CSCI 3104 Fall 2022
Instructors: Prof. Grochow and Chandra Kanth Nagesh

Quiz 5 S14

Due Date ..... Thursday Oct 20, 2022 8pm MT
Name ..... **Tyler Huynh**
Student ID **109603994**
Quiz Code (enter in Canvas to get access to the LaTeX template) ..... **VGHBN**

Contents

Instructions

- You may either type your work using this template, or you may handwrite your work and embed it as an image in this template. **If you choose to handwrite your work, the image must be legible, and oriented so that we do not have to rotate our screens to grade your work.** We have included some helpful LaTeX commands for including and rotating images commented out near the end of the LaTeX template.
- You should submit your work through the **class Gradescope page** only. Please submit one PDF file, compiled using this LaTeX template.
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Problem HC.

- My submission is in my own words and reflects my understanding of the material.
- Any collaborations and external sources have been clearly cited in this document.
- I have not posted to external services including, but not limited to Chegg, Reddit, StackExchange, etc.
- I have neither copied nor provided others solutions they can copy.

I agree to the above, Tyler Huynh. □

14 Standard 14: Analyzing Code—Independent Nested Loops

Problem 14. Analyze the *worst-case* runtime of the following algorithm. Clearly derive the runtime complexity function $T(n)$ for this algorithm, and then find a tight asymptotic bound for $T(n)$ (that is, find a function $f(n)$ such that $T(n) = \Theta(f(n))$). Avoid heuristic arguments from 2270/2824 such as multiplying the complexities of nested loops.

Notice (because they may not be what you think!):

- the upper bound on i in the outer loop, and
- the amount j gets incremented by in the inner loop.

Algorithm 1 Nested Algorithm 1

```
1: procedure FOO2(Integer n)
2:     for i ← 1; i ≤ n^2; i ← i + 1 do
3:         for j ← 1; j ≤ n; j ← j + 2 do
4:             print "Hi"
```

Tyler Huynh

Quiz 5 S14

Inner Loop:
$j \leq n$ - 1 step
$j \leftarrow j+2$ - 2 steps
$j \leftarrow 1$ - 1 step (Outside of loop)
print "Hi" - 1 step

Finding k or the total runtime of the inner loop:

$$1 + 2k \leq n$$
$$2k \leq n-1$$
$$k \leq \frac{n-1}{2}$$

$$1 + \sum_{j=1}^{\frac{n-1}{2}} (1+2+1) = 1 + \sum_{j=1}^{\frac{n-1}{2}} 4$$

$$= 1 + 4\left(\frac{n-1}{2}\right)$$
$$= 1 + \frac{4n-4}{2}$$
$$= 1 + 2n - 2$$
$$= 2n - 1$$

Outer Loop:
Initialize i at 1
$i \leq n^2$ - 1 step
$i \leftarrow i+1$ - 2 steps
The rest of the steps will be from the inner loop.

Answer.

I will now find the total runtime of the outer loop:

$$1 + k \leq n^2$$
$$k \leq n^2 - 1$$

$$1 + \sum_{i=1}^{n^2-1} 1 + 2 = 1 + \sum_{i=1}^{n^2-1} 3 + (2n-1)$$

$$= 1 + \sum_{i=1}^{n^2-1} 2n - 2$$

$$= 1 + (n^2-1)(2n-2)$$

$$= 1 + 2n^3 + 2n^2 - 2n - 2$$

$$= 2n^3 + 2n^2 - 2n - 1$$

From the above we can see that the highest component would be n^3, such that the total runtime of the independent nested loops would be:

$$T(n) = \Theta(n^3)$$

Final answer is $T(n) = \Theta(n^3)$ □