



DeepLearning.AI

Math for Machine Learning

Probability and Statistics

W1 Lesson 1

Introduction to Probability



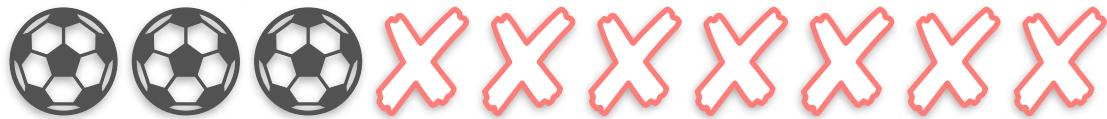
DeepLearning.AI

Introduction to probability

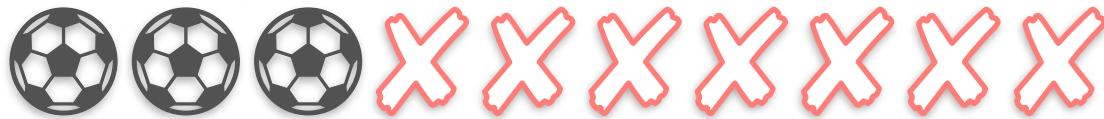
What is Probability?

Introduction to Probability

Introduction to Probability

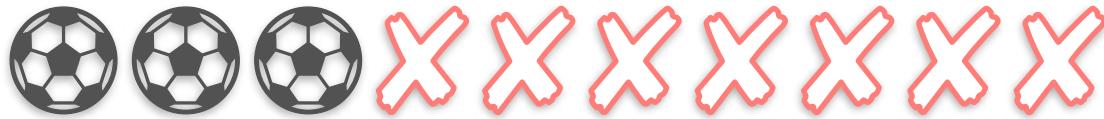


Introduction to Probability



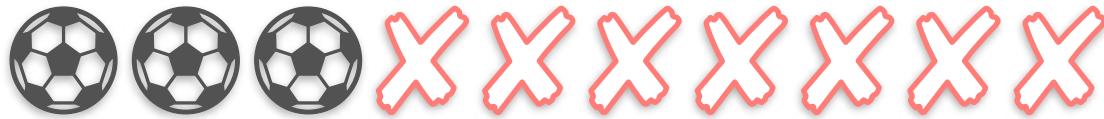
Find the probability that a child picked at random plays soccer.

Introduction to Probability



Find the probability that a child picked at random plays soccer.

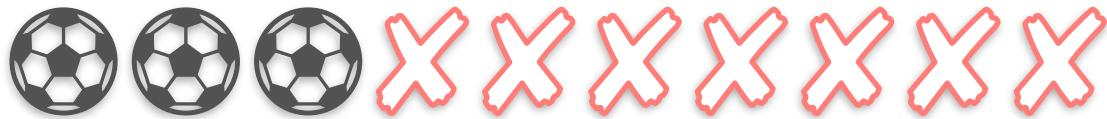
Introduction to Probability



Find the probability that a child picked at random plays soccer.

The probability that a child picked at random plays soccer.

Introduction to Probability

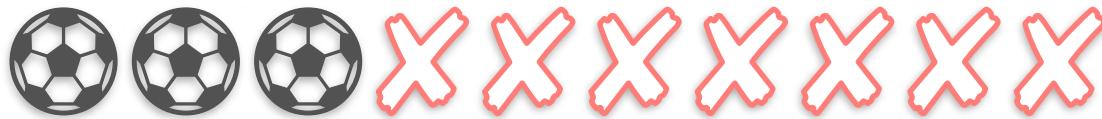


Find the probability that a child picked at random plays soccer.

The probability that a child picked at random plays soccer.

$$P(\text{soccer})$$

Introduction to Probability



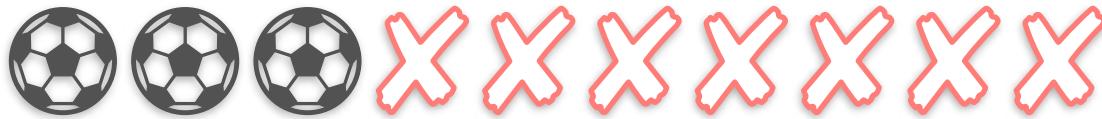
Find the probability that a child picked at random plays soccer.

The probability that a child picked at random plays soccer.

$$P(\text{soccer})$$

A teal curved arrow points from the text "The probability that a child picked at random plays soccer." down to the mathematical expression $P(\text{soccer})$.

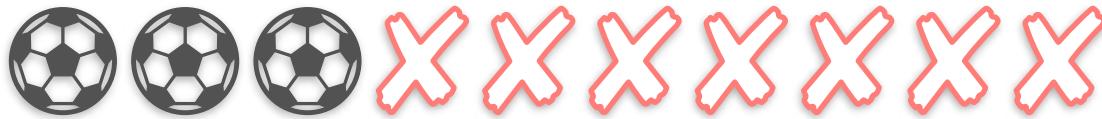
Introduction to Probability



Find the probability that a child picked at random plays soccer.

$$P(\text{soccer})$$

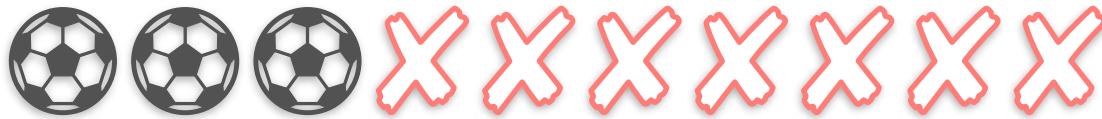
Introduction to Probability



Find the probability that a child picked at random plays soccer.

$$P(\text{soccer}) = \frac{\text{soccer}}{\text{total}}$$

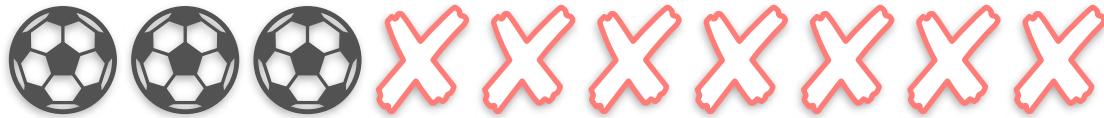
Introduction to Probability



Find the probability that a child picked at random plays soccer.

$$P(\text{soccer}) = \frac{\text{soccer}}{\text{total}} = \underline{\hspace{2cm}}$$

Introduction to Probability

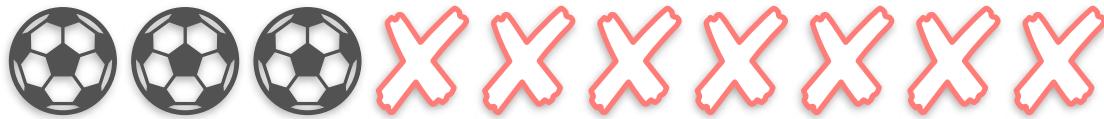


Find the probability that a child picked at random plays soccer.

$$P(\text{soccer}) = \frac{\text{soccer}}{\text{total}} = \frac{3}{10}$$



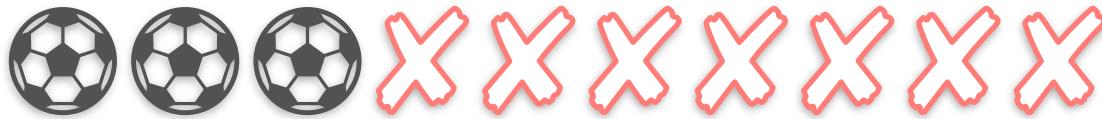
Introduction to Probability



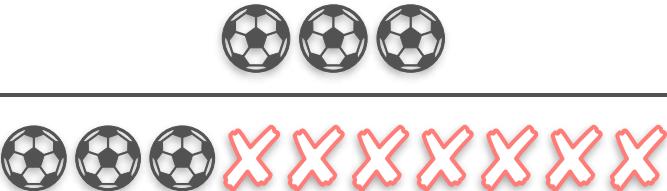
Find the probability that a child picked at random plays soccer.

$$P(\text{soccer}) = \frac{\text{soccer}}{\text{total}} = \frac{\text{soccer balls}}{\text{total items}}$$
The equation shows the probability of picking a soccer ball as a fraction. The numerator is represented by three black soccer balls. The denominator is represented by a horizontal line above which are three black soccer balls, and below which is a sequence of three black soccer balls followed by seven red 'X' marks.

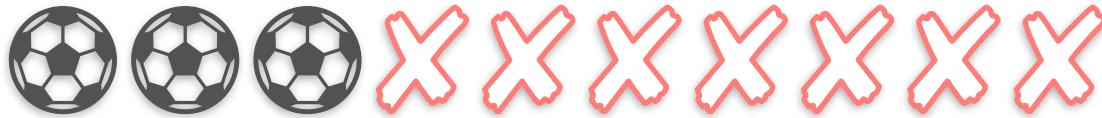
Introduction to Probability



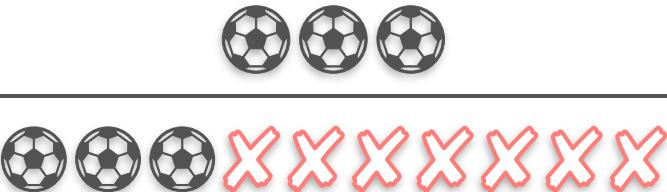
Find the probability that a child picked at random plays soccer.

$$P(\text{soccer}) = \frac{\text{soccer}}{\text{total}} = \frac{3}{10}$$
The fraction is displayed with a horizontal line separating the numerator from the denominator. The numerator is represented by three solid black soccer balls. The denominator is represented by a sequence of ten items: three solid black soccer balls followed by seven red 'X' marks.

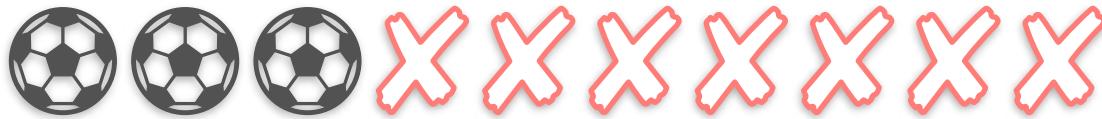
Introduction to Probability



Find the probability that a child picked at random plays soccer.

$$P(\text{soccer}) = \frac{\text{soccer}}{\text{total}} = \frac{3}{10} = 0.3$$
The fraction is displayed with a horizontal line separating the numerator from the denominator. The numerator is represented by three black soccer balls. The denominator is represented by a sequence of 10 items: three black soccer balls followed by seven red 'X' marks.

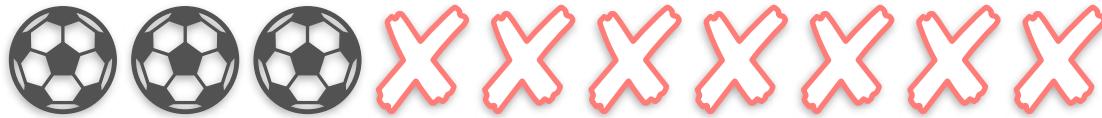
Introduction to Probability



Find the probability that a child picked at random plays soccer.

$$P(\text{soccer}) = \frac{\text{soccer}}{\text{total}} = \frac{3}{10} = 0.3$$
The equation shows the probability of picking a soccer ball (soccer) over the total number of items (total). The numerator is represented by a teal box containing the first three items from the sequence above. The denominator is represented by a teal box containing the first three items from the sequence below. Both sequences consist of three soccer balls followed by seven red 'X' marks.

Introduction to Probability



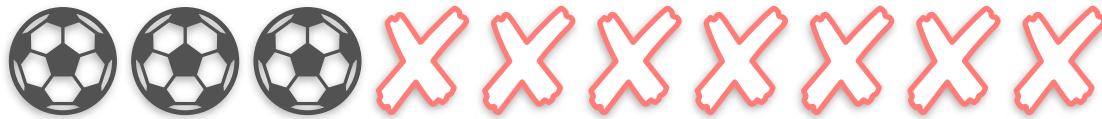
Find the probability that a child picked at random plays soccer.

Event

$$P(\text{soccer}) = \frac{\text{soccer}}{\text{total}} = \frac{\text{Number of soccer balls}}{\text{Total number of items}} = \frac{3}{10} = 0.3$$

The diagram illustrates the calculation of the probability of picking a soccer ball. At the top, a teal box highlights the first three items in the sequence: three solid black soccer balls. Below the sequence, another teal box highlights the first three items again, corresponding to the numerator in the probability formula. The denominator is represented by the total length of the sequence, which includes all ten items (three soccer balls and seven 'X' marks).

Introduction to Probability



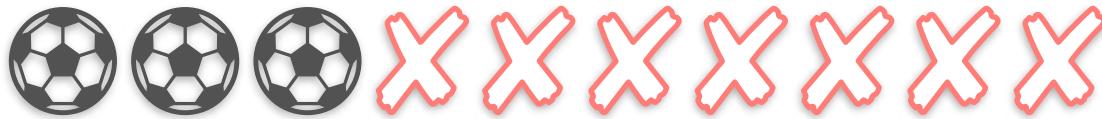
Find the probability that a child picked at random plays soccer.

$$P(\text{soccer}) = \frac{\text{soccer}}{\text{total}} = \frac{\text{Event}}{\text{Total}} = \frac{3}{10} = 0.3$$

The equation shows the probability of picking a soccer ball from a total of 10 items. The term "Event" is highlighted with a teal box and an arrow points to the first three items in the sequence, which are all soccer balls.

The diagram illustrates the probability calculation. A teal box labeled "Event" points to the first three items in the sequence, which are all soccer balls. This visualizes the numerator of the probability fraction. The denominator is represented by the full sequence of 10 items below the fraction line.

Introduction to Probability

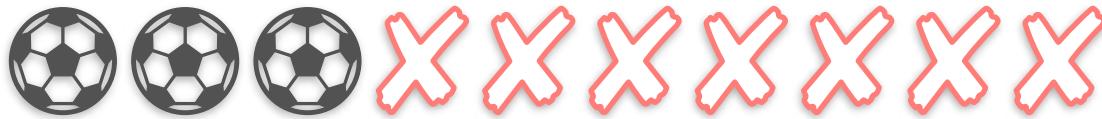


Find the probability that a child picked at random plays soccer.

$$P(\text{soccer}) = \frac{\text{soccer}}{\text{total}} = \frac{\text{Event}}{\text{Total Population}} = \frac{3}{10} = 0.3$$

The diagram illustrates the calculation of probability. A teal bracket labeled "Event" encloses the 3 soccer balls. Another teal bracket labeled "Total Population" encloses all 10 items (3 soccer balls + 7 X's). An arrow points from the word "Event" to the top of the bracket enclosing the soccer balls.

Introduction to Probability

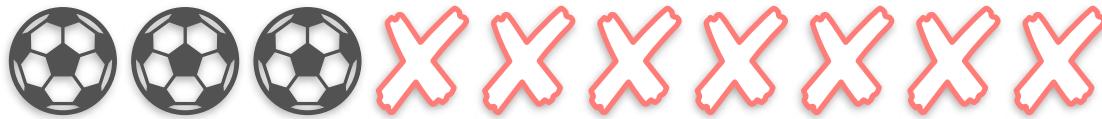


Find the probability that a child picked at random plays soccer.

$$P(\text{soccer}) = \frac{\text{soccer}}{\text{total}} = \frac{\text{Event}}{\text{Sample space}} = \frac{3}{10} = 0.3$$

The diagram illustrates the calculation of probability. A teal bracket labeled "Event" encloses the three soccer balls. Another teal bracket labeled "Sample space" encloses all ten items (three soccer balls and seven 'X' marks).

Introduction to Probability



Find the probability that a child picked at random plays soccer.

$$P(\text{soccer}) = \frac{\text{soccer}}{\text{total}} = \frac{\text{Event}}{\text{Sample space}} = \frac{3}{10} = 0.3$$

The diagram illustrates the calculation of probability. A horizontal bar represents the 'Sample space' containing 10 items: 3 soccer balls and 7 red 'X' marks. Above this bar, a smaller horizontal bar represents the 'Event' containing 3 soccer balls. Arrows point from the labels 'Event' and 'Sample space' to their respective bars.

Introduction to Probability: Venn Diagram

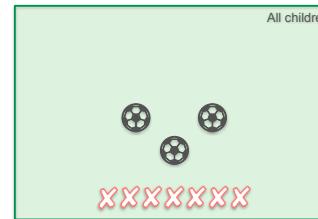
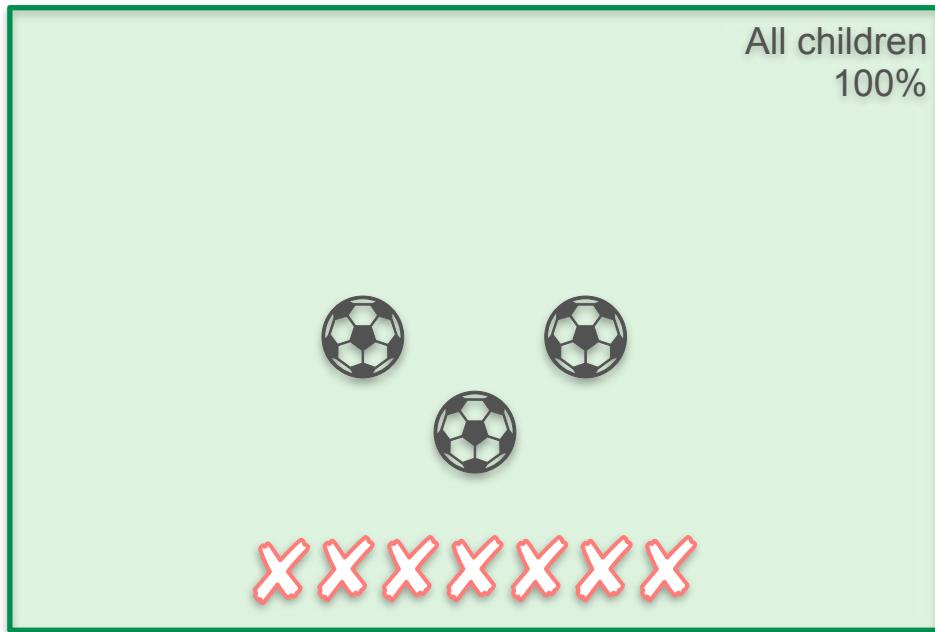


XXXXXX

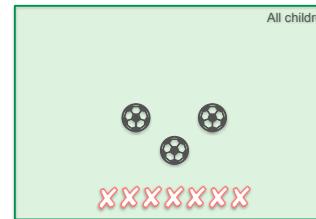
Introduction to Probability: Venn Diagram



Introduction to Probability: Venn Diagram

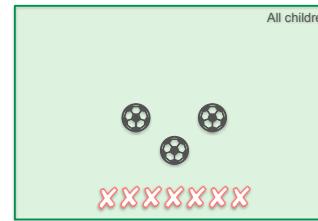
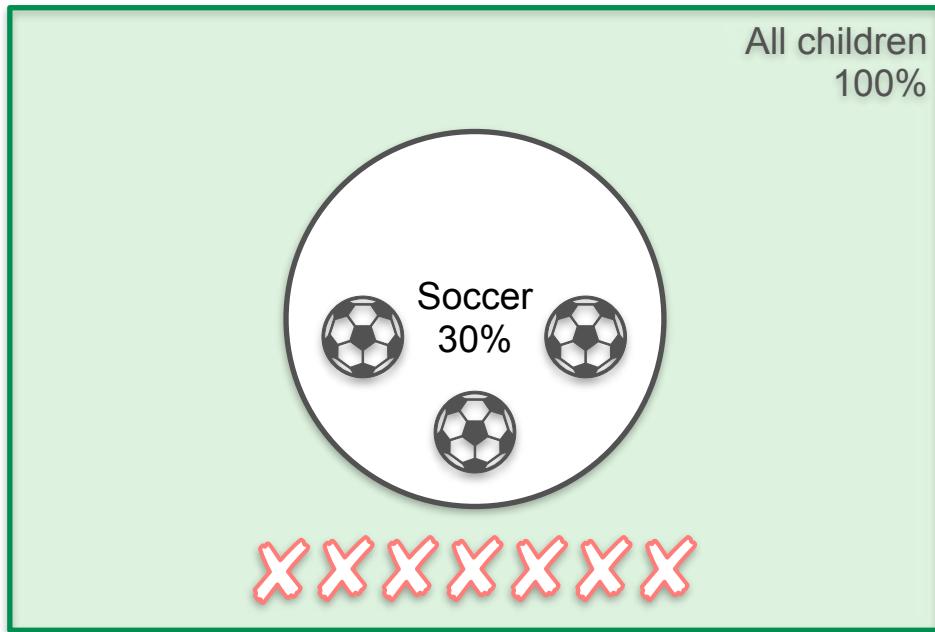


Introduction to Probability: Venn Diagram



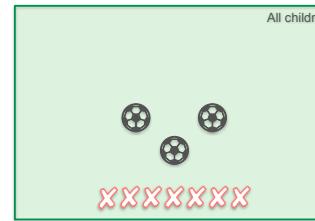
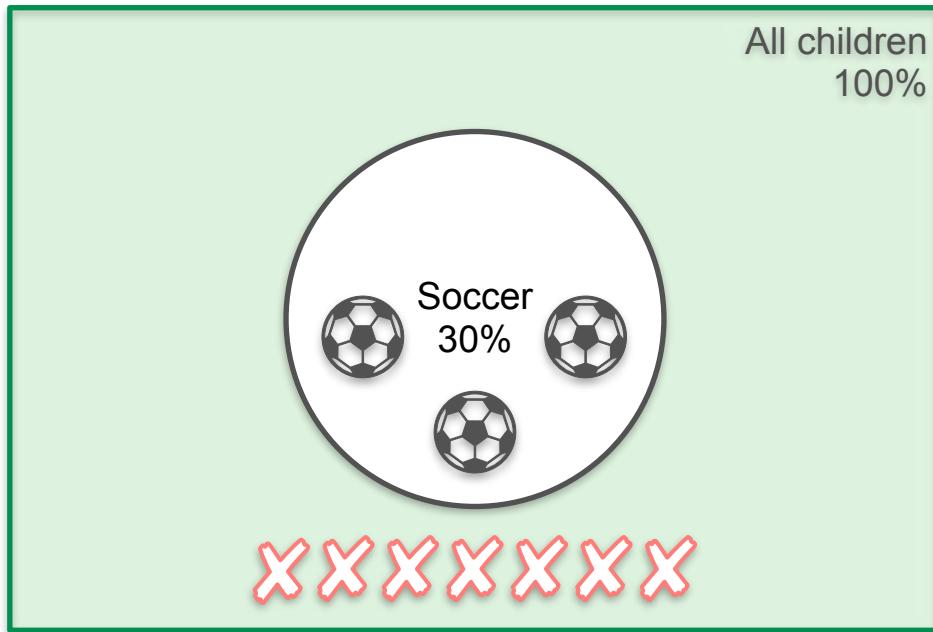
Sample Space

Introduction to Probability: Venn Diagram



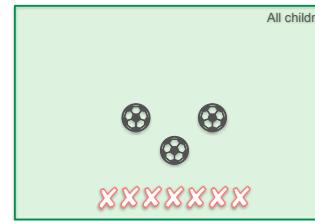
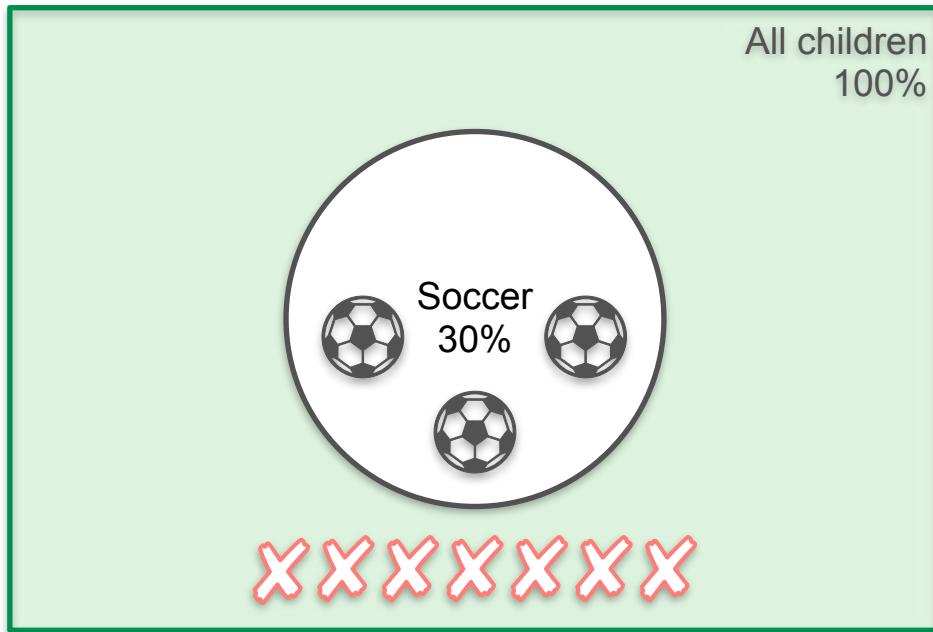
Sample Space

Introduction to Probability: Venn Diagram

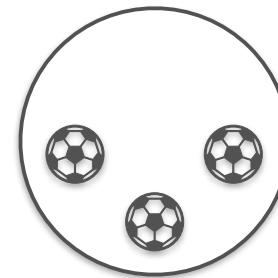


Sample Space

Introduction to Probability: Venn Diagram



Sample Space



Event

Introduction to Probability: Coin Example 1



Introduction to Probability: Coin Example 1



Introduction to Probability: Coin Example 1



Experiment

Introduction to Probability: Coin Example 1



Experiment

Probability of landing on heads

Introduction to Probability: Coin Example 1



Experiment

Probability of landing on heads

$$P(\text{heads})$$

Introduction to Probability: Coin Example 1

Introduction to Probability: Coin Example 1



Introduction to Probability: Coin Example 1



50%

Introduction to Probability: Coin Example 1



50% 50%

Introduction to Probability: Coin Example 1



$$P(\text{heads}) = \underline{\hspace{2cm}}$$

Introduction to Probability: Coin Example 1



50% 50%

$$P(\text{heads}) = \frac{\text{ }}{\text{ }}$$

Introduction to Probability: Coin Example 1



$$P(\text{heads}) = \frac{\text{H}}{\text{H} \quad \text{T}}$$

Introduction to Probability: Coin Example 1



$$P(\text{heads}) = \frac{\text{Number of heads}}{\text{Total number of outcomes}} = \frac{1}{2} = 0.5$$
A fraction is shown with 'P(heads)' as the numerator. The numerator is represented by a single gold coin with 'H' (heads) facing up. The denominator is represented by two gold coins, one with 'H' (heads) and one with 'T' (tails) facing up. A horizontal line separates the numerator from the denominator.

Introduction to Probability: Coin Example 2



50% 50%

Introduction to Probability: Coin Example 2



50% 50%

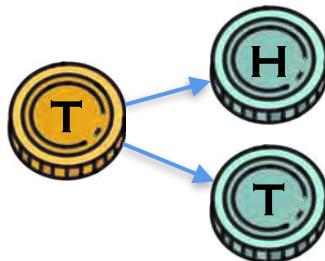
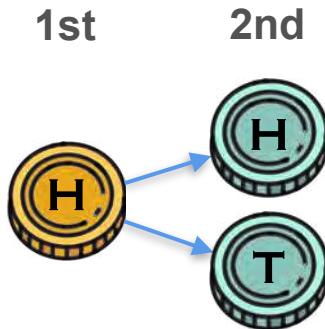
1st



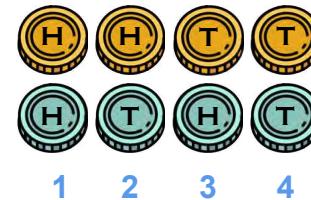
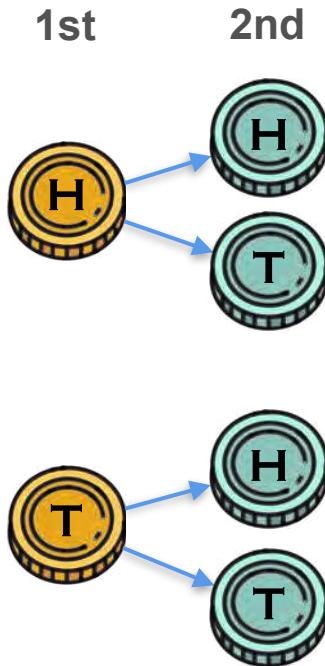
Introduction to Probability: Coin Example 2



50% 50%



Introduction to Probability: Coin Example 2

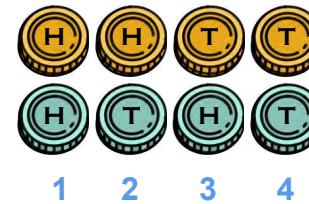
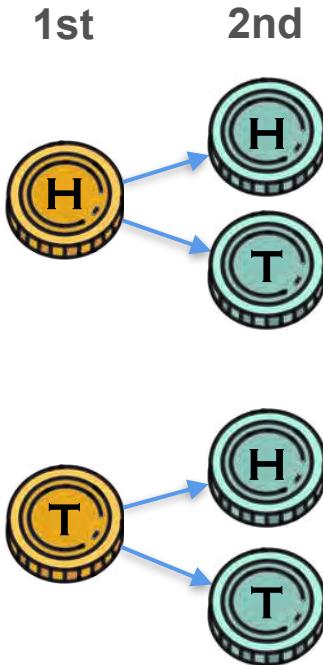


Introduction to Probability: Coin Example 2



50% 50%

What is the probability of landing on heads twice?



Introduction to Probability: Coin Example 2



50% 50%



Introduction to Probability: Coin Example 2



50% 50%

$$P(HH) = \underline{\hspace{2cm}}$$

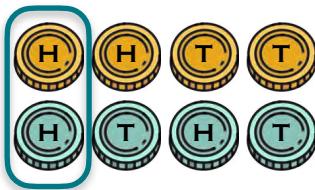


Introduction to Probability: Coin Example 2



50% 50%

$$P(HH) = \underline{\hspace{2cm}}$$



Introduction to Probability: Coin Example 2



50% 50%



$$P(HH) = \underline{\hspace{2cm}}$$



Introduction to Probability: Coin Example 2



50% 50%



$$P(HH) = \frac{\text{Number of HH outcomes}}{\text{Total number of outcomes}}$$

The equation $P(HH) = \frac{\text{Number of HH outcomes}}{\text{Total number of outcomes}}$ is displayed. The numerator is represented by a stack of two coins, one yellow (H) on top of one blue (H). The denominator is represented by a 2x4 grid of eight coins, with four yellow (H) and four blue (T) coins distributed across the four columns.

Introduction to Probability: Coin Example 2



50% 50%



$$P(HH) = \frac{1}{4} = 0.25$$
A 2x4 grid of eight coins representing all possible outcomes of two coin flips. The top row contains two yellow coins with 'H' and two yellow coins with 'T'. The bottom row contains two teal coins with 'H' and two teal coins with 'T'. This visualizes the sample space for the event of getting two heads (HH).

Introduction to Probability: Coin Example 3



50% 50%

Introduction to Probability: Coin Example 3



50% 50%

1st

Introduction to Probability: Coin Example 3



50% 50%

1st



Introduction to Probability: Coin Example 3



50% 50%

1st 2nd

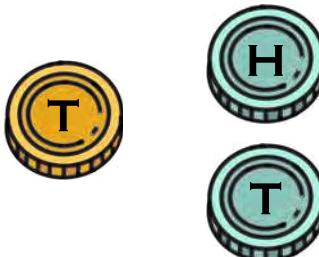
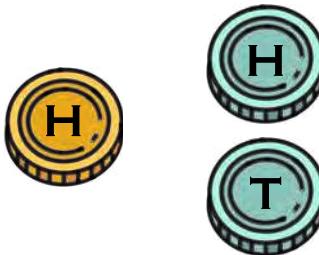


Introduction to Probability: Coin Example 3



50% 50%

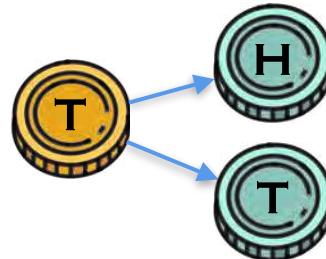
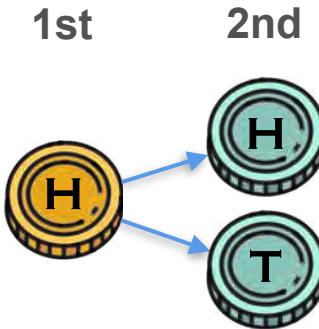
1st 2nd



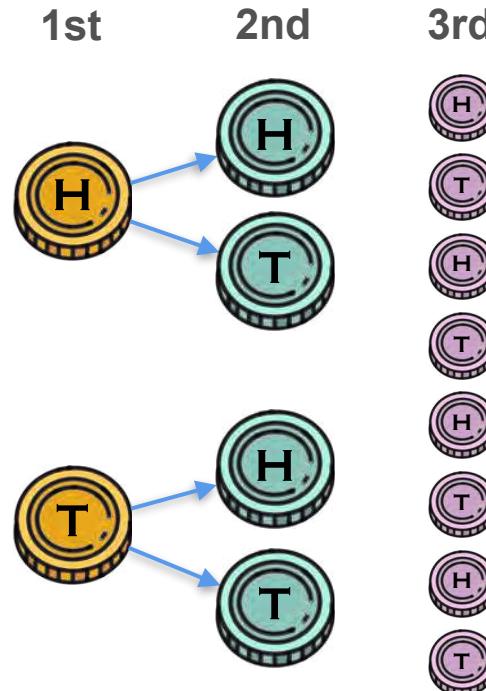
Introduction to Probability: Coin Example 3



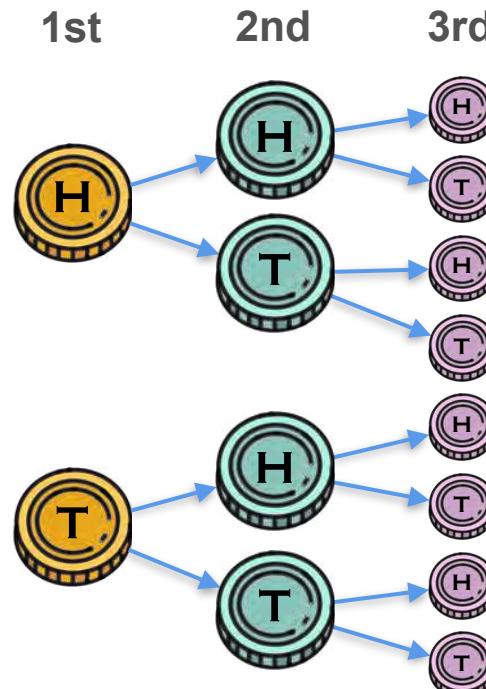
50% 50%



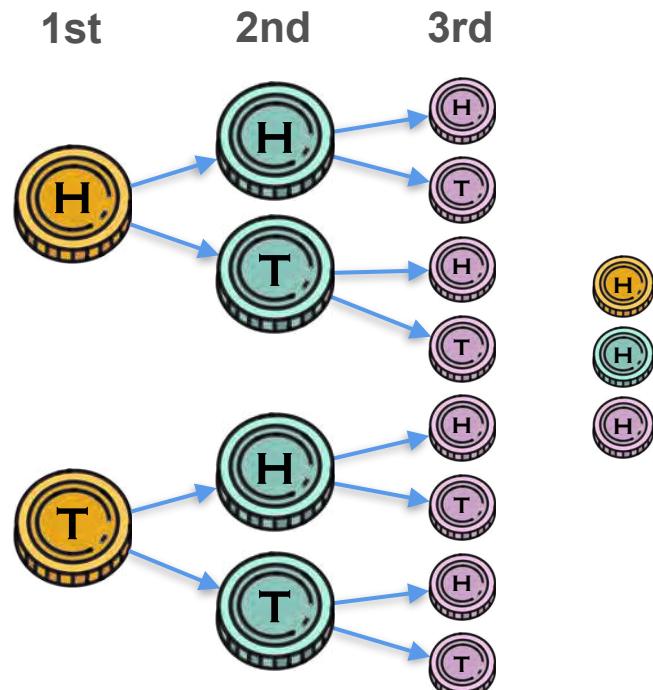
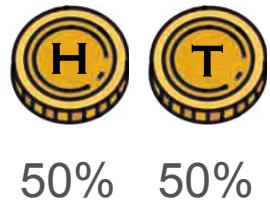
Introduction to Probability: Coin Example 3



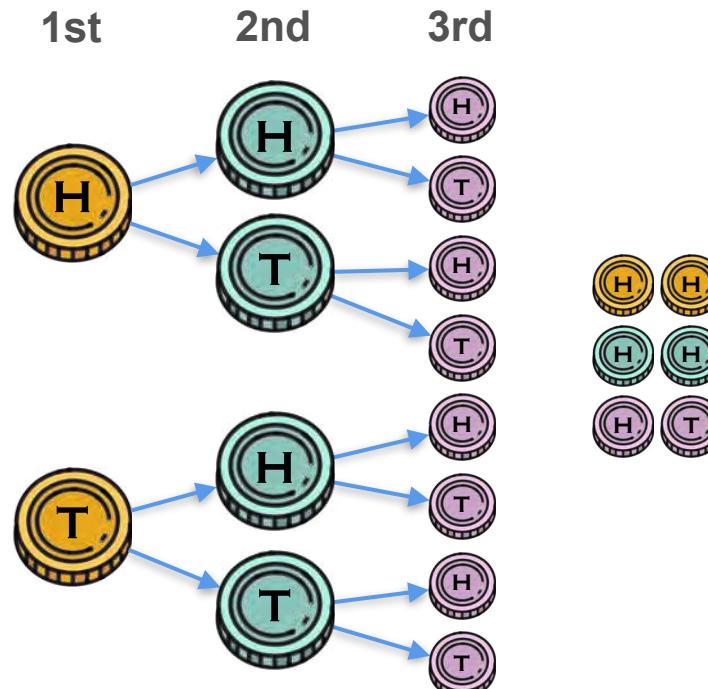
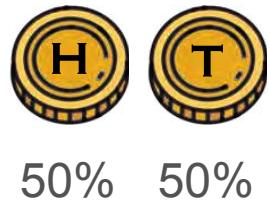
Introduction to Probability: Coin Example 3



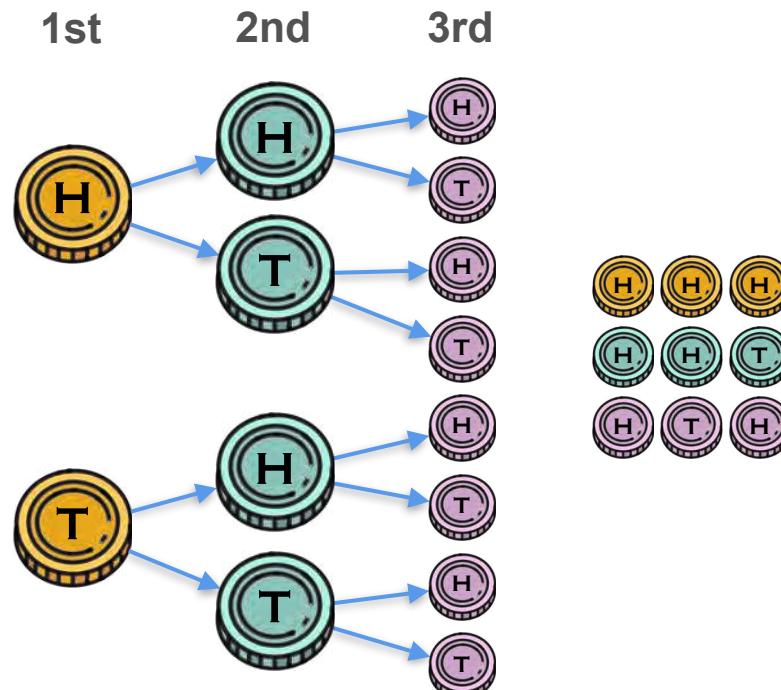
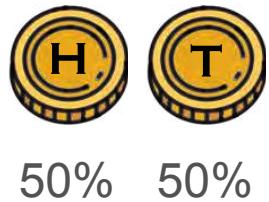
Introduction to Probability: Coin Example 3



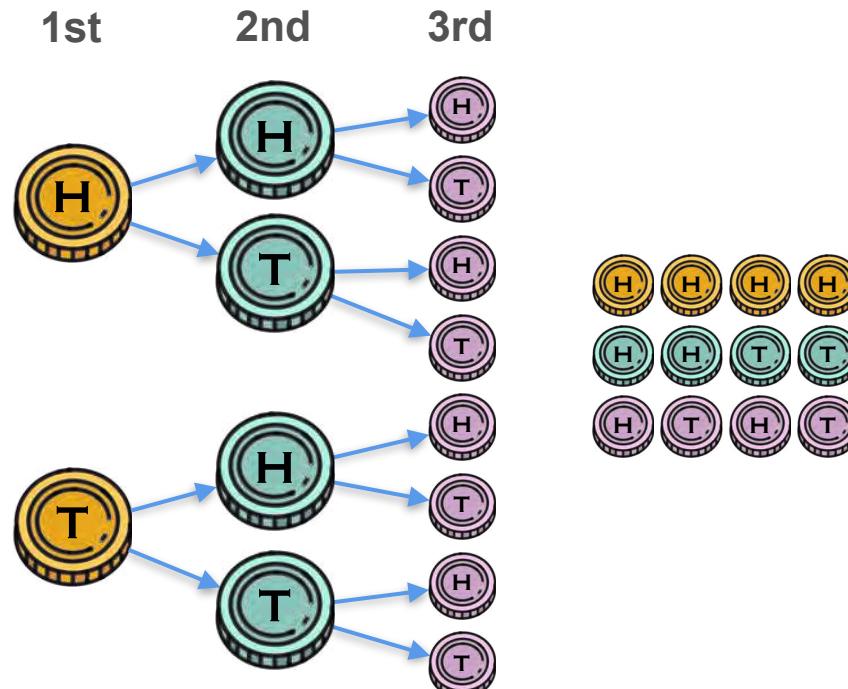
Introduction to Probability: Coin Example 3



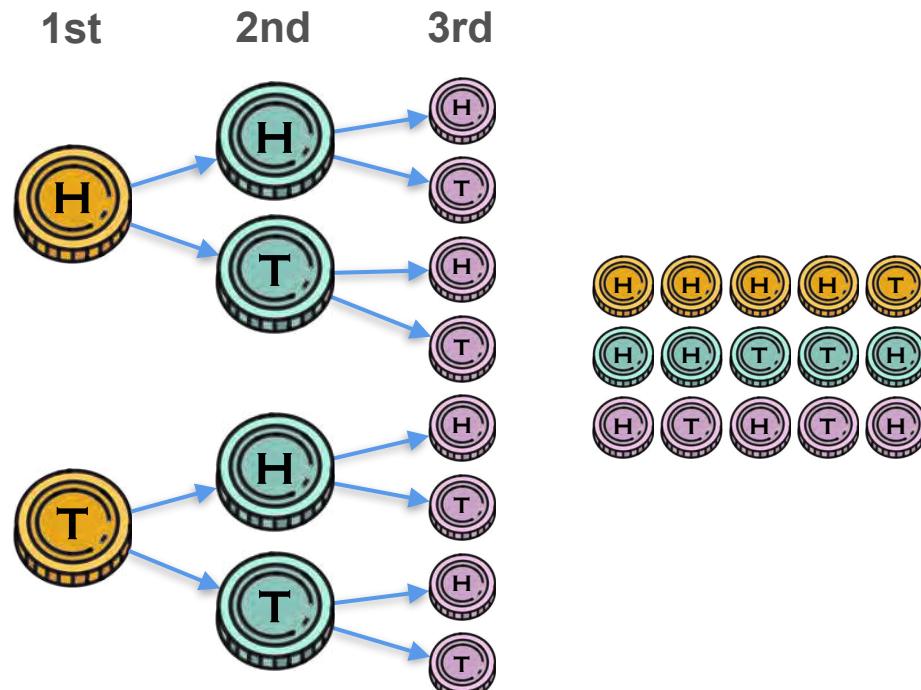
Introduction to Probability: Coin Example 3



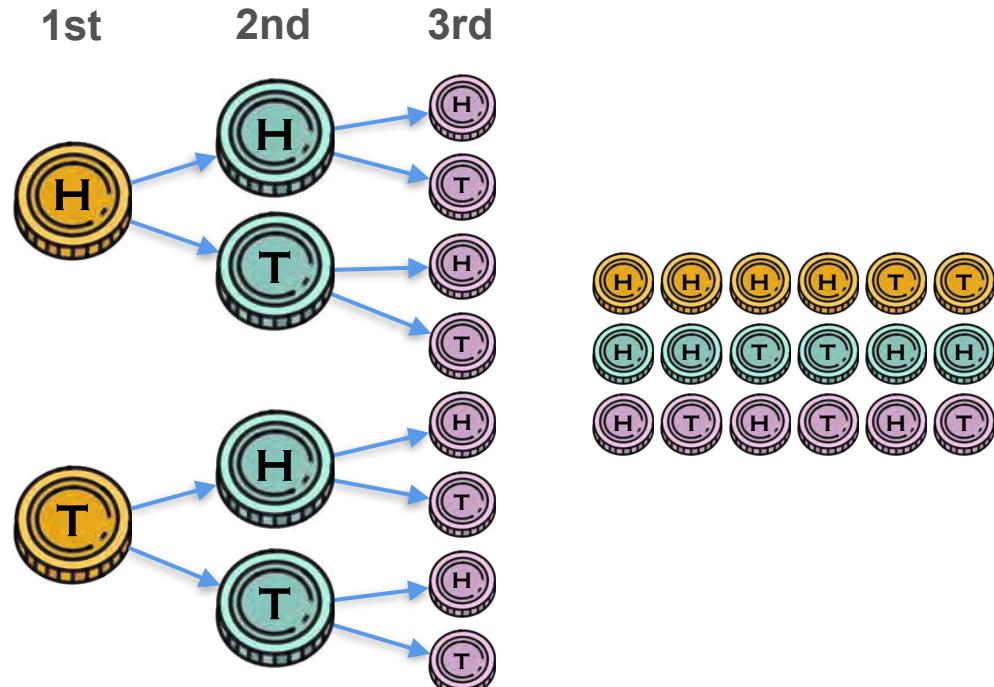
Introduction to Probability: Coin Example 3



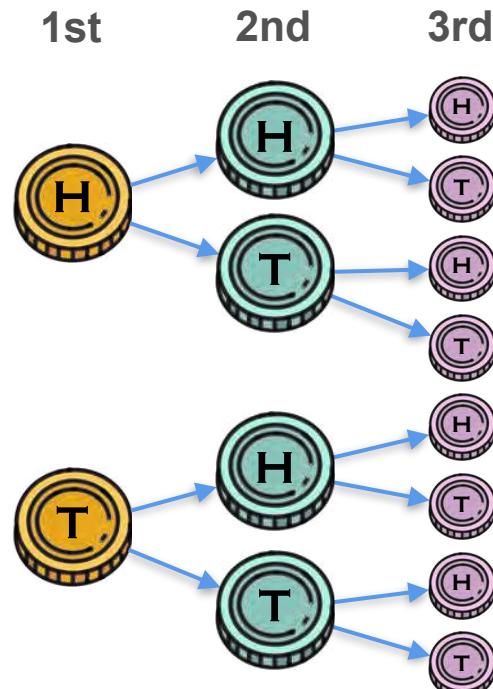
Introduction to Probability: Coin Example 3



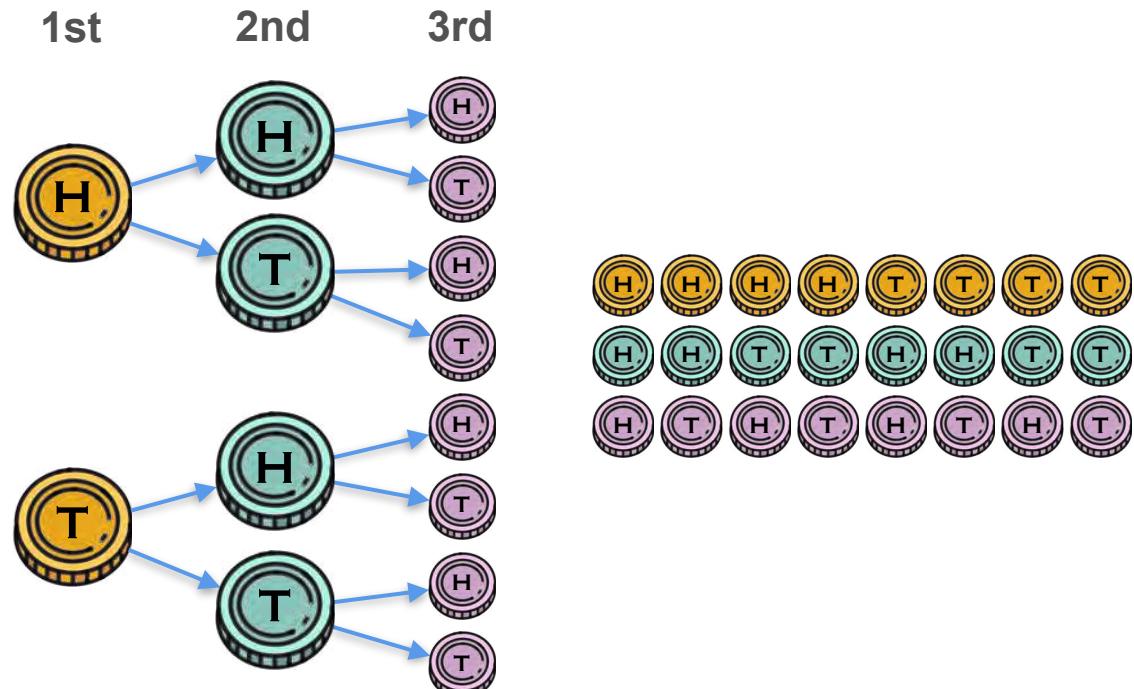
Introduction to Probability: Coin Example 3



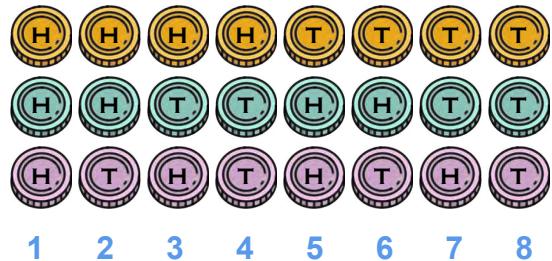
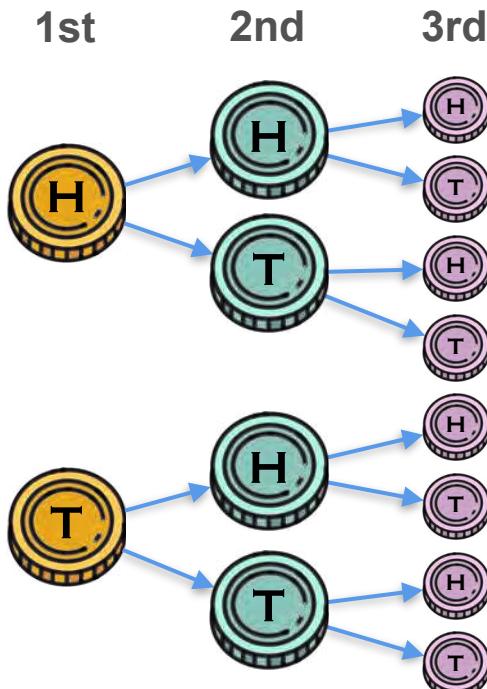
Introduction to Probability: Coin Example 3



Introduction to Probability: Coin Example 3



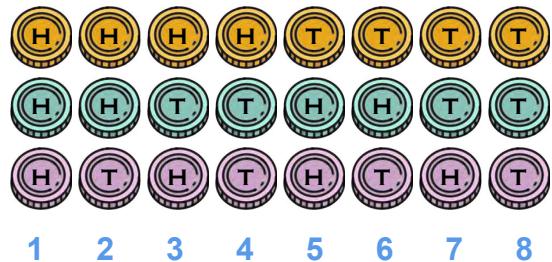
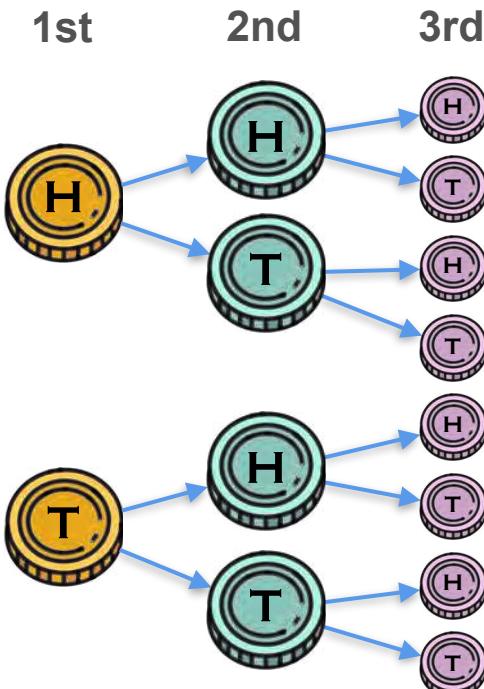
Introduction to Probability: Coin Example 3



Introduction to Probability: Coin Example 3



What is the probability of landing on heads 3 times?



