ECE 278C Assignment 8: Short Baseline Sensing Systems Approximation Techniques

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Abstract

In this assignment, the backward propagation method is used to perform simulation, analysis and estimation of the geolocation of a single target in the active sensing mode using two approximation techniques that differ from the technique seen in Assignment 7. These techniques include the monostatic model and the range and bearing angle model. Moreover, a third technique is implemented which utilizes a simple Fourier Transform to provide a target estimate in both the polar and cartesian coordinate systems. The effectiveness of each approximation technique is analyzed based on target estimation error and computation speed. I find that while the three techniques analyzed in this assignment perform well under certain conditions, the backward propagation technique discussed in Assignment 7 consistently outperforms any other model.

1. Introduction

In Assignment 7, the range estimation technique implemented through the bistatic model offered a perfect estimation of the target location. In the following three sections, three approximation techniques that differ from the bistatic model are discussed and visualized.

First, the monostatic model is implemented by performing range estimation on the data measured at both of the receivers and ignoring the position of the transmitter. However, the locations of the receivers are approximated to be half of their true distances from the transmitter, allowing the model to perform well when the target distance is large or when the separation distance between the receivers is small.

Second, the range and bearing angle model is implemented in two steps. First, the two sets of receiver data are multiplied, allowing for the propagation distance of the two transmitted signals to effectively add. The range estimation of the resulting signal is regarded as about quadruple the distance from the transmitter to the target—scaling this circle by a factor of 4 approximates the target distance. Second, one set of receiver data is multiplied by the conjugate of the other. The bearing angle estimation of the resulting signal reconstructs a hyperbola in the target region which highlights points with the same bearing angle as the received data. The intersection of these two reconstructions gives an approximation of the target location.

Third, the Fourier model is implemented in the same way as the range and bearing angle model. As before, the two sets of receiver data are consolidated in two ways: first by taking a multiplication and second by taking a multiplication with one data set conjugated. However, instead of applying the backward propagation technique on these data sets to perform the reconstructions and obtain the two range profiles, a simple Fourier Transform is performed on the two consolidated data sets to locate the approximate range and bearing angle of the target. These two Fourier Transforms are each characterized by a peak, which offer an approximation of the range and bearing angle of the target when the transforms are properly indexed.

In the final section of this assignment, I compare the performance of the target estimation methods by plotting the estimation error of each method as a function of receiver separation distance, transmission bandwidth, target distance, and target bearing angle.

Unless otherwise stated, I orient my transmitter at the origin and place the receivers at $(-8\lambda_0, 0)$ and $(8\lambda_0, 0)$. I regard the separation of the receivers as $2D = 16\lambda_0$. I set λ_0 to 1 and place the target at $(9\lambda_0, 15\lambda_0)$. This means the distance to the target from the transmitter is $r_0 = 17.5$ with a bearing angle of $\theta = 59$ degrees. For data collection, I transmit an FMCW wave by forward propagating Green's function from the origin to the target and back to each receiver for $n = 128$ step wavelengths centered around λ_0. I measure the phase shift for each of these transmitted waves and store these 128 complex numbers as g_1 for the first receiver and as g_2 for the second receiver.

2. Monostatic Model

In a monostatic model, received data is backward propagated from the receiver to each point in the target region and back to the receiver. This allows the range estimation profile to appear circular. When two transmitter/receiver pairs are implemented, the intersection of the two circular range profiles gives the location of the target. This technique was demonstrated in Assignment 6. However, in my data acquisition process there is one transmitter at the origin and two receivers alongside it at different locations. To apply the monostatic model, I approximate the true physical setup as shown in Figure 1.

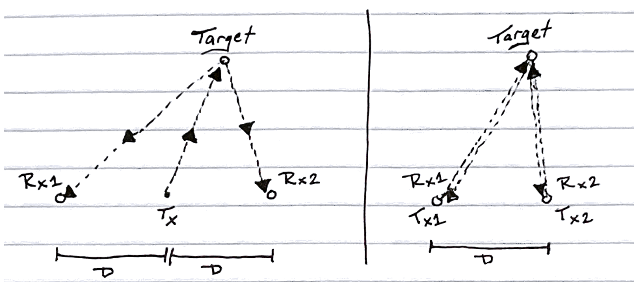

Figure 1. True physical setup (left) and assumed setup to apply the monostatic model (right)

Because this model assumes that there are two transmitters at the locations of the two receivers, the monostatic technique can be applied. It is clear from the image that this model is accurate if the target distance is large or if the separation distance between the receivers is small; in these cases, $2r_1m \approx r_0 + r_1$ and $2r2_m \approx r_0 + r_2$. Figures 2-4 show the target estimation process as n increases.

