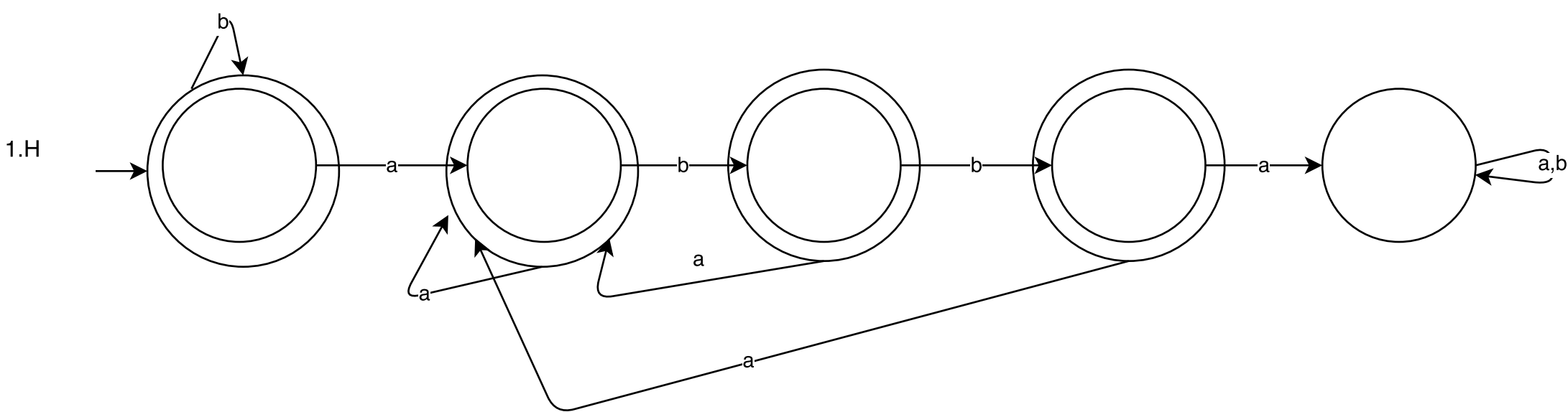
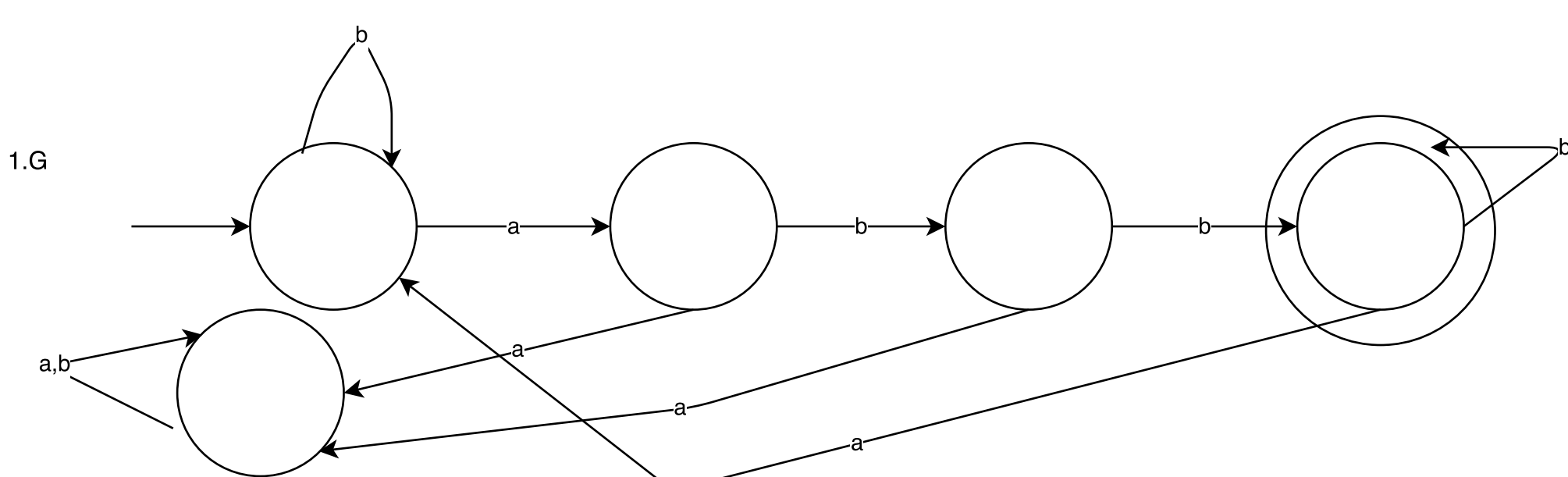