Introduction to Probability: Coin Example 3



50% 50%

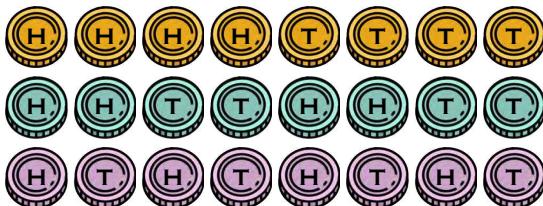


Introduction to Probability: Coin Example 3



50% 50%

$$\mathbf{P}(HHH) = \underline{\hspace{2cm}}$$

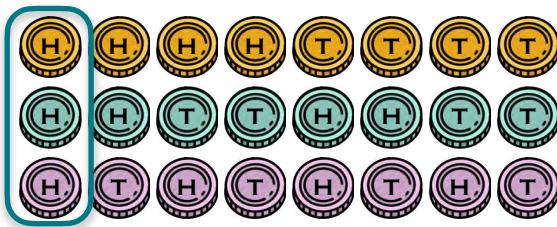


Introduction to Probability: Coin Example 3



50% 50%

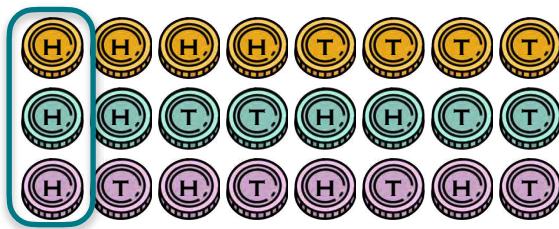
$$\mathbf{P}(HHH) = \underline{\hspace{1cm}}$$



Introduction to Probability: Coin Example 3



50% 50%



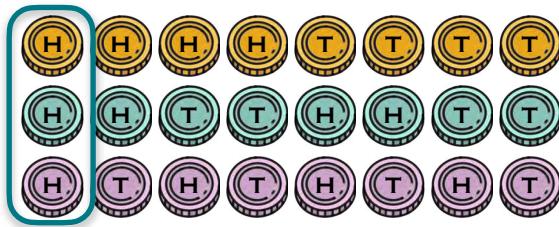
$$\mathbf{P}(HHH) = \underline{\hspace{1cm}}$$



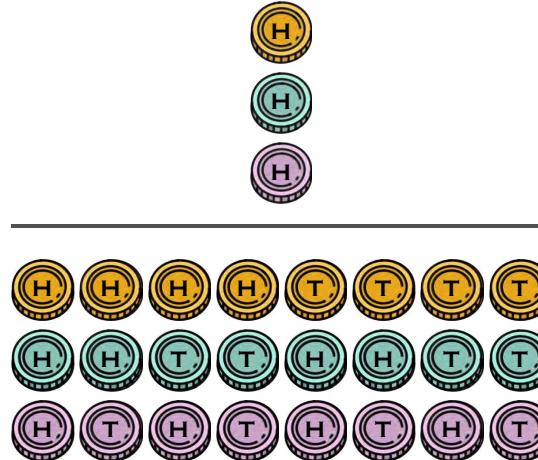
Introduction to Probability: Coin Example 3



50% 50%



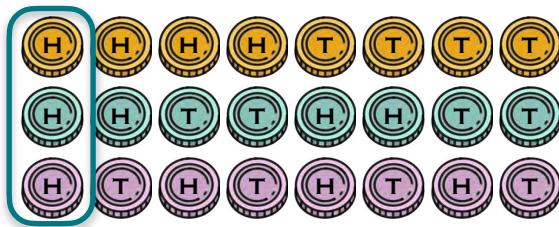
$$P(HHH) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$



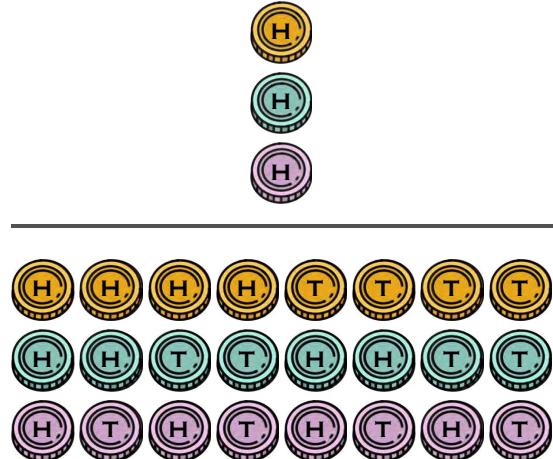
Introduction to Probability: Coin Example 3



50% 50%



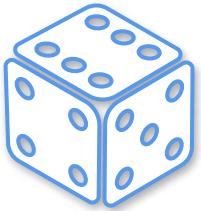
$$P(HHH) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$



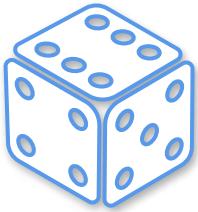
$$= \frac{1}{8} = 0.125$$

Introduction to Probability: Dice Example 1

Introduction to Probability: Dice Example 1

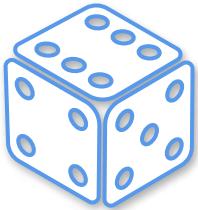


Introduction to Probability: Dice Example 1



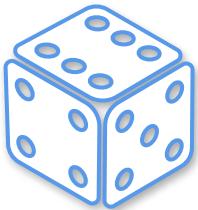
What is the probability of obtaining 6?

Introduction to Probability: Dice Example 1



What is the probability of obtaining 6?

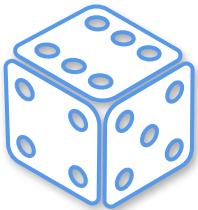
Introduction to Probability: Dice Example 1



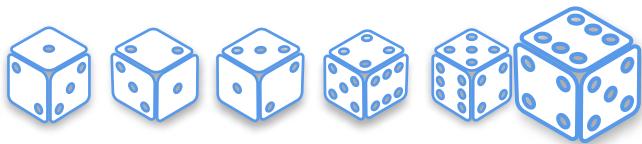
What is the probability of obtaining 6?



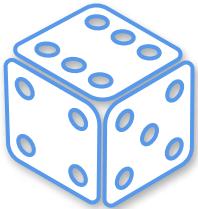
Introduction to Probability: Dice Example 1



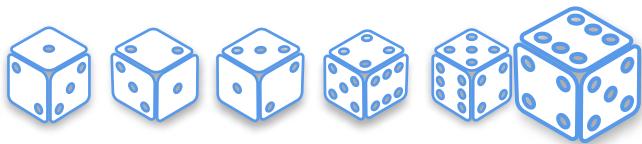
What is the probability of obtaining 6?



Introduction to Probability: Dice Example 1

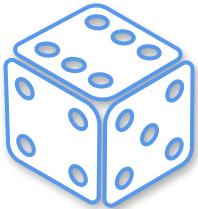


What is the probability of obtaining 6?

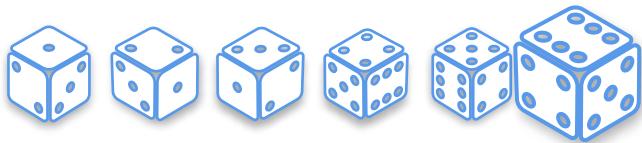


$$P(6) = \underline{\hspace{1cm}}$$

Introduction to Probability: Dice Example 1



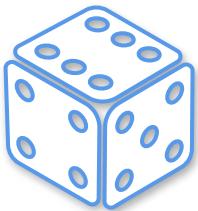
What is the probability of obtaining 6?



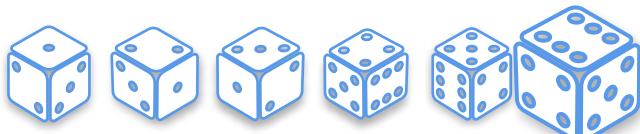
$$P(6) = \underline{\hspace{2cm}}$$



Introduction to Probability: Dice Example 1



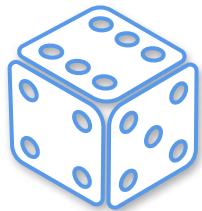
What is the probability of obtaining 6?



$$P(6) = \frac{\text{Number of 6s}}{\text{Total number of dice}}$$

The fraction is shown with a horizontal line separating the numerator from the denominator. Above the numerator is a single die showing a 6. Above the denominator is a row of seven dice, the first six of which show faces with 1 through 5 dots, and the last one showing a 6 dot.

Introduction to Probability: Dice Example 1



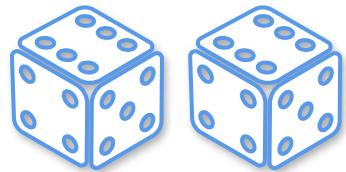
What is the probability of obtaining 6?

$$P(6) = \frac{1}{6}$$

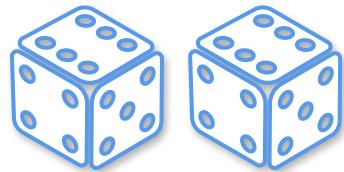
The equation illustrates the probability of rolling a 6 on a single die. The numerator is represented by a single die showing a 6, and the denominator is represented by a row of six dice, where only one die shows a 6.

Introduction to Probability: Dice Example 2

Introduction to Probability: Dice Example 2

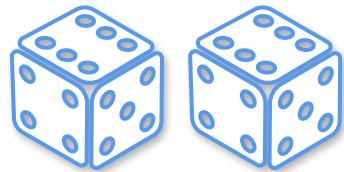


Introduction to Probability: Dice Example 2



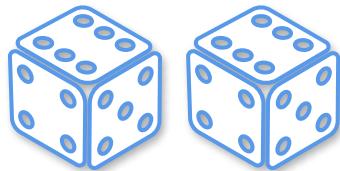
What is the probability of obtaining 6,6?

Introduction to Probability: Dice Example 2



What is the probability of obtaining 6,6?

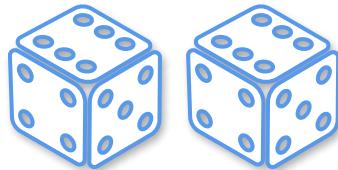
Introduction to Probability: Dice Example 2



What is the probability of obtaining 6,6?



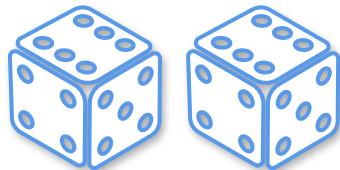
Introduction to Probability: Dice Example 2



What is the probability of obtaining 6,6?

	1,1	1,2	1,3	1,4	1,5	1,6
1,1						
1,2						
1,3						
1,4						
1,5						
1,6						
2,1						
2,2						
2,3						
2,4						
2,5						
2,6						
3,1						
3,2						
3,3						
3,4						
3,5						
3,6						
4,1						
4,2						
4,3						
4,4						
4,5						
4,6						
5,1						
5,2						
5,3						
5,4						
5,5						
5,6						
6,1						
6,2						
6,3						
6,4						
6,5						
6,6						

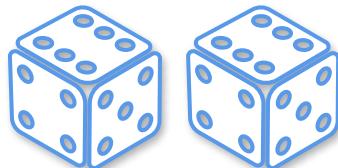
Introduction to Probability: Dice Example 2



What is the probability of obtaining 6,6?

	1,1	1,2	1,3	1,4	1,5	1,6
1,1						
1,2						
1,3						
1,4						
1,5						
1,6						
2,1						
2,2						
2,3						
2,4						
2,5						
2,6						
3,1						
3,2						
3,3						
3,4						
3,5						
3,6						
4,1						
4,2						
4,3						
4,4						
4,5						
4,6						
5,1						
5,2						
5,3						
5,4						
5,5						
5,6						
6,1						
6,2						
6,3						
6,4						
6,5						
6,6						

Introduction to Probability: Dice Example 2

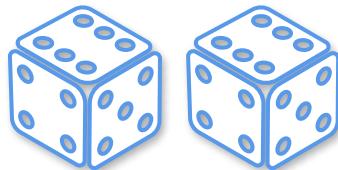


What is the probability of obtaining 6,6?

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6
6,1						

$$P(6,6) = \underline{\hspace{10em}}$$

Introduction to Probability: Dice Example 2

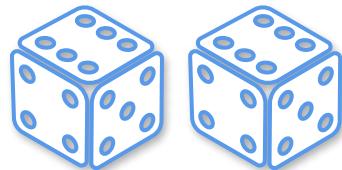


What is the probability of obtaining 6,6?

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
1,2	3,1	3,2	3,3	3,4	3,5	3,6
1,3	4,1	4,2	4,3	4,4	4,5	4,6
1,4	5,1	5,2	5,3	5,4	5,5	5,6
1,5	6,1	6,2	6,3	6,4	6,5	6,6

$$P(6,6) = \frac{1}{36}$$

Introduction to Probability: Dice Example 2



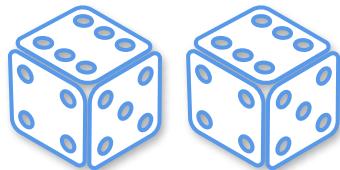
What is the probability of obtaining 6,6?

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6
6,1						

$$P(6,6) = \frac{1}{36}$$

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

Introduction to Probability: Dice Example 2



What is the probability of obtaining 6,6?

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$P(6,6) = \frac{1}{36} = \frac{1}{36}$$

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6



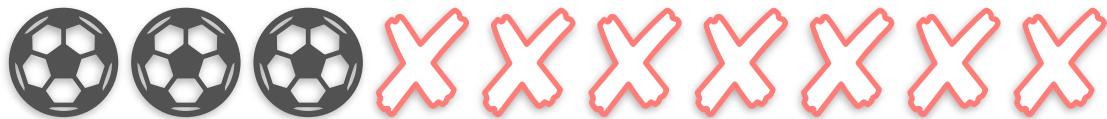
DeepLearning.AI

Introduction to probability

Complement of Probability

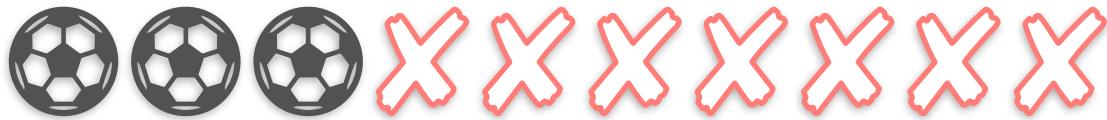
Complement of Probability

Complement of Probability



30%

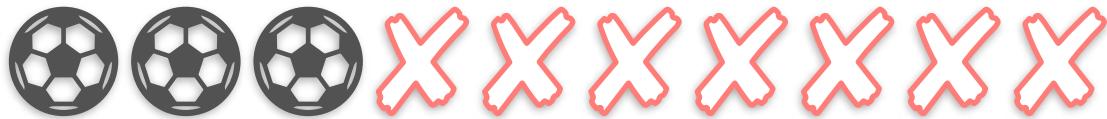
Complement of Probability



30%

What is the probability of a child NOT playing soccer?

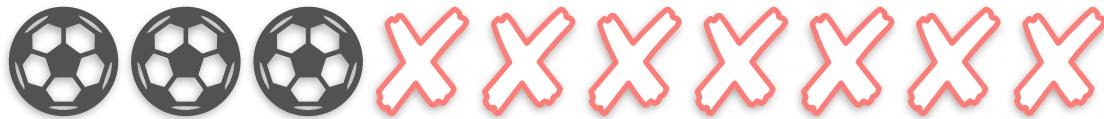
Complement of Probability



30%

What is the probability of a child NOT playing soccer?

Complement of Probability

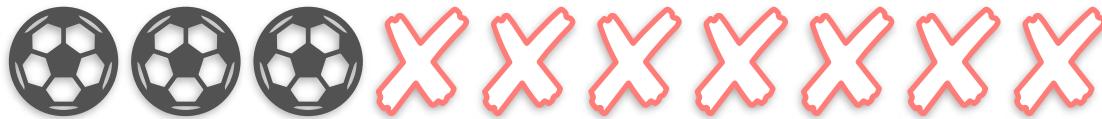


30%

What is the probability of a child NOT playing soccer?

$$P(\text{not soccer}) = \frac{\text{not soccer}}{\text{total}}$$

Complement of Probability

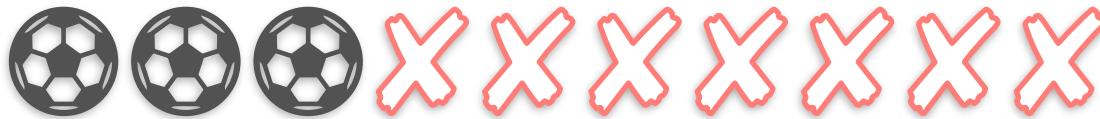


30%

What is the probability of a child NOT playing soccer?

$$P(\text{not soccer}) = \frac{\text{not soccer}}{\text{total}} = \underline{\hspace{2cm}}$$

Complement of Probability

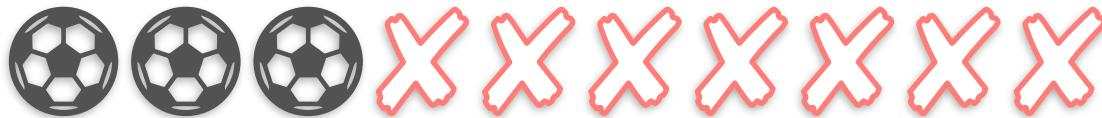


30%

What is the probability of a child NOT playing soccer?

$$P(\text{not soccer}) = \frac{\text{not soccer}}{\text{total}} = \frac{\text{XXXXXX}}{\text{XXXXXX}}$$

Complement of Probability

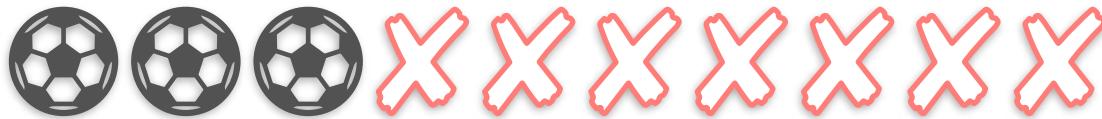


30%

What is the probability of a child NOT playing soccer?

$$P(\text{not soccer}) = \frac{\text{not soccer}}{\text{total}} = \frac{\text{XXXXXXX}}{\text{XXXXXXX}}$$

Complement of Probability

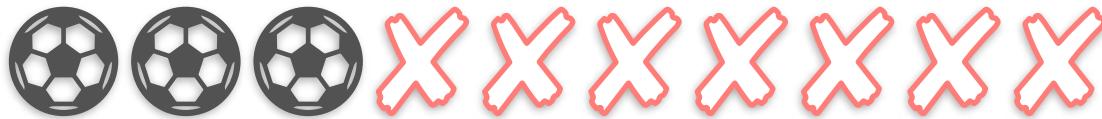


30%

What is the probability of a child NOT playing soccer?

$$P(\text{not soccer}) = \frac{\text{not soccer}}{\text{total}} = \frac{\text{XXXXXXX}}{\text{XXXXXXX}} = \frac{7}{10}$$

Complement of Probability



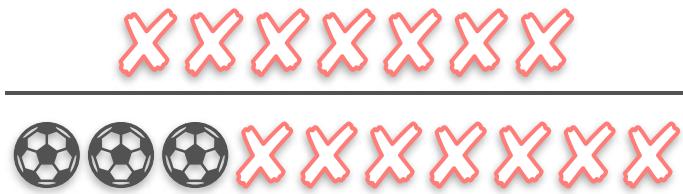
30%

What is the probability of a child NOT playing soccer?

$$P(\text{not soccer}) = \frac{\text{not soccer}}{\text{total}} = \frac{\text{XXXXXXX}}{\text{XXXXXXX}} = \frac{7}{10} = 0.7$$

Complement of Probability

Complement of Probability



$P(\text{not soccer})$

0.7

Complement of Probability

XXXXXX

●●●XXXXX

$P(\text{not soccer})$

0.7

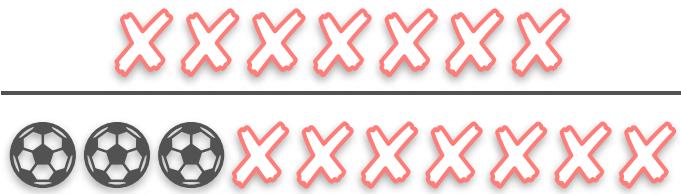
●●●

●●●XXXXX

$P(\text{soccer})$

0.3

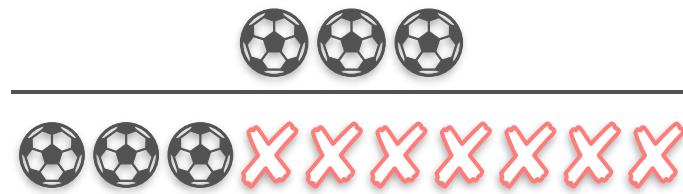
Complement of Probability



$P(\text{not soccer})$

0.7

+

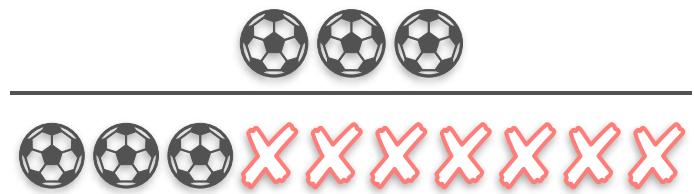
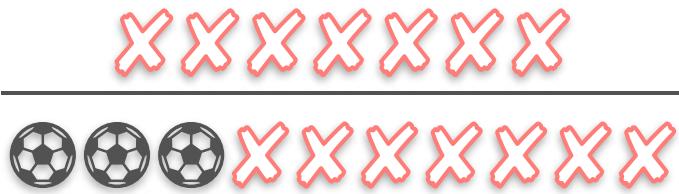


$P(\text{soccer})$

0.3

= 1

Complement of Probability



$P(\text{not soccer})$

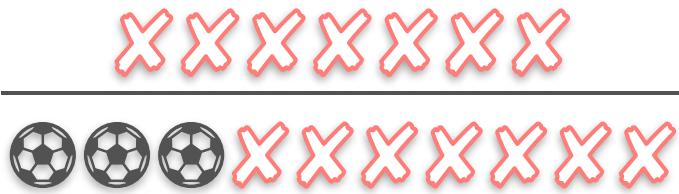
0.7

= 1

$P(\text{soccer})$

0.3

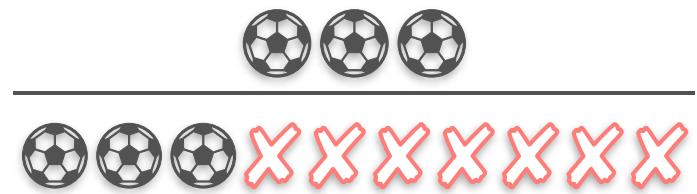
Complement of Probability



$P(\text{not soccer})$

0.7

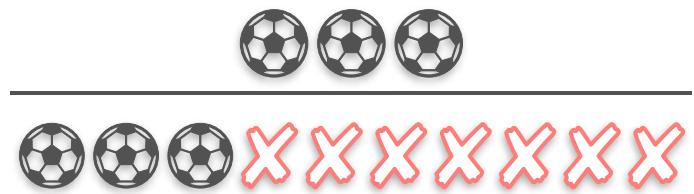
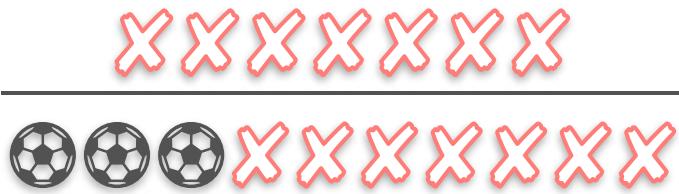
$$= 1 -$$



$P(\text{soccer})$

0.3

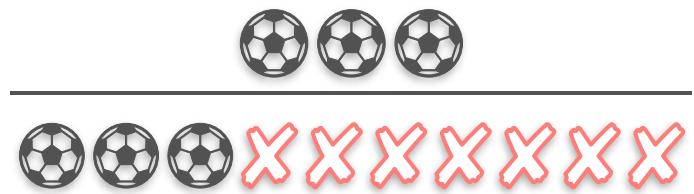
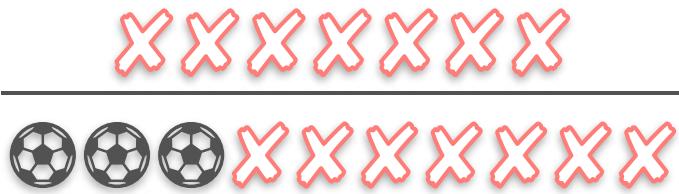
Complement of Probability



$$P(\text{not soccer}) = 1 - P(\text{soccer})$$

$$0.7 = 1 - 0.3$$

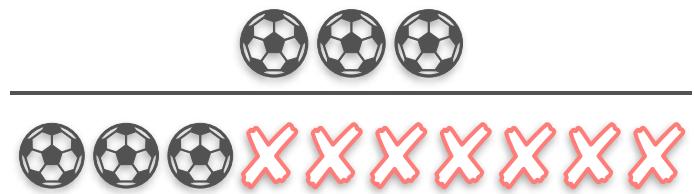
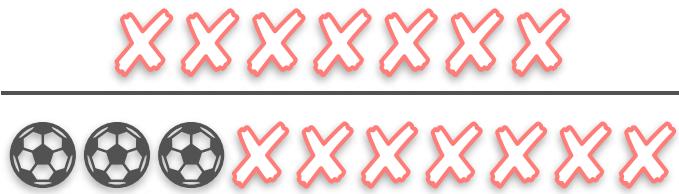
Complement of Probability



$$P(\text{not soccer}) = 1 - P(\text{soccer})$$

$$0.7 = 1 - 0.3$$

Complement of Probability



$$P(\text{not soccer}) = 1 - P(\text{soccer})$$

$$0.7 = 1 - 0.3$$

Complement Rule

Complement of Probability

$$P(\text{not soccer}) = 1 - P(\text{soccer})$$

Complement Rule

Complement of Probability

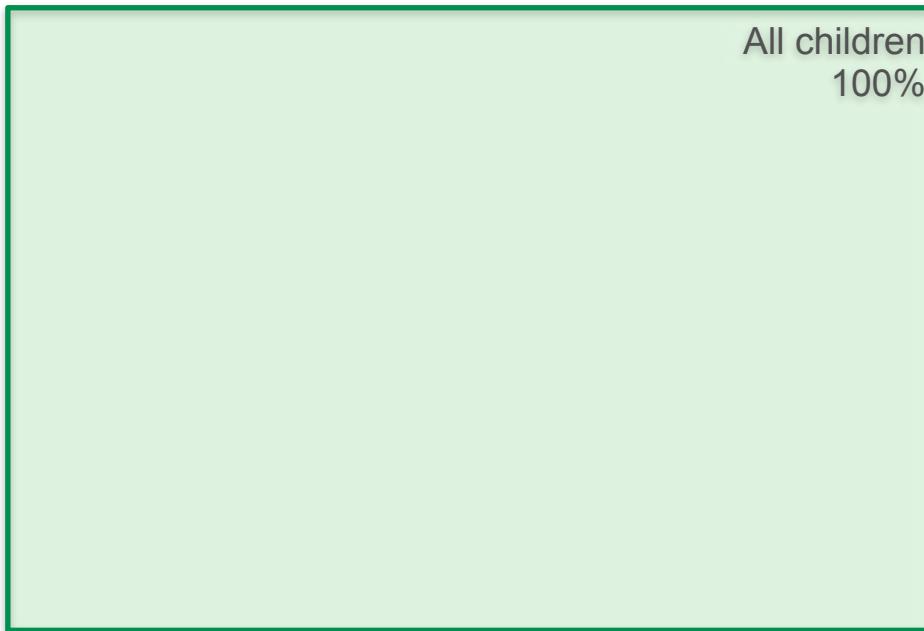
$$P(\text{not soccer}) = 1 - P(\text{soccer})$$

$$P(A') = 1 - P(A)$$

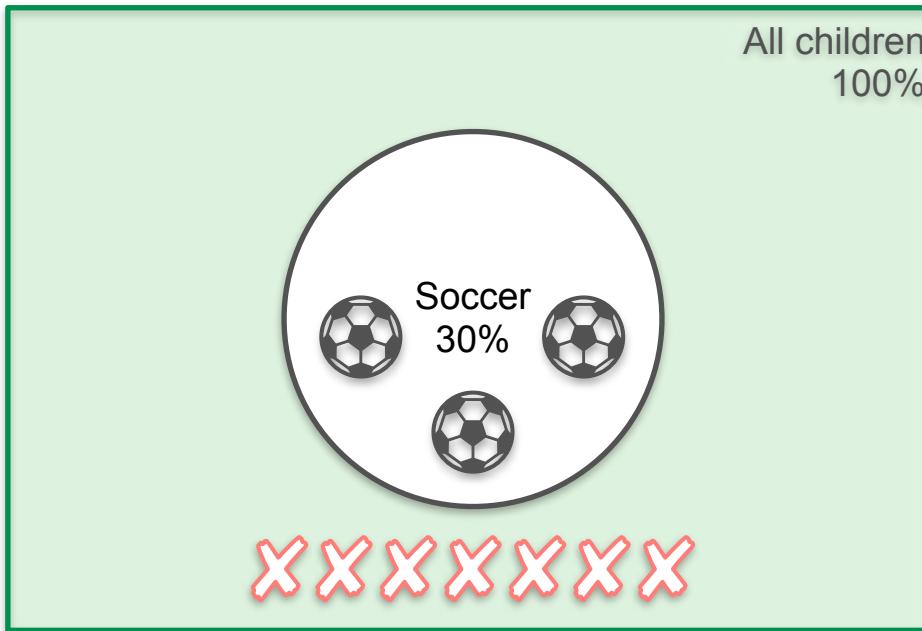
Complement Rule

Complement of Probability: Venn Diagram

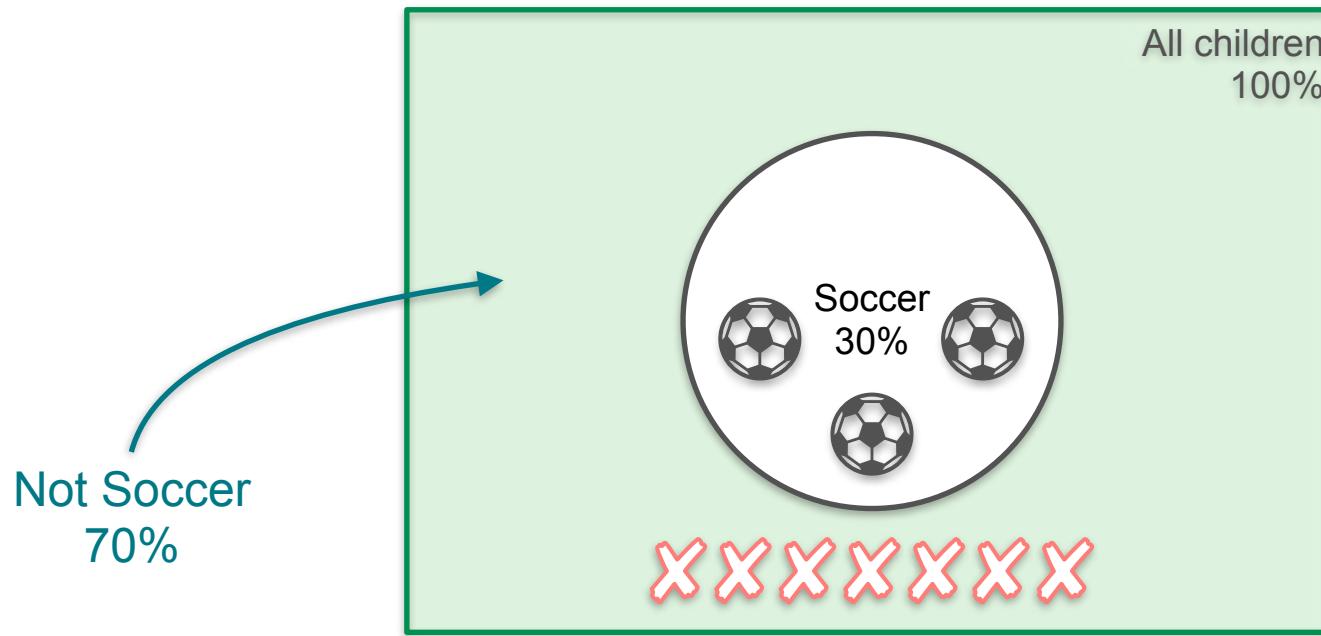
Complement of Probability: Venn Diagram



Complement of Probability: Venn Diagram



Complement of Probability: Venn Diagram



Complement of Probability: Coin Example 1



Complement of Probability: Coin Example 1



What is the probability of not landing on heads 3 times?

Complement of Probability: Coin Example 1



What is the probability of not landing on heads 3 times?

Complement of Probability: Coin Example 1



What is the probability of not landing on heads 3 times?

$$P(\text{not } HHH)$$

Complement of Probability: Coin Example 1



What is the probability of not landing on heads 3 times?

$$P(\text{not } HHH) =$$

Complement of Probability: Coin Example 1



What is the probability of not landing on heads 3 times?

$$P(\text{not } HHH) = 1$$

Complement of Probability: Coin Example 1



What is the probability of not landing on heads 3 times?

$$P(\text{not } HHH) = 1 - P(HHH)$$

Complement of Probability: Coin Example 1



What is the probability of not landing on heads 3 times?

$$\mathbf{P}(\text{not } HHH) = 1 - \mathbf{P}(HHH)$$

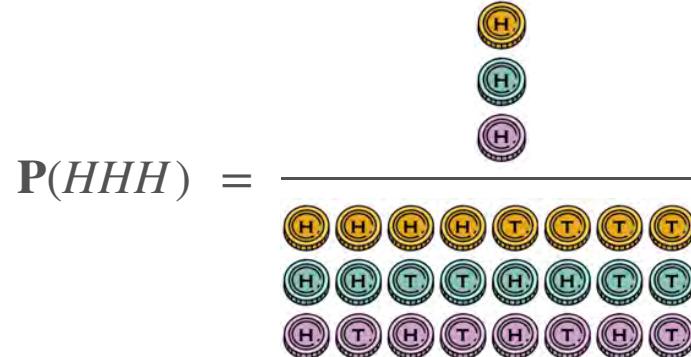
$$\mathbf{P}(HHH) =$$

Complement of Probability: Coin Example 1



What is the probability of not landing on heads 3 times?

$$P(\text{not } HHH) = 1 - P(HHH)$$

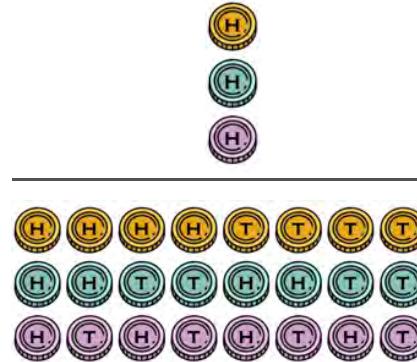


Complement of Probability: Coin Example 1



What is the probability of not landing on heads 3 times?

$$P(\text{not } HHH) = 1 - P(HHH)$$



Complement of Probability: Coin Example 1



What is the probability of not landing on heads 3 times?

$$P(\text{not } HHH) = 1 - P(HHH)$$

$$P(\text{not } HHH) = 1 - \frac{\text{_____}}{\text{_____}}$$

The denominator consists of three rows of 8 coins each, totaling 24 coins. The top row has all heads (H). The middle row has 7 heads and 1 tail (T). The bottom row has 4 heads and 4 tails (T).

Complement of Probability: Coin Example 1



What is the probability of not landing on heads 3 times?

$$P(\text{not } HHH) = 1 - P(HHH)$$

$$P(\text{not } HHH) = 1 - \frac{1}{8}$$

Complement of Probability: Coin Example 1



What is the probability of not landing on heads 3 times?

$$P(\text{not } HHH) = 1 - P(HHH)$$

$$P(\text{not } HHH) = 1 - \frac{1}{8}$$

Complement of Probability: Coin Example 1



What is the probability of not landing on heads 3 times?

$$P(\text{not } HHH) = 1 - P(HHH)$$

$$P(\text{not } HHH) = 1 - \frac{1}{8}$$

$$= \frac{7}{8}$$

Complement of Probability: Coin Example 1

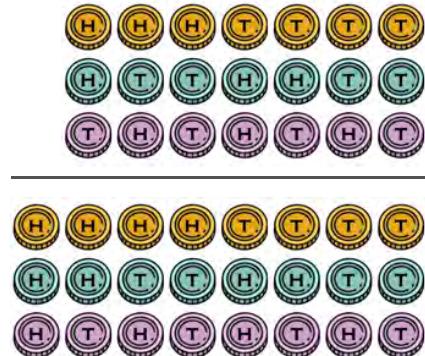


What is the probability of not landing on heads 3 times?

$$P(\text{not } HHH) = 1 - P(HHH)$$

$$P(\text{not } HHH) = 1 - \frac{1}{8}$$

$$= \frac{7}{8}$$



Complement of Probability: Dice Example 1

Complement of Probability: Dice Example 1



Complement of Probability: Dice Example 1



What is the probability of obtaining anything other than 6?

Complement of Probability: Dice Example 1



What is the probability of obtaining anything other than 6?

Complement of Probability: Dice Example 1



What is the probability of obtaining anything other than 6?



Complement of Probability: Dice Example 1



What is the probability of obtaining anything other than 6?



$$P(\text{not } 6) =$$

Complement of Probability: Dice Example 1



What is the probability of obtaining anything other than 6?



$$P(\text{not } 6) = \frac{\text{Number of dice showing not 6}}{\text{Total number of dice}}$$

The equation shows a fraction where the numerator is a row of six dice, all showing faces other than 6 (1, 2, 3, 4, 5). The denominator is another row of six dice, all showing faces with 1 through 6.

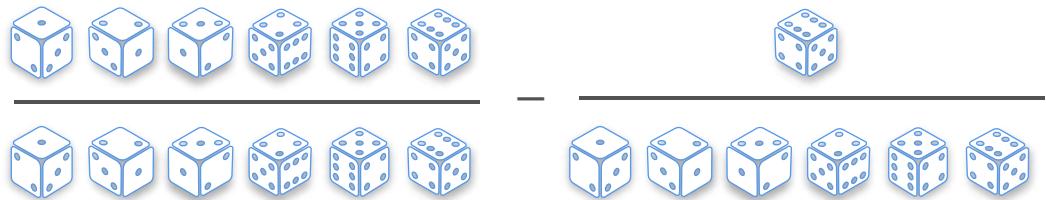
Complement of Probability: Dice Example 1



What is the probability of obtaining anything other than 6?



$$P(\text{not } 6) =$$



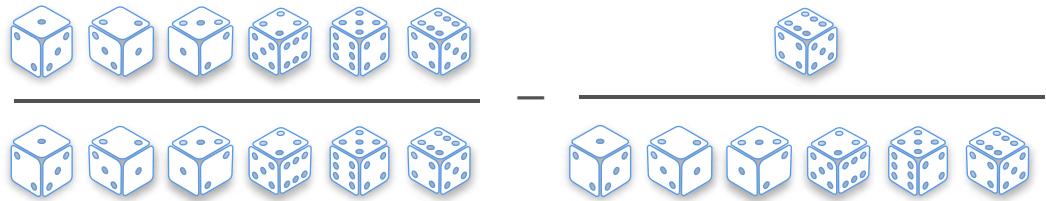
Complement of Probability: Dice Example 1



What is the probability of obtaining anything other than 6?



$$P(\text{not } 6) =$$



=

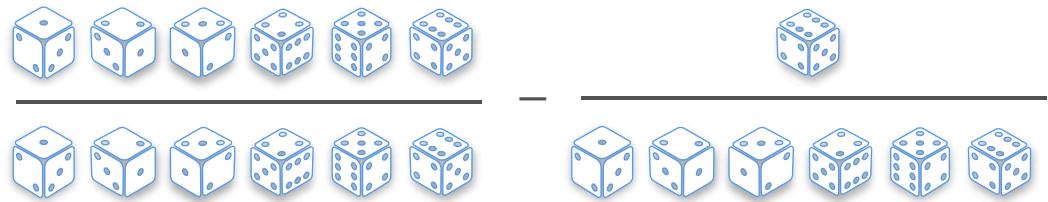
Complement of Probability: Dice Example 1



What is the probability of obtaining anything other than 6?



$$P(\text{not } 6) =$$



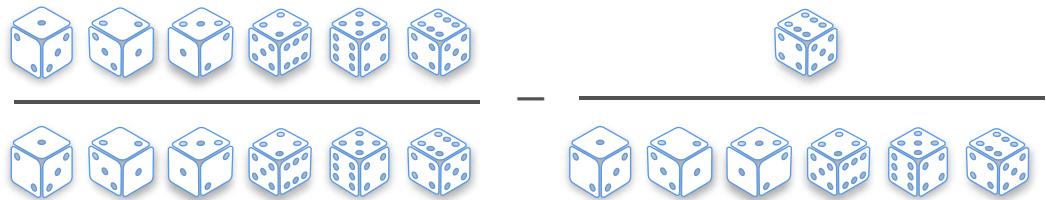
Complement of Probability: Dice Example 1



What is the probability of obtaining anything other than 6?



$$P(\text{not } 6) =$$



$$= \frac{\text{---}}{\text{---}}$$

Complement of Probability: Dice Example 1



What is the probability of obtaining anything other than 6?



$$P(\text{not } 6) =$$

$$\frac{\begin{array}{cccccc} \text{dice} & \text{dice} & \text{dice} & \text{dice} & \text{dice} & \text{dice} \\ \hline \end{array}}{\begin{array}{cccccc} \text{dice} & \text{dice} & \text{dice} & \text{dice} & \text{dice} & \text{dice} \end{array}} - \frac{\begin{array}{cccccc} \text{dice} & \text{dice} & \text{dice} & \text{dice} & \text{dice} & \text{dice} \\ \hline \end{array}}{\begin{array}{cccccc} \text{dice} & \text{dice} & \text{dice} & \text{dice} & \text{dice} & \text{dice} \end{array}}$$
$$= \frac{\begin{array}{cccccc} \text{dice} & \text{dice} & \text{dice} & \text{dice} & \text{dice} & \text{dice} \\ \hline \end{array}}{\begin{array}{cccccc} \text{dice} & \text{dice} & \text{dice} & \text{dice} & \text{dice} & \text{dice} \end{array}}$$

Complement of Probability: Dice Example 1



What is the probability of obtaining anything other than 6?



$$P(\text{not } 6) =$$

$$\frac{\begin{array}{cccccc} \text{dice} & \text{dice} & \text{dice} & \text{dice} & \text{dice} & \text{dice} \\ \hline \end{array}}{\begin{array}{cccccc} \text{dice} & \text{dice} & \text{dice} & \text{dice} & \text{dice} & \text{dice} \end{array}} - \frac{\begin{array}{cccccc} \text{dice} & \text{dice} & \text{dice} & \text{dice} & \text{dice} & \text{dice} \\ \hline \end{array}}{\begin{array}{cccccc} \text{dice} & \text{dice} & \text{dice} & \text{dice} & \text{dice} & \text{dice} \end{array}}$$
$$= \frac{\begin{array}{ccccc} \text{dice} & \text{dice} & \text{dice} & \text{dice} & \text{dice} \\ \hline \end{array}}{\begin{array}{ccccc} \text{dice} & \text{dice} & \text{dice} & \text{dice} & \text{dice} \end{array}}$$

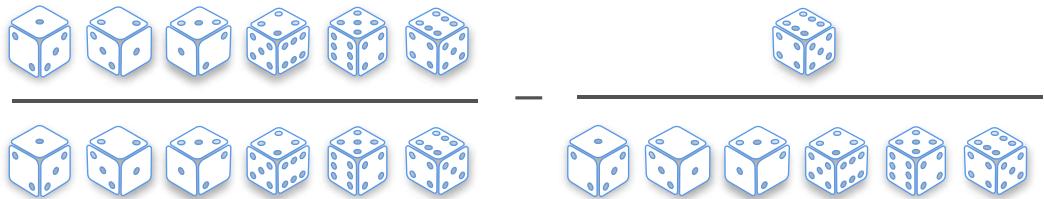
Complement of Probability: Dice Example 1



What is the probability of obtaining anything other than 6?



$$P(\text{not } 6) =$$



$$= \frac{\text{dice not showing 6}}{\text{total dice}}$$

=

Complement of Probability: Dice Example 1



What is the probability of obtaining anything other than 6?

$$\begin{aligned} P(\text{not } 6) &= \frac{\text{Number of outcomes not 6}}{\text{Total number of outcomes}} = \frac{\text{Number of outcomes not 6}}{6} \\ &= \frac{5}{6} \end{aligned}$$

The diagram illustrates the calculation of the probability of not rolling a 6. It shows a total of 6 dice in the numerator, with 5 dice showing faces other than 6 (1, 2, 3, 4, 5). In the denominator, there are 6 dice in total, with all 6 faces showing a 6. A horizontal line separates the numerator from the denominator.



DeepLearning.AI

Introduction to probability

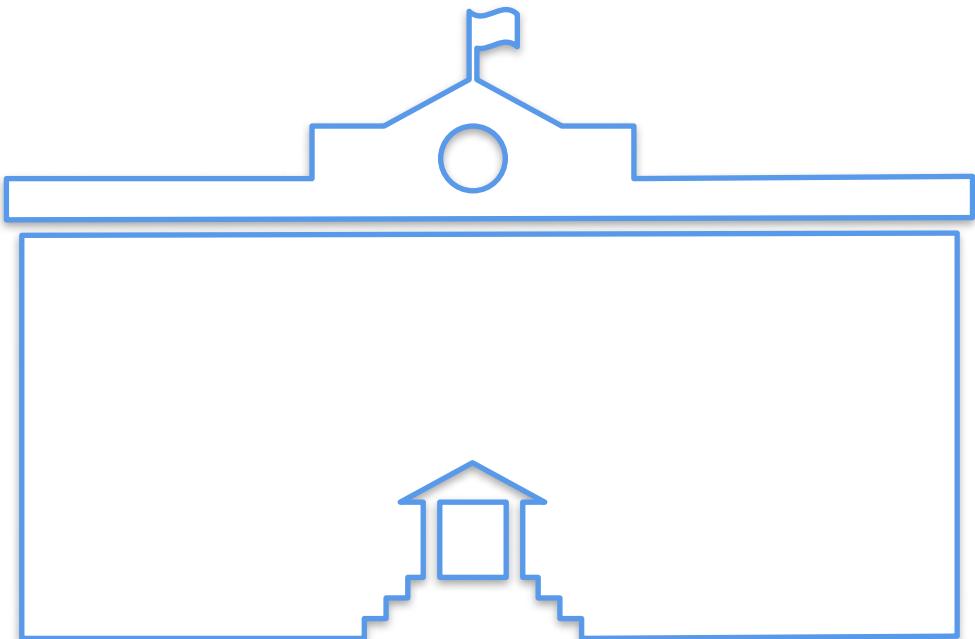
Sum of Probabilities

Sum of Probabilities: Quiz 1

Sum of Probabilities: Quiz 1

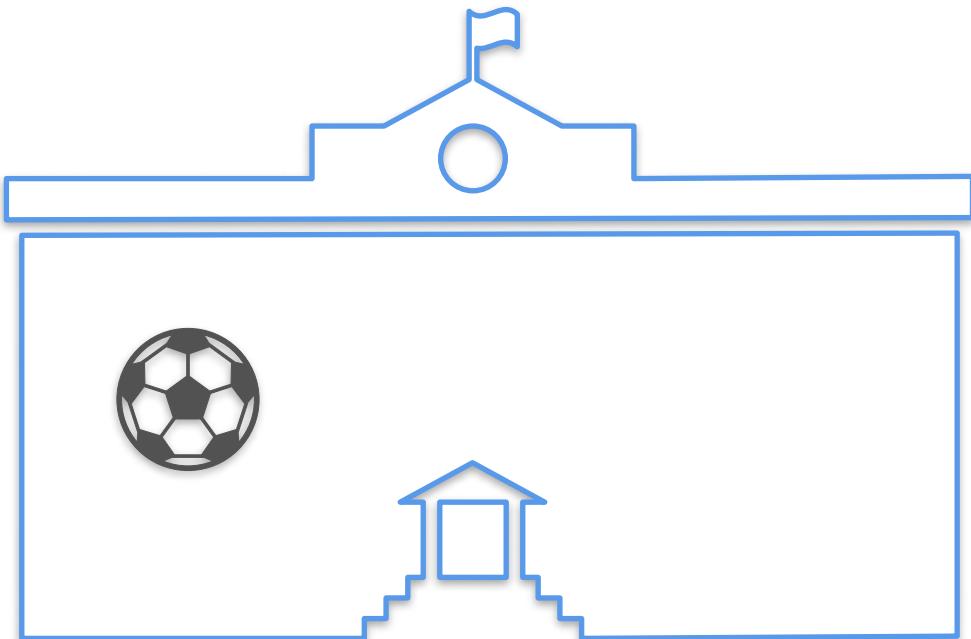
At a school, kids can only play one sport.

Sum of Probabilities: Quiz 1



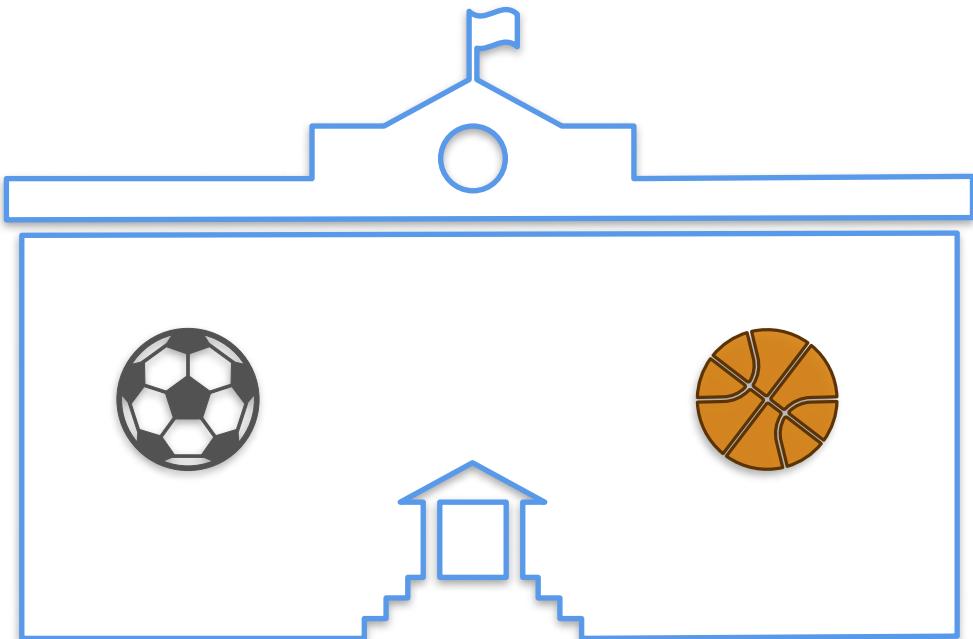
At a school, kids can only play one sport.

Sum of Probabilities: Quiz 1



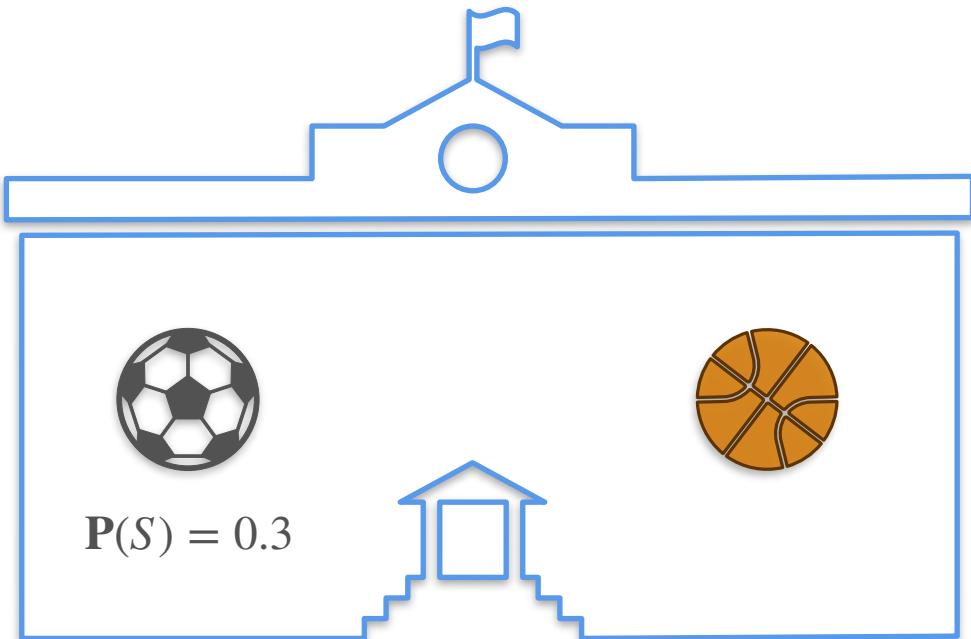
At a school, kids can only play one sport.

Sum of Probabilities: Quiz 1



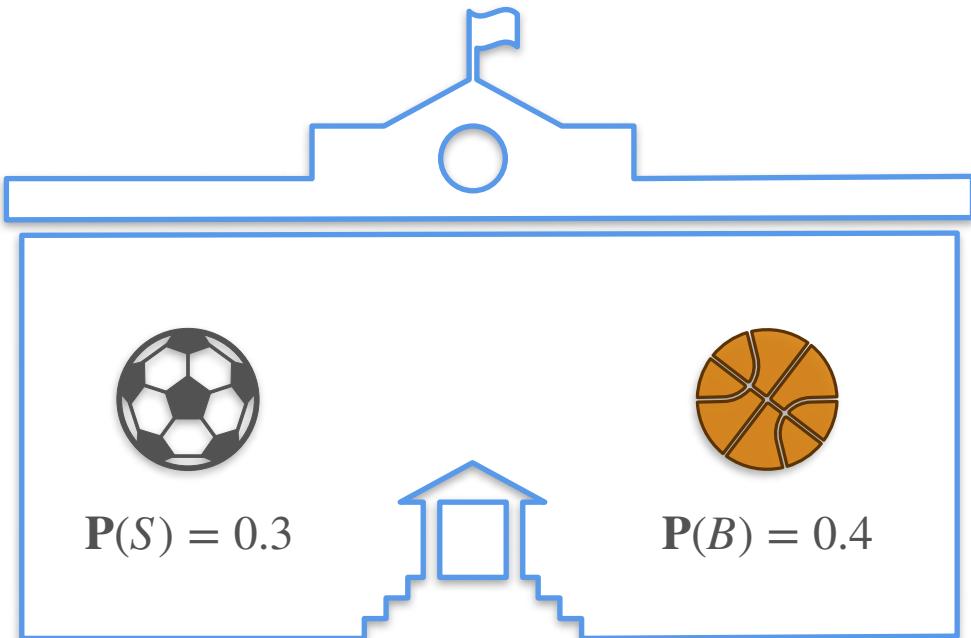
At a school, kids can only play one sport.

Sum of Probabilities: Quiz 1



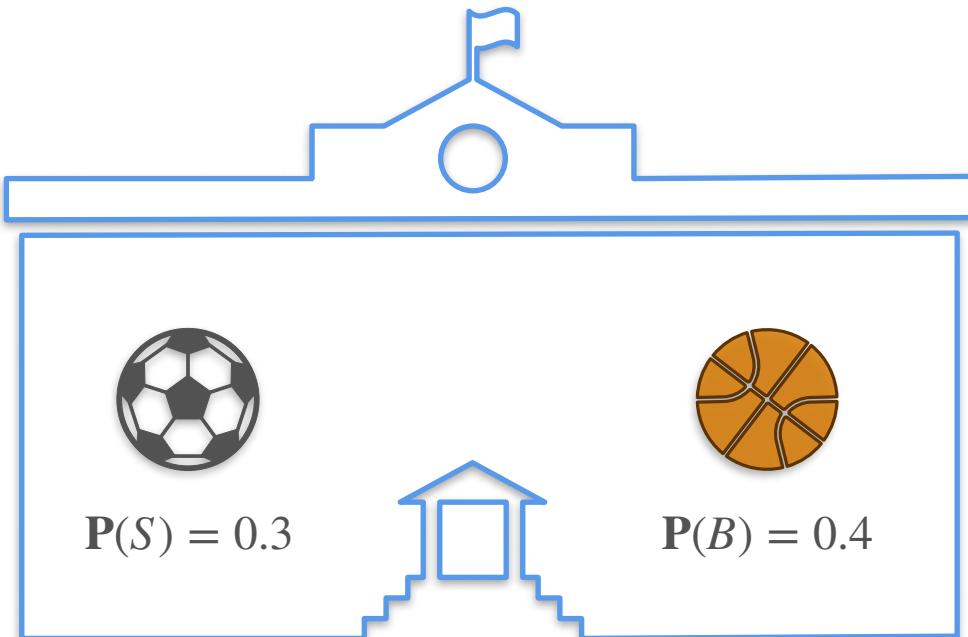
At a school, kids can only play one sport.