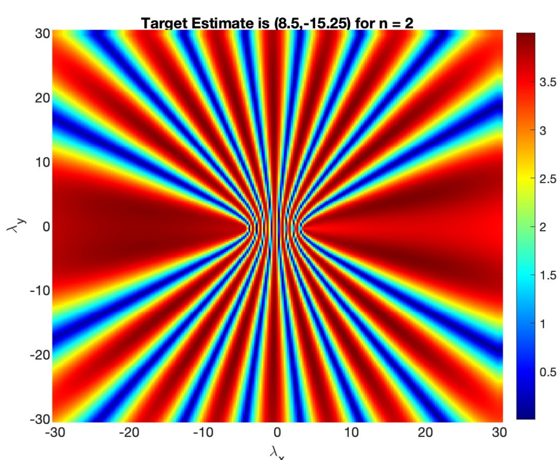

Figure 2. Monostatic model target estimation for $n = 2$

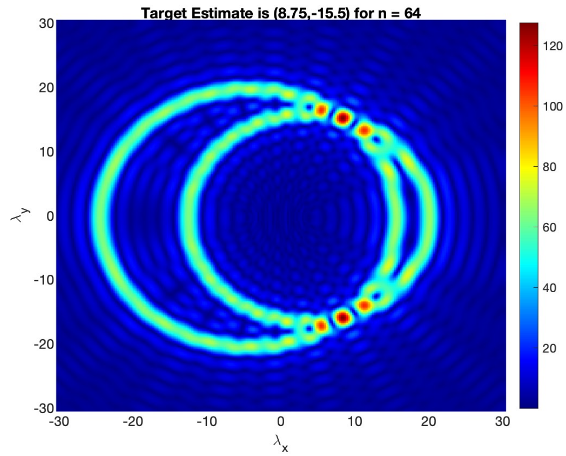

Figure 3. Monostatic model target estimation for $n = 64$

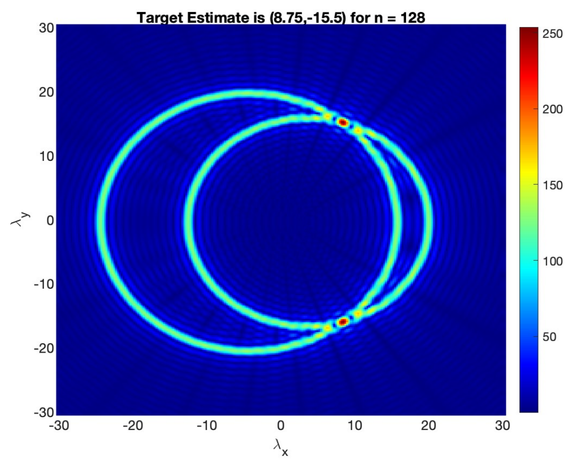

Figure 4. Monostatic model target estimation for $n = 128$

I see that the target approximation becomes clearer as n increases. This makes sense because I am transmitting and receiving a signal with more information as n increases. Looking at the titles of the plots in Figure 2-4, I see that the target estimate gets slightly closer to the true value of $(9, 15)$ as n increases. However, there is still a slight error for $n = 128$. This is expected, as this model is an approximation of the true physical setup.

3. Range and Bearing Angle Model

To apply the range and bearing angle model, I first calculate $g_r = g_1 g_2$ and $g_b = g_1 g_2^*$. g_r represents the data I would have received if the FMCW wave had propagated forward along $r_0 + r_1 + r_0 + r_2 \approx 4r_0$. On the other-hand, g_b represents the data I would have received if the FMCW wave had propagated forward along $r_0 + r_1$ and backward along $r_0 + r_2$. Therefore, the phase shift of the data in g_b reflects the data I would have received if the

FMCW wave had propagated forward along $r_1 - r_2$. To obtain the target range estimation, I perform the backward propagation technique on g_r as if the data was transmitted and received at the origin and scale the resulting circular range profile by a factor of 4. This range estimate is accurate when $r_0 + r_1 + r_0 + r_2 \approx 4r_0$. To obtain the target bearing angle estimation, I backward propagate the data in g_b from the first receiver to every point in the target region but forward propagate the data back to the second receiver. This allows peaks in the resulting reconstruction to arise at points in the target region where $r_{1t} - r_{2t} = r_1 - r_2$, which results in a hyperbolic range profile. The intersection of these two range profiles gives the target estimation. As a whole, this model is generally accurate if the target distance is large or if the separation distance between the receivers is small. Figure 5-7 shows the target estimation process for this model as n increases.

