ECE 278C Assignment 4: Multi-frequency Backward Propagation

Tyler Hattori
University of California, Santa Barbara
thattori@ucsb.edu

Abstract

In this assignment, I perform image formation by applying the backward propagation technique across a wide band of frequencies and combining the result. As the sub-image reconstructions are sequentially added together, the reconstruction becomes a better reconstruction of the source. The technique is applied for the two source regions defined in Assignment 2, where each region is experimentally reconstructed without the magnitude content of Green's function and the resulting spectrum is acquired. I find that the depth information gleaned from the reconstruction in Assignment 2 is still discernable in the wideband operating mode. Moreover, the spectral coverage is greater for a wider band. The requested videos are attached to the submission email.

1. Introduction

In Assignment 2, I saw that the spectrum of a generated wavefield aligns with the linear combination of plane waves that it is characterized by. In Assignment 3, I found that the reconstruction of a source region is made possible by the accumulation-cancellation effect, which relies heavily on the phase terms of the plane waves comprising the reconstruction. For this reason, I can safely omit the magnitude term of Green's function from reconstructions in this Assignment. Moreover, in Assignment 1 I saw that increasing the operating wavelength of a point source increases the radius of its reconstruction spectrum. For this reason, I expect that the wide band operating mode leads to an increase in spectral content and therefore higher accuracy in the reconstruction. In the following section I explain the results of the wideband reconstruction and spectrum for a single point source. In the last section I do the same for a source region with three point sources symmetrically oriented around the origin.

2. Single Point Source

2.1. Single Reconstruction

I perform the same reconstruction procedure from Assignment 2 for a source region characterized by a single point source at the origin. However, I carry out this process 64 times and vary λ according to $\lambda_n = \frac{64\lambda_0}{n+32}$. Figure 1 shows the reconstruction for $n = 1$, $n = 32$, and $n = 64$ alongside its corresponding spectrum.

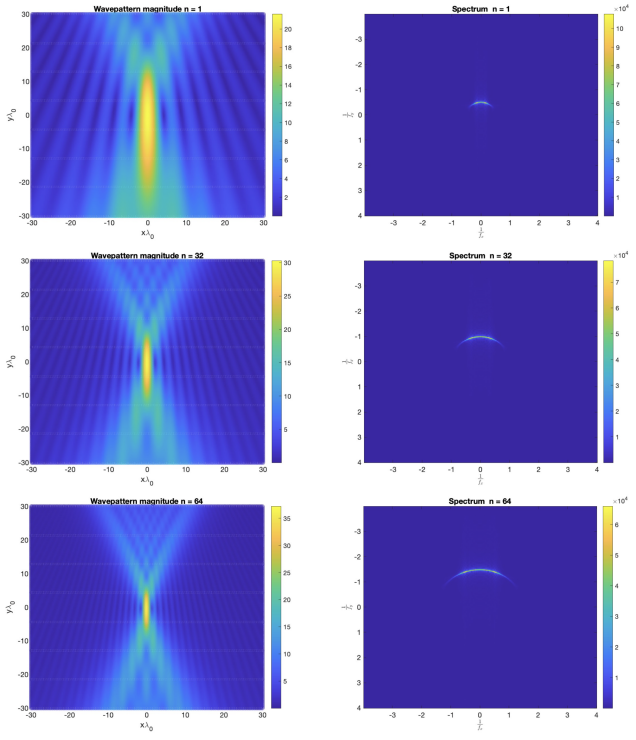

Figure 1. Reconstructions (left) and spectra (right) for $n = 1$, $n = 32$, and $n = 64$ from the top

I see that the radius of the arc in the spectrum grows larger as n increases. Naturally, the width of the arc also grows because the aperture span is unchanged. Moreover, I see that the reconstructed point sources get shorter and skinnier

as n increases. The noise to the left and the right of the point source also fades away. These results show that using a higher operating frequency yields a higher resolution in the reconstruction.

To increase the spectral coverage of the reconstruction, I can sequentially add the reconstructions together and observe the spectra of the results. This is expected to improve the quality of the reconstruction because the accumulation-cancellation effect applies to more vectors. Figure 2 shows the combined reconstructions for $n = 1$, $n = 32$, and $n = 64$ alongside their corresponding spectra.

2.2. Sequential Reconstruction

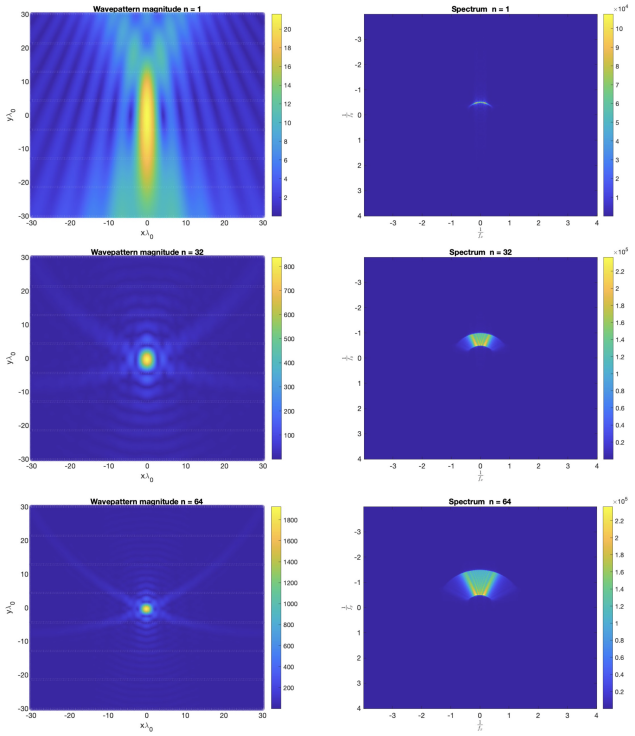

Figure 2. Sum reconstructions (left) and spectra (right) for $n = 1$, $n = 32$, and $n = 64$ from the top

