQR控制；双腿轮式平衡机器人，PID控制，模糊逻辑系统***

I.介绍

近年来，自动机器人被迅速发明出来，以分享工厂、餐厅[1]、机场[2]和配送[3][4]的人力需求。与传统的机器人和人类工人作为独立的工作空间进行操作不同，人机合作被用来完成复杂的任务。因此，在机器人设计中必须考虑人机交互，这意味着机器人和人可以一起共享工作空间 [5]。两轮倒立摆（TWIP）机器人[6]是一个由倒立摆组成的扩展系统和移动机器人，具有紧凑性、移动性和类人功能等优点。这种机器人在物流运输、通勤、导航、自平衡能力等方面有更多的应用。因为TWIP移动平台[7]被归类为一个欠驱动系统实现3-DOF运动俯仰，偏航，和直接运动只有两个执行器输入，高性能运动控制这个机器人是一个非常具有挑战性的任务控制社区和最近许多结果被报道为分类。

当TWIP机器人在平衡点周围以一个较小的俯仰角工作时，已经采用了一些传统的线性控制技术，如PID控制[811]和线性二次调节器（LQR）[8][9][1216]。然而，当机器人由于外部干扰、建模误差或内部机动而在俯仰角较大的非线性区域工作时，线性控制方法的控制性能将会下降。为了缓解这些问题，提高控制性能，我们研究了许多非线性控制方法，如反馈线性化控制[17][18]、滑模控制[19-21]、后退控制[22-24]和模型预测控制 [25][26]。这些方法通常需要一个数学模型来进行设计过程。在实践中，很难准确地确定数学模型，由于老化和外部环境的影响，参数往往随时间而变化。为了应对这些挑战，我们对TWIP机器人进行了自适应控制[19][27-29]、神经网络控制[19][30-32]和模糊控制[33-36]的研究。在[19]中，采用神经网络来估计未知的模型参数，并采用鲁棒自适应控制来补偿两轮自平衡机器人系统中的估计误差和不确定性。在[27]中，在模型参数不确定的情况下，建立了轮式倒立摆系统的自适应后退控制。在[30]中，我们使用自适应神经网络来补偿自平衡机器人输出动力学中的未知项。在[33]中，为两轮自平衡机器人设计了模糊逻辑控制和极点放置状态反馈控制器。采用极点放置状态反馈控制器来保持机器人的平衡，并采用模糊逻辑控制器来控制机器人的位置。在[37]中，采用模糊和PD控制控制具有结构和非结构不确定性的两轮自平衡机器人。采用PD控制控制机器人的平衡，采用模糊PD控制控制机器人的位置。

众所周知，传统的TWIP机器人可以在平坦的道路上快速、稳定地移动。然而，当它们在不平坦的地形上移动时，如沟壑和斜坡时，它们的局限性就出现了。当车轮半径小于障碍物高度或接触点在车轮[38]中心以上时，它们无法克服障碍物。为了应对这一挑战，一些先进的自平衡机器人[38-41]被构造成与人一起工作。[38]的研究设计了一种地形自适应的双腿轮式机器人，其腿部机构可以跳过障碍物。Klemm等人。[39] 描述了Ascento的基本设计两条腿的轮式跳跃机器人，在不平的地形上移动，也通过跳跃爬上楼梯。周等人。[40]提出了一种质心调整控制方法，使机器人具有更高的图示

描述已自动生成移动鲁棒性。研究了一些传统的控制方法[39][42-48]来处理这类机器人的平衡和高度控制问题。在[39]中，采用线性二次调节器（LQR）和PID控制器来控制稳定、驾驶和跳跃。为了稳定和驱动，LQR控制器是由线性化状态空间模型线性化大约10个不同的腿高度设计的。在[42]中，提出了一种双足腿轮机器人的级联PID控制器，以保证其稳定和驾驶。在[49]中，LQR控制器和模糊 PD控制器研究了一种新型的具有平行四杆机构的轮腿机器人，以稳定运动和跳跃障碍物。

