Inductive Step:
Proving for $k+1$, is true

Proving for LHS to obtain RHS:

$$\sum_{i=0}^{k+1} T_i = T_{k+1} + \sum_{i=0}^{k} T_i$$

By our inductive hypothesis we know that $\sum_{i=0}^{k} T_i = T_{k+1} - (k+1)$ will be true, such that

$$\sum_{i=0}^{k+1} = T_{k+1} + T_{k+1} - (k+1)$$

$$= 2T_{k+1} - (k+1)$$

$$= (2T_{k+1} + 1) - k - 1 - 1$$

$$= T_{k+2} - k - 2$$

$$= T_{k+2} - (k+2) \checkmark$$

Thus, by using weak induction we have shown that $\sum_{i=0}^{n} T_i = T_{n+1} - (n+1)$ for every $n \geq 0$.