Sum of Probabilities: Quiz 1



At a school, kids can only play one sport.

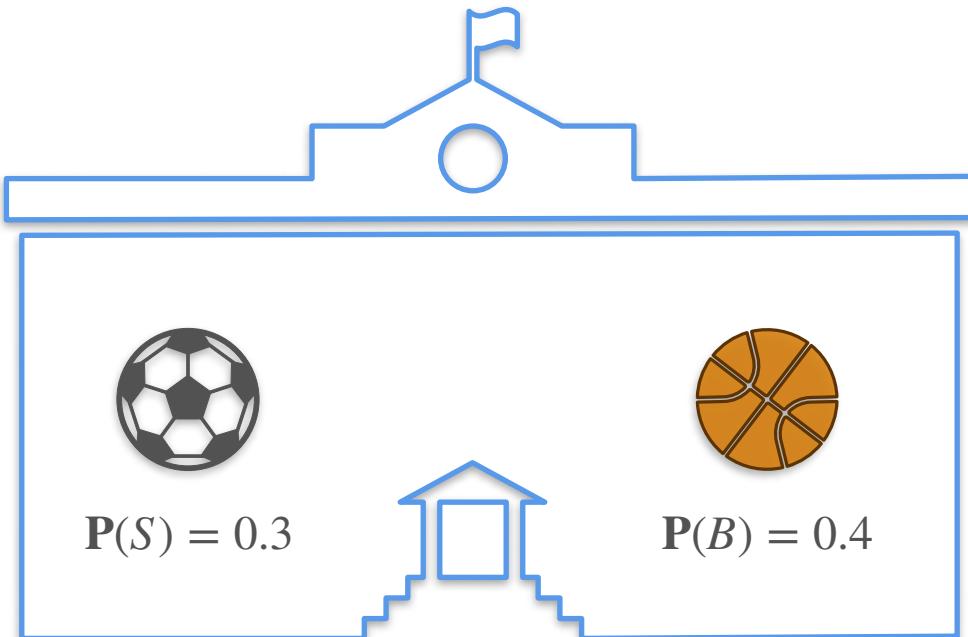
Sum of Probabilities: Quiz 1



At a school, kids can only play one sport.

What is the probability that a kid plays soccer or basketball?

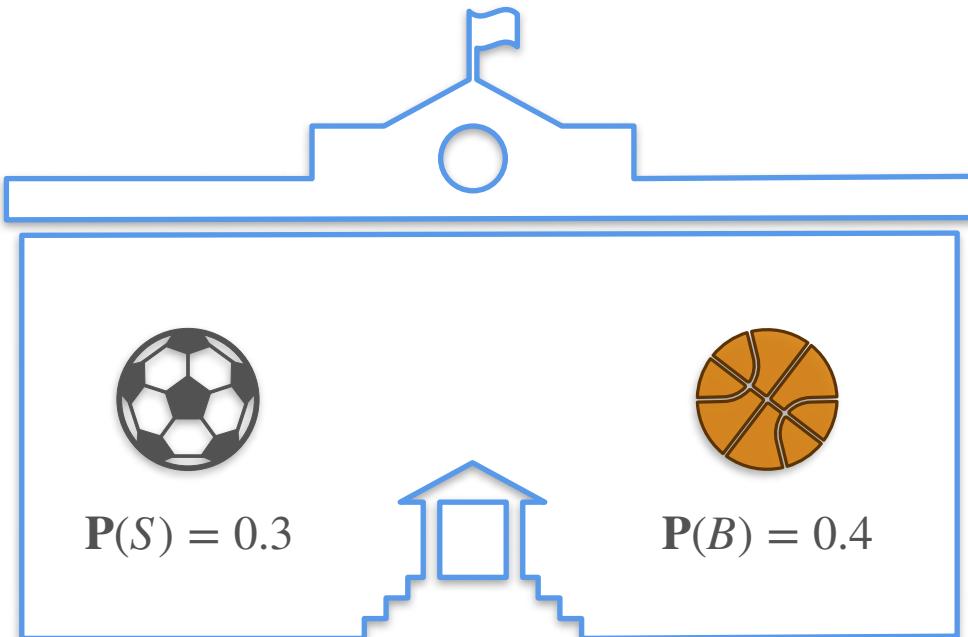
Sum of Probabilities: Quiz 1



At a school, kids can only play one sport.

What is the probability that a kid plays soccer or basketball?

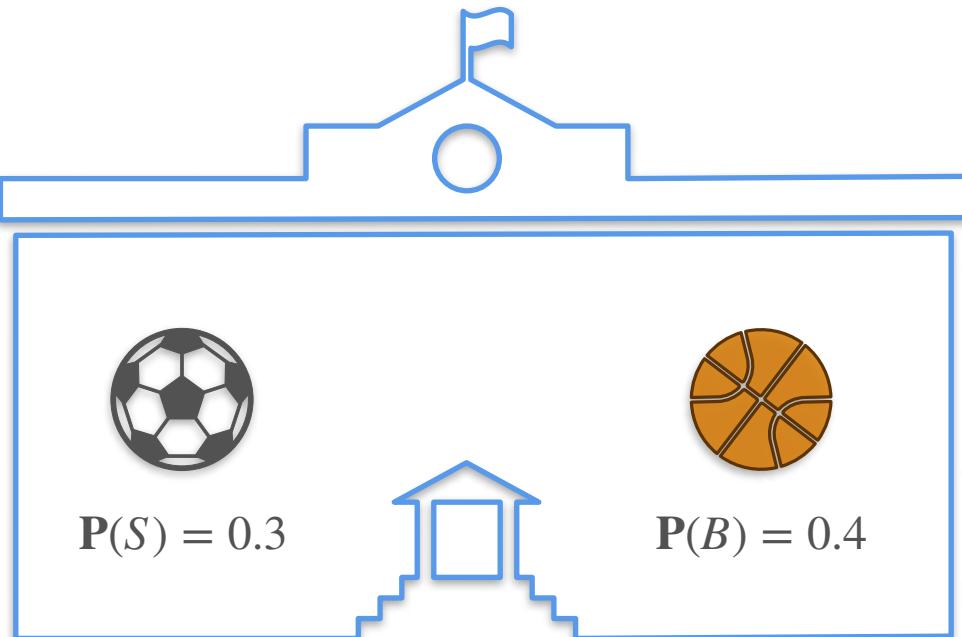
Sum of Probabilities: Quiz 1



At a school, kids can only play one sport.

What is the probability that a kid plays soccer or basketball?

Sum of Probabilities: Quiz 1



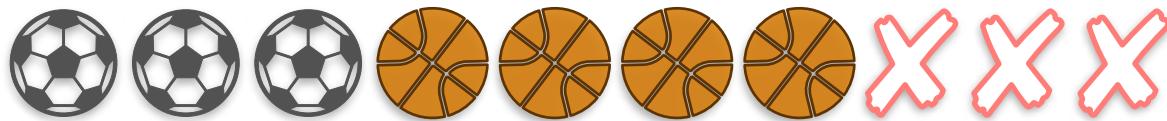
At a school, kids can only play one sport.

What is the probability that a kid plays soccer or basketball?

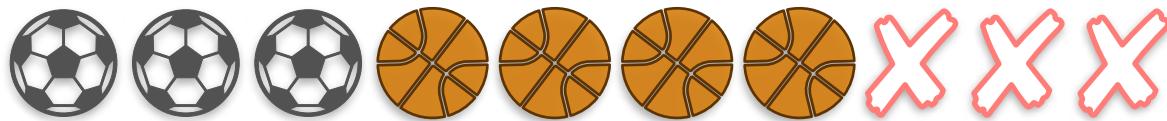
Hint: What if there were only 10 kids?

Sum of Probabilities: Quiz 1 Solution

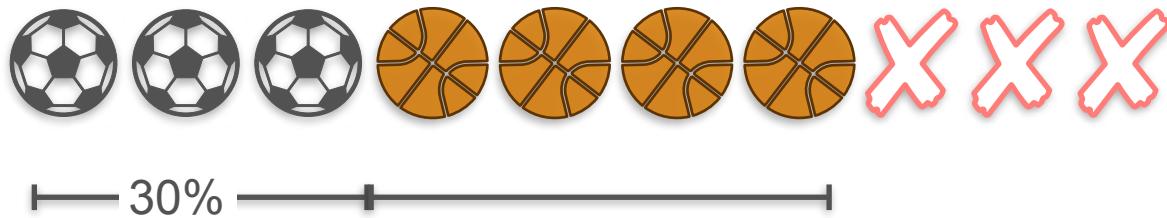
Sum of Probabilities: Quiz 1 Solution



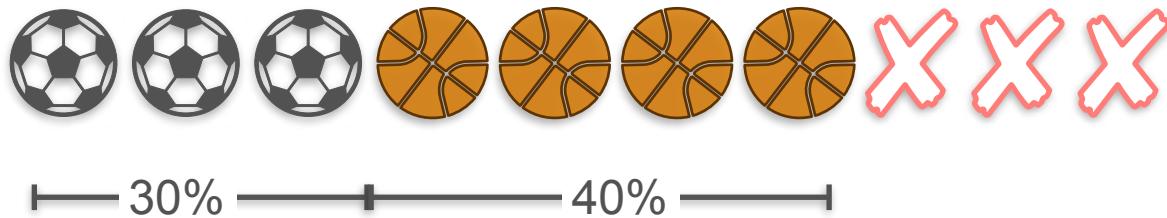
Sum of Probabilities: Quiz 1 Solution



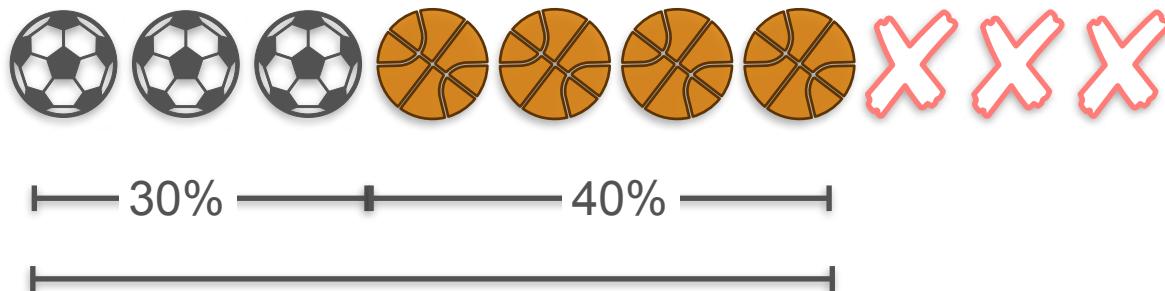
Sum of Probabilities: Quiz 1 Solution



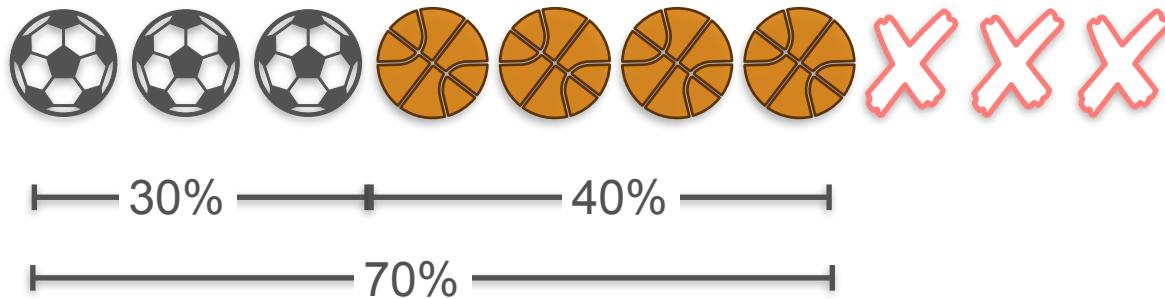
Sum of Probabilities: Quiz 1 Solution



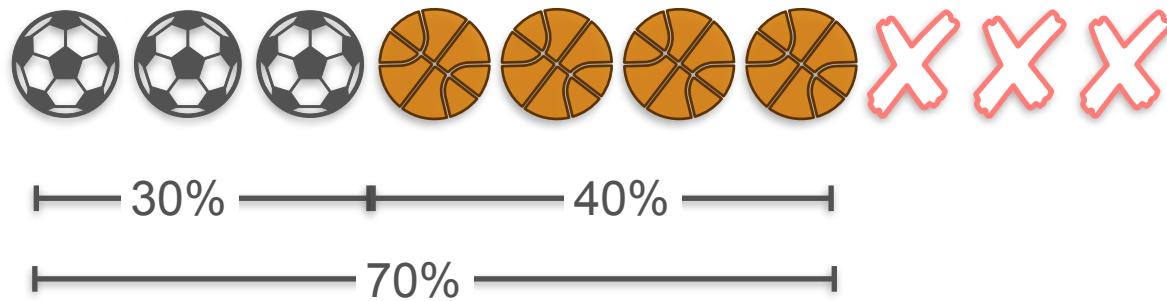
Sum of Probabilities: Quiz 1 Solution



Sum of Probabilities: Quiz 1 Solution

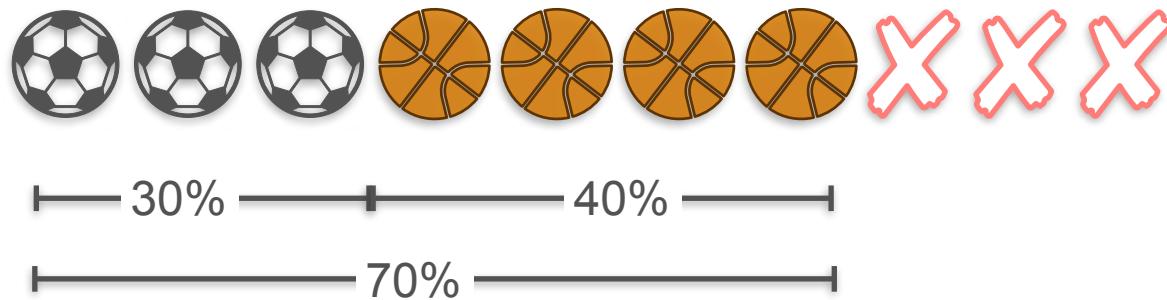


Sum of Probabilities: Quiz 1 Solution



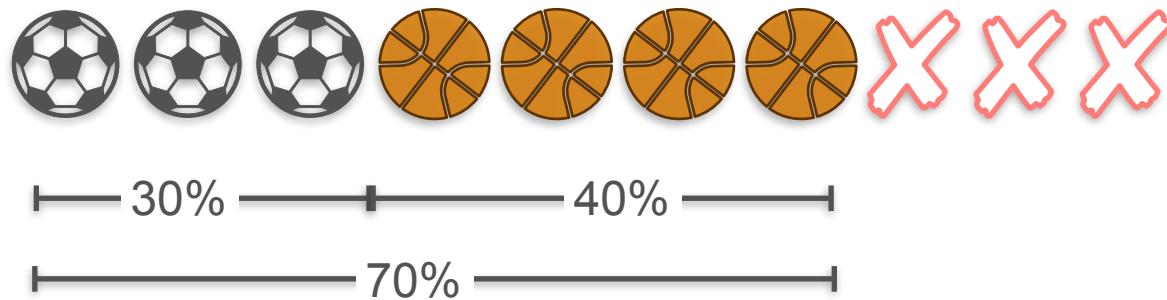
$$P(\text{soccer or basketball}) = \frac{\text{soccer or basketball}}{\text{total}}$$

Sum of Probabilities: Quiz 1 Solution



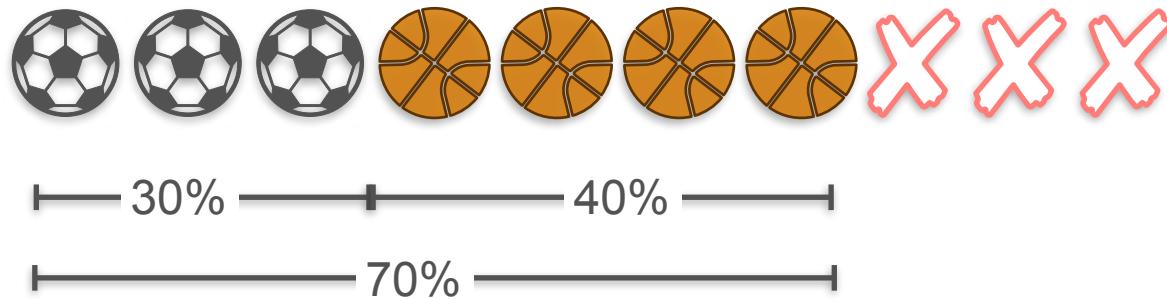
$$P(\text{soccer or basketball}) = \frac{\text{soccer or basketball}}{\text{total}} = \frac{3 + 4}{10}$$

Sum of Probabilities: Quiz 1 Solution



$$P(\text{soccer or basketball}) = \frac{\text{soccer or basketball}}{\text{total}} = \frac{3 + 4}{10} = 0.7$$

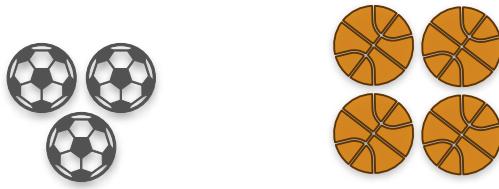
Sum of Probabilities: Quiz 1 Solution



$$P(\text{soccer or basketball}) = \frac{\text{soccer or basketball}}{\text{total}} = \frac{3 + 4}{10} = 0.7$$

$$P(\text{soccer or basketball}) = P(\text{soccer}) + P(\text{basketball})$$

Sum of Probabilities: Quiz 1 Solution



XXX

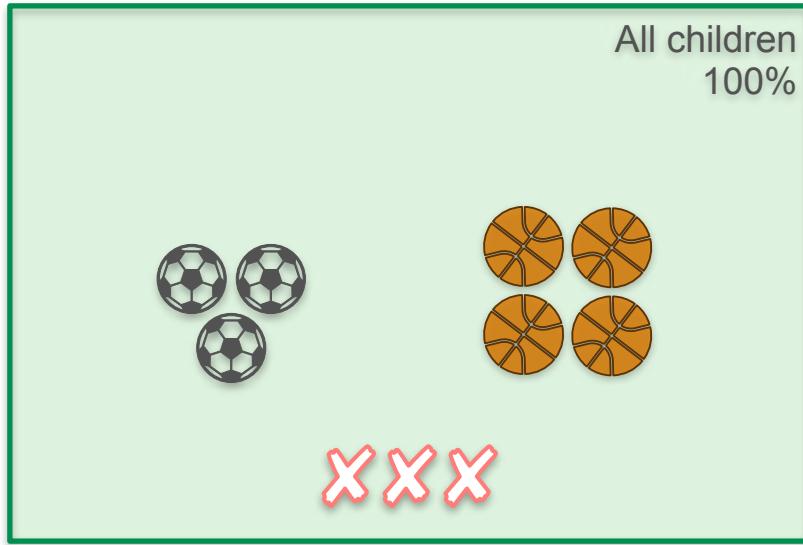
Sum of Probabilities: Quiz 1 Solution



XXX

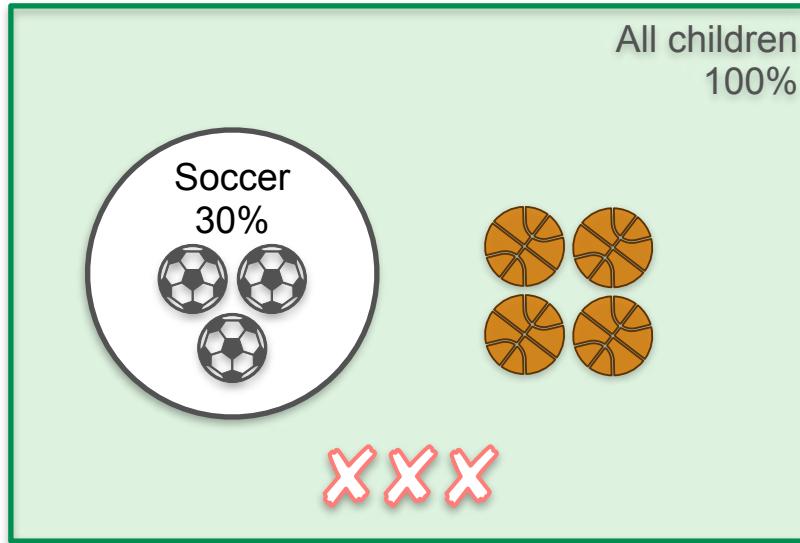
$$P(\text{soccer} \cup \text{basketball}) = P(\text{soccer}) + P(\text{basketball})$$

Sum of Probabilities: Quiz 1 Solution



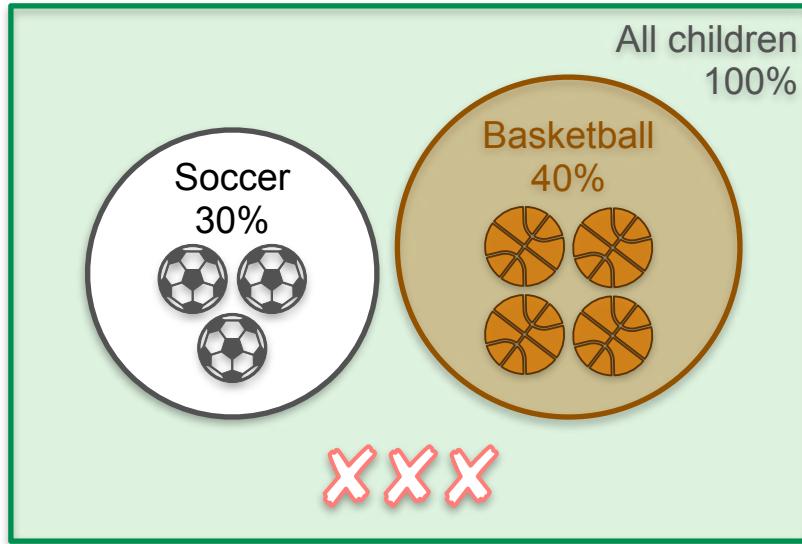
$$P(\text{soccer} \cup \text{basketball}) = P(\text{soccer}) + P(\text{basketball})$$

Sum of Probabilities: Quiz 1 Solution



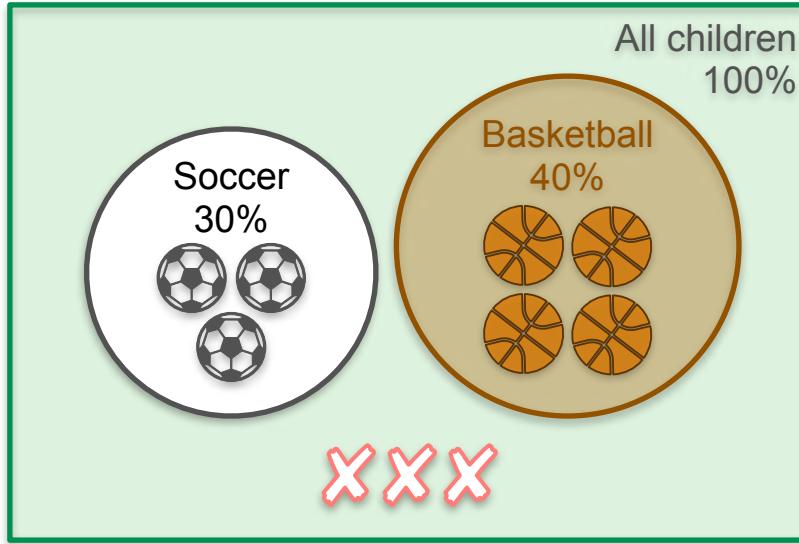
$$P(\text{soccer} \cup \text{basketball}) = P(\text{soccer}) + P(\text{basketball})$$

Sum of Probabilities: Quiz 1 Solution



$$P(\text{soccer} \cup \text{basketball}) = P(\text{soccer}) + P(\text{basketball})$$

Sum of Probabilities: Quiz 1 Solution



$$P(\text{soccer} \cup \text{basketball}) = P(\text{soccer}) + P(\text{basketball})$$

$$P(A \cup B) = P(A) + P(B)$$

Sum of Probabilities: Dice Example 1

Sum of Probabilities: Dice Example 1



Sum of Probabilities: Dice Example 1



What is the probability of obtaining
an even number or a 5?

Sum of Probabilities: Dice Example 1



What is the probability of obtaining
an even number or a 5?

Sum of Probabilities: Dice Example 1



What is the probability of obtaining an even number or a 5?

A

B

Sum of Probabilities: Dice Example 1



What is the probability of obtaining an even number or a 5?

A



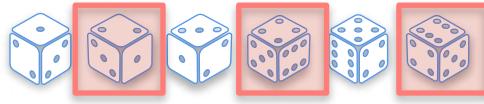
B

Sum of Probabilities: Dice Example 1



What is the probability of obtaining an even number or a 5?

A



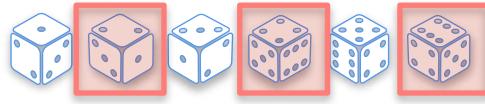
B

Sum of Probabilities: Dice Example 1



What is the probability of obtaining an even number or a 5?

A



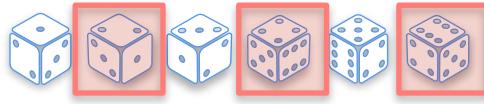
B

Sum of Probabilities: Dice Example 1



What is the probability of obtaining an even number or a 5?

A



B

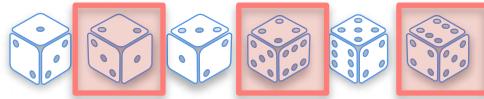


Sum of Probabilities: Dice Example 1



What is the probability of obtaining an even number or a 5?

A



B

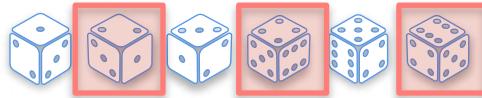


Sum of Probabilities: Dice Example 1

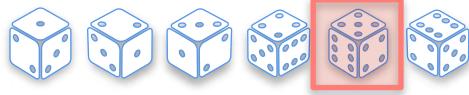


What is the probability of obtaining an even number or a 5?

A



B

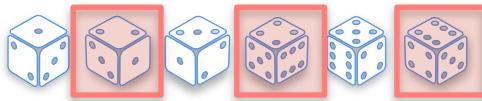


Sum of Probabilities: Dice Example 1



What is the probability of obtaining an even number or a 5?

A



B



+



Sum of Probabilities: Dice Example 1



What is the probability of obtaining an even number or a 5?

A



B



+

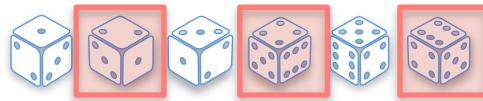


Sum of Probabilities: Dice Example 1

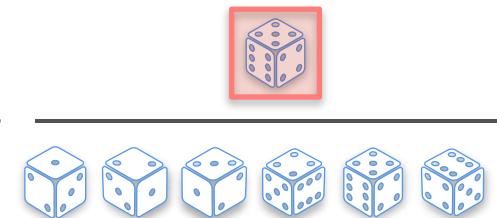
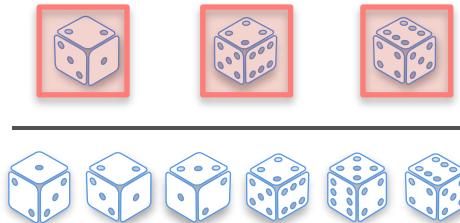


What is the probability of obtaining an even number or a 5?

A



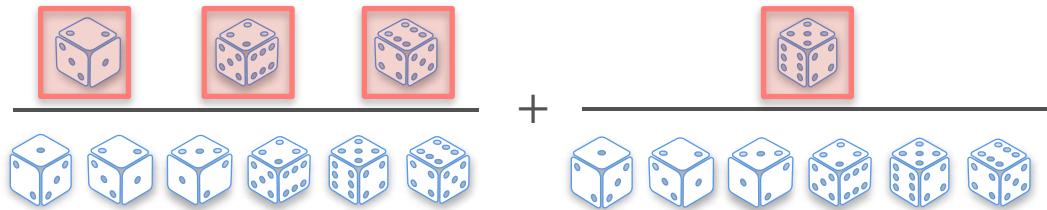
B



Sum of Probabilities: Dice Example 1



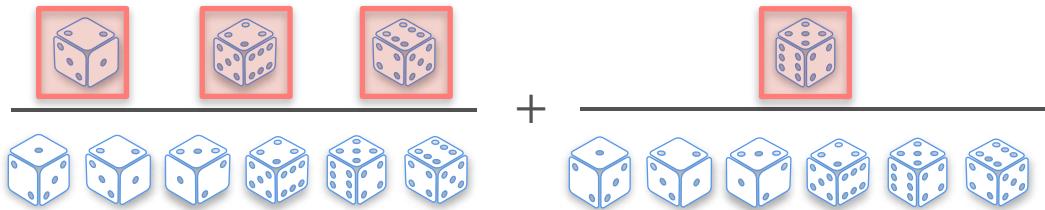
What is the probability of obtaining an even number or a 5?



Sum of Probabilities: Dice Example 1



What is the probability of obtaining an even number or a 5?

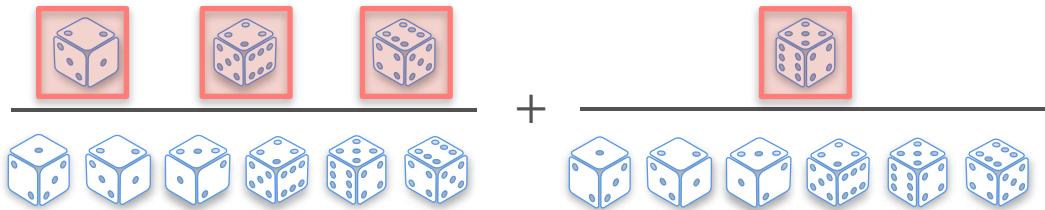


$$P(\text{even number or } 5) =$$

Sum of Probabilities: Dice Example 1



What is the probability of obtaining an even number or a 5?

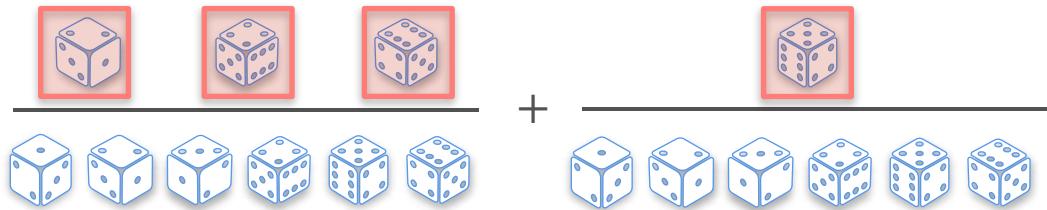


$$P(\text{even number or } 5) = P(\text{even number})$$

Sum of Probabilities: Dice Example 1



What is the probability of obtaining an even number or a 5?

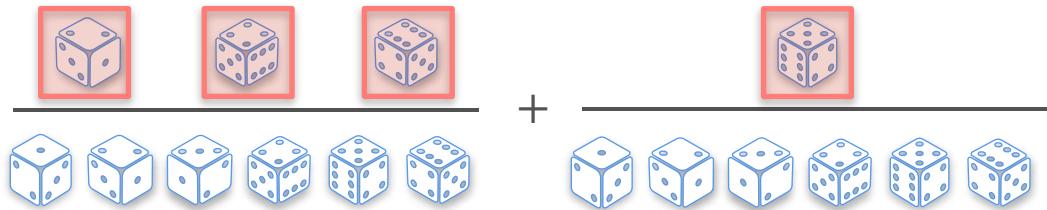


$$P(\text{even number or } 5) = P(\text{even number}) + P(5)$$

Sum of Probabilities: Dice Example 1



What is the probability of obtaining an even number or a 5?

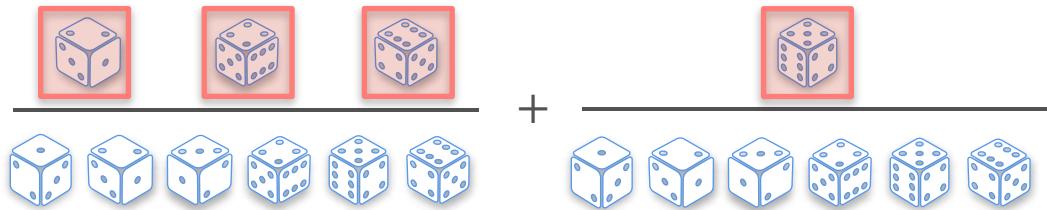


$$P(\text{even number or } 5) = P(\text{even number}) + P(5) = \frac{3}{6}$$

Sum of Probabilities: Dice Example 1



What is the probability of obtaining an even number or a 5?

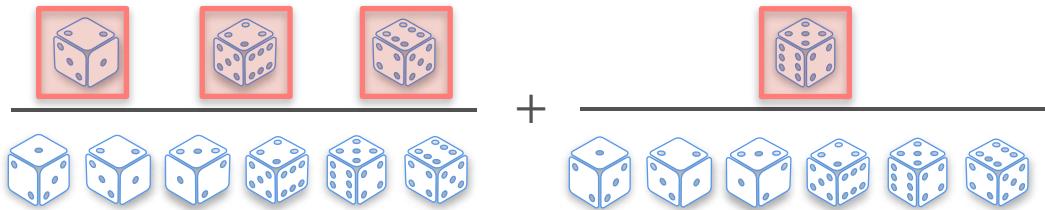


$$P(\text{even number or } 5) = P(\text{even number}) + P(5) = \frac{3}{6} + \frac{1}{6}$$

Sum of Probabilities: Dice Example 1



What is the probability of obtaining an even number or a 5?

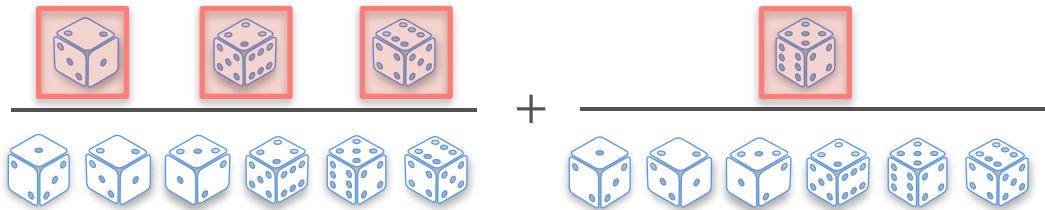


$$P(\text{even number or } 5) = P(\text{even number}) + P(5) = \frac{3}{6} + \frac{1}{6} = \frac{4}{6}$$

Sum of Probabilities: Dice Example 1



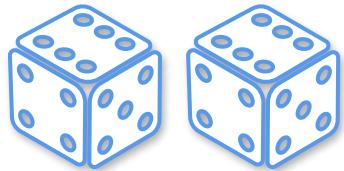
What is the probability of obtaining an even number or a 5?



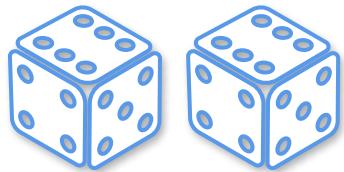
$$P(\text{even number or } 5) = P(\text{even number}) + P(5) = \frac{3}{6} + \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$$

Sum of Probabilities: Dice Example 2

Sum of Probabilities: Dice Example 2



Sum of Probabilities: Dice Example 2



What is the probability of obtaining a sum of 7 or a sum of 10?

Sum of Probabilities: Dice Example 2

Sum of Probabilities: Dice Example 2

A

B

Sum of Probabilities: Dice Example 2

A

sum of 7

B

sum of 10

Sum of Probabilities: Dice Example 2

A

sum of 7

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
1,2	3,1	3,2	3,3	3,4	3,5	3,6
1,3	4,1	4,2	4,3	4,4	4,5	4,6
1,4	5,1	5,2	5,3	5,4	5,5	5,6
1,5	6,1	6,2	6,3	6,4	6,5	6,6
1,6						

B

sum of 10

Sum of Probabilities: Dice Example 2

A

sum of 7

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
1,2	3,1	3,2	3,3	3,4	3,5	3,6
1,3	4,1	4,2	4,3	4,4	4,5	4,6
1,4	5,1	5,2	5,3	5,4	5,5	5,6
1,5	6,1	6,2	6,3	6,4	6,5	6,6

B

sum of 10

Sum of Probabilities: Dice Example 2

A

sum of 7

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

B

sum of 10

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

Sum of Probabilities: Dice Example 2

A or *B*

sum of 7 or sum of 10

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
1,2	3,1	3,2	3,3	3,4	3,5	3,6
1,3	4,1	4,2	4,3	4,4	4,5	4,6
1,4	5,1	5,2	5,3	5,4	5,5	5,6
1,5	6,1	6,2	6,3	6,4	6,5	6,6

Sum of Probabilities: Dice Example 2

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,1	6,2	6,3	6,4	6,5	6,6

Sum of Probabilities: Dice Example 2

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

P(sum of 7 or sum of 10)

Sum of Probabilities: Dice Example 2

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

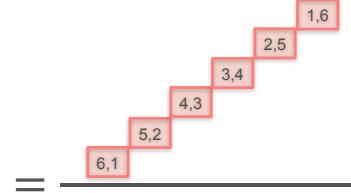
P(sum of 7 or sum of 10)

= _____

Sum of Probabilities: Dice Example 2

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

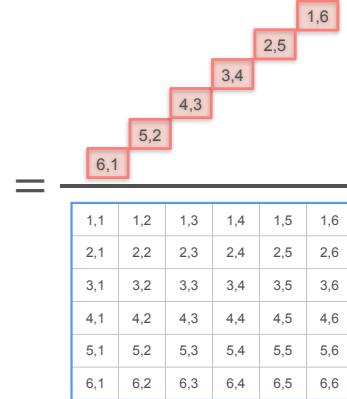
P(sum of 7 or sum of 10)



Sum of Probabilities: Dice Example 2

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

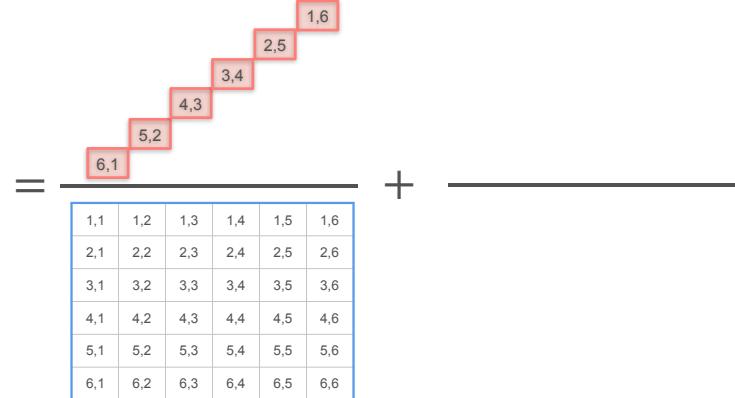
P(sum of 7 or sum of 10)



Sum of Probabilities: Dice Example 2



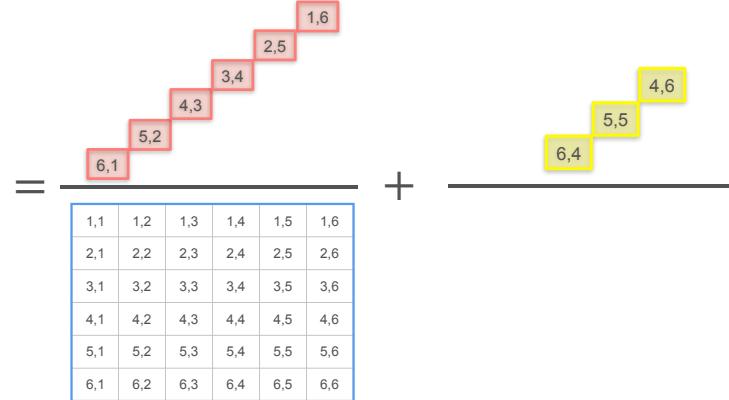
P(sum of 7 or sum of 10)



Sum of Probabilities: Dice Example 2

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

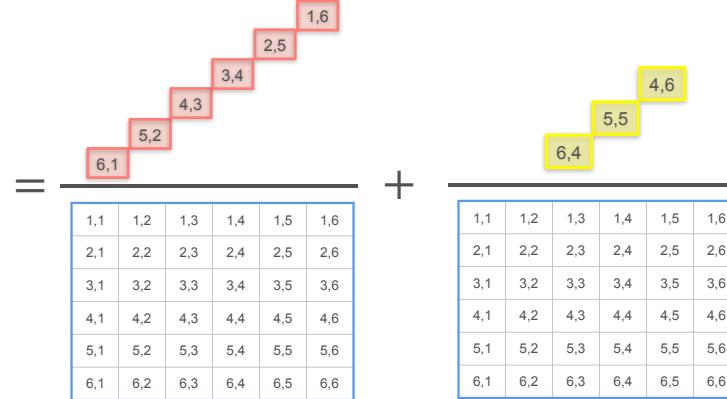
P(sum of 7 or sum of 10)



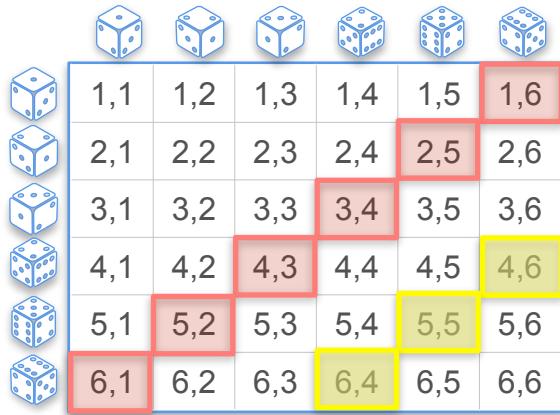
Sum of Probabilities: Dice Example 2

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

P(sum of 7 or sum of 10)



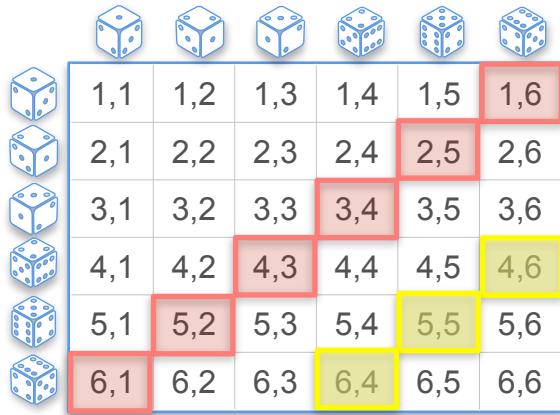
Sum of Probabilities: Dice Example 2



P(sum of 7 or sum of 10)

$$\begin{array}{c}
 \begin{array}{c}
 = - \frac{1}{36} \left[\begin{array}{cccccc}
 1,1 & 1,2 & 1,3 & 1,4 & 1,5 & 1,6 \\
 2,1 & 2,2 & 2,3 & 2,4 & 2,5 & 2,6 \\
 3,1 & 3,2 & 3,3 & 3,4 & 3,5 & 3,6 \\
 4,1 & 4,2 & 4,3 & 4,4 & 4,5 & 4,6 \\
 5,1 & 5,2 & 5,3 & 5,4 & 5,5 & 5,6 \\
 6,1 & 6,2 & 6,3 & 6,4 & 6,5 & 6,6
 \end{array} \right] + \begin{array}{cccccc}
 1,1 & 1,2 & 1,3 & 1,4 & 1,5 & 1,6 \\
 2,1 & 2,2 & 2,3 & 2,4 & 2,5 & 2,6 \\
 3,1 & 3,2 & 3,3 & 3,4 & 3,5 & 3,6 \\
 4,1 & 4,2 & 4,3 & 4,4 & 4,5 & 4,6 \\
 5,1 & 5,2 & 5,3 & 5,4 & 5,5 & 5,6 \\
 6,1 & 6,2 & 6,3 & 6,4 & 6,5 & 6,6
 \end{array} \right] \\
 \end{array}
 \end{array}$$

Sum of Probabilities: Dice Example 2



P(sum of 7 or sum of 10)

$$\begin{array}{c}
 = \frac{6}{36} + \frac{3}{36} \\
 = \frac{9}{36} \\
 = \frac{1}{4}
 \end{array}$$

Sum of Probabilities: Dice Example 2

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

P(sum of 7 or sum of 10)

$$\begin{aligned} &= \frac{\begin{array}{c} 1,6 \\ 2,5 \\ 3,4 \\ 4,3 \\ 5,2 \\ 6,1 \end{array}}{\begin{array}{ccccccc} 1,1 & 1,2 & 1,3 & 1,4 & 1,5 & 1,6 \\ 2,1 & 2,2 & 2,3 & 2,4 & 2,5 & 2,6 \\ 3,1 & 3,2 & 3,3 & 3,4 & 3,5 & 3,6 \\ 4,1 & 4,2 & 4,3 & 4,4 & 4,5 & 4,6 \\ 5,1 & 5,2 & 5,3 & 5,4 & 5,5 & 5,6 \\ 6,1 & 6,2 & 6,3 & 6,4 & 6,5 & 6,6 \end{array}} + \frac{\begin{array}{c} 1,6 \\ 2,5 \\ 3,4 \\ 4,3 \\ 5,2 \\ 6,1 \end{array}}{\begin{array}{ccccccc} 1,1 & 1,2 & 1,3 & 1,4 & 1,5 & 1,6 \\ 2,1 & 2,2 & 2,3 & 2,4 & 2,5 & 2,6 \\ 3,1 & 3,2 & 3,3 & 3,4 & 3,5 & 3,6 \\ 4,1 & 4,2 & 4,3 & 4,4 & 4,5 & 4,6 \\ 5,1 & 5,2 & 5,3 & 5,4 & 5,5 & 5,6 \\ 6,1 & 6,2 & 6,3 & 6,4 & 6,5 & 6,6 \end{array}} \\ &= \frac{6}{36} + \frac{3}{36} \\ &= \frac{9}{36} \end{aligned}$$

Sum of Probabilities: Dice Example 2

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

P(sum of 7 or sum of 10)

$$\begin{aligned} &= \frac{\begin{array}{c} 1,6 \\ 2,5 \\ 3,4 \\ 4,3 \\ 5,2 \\ 6,1 \end{array}}{\begin{array}{ccccccc} 1,1 & 1,2 & 1,3 & 1,4 & 1,5 & 1,6 \\ 2,1 & 2,2 & 2,3 & 2,4 & 2,5 & 2,6 \\ 3,1 & 3,2 & 3,3 & 3,4 & 3,5 & 3,6 \\ 4,1 & 4,2 & 4,3 & 4,4 & 4,5 & 4,6 \\ 5,1 & 5,2 & 5,3 & 5,4 & 5,5 & 5,6 \\ 6,1 & 6,2 & 6,3 & 6,4 & 6,5 & 6,6 \end{array}} + \frac{\begin{array}{c} 1,6 \\ 2,5 \\ 3,4 \\ 4,3 \\ 5,2 \\ 6,1 \end{array}}{\begin{array}{ccccccc} 1,1 & 1,2 & 1,3 & 1,4 & 1,5 & 1,6 \\ 2,1 & 2,2 & 2,3 & 2,4 & 2,5 & 2,6 \\ 3,1 & 3,2 & 3,3 & 3,4 & 3,5 & 3,6 \\ 4,1 & 4,2 & 4,3 & 4,4 & 4,5 & 4,6 \\ 5,1 & 5,2 & 5,3 & 5,4 & 5,5 & 5,6 \\ 6,1 & 6,2 & 6,3 & 6,4 & 6,5 & 6,6 \end{array}} \\ &= \frac{6}{36} + \frac{3}{36} \\ &= \frac{9}{36} = \frac{1}{4} \end{aligned}$$

Sum of Probabilities: Dice Example 2

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

P(sum of 7 or sum of 10)

$$\begin{aligned} &= \boxed{P(\text{sum of 7})} + \frac{\begin{array}{|c|c|c|c|c|c|c|}\hline 1,1 & 1,2 & 1,3 & 1,4 & 1,5 & 1,6 & \\ \hline 2,1 & 2,2 & 2,3 & 2,4 & 2,5 & 2,6 & \\ \hline 3,1 & 3,2 & 3,3 & 3,4 & 3,5 & 3,6 & \\ \hline 4,1 & 4,2 & 4,3 & 4,4 & 4,5 & 4,6 & \\ \hline 5,1 & 5,2 & 5,3 & 5,4 & 5,5 & 5,6 & \\ \hline 6,1 & 6,2 & 6,3 & 6,4 & 6,5 & 6,6 & \\ \hline \end{array}}{36} \\ &= \frac{6}{36} + \frac{3}{36} \\ &= \frac{9}{36} = \frac{1}{4} \end{aligned}$$

Sum of Probabilities: Dice Example 2

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

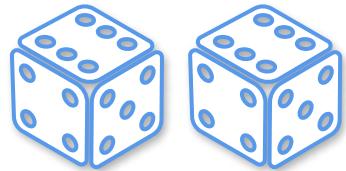
P(sum of 7 or sum of 10)

$$\begin{aligned} &= P(\text{sum of 7}) + P(\text{sum of 10}) \\ &= \frac{6}{36} + \frac{3}{36} \\ &= \frac{9}{36} = \frac{1}{4} \end{aligned}$$

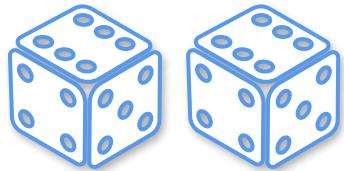
The diagram illustrates the calculation of the probability of rolling a sum of 7 or 10 with two six-sided dice. It shows a 6x6 grid of outcomes. Outcomes where the sum is 7 are highlighted in red boxes (e.g., (1,6), (2,5), (3,4), (4,3), (5,2), (6,1)). Outcomes where the sum is 10 are highlighted in yellow boxes (e.g., (4,6), (5,5), (6,4)). The total number of favorable outcomes is 9, while the total number of possible outcomes is 36.

Sum of Probabilities: Dice Example 3

Sum of Probabilities: Dice Example 3



Sum of Probabilities: Dice Example 3



What is the probability of obtaining
a difference of 2 or a difference of 1?

Sum of Probabilities: Dice Example 3

Sum of Probabilities: Dice Example 3

A

diff = 2

B

diff = 1

Sum of Probabilities: Dice Example 3

A

diff = 2

	1,1	1,2	1,3	1,4	1,5	1,6
1,1						
1,2						
1,3						
1,4						
1,5						
1,6						
2,1	1,1	1,2	1,3	1,4	1,5	1,6
2,2	2,1	2,2	2,3	2,4	2,5	2,6
2,3	3,1	3,2	3,3	3,4	3,5	3,6
2,4	4,1	4,2	4,3	4,4	4,5	4,6
2,5	5,1	5,2	5,3	5,4	5,5	5,6
2,6	6,1	6,2	6,3	6,4	6,5	6,6

B

diff = 1

Sum of Probabilities: Dice Example 3

A

diff = 2

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,1	6,2	6,3	6,4	6,5	6,6

B

diff = 1

Sum of Probabilities: Dice Example 3

A

diff = 2

		dice					
		1,1	1,2	1,3	1,4	1,5	1,6
		2,1	2,2	2,3	2,4	2,5	2,6
		3,1	3,2	3,3	3,4	3,5	3,6
		4,1	4,2	4,3	4,4	4,5	4,6
		5,1	5,2	5,3	5,4	5,5	5,6
		6,1	6,2	6,3	6,4	6,5	6,6

B

diff = 1

		dice					
		1,1	1,2	1,3	1,4	1,5	1,6
		2,1	2,2	2,3	2,4	2,5	2,6
		3,1	3,2	3,3	3,4	3,5	3,6
		4,1	4,2	4,3	4,4	4,5	4,6
		5,1	5,2	5,3	5,4	5,5	5,6
		6,1	6,2	6,3	6,4	6,5	6,6

Sum of Probabilities: Dice Example 3

A

diff = 2

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,1	6,2	6,3	6,4	6,5	6,6

B

diff = 1

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,1	6,2	6,3	6,4	6,5	6,6

Sum of Probabilities: Dice Example 3

A

diff = 2

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,1	6,2	6,3	6,4	6,5	6,6

B

diff = 1

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,1	6,2	6,3	6,4	6,5	6,6

Sum of Probabilities: Dice Example 3

A or *B*

diff = 2 or diff = 1

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6

Dice icons are placed along the top row and left column of the grid.

