CubicSpline

Sunday, November 5, 2017 8:40 AM

on the way to finding g_n-s a matrix was constructed

$$
\begin{array}{c}
1 \\ 2 \\ 3 \\ \vdots \\ N-1 \\ N
\end{array}
\begin{bmatrix}
 & & ? & & & \\
\Delta_2 & 2(\Delta_1+\Delta_2) & \Delta_1 & 0 & 0 & \cdots \\
0 & \Delta_3 & 2(\Delta_2+\Delta_3) & \Delta_2 & 0 & \cdots \\
\vdots & & \ddots & \ddots & \ddots & \\
\cdots & & \Delta_{N-1} & 2(\Delta_{N-2}+\Delta_{N-1}) & \Delta_{N-2} & \\
 & & ? & & &
\end{bmatrix}
\begin{Bmatrix} ? \\ \\ \vec{g} \\ \\ ? \end{Bmatrix}
=
\begin{Bmatrix}
? \\
\frac{3\Delta_2}{\Delta_1}(f_1-f_2)+\frac{3\Delta_1}{\Delta_2}(f_3-f_1) \\
\vdots \\
\frac{3\Delta_{N-1}}{\Delta_{N-2}}(f_{N-2}-f_{N-1})+\frac{3\Delta_{N-2}}{\Delta_{N-1}}(f_N-f_{N-1}) \\
?
\end{Bmatrix}
$$

this matrix however, is missing some information for rows 1, and N. we can use eq'n (5) to find these given "free" Boundary conditions

let $f''_{cs}(x_1) = f''_{cs}(x_N) = 0$

observe also that $t(x_1) = \frac{x_1 - x_1}{x_2 - x_1} = 0$

and $t(x_N) = \frac{x_N - x_{N-1}}{x_N - x_{N-1}} = 1$

then equation 5 Becomes

$$x_1:\ 0 = f_1(\underset{-6}{\cancel{12t-6}}) + f_2(\underset{6}{\cancel{6-12t}}) + \Delta_1\left(g_1(\underset{-4}{\cancel{6t-4}}) + g_2(\underset{-2}{\cancel{6t-2}})\right)$$

$$\rightarrow \boxed{-4\Delta_1 g_1 - 2\Delta_1 g_2 = 6f_1 - 6f_2}$$

$$x_{N-1}:\ 0 = f_{N-1}(\underset{6}{\cancel{12t-6}}) + f_N(\underset{-6}{\cancel{6-12t}}) + \Delta_{N-1}\left(g_{N-1}(\underset{2}{\cancel{6t-4}}) + g_N(\underset{4}{\cancel{6t-2}})\right)$$

$$\rightarrow \boxed{2\Delta_{N-1} g_{N-1} + 4\Delta_{N-1} g_N = 6f_N - 6f_{N-1}}$$

we can now fill in the first and last

rows of our matrix, and solve the linear system to obtain $\vec{g}$.

then having found $\vec{g}$, we can construct $\hat{f}cs_n$ for $n \in [1, N]$