Quiz 5 5/14

Inner Loop:

$j \leq n - 1$ step

$j \leftarrow j + 2$ - 2 steps

$j \leftarrow 1$ - 1 step (Outside of loop)

print "Hi" - 1 step

Finding k or the total runtime of the inner loop:

$$1 + 2k \leq n$$
$$2k \leq n - 1$$
$$\frac{2k}{2} \leq \frac{n-1}{2}$$
$$k \leq \frac{n-1}{2}$$

$$1 + \sum_{j=1}^{\frac{n-1}{2}} (1 + 2 + 1) = 1 + \sum_{j=1}^{\frac{n-1}{2}} 4$$

$$= 1 + 4\left(\frac{n-1}{2}\right)$$
$$= 1 + \frac{4n-4}{2}$$
$$= 1 + 2n - 2$$
$$= 2n - 1$$

Outer Loop:

Initialize i at 1

$i \leq n^2 - 1$ step

$i \leftarrow i + 1$ - 2 steps

The rest of the steps will be from the inner loop.

I will now find the total runtime of the outer loop:

$$1 + k \le n^2$$
$$k \le n^2 - 1$$

$$1 + \sum_{i=1}^{n^2-1} 1 + 2 = 1 + \sum_{i=1}^{n^2-1} 3 + (2n-1)$$

$$= 1 + \sum_{i=1}^{n^2-1} 2n - 2$$

$$= 1 + (n^2-1)(2n-2)$$

$$= 1 + 2n^3 + 2n^2 - 2n - 2$$

$$= 2n^3 + 2n^2 - 2n - 1$$

From the above we can see that the highest component would be n^3, such that the total runtime of the independent nested loops would be:

$$T(n) = \Theta(n^3)$$