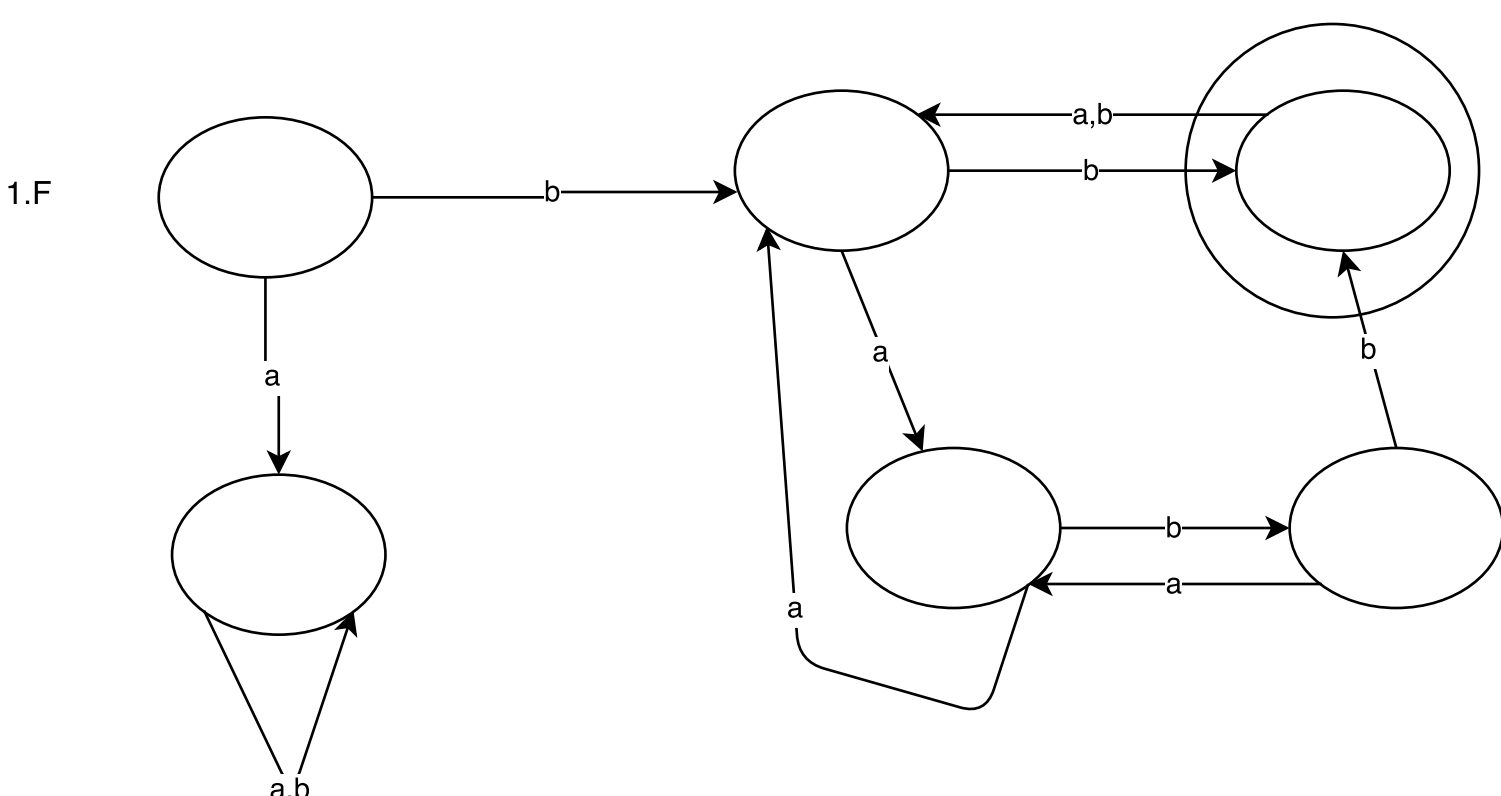
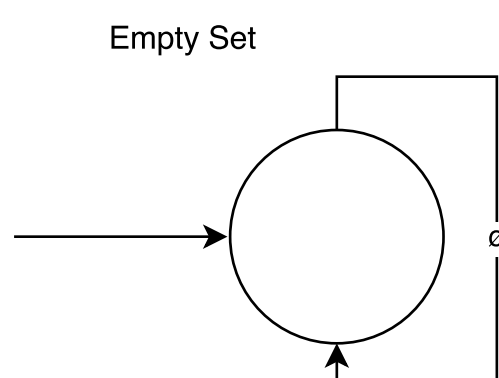


