CSCI 3104 Fall 2022
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# Midterm S1

Due Date .......... Saturday Oct 8, 2022 4pm MT
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Quiz Code (enter in Canvas to get access to the LaTeX template) .......... **hyjPTvJx79**

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## Instructions

- You may either type your work using this template, or you may handwrite your work and embed it as an image in this template. **If you choose to handwrite your work, the image must be legible, and oriented so that we do not have to rotate our screens to grade your work.** We have included some helpful LaTeX commands for including and rotating images commented out near the end of the LaTeX template.
- You should submit your work through the **class Gradescope page** only. Please submit one PDF file, compiled using this LaTeX template.
- You may not need a full page for your solutions; pagebreaks are there to help Gradescope automatically find where each problem is. Even if you do not attempt every problem, please submit this document with no fewer pages than the blank template (or Gradescope has issues with it).
- You **may not collaborate with other students**. **Copying from any source is an Honor Code violation. Furthermore, all submissions must be in your own words and reflect your understanding of the material.** If there is any confusion about this policy, it is your responsibility to clarify before the due date.
- Posting to **any** service including, but not limited to Chegg, Discord, Reddit, StackExchange, etc., for help on an assignment is a violation of the Honor Code.
- You **must** virtually sign the Honor Code. Failure to do so will result in your assignment not being graded.

## Honor Code (Make Sure to Virtually Sign)

**Problem HC.**

- My submission is in my own words and reflects my understanding of the material.
- Any collaborations and external sources have been clearly cited in this document.
- I have not posted to external services including, but not limited to Chegg, Reddit, StackExchange, etc.
- I have neither copied nor provided others solutions they can copy.

*I agree to the above, Tyler Huynh.* □

# 1 Standard 1: Induction

**Problem 1.** A *proper $k$-coloring* of an undirected graph $G = (V, E)$ is a function $c \colon V \to \{1, \ldots, k\}$ (a "coloring") such that no two adjacent vertices receive the same color, that is, for all edges $\{u, v\} \in E$, $c(u) \neq c(v)$.

Given a graph $G$, let $d$ be the maximum degree of any vertex in $G$. **Prove, by induction on the number of vertices of $G$, that there exists a proper $(d+1)$-coloring of $G$.**

*Hint:* In the inductive step, consider a vertex $v \in G$. Your inductive hypothesis should apply to the graph $G' = G \backslash v$, in which $v$ is removed from the vertex set and all the edges including $v$ are removed from the edge set. Once you have a proper $d+1$-coloring of $G'$, how do you know that it can be extended to a proper $(d+1)$-coloring of $G$ by assigning $c(v)$ to be some number in $\{1, \ldots, d+1\}$?

Tyler Huynh

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Proof by Induction (Proof by Strong Induction)

Base Case:

O • A graph of one vertex, only has one coloring thus when $d=0$, since there are no outgoing edges from this vertex, such that $d+1 \rightarrow 0+1=1$

Inductive Hypothesis:

· For this let us consider a graph that is $G' = G \setminus v$ this is the graph $G$ where vertex $v$ and its respective edges are removed. From this the max degree of graph $G'$ is at most $d$. $d$ is the maximum degree of $G$.

From this, it means that it will be a proper $d+1$ coloring of $G'$.

Inductive Step:

Consider the vertex $v$ that does not exist in $G'$, but does exist in $G$.

Now, we shall add $v$ into the graph $G'$, we know that $v$ can have at most $d$ edges (or the maximum degree), thus we can color $v$ as $d+1$ for graph $G'$, such that:

$c(v) = d+1$, if all of it's neighbors are $\{1, \ldots, d+1\}$ because the vertex $v$ has $d$ neighbors, thus $G$ is $d+1$ coloring graph.

*Answer.* □