CSCI 3104 Fall 2022
Instructors: Prof. Grochow and Nagesh

Problem Set 4

Due Date .... September 26, 2022 8pm MT
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Collaborators .... **N/A**

Contents

Instructions

- The solutions **should be typed**, using proper mathematical notation. We cannot accept hand-written solutions. Here's a short intro to LaTeX.
- You should submit your work through the **class Gradescope page** only. Please submit one PDF file, compiled using this LaTeX template.
- You may not need a full page for your solutions; pagebreaks are there to help Gradescope automatically find where each problem is. Even if you do not attempt every problem, please submit this document with no fewer pages than the blank template (or Gradescope has issues with it).
- You are welcome and encouraged to collaborate with your classmates, as well as consult outside resources. You must **cite your sources in this document. Copying from any source is an Honor Code violation. Furthermore, all submissions must be in your own words and reflect your understanding of the material.** If there is any confusion about this policy, it is your responsibility to clarify before the due date.

- Posting to **any** service including, but not limited to Chegg, Reddit, StackExchange, etc., for help on an assignment is a violation of the Honor Code.
- You **must** virtually sign the Honor Code (see Section). Failure to do so will result in your assignment not being graded.

Honor Code (Make Sure to Virtually Sign)

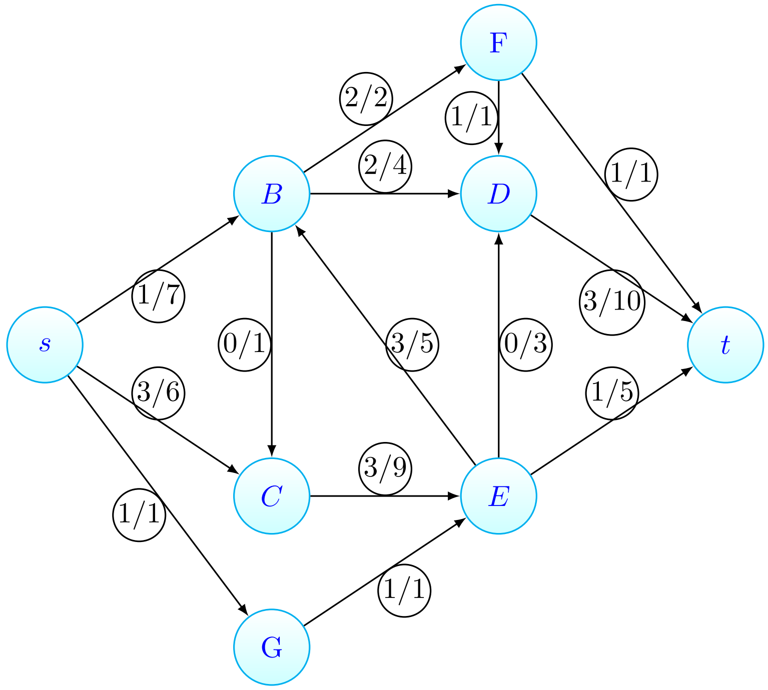
Problem HC.

- My submission is in my own words and reflects my understanding of the material.
- Any collaborations and external sources have been clearly cited in this document.
- I have not posted to external services including, but not limited to Chegg, Reddit, StackExchange, etc.
- I have neither copied nor provided others solutions they can copy.

I agree to the above, Tyler Huynh. □

10 Standard 10 - Flow Networks: Terminology

Problem 10. Consider the following flow network, with the following flow configuration f as indicated below.



Do the following (there are five parts, (a)–(e), continuing on to the next page).

(a) Given the current flow configuration f, what is the maximum *additional* amount of flow that we can push across the edge (B, D) from $B \to D$? Justify using 1-2 sentences.

Answer. The maximum additional flow that we can push across the edge (B, D) from B $\to$ D would be a total flow of 2. We can push at most an additional flow of 2 from B $\to$ D. □

(b) Given the current flow configuration f, what is the maximum amount of flow that B can push backwards to E? Do **not** consider whether E can reroute that flow elsewhere; just whether B can push flow backwards. Justify using 1-2 sentences.

Answer. There is a flow of 3 being pushed back from B $\to$ E. Since there is a total amount of 3 flow being pushed from E $\to$ B, the residual amount of flow coming from B $\to$ E would have a capacity of 0 and a flow of -3 and the flow after some arithmetic would be a total of 3 from B $\to$ E. □

(c) Given the current flow configuration f, what is the maximum amount of flow that D can push backwards to E? Do **not** consider whether D can reroute that flow elsewhere; just whether D can push flow backwards. Justify using 1-2 sentences.