在此基础上，本文提出了一种双腿轮式机器人（TLWR）的模糊LQR PID控制方法，以保持其对不确定性和不同高度的稳定性。基于模糊监控器、LQR和PID设计了该控制方法。因此，这种方法不需要严格的数学模型。利用模糊LQR，包括LQR控制器和模糊监控器，来控制变姿态机器人的稳定性和运动。根据机器人的高度，模糊监督器将估计LQR控制的增益。主管由一个输入和一个输出组成，它们分别有三个成员函数，对机器人的三个姿势。PID控制件用于控制机器人的姿态。为了验证该控制的有效性，构建了实用的机器人，并将该控制嵌入控制板。此外，还进行了两个实验来验证机器人的平衡和移动能力。由于传感器的位移平面根据机器人的姿态而倾斜，因此进行了一些计算来补偿这个倾斜角度。本文的主要贡献总结如下：

1. 基于模糊监控器和三个LQR控制器，构造了三种控制器。因此，在控制设计中降低了TLWR的复杂性。
2. 在一个实际测试台上验证了该控制方法的有效性，并讨论了在实际测试台中测量机器人高度和俯仰角的补偿角所面临的挑战。

本文构造如下：第二节讨论了变高度双腿轮式平衡机器人的问题公式，包括等效质心计算、TLWR建模和线性状态空间模型。第3节设计了由PID控制和模糊LQR控制组成的建议控制器。在第4节中，在实际机器人中进行了一些实验，并将该控制的结果与另一个控制的结果进行了比较。最后，在第5节中提到了一些结论和未来的工作。

**II.** 双腿轮式机器人的描述与建模

自平衡双腿轮式机器人的结构如图所示。1 . 该机器人系统配备了一个控制板，一个惯性测量单元（IMU），一个 Zigbee模块，三个包括编码器的直流电机，和一个12V可充电铅酸电池。控制板设计为主控制器，IMU用于计算平台倾斜度的速率和角度。此外，控制板还可以驱动机器人平台的偏航控制装置。在机器人的脚上安装了两个电机，包括编码器来驱动机器人的运动。另一个被添加在臀部来调整机器人的高度。为了节省髋关节电机的能量，在机器人膝关节的内关节处安装了扭转弹簧。推算推算计算是基于安装在驱动电机中的两个光学编码器的信息来操作的。

图 1. 机器人的 CAD 模型

备注1：在本文中，我们限制了上半身对矢状面运动的应用。机器人的偏航控制由两个轮子的差动运动实现，躯干的俯仰角由髋关节控制。TLWR的滚动角度可以通过调整两条腿的高度来控制，但在本文中，两条腿的运动是相同的，所以滚动角度始终保持在零。此外，本文中使用的 TLWR的符号总结见表一。

TABLE I. THE PARAMETERS OF THE TLWR

|  |  |
| --- | --- |
| **符号** | **定义** |
| 𝜃𝑤 (rad) | 车轮的旋转角度 |
| 𝜃𝑏 (rad) | 等效WIP模型本体的倾斜角度 |
| *r* | 驱动轮的半径 |
| 𝐼𝑏 | 在TLWR模型中，驱动轮上方的总惯性矩 |
| 𝑙𝑐 | TLWR模型中的质心（CoM）位置与车轮轴之间的距离 |
| 𝑑 | 两个车轮之间的距离 |
| 𝑚1 | 柄的质量 |
| 𝑚2 | 大腿下部的重量 |
| 𝑚3 | 大腿上部的重量 |
| 𝑚4 | 身体重量 |
| 𝑙1 | 柄的长度 |
| 𝑙2 | 大腿下部的长度 |
| 𝑙3 | 大腿上部的长度 |
| 𝑙4 | 第四第五坐标系之间的距离 |
| 𝑙5 | 第五第六坐标系之间的距离 |
| 𝜏𝐿 | 左轮的扭矩 |
| 𝜏𝑅 | 右轮的扭矩 |
| 𝑙𝑐1 | 柄部基线的位置 |
| 𝑙𝑐2 | 大腿下缘的位置 |
| 𝑙𝑐3 | 大腿上缘的位置 |
| 𝑙𝑐4 | 车身中心点的位置 |
| 𝜃1 | 第 0 和第 1 坐标系之间的夹角 |
| 𝜃2 | 坐标系 1 和坐标系 2 之间的夹角 |
| 𝜃3 | 坐标系 2 和坐标系 3 之间的夹角 |
| 𝜃4 | 坐标系 2 和坐标系 4 之间的夹角 |
| 𝜃5 | 坐标系 4 和 5 之间的连接角 |
| 𝜃6 | 坐标系 5 和 6 之间的连接角 |
| 𝑎 | 第 2 和第 3 坐标系之间的距离 |

