Instructions

(1) This assignment is due on Wednesday November 2nd.
(2) Please submit your written solutions to crowdmark with each problem started on a separate page.
(3) Please list your collaborators on your assignment. It's important to give credit to those you have worked with.

Question 1 (Adding Continuous Random Variables). Let M be the mass of a single raspberry in grams, and suppose that M is distributed as

$$(1) \qquad f(t) = \begin{cases} 0 & t \leq 0 \\ \frac{t}{2} e^{-\frac{t}{2}} & t > 0 \end{cases}$$

    a. What is the expected mass of a raspberry?
    b. Find the pdf for the mass of two raspberries.
    c. What is the probability that two raspberries have a mass less than 1g?

Question 2 (The Normal Distribution). The Janka hardness test for wood measures the amount of force required to embed a 11mm steel ball halfway into a sample of the wood. The hardness of white oak is normally distributed with mean 6000N and a standard deviation of 250N.

    a. What is the probability that a randomly selected piece of white oak has hardness less than 5500N?
    b. Suppose I select 8 independent white oak boards. What is the probability that at least one of these boards has a hardness less than 5000N?

Question 3 (Hypothesis testing: Discrete Random Variables). I bought some high quality chalk, which was shipped here from Japan. The company claims that their packaging protects over 99% of chalk from breaking. In the box of 70 pieces of chalk I received, there were two pieces of broken chalk. Conduct a hypothesis test based on my box of chalk to determine if their quoted 99% figure is correct. How confident are you in your assessment?

Question 4 (Hypothesis Testing: Continuous Random Variables). The paint used to mark lines on roads must reflect enough light to be clearly visible at night. It is required that the mean of reflectometer readings is at least 20. Let μ denote the true average reflectometer reading for a new type of paint under consideration. Suppose the standard deviation of reflectometer readings of this paint is $\sigma = 3$.

   36 stripes are painted on the highway and tested. The sample mean is $\bar{X} = 19.2$. The inspection team decides to test the null hypothesis $H_0 : \mu \geq 20$ against an alternate hypothesis.

    a. What alternate hypothesis should the team use?

    b. Describe what type I and type II errors are in this context. Which do you think would be more serious, and why?

    c. If the inspection team observes the sample mean $\bar{X} = 19.2$ and rejects H_0, what is the probability of making a type I error.

    d. If the true mean is $\mu = 19$, what is the probability of a type II error?

    e. Find the cutoff value and the rejection region for a type I error with $\alpha = 0.1$.