Queen's University
Faculty of Arts and Science
Department of Mathematics and Statistics

MTHE 224 - T. Meadows
December 20th, 2022

- No aids other than your calculator (Casio 991 series) and an 8.5" ×11" formula sheet are allowed.
- For full marks, you must show all your work and explain how you arrived at your answers, unless explicitly told to do otherwise.
- **Please note:** Proctors are unable to respond to queries about the interpretation of exam questions. Do your best to answer exam questions as written.

Question 1 (6 pts).

a. Show that $\{(1+t),(1-t),(2+t)\}$ is a linearly dependent set on the interval $(-\infty,\infty)$.

b. Show that $\{(1-t)^2,(1+t)^2,(2+t)^2\}$ is a linearly independent set on the interval $(-\infty,\infty)$.

Question 2 (6 pts)**.**
Find the solution to the initial value problem

$$t\frac{dy}{dt} + (t-1)y = 0, \qquad\qquad y(1) = 2$$

Question 3 (6 pts)**.** Consider the following initial value problem:

$$\frac{dy}{dt} = t^2 y \qquad\qquad y(0) = 1 \tag{1}$$

Use Euler's method with a step size of $1/4$ to approximate $y(1)$

Question 4 (8 pts)**.** Consider the differential equation

$$t\frac{d^2y}{dt^2} - (t+1)\frac{dy}{dt} + y = 0 \tag{2}$$

a. Verify that $y_1(t) = e^t$ is a fundamental solution.

b. Find a second, linearly independent fundamental solution.

Question 5 (8 pts)**.** A sandwich beam is a beam made of three pieces: two thin face sheets, and a thicker core material (think corrugated cardboard). In appropriate units, the deflection, $\omega(x)$, of a sandwich beam can be described by the differential equation

$$\frac{d^4\omega}{dx^4} - \left(\frac{1+\alpha}{\beta}\right)\frac{d^2\omega}{dx^2} = \frac{M}{\beta D^{beam}} - \frac{q(x)}{D^{face}},$$

where $\alpha, \beta, M, D^{beam}$, and D^{face} are positive parameters and $q(x)$ is the load on the beam at distance x from the support. For simplicity, assume that $M = D^{beam}$, $\beta = 1$, $\alpha = 8$, and $D^{face} = 1$. Find the general solution for the deflection of the sandwich beam if $q(x) = x$.

Question 6 (8 pts)**.** Find the solution to the initial value problem

$$\frac{d}{dt}\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 5 & 5 \\ 0 & -1 & 3 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \qquad \begin{bmatrix} y_1(0) \\ y_2(0) \\ y_3(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \tag{3}$$

Question 7 (8 pts)**.** Consider the linear differential operator

$$\mathcal{L}[y] = \frac{d^2y}{dt^2} + (2e^t - 1)\frac{dy}{dt} + e^{2t}y \tag{4}$$

a. Find $\mathcal{L}[e^t]$

b. Use the chain rule to find $\mathcal{L}[u(e^t)]$.

c. Use your answer from part b to find the general solution of $\mathcal{L}[y] = 0$.

Room for extra work

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