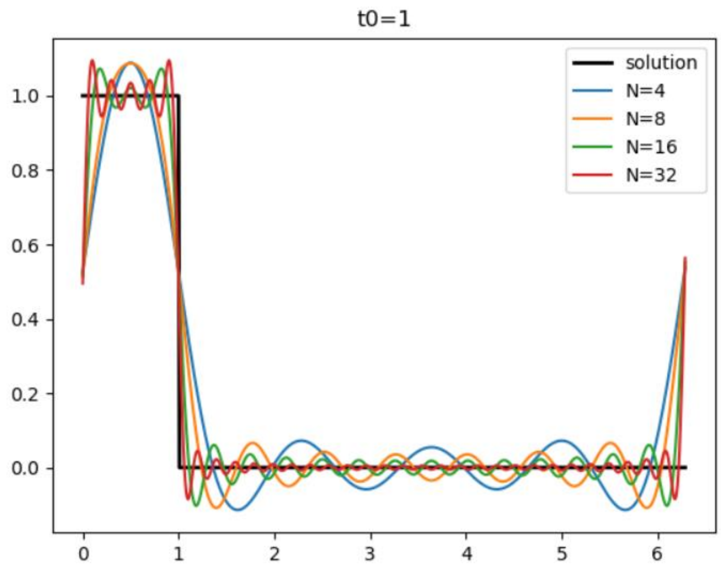
**TYLER GABB**

**HOMEWORK 3**

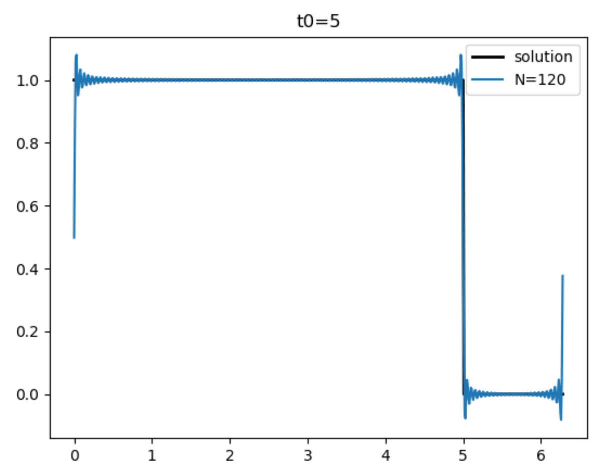
## Q1:

The arithmetic involving the validity of the formula is in the attached file Analysis.pdf.

Below is the plot requested



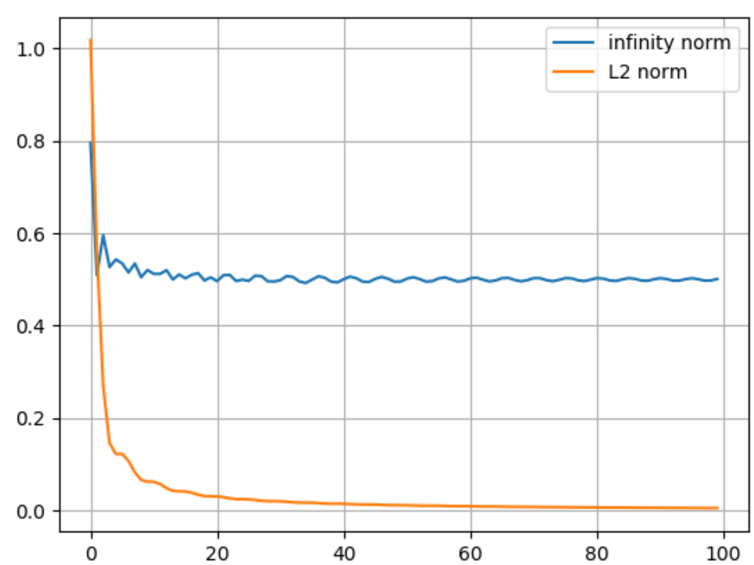

I notice that the solutions are periodic over the interval $2\pi$. End points are tied to $x(n2\pi) = x\big((n+1)2\pi\big) = 0.5$ for some n in the non-negative integers, no matter how many terms were summed to find the solution.




As you can see there is still a bit of overshoot even for N = 120, and it appears that no matter where the discontinuity is, the endpoints of the function are always tied to $x = 0.5$.

## Q2:

To the right is a plot overlaying the infinity norm and L2 norms.

