

Solving for the runtime complexity of $T(n)$:

$$T(n) = (\text{cost of base case}) (\# \text{ of times base case is reached}) + \sum_{i=0}^{\log_5 n - 2} 3^i \left(\frac{n}{5}\right)^2$$

$$= (1) (3^{\log_5 n - 1}) + \sum_{i=0}^{\log_5 n - 2} 3^i \frac{n^2}{5^{2i}}$$

$$= 3^{\log_5 n - 1} + \sum_{i=0}^{\log_5 n - 2} n^2 \frac{3^i}{25^i}$$

$$= 3^{\log_5 n - 1} + n^2 \sum_{i=0}^{\log_5 n - 2} \left(\frac{3}{25}\right)^i \leftarrow \text{using geo series}$$

$$= 3^{\log_5 n - 1} + n^2 \left(\frac{1 - \left(\frac{3}{25}\right)^{\log_5 n - 1}}{1 - \frac{3}{25}} \right)$$

$$\log_5 n = \frac{\log(3/25)^n}{\log(3/25)}$$

$$= 3^{\log_5 n - 1} + n^2 \left(\frac{1 - \left(\frac{3}{25}\right)^{\log_5 n - 1}}{\frac{22}{25}} \right)$$

$$= 3^{\log_5 n - 1} + \frac{25n^2}{22} \left(1 - \left(\frac{3}{25}\right)^{\log_5 n - 1} \right)$$

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$$= 3^{\log_5 n - 1} + \frac{25}{22} \left(n^2 - \left(\frac{3}{25}\right)^{\log_5 n - 1} n^2 \right)$$

From the above we know that $2 + \frac{1}{\log_{3/25} 5} < 2$, such that the highest component is n^2 , such that

$$T(n) = \Theta(n^2)$$