Chapter 1

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Notes

Exercises

Exercise 1.1.

Let $\{M_i\}_{i\in I}$, $\{N_j\}_{j\in J}$ be two families of modules over a ring A. Show that

$$\left(\bigoplus_{i\in I} M_i\right) \otimes_A \left(\bigoplus_{j\in J} N_j\right) \cong \bigoplus_{(i,j)\in I\times J} (M_i \otimes_A N_j).$$

Proof.

Undone.

Exercise 1.2.

Show that there exists a unique A-linear map

$$f : \mathrm{Hom}_A(M, M') \otimes_A \mathrm{Hom}_A(N, N') \to \mathrm{Hom}_A(M \otimes_A N, M' \otimes_A N')$$

such that $f(u \otimes v) = u \otimes v$.

Proof.

Undone.

Exercise 1.6.

Let $(N_i)_{i\in I}$ be a direct system of A-modules. Then for any A-module M, there exists a canonical isomorphism

$$\varinjlim(N_i \otimes_A M) \cong (\varinjlim N_i) \otimes_A M.$$

(Hint: show that $\varinjlim(N_i \otimes_A M)$ verifies the universal property of the tensor product $(\varinjlim N_i) \otimes_A M$.)

Proof.

Undone.

Exercise 1.7.

Let B be an A-algebra, and let M, N be B-modules. Show that there exists a canonical surjective homomorphism

$$M \otimes_A N \to M \otimes_B N.$$

Proof.

Undone.