CSCI 3104 Fall 2022
Instructors: Prof. Grochow and Chandra Kanth Nagesh

Midterm S17

Due Date .... Saturday Nov 19, 2022 4pm MT
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Quiz Code (enter in Canvas to get access to the LaTeX template) **1cHavsfdh0**

Contents

Instructions

- You may either type your work using this template, or you may handwrite your work and embed it as an image in this template. **If you choose to handwrite your work, the image must be legible, and oriented so that we do not have to rotate our screens to grade your work.** We have included some helpful LaTeX commands for including and rotating images commented out near the end of the LaTeX template.
- You should submit your work through the **class Gradescope page** only. Please submit one PDF file, compiled using this LaTeX template.
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Problem HC.

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- I have neither copied nor provided others solutions they can copy.

I agree to the above, Tyler Huynh. □

17 Standard 17: Solving Recurrences Using Unrolling

Problem 17. Using the unrolling method, find a suitable function $f(n)$ such that $T(n) \in \Theta(f(n))$. Show all work.

$$T(n) = \begin{cases} 1 & : n < 5, \\ 3T(n/5) + n^2 & : n \geq 5. \end{cases}$$

Answer. □

Tyler Huynh

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1.)

$$T(n) = \begin{cases} 1 & : n < 5, \\ 3T(\frac{n}{5}) + n^2 & : n \geq 5 \end{cases}$$

Finding how many times T(n) will run through the unrolling method

$T(n) = 3T(\frac{n}{5}) + n^2$
$T(\frac{n}{5}) = 3T(\frac{n}{25}) + (\frac{n}{5})^2$
$T(\frac{n}{25}) = 3T(\frac{n}{125}) + (\frac{n}{25})^2$

$T(n) = 3T(\frac{n}{5}) + n^2$
$= 3[3T(\frac{n}{25}) + (\frac{n}{5})^2] + n^2$ ← unrolling
$= 3[3[3T(\frac{n}{125}) + (\frac{n}{25})^2] + (\frac{n}{5})^2] + n^2$ ← unrolling
$= 3^3 T(\frac{n}{125}) + 3^2(\frac{n}{25})^2 + 3(\frac{n}{5})^2 + n^2$ ← simplify
$= 3^k T(\frac{n}{5^k}) + \sum_{i=0}^{k-1} 3^i \cdot (\frac{n}{5^i})^2$

Finding how many times of k until we reach the base case:

$\frac{n}{5^k} < 5$

$n < 5(5^k)$

$\log_5 n < \log_5 (5^k \cdot 5)$

$\log_5 n < (\log_5 5^k) + (\log_5 5)$

$\log_5 n < k + 1$

$k > \log_5 n - 1$ ← this will be how many times k will run until we reach the base case.

Solving for the runtime complexity of $T(n)$:

$T(n) = $ (cost of base case)(# of times base case is reached) $+ \sum_{i=0}^{\log_5 n - 2} 3^i \left(\frac{n}{5^i}\right)^2$

$$= (1)\left(3^{\log_5 n - 1}\right) + \sum_{i=0}^{\log_5 n - 2} 3^i \frac{n^2}{5^{2i}}$$

$$= 3^{\log_5 n - 1} + \sum_{i=0}^{\log_5 n - 2} n^2 \frac{3^i}{25^i}$$

$$= 3^{\log_5 n - 1} + n^2 \sum_{i=0}^{\log_5 n - 2} \left(\frac{3}{25}\right)^i \leftarrow \text{using geo series}$$

$$= 3^{\log_5 n - 1} + n^2 \left(\frac{1 - \left(\frac{3}{25}\right)^{\log_5 n - 1}}{1 - \frac{3}{25}}\right) \qquad \log_5 n = \frac{\log_{(3/25)} n}{\log_{(3/25)} 5}$$

$$= 3^{\log_5 n - 1} + n^2 \left(\frac{1 - \left(\frac{3}{25}\right)^{\log_5 n} \left(\frac{3}{25}\right)^{-1}}{\frac{22}{25}}\right)$$

$$= 3^{\log_5 n - 1} + \frac{25n^2}{22}\left(1 - \left(\frac{3}{25}\right)^{\frac{\log_{3/25} n}{\log_{3/25} 5}} \left(3/25\right)^{-1}\right)$$

$$= 3^{\log_5 n - 1} + \frac{25n^2}{22}\left(1 - \left(3/25^{\log_{3/25} n}\right)^{1/\log_{3/25} 5} \left(3/25\right)^{-1}\right)$$

$$= 3^{\log_5 n - 1} + \frac{25n^2}{22}\left(1 - (n)^{1/\log_{3/25} 5} \left(25/3\right)\right)$$

$$= 3^{\log_5 n - 1} + \frac{25}{22}\left(n^2 - (n)^{2 + 1/\log_{3/25} 5} \left(\frac{25}{3}\right)\right)$$

From the above we know that $2 + \frac{1}{\log_{3/25} 5} < 2$, such that the highest component is n^2, such that

$$T(n) = \Theta(n^2)$$