Sum of Probabilities: Dice Example 3

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6
6,1						

Sum of Probabilities: Dice Example 3

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$P(\text{diff} = 2 \text{ or } \text{diff} = 1)$

Sum of Probabilities: Dice Example 3

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

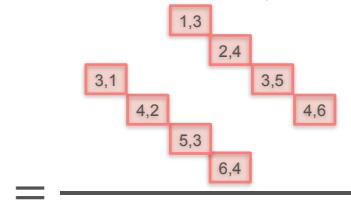
$P(\text{diff} = 2 \text{ or } \text{diff} = 1)$

= _____

Sum of Probabilities: Dice Example 3

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

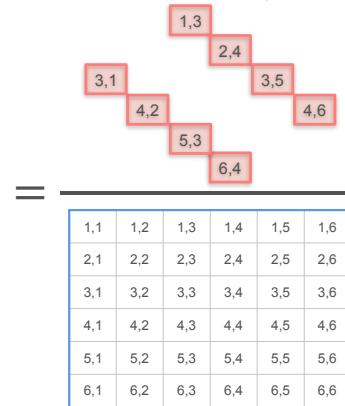
$P(\text{diff} = 2 \text{ or } \text{diff} = 1)$



Sum of Probabilities: Dice Example 3

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$P(\text{diff} = 2 \text{ or } \text{diff} = 1)$



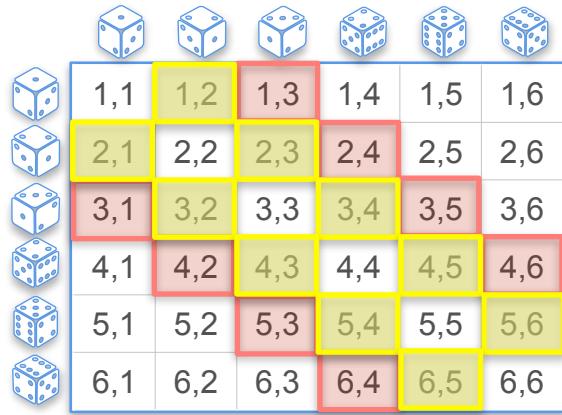
Sum of Probabilities: Dice Example 3

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$P(\text{diff} = 2 \text{ or } \text{diff} = 1)$

$$= \boxed{\begin{array}{ccccccc} & & 1,3 & & 2,4 & & \\ & & 3,1 & 4,2 & & 5,3 & 6,4 \\ & & & & 5,3 & & \\ & & & & & 6,4 & \\ \hline 1,1 & 1,2 & 1,3 & 1,4 & 1,5 & 1,6 & \\ 2,1 & 2,2 & 2,3 & 2,4 & 2,5 & 2,6 & \\ 3,1 & 3,2 & 3,3 & 3,4 & 3,5 & 3,6 & \\ 4,1 & 4,2 & 4,3 & 4,4 & 4,5 & 4,6 & \\ 5,1 & 5,2 & 5,3 & 5,4 & 5,5 & 5,6 & \\ 6,1 & 6,2 & 6,3 & 6,4 & 6,5 & 6,6 & \end{array}} + \boxed{\begin{array}{ccccccc} & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \end{array}}$$

Sum of Probabilities: Dice Example 3

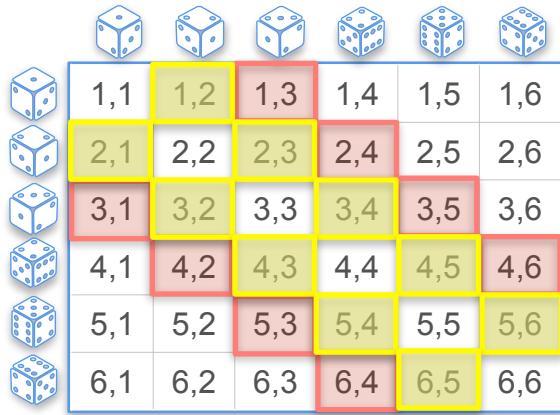


$P(\text{diff} = 2 \text{ or } \text{diff} = 1)$

$$= \begin{array}{c} \text{red boxes: } (1,3), (2,4), (3,5), (4,6), (5,3), (6,4) \\ \text{yellow boxes: } (1,2), (2,3), (3,2), (4,3), (5,4), (6,5) \\ \text{orange boxes: } (2,1), (3,1), (4,2), (5,2), (6,1) \\ \text{purple boxes: } (1,1), (2,2), (3,3), (4,4), (5,5), (6,6) \end{array} + \begin{array}{c} \text{red boxes: } (1,3), (2,4), (3,5), (4,6) \\ \text{yellow boxes: } (1,2), (2,3), (3,4), (4,5), (5,6), (6,5) \\ \text{orange boxes: } (2,1), (3,2), (4,3), (5,4), (6,1) \\ \text{purple boxes: } (1,1), (2,2), (3,3), (4,4), (5,5), (6,6) \end{array}$$

=

Sum of Probabilities: Dice Example 3

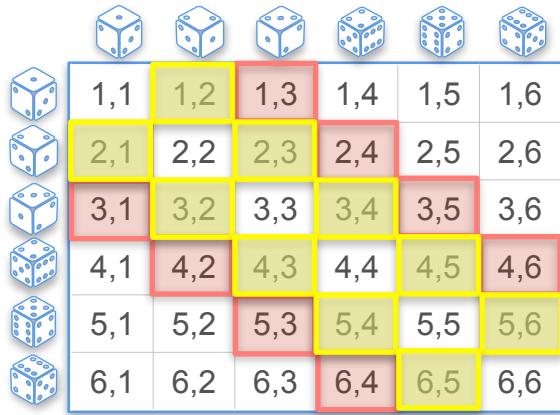


$P(\text{diff} = 2 \text{ or } \text{diff} = 1)$

$$= \begin{array}{c} \begin{array}{ccccccc} & & 1,3 & & 2,4 & & 3,5 & \\ & & 3,1 & 4,2 & & 5,3 & 6,4 & \\ & & 4,1 & & 5,4 & & 6,5 & \\ & & 5,1 & 5,2 & 5,3 & 5,4 & 5,5 & 5,6 & \\ & & 6,1 & 6,2 & 6,3 & 6,4 & 6,5 & 6,6 & \end{array} \\ + \end{array} \begin{array}{c} \begin{array}{ccccccc} 1,2 & & 2,3 & & 3,4 & & 4,5 & \\ 2,1 & & 2,2 & 3,3 & & 4,4 & 5,5 & 6,6 \\ 3,2 & & 3,3 & 3,4 & 4,5 & & 5,6 & \\ 4,3 & & 4,4 & 4,5 & 4,6 & 5,6 & & 6,5 \\ 5,4 & & 5,5 & 5,6 & & 6,6 & & \end{array} \end{array}$$

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

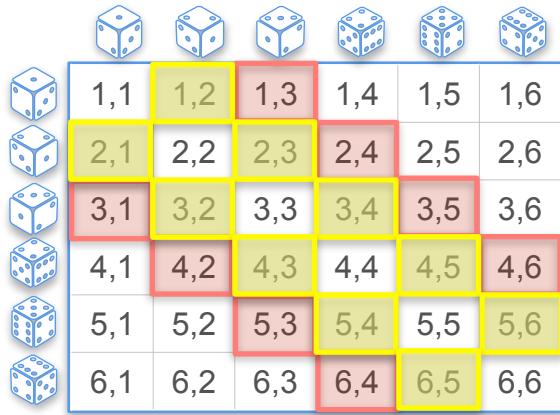
Sum of Probabilities: Dice Example 3



$P(\text{diff} = 2 \text{ or } \text{diff} = 1)$

$$= \frac{\begin{array}{c} 1,3 \\ 2,4 \\ 3,5 \\ 4,6 \\ 5,3 \\ 6,4 \end{array}}{36} + \frac{\begin{array}{c} 1,2 \\ 2,3 \\ 3,4 \\ 4,5 \\ 5,6 \\ 6,5 \end{array}}{36}$$
$$= \frac{8}{36}$$

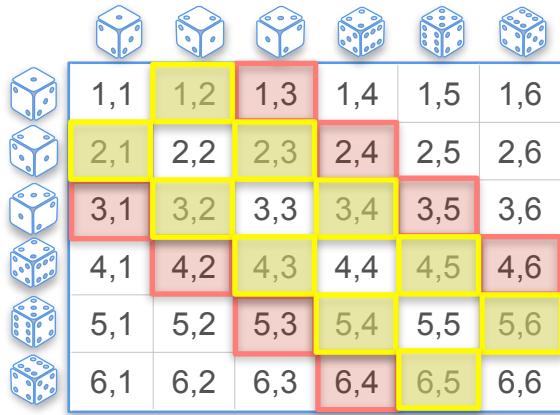
Sum of Probabilities: Dice Example 3



$P(\text{diff} = 2 \text{ or } \text{diff} = 1)$

$$\begin{aligned} &= \frac{\begin{array}{c} 1,3 \\ 2,4 \\ 3,5 \\ 4,6 \\ 5,3 \\ 6,4 \end{array}}{\begin{array}{c} 1,1 \\ 1,2 \\ 1,3 \\ 1,4 \\ 1,5 \\ 1,6 \\ 2,1 \\ 2,2 \\ 2,3 \\ 2,4 \\ 2,5 \\ 2,6 \\ 3,1 \\ 3,2 \\ 3,3 \\ 3,4 \\ 3,5 \\ 3,6 \\ 4,1 \\ 4,2 \\ 4,3 \\ 4,4 \\ 4,5 \\ 4,6 \\ 5,1 \\ 5,2 \\ 5,3 \\ 5,4 \\ 5,5 \\ 5,6 \\ 6,1 \\ 6,2 \\ 6,3 \\ 6,4 \\ 6,5 \\ 6,6 \end{array}} + \frac{\begin{array}{c} 1,2 \\ 2,3 \\ 3,4 \\ 4,5 \\ 5,6 \\ 6,5 \end{array}}{\begin{array}{c} 1,1 \\ 1,2 \\ 1,3 \\ 1,4 \\ 1,5 \\ 1,6 \\ 2,1 \\ 2,2 \\ 2,3 \\ 2,4 \\ 2,5 \\ 2,6 \\ 3,1 \\ 3,2 \\ 3,3 \\ 3,4 \\ 3,5 \\ 3,6 \\ 4,1 \\ 4,2 \\ 4,3 \\ 4,4 \\ 4,5 \\ 4,6 \\ 5,1 \\ 5,2 \\ 5,3 \\ 5,4 \\ 5,5 \\ 5,6 \\ 6,1 \\ 6,2 \\ 6,3 \\ 6,4 \\ 6,5 \\ 6,6 \end{array}} \\ &= \frac{8}{36} + \frac{10}{36} \end{aligned}$$

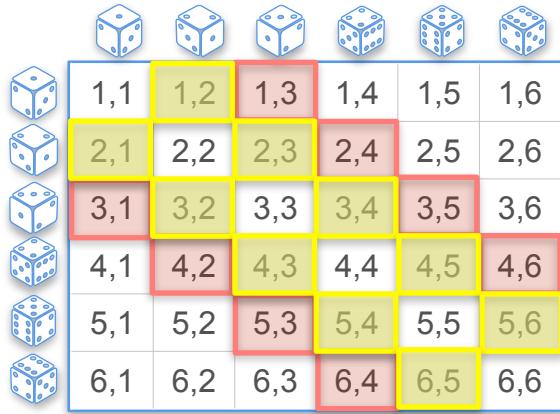
Sum of Probabilities: Dice Example 3



$P(\text{diff} = 2 \text{ or } \text{diff} = 1)$

$$\begin{aligned} &= \frac{\begin{array}{c} 1,3 \\ 2,4 \\ 3,5 \\ 4,6 \\ 5,3 \\ 6,4 \end{array}}{36} + \frac{\begin{array}{c} 1,2 \\ 2,3 \\ 3,4 \\ 4,5 \\ 5,6 \\ 6,5 \end{array}}{36} \\ &= \frac{8}{36} + \frac{10}{36} \\ &= \frac{18}{36} \end{aligned}$$

Sum of Probabilities: Dice Example 3

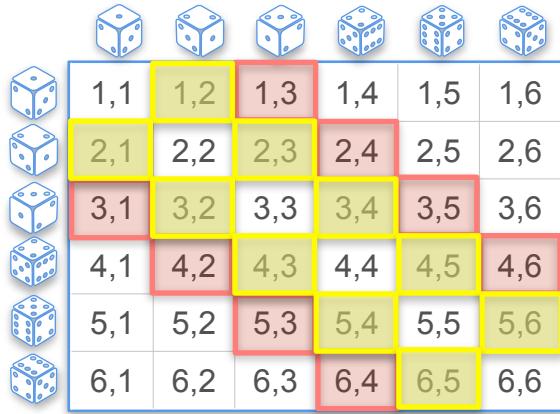


$P(\text{diff} = 2 \text{ or } \text{diff} = 1)$

$$\begin{aligned} &= P(\text{diff} = 2) + \\ &\quad \begin{array}{c} \text{---} \\ \begin{matrix} 1,2 \\ 2,1 \\ 3,2 \\ 4,3 \\ 5,4 \\ 6,5 \end{matrix} \end{array} \\ &= \frac{8}{36} + \frac{10}{36} \\ &= \frac{18}{36} \end{aligned}$$

The diagram illustrates the probability calculation. It shows the total sample space of 36 outcomes as a 6x6 grid. The outcomes where the difference is 2 are highlighted in yellow. The outcomes where the difference is 1 are highlighted in red. The intersection of these two sets (outcomes where the difference is both 1 and 2) is shown in white, indicating they are counted in both sets.

Sum of Probabilities: Dice Example 3



$P(\text{diff} = 2 \text{ or } \text{diff} = 1)$

$$= P(\text{diff} = 2) + P(\text{diff} = 1)$$

$= \frac{8}{36} + \frac{10}{36}$

$= \frac{18}{36}$

The diagram illustrates the calculation of the probability of the difference being 2 or 1. It shows two sets of outcomes highlighted in boxes:

- P(diff = 2):** Outcomes where the absolute difference between the two dice rolls is 2. These are: (1,3), (2,4), (3,5), (4,6), (2,1), (3,2), (4,3), (5,4), (1,5), (2,6), (3,1), (4,2), (5,3), (6,1), (1,7), (2,8), (3,9), (4,10), (5,11), and (6,12). There are 8 such outcomes.
- P(diff = 1):** Outcomes where the absolute difference between the two dice rolls is 1. These are: (1,2), (2,1), (1,4), (4,1), (2,3), (3,2), (1,6), (6,1), (2,5), (5,2), (3,4), (4,3), (3,6), (6,3), (4,5), (5,4), (4,2), (2,4), (3,1), (1,3), (5,1), (1,5), (6,2), (2,6), (3,5), (5,3), (4,6), (6,4), (5,2), (2,5), (3,4), (4,3), (5,1), (1,4), (4,1), (2,1), (1,2), (3,6), (6,3), (4,5), (5,4), (3,2), (2,3), (4,1), (1,4), (5,3), (3,5), (6,1), (1,6), (6,1), (2,4), (4,2), (3,1), (1,3), (5,2), (2,5), (4,6), (6,4), (5,1), (1,5), (6,2), (2,6), (3,4), (4,3), (5,2), (2,5), (3,1), (1,3), (5,1), (1,5), (6,1), (1,7), (7,1), (2,8), (8,2), (3,9), (9,3), (4,10), (10,4), (5,11), (11,5), and (6,12). There are 10 such outcomes.



DeepLearning.AI

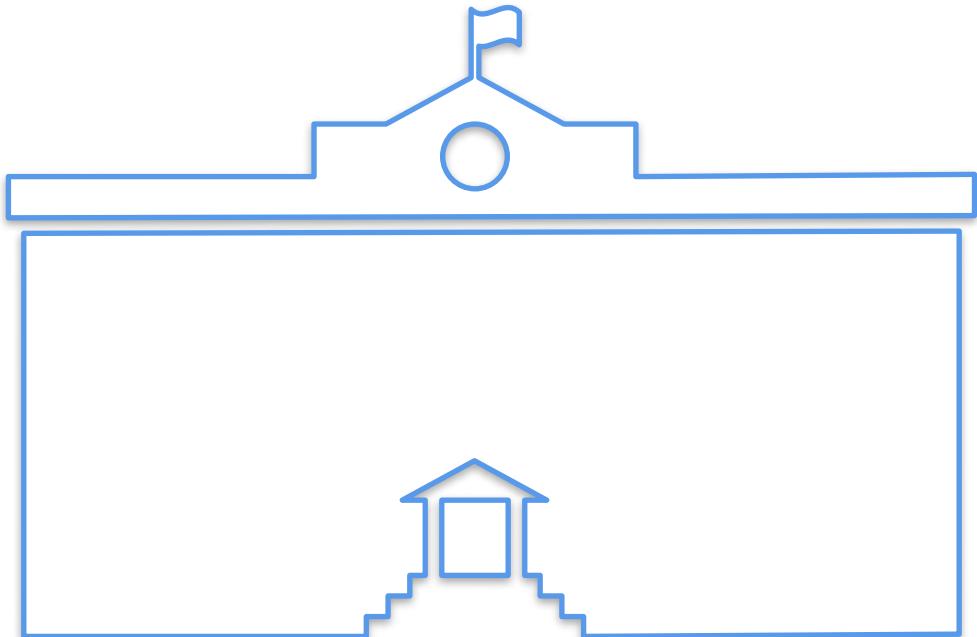
Introduction to probability

**Sum of Probabilities
(Joint Events)**

Sum of Probabilities (Joint Events): Quiz 1

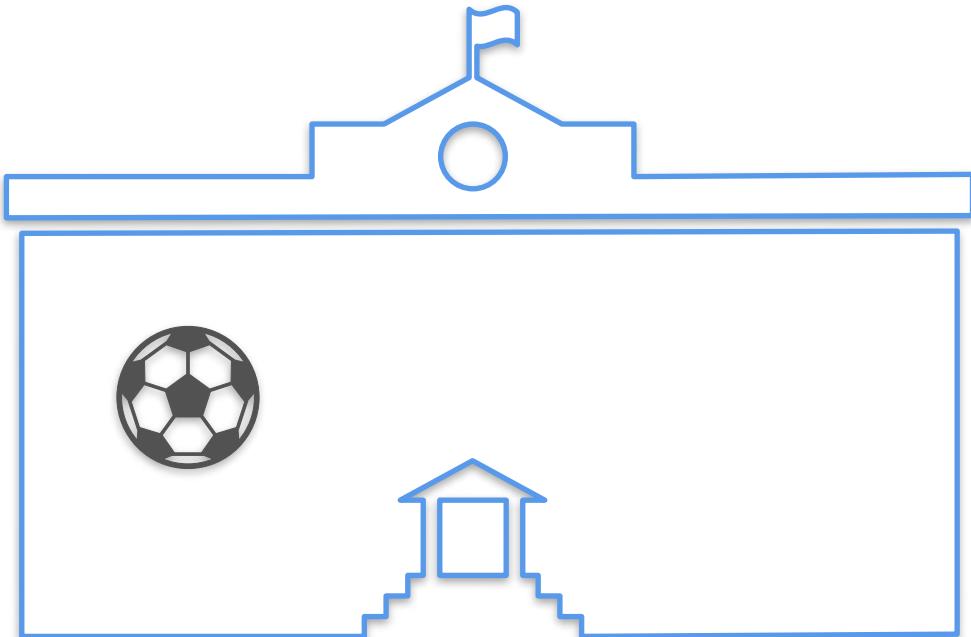
At a school, kids can play as many sports as they want.

Sum of Probabilities (Joint Events): Quiz 1



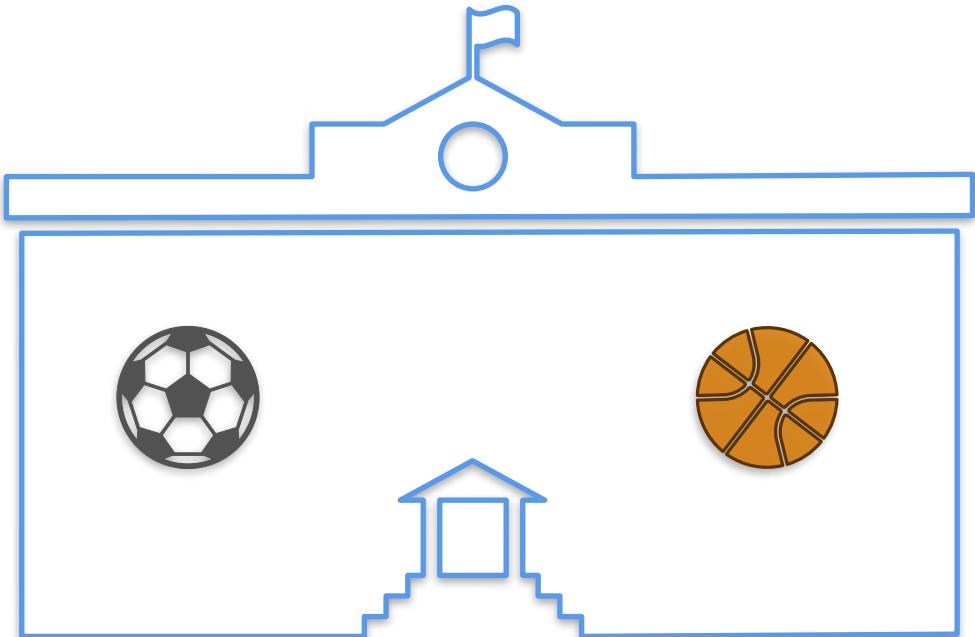
At a school, kids can play as many sports as they want.

Sum of Probabilities (Joint Events): Quiz 1



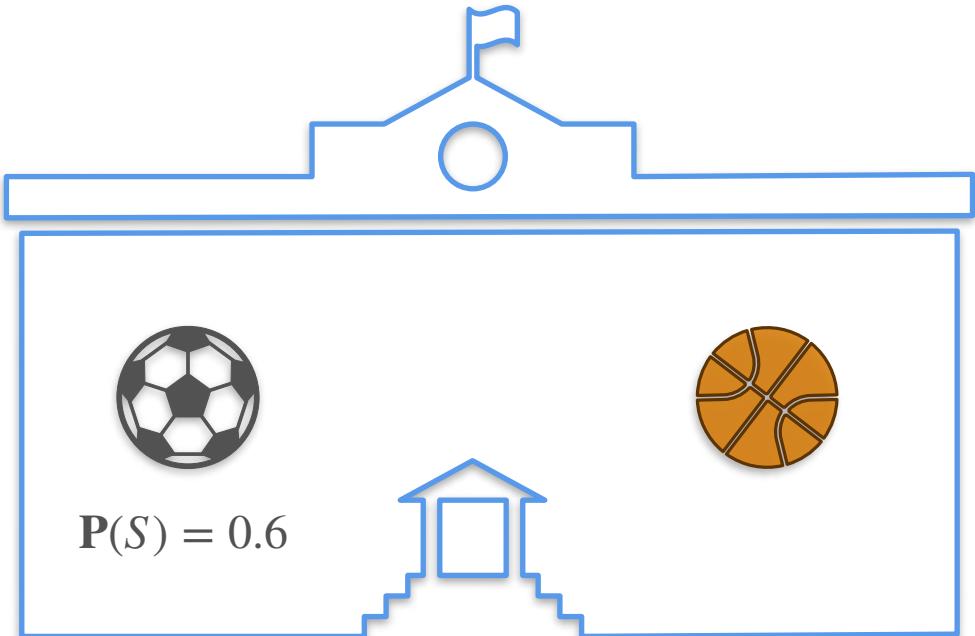
At a school, kids can play as many sports as they want.

Sum of Probabilities (Joint Events): Quiz 1



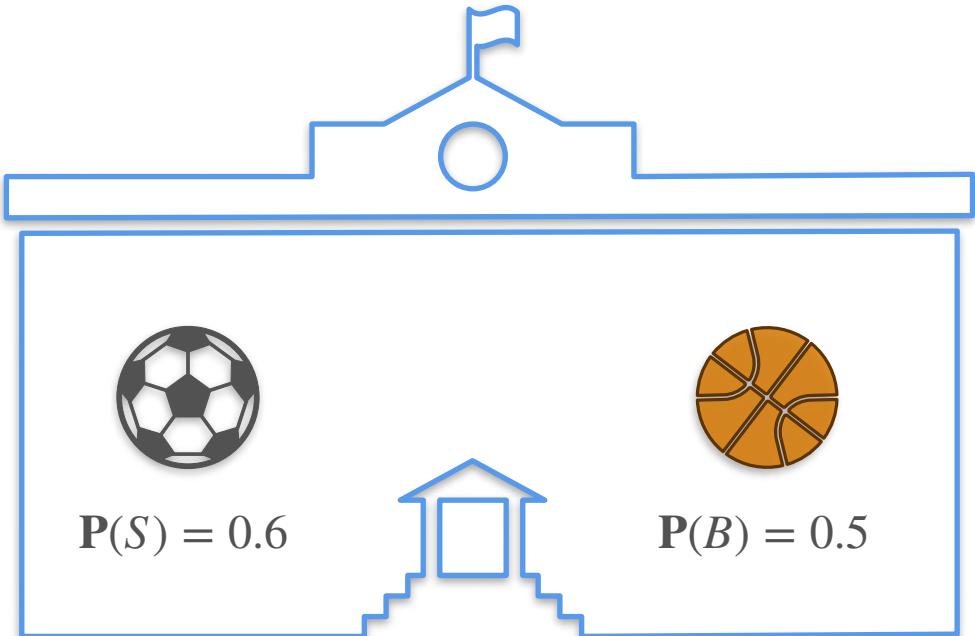
At a school, kids can play as many sports as they want.

Sum of Probabilities (Joint Events): Quiz 1



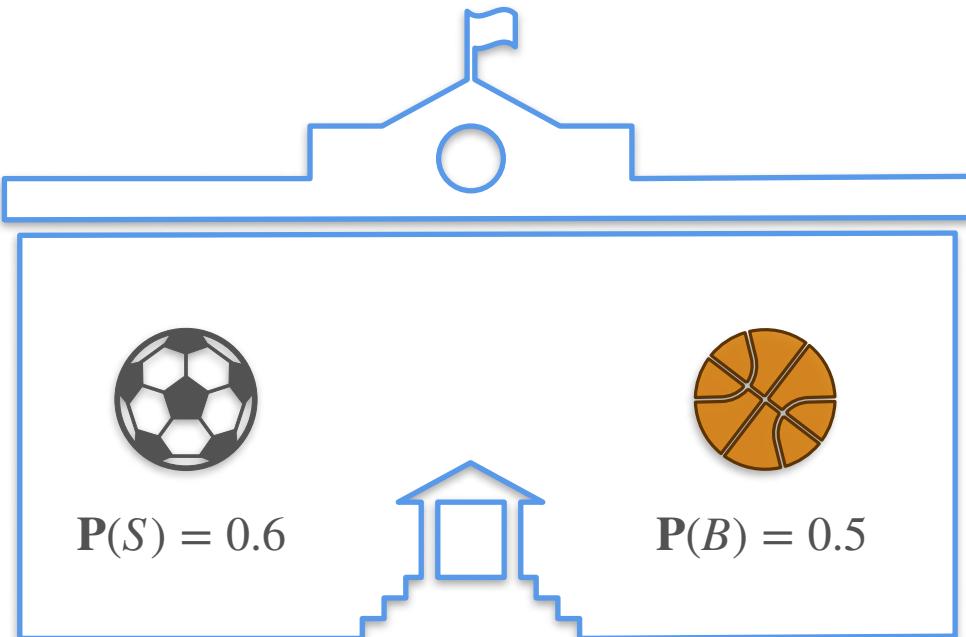
At a school, kids can play as many sports as they want.

Sum of Probabilities (Joint Events): Quiz 1



At a school, kids can play as many sports as they want.

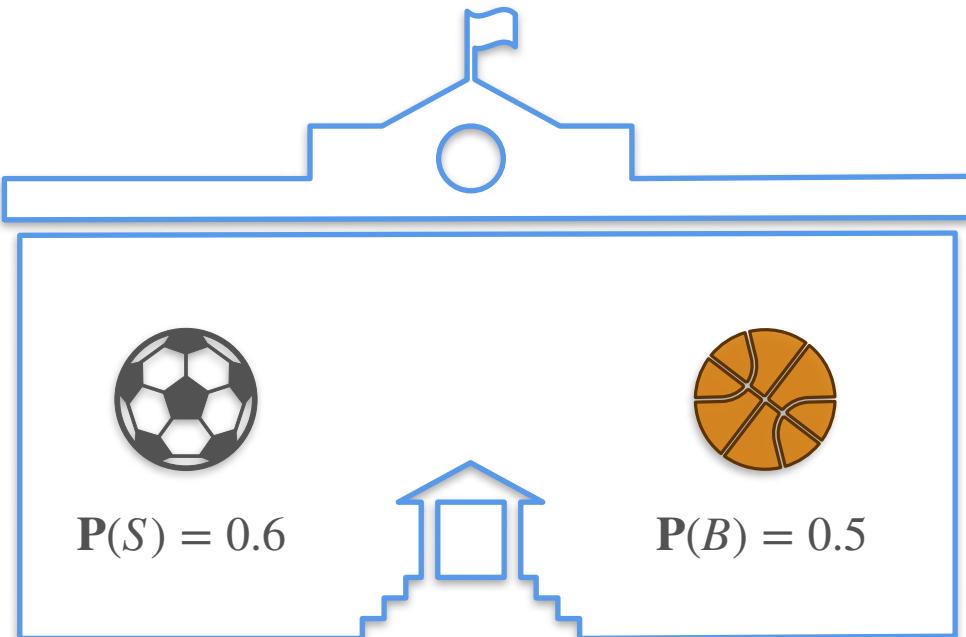
Sum of Probabilities (Joint Events): Quiz 1



At a school, kids can play as many sports as they want.

What is the probability that a kid plays soccer or basketball?

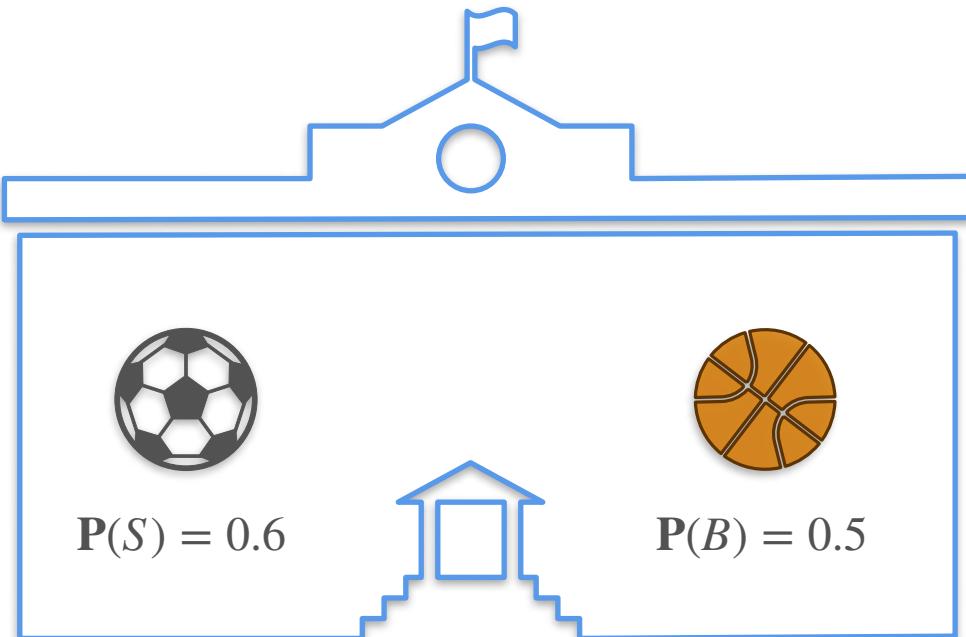
Sum of Probabilities (Joint Events): Quiz 1



At a school, kids can play as many sports as they want.

What is the probability that a kid plays soccer or basketball?

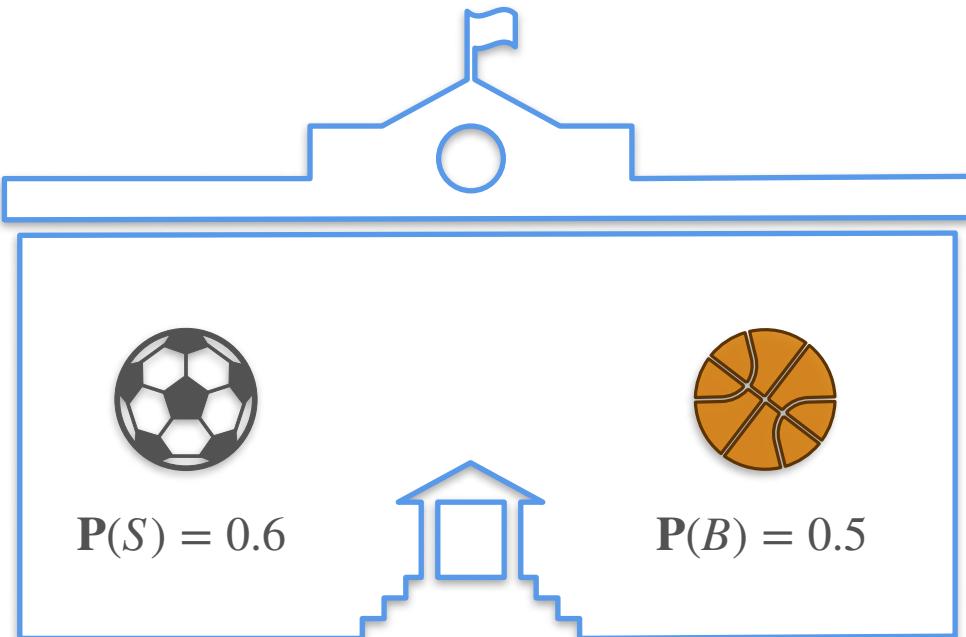
Sum of Probabilities (Joint Events): Quiz 1



At a school, kids can play as many sports as they want.

What is the probability that a kid plays soccer or basketball?

Sum of Probabilities (Joint Events): Quiz 1



At a school, kids can play as many sports as they want.

What is the probability that a kid plays soccer or basketball?

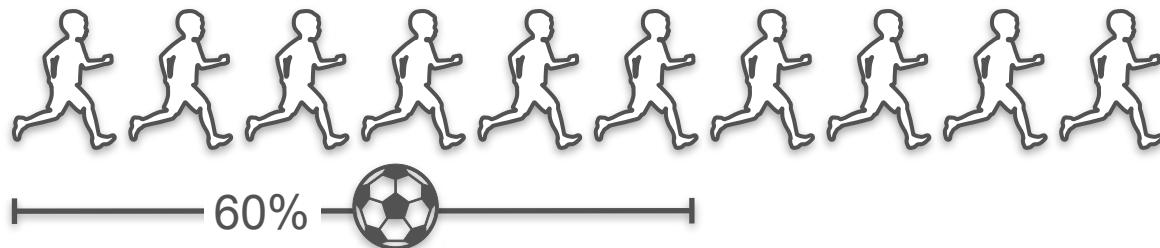
Hint: What if there were only 10 kids?

Sum of Probabilities (Joint Events): Quiz 1 Solution

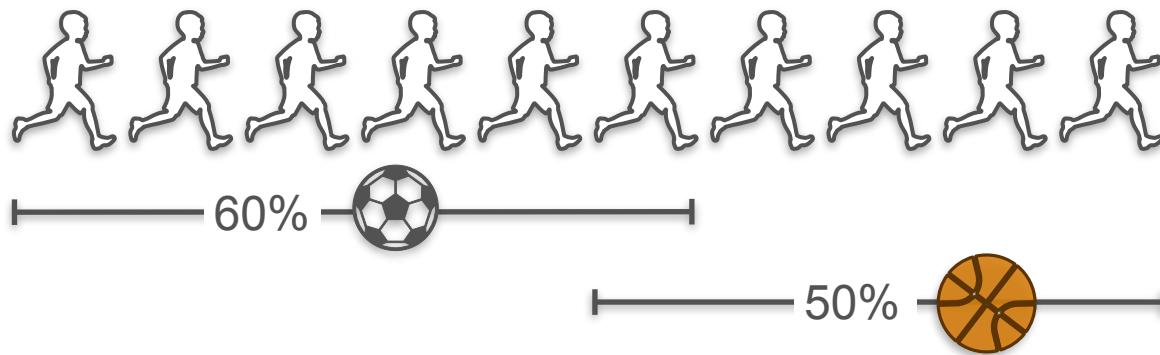
Sum of Probabilities (Joint Events): Quiz 1 Solution



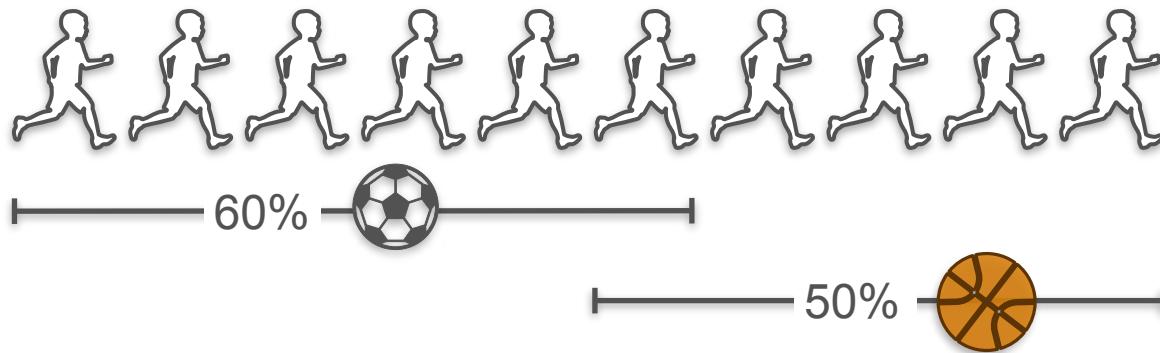
Sum of Probabilities (Joint Events): Quiz 1 Solution



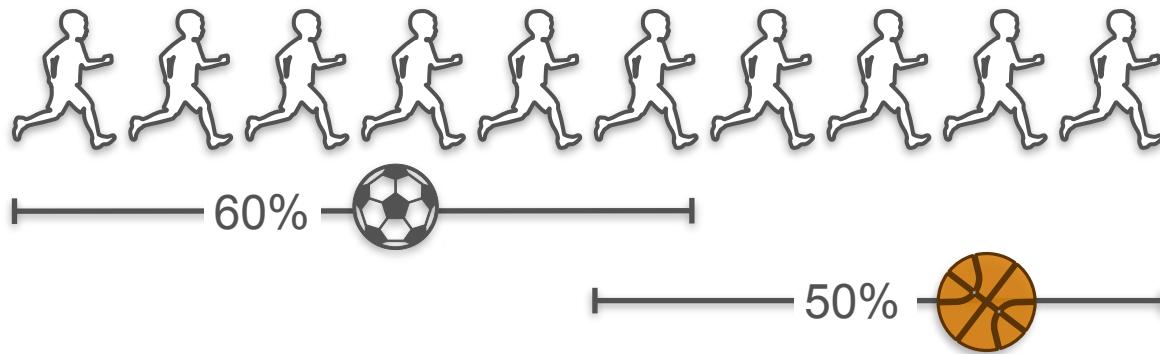
Sum of Probabilities (Joint Events): Quiz 1 Solution



Sum of Probabilities (Joint Events): Quiz 1 Solution

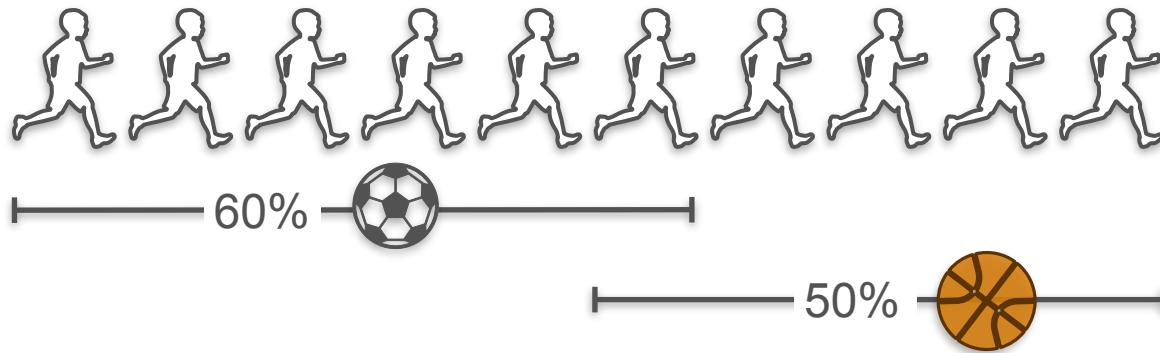


Sum of Probabilities (Joint Events): Quiz 1 Solution



$$P(\text{soccer} \cup \text{basketball}) = ?$$

Sum of Probabilities (Joint Events): Quiz 1 Solution



$$P(\text{soccer} \cup \text{basketball}) = ?$$

We don't know how many children play multiple sports

Sum of Probabilities (Joint Events): Quiz 1 Solution

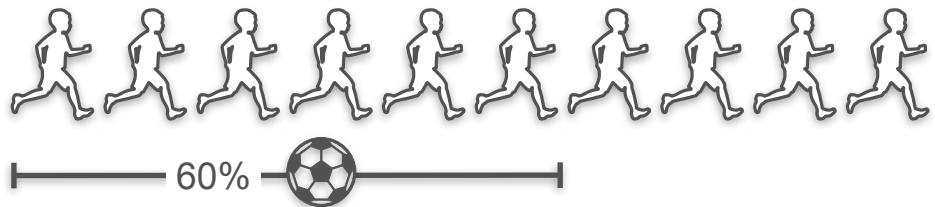
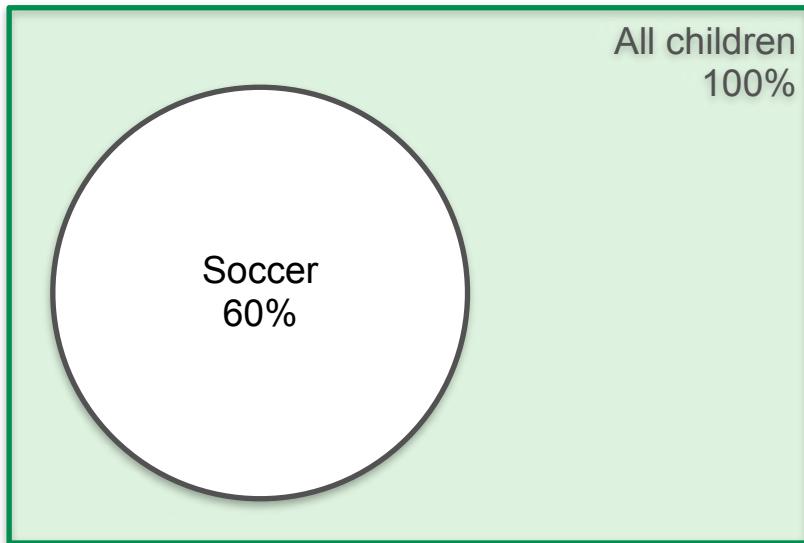


Sum of Probabilities (Joint Events): Quiz 1 Solution

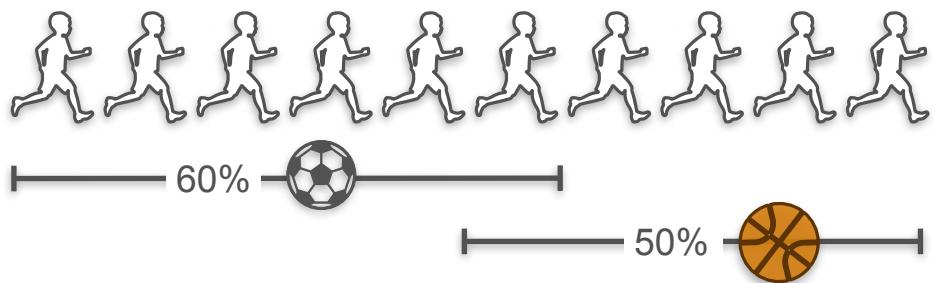
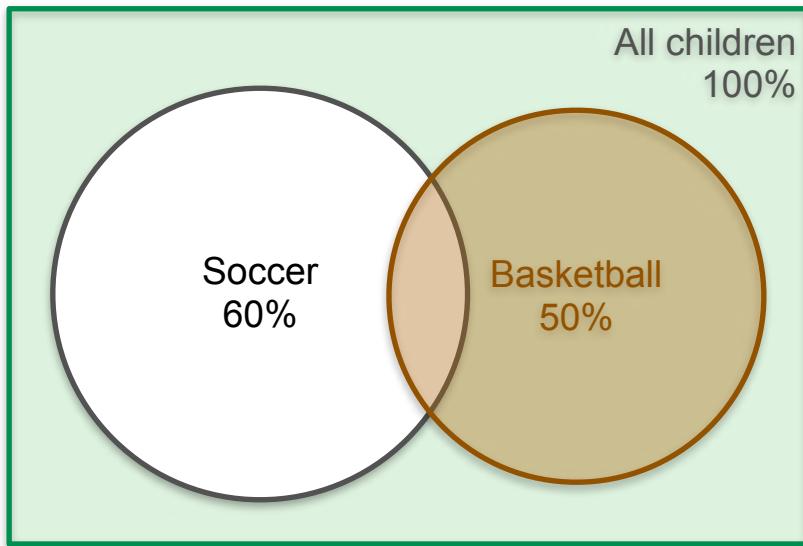
All children
100%



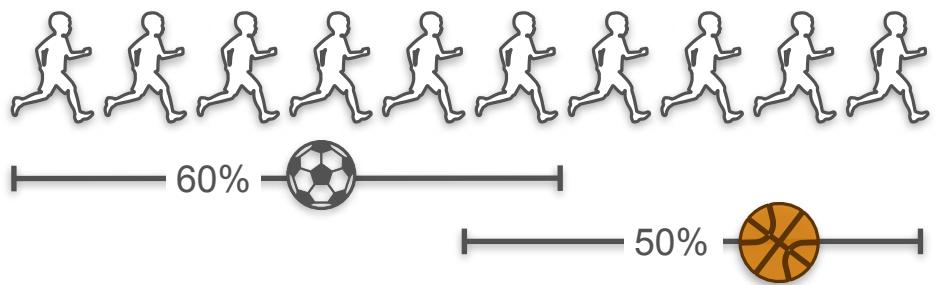
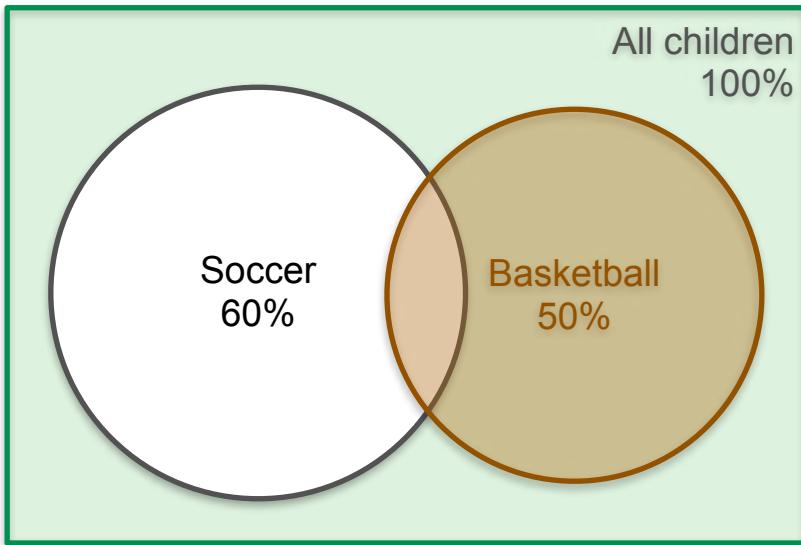
Sum of Probabilities (Joint Events): Quiz 1 Solution



Sum of Probabilities (Joint Events): Quiz 1 Solution

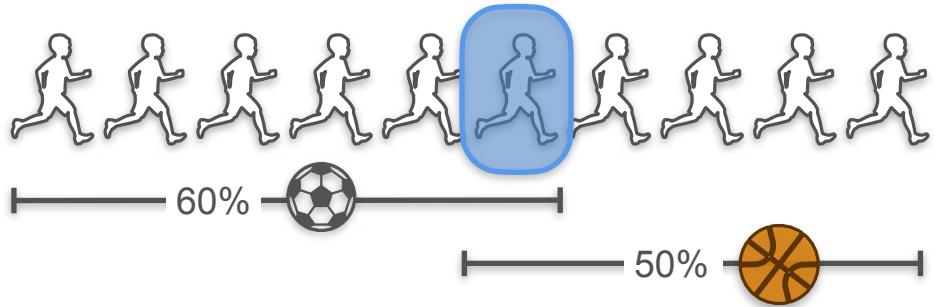
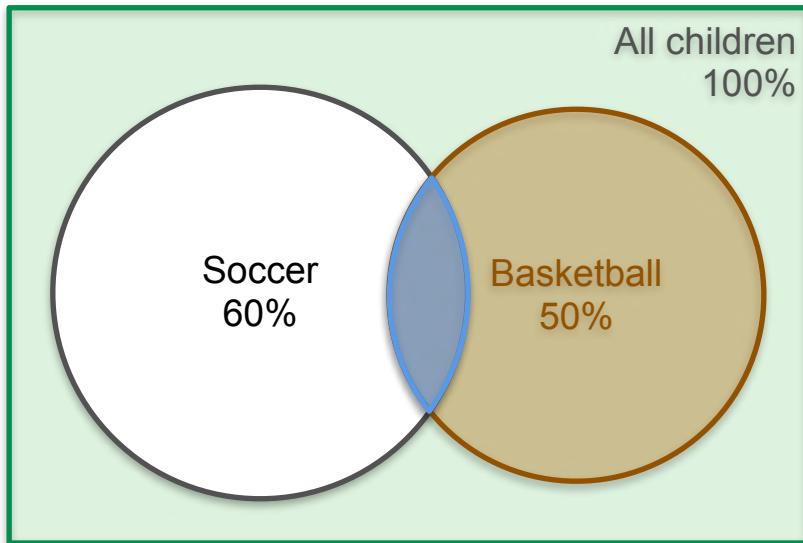


Sum of Probabilities (Joint Events): Quiz 1 Solution



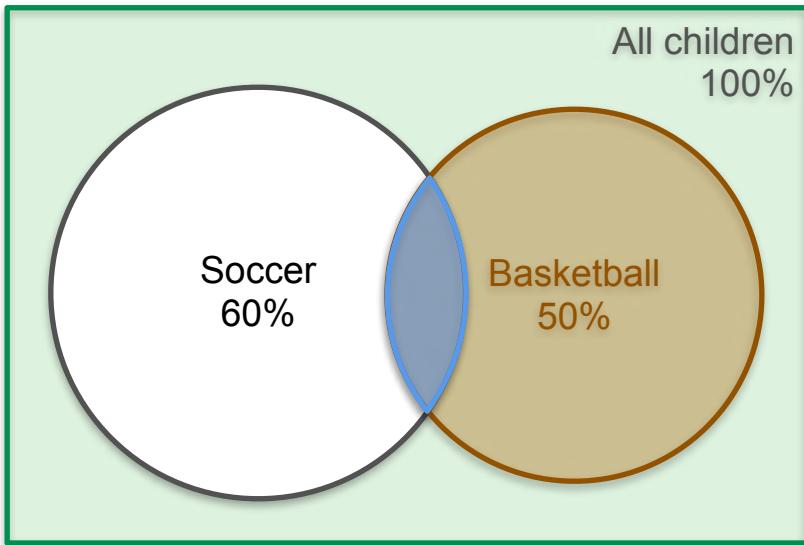
$$P(S \cup B) = P(S) + P(B)$$

Sum of Probabilities (Joint Events): Quiz 1 Solution

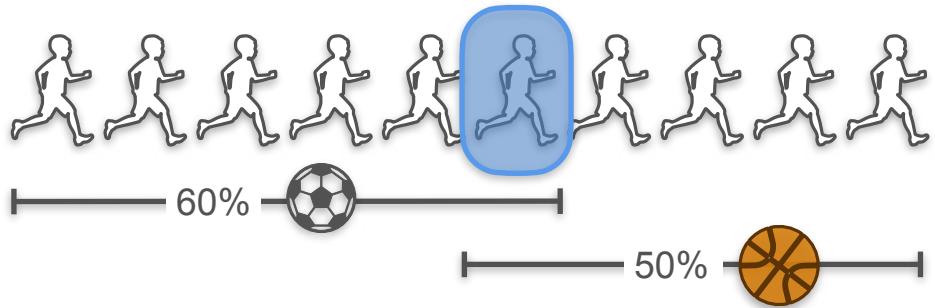


$$P(S \cup B) = P(S) + P(B)$$

Sum of Probabilities (Joint Events): Quiz 1 Solution

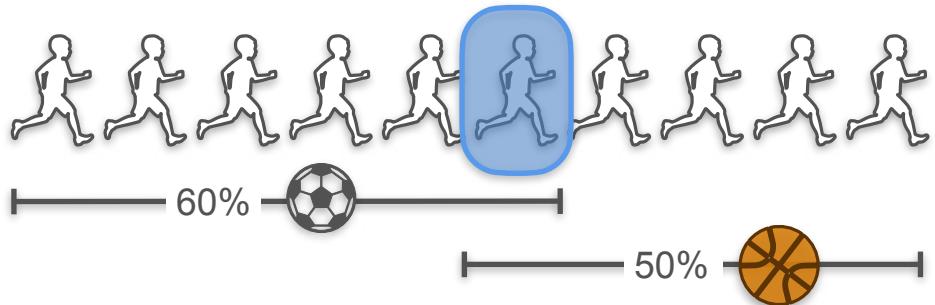
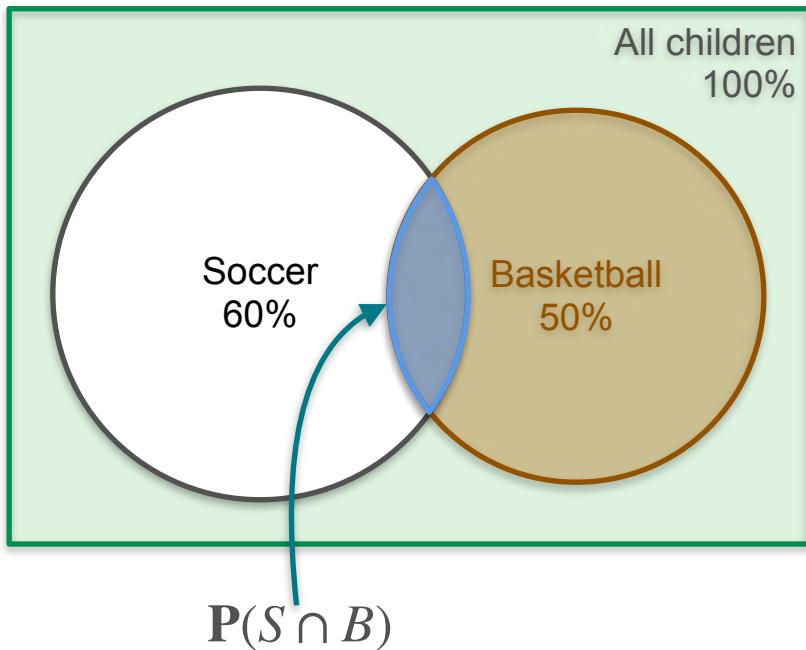


$$\mathbf{P}(S \cap B)$$



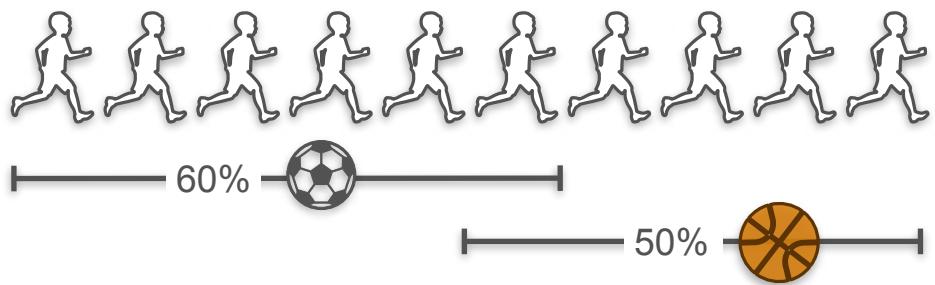
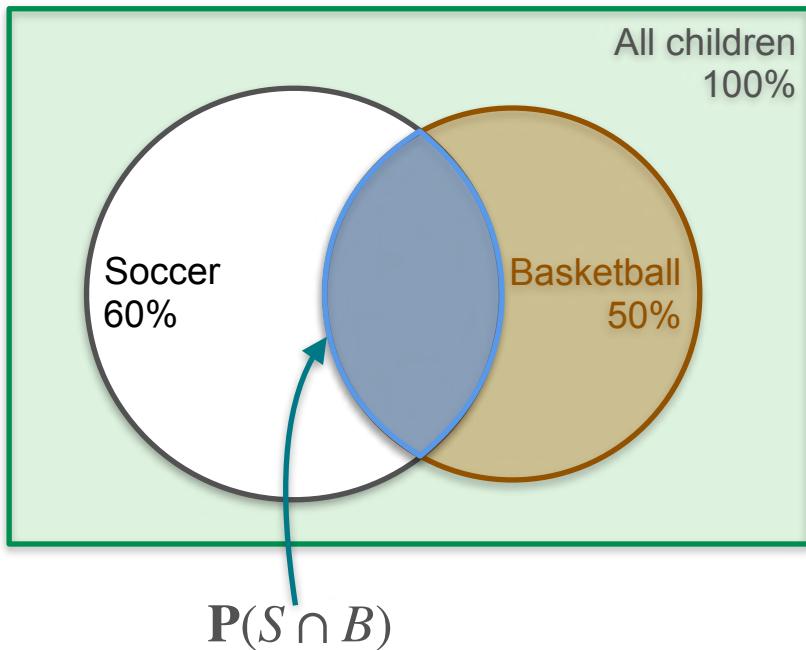
$$\mathbf{P}(S \cup B) = \mathbf{P}(S) + \mathbf{P}(B)$$

