Instructions

(1) This assignment is due on Friday October 21st.
(2) Please submit your written solutions to crowdmark with each problem started on a separate page.
(3) Please list your collaborators on your assignment. It's important to give credit to those you have worked with.

Question 1 (Random Variables). For each of the following random variables, determine whether it is continuous or discrete, and determine its range (list all possible values). Explain your reasoning.

a) The number of cars that travel over the Confederation bridge in a given year.
b) The volume of wastewater treated during a 24 hour period at the Ravensview wastewater treatment facility in Kingston's east end.
c) The balance of your bank account on a randomly chosen day.

Question 2 (Discrete Probability Distributions). Let X be a discrete random variable with probability mass function

(1)
$$f(x) = \begin{cases} 1/2 & x = 0 \\ 1/3 & x = 1 \\ 1/6 & x = 2 \end{cases}.$$

a) Calculate $\mathbb{E}(X)$.
b) Calculate $\mathbb{V}(X)$.
c) Calculate $\mathbb{E}(X^5)$.

Question 3 (The Binomial Distribution). Let X_1 and X_2 be independent binomially distributed random variables with n trials and probability of success p. Show that $Z = X_1 + X_2$ is a binomially distributed random variable with $2n$ trials and probability of success p. The identity

$$\sum_{x=0}^{n} \binom{n}{x}\binom{n}{z-x} = \binom{2n}{z}$$

should be useful.

Question 4 (The Poisson Distribution). Assume that the number of goals that Auston Matthews scores in one hockey game is Poisson distributed with rate $\lambda = 0.636 \frac{\text{goals}}{\text{game}}$ and that the number of goals scored in each game is independent.

a) What is the probability he scores 3 or more goals in one game?
b) What is the probability that he goes 3 games without scoring any goals?
c) What is the probability he scores 3 or more goals over the course of 2 games?

d) **Bonus:** Assuming Auston plays all 82 games this year, what is the probability that he scores 60 or more goals?

Question 5 (Continuous Probability Distributions)**.** The Pareto distribution has probability density function

(2)
$$f(x; k, \phi) = \begin{cases} \frac{k\phi^k}{x^{k+1}} & \text{if } x \geq \phi \\ 0 & \text{if } x < \phi \end{cases}.$$

a) Assuming $k > 1$, find $\mathbb{E}(X)$. What goes wrong if $k \leq 1$?
b) Assuming $k > 2$, find $\mathbb{V}(X)$.

Question 6 (The Exponential Distribution)**.** The shelf life of nickle-cadmium batteries is exponentially distributed with a mean shelf life of 3 years.

a) What is the probability that a randomly selected battery lasts for at least 2 years.
b) You buy a 2 year old pack of four batteries. What is the probability that all four batteries still work?