A. 等效中心点计算

本文涉及的坐标系分别为万向框架的𝑂ᵃ 𝑥𝐺 𝑦𝐺 𝑧ᵃ和轮轴框架的𝑂𝑤𝑥𝑤𝑦𝑤𝑧𝑤。在解耦过程中，如图 2 所示，五连杆多刚体系统等效为一个块状质量点。等效中心点的位置由各个链节的质量及其中心点位置加权得出。为了确定该质量中心与车轴坐标系之间的关系，我们采用了德纳维特-哈顿伯格（D-H）惯例来建立运动模型。通过设置如图 3 所示的坐标，TLWR 的 D-H 参数如表 II 所示。

图示

描述已自动生成

图 2. 不同类型机器人动态模型示意图 (a) 两腿轮式机器人，(b) 解耦等效 WIP 模型

图示

描述已自动生成

图 3. TLWR 的运动学参数

表 II. TLWR 的 DH 参数

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Coordinate** | 𝒂𝒊−𝟏 | 𝜶𝒊−𝟏 | 𝒅𝒊 | 𝜽𝒊 |
| 1 | 0 | 0 | 0 | 𝜃1 |
| 2 | 𝑙1 − 𝑎 | 0 | 0 | 𝜃2 |
| 3 | 𝑙2 | 0 | 0 | 𝜃4 |
| 4 | 𝑙4 | 0 | 0 | 𝜃5 |
| 5 | 𝑙5 | 0 | 0 | 𝜃6 |

从第 i 个坐标到第 i-1 个坐标的同质变换矩阵为 (1)。

坐标系 i 和轴坐标系之间的同质变换矩阵为 (2)。

上半身 CoM 相对于世界坐标系的位置可表示为 (3)。

其中 𝑚𝑖 是第 i 个链节的质量， △𝑏 = [△𝑏1, . . 𝑇 是实际角度， 𝑃𝑤𝐶 = [𝑥𝑐 , 𝑧𝑐 ]𝑇 是等效 CoM 相对于车轴坐标系的位置坐标、 𝑃𝑤𝐶𝑖是第 𝑃 个链接的 CoM 在轮轴坐标系中的位置坐标，𝑃 𝑖 𝐶𝑖是第 𝑃 个链接的 CoM 在本地坐标系中的位置。根据 CoM 的坐标，可以得到倒立摆的摆长𝑙𝑐和倾角𝜃，如式（4）和（5）所示。

*B. TLWR Modelling*

假设 2：驱动轮受滚动约束，轮子与地面之间没有滑动。

本研究采用（4）中的摆长来分析倒立摆的动力学特性。车轮倒立摆的状态变量选取为 △𝑤 = [𝑥 𝜃 𝜑]𝑇 。利用欧拉-拉格朗日方法，TLWR 的动态方程可表示为 (6)。其中，𝑀（△𝑤）∈ 𝑅3×3，𝑉（△𝑤，△𝑤）∈ 𝑅3，𝐺（△𝑤）∈ 𝑅3分别为惯性矩阵、科里奥利力和向心力矢量、重力矢量； 𝐵为输入矩阵，𝜏𝑥𝑡为包括建模误差和外部力矩在内的不确定性，𝜏𝑤 = [𝜏𝐿 𝜏 𝑅 ]𝑇为车轮驱动力矩。在差动驱动移动机器人中，输入矩阵定义为

C. 线性状态空间模型

本研究在控制设计中考虑了图 4 所示 TLWR 的三种姿态。其线性状态空间模型如 (7 ) 所示。

其中，𝑥 = [𝑥 𝜃 𝜑 𝑥 ̇ 𝜃 𝜑]𝑇∈ 𝑅6×1 为状态空间，𝑢 = 𝜏 ∈ 𝑅2×1 为控制输入。𝐴𝑖 , 𝐵𝑖 , 𝐶𝑖 与 𝑖 = 1,2,3 分别是 TLWR 三个姿态的矩阵，计算公式为 [50]。