Sum of Probabilities (Joint Events): Quiz 1 Solution



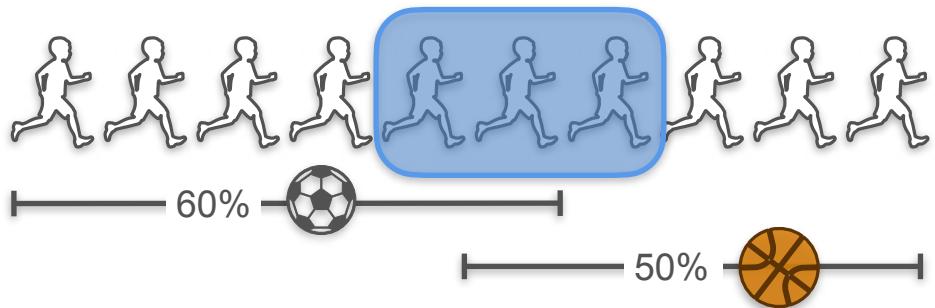
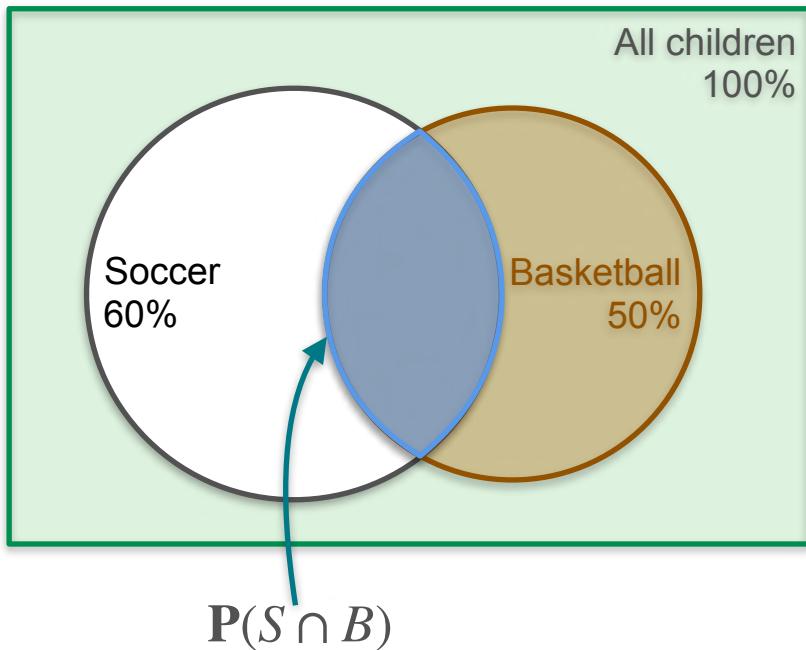
$$P(S \cup B) = P(S) + P(B)$$

Sum of Probabilities (Joint Events): Quiz 1 Solution



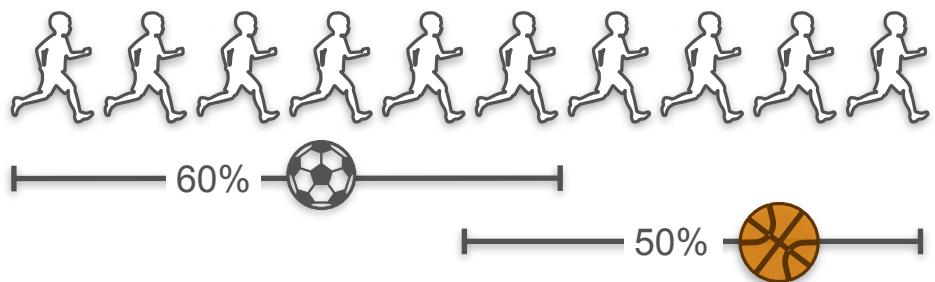
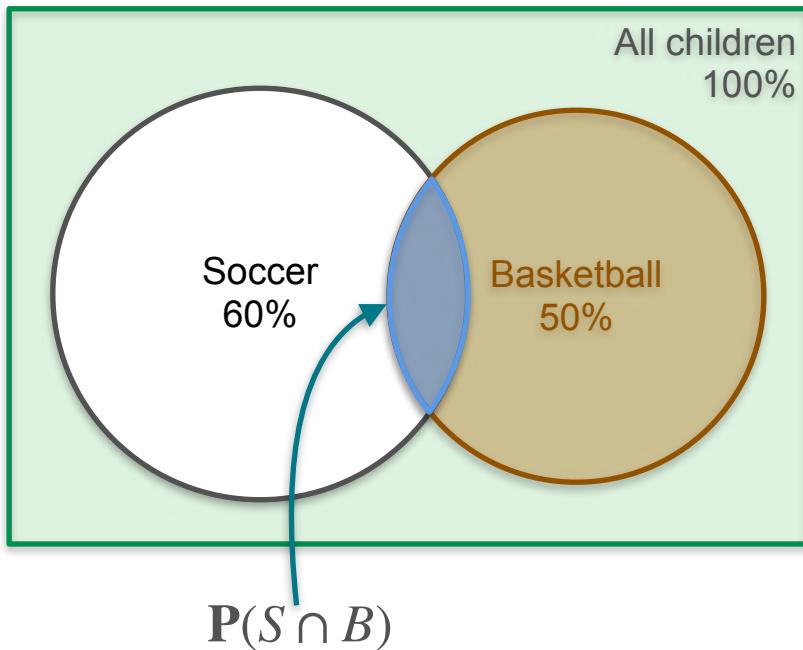
$$P(S \cup B) = P(S) + P(B)$$

Sum of Probabilities (Joint Events): Quiz 1 Solution



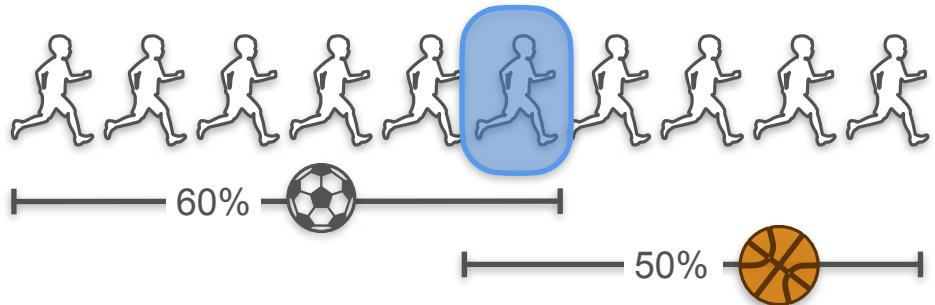
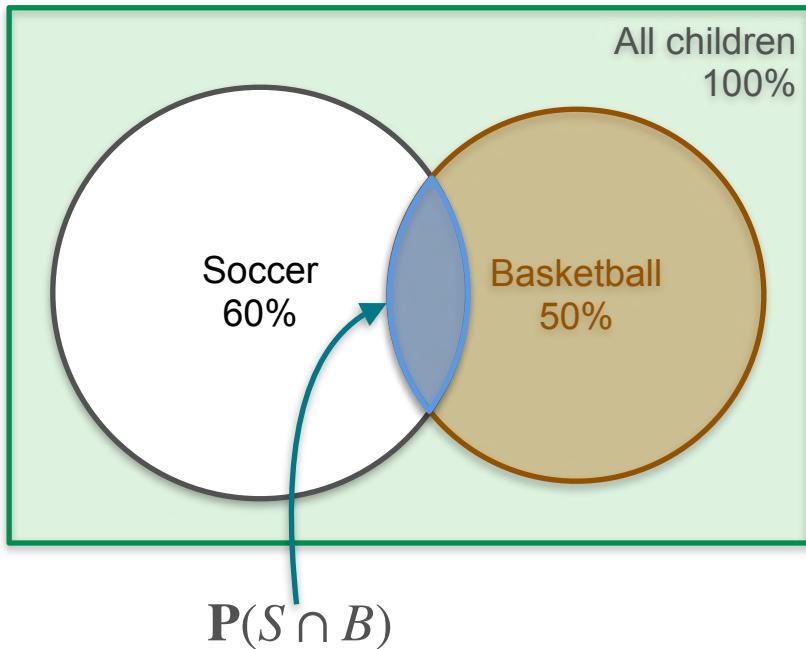
$$P(S \cup B) = P(S) + P(B)$$

Sum of Probabilities (Joint Events): Quiz 1 Solution



$$P(S \cup B) = P(S) + P(B)$$

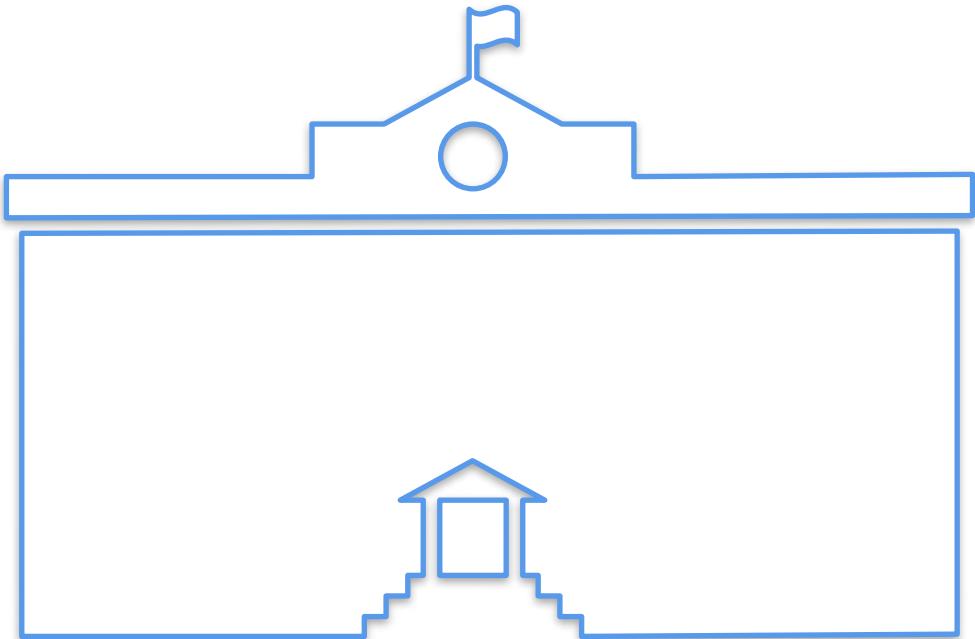
Sum of Probabilities (Joint Events): Quiz 1 Solution



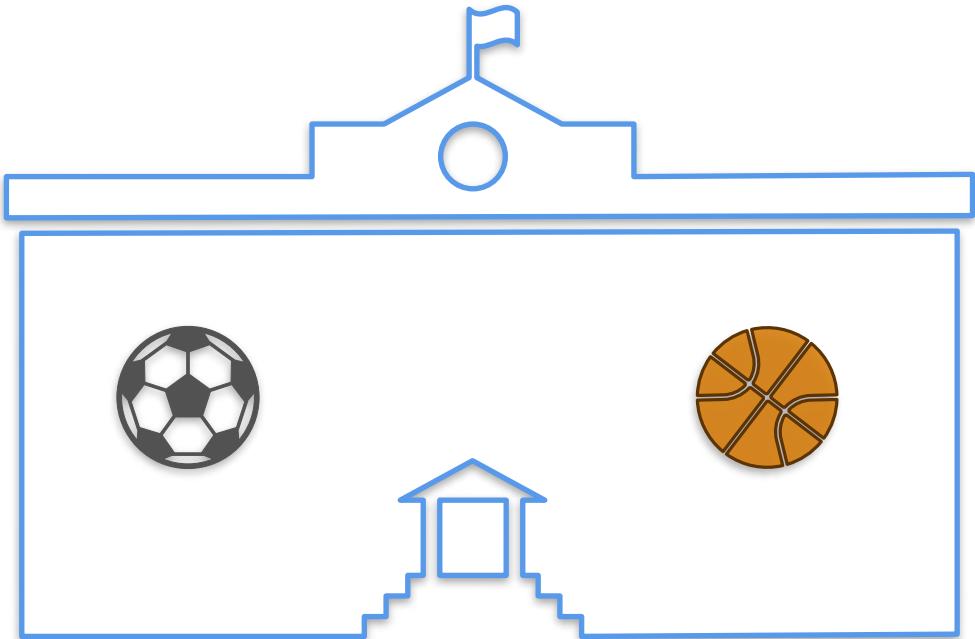
$$P(S \cup B) = P(S) + P(B)$$

Sum of Probabilities (Joint Events): Quiz 2

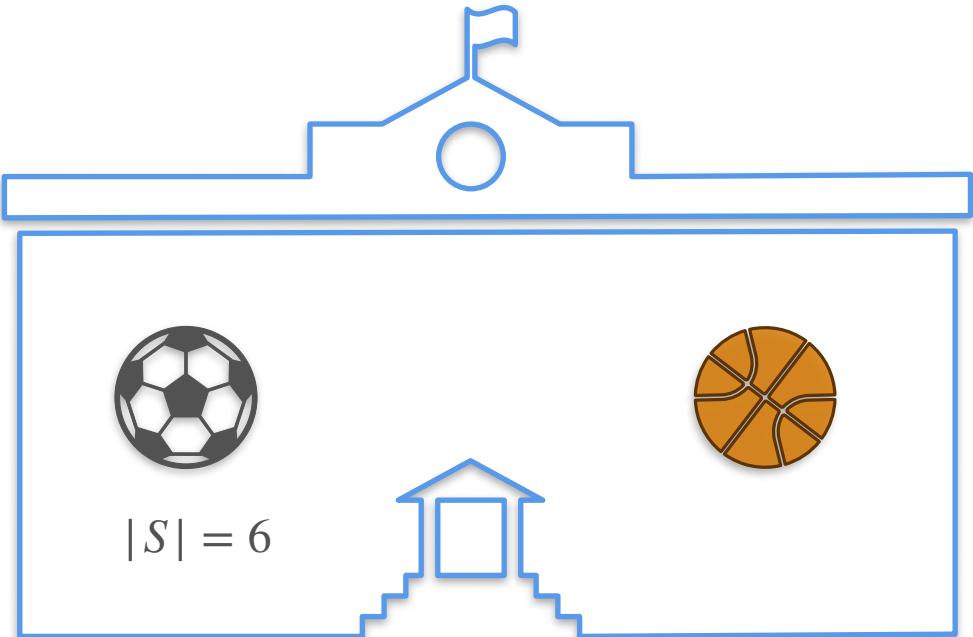
Sum of Probabilities (Joint Events): Quiz 2



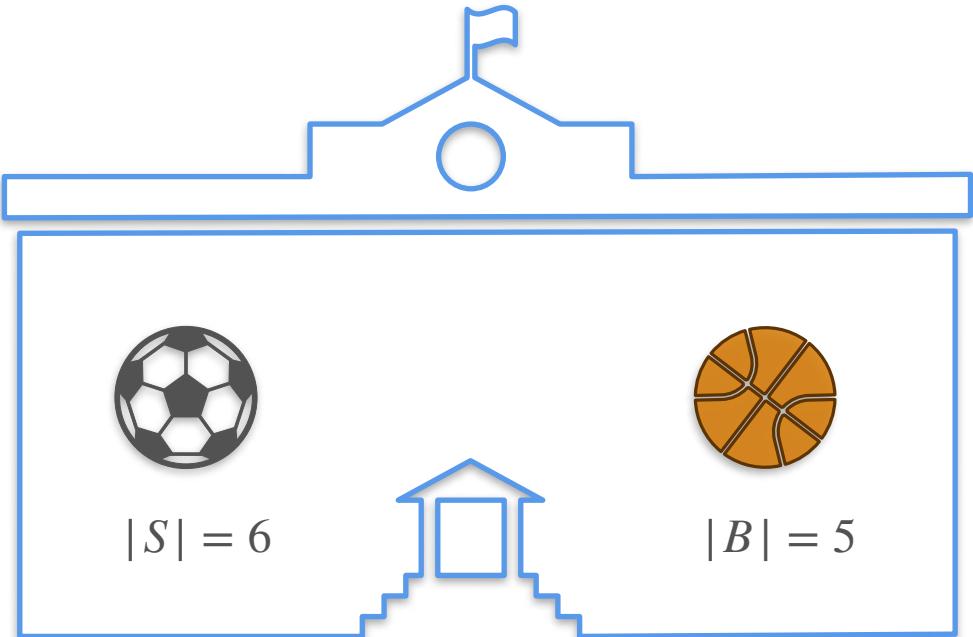
Sum of Probabilities (Joint Events): Quiz 2



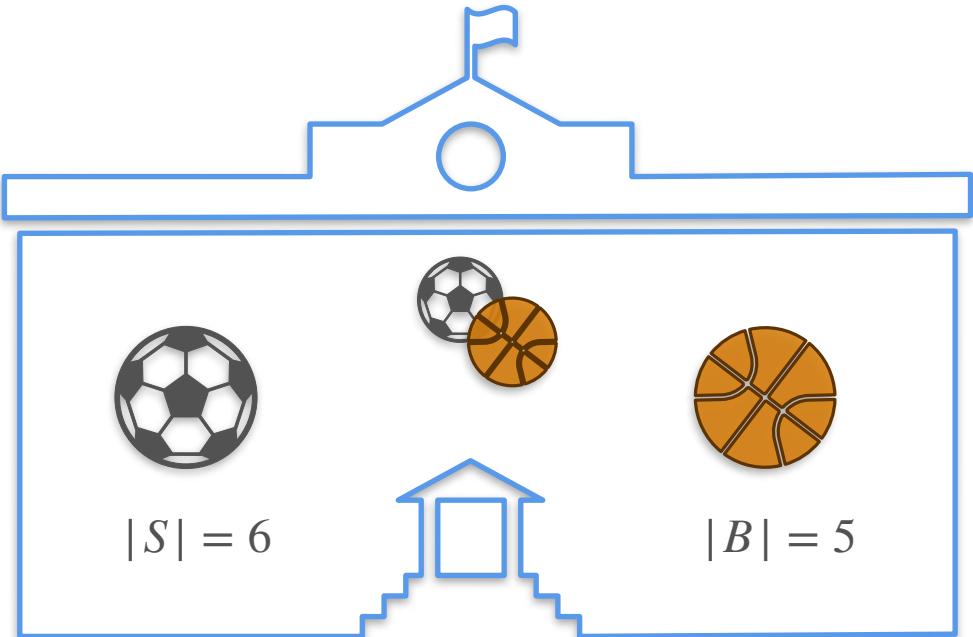
Sum of Probabilities (Joint Events): Quiz 2



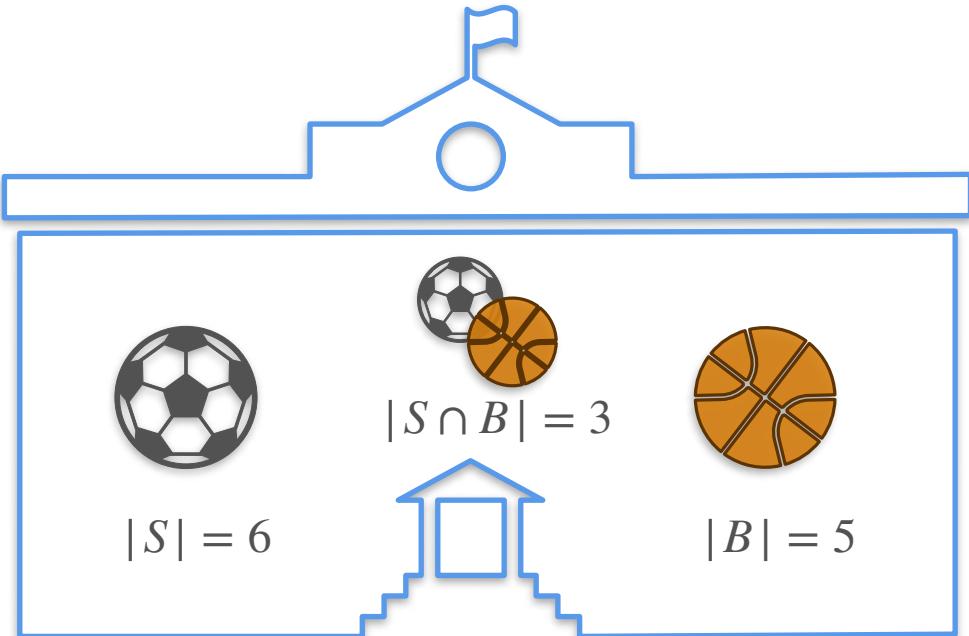
Sum of Probabilities (Joint Events): Quiz 2



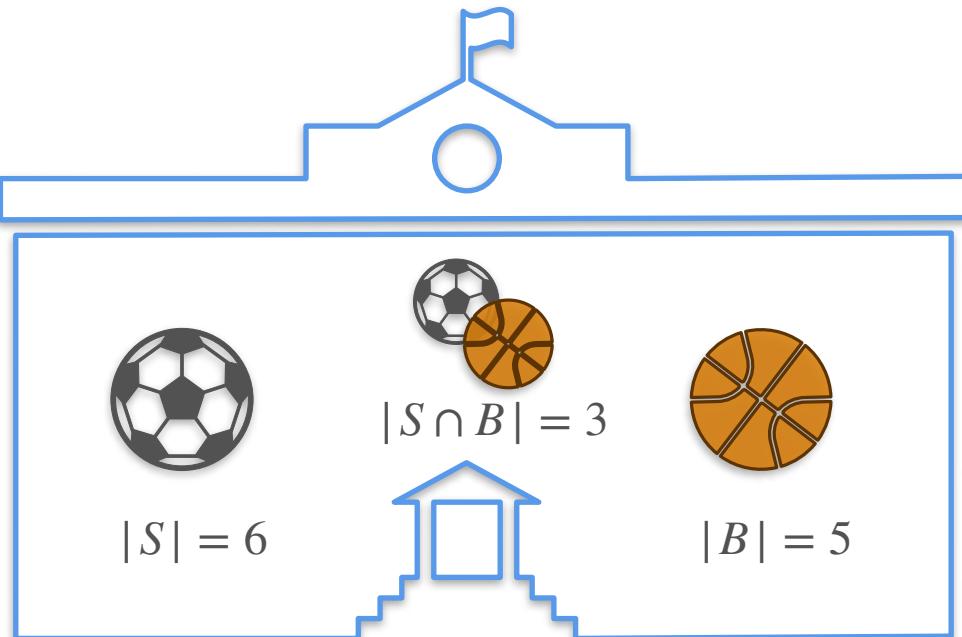
Sum of Probabilities (Joint Events): Quiz 2



Sum of Probabilities (Joint Events): Quiz 2

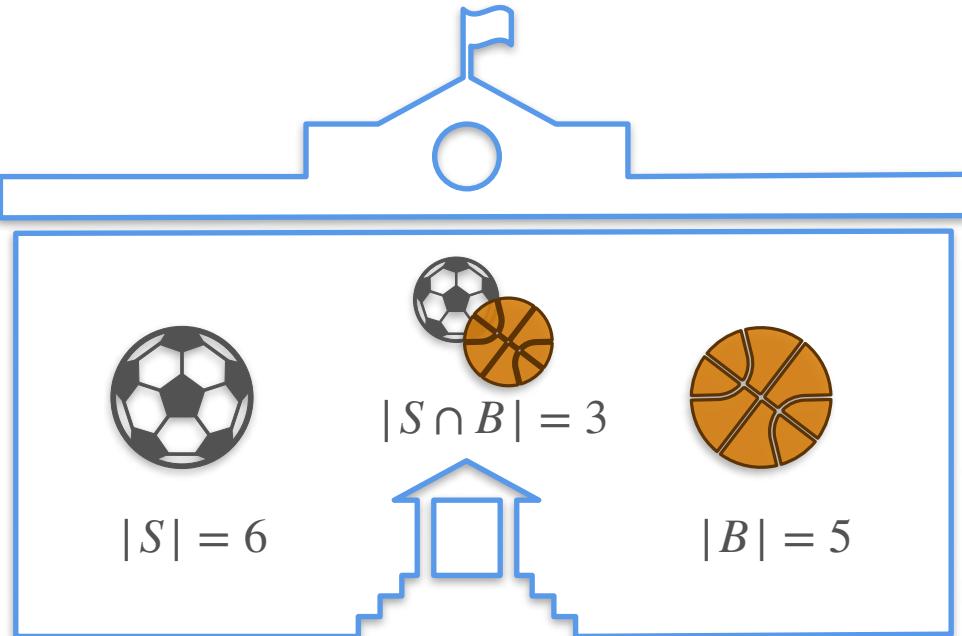


Sum of Probabilities (Joint Events): Quiz 2



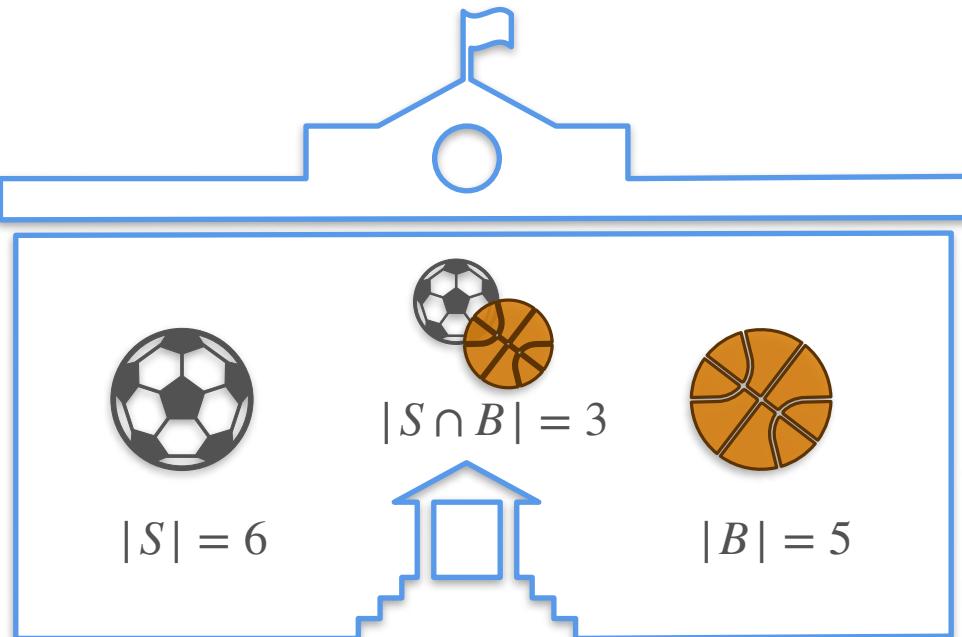
How many kids play soccer or basketball?

Sum of Probabilities (Joint Events): Quiz 2



How many kids play soccer or basketball?

Sum of Probabilities (Joint Events): Quiz 2



How many kids play soccer or basketball?

Hint: What if there were only 10 kids?

Sum of Probabilities (Joint Events): Quiz 2 Solution

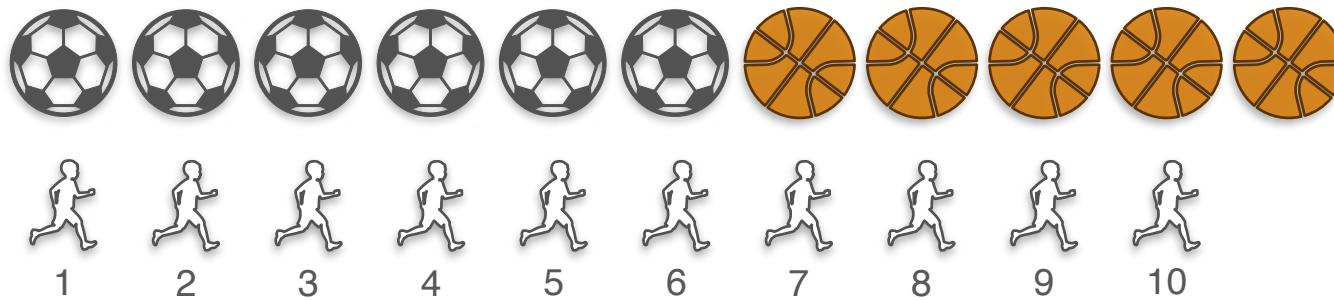
Sum of Probabilities (Joint Events): Quiz 2 Solution



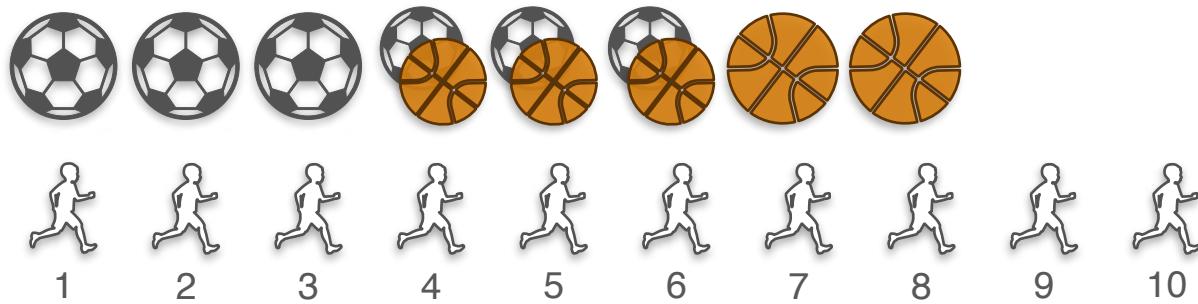
Sum of Probabilities (Joint Events): Quiz 2 Solution



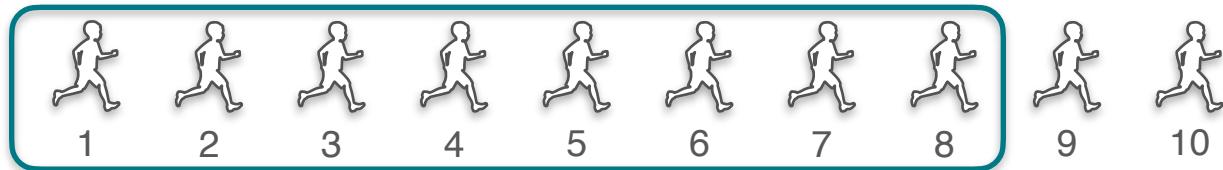
Sum of Probabilities (Joint Events): Quiz 2 Solution



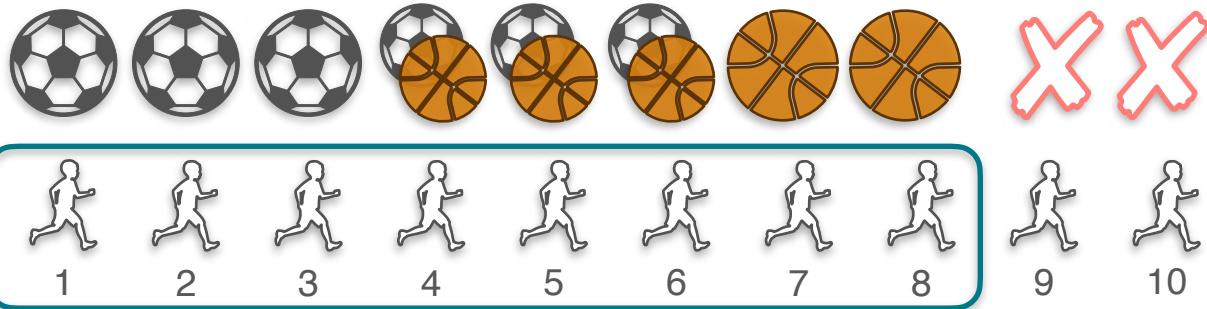
Sum of Probabilities (Joint Events): Quiz 2 Solution



Sum of Probabilities (Joint Events): Quiz 2 Solution



Sum of Probabilities (Joint Events): Quiz 2 Solution

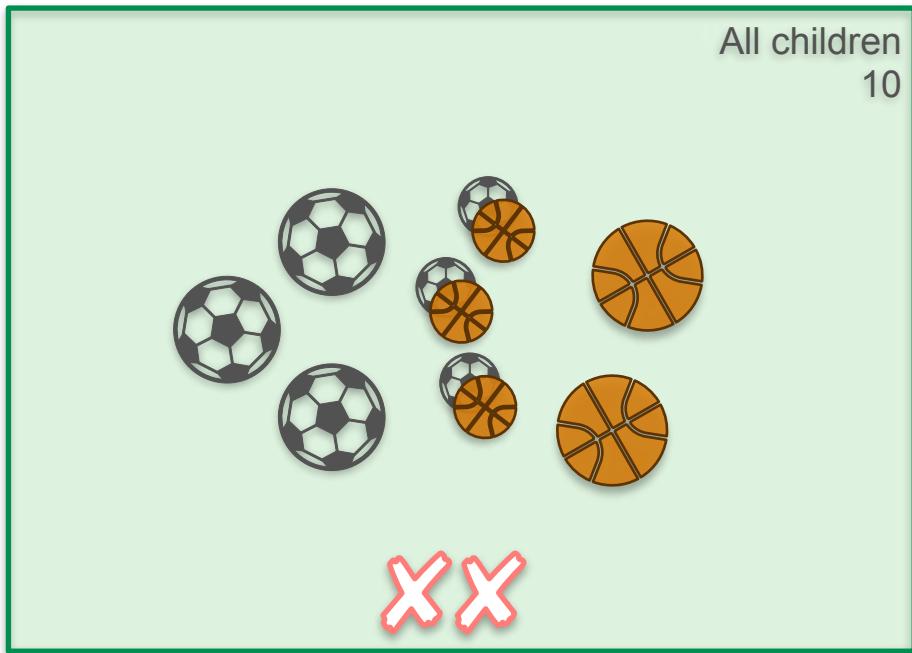


Sum of Probabilities (Joint Events): Venn Diagram

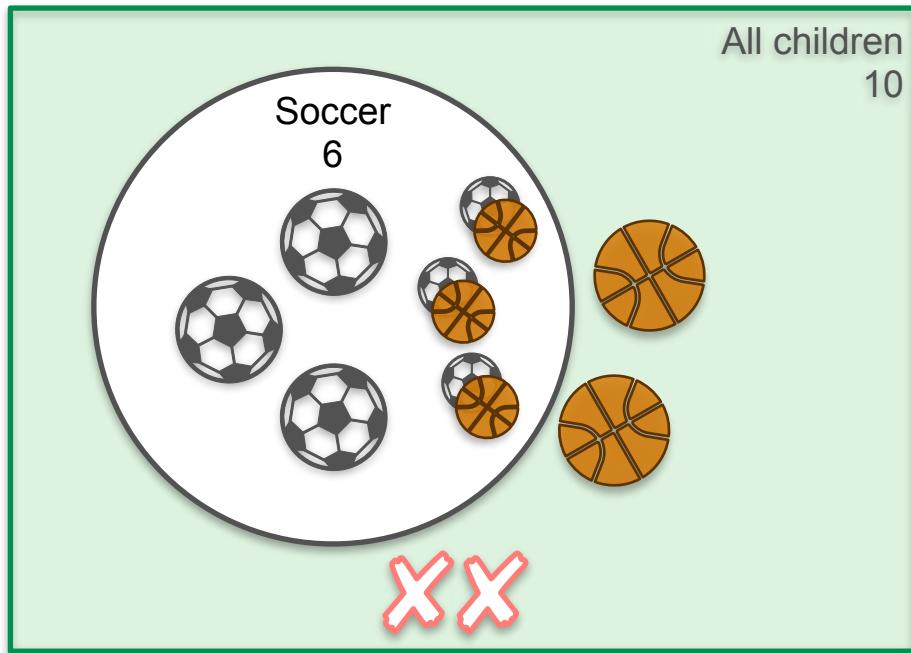


XX

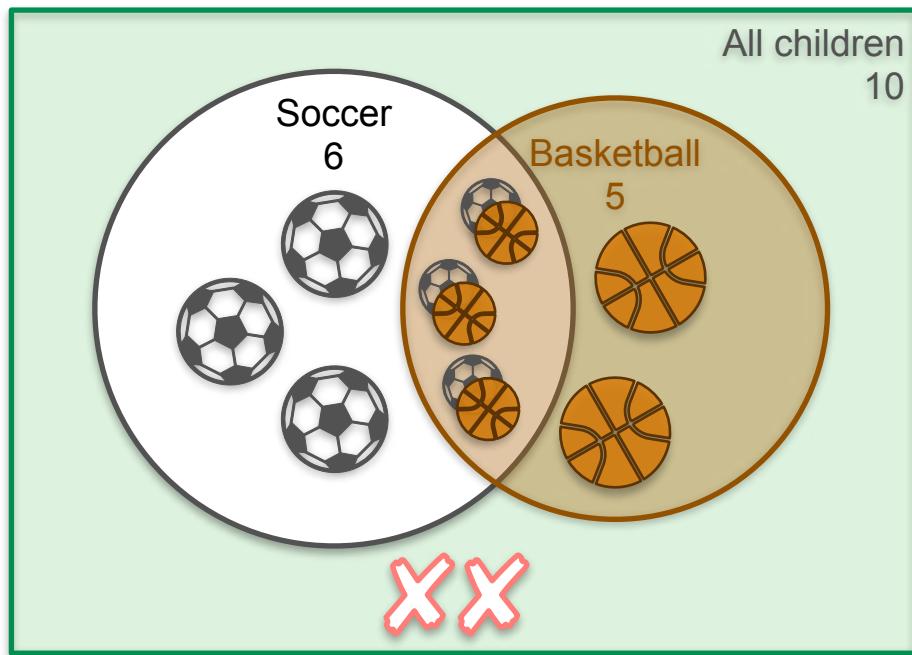
Sum of Probabilities (Joint Events): Venn Diagram



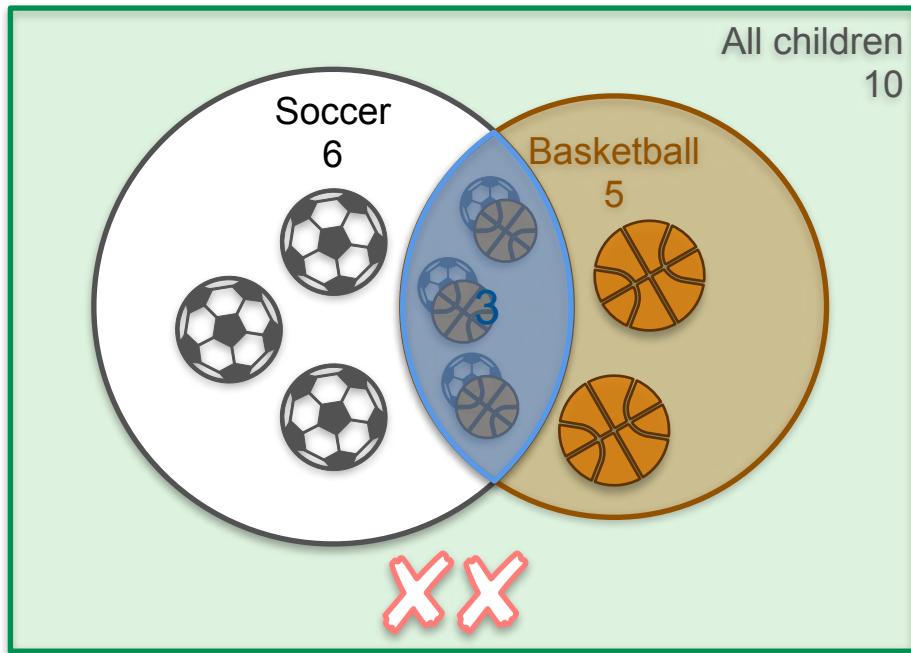
Sum of Probabilities (Joint Events): Venn Diagram



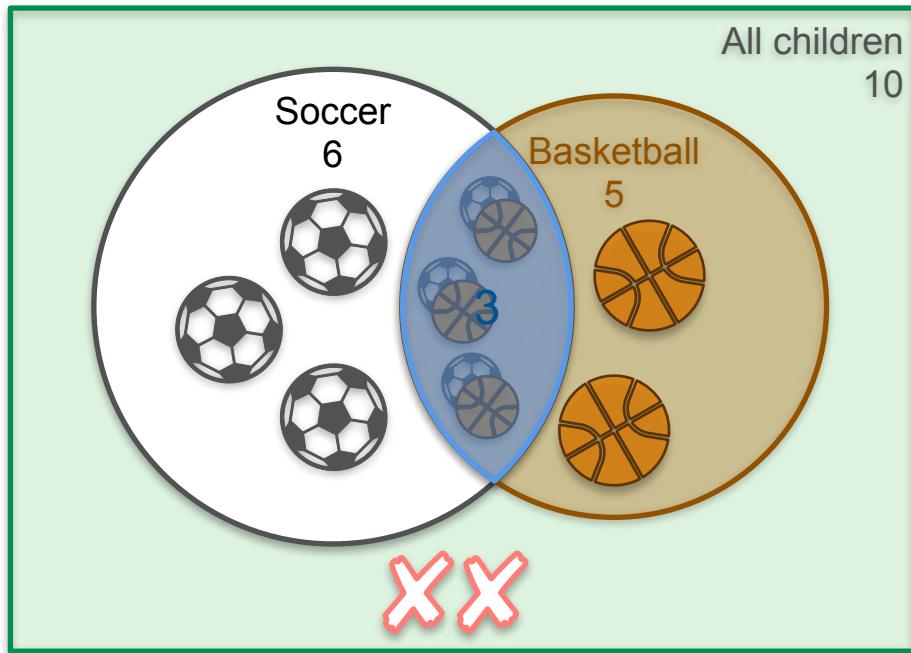
Sum of Probabilities (Joint Events): Venn Diagram



Sum of Probabilities (Joint Events): Venn Diagram

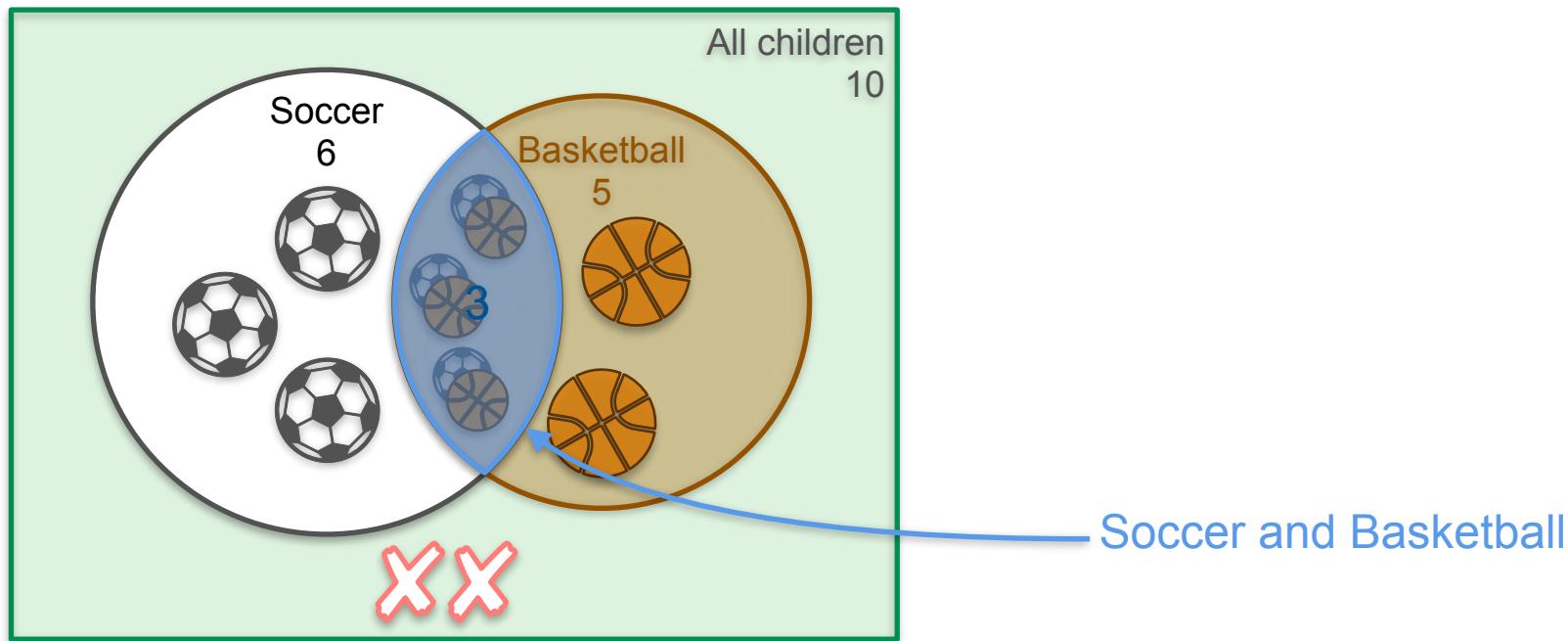


Sum of Probabilities (Joint Events): Venn Diagram

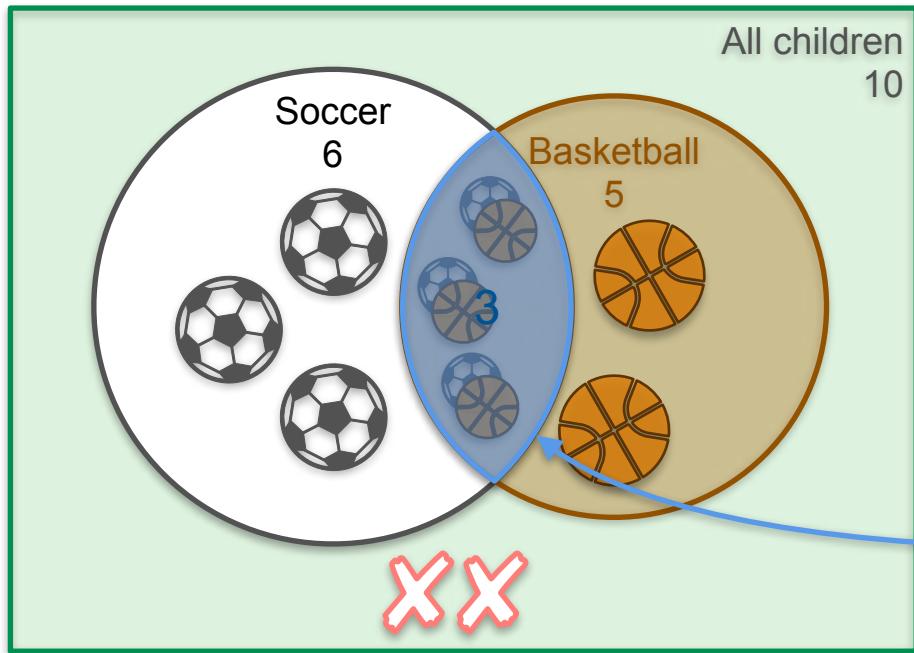


Soccer and Basketball

Sum of Probabilities (Joint Events): Venn Diagram



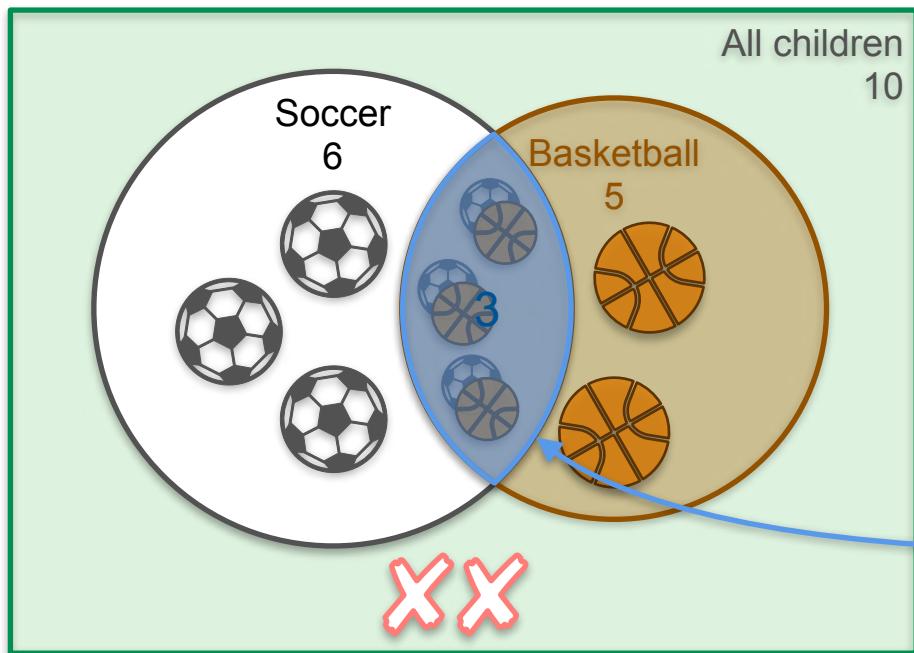
Sum of Probabilities (Joint Events): Venn Diagram



$$|S \cup B| =$$

Soccer and Basketball

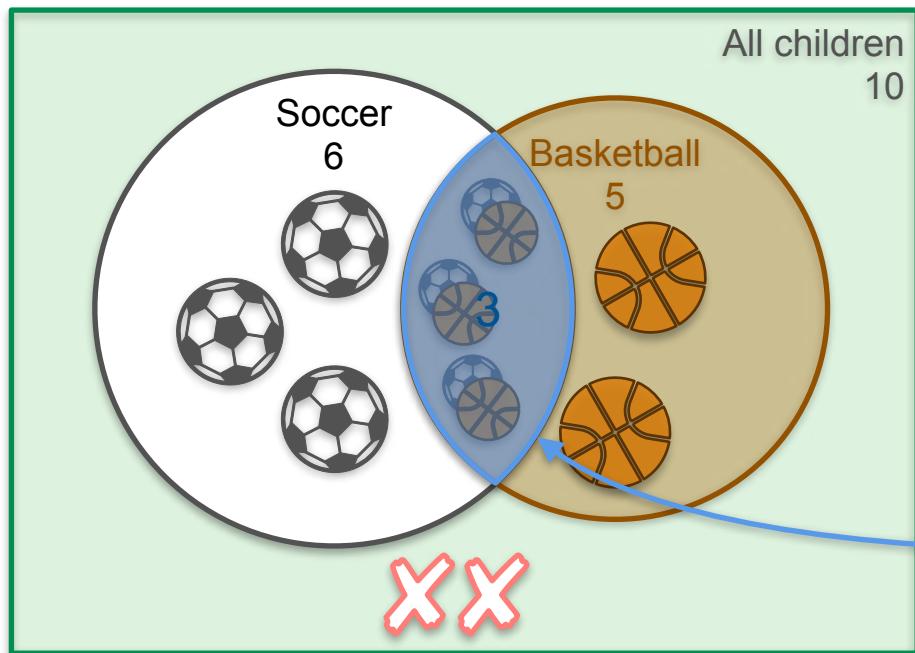
Sum of Probabilities (Joint Events): Venn Diagram



$$|S \cup B| = |S|$$

Soccer and Basketball

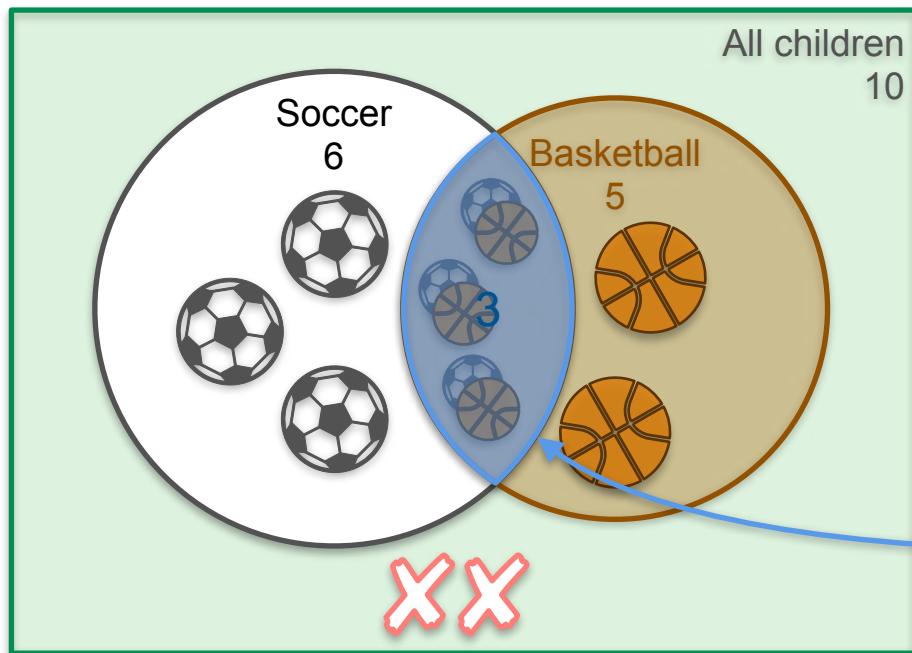
Sum of Probabilities (Joint Events): Venn Diagram



$$|S \cup B| = |S|$$

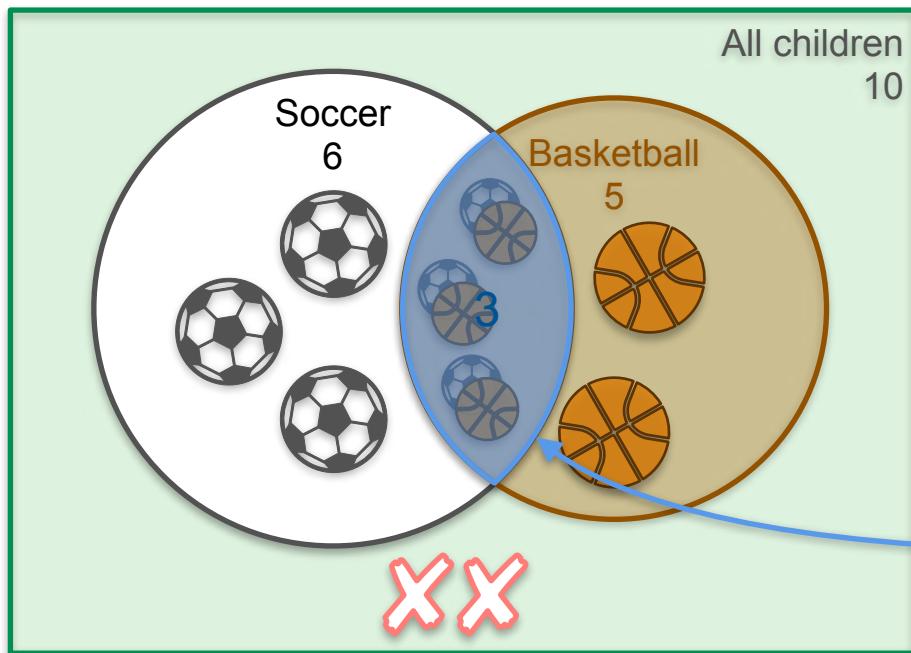
Soccer and Basketball

Sum of Probabilities (Joint Events): Venn Diagram



$$|S \cup B| = |S| + |B|$$

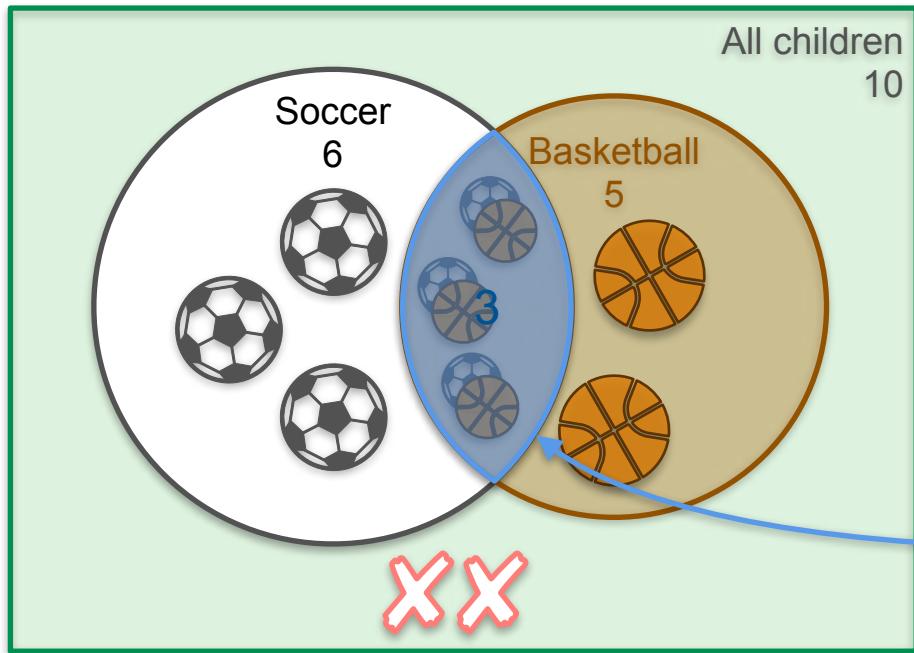
Sum of Probabilities (Joint Events): Venn Diagram