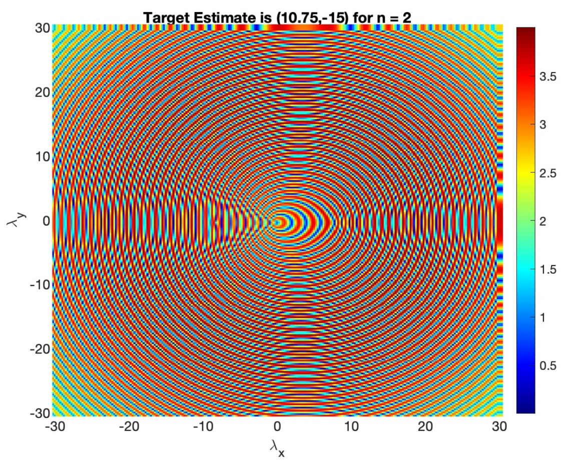

Figure 5. Range and bearing angle model target estimation for $n = 2$

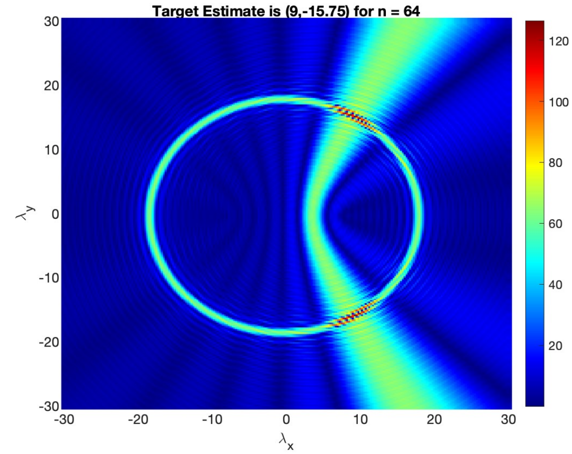

Figure 6. Range and bearing angle model target estimation for $n = 64$

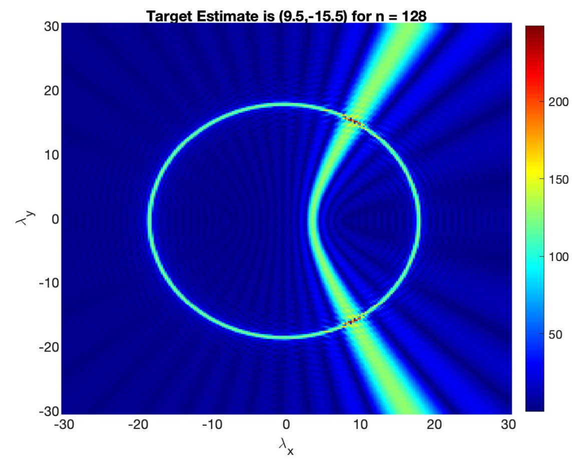

Figure 7. Range and bearing angle model target estimation for $n = 128$

Again, I see that the target approximation becomes clearer as n increases. Moreover, I see that the target estimate is slightly different that the estimate found in the previous section. This is expected, as this model implements a different approximation model. The hyperbolic shape of the bearing angle estimation sheds light on the fact that the model becomes more accurate in the far-field regime. This makes sense, as the separation distance of the receivers becomes relatively small for far-field targets, which improves the approximation.

4. Fourier Model

So far, every model has used the backward propagation technique. Now, to reduce on computation, the target range and bearing angle estimations are found by taking 1-dimensional Fourier Transforms of g_r and g_b. The peak that occurs on G_r is scaled by a factor of 4 to produce the range estimate, while the peak that occurs on G_b is scaled by a factor of the separation distance between the receivers to produce the bearing angle estimate. The scaling for the bearing angle approximation does not require the expected sine operation because of the small-angle approximation. By "smearing" the Fourier peaks across the range and bearing angle axes in the polar coordinate format, the intersection of the two smeared bands can be regarded as the target location in polar coordinates. To obtain the cartesian estimate, each point in the polar coordinate system can be converted from polar coordinates to cartesian coordinates. The smeared band of constant range appears as a band of circles in the cartesian plane, while the smeared band of constant bearing angle appears as a band of straight lines. Figures 8-10 show the target estimation process in polar coordinates and Figures 11-13 shows the target estimation process in cartesian coordinates.

