Midterm 1 S1

Proof by Induction (Proof by Strong Induction)

Base Case:

- A graph of one vertex, only has one coloring thus when $d=0$, since there are no outgoing edges from this vertex, such that $d+1 \rightarrow 0+1=1$

Inductive Hypothesis:

- For this let us consider a graph that is $G' = G|v$ this is the graph G where vertex v and its respective edges are removed. From this the max degree of graph G' is at most d. d is the maximum degree of G.

From this, it means that it will be a proper $d+1$ coloring of G'.

Inductive Step:

Consider the vertex v that does not exist in G', but does exist in G.

Now, we shall add v into the graph G', we know that v can have at most d edges (or the maximum degree), thus we can color v as $d+1$ for graph G', such that:

$c(v) = d+1$, if all of it's neighbors are $\{1, \ldots, d+1\}$ because the vertex v has d neighbors, thus G is $d+1$ coloring graph.