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Midterm 2 517

1.)

$$T(n) = \begin{cases} 1 & : n < 5, \\ 3T(\frac{n}{5}) + n^2 & : n \geq 5 \end{cases}$$

Finding how many times T(n) will run through the unrolling method

$T(n) = 3T(\frac{n}{5}) + n^2$
$T(\frac{n}{5}) = 3T(\frac{n}{25}) + (\frac{n}{5})^2$
$T(\frac{n}{25}) = 3T(\frac{n}{125}) + (\frac{n}{25})^2$

$$\begin{aligned} T(n) &= 3T(\tfrac{n}{5}) + n^2 \\ &= 3[3T(\tfrac{n}{25}) + (\tfrac{n}{5})^2] + n^2 \quad \leftarrow \text{unrolling} \\ &= 3[3[3T(\tfrac{n}{125}) + (\tfrac{n}{25})^2] + (\tfrac{n}{5})^2] + n^2 \quad \leftarrow \text{unrolling} \\ &= 3^3 T(\tfrac{n}{125}) + 3^2(\tfrac{n}{25})^2 + 3(\tfrac{n}{5})^2 + n^2 \quad \leftarrow \text{simplify} \\ &= 3^k T(\tfrac{n}{5^k}) + \sum_{i=0}^{k-1} 3^i \cdot (\tfrac{n}{5^i})^2 \end{aligned}$$

Finding how many times of k until we reach the base case:

$\frac{n}{5^k} < 5$

$n < 5(5^k)$

$\log_5 n < \log_5 (5^k \cdot 5)$

$\log_5 n < (\log_5 5^k) + (\log_5 5)$

$\log_5 n < k + 1$

$k > \log_5 n - 1$ ← this will be how many times k will run until we reach the base case.