Although I am not using a magnitude term in the convolution kernel, the accumulation-cancellation effect still characterizes the reconstruction. The quality of the reconstruction is much higher at $n = 64$, as expected. However, I can still distinguish the endpoints of the aperture from the faint traces of the plane waves back propagated at the edges of the aperture. These waves were not completely cancelled by the accumulation-cancellation effect. Moreover, the spectrum content increases, as expected. It makes intuitive sense that obtaining more fourier coefficients increased the resolution of the reconstruction. The spectrum has higher magnitude towards the DC value and its width aligns with the size of the aperture.

3. Three Point Sources

3.1. Single Reconstruction

I perform the reconstruction for a source region characterized by three point sources symmetrically oriented around the origin. Figure 3 shows this reconstruction and its corresponding spectrum as n increases.

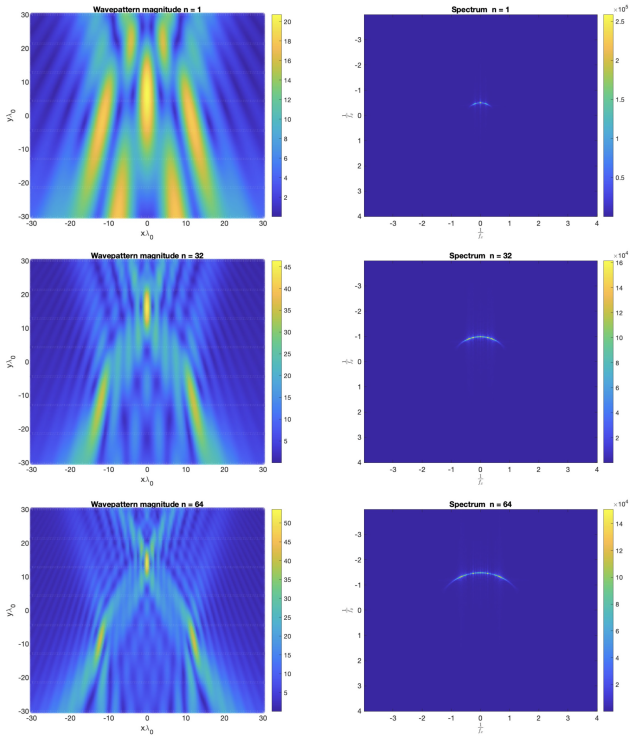

Figure 3. Reconstructions (left) and spectra (right) for $n = 1$, $n = 32$, and $n = 64$ from the top

Again, I see that the resolution of the reconstruction improves as the frequency increases. As seen in my last assignments, the depth of the point sources can be distinguished by the bluriness and the slanted magnitude pattern of the bottom two point sources. The spectrum appears distorted on the arc because the operating wavelength of each point source is the same.

3.2. Sequential Reconstruction

Figure 4 shows the sequential combination of the reconstruction for the three point sources and its corresponding spectrum.

I see that the resolution of the reconstruction improves as n increases. Moreover, the depth information of the three point sources is maintained. The spectrum width is the same

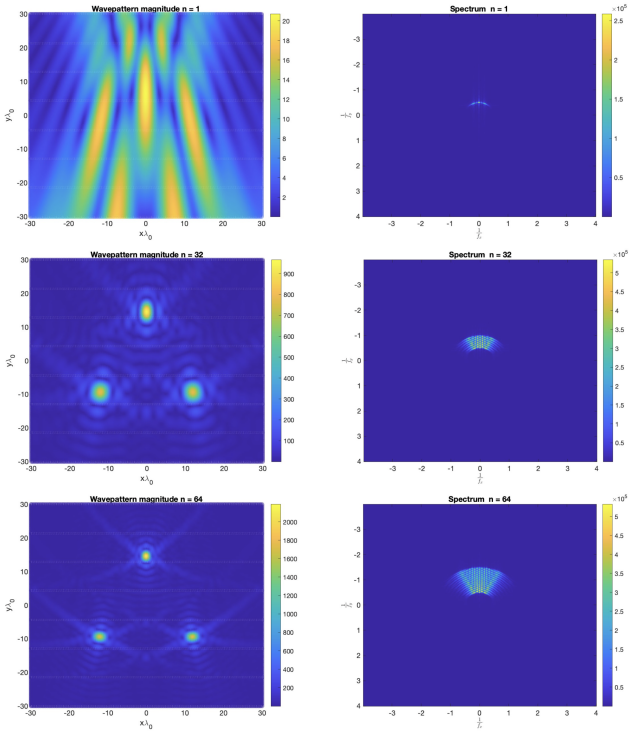

Figure 4. Sum reconstructions (left) and spectra (right) for $n = 1$, $n = 32$, and $n = 64$ from the top

as in the last section, as the aperture size is constant. However, in the wideband operating mode I can now see that the distortion on the spectrum's arcs create a pattern clearly emphasizing the directionality of the three point sources from the aperture. These lines of higher magnitude on the spectrum reveal one benefit of the wideband operating mode.

4. Conclusion

I have shown that the wideband operating mode reveals higher resolution in the reconstructed image and increases the coverage of the spectrum. Moreover, I saw that the depth information in the reconstruction was maintained in the wideband operating mode and that the span of the aperture directly impacts the width of the spectrum. In Assignment 2, I effectively implemented a simple form of holography; in this Assignment, I saw how the wideband operating mode implements tomography.