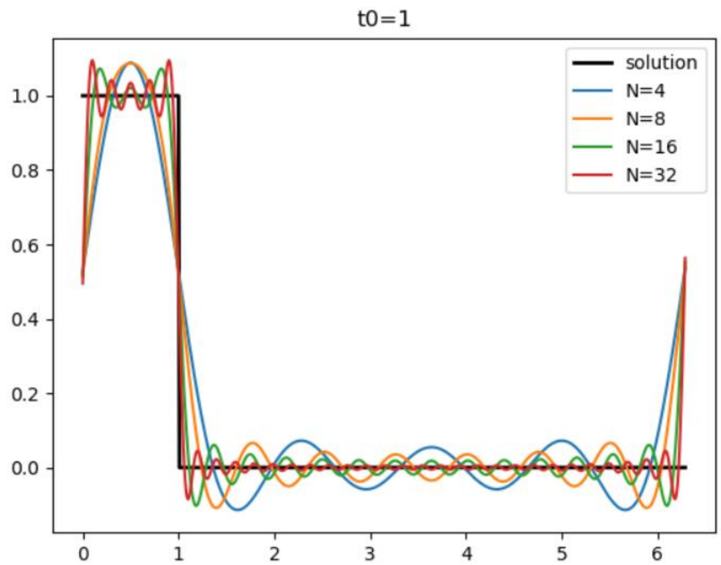
TYLER GABB

HOMEWORK 3

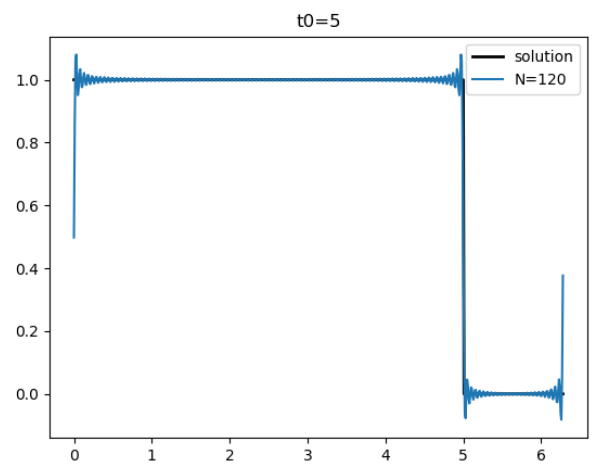
Q1:

The arithmetic involving the validity of the formula is in the attached file Analysis.pdf.

Below is the plot requested



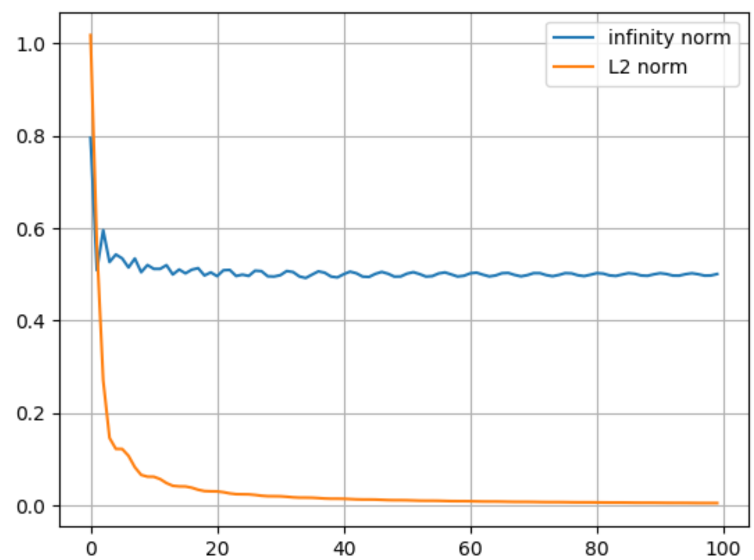
I notice that the solutions are periodic over the interval 2π. End points are tied to $x(n2\pi) = x\big((n+1)2\pi\big) = 0.5$ for some n in the non-negative integers, no matter how many terms were summed to find the solution.



As you can see there is still a bit of overshoot even for N = 120, and it appears that no matter where the discontinuity is, the endpoints of the function are always tied to $x = 0.5$.

Q2:

To the right is a plot overlaying the infinity norm and L2 norms.