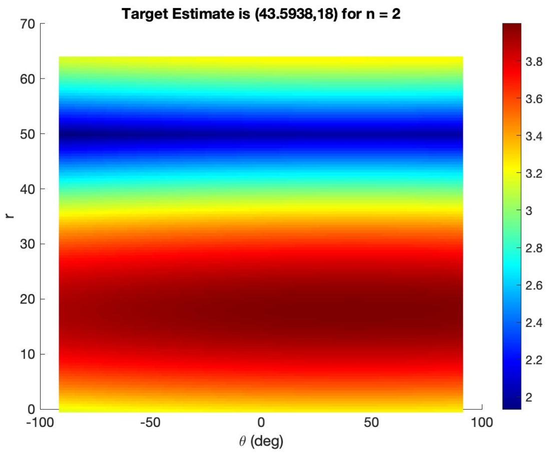

Figure 8. Fourier model target estimation in the polar regime for $n = 2$

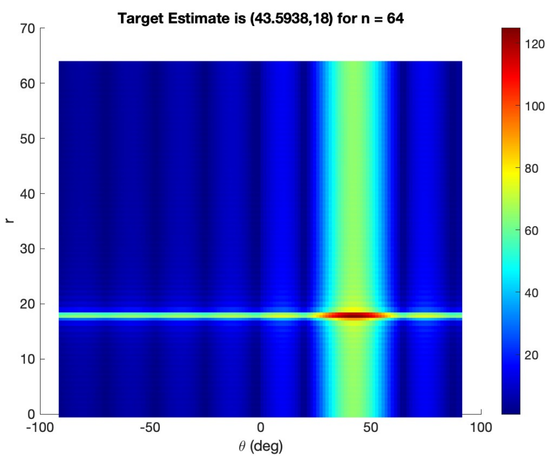

Figure 9. Fourier model target estimation in the polar regime for $n = 64$

Again, I see that the target approximation becomes clearer as n increases. However, I see that the cartesian target estimation is much farther off the true target value of $(9, 15)$ than the previously seen models. This illustrates the natural trade-off between computation time and accuracy. The polar target estimation is also off the true target value of $(60deg, 17.5)$, but the bearing angle is much farther off than the range. This is because this model has especially poor performance at diagonal angles—especially in the near-field regime. This concept is well-illustrated by the plots in Figures 5-7; the hyperbolic shape of the bearing angle profile is more accurate for large target distances.