1.E Can't be even and odd.



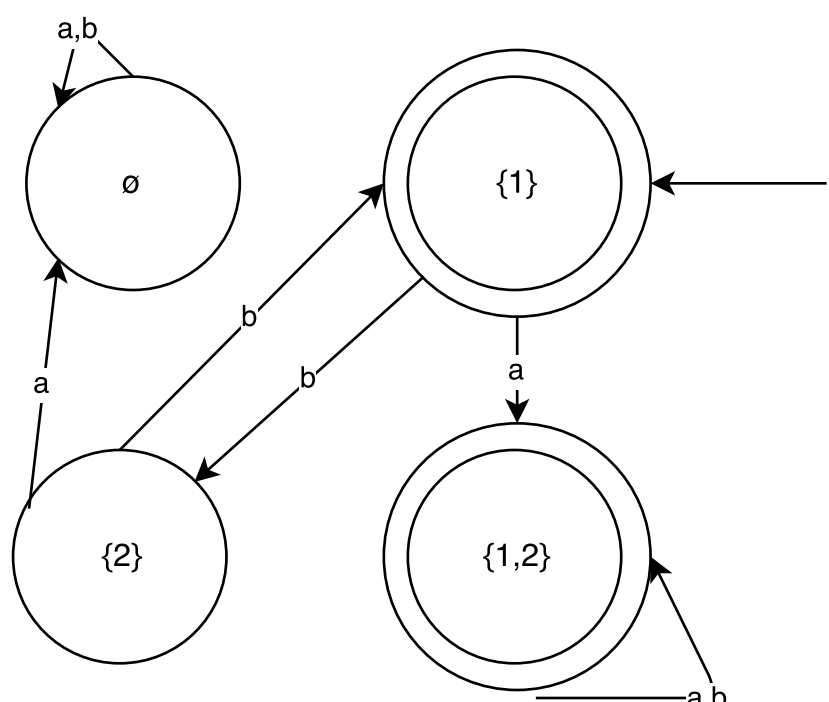
2.
  - Every state M is a set of states of N.
  - DFA transition function equals the union of all states of the NFA transition function.