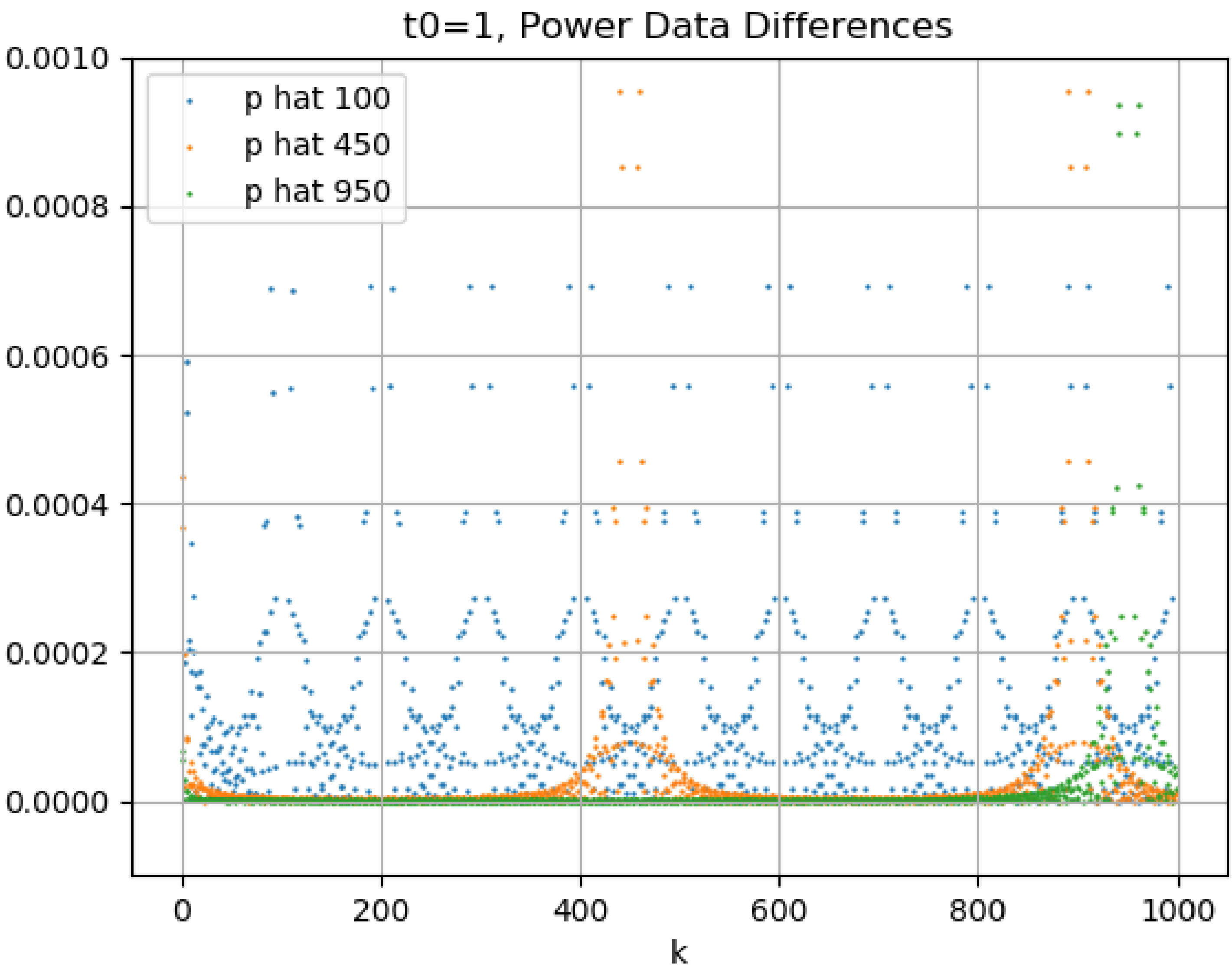


The infinity norm saturates to a value of 0.5 as $N \to \infty$ which makes sense because the maximum of the error will always be 0.5 because of the behavior at the bounds of the interval.

It makes sense that the $L2$ norm would decrease to zero as $N \to \infty$ because it is like the area between the solution and approximated solution in the interval $[0,2\pi]$. As can be seen in the plots from Q1, more terms in the summation lead to a tighter approximation for the solution, which means less area between them.

I really wanted to carry these out analytically, however, I chose the easy route because I did not know where to start. I used Simpson's rule for a given N to approximate the integral for $\|x\|_2$, and just enumerated $|x(t) - x_N(t)|$ for some partition of $[0,2\pi]$ and found the maximum value programmatically for $\|x\|_\infty$

Q3:



The above plot is a plot of $|\tilde{p} - \hat{p}|$ as a function of k. $\hat{p}$ was found for N values 100, 450 and 900. For higher values of N the plot points stayed closer to zero for longer. For smaller values of N, the periodicity property can be clearly seen. Smaller values of N also cause the plot points to stay further away from the solution for longer.