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Prove the following: $1^k + 2^k + 3^k + \ldots + n^k = O(n^{(k+1)})$

1) This can be rewritten as:
    a) $1^k + 2^k + 3^k + \ldots + n^k \le n^{(k+1)}$
2) Using the additive property for BigO, which is $O(A) + O(B) = \max(O(A), O(B))$
   we can simplify the left side of the equation to the following:
    a) $O(1^k + 2^k + 3^k + \ldots) + O(n^k) = O(n^k)$
    b) We can do this because the slower parts of the algorithm will always dominate the faster parts. We could also see this as a group of comparisons like the following:
        i) $O(1^k) + O(2^k) = O(2^k)$
        ii) $O(2^k) + O(3^k) = O(3^k)$
        iii) ...
        iv) $O((n-1)^k) + O(n^k) = O(n^k)$
3) So we can simplify to the following:
    a) $n^k \le n^{(k+1)}$
    b) can be rewritten as $n^k \le n * n^k$
4) $n^k \le n * n^{(k)}$ is always true for all real numbers and therefore shows that
   $1^k + 2^k + 3^k + \ldots + n^k = O(n^{(k+1)})$ is true.