CSCI 3104 Fall 2022
Instructors: Prof. Grochow and Chandra Kanth Nagesh

Quiz 5 S15

Due Date ..... Thursday Oct 20, 2022 8pm MT
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Quiz Code (enter in Canvas to get access to the LaTeX template) ..... **DSGZH**

Contents

Instructions

- You may either type your work using this template, or you may handwrite your work and embed it as an image in this template. **If you choose to handwrite your work, the image must be legible, and oriented so that we do not have to rotate our screens to grade your work.** We have included some helpful LaTeX commands for including and rotating images commented out near the end of the LaTeX template.
- You should submit your work through the **class Gradescope page** only. Please submit one PDF file, compiled using this LaTeX template.
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Problem HC.

- My submission is in my own words and reflects my understanding of the material.
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I agree to the above, Tyler Huynh. □

15 Standard 15: Analyzing Code—Nested Dependent Loops

Problem 15. Analyze the *worst-case* runtime of the following algorithm. Clearly derive the runtime complexity function $T(n)$ for this algorithm, and then find a tight asymptotic bound for $T(n)$ (that is, find a function $f(n)$ such that $T(n) = \Theta(f(n))$). Avoid heuristic arguments from 2270/2824 such as multiplying the complexities of nested loops.

Notice (because they may not be what you think!):

- the lower bound on j in the inner loop, and
- the upper bound on j in the inner loop

Algorithm 1 Nested Algorithm 2

```
1: procedure Foo7(Integer n)
2:     for i ← 1; i ≤ n; i ← i + 1 do
3:         for j ← i; j ≤ n − i; j ← j + 1 do
4:             print "Hi"
```

Answer.

Tyler Hayes

Quiz 5 S14

Inner loop:
$j \leftarrow i$ = 1 step (outside of loop)
$j \leq n-i$ = 1 step
$j \leftarrow j+1$ = 2 steps
print "Hi" = 1 step

I will now find the runtime of the inner loop:

$$i + k \leq n - i$$
$$k \leq n - 2i$$

$$1 + \sum_{j=i}^{n-2i} (1+2+1) = 1 + \sum_{j=1}^{n-2i} 4$$
$$= 1 + 4(n-2i)$$
$$= 1 + 4n - 8i$$

Outer loop:

i will be initialized at 1 (which will take 1 step outside of the loop).
$i \leq n$ = 1 step
$i \leftarrow i+1$ = 2 steps
The rest of the steps will be from the inner loop.

I will now find the runtime of the outer loop:

$$1 + k \leq n$$
$$k \leq n - 1$$

I will now find the total runtime complexity of both nested loops:

$$1 + \sum_{i=1}^{n-1} (1+2) + 1 + 4n - 8i = 1 + \sum_{i=1}^{n-1} 4 + 4n - 8i$$

$$= 1 + \sum_{i=1}^{n-1} 4 + \sum_{i=1}^{n-1} 4n - 8i$$

$$= 1 + 4(n-1) + 4\sum_{i=1}^{n-1} n - 8\sum_{i=1}^{n-1} i$$

$$= 1 + 4n - 4 + 4n(n-1) + 8\left(\frac{(n-1)((n-1)+1)}{2}\right)$$

$$= 1 + 4n - 4 + 4n^2 - 4n + \left(\frac{8(n-1)(n)}{2}\right)$$

$$= 1 + 4n - 4 + 4n^2 - 4n + 4(n^2 - n)$$

$$= 1 + \cancel{4n} - 4 + 4n^2 - \cancel{4n} + 4n^2 - 4n$$

$$= 8n^2 - 4n - 3$$

From the above we can see that the highest component would be n^2, such that the total runtime complexity of the dependent nested loops is:

$$T(n) = \Theta(n^2)$$

The final answer is $T(n) = \Theta(n^2)$ □