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$$T_n = \begin{cases} 0 & : n=0 \\ 1 & : n=1 \\ 2T_{n-1}+1 & : n \geq 2 \end{cases}$$

Prove by induction that every $n \geq 0$,

$$\sum_{i=0}^{n} T_i = T_{n+1} - (n+1)$$

Base Case: (Proof by weak induction)

For $n=0$:

(LHS) (RHS)

$$\sum_{i=0}^{0} T_i = T_0 = 0 \qquad T_{0+1} - (0+1) = 1 - 1 = 0$$

For $n=1$:

(LHS) (RHS)

$$\sum_{i=0}^{0} T_i = T_0 + T_1 = 1 \qquad T_{1+1} - (1+1) = T_2 - 2 = 3 - 2 = 1$$

$$T_2 = 2T_1 + 1$$
$$= 2(1)+1$$
$$= 3$$

Inductive Hypothesis:

- For some $k \geq 2$, that $\sum_{i=0}^{k} T_i = T_{k+1} - (k+1)$, such that it will be true.

Inductive Step:
Proving for $k+1$, is true

Proving for LHS to obtain RHS:

$$\sum_{i=0}^{k+1} T_i = T_{k+1} + \underbrace{\sum_{i=0}^{k} T_i}$$

By our inductive hypothesis we know that $\sum_{i=0}^{k} T_i = T_{k+1} - (k+1)$ will be true, such that

$$\sum_{i=0}^{k+1} = T_{k+1} + T_{k+1} - (k+1)$$

$$= 2T_{k+1} - (k+1)$$

$$= (2T_{k+1} + 1) - k - 1 - 1$$

$$= T_{k+2} - k - 2$$

$$= T_{k+2} - (k+2) \checkmark$$

Thus, by using weak induction we have shown that $\sum_{i=0}^{n} T_i = T_{n+1} - (n+1)$ for every $n \geq 0$.