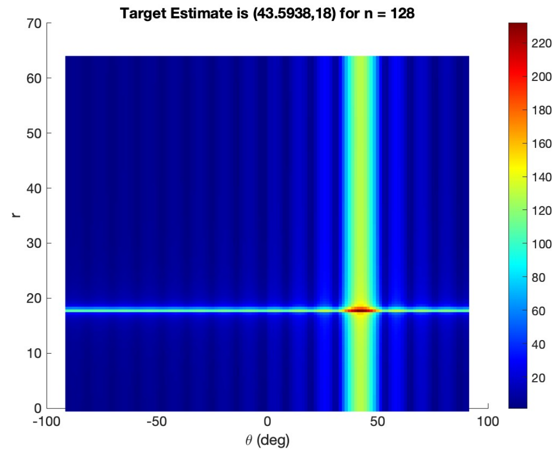

Figure 10. Fourier model target estimation in the polar regime for $n = 128$

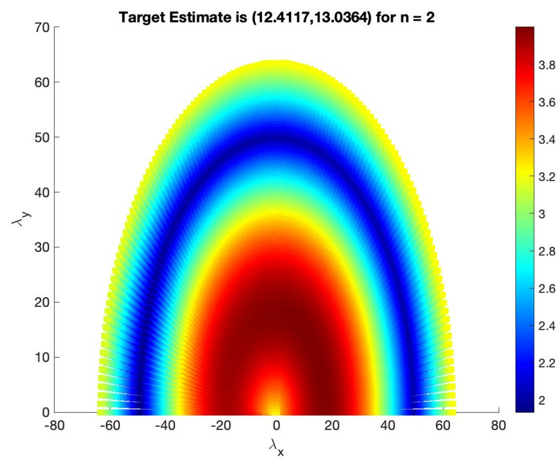

Figure 11. Fourier model target estimation in the cartesian regime for $n = 2$

5. Accuracy Analysis

In the previous sections, it is clear that the target location estimate becomes more obvious as n increases. In this section, I quantitatively evaluate the effectiveness of each approximation by plotting the error of each model's estimation with respect to four key design parameters. These parameters are (1) the separation distance between receivers, (2) the bandwidth of the FMCW waveform, (3) the distance from the transmitter to the target, and (4) the bearing angle of the target with respect to the transmitter. In this section, a bearing angle of 0 degrees corresponds to a target along the positive y-axis and a positive bearing angle refers to a target in the first quadrant.

5.1. Separation Distance

Figure 14 shows the error of each model as a function of the total separation distance between the two receivers.

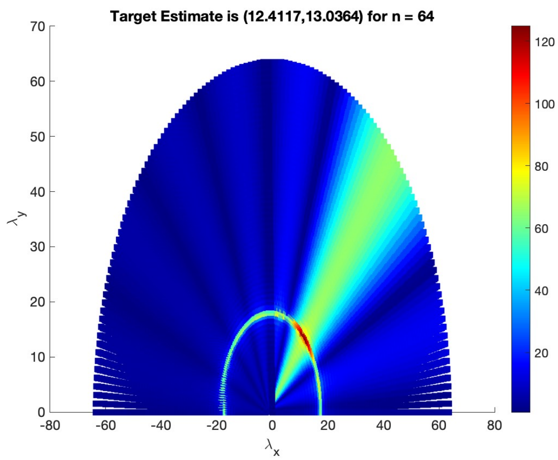

Figure 12. Fourier model target estimation in the cartesian regime for $n = 64$

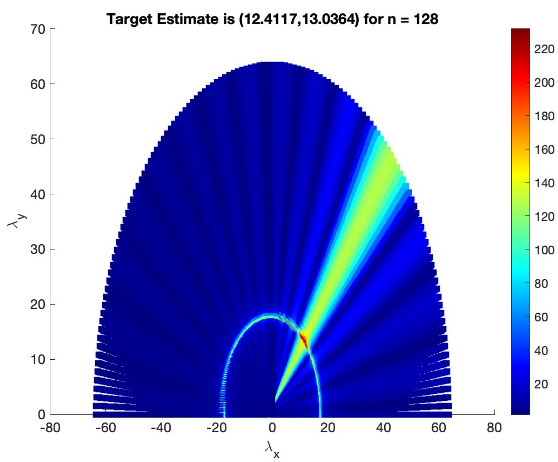

Figure 13. Fourier model target estimation in the cartesian regime for $n = 128$

The other 3 design parameters have the same values as they were used in previous sections.
As expected, the monostatic and range and bearing angle models perform poorly as the separation distance increases. Moreover, the bistatic model has no error and the Fourier model performs poorly for all separation distances.

5.2. Frequency Bandwidth

Figure 15 shows the error of each model as a function of the number of wavelengths in the transmitted FMCW waveform. The other 3 design parameters have the same values as they were used in previous sections.
As expected, the estimation of the monostatic and range and bearing angle models barely change as the FMCW bandwidth increases. This aligns with my observations from previous sections. Moreover, the bistatic model has no error and the Fourier model performs worse than any other

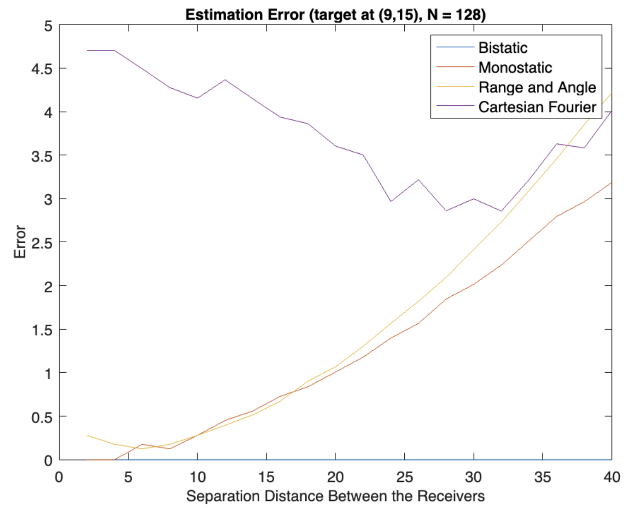

Figure 14. Target estimation error of each model as a function of the separation distance between the receivers