$$|S \cup B| = |S| + |B|$$

Soccer and Basketball

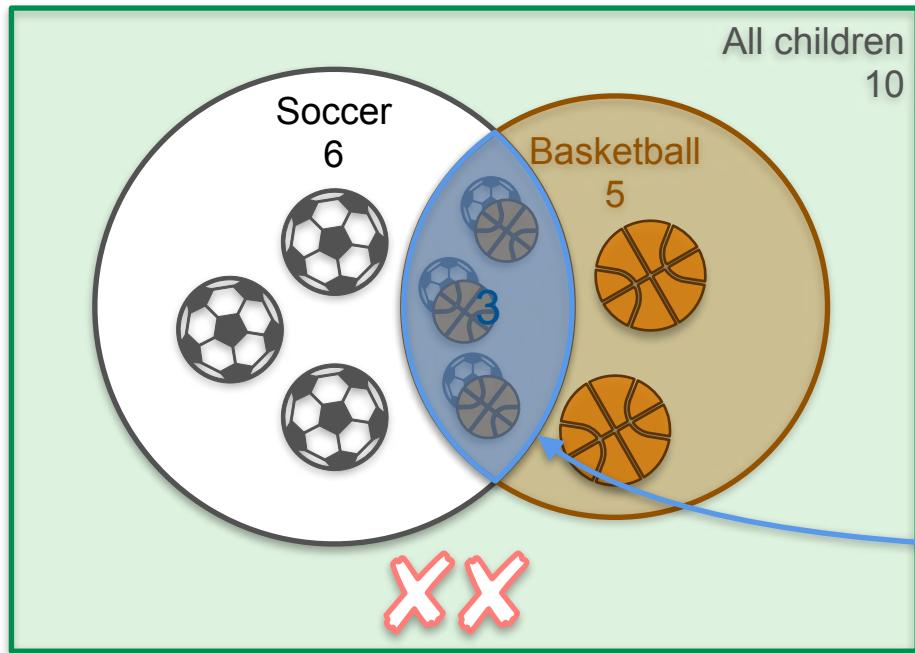
Sum of Probabilities (Joint Events): Venn Diagram



$$|S \cup B| = |S| + |B| - |S \cap B|$$

Soccer and Basketball

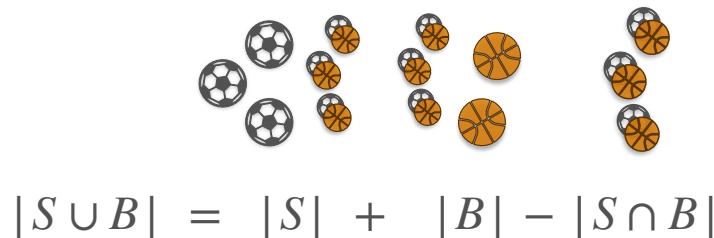
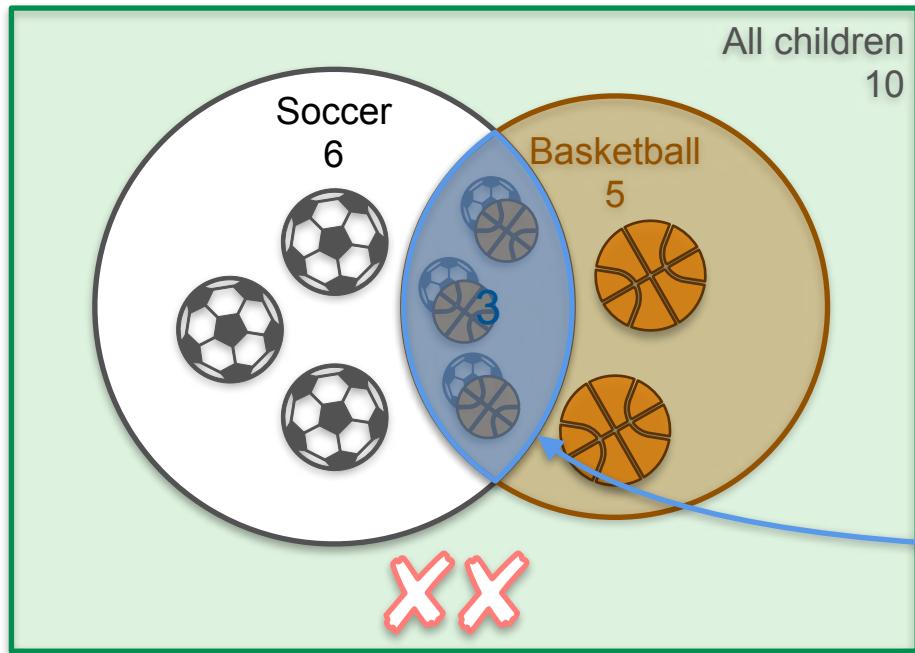
Sum of Probabilities (Joint Events): Venn Diagram



$$|S \cup B| = |S| + |B| - |S \cap B|$$

Soccer and Basketball

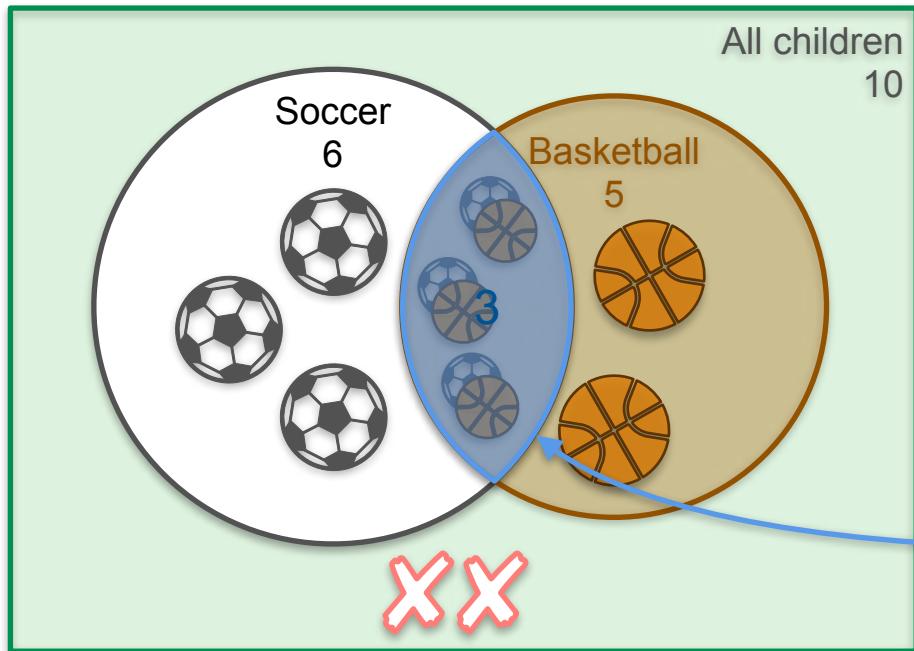
Sum of Probabilities (Joint Events): Venn Diagram



$$|S \cup B| = |S| + |B| - |S \cap B|$$
$$=$$

Soccer and Basketball

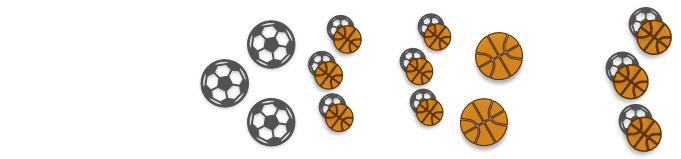
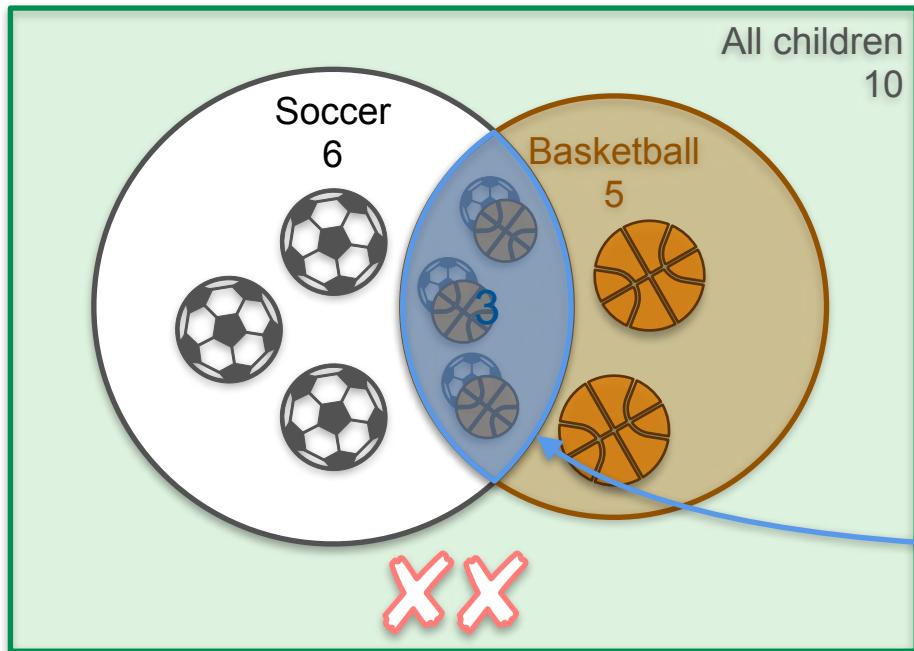
Sum of Probabilities (Joint Events): Venn Diagram



$$\begin{aligned}|S \cup B| &= |S| + |B| - |S \cap B| \\&= 6 + 5 - 3\end{aligned}$$

Soccer and Basketball

Sum of Probabilities (Joint Events): Venn Diagram

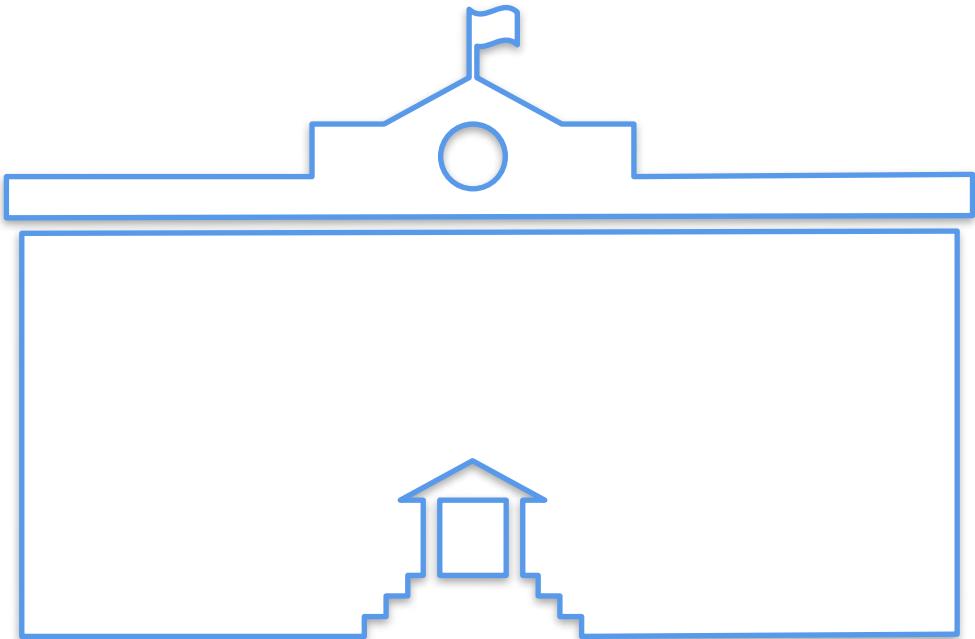


$$\begin{aligned}|S \cup B| &= |S| + |B| - |S \cap B| \\&= 6 + 5 - 3 \\&= 8\end{aligned}$$

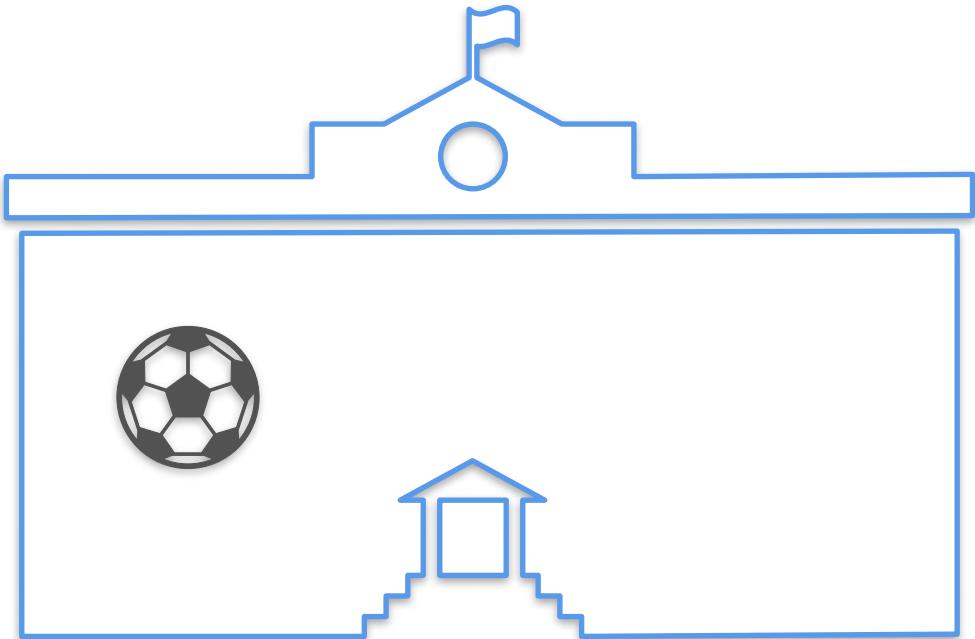
Soccer and Basketball

Sum of Probabilities (Joint Events): Quiz 3

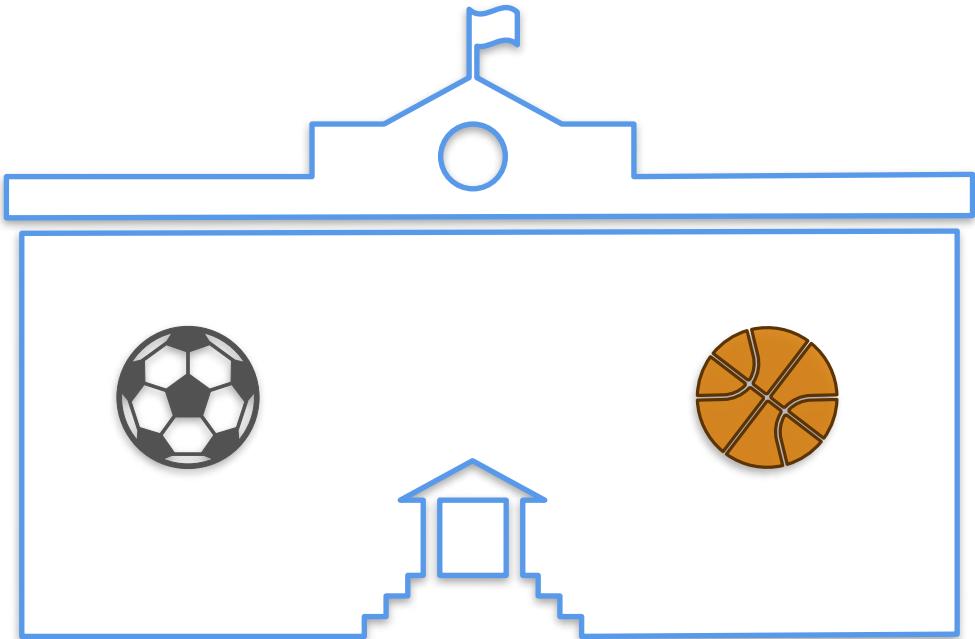
Sum of Probabilities (Joint Events): Quiz 3



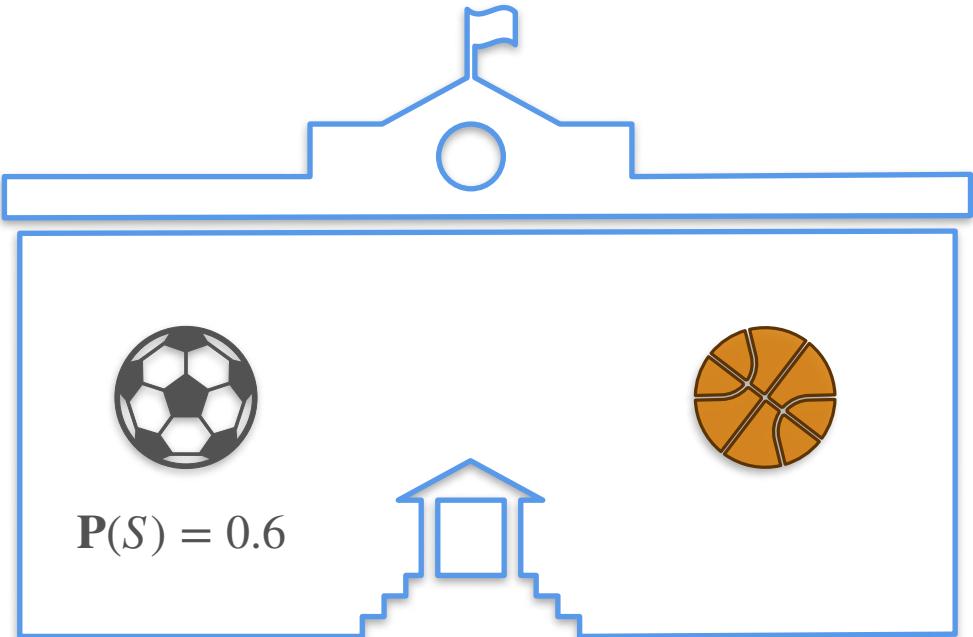
Sum of Probabilities (Joint Events): Quiz 3



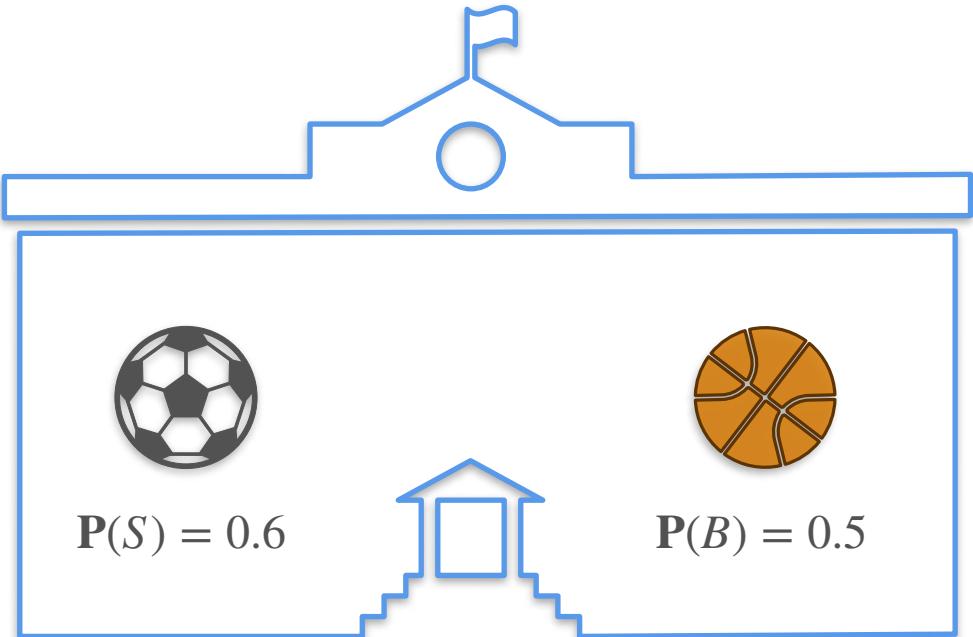
Sum of Probabilities (Joint Events): Quiz 3



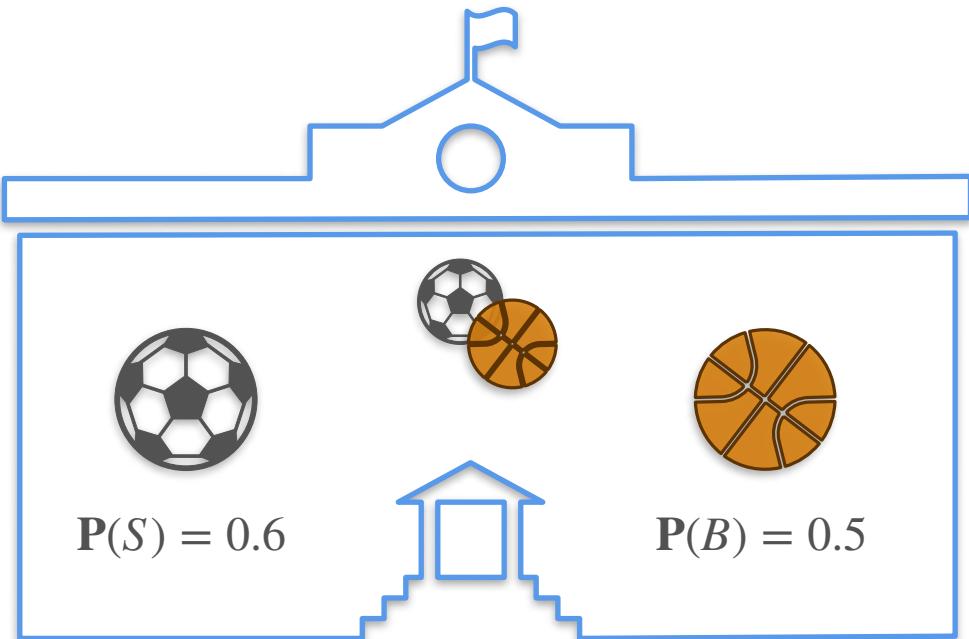
Sum of Probabilities (Joint Events): Quiz 3



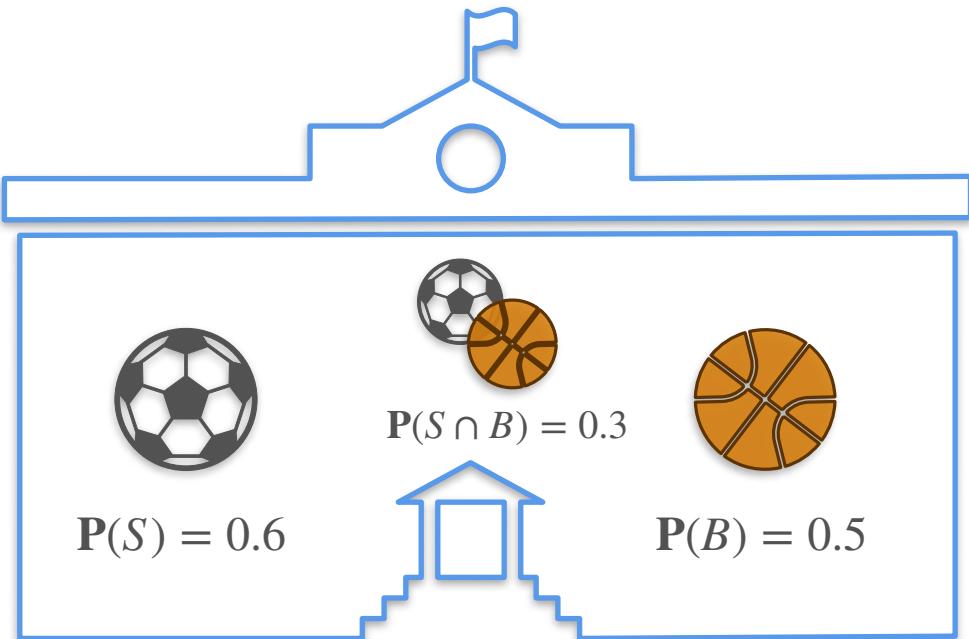
Sum of Probabilities (Joint Events): Quiz 3



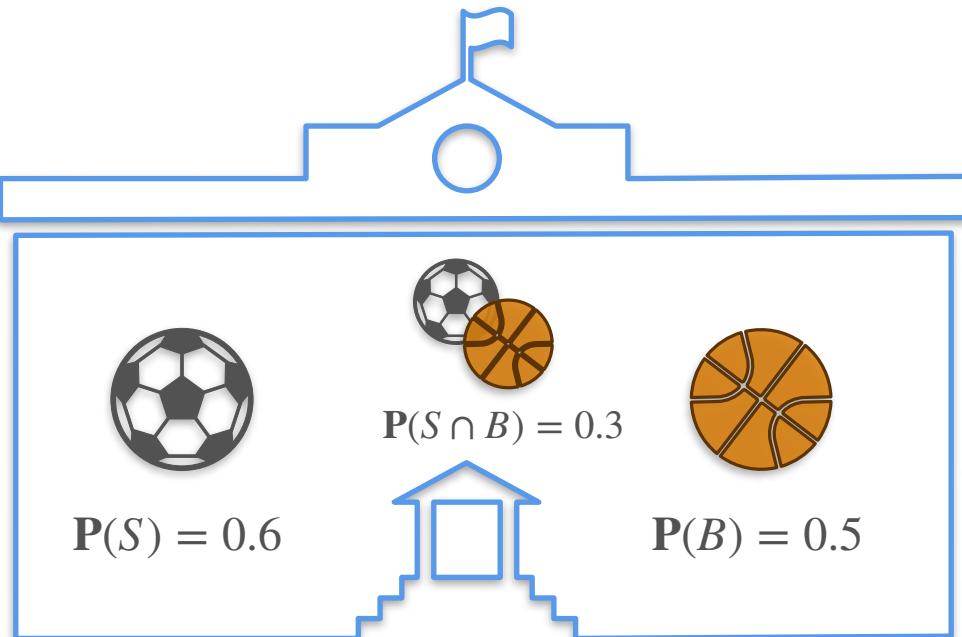
Sum of Probabilities (Joint Events): Quiz 3



Sum of Probabilities (Joint Events): Quiz 3



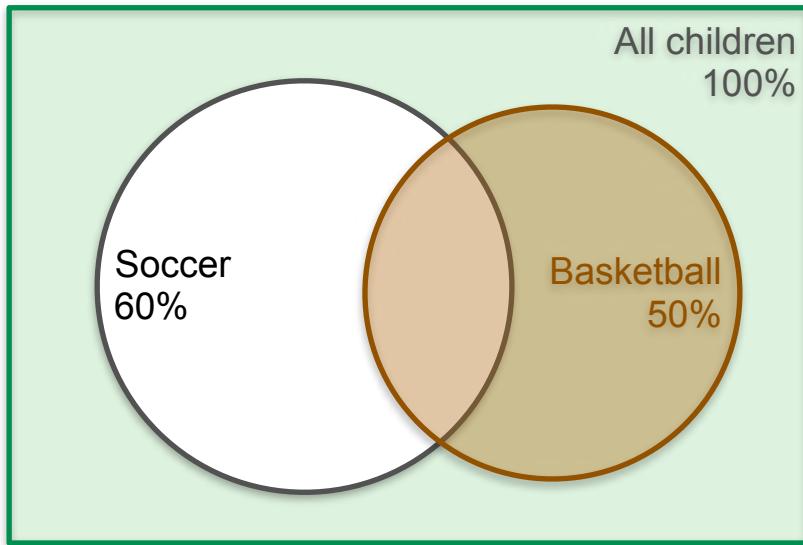
Sum of Probabilities (Joint Events): Quiz 3



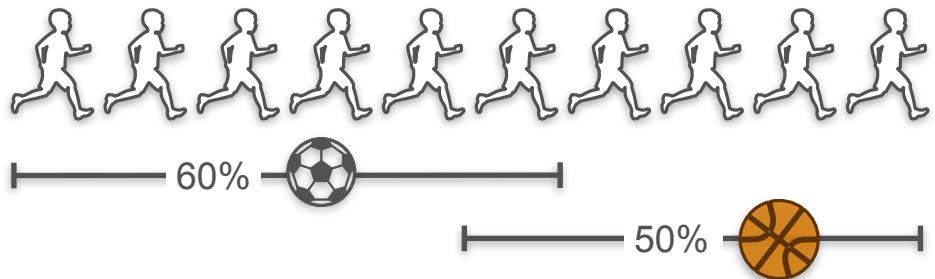
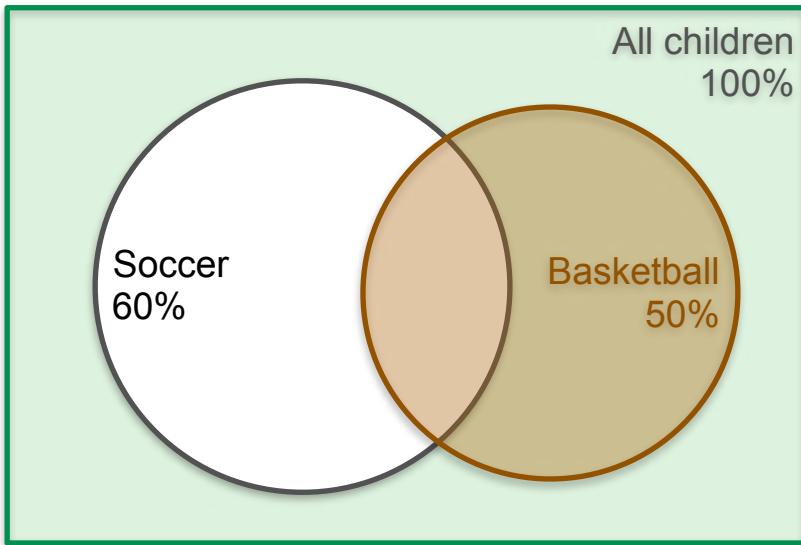
What is the probability that a child plays soccer or basketball?

Sum of Probabilities (Joint Events): Quiz 3 Solution

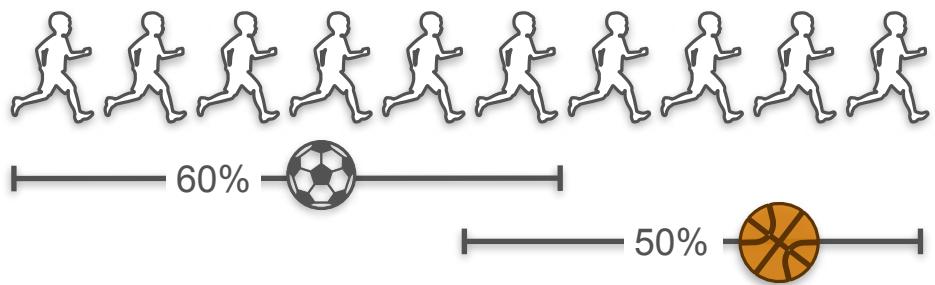
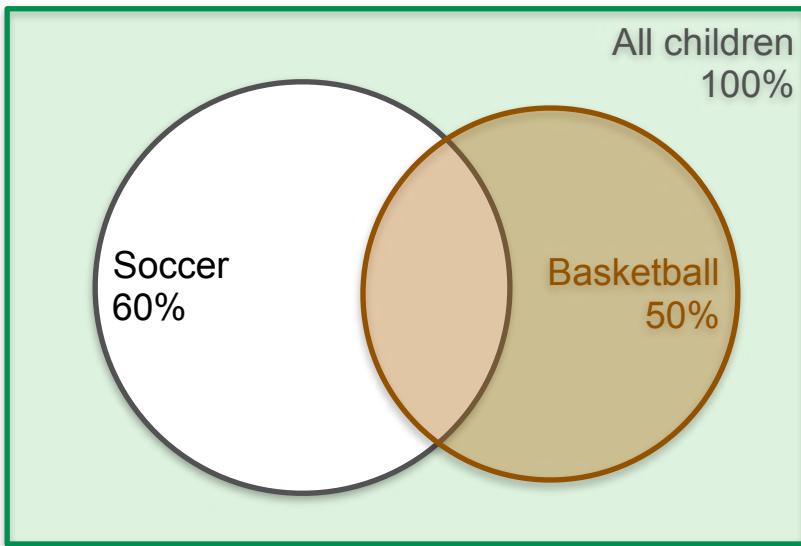
Sum of Probabilities (Joint Events): Quiz 3 Solution



Sum of Probabilities (Joint Events): Quiz 3 Solution

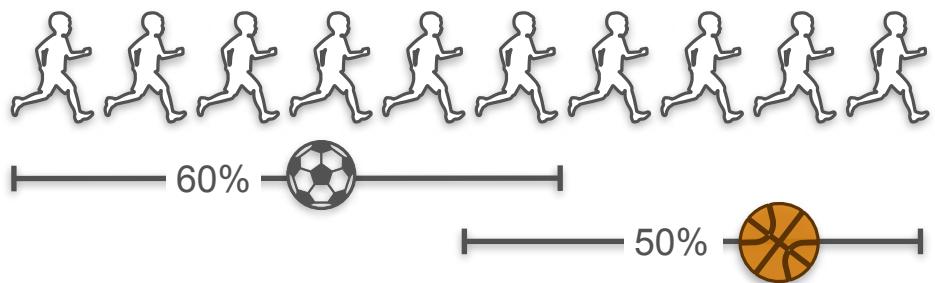
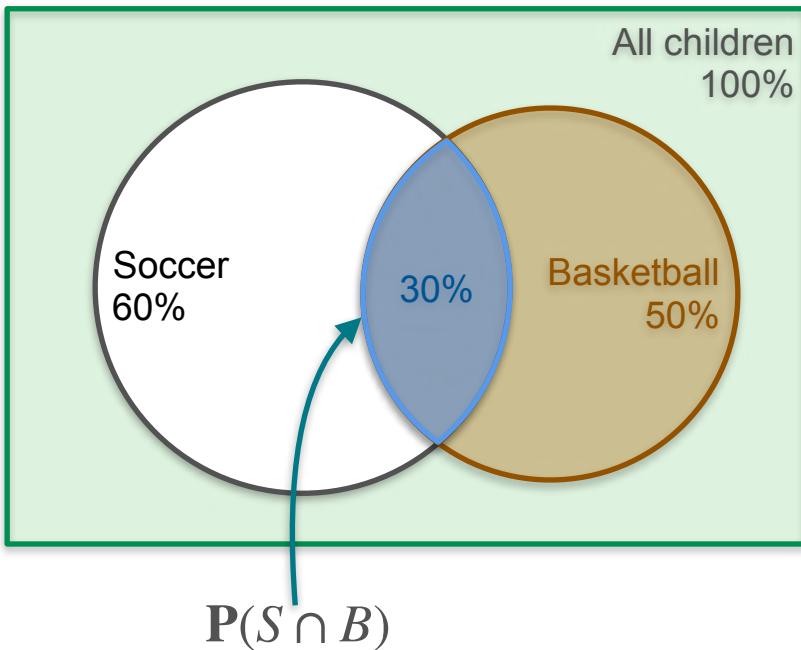


Sum of Probabilities (Joint Events): Quiz 3 Solution



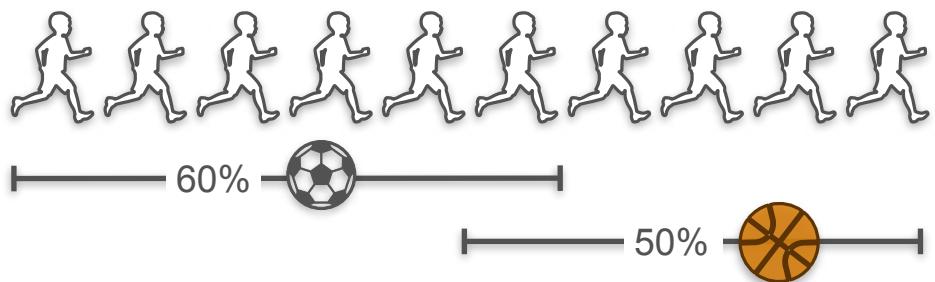
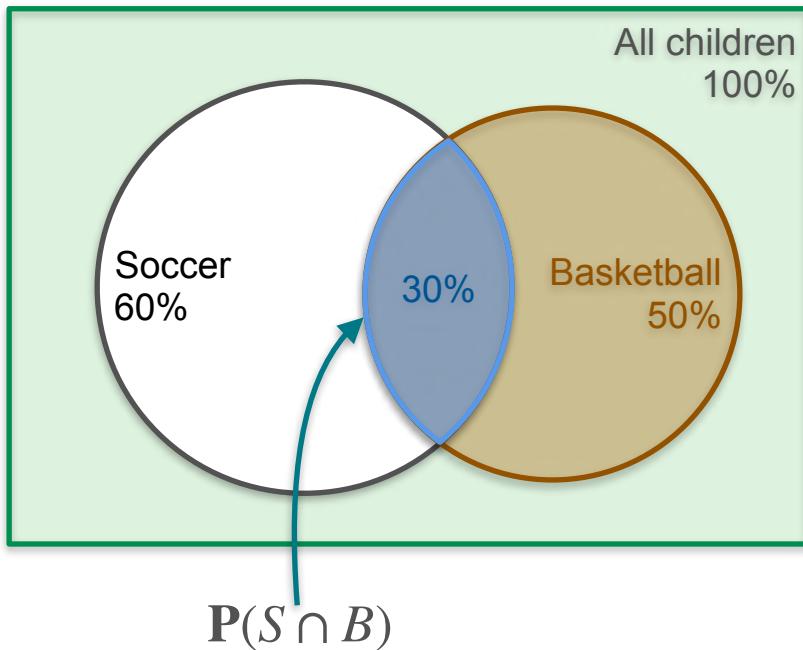
$$P(S \cup B) = P(S) + P(B)$$

Sum of Probabilities (Joint Events): Quiz 3 Solution



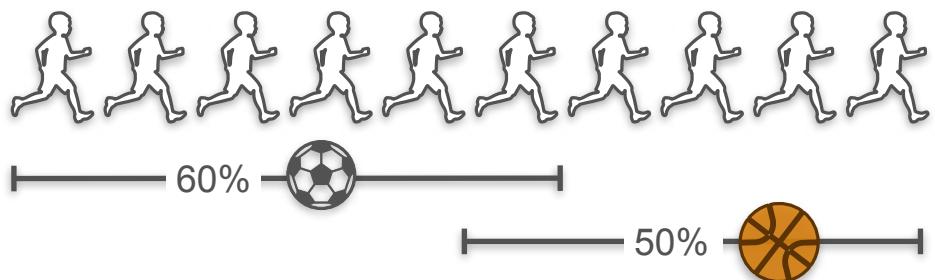
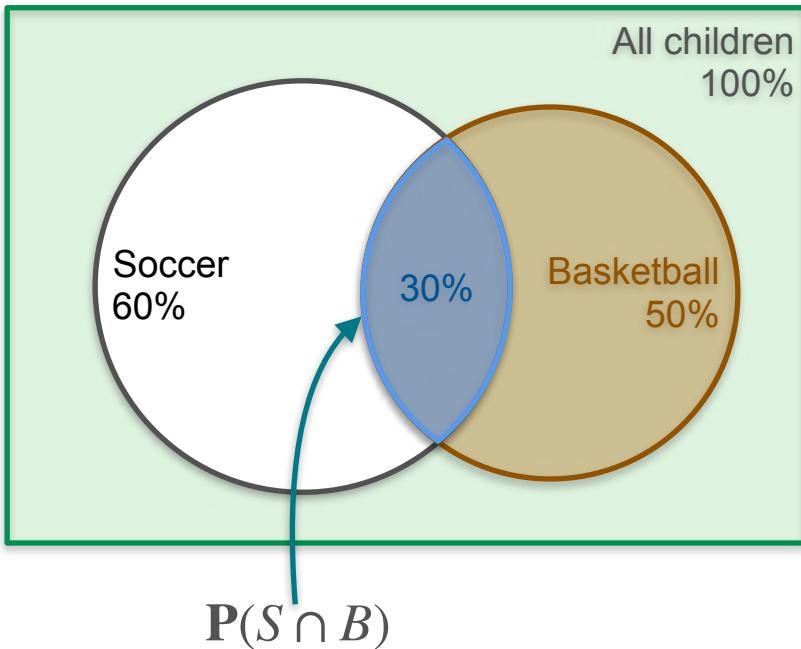
$$\mathbf{P}(S \cup B) = \mathbf{P}(S) + \mathbf{P}(B)$$

Sum of Probabilities (Joint Events): Quiz 3 Solution



$$\mathbf{P}(S \cup B) = \mathbf{P}(S) + \mathbf{P}(B) - \mathbf{P}(S \cap B)$$

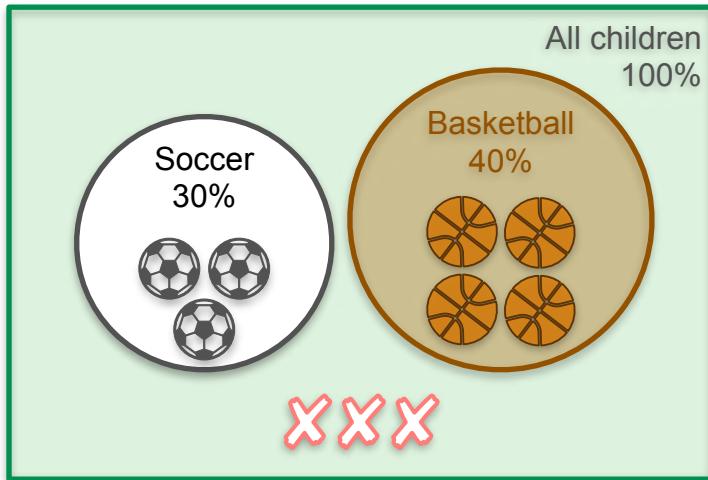
Sum of Probabilities (Joint Events): Quiz 3 Solution



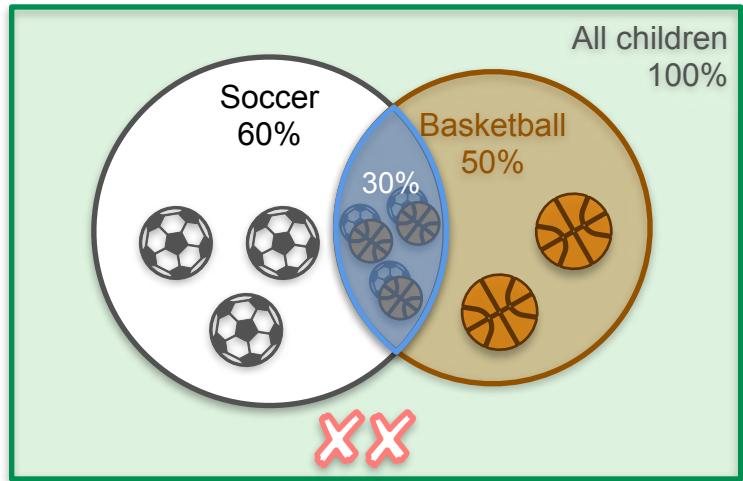
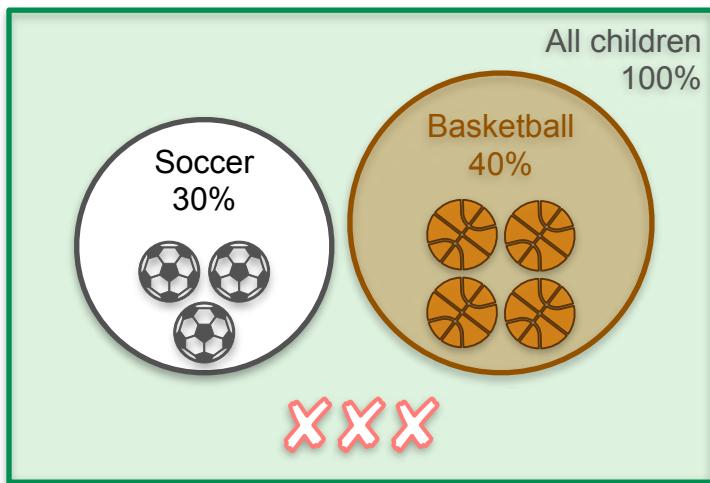
$$\begin{aligned}\mathbf{P}(S \cup B) &= \mathbf{P}(S) + \mathbf{P}(B) - \mathbf{P}(S \cap B) \\ &= 0.6 + 0.5 - 0.3 \\ &= 0.8\end{aligned}$$

Disjoint Events Vs Joint Events

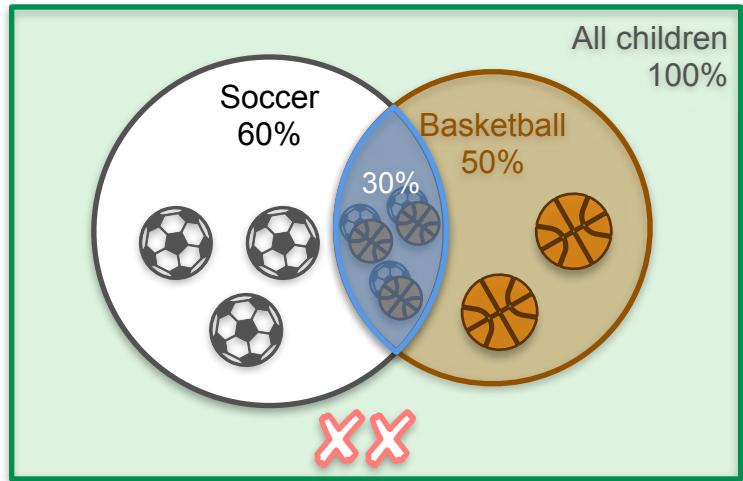
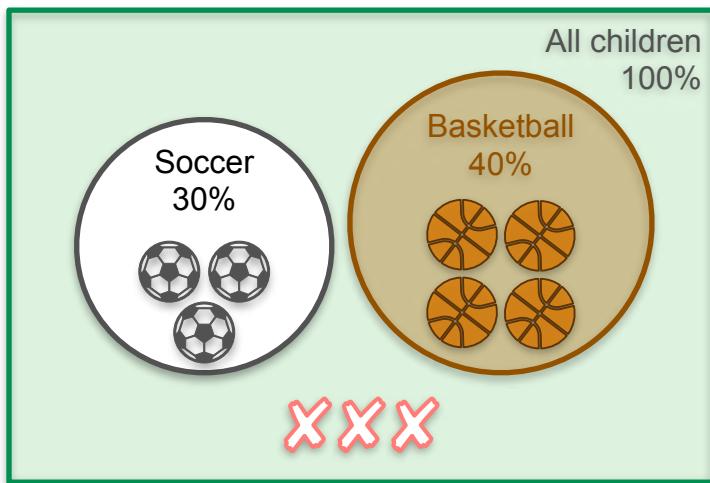
Disjoint Events Vs Joint Events



Disjoint Events Vs Joint Events

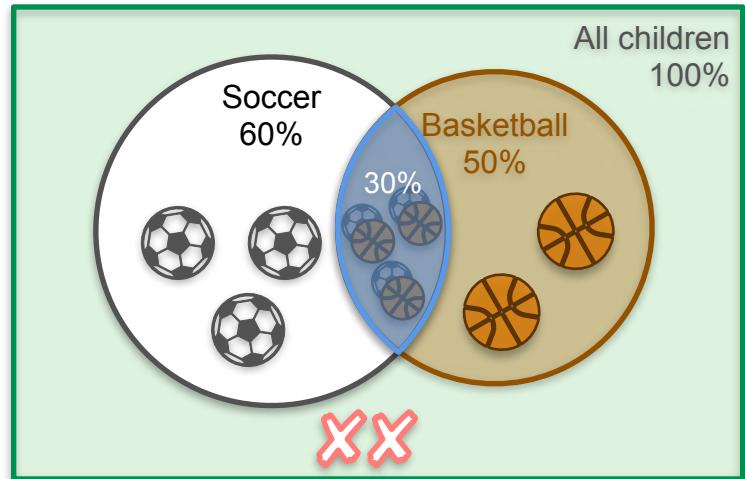
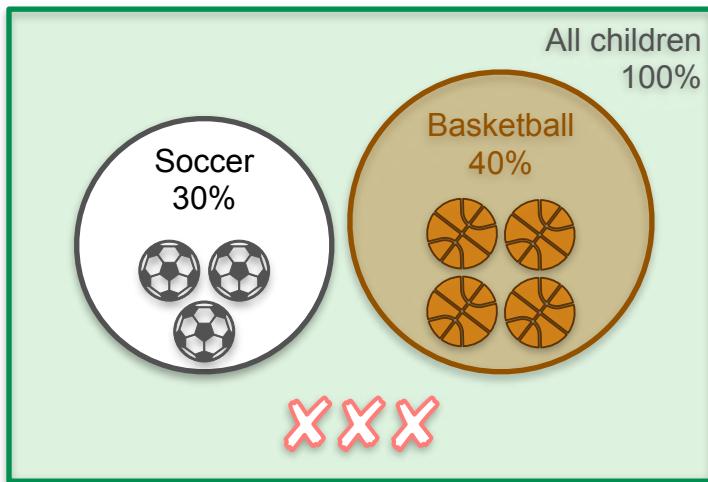


Disjoint Events Vs Joint Events



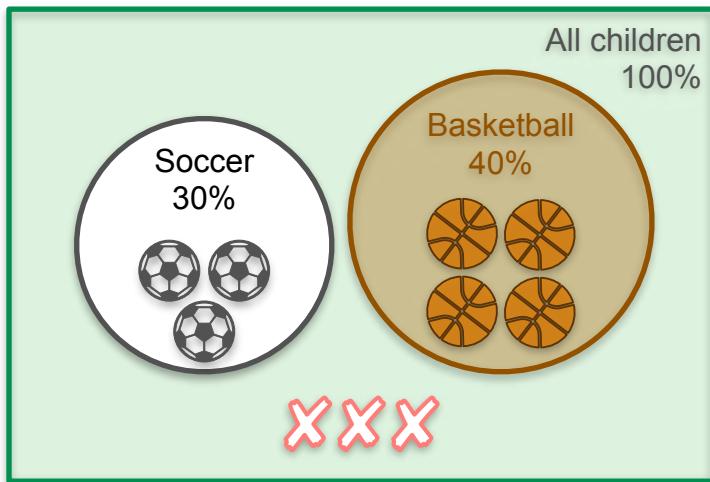
Disjoint Events Vs Joint Events

Disjoint

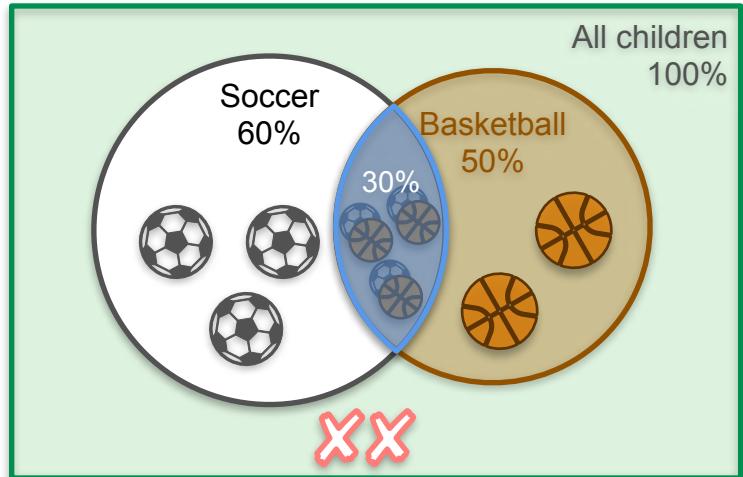


Disjoint Events Vs Joint Events

Disjoint

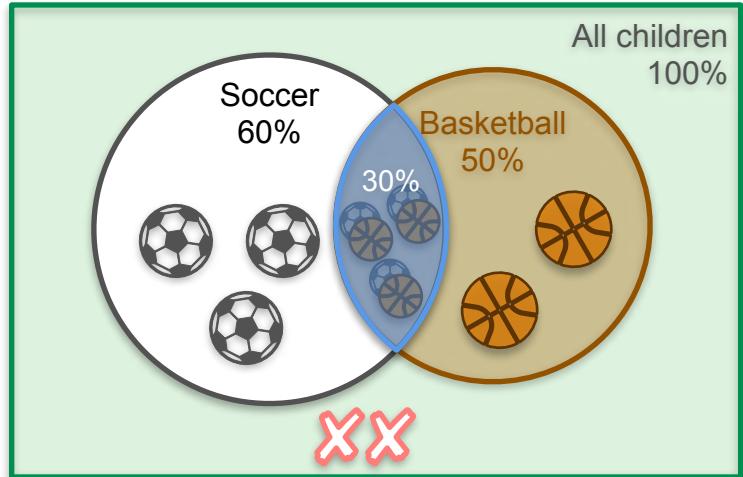
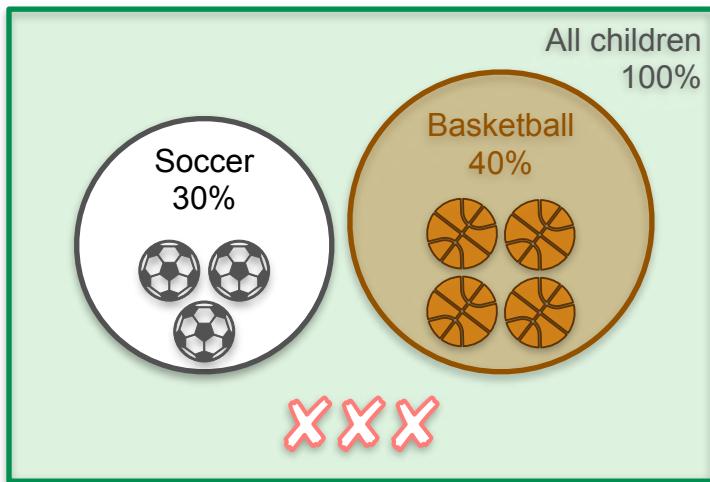