Answer. D cannot push and flow backward from D $\to$ E. This is due to the fact that there is no flow coming from the vertex of E, thus we cannot push back any flow. □

(d) How much *additional* flow can be pushed along the flow-augmenting path $s \to B \to E \to t$? Do not include the current flow along these edges. Justify using 1-2 sentences.

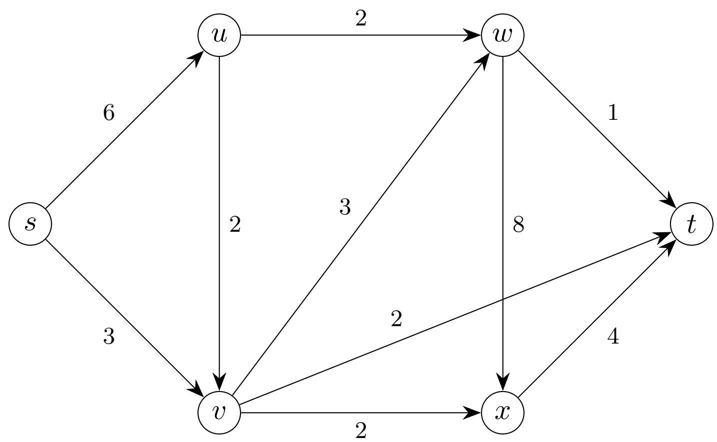
Answer. The additional flow that can be pushed along the flow-augmenting path from $s \to B \to E \to t$ would be a total of 3. On the edge of $s \to B$ we are able to push an additional 6 flow. From here we see that there is a flow of 3 from $E \to B$, the maximum amount of flow that we can push back to E on the edge of $B \to E$ would be 3. On the edge of $E \to t$ we can push an additional flow of 4. However since we can only push a maximum flow of 3 on the edge of $B \to E$ the additional amount of flow we can push is 3, thus the total amount of additional flow would be 3. □

(e) Find a second flow-augmenting path and indicate the maximum amount of additional flow that can be pushed along the path. Assume that the flow-augmenting path from part (d) has **not** been applied. Justify using 1-2 sentences.

Answer. A second flow-augmenting path would be the from $s \to C \to E \to t$, for an additional flow of 3. This is due to the fact that from $S \to C$ we can only push a flow of 3 as the capacity is only 6 with a total flow of 3 already. On the edge of $C \to E$ we can push an additional flow of 6 and on the edge of $E \to t$ we can push an additional flow of 4. On the edge from $S \to C$ we can only push a flow of 3 as the capacity is only 6 with a total flow of 3 already. Thus our additional flow would be 3 on this flow-augmenting path from $s \to C \to E \to t$. □

11 Standard 11 - Flow Networks: Ford–Fulkerson

Problem 11. For Problem 11, consider the following flow network.

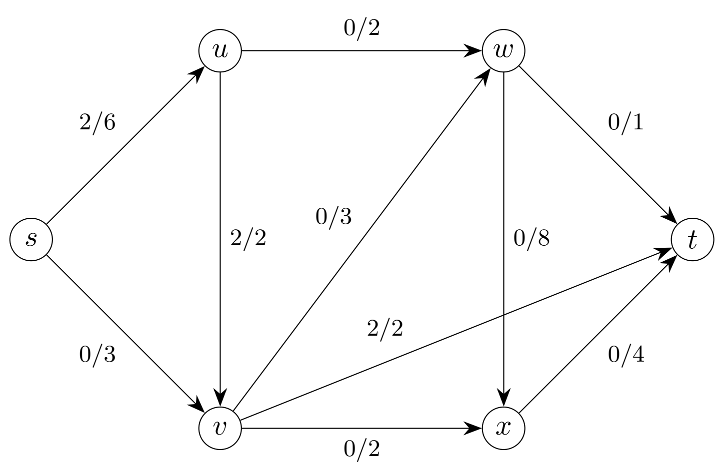


Do the following.

11.1 Problem 11(a)

(a) Consider the flow-augmenting path $s \to u \to v \to t$. Push as much flow through the flow-augmenting path and draw the updated flow network below (we have provided a tikzpicture in the LaTeX comments that you can use and just modify the labels on the edges, or you may hand-draw and embed an image).