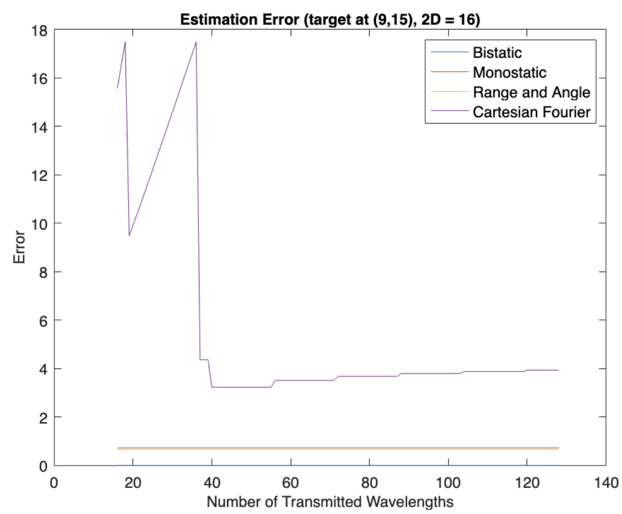

Figure 15. Target estimation error of each model as a function of the number of transmitted wavelengths

model.

5.3. Target Distance

Figure 16 shows the error of each model as a function of the distance between the target and the transmitter. The other 3 design parameters have the same values as they were used in previous sections. As expected, the estimation of the monostatic and range and bearing angle models improves as the target distance from the transmitter increases. Moreover, the bistatic model has no error and the Fourier model performs worse than any other model.
It may be surprising to see that the Fourier model estimations gets worse as the target distance increases, as the bearing angle approximation is expected to improve. It is impor-

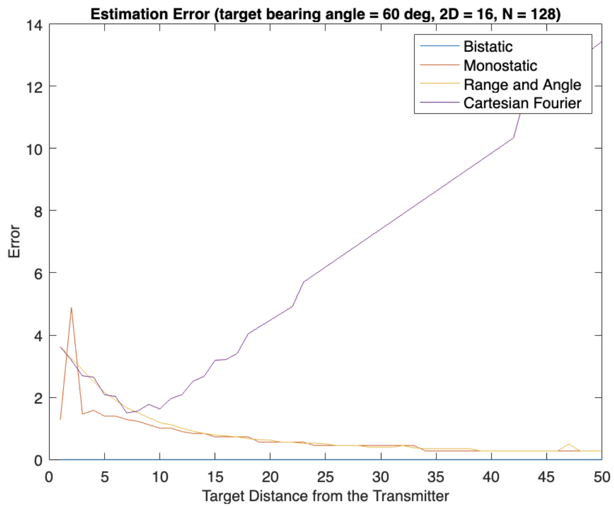

Figure 16. Target estimation error of each model as a function of the target distance

tant to note that this error is the cartesian error of the Fourier model. Therefore, to examine this trend further I individually plot the range and bearing angle estimation errors from the polar Fourier model in Figure 17.

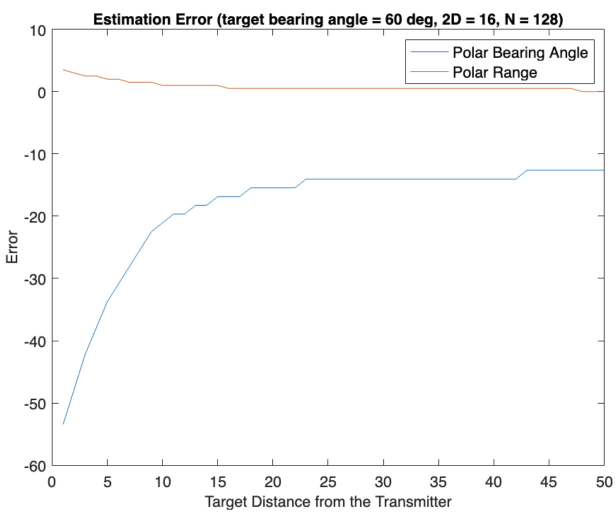

Figure 17. Range and bearing angle estimation error of the polar Fourier model as a function of the target distance

As expected, I see that the bearing angle estimation greatly improves as the target distance increases. The reason the cartesian error increases is simply because the values for x and y cartesian estimates are growing, so the cartesian distance between the estimate and the true value also grows even when the bearing angle approximation improves. The reason this model may still be helpful is because there are some situations where you first want to rotate your system to face the target. This application was discussed in lecture for the submarine warfare application. Of course, the other models will always outperform the Fourier model, as a bearing angle can simply be extrapolated from a cartesian estimate anyways.

