Math475a, Fall 2017. Assignment 4. Due October 8th

Recall that Lagrange interpolating polynomial for a set of N points, (x_n, f_n), $n = 1, \ldots, N$, is given by

$$P^{(N-1)}(x) = \sum_{n=1}^{N} f_n P_n^{(N-1)}(x), \quad \text{where} \quad P_n^{(N-1)}(x) = \prod_{\substack{k=1 \\ k \neq n}}^{N} \frac{x - x_k}{x_n - x_k}.$$

Write a code that implements polynomial interpolation and run the tests described below.

Different Orders. Let $f(x) = \arctan(x)$. Construct polynomial approximations to $f(x)$ for $x \in [-10, 10]$. Use $N = 2, 4, 8, 16, 32$: distribute your N sample points in $[-10, 10]$ uniformly, i.e, setting (for every N)

$$x_n = -10 + 20(n-1)/(N-1), \quad f_n = f(x_n); \quad n = 1, \ldots, N.$$

Plot the graphs of the original function and the approximations that you constructed in the same graph for $x \in [-11, 11]$. Discuss your observations.

Different Point Locations. Consider the same function $f(x) = \arctan(x)$. Suppose we just want to use a fifth order polynomial (i.e., $N = 6$). Is there a way to place the sample points x_n, $n = 1, \ldots, 6$ in some optimal way? Try a few possibilities (one of them is the uniform sampling as described above); look at the errors in max-norm, i.e.,

$$\max_{x \in [-10,10]} |f(x) - \hat{f}(x)|,$$

where $\hat{f}(x)$ is the polynomial approximation that you constructed. Estimate the max-norm error numerically by sampling the value of $|f(x) - \hat{f}(x)|$ at a sufficiently fine resolution (mesh size about 0.1 should suffice). Plot the graphs of your approximations. Discuss your findings.