CSCI 3104 Fall 2022
Instructor: Prof. Grochow and Chandra Kanth Nagesh

Quiz 2 - Standard 5

Due Date ..... Thursday Sep 22, 8pm MT
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Quiz Code (enter in Canvas to get access to the LaTeX template) ..... **JUHGD**

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Instructions

- You may either type your work using this template, or you may handwrite your work and embed it as an image in this template. **If you choose to handwrite your work, the image must be legible, and oriented so that we do not have to rotate our screens to grade your work.** We have included some helpful LaTeX commands for including and rotating images commented out near the end of the LaTeX template.
- You should submit your work through the **class Gradescope page** only. Please submit one PDF file, compiled using this LaTeX template.
- You may not need a full page for your solutions; pagebreaks are there to help Gradescope automatically find where each problem is. Even if you do not attempt every problem, please submit this document with no fewer pages than the blank template (or Gradescope has issues with it).
- You **may not collaborate with other students**. **Copying from any source is an Honor Code violation. Furthermore, all submissions must be in your own words and reflect your understanding of the material.** If there is any confusion about this policy, it is your responsibility to clarify before the due date.
- Posting to **any** service including, but not limited to Chegg, Discord, Reddit, StackExchange, etc., for help on an assignment is a violation of the Honor Code.
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Honor Code (Make Sure to Virtually Sign)

Problem HC.

- My submission is in my own words and reflects my understanding of the material.
- Any collaborations and external sources have been clearly cited in this document.
- I have not posted to external services including, but not limited to Chegg, Reddit, StackExchange, etc.
- I have neither copied nor provided others solutions they can copy.

I agree to the above, Tyler Huynh. □

5 Standard 5 - Exchange Arguments

Problem 5. Consider the interval scheduling problem from class. You are given a set of intervals $\mathcal{I}$, where each interval has a start and finish time $[s_i, f_i]$. Your goal is to select a subset S of the given intervals such that (i) no two intervals in S overlap, and (ii) S contains as many intervals as possible subject to condition (i).

Suppose we have two intervals with the same start time but different finish times. That is, let $I_1 = [s, f_1]$ and $I_2 = [s, f_2]$ with $f_2 > f_1$.

(a) Let overlap($[s, f]$) denote the number of intervals of $\mathcal{I}$ (other than $[s, f]$) with which $[s, f]$ overlaps. Explain carefully why overlap(I_1) $\leq$ overlap(I_2).

Answer. The number of overlapping intervals of, Overlap(I_1) $\leq$ Overlap(I_2) is true because within the I_1 interval it would have different finish time that is less than the finish time of I_2. Thus, from this since the finish time of I_2 is greater than the finish time of I_1, it will yield for a larger or equal amount of overlaps to occur within the set of intervals I. This is because an interval can overlap between the finish times of $[f_1, f_2]$, causing an overlap with I_2, but will not cause an overlap in I_1. □

(b) Suppose that overlap(I_1) $<$ overlap(I_2). Suppose $S \subseteq \mathcal{I}$ is a non-overlapping set of intervals containing I_2. Explain carefully why $(S \backslash \{I_2\}) \cup \{I_1\}$ is another set of non-overlapping intervals, no smaller than S.

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b.)

$S' = S/I_2 \cup I_1$

$|S'|$ not smaller than $|s|$ and is also a valid set of non-overlapping intervals.

We will first start with the S that contains non-overlapping intervals. We will now remove I_2, thus the number of intervals within S' will become the $|s|-1$.

The above will still be a valid set of non-overlapping intervals.

If we now do exchange I_2 with I_1, the number of non-overlapping intervals within S' will become $|s|$.

This is still a valid set of non-overlapping intervals because any intervals that does not overlap with the interval of I_2, does not overlap with the interval of I_1, since I_1 is during the timeframe of I_2.

Since the overlap of $(I_1) \subseteq$ overlap of (I_2) we are able to add another non-overlapping interval into our set, thus it is possible that $|S'| \geq |S|$, meaning it will never be smaller than S.

Answer. □