CSCI 3104 Fall 2022
Instructors: Prof. Grochow and Chandra Kanth Nagesh

Midterm S15

Due Date .... Saturday Nov 19, 2022 4pm MT
Name .... **Tyler Huynh**
Student ID **109603994**
Quiz Code (enter in Canvas to get access to the LaTeX template) **RgZkSGYU5X**

Contents

Instructions

- You may either type your work using this template, or you may handwrite your work and embed it as an image in this template. **If you choose to handwrite your work, the image must be legible, and oriented so that we do not have to rotate our screens to grade your work.** We have included some helpful LaTeX commands for including and rotating images commented out near the end of the LaTeX template.
- You should submit your work through the **class Gradescope page** only. Please submit one PDF file, compiled using this LaTeX template.
- You may not need a full page for your solutions; pagebreaks are there to help Gradescope automatically find where each problem is. Even if you do not attempt every problem, please submit this document with no fewer pages than the blank template (or Gradescope has issues with it).
- You **may not collaborate with other students**. **Copying from any source is an Honor Code violation. Furthermore, all submissions must be in your own words and reflect your understanding of the material.** If there is any confusion about this policy, it is your responsibility to clarify before the due date.
- Posting to **any** service including, but not limited to Chegg, Discord, Reddit, StackExchange, etc., for help on an assignment is a violation of the Honor Code.
- You **must** virtually sign the Honor Code. Failure to do so will result in your assignment not being graded.

Honor Code (Make Sure to Virtually Sign)

Problem HC.

- My submission is in my own words and reflects my understanding of the material.
- Any collaborations and external sources have been clearly cited in this document.
- I have not posted to external services including, but not limited to Chegg, Reddit, StackExchange, etc.
- I have neither copied nor provided others solutions they can copy.

I agree to the above, Tyler Huynh. □

15 Standard 15: Analyzing Code—Dependent Nested Loops

Problem 15. Analyze the *worst-case* runtime of the following algorithm. Clearly derive the runtime complexity function $T(n)$ for this algorithm, and then find a tight asymptotic bound for $T(n)$ (that is, find a function $f(n)$ such that $T(n) = \Theta(f(n))$). Avoid heuristic arguments from 2270/2824 such as multiplying the complexities of nested loops.

Notice (because it may not be what you think!):

- the amount j gets incremented by in the inner loop.

You may find useful the equality $\sum_{i=1}^{n} 1/i = \Theta(\log n)$.

Algorithm 1 Nested Algorithm 1

```
procedure MID(Integer n)
    for i ← 1; i ≤ n; i ← i + 1 do
        for j ← 1; j ≤ n; j ← j + i do
            print "Hi"
```

Tyler Huynh

Midterm 2 S15

I will first find the steps of the inner loop:
The j-loop will take 1-step to initialize.
The comparison of $j \le n$ will take 1-step.
On line 3 the j loop will take 2-steps for the the evaluation of $j+i$ and the assignment of $j \leftarrow$
It will also take 1 step to print "Hi"
From this I will find the total steps of j of the inner loop:
We can see a pattern for each iteration of our inner loop:

$$\left.\begin{array}{llllll} j = 1, & 1+i, & 1+2i, & 1+3i, & \dots & n \\ k = 1 & 2 & 3 & 4 & & \end{array}\right\} 1+(k-1)i$$

$$1+(k-1)i \le n$$
$$(k-1)i \le n-1$$
$$ki - i \le n-1$$
$$\frac{ki}{i} \le \frac{n-1+i}{i}$$
$$k \le \frac{n-1+i}{i}$$
$$k \le \frac{n-1}{i} + 1$$

$$1 + \sum_{j=1}^{\frac{n-1}{i}+1} (1+2+1) = 1 + \sum^{\frac{n-1}{i}+1} 4$$
$$= 1 + 4\left(\frac{n-1}{i}+1\right) \leftarrow$$ for the inner loop runtime

Answer.

Outer loop:
The outer loop will take 1 step to $i \le n$
It will take 2 steps for $i \leftarrow i+1$ for the evaluation and assignment
It will also take 1 step for initializing i
The rest of the steps will be from the inner loop, such that:
The outer loop will run "n" times due to it being from $1 \rightarrow n$ and incrementing by 1 each time.

$$1 + \sum_{i=1}^{n} 1 + 2 + \left(1 + 4\left(\frac{n-1}{i} + 1\right)\right)$$

$$1 + \sum_{i=1}^{n} 3 + \left(1 + \frac{4n-4}{i} + 4\right)$$

$$1 + \sum_{i=1}^{n} 8 + \frac{4n-4}{i}$$

$$1 + \left(\sum_{i=1}^{n} 8 + \sum_{i=1}^{n} \frac{4n-4}{i}\right)$$

$$1 + \left(\sum_{i=1}^{n} 8 + \sum_{i=1}^{n} \frac{4n}{i} - \sum_{i=1}^{n} \frac{4}{i}\right)$$

let $\Theta(\log n) = c \log n$

$$1 + \left(8n + 4n \sum_{i=1}^{n} \frac{1}{i} - 4 \sum_{i=1}^{n} \frac{1}{i}\right)$$

$$1 + \left(8n + 4n(\Theta(\log n)) - 4(\Theta(\log n))\right)$$

$$1 + \left(8n + 4n(c \log n) - 4(c \log n)\right)$$

$$1 + (8n + 4nc \log n - 4c \log n)$$

$$1 + (8n + 4n\,\Theta(\log n) - 4\,\Theta(\log n)) \rightarrow 1 + (8n + 4\,\Theta(n \log n) - 4\,\Theta(\log n))$$

From the above we can see that the highest component would be $n \log n$ such that the total runtime complexity of the dependent nested loop is:

$$T(n) = \Theta(n \log n)$$

The final runtime is $T(n) = \Theta(n \log n)$ $\square$