2.A

NFA states = {1,2}

DFA states thus = { $\emptyset$ , {1}, {2}, {1,2}}

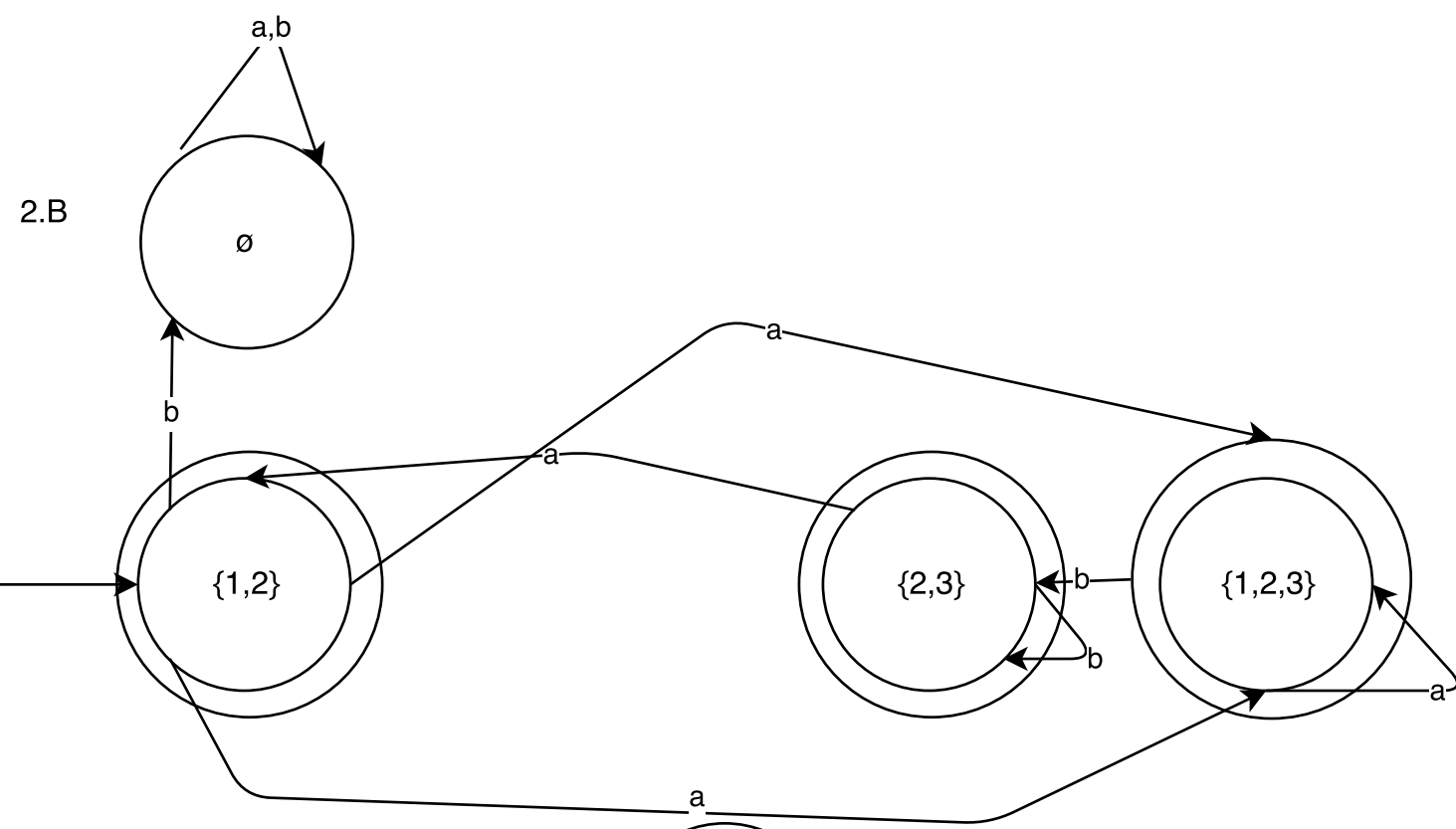
Next determine start and accept states of DFA.



Start state is E({1}), since there are no empty sets the start state remains the same, as 1. New Accept states are {{1}, {1,2}}, since NFA accept state is {1}.

Now we determine transition function for the new DFA. Each state goes from one place for input a, and one for input b.

