Math475a, Fall 2017. Assignment 1. Due September 7

Problem 1: Error estimate for Midpoint Rule. We can approximate a definite integral of a function using the so-called *mid-point rule,*

$$\int_a^b f(x)\,\mathrm{d}x \approx \Delta \sum_{n=1}^{N} f(a+(n-1/2)\Delta),$$

where $\Delta = (b-a)/N$, N is the number of (equal) partitions of the interval $[a,b]$. Assume that $f \in \mathbf{C}^1[a,b]$. Derive the best bound for the approximation error that you can, i.e., find the smallest possible C in the inequality,

$$\left| \int_a^b f(x)\,\mathrm{d}x \;-\; \Delta \sum_{n=1}^{N} f(a+(n-1/2)\Delta) \right| \le C.$$

Note that your estimate should be sufficiently good to demonstrate that the error tends to zero as $\Delta \to 0$ (or as $N \to \infty$), otherwise the midpoint method would not be suitable for computing integrals.

Hint: split the integration interval $[a,b]$ into N sub-intervals, $[a+\Delta(n-1), a+\Delta n]$ and show that for each sub-interval,

$$\left| \int_{a+\Delta(n-1)}^{a+\Delta n} f(x)\,\mathrm{d}x - f(a+(n-1/2)\Delta)\Delta \right| \le \frac{M_1 \Delta^2}{2},$$

where $M_1 = \max_{x\in[a,b]} |f'(x)|$.

Extra: Can you improve the error estimate if you assume that $f \in \mathbf{C}^2[a,b]$?

Problem 2: Binary representation of numbers. Write a code that reads (line by line) real numbers from a text file, e.g., *input.txt*, and writes strings (also one per line) with their binary representation into another file, e.g., *output.txt*. For example, if *input.txt* looks like this

3
109
-4.7

your code should produce

11
1101101
-100.1011001100...

If the fractional part is infinite, truncate it, e.g., after ten digits. Submit both files (make up 5-6 your own numbers to convert to binary).