Answer. This would be the maximum amount of flow that can be pushed through on this flow-augmenting path. This is due to the fact that on the edges of $u \to v, v \to t$ the maximum amount of flow that can be pushed on these edges is 2.

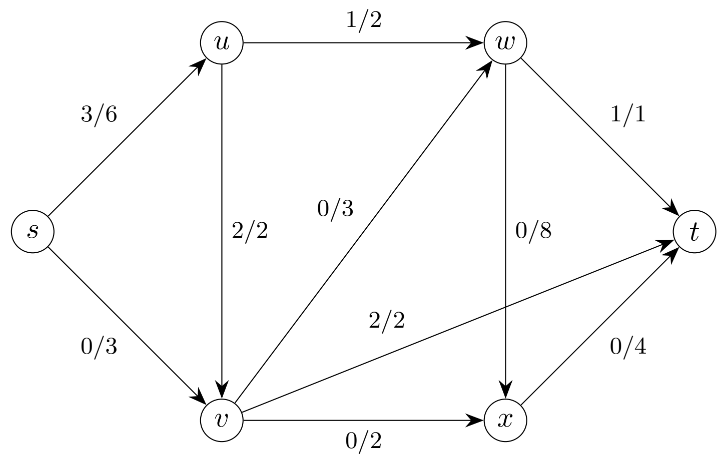


□

11.2 Problem 11(b)

(b) Find a flow-augmenting path starting from the updated flow configuration from your answer to part (a). Then do the following: (i) clearly identify both the new flow-augmenting path and the maximum amount of flow that can be pushed through said path; and then (ii) push as much flow through the flow-augmenting path and draw the updated flow network below.

Answer. The new flow-augmenting path would be from $s \to u \to w \to t$. This path would have a maximum amount of flow of 1. It would have a maximum amount of flow of one because on the edge from $w \to t$ the capacity is only 1, so thus it restricts the rest of our path to only allow us to push a maximum flow of 1 through this path.

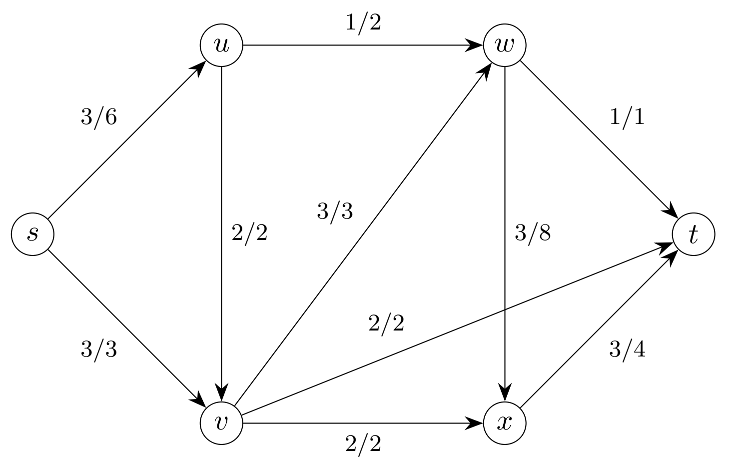


□

11.3 Problem 11(c)

(c) Find a flow-augmenting path starting from the updated flow configuration from your answer to part (b). Then do the following: (i) clearly identify both the new flow-augmenting path and the maximum amount of flow that can be pushed through said path; and then (ii) push as much flow through the flow-augmenting path and draw the updated flow network below.

Answer. The new flow-augmenting path would be $s \to v \to w \to x \to t$, with a maximum amount of flow being 3. The maximum amount of flow is only 3 because from the edges of $s \to v, v \to w$ the capacity for these edges are only 3, thus we can only push a maximum flow of 3 through this path.

