

$$T(n) = (\text{cost of base case}) \cdot (\# \text{ of times base case reached}) + \sum_{i=0}^{\log_5 n - \log_5 4} (3^i) \left(\frac{n}{5^i}\right)^2$$

$$= (3)(3^k) + \sum_{i=0}^{\log_5 n - \log_5 4} (3^i) \left(\frac{n}{5^i}\right)^2$$

$$= 3^{k+1} + n^2 \sum_{i=0}^{\log_5 n - \log_5 4} \left(\frac{3}{25}\right)^i$$

$$= 3^{k+1} + n^2 \sum_{i=0}^{\log_5 n - \log_5 4} \left(\frac{3}{25}\right)^i \quad \leftarrow \text{using geometric sum formula}$$

$$= 3^{k+1} + n^2 \left( \frac{1 - \left(\frac{3}{25}\right)^{\log_5 n - \log_5 4 + 1}}{1 - \frac{3}{25}} \right)$$

$$= 3^{k+1} + n^2 \left( \frac{1 - \frac{3}{25}^{\log_5 n - \log_5 4 + \log_5 5}}{\frac{22}{25}} \right)$$

$$= 3^{k+1} + \frac{25n^2}{22} \left( 1 - \frac{3}{25}^{\log_5 \left(\frac{5n}{4}\right)} \right) \quad \text{log base change: } \log_5 \left(\frac{5n}{4}\right) = \frac{\log_{3/25} \left(\frac{5n}{4}\right)}{\log_{3/25} 5}$$

$$= 3^{k+1} + \frac{25n^2}{22} \left( 1 - \frac{3/25^{\log_{3/25} \left(\frac{5n}{4}\right)}}{\log_{3/25} 5} \right)$$

$$= 3^{k+1} + \frac{25n^2}{22} \left( 1 - \frac{1}{4}^{\frac{1}{\log_{3/25} 5}} \right)$$

$$= 3^{k+1} + \frac{25n^2}{22} - \frac{125n^{2 + \frac{1}{\log_{3/25} 5}}}{88}$$

I know that  $2 + \frac{1}{\log_{3/25} 5}$  is smaller than 2, such that our highest term is  $n^2$ , thus our final answer will be:

$$T(n) = \Theta(n^2)$$