CS 325 HW 4 – SP17

1. Shortest Paths using LP: **(5 points)** Shortest paths can be cast as an LP using distances dv from the source s to a particular vertex v as variables.

- We can compute the shortest path from s to t in a weighted directed graph by solving.

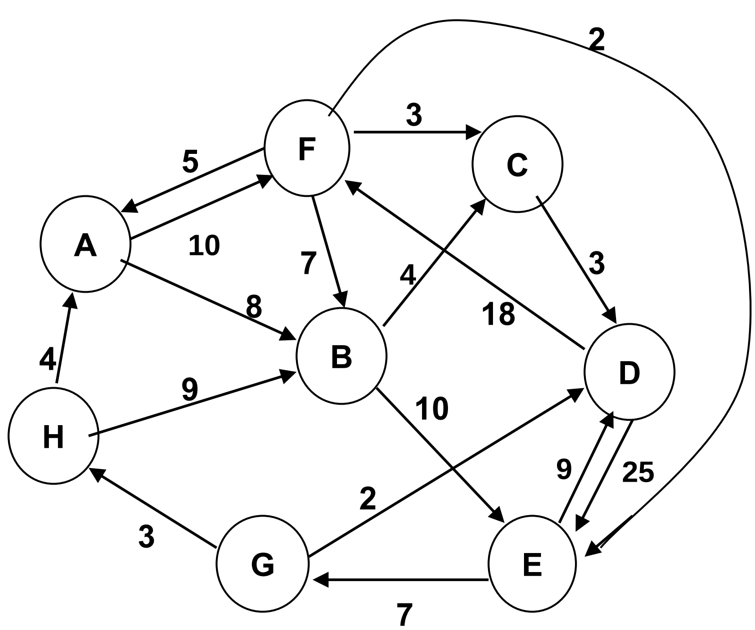
max dt
subject to
$ds = 0$
$dv - du \leq w(u,v)$ for all $(u,v) \in E$

- We can compute the single-source by changing the objective function to

max $\sum_{v \in V} dv$

Use linear programming to answer the questions below. Submit a copy of the LP code and output.

a) Find the distance of the shortest path from G to C in the graph below.
Shortest distance is 16. Check submitted files (HW4 Question 1 Part A.txt and HW4 Question 1 Part A Output.txt) for code and output. I used LINDO, so you should be able to copy and paste HW4 Question 1 Part A.txt into LINDO and get the correct output if you need to verify.
b) Find the distances of the shortest paths from G to all other vertices.
Distance from G to all other vertices is as follows:
G → C = 16
G → A = 7
G → B = 12
G → D = 2
G → E = 19
G → F = 17
G → H = 3
Check submitted files (HW4 Question 1 Part B.txt and HW4 Question 1 Part B Output.txt) for code and output. I used LINDO for this as well and only needed to change the objective function.



2. <u>Product Mix</u>: **(10 points)** Acme Industries produces four types of men's ties using three types of material. Your job is to determine how many of each type of tie to make each month. The goal is to maximize profit, where profit per tie = selling price - labor cost – material cost. Labor cost is $0.75 per tie for all four types of ties. The material requirements and costs are given below.

Material	**Cost per yard**	**Yards available per month**
Silk	$20	1,000
Polyester	$6	2,000
Cotton	$9	1,250

Product Information	**Type of Tie**			
	Silk	**Polyester**	**Blend 1**	**Blend 2**
Selling Price per tie	$6.70	$3.55	$4.31	$4.81
Monthly Minimum units	6,000	10,000	13,000	6,000
Monthly Maximum units	7,000	14,000	16,000	8,500

Material Information in yards	**Type of Tie**			
	Silk	**Polyester**	**Blend 1 (50/50)**	**Blend 2 (30/70)**
Silk	0.125	0	0	0
Polyester	0	0.08	0.05	0.03
Cotton	0	0	0.05	0.07

a) Formulate the problem as a linear program with an objective function and all constraints.
Objective function:

We want to maximize the profit while minimizing the costs. We have to factor in the cost of materials for each tie as well as the cost of labor to produce each tie. So for the all silk tie, it uses 0.125 yards of silk and costs $0.75 to produce (it costs $0.75 for every tie, so I will leave this explanation out when explaining the profit of each subsequent tie). Silk is $20 per yard so 0.125 * 20 = 2.5. So the profit for the silk tie is $6.70 - $2.50 - $0.75 = $3.45. The polyester tie uses .08 yards of polyester and a yard of polyester is $6 so .08 * 6 = .48. $3.55 - $0.48 - $0.75 = $2.32. Blend 1 is half polyester and half cotton: .05 * 6 + .05 * 9 = 0.75. $4.31 - $0.75 - $0.75 = $2.81. Finally, we have Blend 2: .03 * 6 + .07 * 9 = 0.81. $4.81 - $0.81 - $0.75 = $3.25. Now we have the actual profit from each tie and we can write the objective function:

Max $\$3.45*S + \$2.32*P + \$2.81*B_1 + \$3.25*B_2$

Constraints:

There are several constraints we can get from the tables provided. We know we have a limited amount of resources available, so we need to account for that in our constraints.

$0.125*S \leq 1000$ (Silk is only used in the all silk tie and the total amount of silk used must be less than the amount of silk available each month)

$.08*P + .05*B_1 + .03 * B_2 \leq 2000$ (Polyester is used in three ties and the amount of yards used in each tie must be less than the total amount of yards available each month)

$.05*B_1 + .07*B_2 \leq 1250$ (Similar to polyester. Cotton is used in two ties and we can't exceed the total amount of cotton available each month).

We also have a minimum number of ties that need to be sold and a maximum numbers of ties that can be sold for each tie:

$S \geq 6000$

$S \leq 7000$

$P \geq 10000$

$P \leq 14000$

$B_1 \geq 13000$

$B_1 \leq 16000$

$B_2 \geq 6000$

$B_2 \leq 8500$

b) Determine the optimal solution for the linear program using any software you want. Include a copy of the code and output.
I used LINDO to answer this as well. Check submitted files: HW4 Question 2 Part B.txt and HW4 Question 2 Part B Output.txt to see the code and output, respectively.

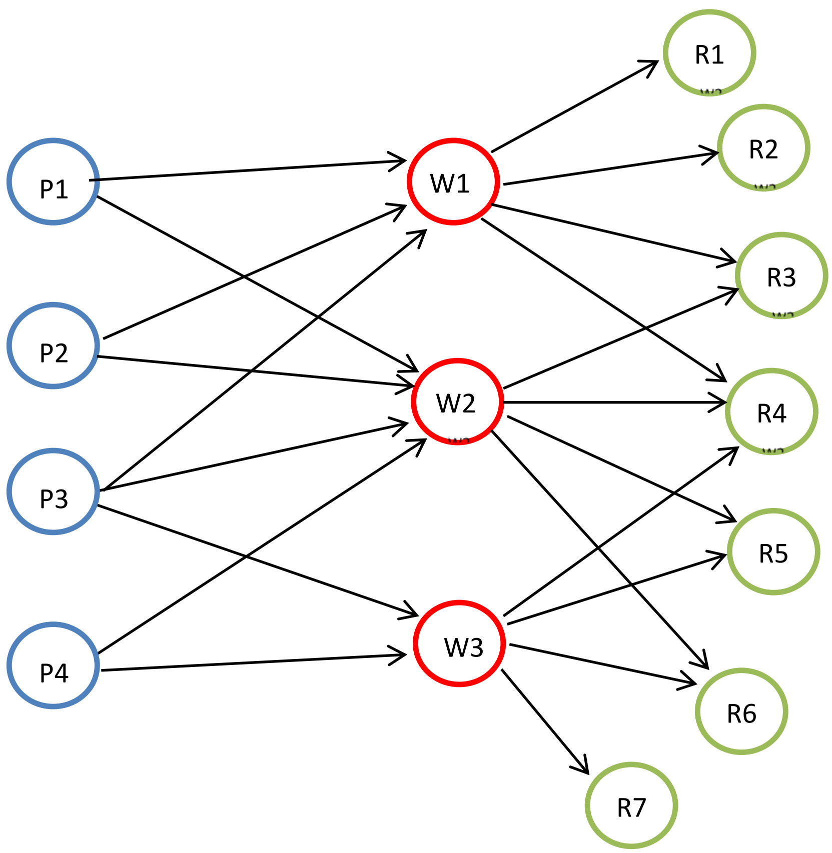
c) What are the optimal numbers of ties of each type to maximize profit?

The optimal number of ties is: 7000 silk ties, 13,625 polyester ties, 13,100 blend 1 ties, and 8,500 blend 2 ties. The maximum profit is $97,253.50. Again, you can check my submitted files for Part B to verify.