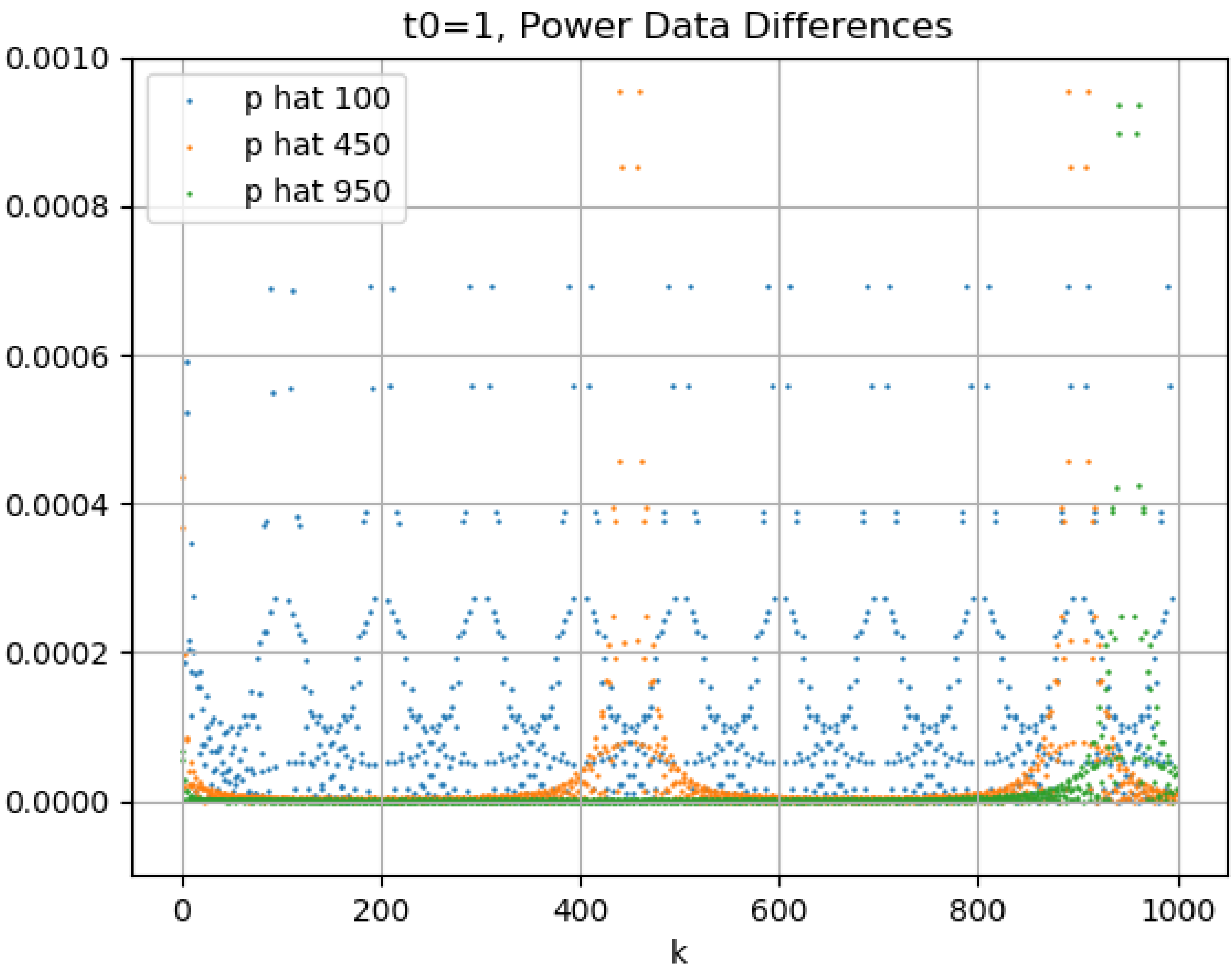



The infinity norm saturates to a value of 0.5 as $N \to \infty$ which makes sense because the maximum of the error will always be 0.5 because of the behavior at the bounds of the interval.

It makes sense that the $L2$ norm would decrease to zero as $N \to \infty$ because it is like the area between the solution and approximated solution in the interval $[0,2\pi]$. As can be seen in the plots from Q1, more terms in the summation lead to a tighter approximation for the solution, which means less area between them.