图片包含 雷达图

描述已自动生成

图 4. TLWR 的三种姿态

III. 控制设计

如图 5 所示，拟议的控制结构包括模糊 LQR 控制和姿态控制。姿态控制由 PID 控制设计而成，用于调整机器人的臀部角度。机器人的高度由校准 1 计算得出，校准 1 显示了髋关节角度与机器人高度之间的关系。模糊 LQR 控制结合了模糊监管器和 LQR 控制。模糊监管器包括一个输入和六个输出，它们是 LQR 控制的控制增益。模糊输入有三个成员函数，分别代表机器人的低、中、高姿态。随着姿势的不同，机器人的质心也不同，因此机器人的动态也会发生变化。由于 LQR 的控制增益取决于机器人的动态，因此在不同的姿态下，控制增益也不同。模糊监控器旨在计算 LQR 的控制增益与机器人高度的关系。当机器人的姿势发生变化时，IMU 传感器的倾斜角也会发生变化。因此，IMU 传感器计算出的倾斜角应通过校准 2 进行调整。根据机器人的运动学原理，计算出与机器人高度相关的辅助角度。输入转换矩用于将拟议控制计算出的控制输入转换为每个车轮电机的输入电压。

图示

描述已自动生成

A. PID 控制

为了控制 TLWR 的姿态，根据姿态曲线与 TLWR 真实姿态的差异，采用 PID 控制来驱动髋部电机。根据图 3 中 TLWR 的几何结构，可以按照臀部角度计算出摆锤长度。校准 1 用于根据臀部角度计算机器人高度。

根据臀部角度计算机器人高度。PID 控制法则如 (8) 所示。

其中，𝑝 是比例增益，𝑖 是积分增益，而𝑑 是微分增益。

B. 线性夸脱调节器（LQR）

在 LQR 控制器中，根据成本函数 (𝐽)计算出最佳控制增益 K，从而优化系统 (9) 的状态 𝑥(𝑡) 和控制信号 𝑢(𝑡)。控制信号𝑢(𝑡) 和成本函数𝐽 如 (9) 和 (10) 所示。@@

@@

其中，𝑄 和 𝑅 是正半定矩阵。为了使成本函数 𝐽 最小化，控制增益 𝑄 的计算公式为 (11)。

@@

其中，𝑃 是里卡提微分方程 (12) 的解。

@@

备注 2：LQR 控制器的性能取决于权重矩阵和线性化矩阵 A 和 B 的选择。因此，不同的姿态会给出不同的最优控制增益，即低、中、高姿态下的分别为 1、2 和 3。

备注 3：当 TLWR 的高度随姿势变化时，传感器平面将相应倾斜一个角度。校准 2 用于补偿传感器的倾斜角度。校准 2 中的方程是通过代数求解技术计算得出的。第 6 协调系统与原点协调之间的变换矩阵可通过公式 (13) 计算得出。@@

通过一些操作，变换矩阵 𝑇 6 0 将只取决于臀部角度。由于两条腿的高度是同时调整的，因此变换矩阵𝑇 6 0 的方向矩阵是绕 Z 轴的旋转操作。因此，该矩阵可表示为 (14)。

@@

传感器的补偿角计算公式为 (15)。

@@

校准的输出结果如 (16) 所示。

@@

**C. 模糊监管器**

根据髋关节角度估算出的机器人高度，模糊上位机将分别计算 LQR 的控制增益。图 6 显示了模糊上位机的结构，包括 1 个输入和 8 个输出，即 LQR 的控制增益。在图 7 中，输入的三个成员函数分别为低、中和高。

图示

描述已自动生成

图 6. 模糊监控器的结构 折线图

描述已自动生成

图 7. 模糊监督器的输入成员函数

* 如果它们是 "低"，则𝑲 = 𝑲1、
* 如果参数值为中等，则 𝑲 = 𝑲2、
* 如果数值为高，则 𝑲 = 𝑲3。

其中，遐是机器人的高度；𝑲𝑖(𝑖=1,2,3) 分别是低、中、高姿态的控制增益。

采用 MAX - PROD 聚合法和 "中心点 "模糊化法。控制增益可按 (17) 计算。@@

IV. 实验讨论

A. 试验台说明

图 8 所示的实用 TLWR 机器人包括控制板、带编码器的双轮电机、臀部电机、臀部编码器和 IMU 传感器。采用 ATmega2560 开发的控制板用于控制机器人的稳定性、姿态和运动。双轮电机是两个直流电机（JGB37-520），转速为每分钟 333 转（RPM），与每转 11 个脉冲的增量式编码器相连。根据控制板发出的控制信号，它们将保持机器人的稳定或驱动机器人运动。为了调整机器人的姿态，在臀部安装了高扭矩 5840-31ZT 电机和编码器 AMT332D-V。IMU 传感器 MPU9250 用于计算机器人的姿态。整个系统的电源由 12 V 锂电池提供。经过实际测量，机器人的重量为 3.15 千克，电池寿命约为 2.5 小时。