3. Transshipment Model **(10 points)**

This is an extension of the transportation model. There are now intermediate transshipment points added between the sources (plants) and destinations (retailers). Items being shipped from a Plant (p_i) must be shipped to a Warehouse (w_j) before being shipped to the Retailer (r_k). Each Plant will have an associated supply (s_i) and each Retailer will have a demand (d_k). The number of plants is n, number of warehouses is q and the number of retailers is m. The edges (i,j) from plant (p_i)to warehouse (w_j) have costs associated denoted cp(i,j). The edges (j,k) from a warehouse (w_j)to a retailer (r_k) have costs associated denoted cw(j,k).

The graph below shows the transshipment map for a manufacturer of refrigerators. Refrigerators are produced at four plants and then shipped to a warehouse (weekly) before going to the retailer.



Below are the costs of shipping from a plant to a warehouse and then a warehouse to a retailer. If it is impossible to ship between the two locations an X is placed in the table.

cost	**W1**	**W2**	**W3**
P1	\$10	\$15	X
P2	\$11	\$8	X
P3	\$13	\$8	\$9
P4	X	\$14	\$8

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cost	R1	R2	R3	R4	R5	R6	R7
W1	$5	$6	$7	$10	X	X	X
W2	X	X	$12	$8	$10	$14	X
W3	X	X	X	$14	$12	$12	$6

The tables below give the capacity of each plant (supply) and the demand for each retailer (per week).

	P1	P2	P3	P4
Supply	150	450	250	150

	R1	R2	R3	R4	R5	R6	R7
Demand	100	150	100	200	200	150	100

Determine the number of refrigerators to be shipped plants to warehouses and then warehouses to retailers to minimize the cost.

a) Formulate the problem as a linear program with an objective function and all constraints.
The objective is to minimize the total shipping costs while still keeping the correct supply and demand (supply and demand will be covered in constraints). So we will be taking the minimum of the cost to deliver from each plant to a warehouse multiplied by the number of refrigerators plus the cost to deliver from a warehouse to each retailer multiplied by the number of refrigerators. So the objective function is:

Min 10*P1W1 + 15*P1W2 + 11*P2W1 + 8*P2W2 + 13*P3W1 + 8*P3W2 + 9*P3W3 + 14*P4W2 + 8*P4W3 + 5 * W1R1 + 6*W1R2 + 7*W1R3 + 10*W1R4 + 12*W2R3 + 8*W2R4 + 10*W2R5 + 14*W2R6 + 14*W3R4 + 12*W3R5 + 12*W3R6 + 6*W3R7

Constraints:
We need to make sure the total number of refrigerators shipped from each plant is ≥ 0:

P1W1 + P2W1 + P3W1 – W1R1 – W1R2 – W1R3 – W1R4 ≥ 0
P1W2 + P2W2 + P3W2 + P4W2 – W2R3 – W2R4 – W2R5 – W2R6 ≥ 0
P3W3 + P4W3 – W3R4 – W3R5 – W3R6 – W3R7 ≥ 0

We also know there's a limited amount of refrigerators available to be shipped from each plant:

P1W1 + P1W2 ≤ 150
P2W1 + P2W2 ≤ 450
P3W1 + P3W2 + P3W3 ≤ 250
P4W2 + P4W3 ≤ 150

Finally, we have to satisfy the demand for each retailer.

W1R1 ≥ 100
W1R2 ≥ 150

W1R3 + W2R3 ≥ 100
W1R4 + W2R4 + W3R4 ≥ 200
W2R5 + W3R5 ≥ 200
W2R6 + W3R6 ≥ 150
W3R7 ≥ 100

b) Determine the optimal solution for the linear program using any software you want. Include a copy of the code and output.
I again used LINDO. Check HW4 Question 3 Part B.txt and HW4 Question 3 Part B Output.txt for the code and the output.

c) What are the optimal shipping routes and minimum cost?
Optimal routes:
150 fridges from P1 to W1
200 fridges from P2 to W1
250 fridges from P2 to W2
150 fridges from P3 to W2
100 fridges from P3 to W3
150 fridges from P4 to W3
100 fridges from W1 to R1
150 fridges from W1 to R2
100 fridges from W1 to R3
200 fridges from W2 to R4
200 fridges from W2 to R5
150 fridges from W3 to R6
100 fridges from W3 to R7

The total minimum cost of all of this shipping is: $17,100.00.