I really wanted to carry these out analytically, however, I chose the easy route because I did not know where to start. I used Simpson's rule for a given $N$ to approximate the integral for $\|x\|_2$, and just enumerated $|x(t) - x_N(t)|$ for some partition of $[0,2\pi]$ and found the maximum value programmatically for $\|x\|_\infty$

**Q3:**

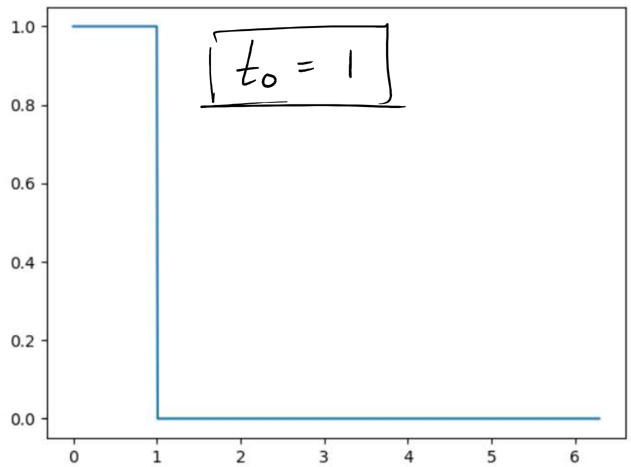



The above plot is a plot of $|\tilde{p} - \hat{p}|$ as a function of $k$. $\hat{p}$ was found for $N$ values 100, 450 and 900. For higher values of $N$ the plot points stayed closer to zero for longer. For smaller values of $N$, the periodicity property can be clearly seen. Smaller values of $N$ also cause the plot points to stay further away from the solution for longer.

# HW 03

Saturday, March 17, 2018 8:15 AM

$$X(t) = \mathbb{1}_{[0,t_0]}$$




Our Fourier coefficients

$$\tilde{X}_K = \frac{1}{T}\int_0^T e^{-ikt} x(t)$$

$$= \frac{1}{2\pi}\int_0^{t_0} e^{-ikt}$$

$$= \frac{1}{2\pi i K}\left(1 - e^{-it_0 K}\right)$$

for $K \neq 0$

$$\boxed{\tilde{X}_0} = \frac{1}{2\pi}\int_0^{t_0} \cancel{e^{-i0t}}\, 1 = \boxed{\frac{t_0}{2\pi}}$$

for $K = 0$

$$X(t) = \sum_{K\in[-N,N]} \tilde{X}_K e^{ikt} = \sum_{K\in[-N,-1]} \tilde{X}_K e^{ikt} + \tilde{X}_0 + \sum_{K\in[1,N]} \tilde{X}_K e^{ikt}$$

$$\sum_{K\in[-N,1]} \underbrace{\frac{1}{2\pi i K}\left(1 - e^{-ikt_0}\right)}_{\tilde{X}_K} e^{ikt} = \sum_{K\in[1,N]} \frac{-1}{2\pi i K}\left(1 - e^{ikt_0}\right) e^{-ikt}$$

take neg side $\longrightarrow$ turn into sum from $1 \to N$

Combine sums $\sum_{K\in S} f(k) + \sum_{K\in S} g(k) = \sum_{K\in S} f(x) + g(k)$

$$\sum_{K\in S}\left(\frac{1}{2\pi i K}\left(1 - e^{-ikt_0}\right) e^{ikt} - \frac{1}{2\pi i K}\left(1 - e^{ikt_0}\right) e^{-ikt}\right) =$$

$$e^{-ikt} \quad e^{ik(t_0-t)}))$$

$k \in S\ (2\pi i k\ ($

$$\frac{1}{2\pi i k}\left(e^{ikt} - e^{ik(t-t_0)} - \left(e^{-ikt} - e^{ik(t_0-t)}\right)\right) =$$

$$\frac{1}{2\pi i k}\left(e^{ikt} - e^{-ikt} - e^{ik(t-t_0)} + e^{-ik(t-t_0)}\right)$$

use euler's formula

$$\cancel{\cos(kt)} + i\sin(kt) - \left(\cancel{\cos(kt)} - i\sin(kt)\right) +$$
$$\cancel{\cos(k(t-t_0))} + i\sin(k(t-t_0)) - \left(\cancel{\cos(k(t-t_0))} - i\sin(k(t-t_0))\right)$$

$$= 2i\sin(kt) + 2i\sin(k(t-t_0))$$

so each term looks like this:

$$\frac{1}{\pi k}\left(\sin(kt) + \sin(k(t-t_0))\right)$$

and the entire solution...

$$X_N(t) = \frac{t_0}{2\pi} + \frac{1}{\pi}\sum_{k \in [1,N]} \sin(kt) + \sin(k(t-t_0))$$