图示

描述已自动生成

图 8. 双腿轮式机器人在实践中可以改变高度

测试台的硬件连接图如图 9 所示。在笔记本电脑上使用 Visual studio C# 构建图形用户界面，获取机器人的位置、俯仰角、髋关节和偏航角以及控制信号，并通过无线射频（RF）收发器 Zigbee C2530 将控制增益实时发送给机器人。控制板是根据从 IMU 传感器和编码器获取的信息来进行控制的。控制板计算出控制信号后，将其提供给臀部电机和腿部电机的驱动器。

日程表

中度可信度描述已自动生成

图 9. 硬件连接图

备注 4：表 III 中的 TLWR 参数，如长度、质量、惯性矩和车轮半径，是通过 SOLIDWORK 中的质量属性指定和计算的。

TABLE III. PARAMETERS OF THE TWO-LEGGED WHEELED ROBOT

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **符号** | **价值** | **单位** | **符号** | **价值** | **单位** |
| 𝑎 | 0.035 | m | 𝑙2 | 0.145 | m |
| *r* | 0.0425 | m | 𝑙3 | 0.145 | m |
| 𝑑 | 0.276 | m | 𝑙4 | 0.055 | m |
| 𝑚1 | 0.56 | kg | 𝑙5 | 0.09275 | m |
| 𝑚2 | 0.44 | kg | 𝑙𝑐1 | 0.09636 | m |
| 𝑚3 | 0.19 | kg | 𝑙𝑐2 | 0.07251 | m |
| 𝑚4 | 1.96 | kg | 𝑙𝑐3 | 0.07251 | m |
| 𝑙1 | 0.19 | m | 𝑙𝑐4 | 0.028 | m |

备注 5：在本研究中，提议的控制是在实用的双腿轮式机器人上进行的。本研究的步骤概述如下： 第 1 步：从需求中选择设备；第 2 步：设计两足轮式机器人；第 3 步：构建真实模型；第 4 步：评估执行器、传感器和机械结构的性能；第 5 步：在实际模式中使用不同姿态的 LQR 控制器；最后，第 6 步：在机器人上实现建议的控制。

B. 实验说明

为了验证所提出的控制方法的有效性，我们将其与模糊 LQR 控制方法进行了比较，后者在模糊监督器中设计了两个成员函数，被命名为具有两个成员函数的模糊 LQR 控制方法（具有 TMF 的 FLQR）。这种控制是在高位和低位 LQR 控制器的基础上设计的。PID 和 LQR 控制器在不同姿态下的控制参数如表 IV 所示。

1. TABLE IV. CONTROL PARAMETERS

|  |  |
| --- | --- |
| **控制器** | **参数** |
| PID | 𝐾𝑃 = 40, 𝐾𝐼 = 3, 𝐾𝐷 = 0.02 |
| LQR Low | 𝑲1 = [12.6 18.3 680 276 24 47 12.6 18.3 680 276 −24 −47] |
| LQR Medium | 𝑲2 = [13.5 19.5 700 286 26 50 13.5 19.5 700 286 −26 −50] |
| LQR High | 𝑲3 = [14.12 22 756 296 28 53 14.12 22 756 296 −28 −53] |

**备注 6：由于在机器人的姿态变化过程中无法保持平衡，因此在低姿、中姿、高姿态下分别采用 LQR 控制。为了进行比较，我们采用了具有两个成员函数的模糊 LQR 控制。**

**为了评估所提出的控制器与机器人的平衡和移动性能，我们进行了两次不同场景的实验，以评估所提出的控制的优越性。在第一个实验中，机器人在原地保持平衡，同时其高度在 60 秒内从高变低、从低变高。在第二种情况下，机器人在 60 秒内保持平衡、前后移动并同时改变姿势。机器人的姿势随时间变化，如图 10 所示。**

