CSCI 3104 Fall 2022
Instructors: Prof. Grochow and Chandra Kanth Nagesh

Quiz 7 S19

Due Date .... Thursday Nov 3, 2022 8pm MT
Name .... **Tyler Huynh**
Student ID **109603994**
Quiz Code (enter in Canvas to get access to the LaTeX template) .... **KASNM**

Contents

Instructions

- You may either type your work using this template, or you may handwrite your work and embed it as an image in this template. **If you choose to handwrite your work, the image must be legible, and oriented so that we do not have to rotate our screens to grade your work.** We have included some helpful LaTeX commands for including and rotating images commented out near the end of the LaTeX template.
- You should submit your work through the **class Gradescope page** only. Please submit one PDF file, compiled using this LaTeX template.
- You may not need a full page for your solutions; pagebreaks are there to help Gradescope automatically find where each problem is. Even if you do not attempt every problem, please submit this document with no fewer pages than the blank template (or Gradescope has issues with it).
- You **may not collaborate with other students**. **Copying from any source is an Honor Code violation. Furthermore, all submissions must be in your own words and reflect your understanding of the material.** If there is any confusion about this policy, it is your responsibility to clarify before the due date.
- Posting to **any** service including, but not limited to Chegg, Discord, Reddit, StackExchange, etc., for help on an assignment is a violation of the Honor Code.
- You **must** virtually sign the Honor Code (see Section). Failure to do so will result in your assignment not being graded.

Honor Code (Make Sure to Virtually Sign)

Problem HC.

- My submission is in my own words and reflects my understanding of the material.
- Any collaborations and external sources have been clearly cited in this document.
- I have not posted to external services including, but not limited to Chegg, Reddit, StackExchange, etc.
- I have neither copied nor provided others solutions they can copy.

I agree to the above, Tyler Huynh. □

19 Standard 19: Solving Recurrences with the Tree Method

Problem 19. Using the tree method, find a suitable function $f(n)$ such that $T(n) = \Theta(f(n))$. Show all work. You may without loss of generality assume that n is a power of 4; that is, $n = 4^k$ for some integer $k \geq 0$.

$$T(n) = \begin{cases} 3 & : n < 4, \\ 3T(n/5) + n^2 & : n \geq 4. \end{cases}$$

Tyler [illegible]

Quiz 7 S19

$$T(n) = \begin{cases} 3 & : n < 4, \\ 3T\left(\frac{n}{5}\right) + n^2 & : n \geq 4. \end{cases}$$

Finding k, how many times it will run, until we reach the base case:

$$\frac{n}{5^k} < 4$$

$$n < 4(5^k)$$

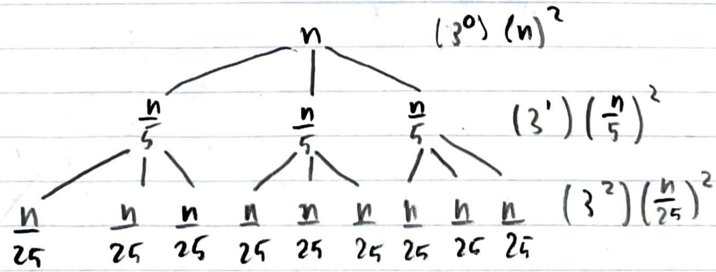
$$\frac{n}{4} < 5^k$$

$$\log_5\left(\frac{n}{4}\right) < \log_5(5^k)$$

$$\log_5\left(\frac{n}{4}\right) < k$$

$$\log_5 n - \log_5 4 < k$$

I will now find the tree of T(n):



Answer. .

$$T(n) = (\text{cost of base case}) \cdot (\text{\# of times base case reached}) + \sum_{i=0}^{\log_5 n - \log_5 4} (3^i)\left(\frac{n}{5^i}\right)^2$$

$$= (3)(3^k) + \sum_{i=0}^{\log_5 n - \log_5 4} (3^i)\left(\frac{n}{5^i}\right)^2$$

$$= 3^{k+1} + n^2 \sum_{i=0}^{\log_5 n - \log_5 4} \left(\frac{3^i}{25^i}\right)$$

$$= 3^{k+1} + n^2 \sum_{i=0}^{\log_5 n - \log_5 4} \left(\frac{3}{25}\right)^i \leftarrow \text{using geometric sum formula}$$

$$= 3^{k+1} + n^2 \left(\frac{1 - \frac{3}{25}^{\log_5 n - \log_5 4 + 1}}{1 - \frac{3}{25}}\right)$$

$$= 3^{k+1} + n^2 \left(\frac{1 - \frac{3}{25}^{\log_5 n - \log_5 4 + \log_5 5}}{\frac{22}{25}}\right)$$

$$= 3^{k+1} + \frac{25n^2}{22}\left(1 - \frac{3}{25}^{\log_5 \left(\frac{5n}{4}\right)}\right)$$

log base change: $\log_5\left(\frac{5n}{4}\right) = \frac{\log_{3/25}\left(\frac{5n}{4}\right)}{\log_{3/25} 5}$

$$= 3^{k+1} + \frac{25n^2}{22}\left(1 - \frac{3}{25}^{\frac{\log_{3/25}\left(\frac{5n}{4}\right)}{\log_{3/25} 5}}\right)$$

$$= 3^{k+1} + \frac{25n^2}{22}\left(1 - \frac{5n}{4}^{\frac{1}{\log_{3/25} 5}}\right)$$

$$= 3^{k+1} + \frac{25n^2}{22} - \frac{125}{88} n^{2 + \frac{1}{\log_{3/25} 5}}$$

I know that $2 + \frac{1}{\log_{3/25} 5}$ is smaller than 2, such that our highest term is n^2, thus our final answer will be:

$$T(n) = \Theta(n^2)$$

$\square$