Midterm S20

Due Date .... Saturday Nov 19, 2022 4pm MT
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Quiz Code (enter in Canvas to get access to the LaTeX template) **7X0vUY9sMh**

Contents

Instructions

- You may either type your work using this template, or you may handwrite your work and embed it as an image in this template. **If you choose to handwrite your work, the image must be legible, and oriented so that we do not have to rotate our screens to grade your work.** We have included some helpful LaTeX commands for including and rotating images commented out near the end of the LaTeX template.
- You should submit your work through the **class Gradescope page** only. Please submit one PDF file, compiled using this LaTeX template.
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Problem HC.

- My submission is in my own words and reflects my understanding of the material.
- Any collaborations and external sources have been clearly cited in this document.
- I have not posted to external services including, but not limited to Chegg, Reddit, StackExchange, etc.
- I have neither copied nor provided others solutions they can copy.

I agree to the above, Tyler Huynh. □

20 Standard 20: QuickSort

Problem 20. Consider a modified version of QuickSort, in which the Partition subroutine *always chooses the* $(2/3)n$*-th largest element of the list.* You may assume Partition still takes $\Theta(n)$ time on lists of length n.

Do the following **three** parts of the question (note the third part on the next page).

1. **Write down** the recurrence relation for the runtime of this modified version of QuickSort. **Justify your recurrence relation in 1–2 sentences.**

Answer.

Tyler [illegible]

Midterm 2 S20

P20.)

1.)

The recurrence relation for the runtime of Quicksort.

$$T(n) = \begin{cases} \Theta(1) & : n \leq 3 \\ T(\frac{n}{3}) + T(\frac{2n}{3}) + \Theta(n) & : n > 3 \end{cases}$$

This is my recurrence relation for this problem because $\Theta(1)$ represents the runtime of $T(n)$, when we reach the base case.

$T(\frac{n}{3}) + T(\frac{2n}{3}) + \Theta(n)$ represents the partition subroutine choosing the $(\frac{2}{3})n^{th}$ largest element of the list, this subroutine will always take $\Theta(n)$ time on lists of length n.

□

2. Solve your recurrence relation from part (1), by the tree method. **Show your work.** Find a function $f(n)$ such that the runtime $T(n)$ is $T(n) = \Theta(f(n))$.

2.)

I will solve my recurrence relation from part 1 by using the tree method.

The tree

$$n \qquad \text{level } 0 = n$$

$$\frac{n}{3} \quad \frac{2n}{3} \qquad \text{level } 1 = n$$

$$\frac{n}{9} \quad \frac{2n}{9} \quad \frac{2n}{9} \quad \frac{4n}{9} \qquad \text{level } 2 = n$$

Work done per level: n

I will now find $\underline{k}$ where it will represent the amount of times it will take us to reach the base case:

I will compare $n/3$ and $2n/3$ to see which ks value will terminate last:

$$T\left(\frac{n}{3}\right) = \frac{n}{3^{k_1}} \le 3$$

$$n \le 3(3^{k_1})$$

$$n \le 3^{k_1+1}$$

$$\log_3 n \le \log_3 (3^{k_1+1})$$

$$\log_3 n < k_1+1$$

$$k_1 > \log_3 n - 1$$

$$T\left(\frac{2n}{3}\right) = \left(\frac{2}{3}\right)^{k_2} n \le 3$$

$$\left(\frac{2}{3}\right)^{k_2} \le \frac{3}{n}$$

$$\log_{\frac{2}{3}} \left(\frac{2}{3}\right)^{k_2} \le \log_{\frac{2}{3}} \frac{3}{n}$$

$$\underbrace{k_2 \le \log_{\frac{2}{3}} \frac{3}{n}}$$

This value will be greater than k_1

Answer.

$$T(n) = (\text{cost of base case})(\#\text{ of times base case is reached}) + \sum_{i=0}^{\log_{\frac{2}{3}} \frac{3}{n}} n$$

$$= (\Theta(1))\left(1^{\log_{\frac{2}{3}} \frac{3}{n}}\right) + n\left(\log_{\frac{2}{3}} \frac{3}{n}\right)$$

$$= (\Theta(1)) + n\left(\log_{\frac{2}{3}} 3 - \log_{\frac{2}{3}} n\right)$$

$$= (\Theta(1)) + n\log_{\frac{2}{3}} 3 - n\log_{\frac{2}{3}} n$$

From the above we can see that the highest element is $n\log n$, such that our runtime complexity for $T(n)$ will be:

$$T(n) = \Theta(n \log n)$$

$\square$

3. Write down the following three functions and put them in order from smallest to largest. If two are asymptotically equal (Θ of one another), you should clearly indicate this.

   - the best-case running time of QuickSort (when it always chooses the best possible pivot),
   - the worst-case running time of QuickSort (when it always chooses the worst possible pivot),
   - your answer from part (2) above.

   You do not need to recalculate the best- and worst-case runtimes, those you can get from class notes.

3.)

Best-case for quicksort: chooses median for the pivot, splitting the list in half, such that:

$$2T\left(\frac{n}{2}\right) + \Theta(n) = \Theta(n\lg n)$$

Worst-case for quicksort: chooses the last or first element in a list as the pivot, such that:

$$T(n-1) + T(1) + \Theta(n) = \Theta(n^2)$$

Runtime for our recurrence relation, choosing $\left(\frac{2}{3}\right)$ nth largest element in a list, such that:

$$T\left(\frac{n}{3}\right) + T\left(\frac{2n}{3}\right) + \Theta(n) = \Theta(n\lg n)$$

Note the runtime for our recurrence and the best-case for quicksort are asymptotically equal to one another.

Ordering the functions from smallest to greatest:

$$(\Theta(n\lg n)), (\Theta(n\lg n)) = (\text{Best-case, our recurrence relation})$$

$$\Theta(n^2) = \text{worst-case}$$

Answer. ∎

□