Instructions

(1) Please submit your written solutions to crowdmark with each problem started on a separate page.
(2) Please list your collaborators on your assignment. It's important to give credit to those you have worked with.

Question 1 (Reduction of Order)**.** Consider the differential equation

$$t\frac{d^2y}{dt^2} - (t+1)\frac{dy}{dt} + y = 0$$

a. Verify that $y_1(t) = e^t$ is a fundamental solution.
b. Find a second, linearly independent fundamental solution.

Question 2 (Undetermined Coefficients)**.** A sandwich beam is a beam made of three pieces: two thin face sheets, and a thicker core material (think corrugated cardboard). In appropriate units, the deflection, $\omega(x)$, of a sandwich beam can be described by the differential equation

$$\frac{d^4\omega}{dx^4} - \left(\frac{1+\alpha}{\beta}\right)\frac{d^2\omega}{dx^2} = \frac{M}{\beta D^{beam}} - \frac{q(x)}{D^{face}},$$

where $\alpha, \beta, M, D^{beam}$, and D^{face} are positive parameters and $q(x)$ is the load on the beam at distance x from the support. For simplicity, assume that $M = D^{beam}$, $\beta = 1,\ \alpha = 8$, and $D^{face} = 1$. Find the general solution for the deflection of the sandwich beam if $q(x) = x$.

Question 3 (Variation of Parameters)**.** Find the general solution to the inhomogeneous Cauchy-Euler equation

$$t^2\frac{d^2y}{dt^2} + t\frac{dy}{dt} - y = \sqrt{t}$$

for $t \in (0, \infty)$.