Math475a, Fall 2017. Assignment 6. Due December 6th

1. The Problem

Denote $F_1(x) = |x - \pi/10|$; $F_2(x) = (x-1)^2$; $F_3(x) = e^x$. In this problem your task is to study the rates of convergence of numerical approximations to

$$\int_{-1}^{1} F_k(x)\,dx \approx \cdots$$

Split the interval $[-1, 1]$ into N equal parts, let $x_n = -1 + 2n/N$, and consider the following three methods:

$$\cdots \approx \frac{2}{N} \sum_{n=1}^{N} F_k(x_n) \qquad \text{(right sums);} \tag{1a}$$

$$\cdots \approx \frac{1}{N} \sum_{n=1}^{N} \big[F_k(x_{n-1}) + F_k(x_n)\big] \qquad \text{(trapezoidal rule);} \tag{1b}$$

$$\cdots \approx \frac{1}{3N} \sum_{n=1}^{N} \big[F_k(x_{n-1}) + 4F_k\big((x_{n-1} + x_n)/2\big) + F_k(x_n)\big] \qquad \text{(Simpson's method).} \tag{1c}$$

Use all three methods for each of the three functions and study (plot or tabulate) the approximation errors for various values of N (between, e.g., 10 and 10^6). How fast do the errors tend to zero (linearly, quadratically, etc) with respect to N? Explain the results.

2. Convergence for periodic functions.

Now consider

$$\int_{-\pi}^{\pi} \sin^2 x\,dx.$$

Compare convergence rates of the same three methods for this problem. What do you observe?