5.4. Target Bearing Angle

Figure 14 shows the error of each model as a function of the bearing angle between the transmitter and the target. The other 3 design parameters have the same values as they were used in previous sections. As expected, the Fourier

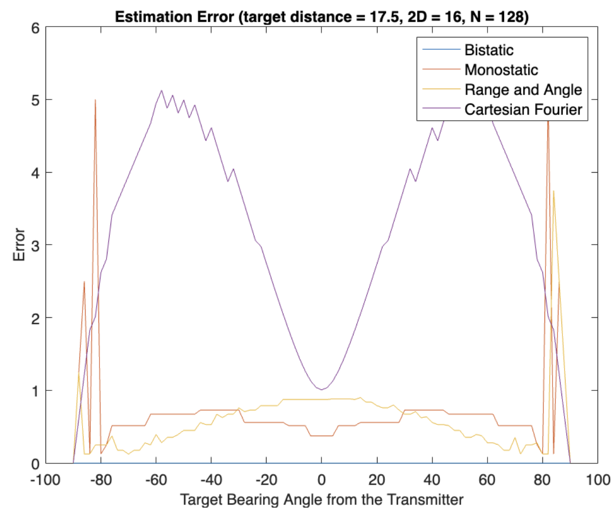

Figure 18. Target estimation error of each model as a function of the target bearing angle

model performs worse at diagonal bearing angles, which aligns with my observations from the previous section. It is interesting to note that the monostatic model has poor performance when the target is placed along the x axis. This makes sense because these are the cases where the approximations $2r_1m \approx r_0 + r_1$ and $2r2_m \approx r_0 + r_2$ hold the least. The same logic is applied to the range and bearing angle model, as the approximation $r_0 + r_1 + r_0 + r_2 \approx 4r_0$ holds the least for targets along the x axis as well.
Figure 19 shows the range and bearing angle estimation errors for the Fourier model in the polar regime.

Again, I see that the bearing angle estimation performs worse at diagonal angles.

6. Conclusion

I have shown three target estimation techniques that rely on mathematical approximations of the physical setup of the system. I find that the two techniques that utilize the backward propagation technique consistently outperform the Fourier model in terms of target estimation accuracy. Moreover, I find that these approximation techniques gener-

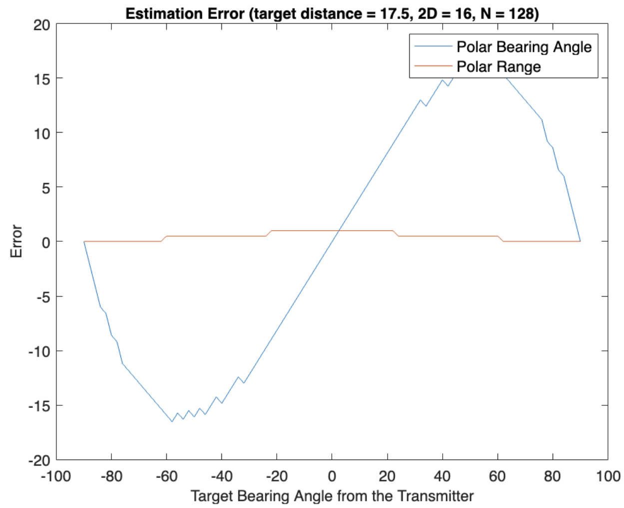

Figure 19. Range and bearing angle estimation error of the polar Fourier model as a function of the target distance

ally perform better when the target distance from the transmitter/receiver setup is large and the separation distance between the receivers is small. This is because the mathematical approximations made with respect to the setup of the system generally hold under these conditions. In every case, utilizing an FMCW waveform with a high bandwidth improves the accuracy of the estimation. Lastly, while the Fourier model is not encouraged, it is able to estimate the range of the target with high accuracy. The problem with the Fourier model is that it is unable to reliably estimate the bearing angle of the target when the target is diagonal to the transmitter. If the Fourier model were to be used, the system would need to be gradually rotated to face the target so that the bearing angle becomes 0. In this case, however, the other models would still perform well if you are willing to sacrifice computation time.