Mutually exclusive



Disjoint Events Vs Joint Events

Disjoint

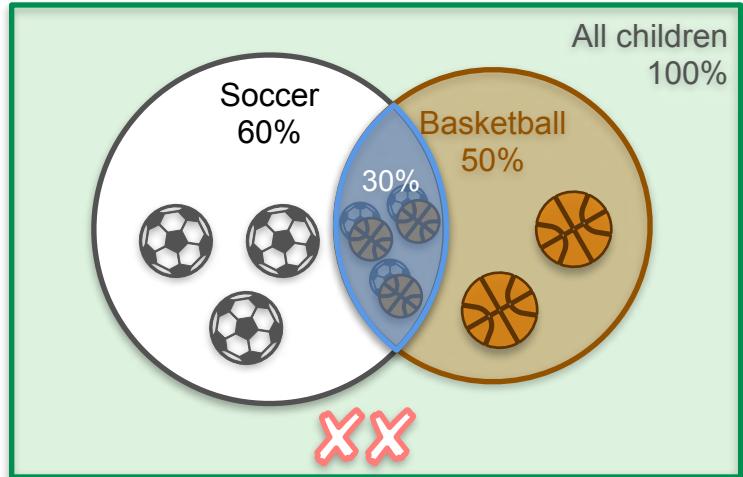
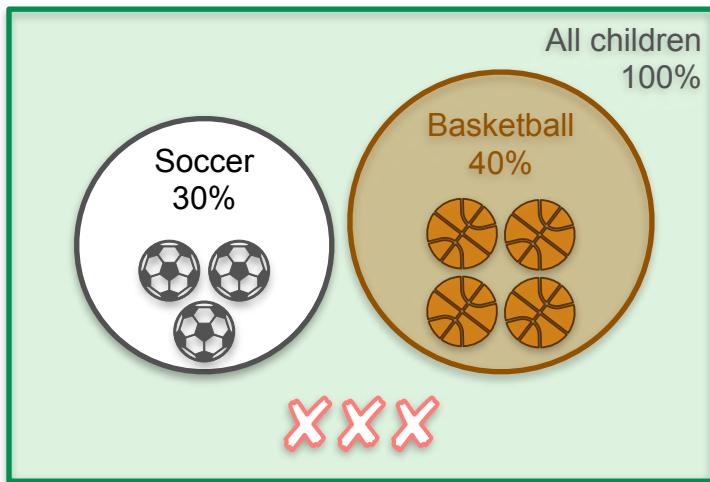


Mutually exclusive

$$P(S \cup B) = P(S) + P(B)$$

Disjoint Events Vs Joint Events

Disjoint

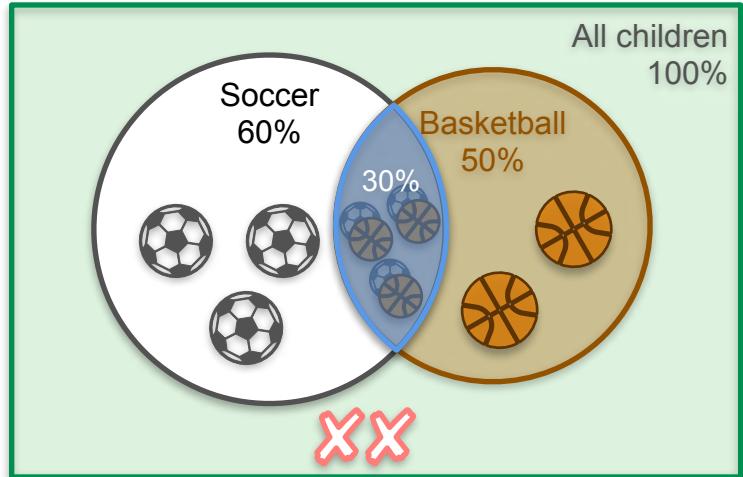
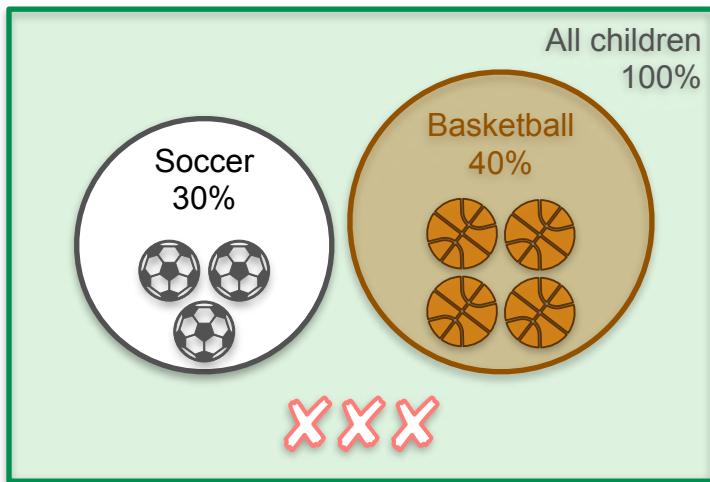


Mutually exclusive

$$P(S \cup B) = P(S) + P(B)$$

Disjoint Events Vs Joint Events

Disjoint

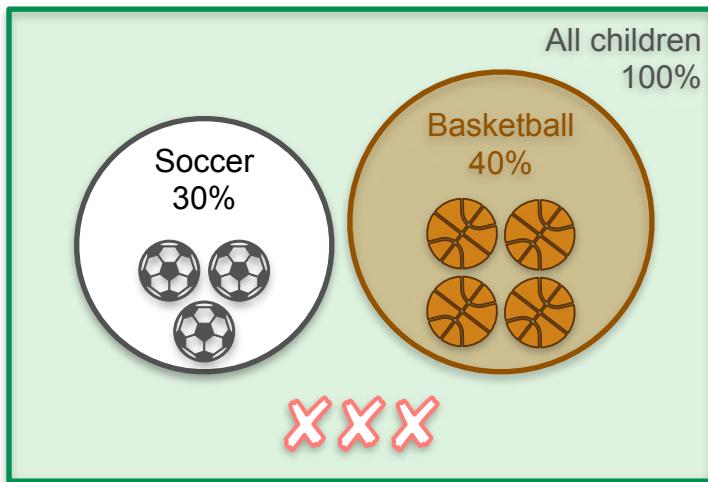


Mutually exclusive

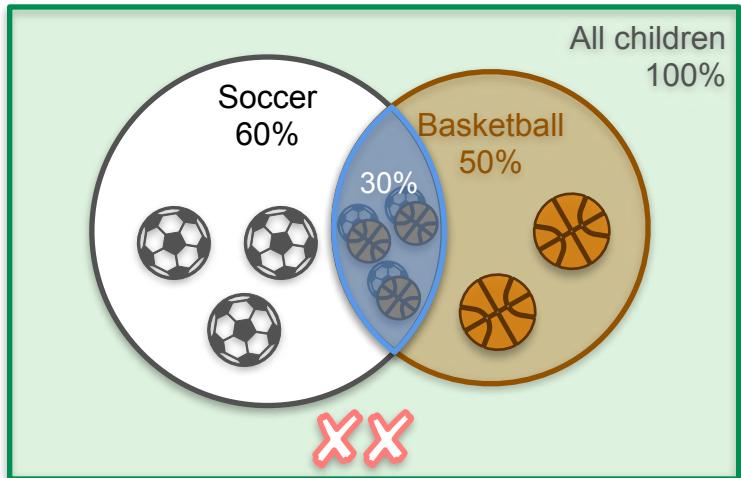
$$P(S \cup B) = P(S) + P(B)$$

Disjoint Events Vs Joint Events

Disjoint



Joint

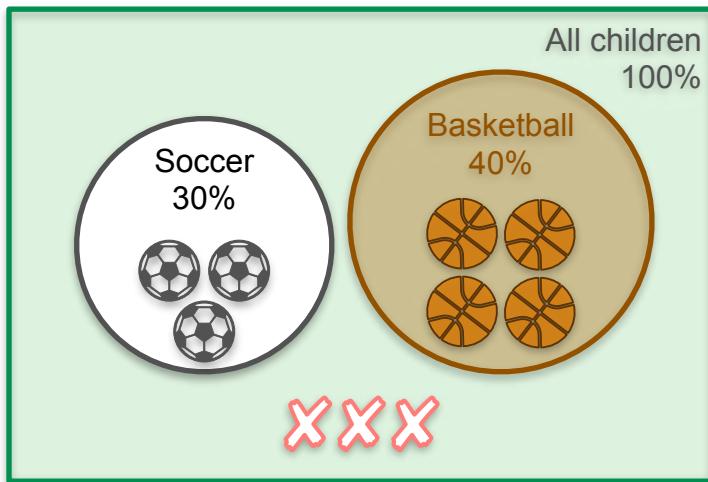


Mutually exclusive

$$P(S \cup B) = P(S) + P(B)$$

Disjoint Events Vs Joint Events

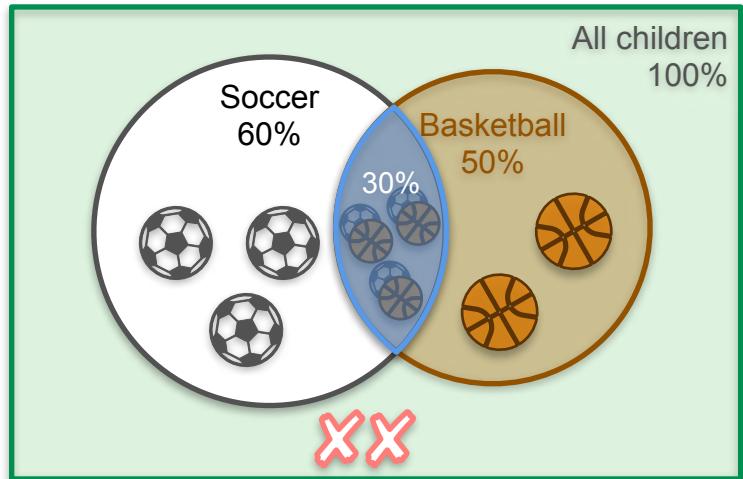
Disjoint



Mutually exclusive

$$P(S \cup B) = P(S) + P(B)$$

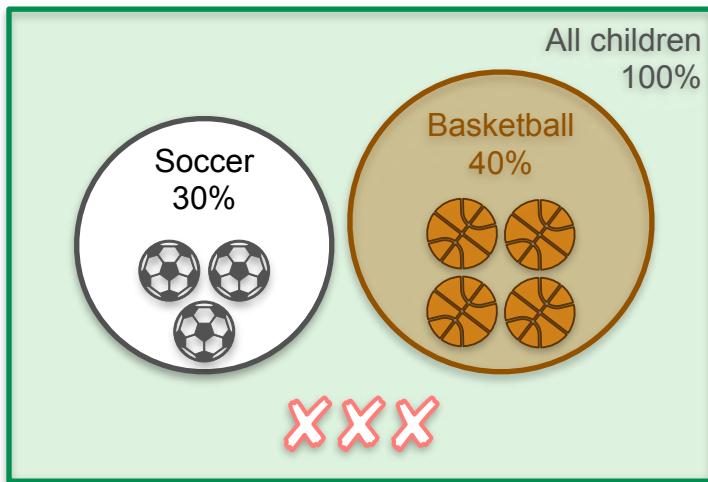
Joint



Non-mutually exclusive

Disjoint Events Vs Joint Events

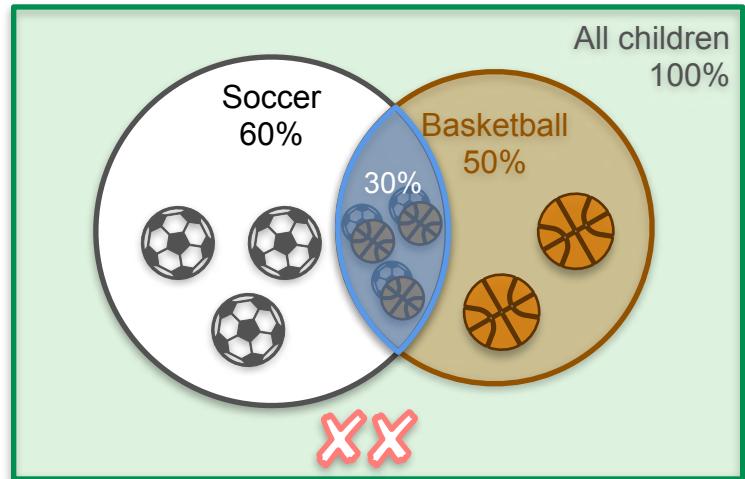
Disjoint



Mutually exclusive

$$P(S \cup B) = P(S) + P(B)$$

Joint

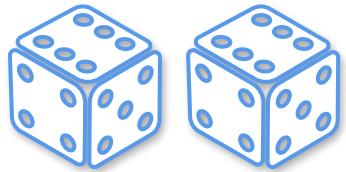


Non-mutually exclusive

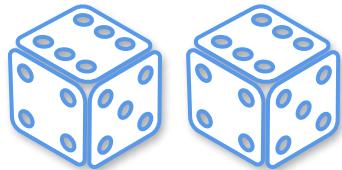
$$P(S \cup B) = P(S) + P(B) - P(S \cap B)$$

Sum of Probabilities (Joint Events): Dice Example 1

Sum of Probabilities (Joint Events): Dice Example 1



Sum of Probabilities (Joint Events): Dice Example 1



What is the probability of obtaining a sum of 7 or a difference of 1?

Sum of Probabilities (Joint Events): Dice Example 1

Sum of Probabilities (Joint Events): Dice Example 1

sum = 7

diff = 1

Sum of Probabilities (Joint Events): Dice Example 1

sum = 7

diff = 1

	1,1	1,2	1,3	1,4	1,5	1,6
1,1						
1,2						
1,3						
1,4						
1,5						
1,6						
2,1	2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,1	6,2	6,3	6,4	6,5	6,6

Sum of Probabilities (Joint Events): Dice Example 1

sum = 7

diff = 1

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6

Sum of Probabilities (Joint Events): Dice Example 1

sum = 7

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6

diff = 1

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6

Sum of Probabilities (Joint Events): Dice Example 1

sum = 7

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,1	6,2	6,3	6,4	6,5	6,6

diff = 1

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,1	6,2	6,3	6,4	6,5	6,6

Sum of Probabilities (Joint Events): Dice Example 1

A

sum = 7

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,1	6,2	6,3	6,4	6,5	6,6

B

diff = 1

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,1	6,2	6,3	6,4	6,5	6,6

Sum of Probabilities (Joint Events): Dice Example 1

A or B

sum = 7 or diff = 1

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6

Sum of Probabilities (Joint Events): Dice Example 1

A or B

sum = 7 or diff = 1

	1,1	1,2	1,3	1,4	1,5	1,6
1,2	2,1	2,2	2,3	2,4	2,5	2,6
2,3	3,1	3,2	3,3	3,4	3,5	3,6
3,4	4,1	4,2	4,3	4,4	4,5	4,6
4,5	5,1	5,2	5,3	5,4	5,5	5,6
5,6	6,1	6,2	6,3	6,4	6,5	6,6

sum = 7 and diff = 1

Sum of Probabilities (Joint Events): Dice Example 1

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

Sum of Probabilities (Joint Events): Dice Example 1

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

P(sum = 7 or diff = 1)

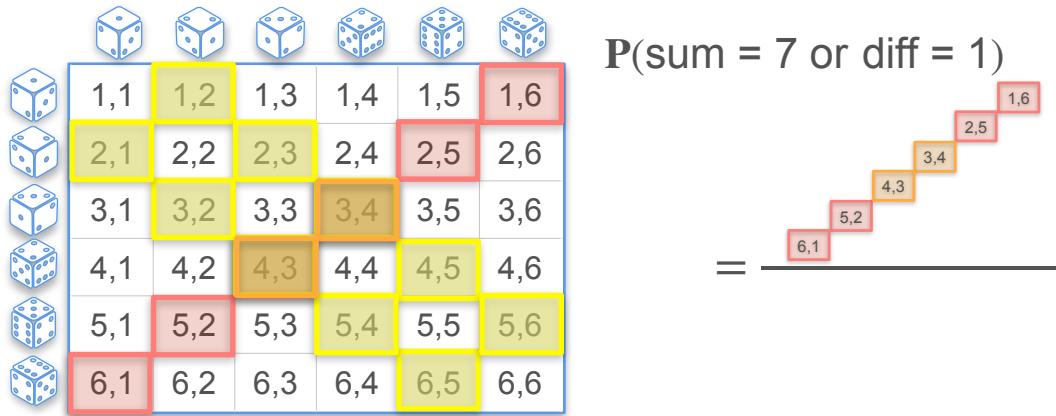
Sum of Probabilities (Joint Events): Dice Example 1

					
1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

P(sum = 7 or diff = 1)

= _____

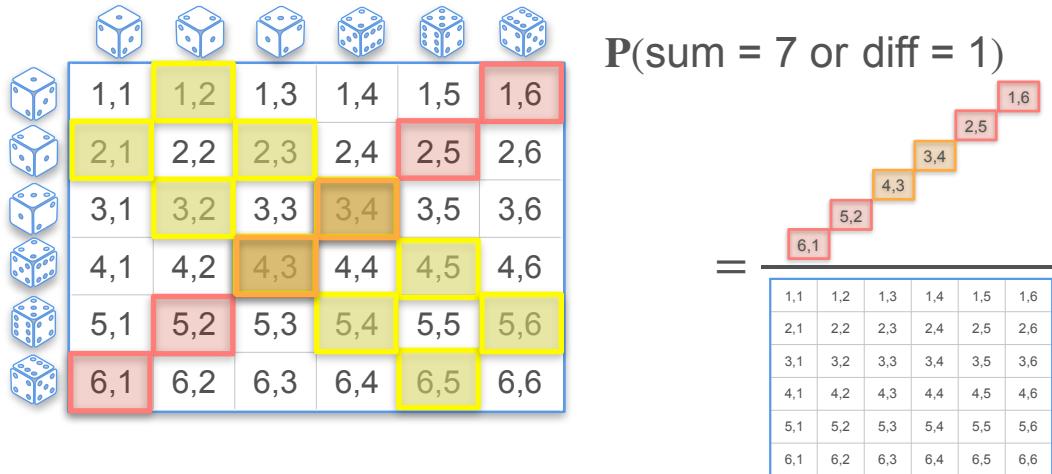
Sum of Probabilities (Joint Events): Dice Example 1



$$= \underline{\hspace{1cm}}$$

A sequence of dice rolls is shown as a staircase path from bottom-left to top-right, representing the joint events that satisfy the conditions of the problem. The rolls are: (6,1), (5,2), (4,3), (3,4), (2,5), and (1,6).

Sum of Probabilities (Joint Events): Dice Example 1



Sum of Probabilities (Joint Events): Dice Example 1

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

P(sum = 7 or diff = 1)

$$= \frac{\begin{array}{ccccccc} & & & & & 1,6 \\ & & & & & 2,5 \\ & & & & & 3,4 \\ & & & & & 4,3 \\ & & & & & 5,2 \\ & & & & & 6,1 \\ \hline 1,1 & 1,2 & 1,3 & 1,4 & 1,5 & 1,6 \end{array}}{\begin{array}{ccccccc} 1,1 & 1,2 & 1,3 & 1,4 & 1,5 & 1,6 \\ 2,1 & 2,2 & 2,3 & 2,4 & 2,5 & 2,6 \\ 3,1 & 3,2 & 3,3 & 3,4 & 3,5 & 3,6 \\ 4,1 & 4,2 & 4,3 & 4,4 & 4,5 & 4,6 \\ 5,1 & 5,2 & 5,3 & 5,4 & 5,5 & 5,6 \\ 6,1 & 6,2 & 6,3 & 6,4 & 6,5 & 6,6 \end{array}} + \underline{\hspace{10em}}$$

Sum of Probabilities (Joint Events): Dice Example 1

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$P(\text{sum} = 7 \text{ or } \text{diff} = 1)$

$$= \frac{\begin{array}{ccccccc} & 1,2 & & & & & \\ & 2,1 & 2,3 & & & & \\ & 3,2 & 3,3 & 3,4 & & & \\ & 4,3 & 4,4 & 4,5 & 4,6 & & \\ & 5,2 & 5,3 & 5,4 & 5,5 & 5,6 & \\ & 6,1 & & & & & \\ \hline 1,1 & 1,2 & 1,3 & 1,4 & 1,5 & 1,6 & \\ 2,1 & 2,2 & 2,3 & 2,4 & 2,5 & 2,6 & \\ 3,1 & 3,2 & 3,3 & 3,4 & 3,5 & 3,6 & \\ 4,1 & 4,2 & 4,3 & 4,4 & 4,5 & 4,6 & \\ 5,1 & 5,2 & 5,3 & 5,4 & 5,5 & 5,6 & \\ 6,1 & 6,2 & 6,3 & 6,4 & 6,5 & 6,6 & \end{array}}{36} + \frac{\begin{array}{ccccccc} & 1,2 & & & & & \\ & 2,1 & & 2,3 & & & \\ & 3,2 & & 3,4 & & & \\ & 4,3 & & 4,5 & & & \\ & 5,4 & & 5,6 & & & \\ & 6,5 & & & & & \\ \hline 1,1 & 1,2 & 1,3 & 1,4 & 1,5 & 1,6 & \\ 2,1 & 2,2 & 2,3 & 2,4 & 2,5 & 2,6 & \\ 3,1 & 3,2 & 3,3 & 3,4 & 3,5 & 3,6 & \\ 4,1 & 4,2 & 4,3 & 4,4 & 4,5 & 4,6 & \\ 5,1 & 5,2 & 5,3 & 5,4 & 5,5 & 5,6 & \\ 6,1 & 6,2 & 6,3 & 6,4 & 6,5 & 6,6 & \end{array}}{36}$$

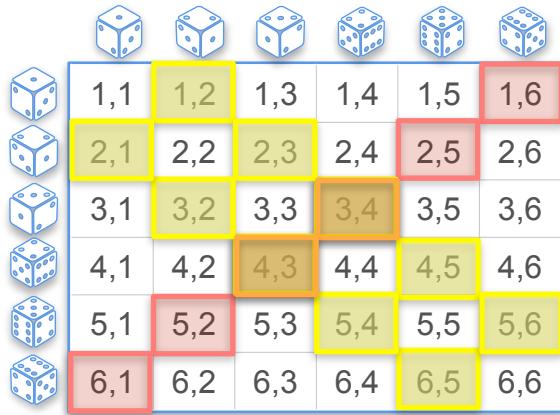
Sum of Probabilities (Joint Events): Dice Example 1

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

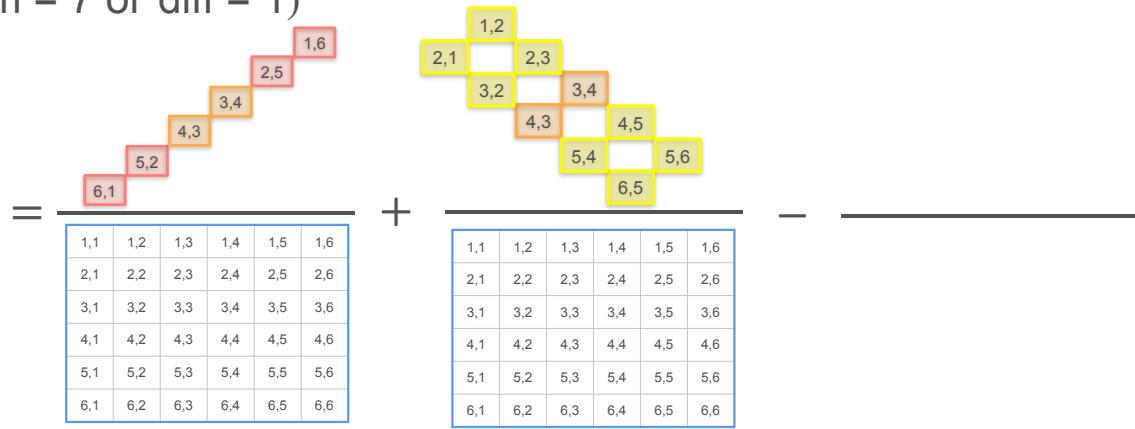
$P(\text{sum} = 7 \text{ or } \text{diff} = 1)$

$$= \frac{\begin{array}{c} 1,2 \\ 2,1 \\ 3,2 \\ 4,3 \\ 5,2 \\ 6,1 \end{array}}{\begin{array}{ccccccc} 1,1 & 1,2 & 1,3 & 1,4 & 1,5 & 1,6 & \\ 2,1 & 2,2 & 2,3 & 2,4 & 2,5 & 2,6 & \\ 3,1 & 3,2 & 3,3 & 3,4 & 3,5 & 3,6 & \\ 4,1 & 4,2 & 4,3 & 4,4 & 4,5 & 4,6 & \\ 5,1 & 5,2 & 5,3 & 5,4 & 5,5 & 5,6 & \\ 6,1 & 6,2 & 6,3 & 6,4 & 6,5 & 6,6 & \end{array}} + \frac{\begin{array}{c} 1,2 \\ 2,3 \\ 3,4 \\ 4,5 \\ 5,6 \\ 6,5 \end{array}}{\begin{array}{ccccccc} 1,1 & 1,2 & 1,3 & 1,4 & 1,5 & 1,6 & \\ 2,1 & 2,2 & 2,3 & 2,4 & 2,5 & 2,6 & \\ 3,1 & 3,2 & 3,3 & 3,4 & 3,5 & 3,6 & \\ 4,1 & 4,2 & 4,3 & 4,4 & 4,5 & 4,6 & \\ 5,1 & 5,2 & 5,3 & 5,4 & 5,5 & 5,6 & \\ 6,1 & 6,2 & 6,3 & 6,4 & 6,5 & 6,6 & \end{array}}$$

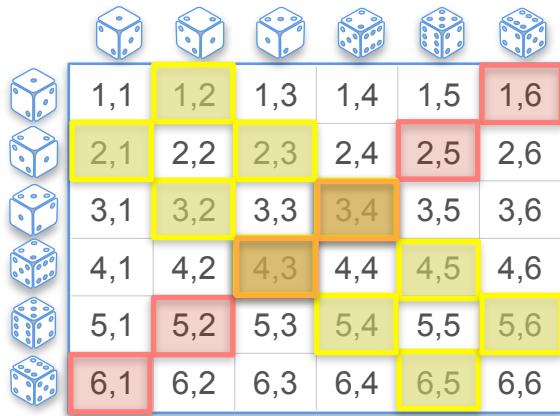
Sum of Probabilities (Joint Events): Dice Example 1



$P(\text{sum} = 7 \text{ or } \text{diff} = 1)$



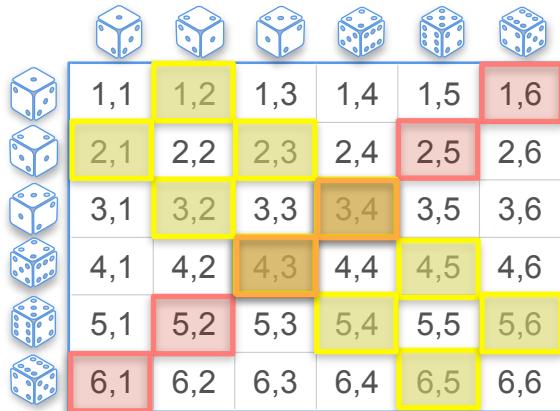
Sum of Probabilities (Joint Events): Dice Example 1



$P(\text{sum} = 7 \text{ or } \text{diff} = 1)$

$$= \frac{\begin{array}{c} 1,2 \\ 2,1 \\ 3,2 \\ 4,3 \\ 5,2 \\ 6,1 \end{array}}{\begin{array}{ccccccc} 1,1 & 1,2 & 1,3 & 1,4 & 1,5 & 1,6 \\ 2,1 & 2,2 & 2,3 & 2,4 & 2,5 & 2,6 \\ 3,1 & 3,2 & 3,3 & 3,4 & 3,5 & 3,6 \\ 4,1 & 4,2 & 4,3 & 4,4 & 4,5 & 4,6 \\ 5,1 & 5,2 & 5,3 & 5,4 & 5,5 & 5,6 \\ 6,1 & 6,2 & 6,3 & 6,4 & 6,5 & 6,6 \end{array}} + \frac{\begin{array}{c} 1,2 \\ 2,3 \\ 3,4 \\ 4,5 \\ 5,6 \\ 6,5 \end{array}}{\begin{array}{ccccccc} 1,1 & 1,2 & 1,3 & 1,4 & 1,5 & 1,6 \\ 2,1 & 2,2 & 2,3 & 2,4 & 2,5 & 2,6 \\ 3,1 & 3,2 & 3,3 & 3,4 & 3,5 & 3,6 \\ 4,1 & 4,2 & 4,3 & 4,4 & 4,5 & 4,6 \\ 5,1 & 5,2 & 5,3 & 5,4 & 5,5 & 5,6 \\ 6,1 & 6,2 & 6,3 & 6,4 & 6,5 & 6,6 \end{array}} - \frac{\begin{array}{c} 3,4 \\ 4,3 \end{array}}{\begin{array}{ccccccc} 1,1 & 1,2 & 1,3 & 1,4 & 1,5 & 1,6 \\ 2,1 & 2,2 & 2,3 & 2,4 & 2,5 & 2,6 \\ 3,1 & 3,2 & 3,3 & 3,4 & 3,5 & 3,6 \\ 4,1 & 4,2 & 4,3 & 4,4 & 4,5 & 4,6 \\ 5,1 & 5,2 & 5,3 & 5,4 & 5,5 & 5,6 \\ 6,1 & 6,2 & 6,3 & 6,4 & 6,5 & 6,6 \end{array}}$$

Sum of Probabilities (Joint Events): Dice Example 1



$P(\text{sum} = 7 \text{ or } \text{diff} = 1)$

$$\begin{array}{c} \text{Diagram showing the decomposition of the joint event into two disjoint sets:} \\ \begin{array}{c} \text{Set 1: } \text{sum} = 7 \\ \text{Set 2: } \text{diff} = 1 \end{array} \\ = \frac{\text{Set 1}}{\text{Total Outcomes}} + \frac{\text{Set 2}}{\text{Total Outcomes}} - \frac{\text{Intersection}}{\text{Total Outcomes}} \end{array}$$

The diagram illustrates the decomposition of the joint event into two disjoint sets:

- Set 1: $\text{sum} = 7$** (represented by yellow boxes)
- Set 2: $\text{diff} = 1$** (represented by red boxes)

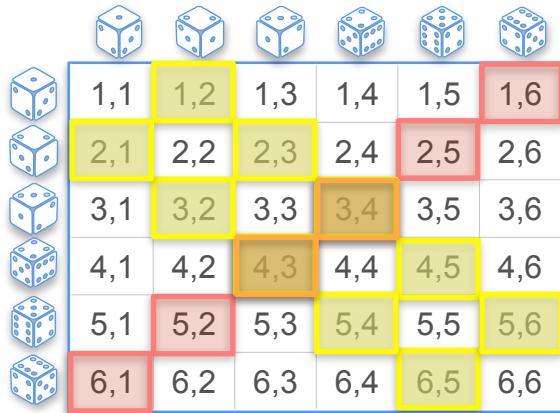
The total number of outcomes is 36. The intersection of the two sets (outcomes where both sum=7 and diff=1) is shown in orange boxes. The final result is the sum of the probabilities of the two sets minus the probability of their intersection.

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

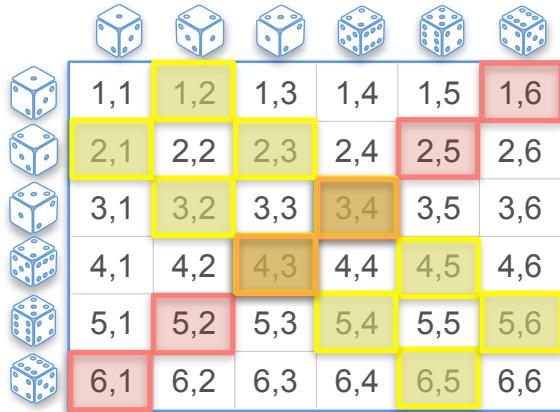
Sum of Probabilities (Joint Events): Dice Example 1



$P(\text{sum} = 7 \text{ or } \text{diff} = 1)$

$$\begin{aligned} &= \frac{\begin{array}{ccccccc} 1,1 & 1,2 & 1,3 & 1,4 & 1,5 & 1,6 \\ 2,1 & 2,2 & 2,3 & 2,4 & 2,5 & 2,6 \\ 3,1 & 3,2 & 3,3 & 3,4 & 3,5 & 3,6 \\ 4,1 & 4,2 & 4,3 & 4,4 & 4,5 & 4,6 \\ 5,1 & 5,2 & 5,3 & 5,4 & 5,5 & 5,6 \\ 6,1 & 6,2 & 6,3 & 6,4 & 6,5 & 6,6 \end{array}}{36} + \frac{\begin{array}{ccccccc} 1,2 & & & & & & \\ 2,1 & 2,3 & & & & & \\ 3,2 & 3,4 & & & & & \\ 4,3 & 4,5 & & & & & \\ 5,4 & 5,6 & & & & & \\ 6,5 & & & & & & \end{array}}{36} - \frac{\begin{array}{ccccc} 3,4 & & & & \\ 4,3 & & & & \\ 5,4 & & & & \\ 6,3 & & & & \end{array}}{36} \\ &= \frac{6}{36} \end{aligned}$$

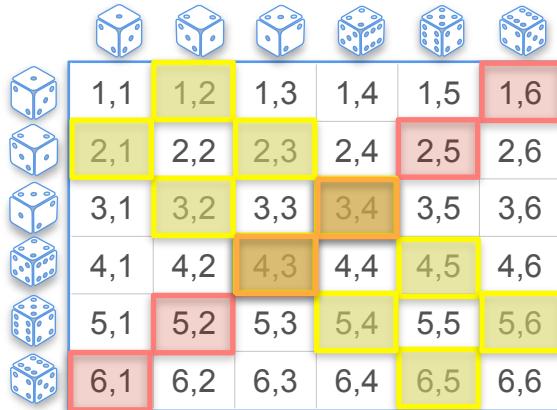
Sum of Probabilities (Joint Events): Dice Example 1



$P(\text{sum} = 7 \text{ or } \text{diff} = 1)$

$$\begin{aligned} &= \frac{\begin{array}{ccccccc} 1,1 & 1,2 & 1,3 & 1,4 & 1,5 & 1,6 \\ 2,1 & 2,2 & 2,3 & 2,4 & 2,5 & 2,6 \\ 3,1 & 3,2 & 3,3 & 3,4 & 3,5 & 3,6 \\ 4,1 & 4,2 & 4,3 & 4,4 & 4,5 & 4,6 \\ 5,1 & 5,2 & 5,3 & 5,4 & 5,5 & 5,6 \\ 6,1 & 6,2 & 6,3 & 6,4 & 6,5 & 6,6 \end{array}}{36} + \frac{\begin{array}{ccccccc} 1,1 & 1,2 & 1,3 & 1,4 & 1,5 & 1,6 \\ 2,1 & 2,2 & 2,3 & 2,4 & 2,5 & 2,6 \\ 3,2 & 3,3 & 3,4 & 3,5 & 3,6 & \\ 4,3 & 4,4 & 4,5 & 4,6 & \\ 5,4 & 5,5 & 5,6 & \\ 6,5 & \end{array}}{36} - \frac{\begin{array}{ccccccc} 1,1 & 1,2 & 1,3 & 1,4 & 1,5 & 1,6 \\ 2,1 & 2,2 & 2,3 & 2,4 & 2,5 & 2,6 \\ 3,1 & 3,2 & 3,3 & 3,4 & 3,5 & 3,6 \\ 4,1 & 4,2 & 4,3 & 4,4 & 4,5 & 4,6 \\ 5,1 & 5,2 & 5,3 & 5,4 & 5,5 & 5,6 \\ 6,1 & 6,2 & 6,3 & 6,4 & 6,5 & 6,6 \end{array}}{36} \\ &= \frac{6}{36} + \frac{10}{36} \end{aligned}$$

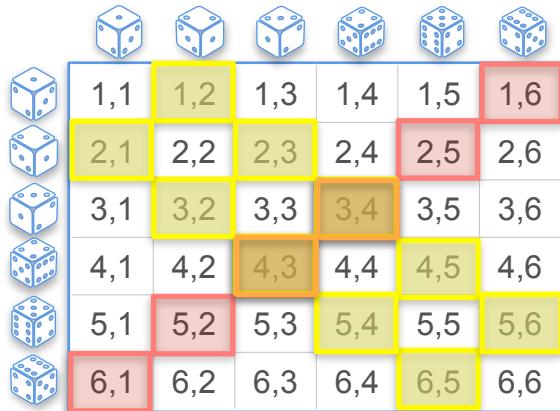
Sum of Probabilities (Joint Events): Dice Example 1



$P(\text{sum} = 7 \text{ or } \text{diff} = 1)$

$$\begin{aligned} &= \frac{\begin{array}{ccccccc} 1,1 & 1,2 & 1,3 & 1,4 & 1,5 & 1,6 \\ 2,1 & 2,2 & 2,3 & 2,4 & 2,5 & 2,6 \\ 3,1 & 3,2 & 3,3 & 3,4 & 3,5 & 3,6 \\ 4,1 & 4,2 & 4,3 & 4,4 & 4,5 & 4,6 \\ 5,1 & 5,2 & 5,3 & 5,4 & 5,5 & 5,6 \\ 6,1 & 6,2 & 6,3 & 6,4 & 6,5 & 6,6 \end{array}}{36} + \frac{\begin{array}{ccccccc} 1,2 & & & & & & \\ 2,1 & 2,3 & & & & & \\ 3,2 & 3,4 & & & & & \\ 4,3 & & 4,5 & & & & \\ 5,4 & & 5,6 & & & & \\ 6,5 & & & & & & \end{array}}{36} - \frac{\begin{array}{ccccccc} 3,4 & & & & & & \\ 4,3 & & & & & & \\ 5,4 & & & & & & \end{array}}{36} \\ &= \frac{6}{36} + \frac{10}{36} - \frac{2}{36} \end{aligned}$$

Sum of Probabilities (Joint Events): Dice Example 1

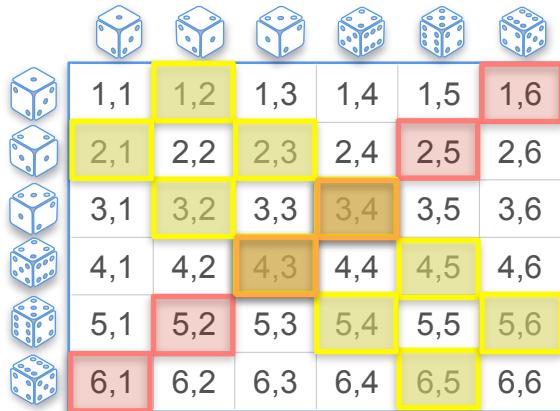


$P(\text{sum} = 7 \text{ or } \text{diff} = 1)$

$$\begin{aligned}
 &= \frac{\begin{array}{c} 1,2 \\ 2,1 \\ 3,2 \\ 4,3 \\ 5,2 \\ 6,1 \end{array}}{\begin{array}{ccccccc} 1,1 & 1,2 & 1,3 & 1,4 & 1,5 & 1,6 \\ 2,1 & 2,2 & 2,3 & 2,4 & 2,5 & 2,6 \\ 3,1 & 3,2 & 3,3 & 3,4 & 3,5 & 3,6 \\ 4,1 & 4,2 & 4,3 & 4,4 & 4,5 & 4,6 \\ 5,1 & 5,2 & 5,3 & 5,4 & 5,5 & 5,6 \\ 6,1 & 6,2 & 6,3 & 6,4 & 6,5 & 6,6 \end{array}} + \frac{\begin{array}{c} 1,2 \\ 2,3 \\ 3,4 \\ 4,5 \\ 5,6 \\ 6,5 \end{array}}{\begin{array}{ccccccc} 1,1 & 1,2 & 1,3 & 1,4 & 1,5 & 1,6 \\ 2,1 & 2,2 & 2,3 & 2,4 & 2,5 & 2,6 \\ 3,1 & 3,2 & 3,3 & 3,4 & 3,5 & 3,6 \\ 4,1 & 4,2 & 4,3 & 4,4 & 4,5 & 4,6 \\ 5,1 & 5,2 & 5,3 & 5,4 & 5,5 & 5,6 \\ 6,1 & 6,2 & 6,3 & 6,4 & 6,5 & 6,6 \end{array}} - \frac{\begin{array}{c} 3,4 \\ 4,3 \\ 5,4 \end{array}}{\begin{array}{ccccccc} 1,1 & 1,2 & 1,3 & 1,4 & 1,5 & 1,6 \\ 2,1 & 2,2 & 2,3 & 2,4 & 2,5 & 2,6 \\ 3,1 & 3,2 & 3,3 & 3,4 & 3,5 & 3,6 \\ 4,1 & 4,2 & 4,3 & 4,4 & 4,5 & 4,6 \\ 5,1 & 5,2 & 5,3 & 5,4 & 5,5 & 5,6 \\ 6,1 & 6,2 & 6,3 & 6,4 & 6,5 & 6,6 \end{array}}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{6}{36} + \frac{10}{36} - \frac{2}{36} \\
 &= \frac{14}{36}
 \end{aligned}$$

Sum of Probabilities (Joint Events): Dice Example 1

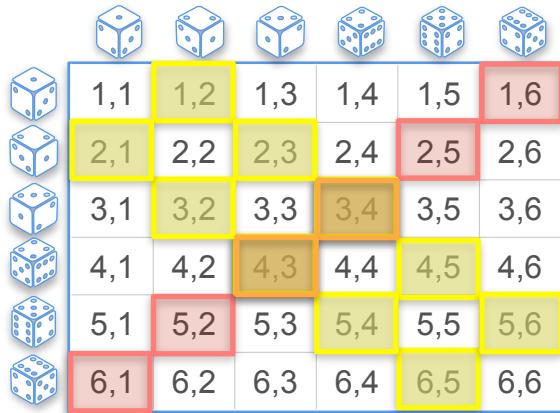


$P(\text{sum} = 7 \text{ or } \text{diff} = 1)$

$$\begin{aligned}
 &= P(\text{sum} = 7) + \frac{\text{Count of highlighted cells}}{\text{Total number of outcomes}} - \frac{\text{Count of double-highlighted cells}}{\text{Total number of outcomes}} \\
 &= \frac{6}{36} + \frac{10}{36} - \frac{2}{36} \\
 &= \frac{14}{36}
 \end{aligned}$$

The diagram illustrates the calculation of the probability of sum=7 or difference=1. It shows three sets of highlighted cells: yellow for sum=7, red for difference=1, and orange for both. The first term, $P(\text{sum} = 7)$, is highlighted with a green border. The second term is the count of yellow cells divided by 36. The third term is the count of cells highlighted in both yellow and red divided by 36.

Sum of Probabilities (Joint Events): Dice Example 1



$P(\text{sum} = 7 \text{ or } \text{diff} = 1)$

$$= P(\text{sum} = 7) + P(\text{diff} = 1) - \frac{2}{36}$$

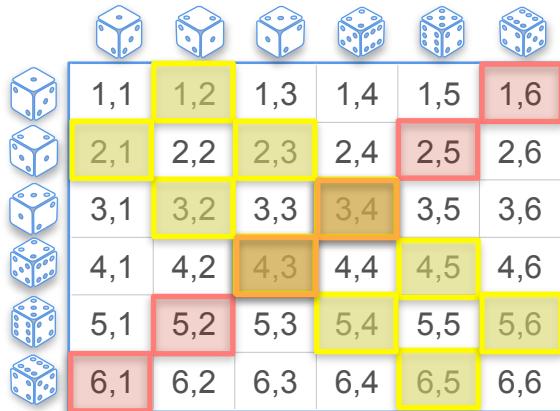
The equation shows the probability of the sum being 7 plus the probability of the difference being 1 minus the probability of both events occurring together. The first term is calculated from a 6x6 grid where the sum of each row and column is 7. The second term is calculated from a 6x6 grid where the absolute difference between each row and column is 1. The third term is the intersection of these two sets, which contains 2 elements: (1,6) and (6,1).

$= \frac{6}{36}$

$+ \frac{10}{36}$

$- \frac{2}{36}$

Sum of Probabilities (Joint Events): Dice Example 1



$P(\text{sum} = 7 \text{ or } \text{diff} = 1)$

$$\begin{aligned}
 &= P(\text{sum} = 7) + P(\text{diff} = 1) - P(\text{sum} = 7 \cap \text{diff} = 1) \\
 &= \frac{6}{36} + \frac{10}{36} - \frac{2}{36} \\
 &= \frac{14}{36}
 \end{aligned}$$