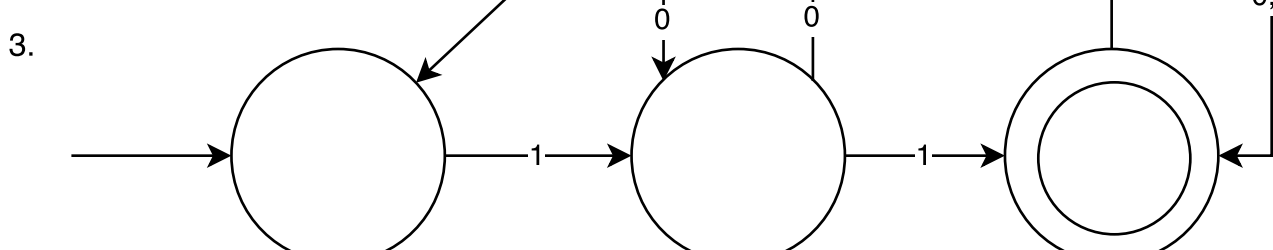
Since 2 has no outgoing a it goes to the empty set. State 2 has an outgoing b that goes to state 1. both State 1 and 2 have incoming a's and b's thus it goes to itself. State 1 has an outgoing a to itself and 2 thus it goes to the state {1,2}. State 1 has an outgoing b to state 2.



NFA states = {1,2,3}

DFA states = { $\emptyset$ , {1}, {2}, {3}, {1,2}, {1,3}, {2,3}, {1,2,3}}

Start state for NFA is {1}, but it has an empty set, thus the start state is {1,2}. Accept state is 2, thus DFA accept states are {{2}, {1,2}, {2,3}, {1,2,3}}.



4. A

The formal definition for a regular language is the following:  
 The empty language  $\emptyset$ , and the empty string  $\{e\}$  are regular languages.  
 For each  $a \in \Sigma$ , the singleton language  $\{a\}$  is a regular language.  
 If A and B are regular languages, then  $A \cup B$ ,  $A \cdot B$  and  $A^*$  are regular languages.  
 - All finite languages are regular.

For any language A, let  $A^R = \{W^R \mid w \in A\}$ . Let  $W = s_1 \dots s_n$  and  $W^R = s_n \dots s_1$

If A is regular there exists a DFA M, such that  $A(M) = A$ .  
 We can show  $A^R$  is regular by constructing a DFA M' that exemplifies the relationship A and  $A^R$  hold, where  $A^R$  accepts all reversed strings that A accepts.

Let  $w^R = a_0 a_1 \dots a_n$  be a string over an alphabet  $A^R$ . The DFA M' accepts the state sequence  $r_0 r_1 \dots r_n$  if w is a string in DFA A.

1. Reverse the arrows of M using the transition function for M's arrows.
2. Make the start state an accepting state.
3. create a start state that points to the old accept states with pointers having empty string.

Now that  $A^R$  has a DFA M', it is a regular language.

4.B

$B(M) = M$ , M is a DFA of B  
 $B^R(M') = B^R$ , M' is a DFA of  $B^R$

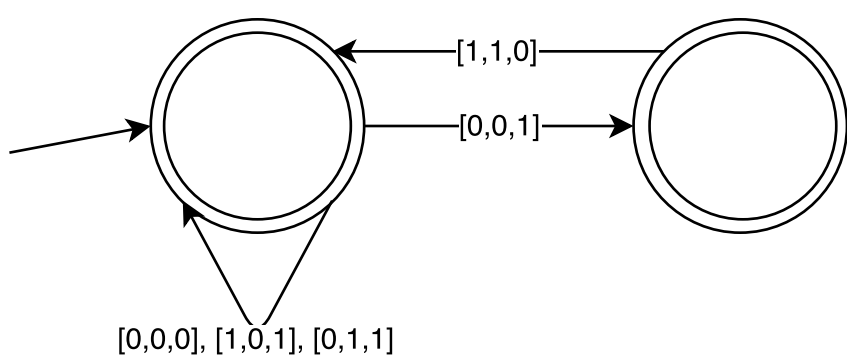
From the prior proof 4.A, we can see that if A is regular,  $A^R$  is regular. Thus, the opposite is true, if  $A^R$  is regular, A is regular.

If  $B^R$  has a DFA M', then it is regular.

In order for M' to be a valid DFA it needs a start state, transition function, and set of accept states.

Using the DFA description of M' in 4.A, we can see that M' will follow the same characteristics.

Now that  $B^R$  has a DFA M', B is a regular language.



Since this NFA Exists we can say that B is regular.