图表

中度可信度描述已自动生成

C. 实验结果

在第一种情况下，我们进行了实验来验证所提出的控制方法是否能让机器人在不同姿态下保持平衡。机器人一开始保持高姿态。从第 15 秒到第 25 秒，机器人的姿态将变为低姿态。第 40 秒至第 50 秒，机器人再次变为高姿态，并保持 15 秒。最后，机器人将一直保持这种姿势。在机器人的姿态变化过程中，机器人保持原地平衡。图 11 显示了机器人的输出响应，即机器人位置、俯仰角、旋转角和高度，其中黑线为参考值，蓝线为带有 TMF 的 FLQR，红线为建议的控制。结果表明，当姿势随时间变化时，两个控制器都能很好地保持机器人的平衡。图 12 显示了机器人的误差表现，即带 TMF 的 FLQR 控制器的蓝线和拟议控制器的红线所显示的机器人位置、俯仰角、旋转角和高度。图 13 显示了左、右和臀部电机控制器的控制信号，图中蓝线为带 TMF 的 FLQR 控制信号，红线为建议的控制信号。电机的颤振效应非常明显。

图表

描述已自动生成

图 11. 使用两个控制器的双足轮式机器人在 (a) 机器人位置；(b) 俯仰角；(c) 旋转角；(d) 机器人高度时的输出响应 图形用户界面, 图表

中度可信度描述已自动生成

图 12. 两个控制器在 (a) 机器人位置；(b) 俯仰角；(c) 旋转角；(d) 机器人高度方面的误差表现

在第二种情况下，实验的目的是评估建议的控制在机器人前后移动时保持不同姿态平衡的有效性。机器人在初始时保持高姿态，然后从第 10 秒到第 20 秒开始降低高度至低姿态。在 15 秒内保持低姿势，在第 35 秒至第 45 秒时增加高度至高姿势。

图示

描述已自动生成

图 13. 使用两个控制器的双足轮式机器人的控制信号响应：（a）左电机；（b）右电机；（c）臀部电机

最后，它将在最后一次保持这一状态。除了姿态随时间变化外，机器人还从第 10 秒到第 20 秒向前移动到距离原点 1.6 米的设定点。然后，它在这个位置停留 15 秒，再从第 35 秒到第 45 秒向后移动到原点位置。

最后，它将停留在最后一次的原点位置。图 14 显示了机器人的输出响应，即机器人的位置、俯仰角、旋转角和高度，其中黑线为参考值，蓝线为带有 TMF 的 FLQR，红线为建议的控制。结果表明，当机器人以不同姿势前后移动时，两个控制器都能很好地保持机器人的平衡。

图表, 折线图

描述已自动生成

图 14. 使用两个控制器的双足轮式机器人在 (a) 机器人位置；(b) 俯仰角；(c) 旋转角；(d) 机器人高度时的输出响应

此外，与带 TMF 的 FLQR 相比，带模糊监督器的拟议控制具有更好的性能。图 15 显示了机器人的误差性能，即机器人位置、俯仰角、旋转角和高度，图中蓝线为带 TMF 的 FLQR 控制，红线为建议的控制。在本案例研究中，电机的颤振效应也很明显。

图形用户界面, 图示

描述已自动生成

图 15. 两个控制器在 (a) 机器人位置；(b) 俯仰角；(c) 旋转角；(d) 机器人高度方面的误差表现

图示

描述已自动生成

图 16. 使用两个控制器的双足轮式机器人的控制信号响应 (a) 左电机；(b) 右电机；(c) 臀部电机

**V. 结论与未来工作**

本文介绍了一种针对双足轮式平衡机器人的模糊 LQR PID 控制，以保持其在不确定性和不同高度下的稳定性。所提出的控制包括模糊控制、LQR 和 PID。模糊 LQR 用于控制机器人在不同姿态下的稳定性和运动。模糊监督器用于根据机器人的高度调整 LQR 控制。它由一个输入和一个输出组成。输入和输出有三个成员函数，分别代表机器人的三种姿态。PID 控制用于控制机器人的姿态。为了验证所提控制的有效性，我们制作了实用机器人，并将所提控制嵌入到控制板中。还进行了两次实验来验证机器人的平衡和移动能力。在实际应用中，当机器人的姿势发生变化时，传感器安装平面会相对于机器人的高度倾斜一个角度。因此，需要进行校准，用 IMU 传感器计算出的俯仰角进行补偿。

本研究的未来工作将集中在：1）抑制颤振效应；2）改进机器人试验台，分别控制两条腿的长度；3）应用 2 型模糊系统管理系统中的不确定性；4）利用激光雷达和 CCD 图像研究一些先进的导航方法。

**致谢**

本研究课题得到了由胡志明共青团青年促进科技中心和胡志明市科技局管理的青年科技孵化器计划的支持，合同编号为 "22/2022/HĐ-KHCNT-VU"，签订于 2022 年 12 月 30 日。

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