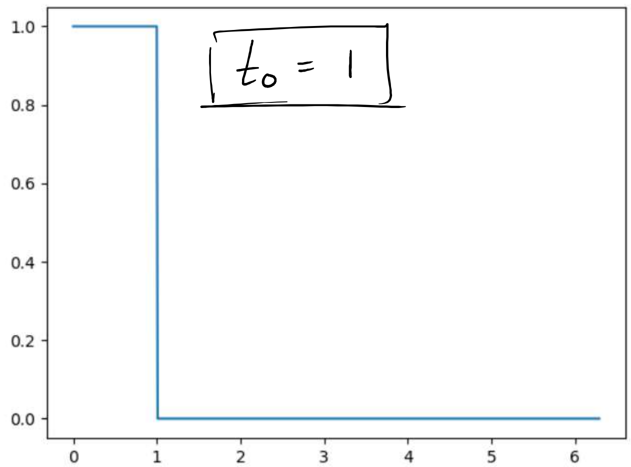
HW 03

Saturday, March 17, 2018 8:15 AM

$$X(t) = \mathbb{1}_{[0,t_0]}$$



our Fourier coefficients

$$\tilde{X}_K = \frac{1}{T}\int_0^T e^{-ikt} x(t)$$

$$= \frac{1}{2\pi}\int_0^{t_0} e^{-ikt}$$

$$= \frac{1}{2\pi i K}\left(1 - e^{-it_0 K}\right)$$

for $K \neq 0$

$$\boxed{\tilde{X}_0} = \frac{1}{2\pi}\int_0^{t_0} \cancel{e^{-i0t}}\, 1 = \boxed{\frac{t_0}{2\pi}}$$

for $K = 0$

$$X(t) = \sum_{K\in[-N,N]} \tilde{X}_K e^{ikt} = \sum_{K\in[-N,-1]} \tilde{X}_K e^{ikt} + \tilde{X}_0 + \sum_{K\in[1,N]} \tilde{X}_K e^{ikt}$$

$$\sum_{K\in[-N,1]} \underbrace{\frac{1}{2\pi i K}\left(1 - e^{-ikt_0}\right)}_{\tilde{X}_K} e^{ikt} = \sum_{K\in[1,N]} \frac{-1}{2\pi i K}\left(1 - e^{ikt_0}\right)e^{-ikt}$$

take neg side ⟶ turn into sum from $1 \to N$

Combine sums $\sum_{K\in S} f(k) + \sum_{K\in S} g(k) = \sum_{K\in S} f(x) + g(k)$

$$\sum_{K\in S}\left(\frac{1}{2\pi i K}\left(1 - e^{-ikt_0}\right)e^{ikt} - \frac{1}{2\pi i K}\left(1 - e^{ikt_0}\right)e^{-ikt}\right) =$$

$$e^{-ikt} \quad e^{ik(t_0 - t)}))$$

$k \in S \ (2\pi i k \ ($

$$\frac{1}{2\pi i k}\left(e^{ikt} - e^{ik(t-t_0)} - \left(e^{-ikt} - e^{ik(t_0-t)}\right)\right) =$$

$$\frac{1}{2\pi i k}\left(e^{ikt} - e^{-ikt} - e^{ik(t-t_0)} + e^{-ik(t-t_0)}\right)$$

use euler's formula

$$\cancel{\cos(kt)} + i\sin(kt) - \left(\cancel{\cos(kt)} - i\sin(kt)\right) +$$
$$\cancel{\cos(k(t-t_0))} + i\sin(k(t-t_0)) - \left(\cancel{\cos(k(t-t_0))} - i\sin(k(t-t_0))\right)$$

$$= 2i\sin(kt) + 2i\sin(k(t-t_0))$$

so each term looks like this:

$$\frac{1}{\pi k}\left(\sin(kt) + \sin(k(t-t_0))\right)$$

and the entire solution...

$$X_N(t) = \frac{t_0}{2\pi} + \frac{1}{\pi}\sum_{k\in[1,N]} \sin(kt) + \sin(k(t-t_0))$$