Instructions

(1) This assignment is due on Friday November 25th.
(2) Please submit your written solutions to crowdmark with each problem started on a separate page.
(3) Please list your collaborators on your assignment. It's important to give credit to those you have worked with.

Question 1 (Linear Independence)**.** Determine whether or not each set of functions is linearly independent on the interval provided.

a. $\{1, \cos(2t), \sin(2t)\}$ for $t \in (-\infty, \infty)$.
b. $\{1, t, \ln(t), t\ln(t)\}$ for $t \in (0, \infty)$.
c. $\{e^t, e^{2t}, e^{3t}\}$ for $t \in (-\infty, \infty)$.

Question 2 (Linear Homogeneous ODEs)**.** For each ODE, find the solution to the initial value problem.

a.
$$2\frac{d^2y}{dt^2} - 5\frac{dy}{dt} - 3y = 0, \qquad y(0) = 0, \quad y'(0) = 1$$

b.
$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 5y = 0, \qquad y(0) = 1, \quad y'(0) = 1$$

c.
$$\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 9y = 0, \qquad y(0) = 1, \quad y'(0) = 0$$

Question 3 (Inhomogeneous Linear ODEs)**.** Find the general solution for each ODE

a.
$$\frac{d^2y}{dt^2} + 4y = e^{2t}$$

b.
$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = te^t$$

c.
$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 2y = t\sin(t)$$

Question 4 (Reduction of Order)**.** Consider the linear differential equation

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0. \tag{1}$$

Given that $y_1(x) = \cos(\ln(x))$ is a solution to this equation for $x \in (0, \infty)$, find another linearly independent solution for $x \in (0, \infty)$.

Question 5. A sandwich beam is a beam made of three pieces: two thin face sheets, and a thicker core material (think corrugated cardboard). In appropriate units, the deflection, $\omega(x)$, of a sandwich beam can be described by the differential equation

$$\frac{d^4\omega}{dx^4} - \left(\frac{1+\alpha}{\beta}\right)\frac{d^2\omega}{dx^2} = \frac{M}{\beta D^{beam}} - \frac{q(x)}{D^{face}},$$

where $\alpha, \beta, M, D^{beam}$, and D^{face} are positive parameters and $q(x)$ is the load on the beam at distance x from the support.

For simplicity, assume that $M = D^{beam}$, $\beta = 1$, $\alpha = 3$, and $D^{face} = 1$. Find the deflection of the sandwich beam if $q(x) = 0$, and the beam is fixed at both ends.

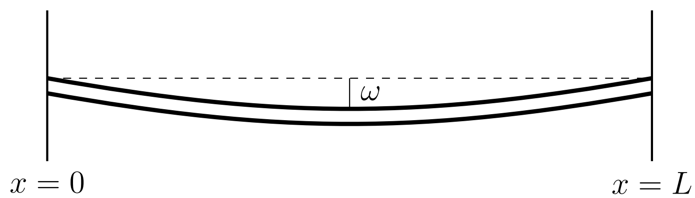

FIGURE 1. A sandwich beam fixed at both ends.