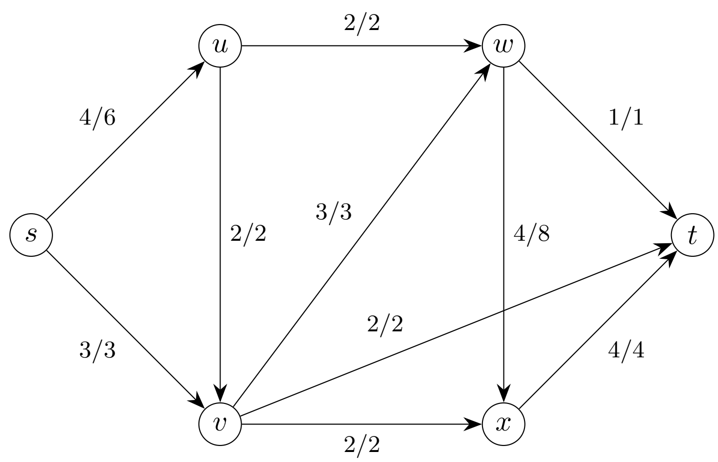


$\square$

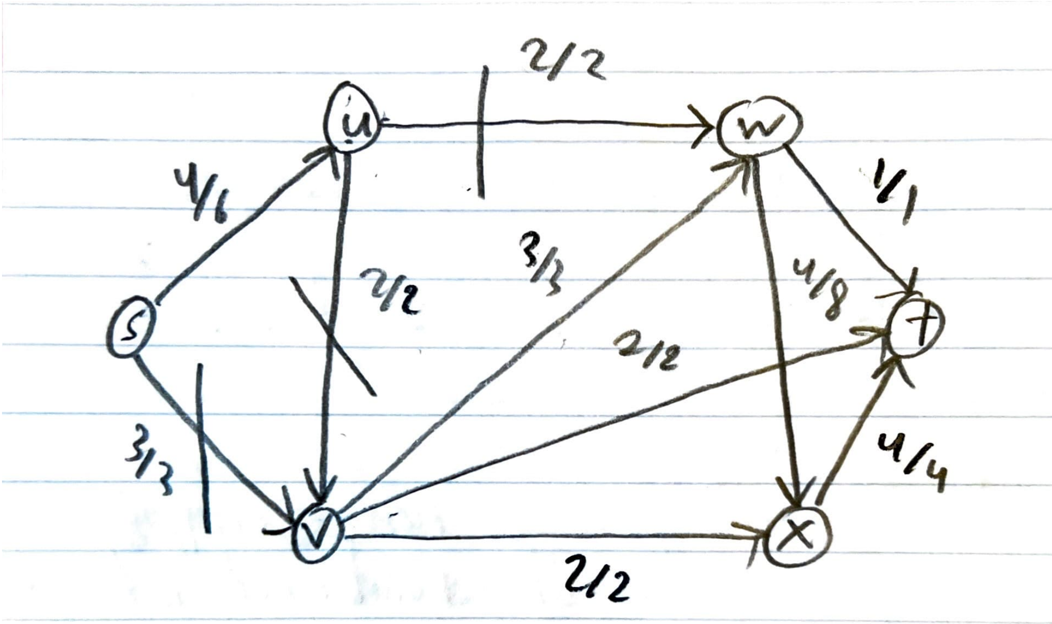
11.4 Problem 11(d)

(d) Using the flow configuration from part (c), finish executing the Ford–Fulkerson algorithm. Include the following here: (i) your flow network, reflecting the maximum-valued flow configuration you found, and (ii) the corresponding minimum capacity cut. There may be multiple minimum capacity cuts, but you should identify the one corresponding to your maximum-valued flow configuration. Then (iii) finally, compare the value of your flow to the capacity of the cut.

Answer. **Used Michael Levet notes to help formulate an answer.**
Our final flow-augmenting path would be from $s \to u \to w \to x \to t$, with a maximum capacity of 1. This path would only have a maximum capacity of 1 because from the edge $u \to w$ we have already pushed a flow of 1 through it thus, it can only hold a maximum flow of 1. Thus, we have now finished the Ford-Fulkerson algorithm as there are no more flow augmenting paths in the flow network.



From the source vertex of s we can only traverse to the vertex of u, such that we can push positive flow on the edge from $\{s, u\}$. We will make cuts on the flow network on the edges of $c\{u, w\}$, $c\{u, v\}$, and $c\{s, v\}$ as these are the edges that are directly adjacent to our source vertex of s and the vertex of u where these edge's capacities are full not allowing for any additional positive flow to go, making the total value of our minimum cut 7 such that $2 + 2 + 3 = 7$. Below we have the image with the corresponding minimum cuts of the flow network.



The value of our flow after making these cuts, would be 7 as we are pushing a total of 4 flow through the edge of $\{s, u\}$ and 3 through the edge of $\{s, v\}$. The total capacity of our minimum cut is 7. Since our max flow on this network is 7 from the edges of $\{s, u\}$ and $\{s, v\}$ and our total capacity of our minimum cut is 7 this will then satisfy the max flow-min cut theorem since the total flow of our network and the total capacity of our minimum cut are equal to one another. □