DeepLearning.AI

Introduction to probability

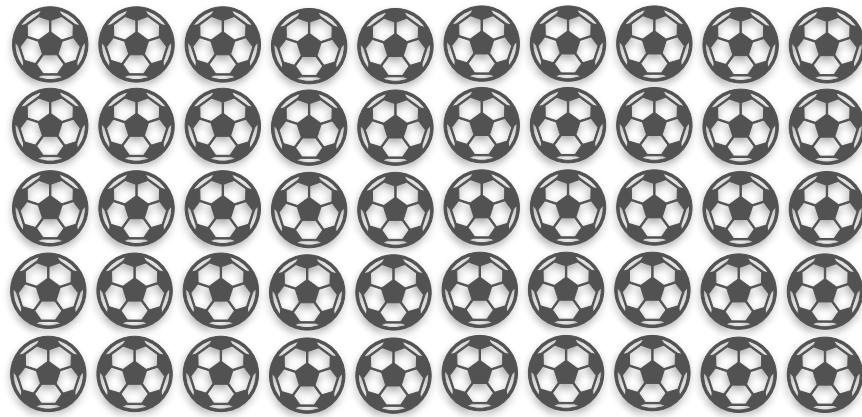
Independence

Independence: Quiz 1

Independence: Quiz 1

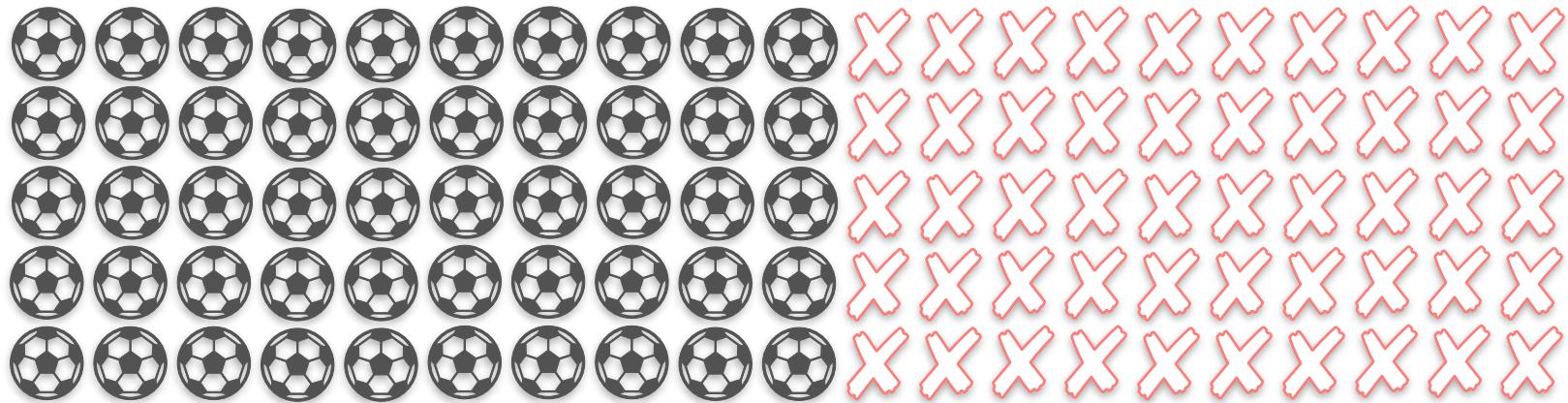
100 kids

Independence: Quiz 1



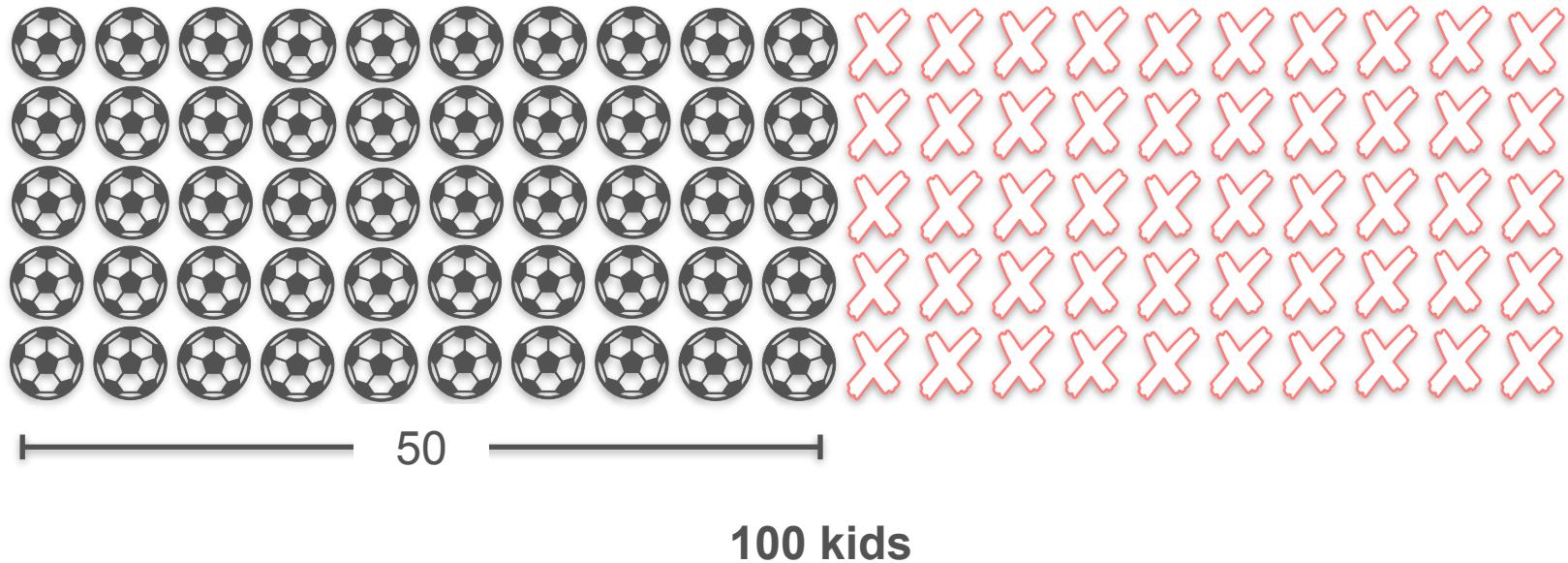
100 kids

Independence: Quiz 1

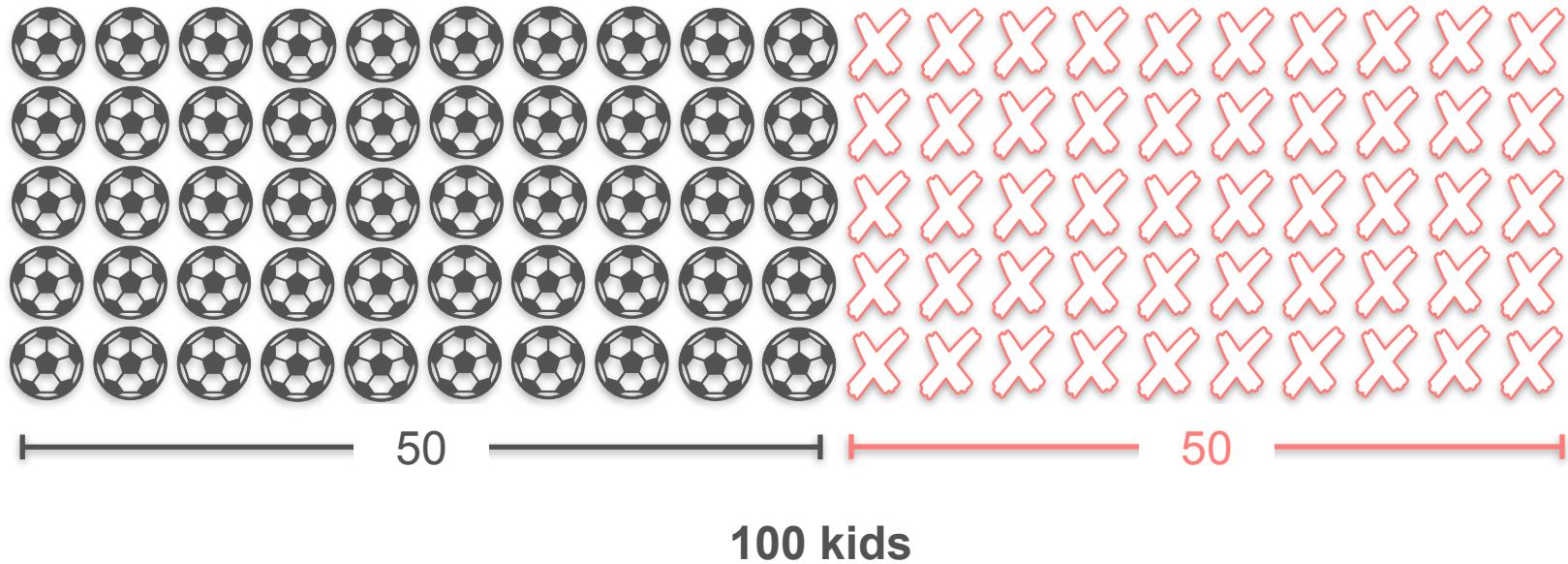


100 kids

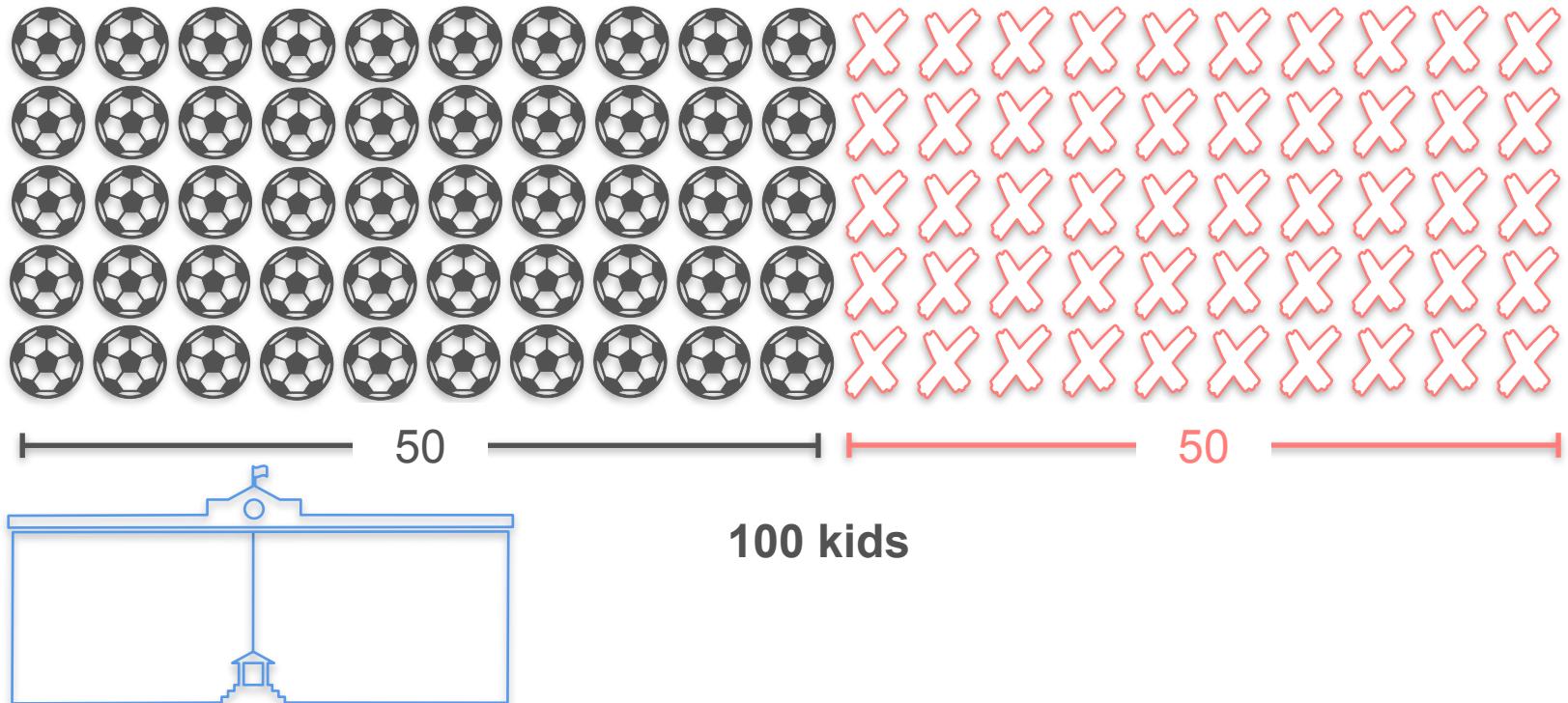
Independence: Quiz 1



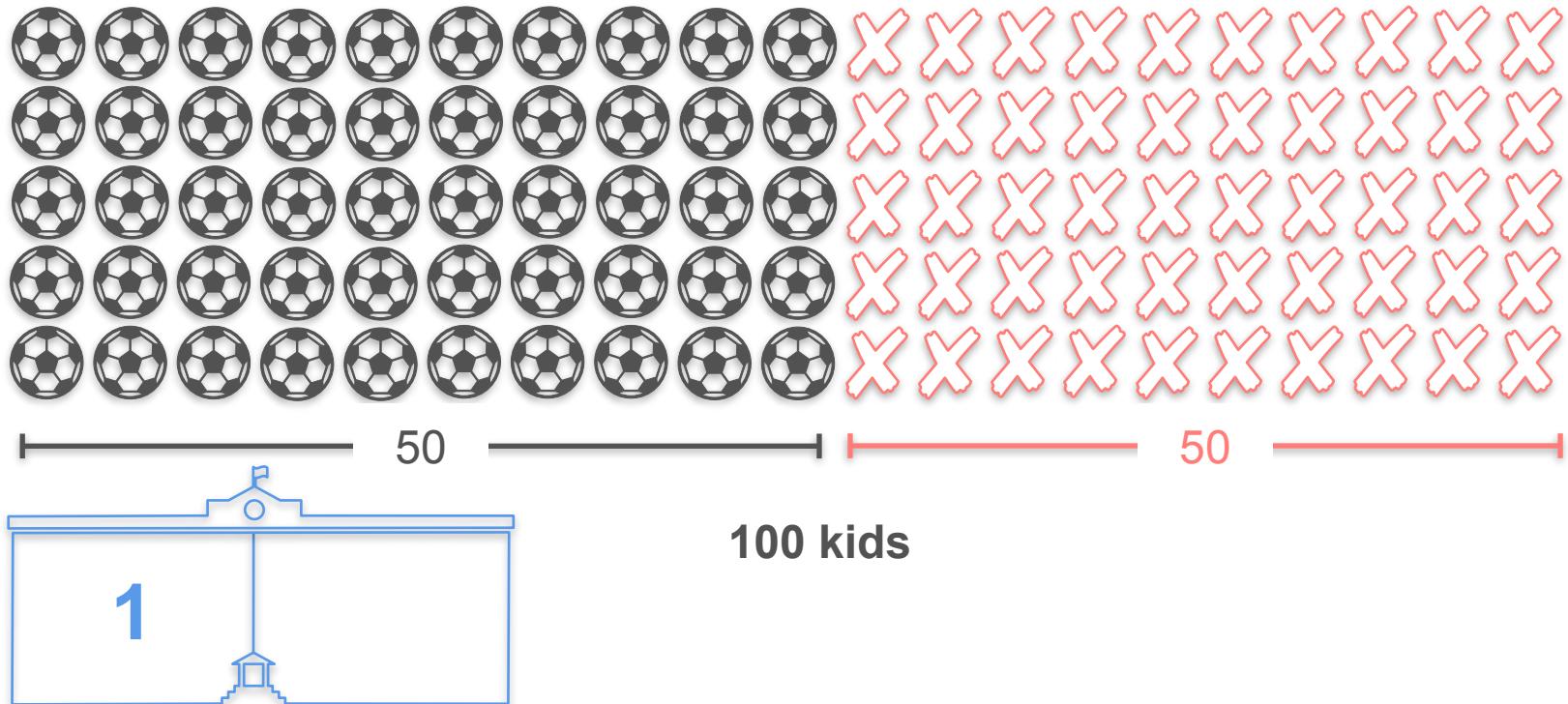
Independence: Quiz 1



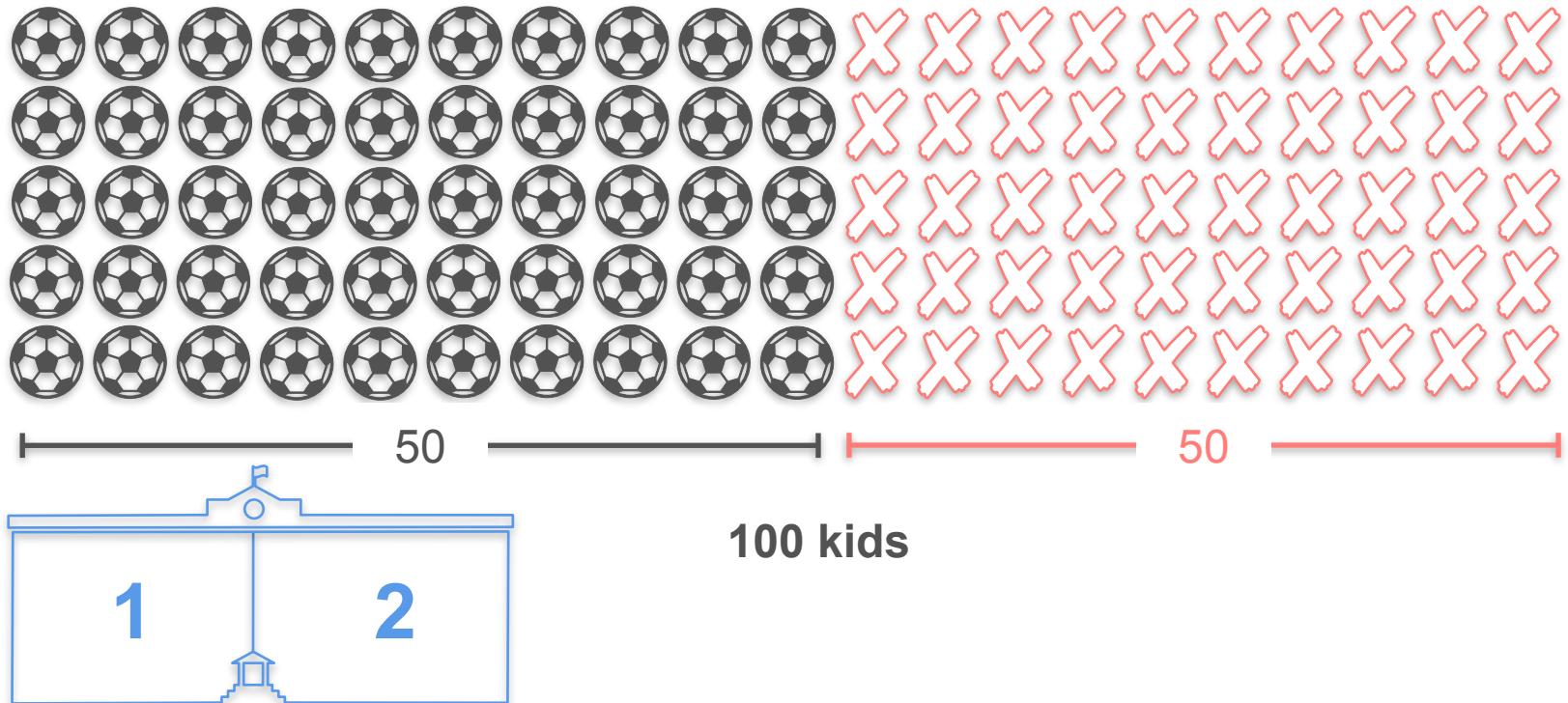
Independence: Quiz 1



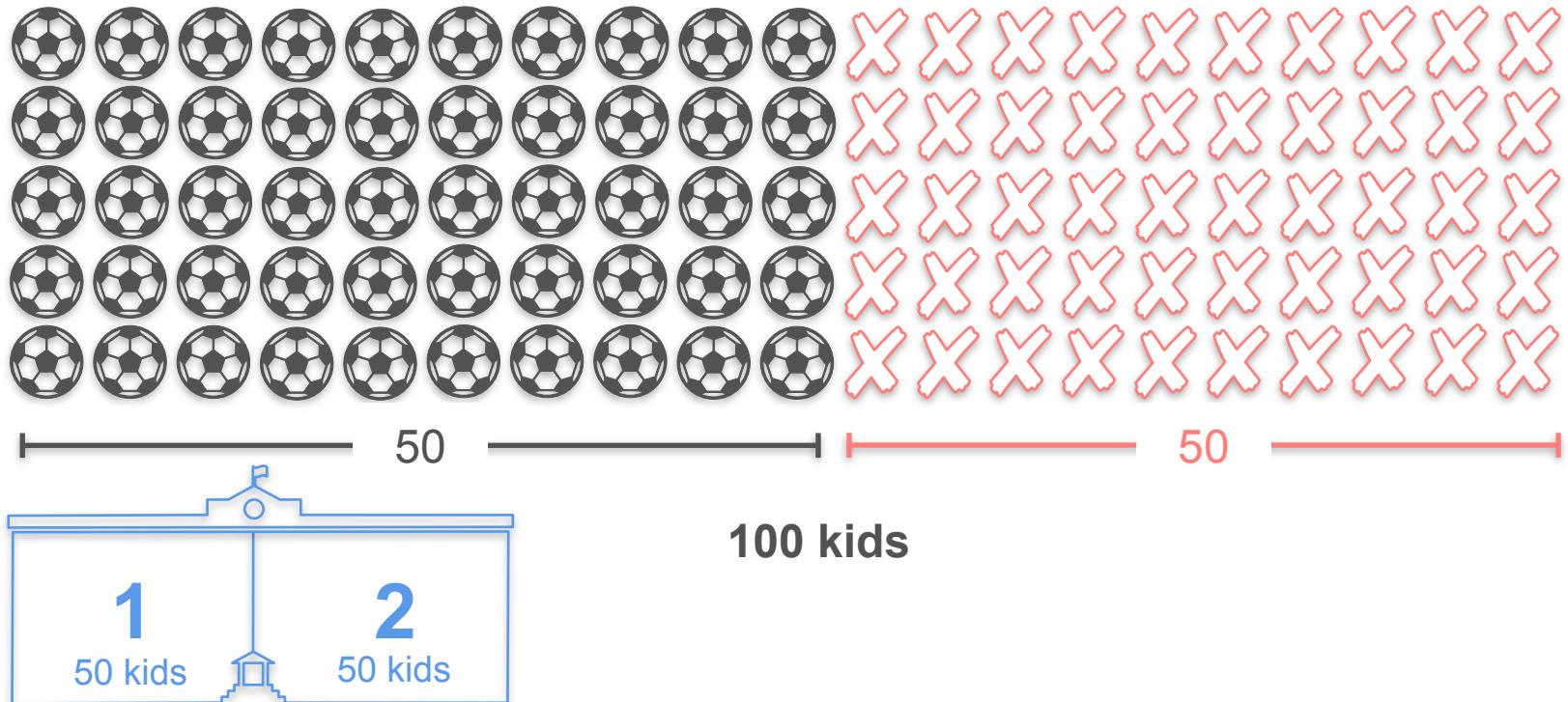
Independence: Quiz 1



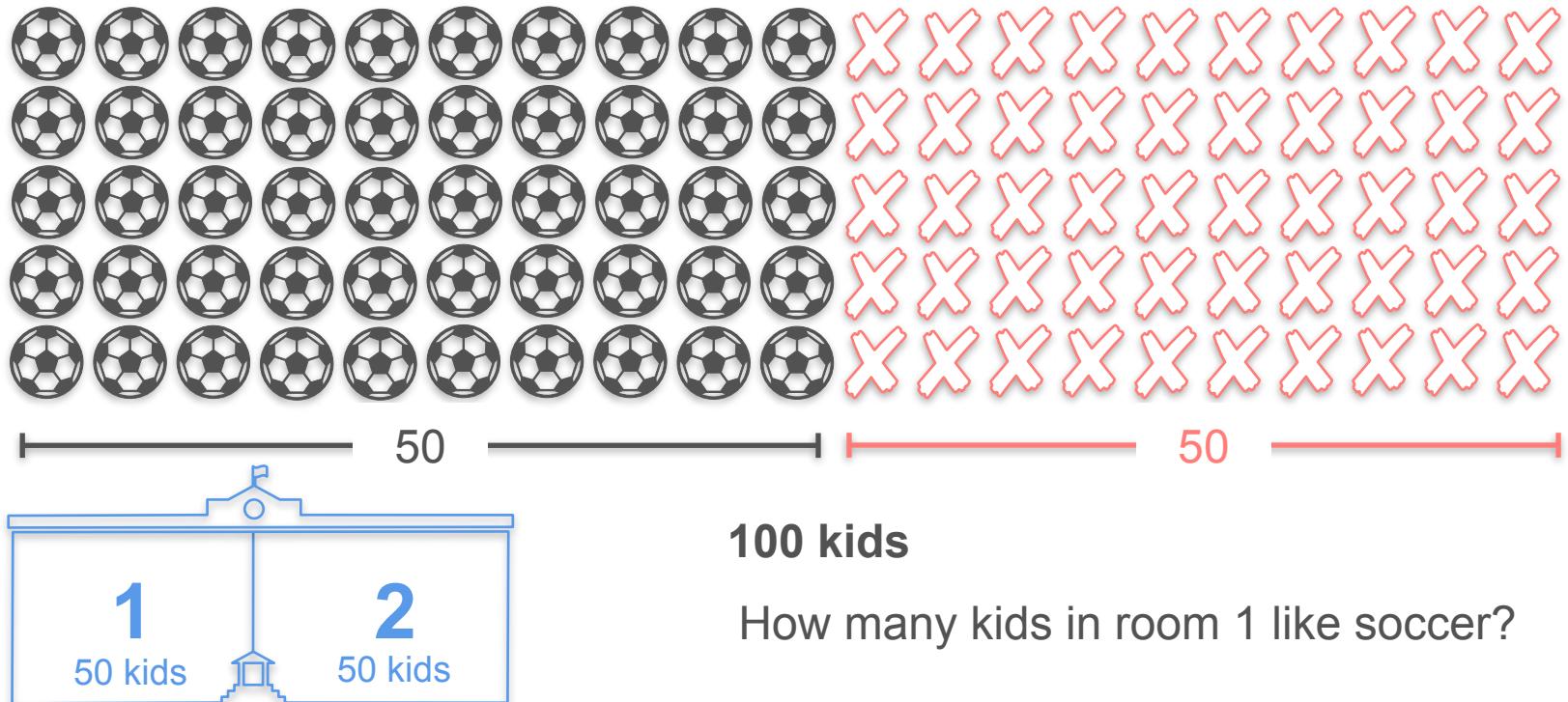
Independence: Quiz 1



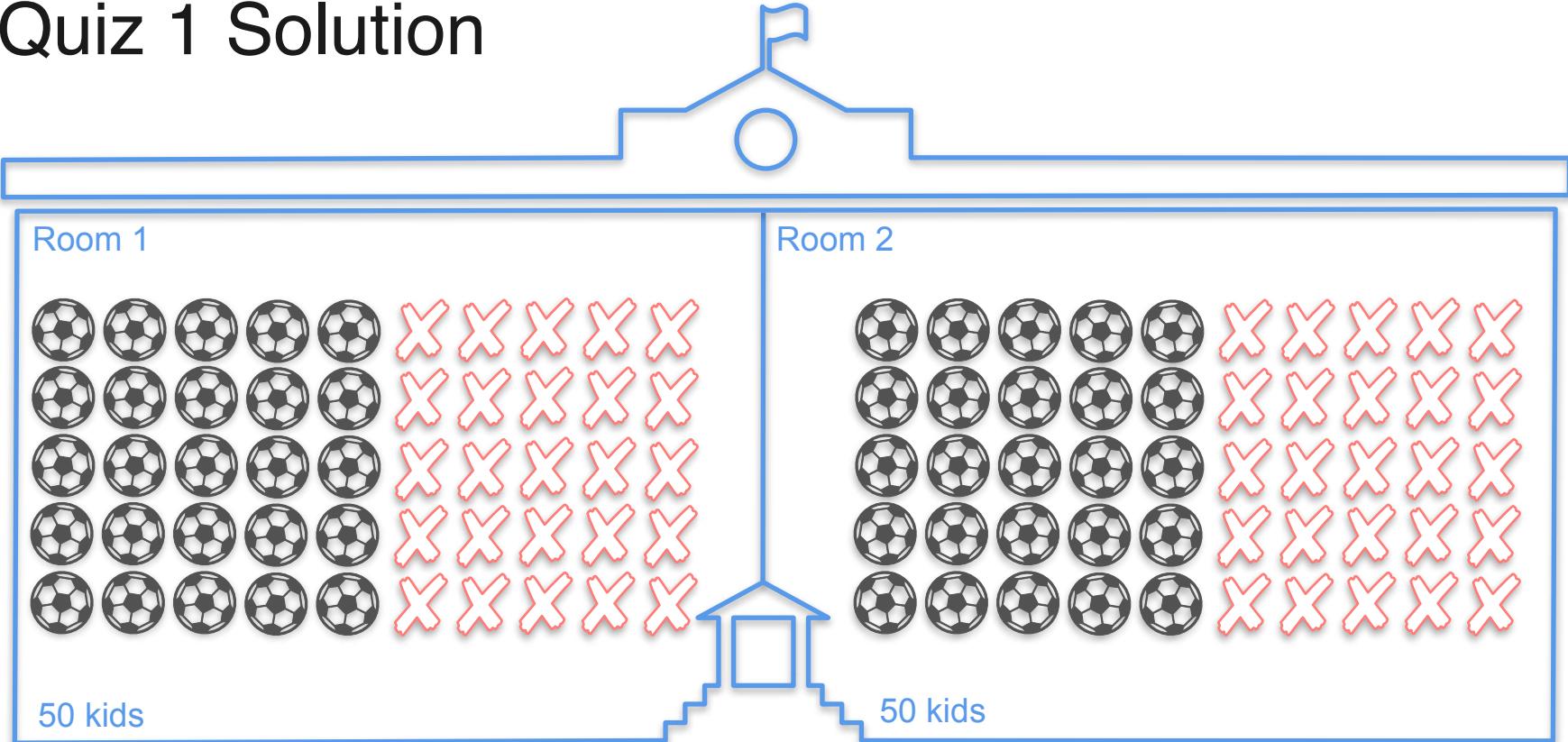
Independence: Quiz 1



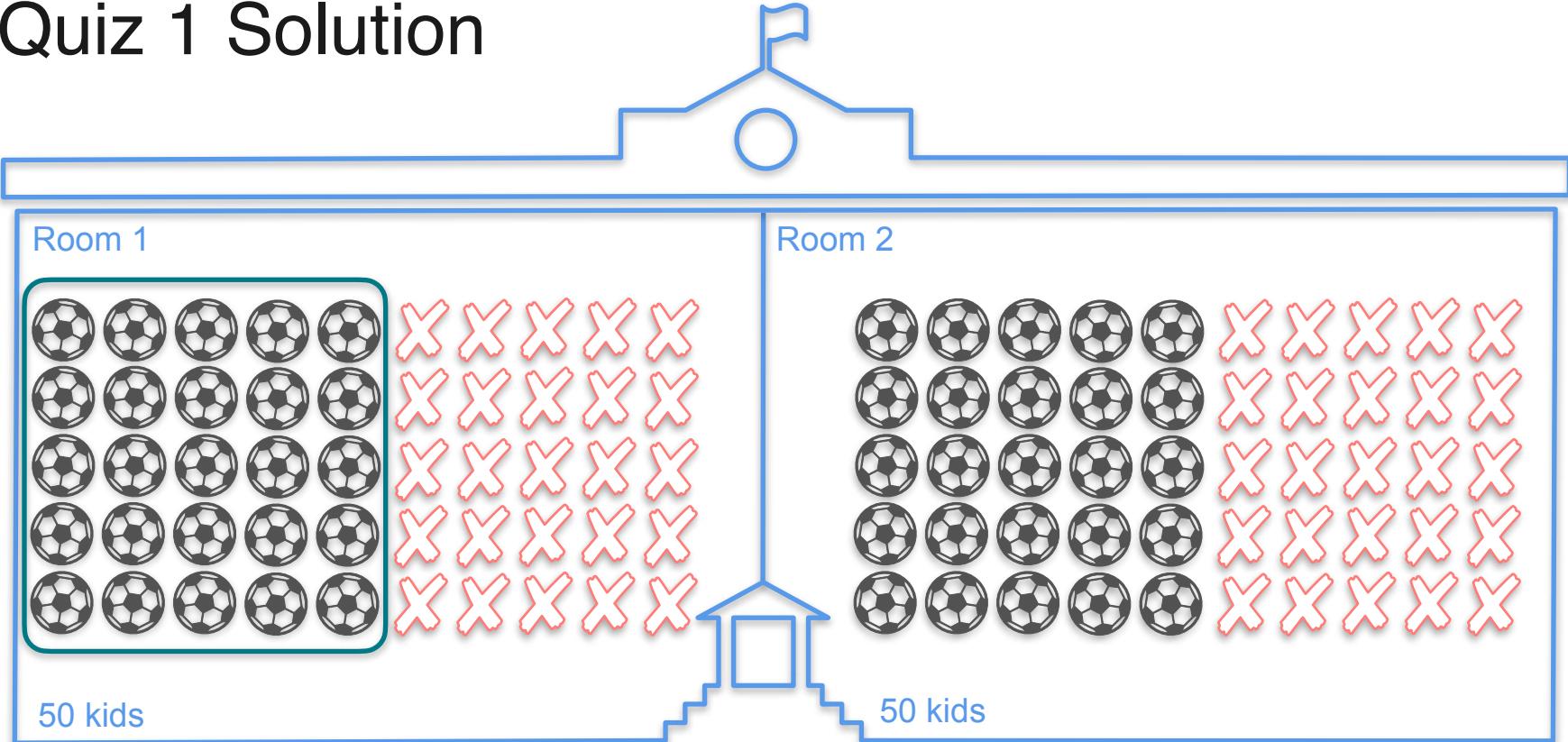
Independence: Quiz 1



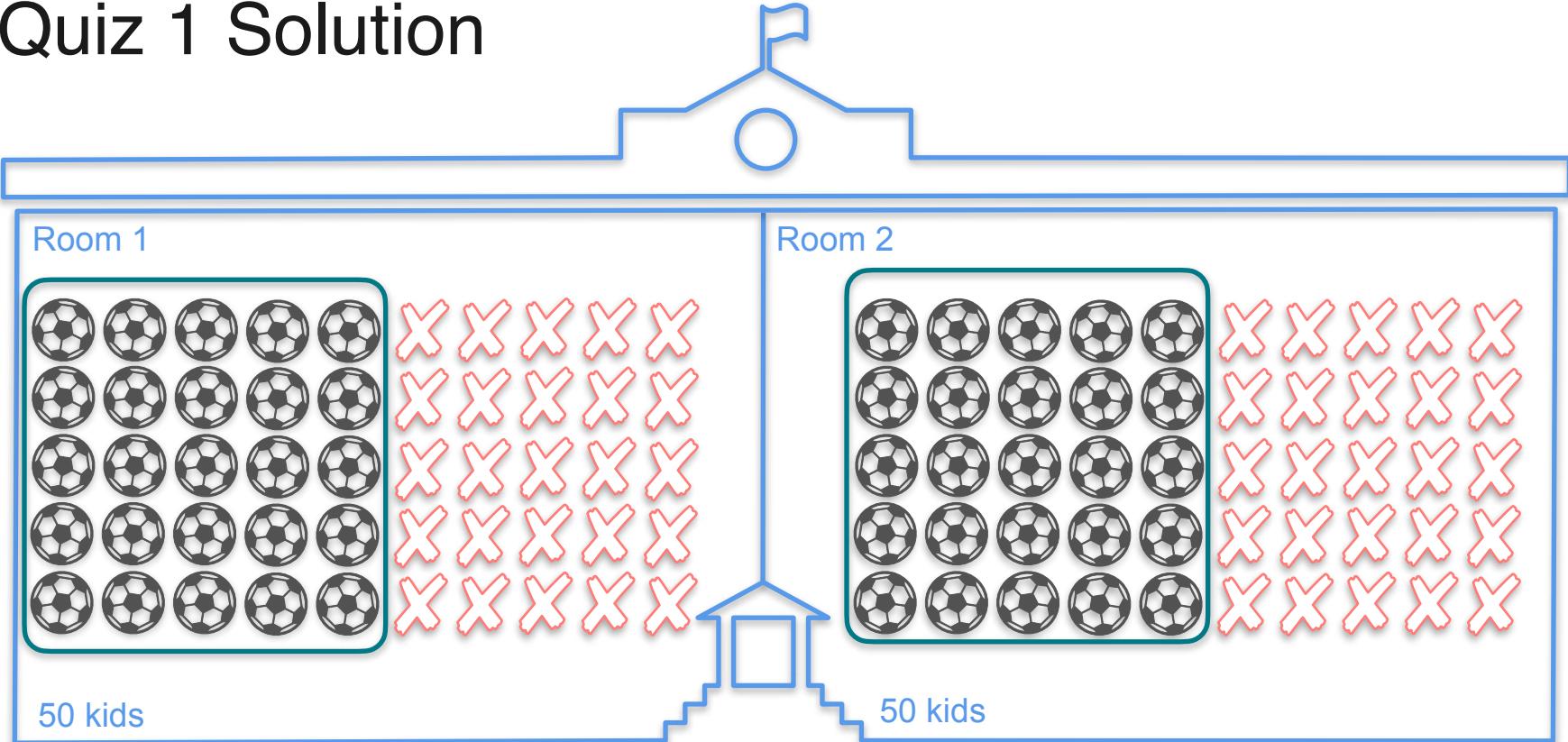
Quiz 1 Solution



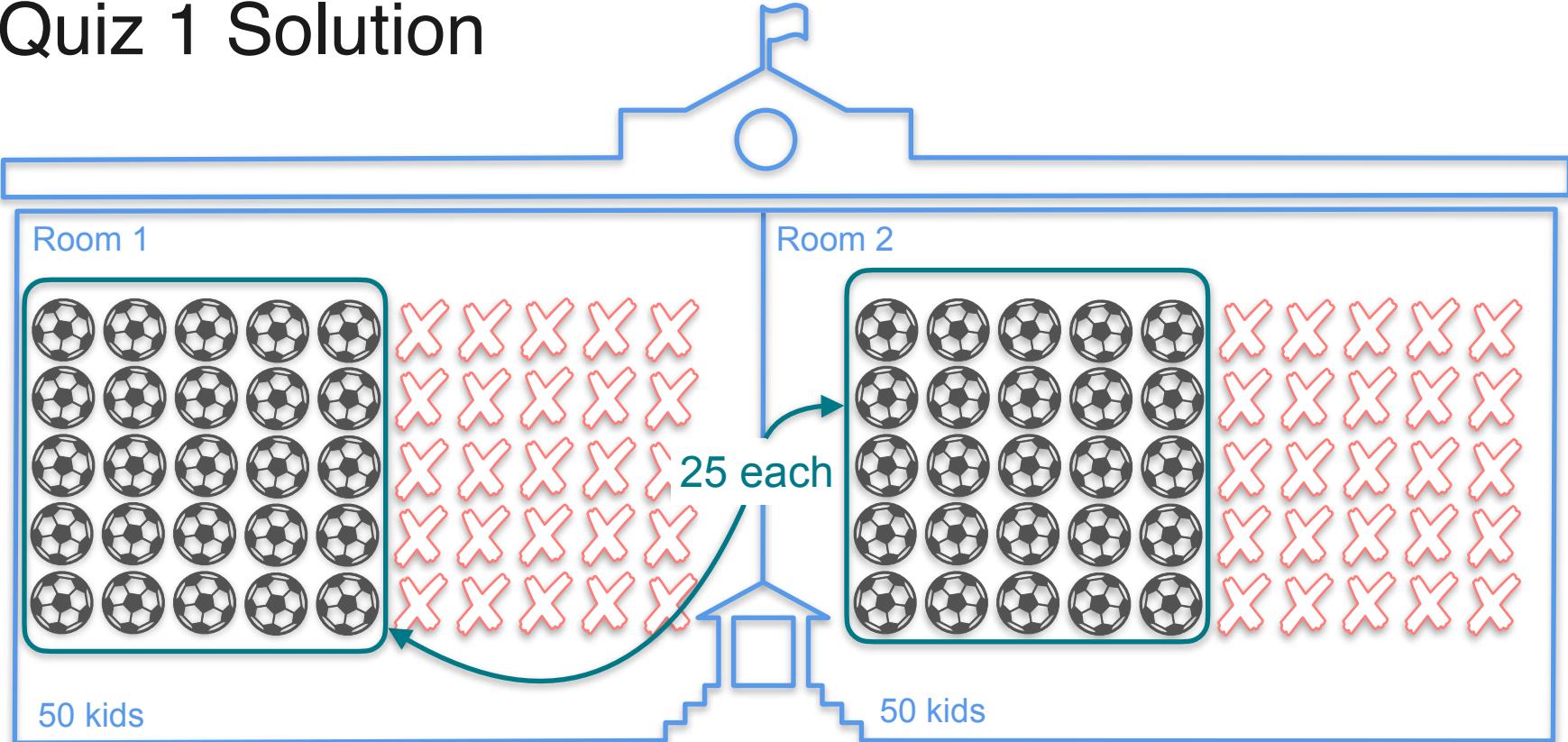
Quiz 1 Solution



Quiz 1 Solution

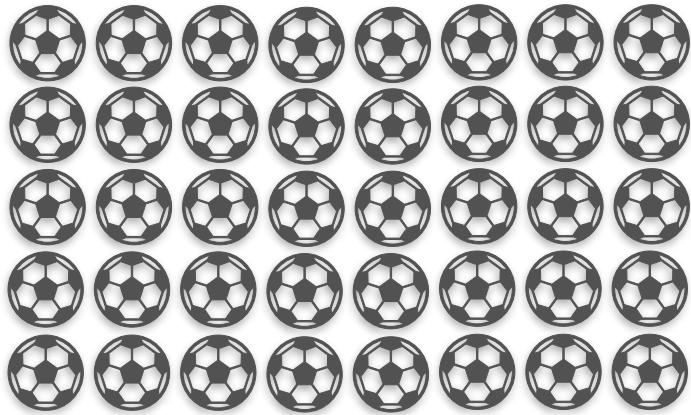


Quiz 1 Solution

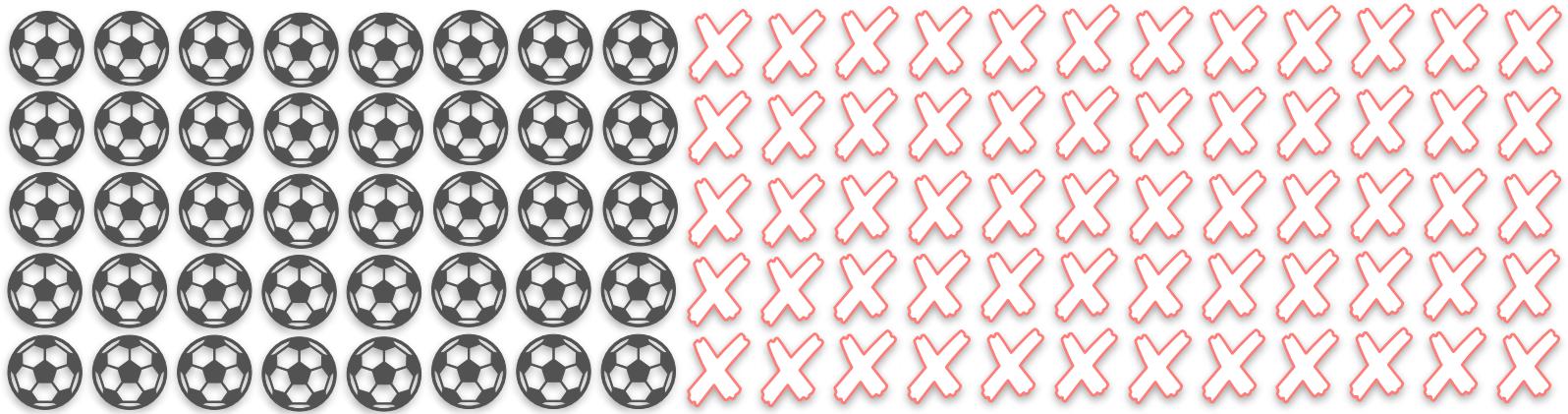


Independence: Quiz 2

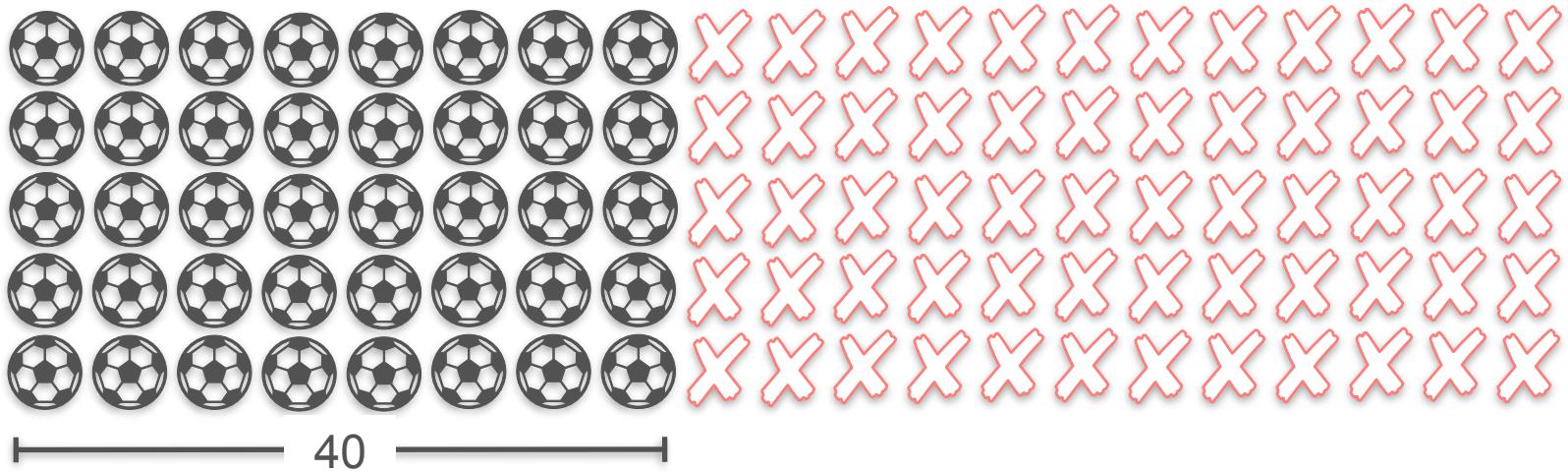
Independence: Quiz 2



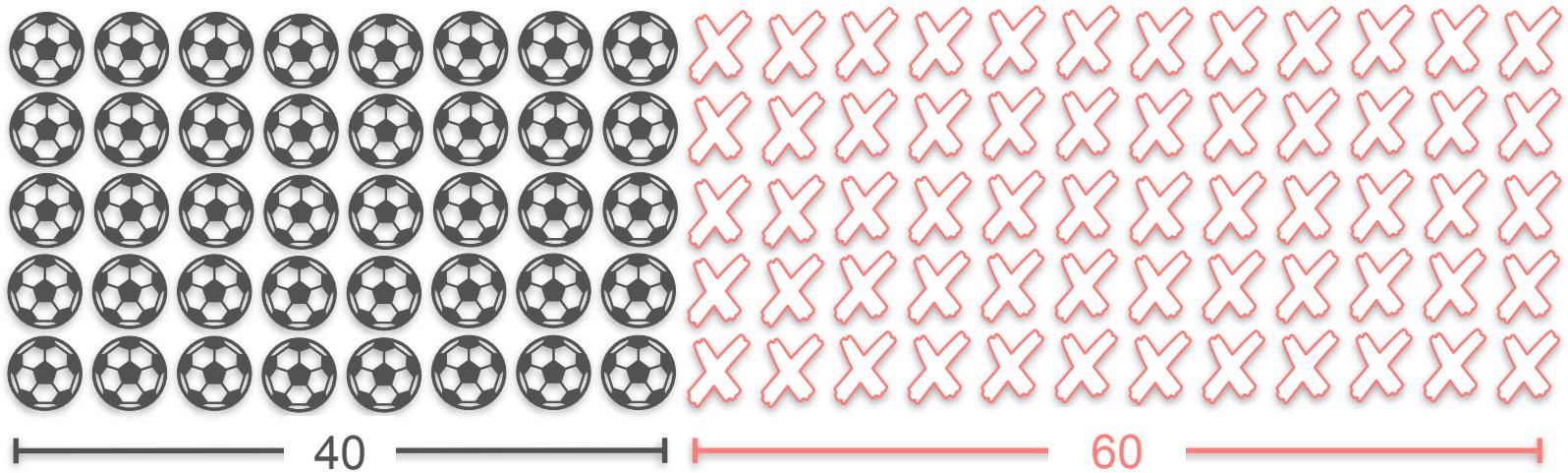
Independence: Quiz 2



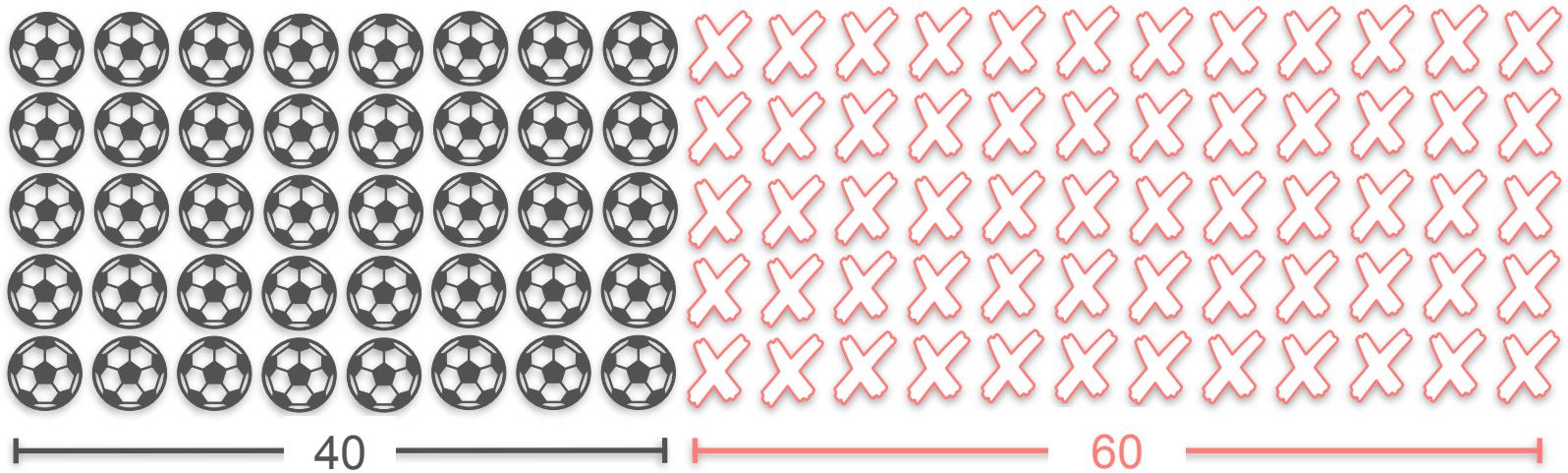
Independence: Quiz 2



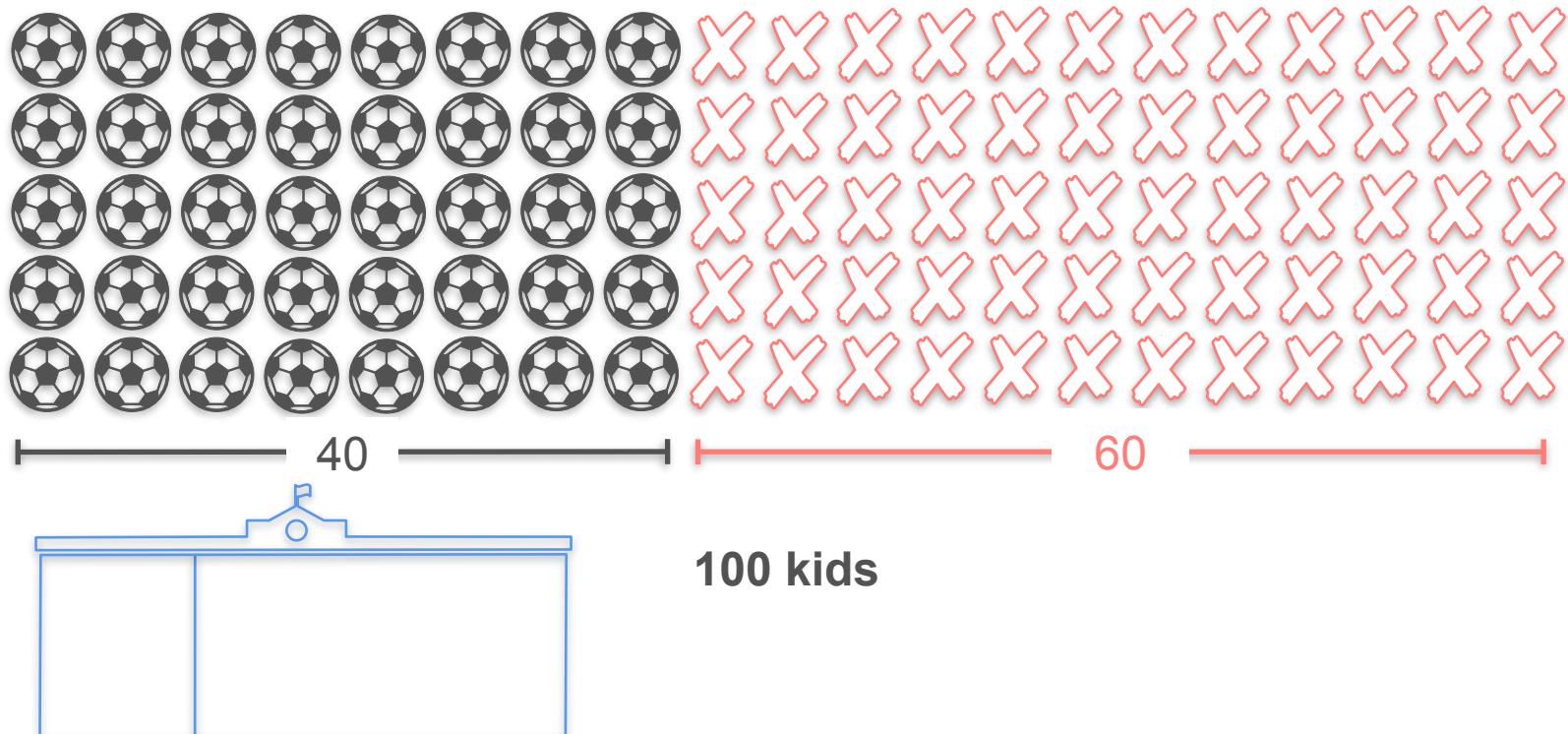
Independence: Quiz 2



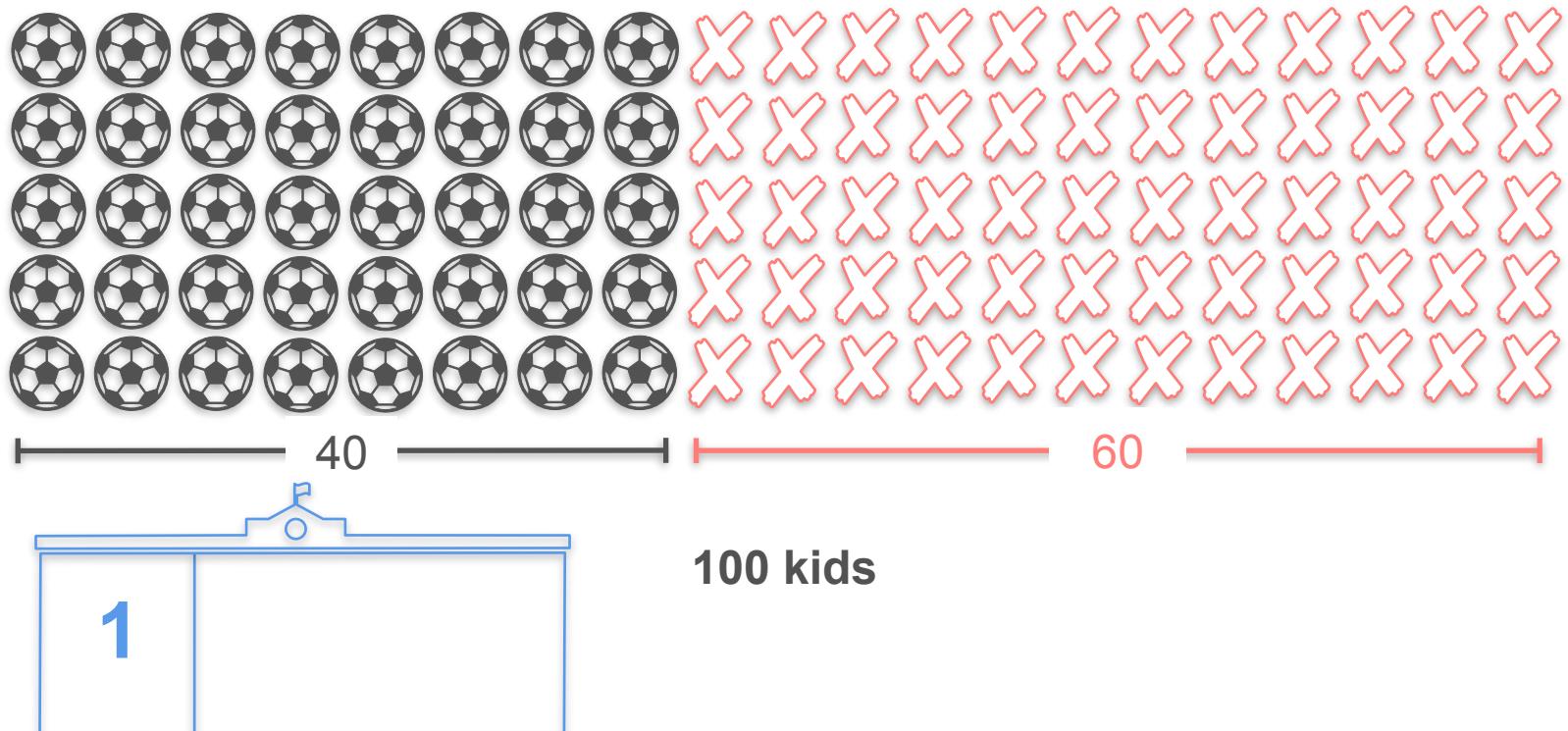
Independence: Quiz 2



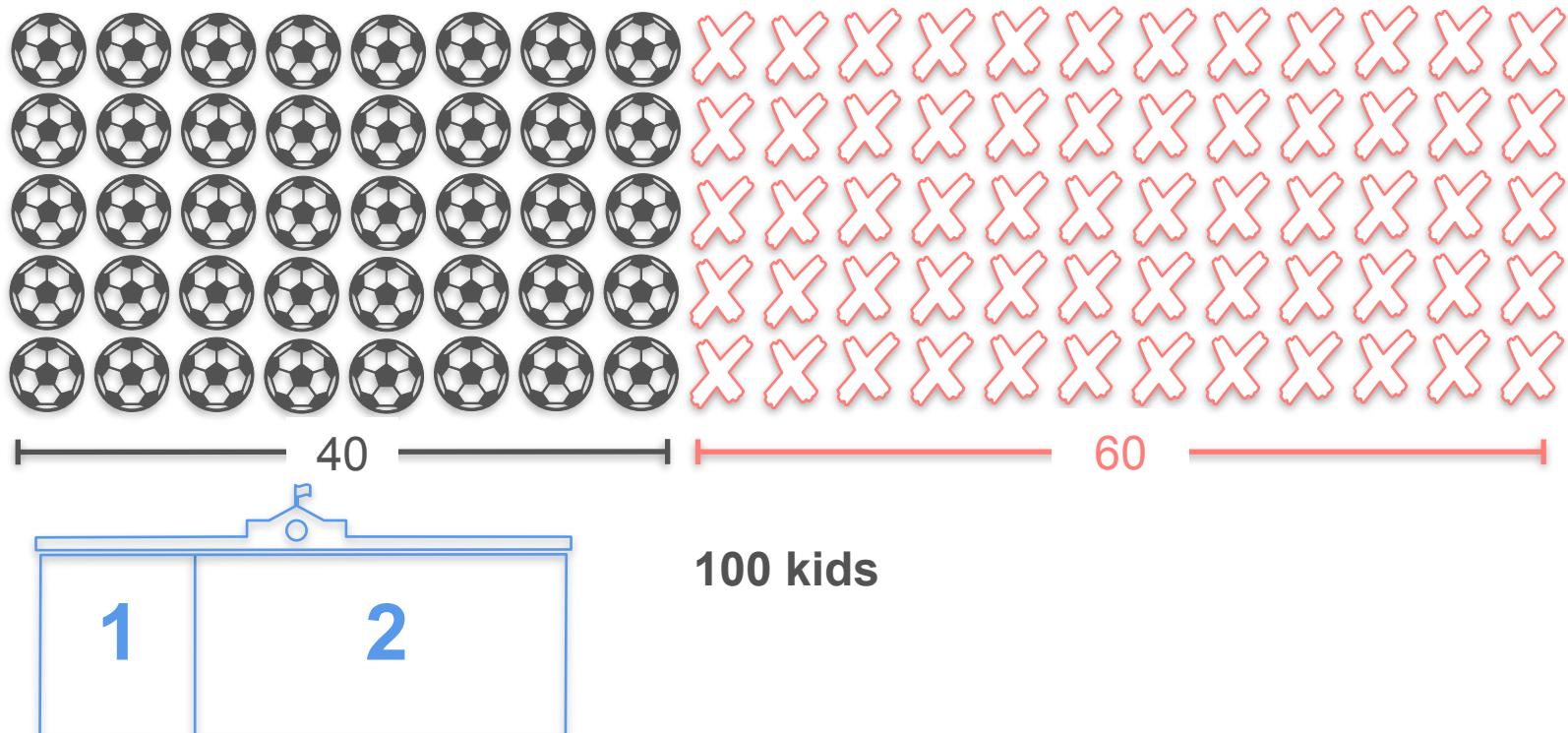
Independence: Quiz 2



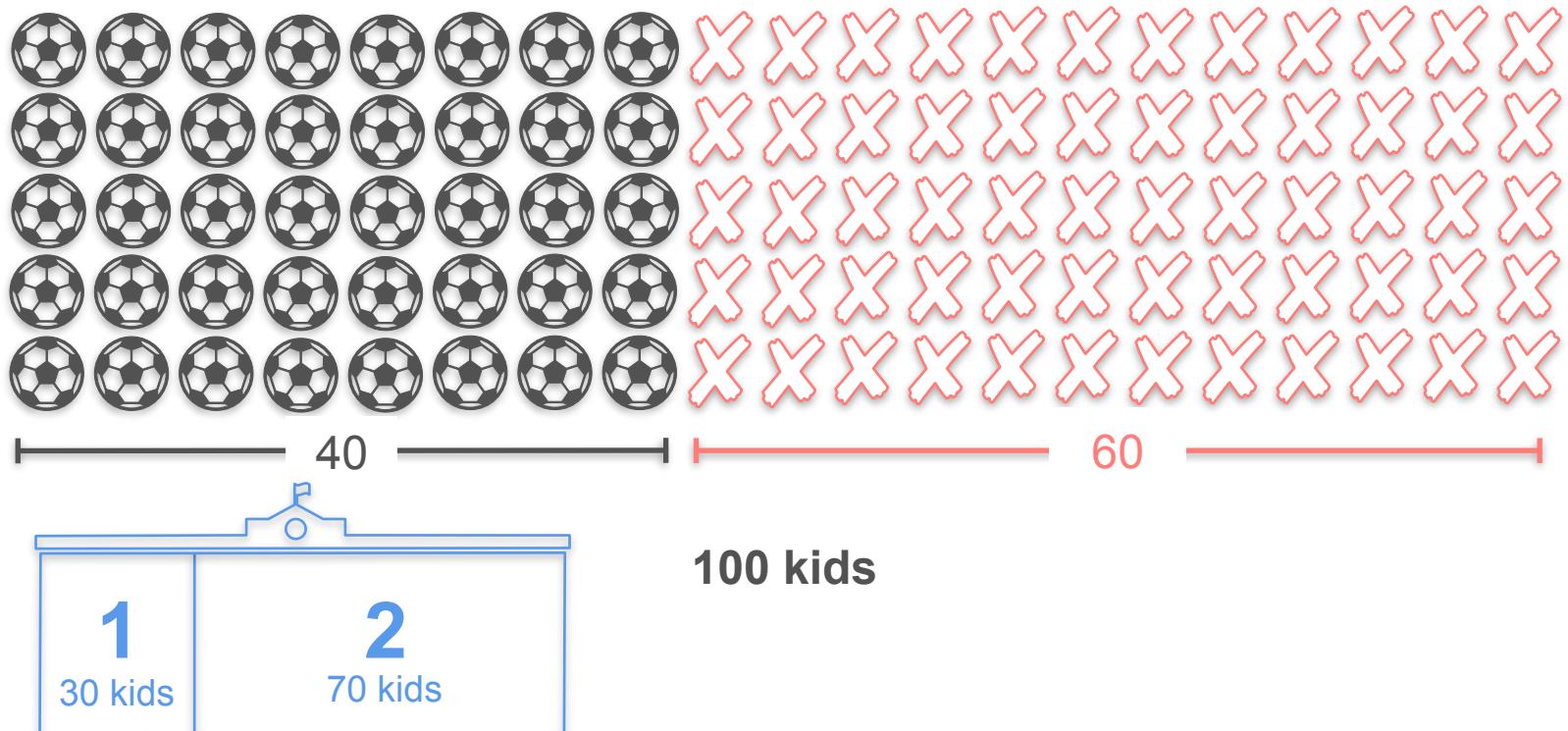
Independence: Quiz 2



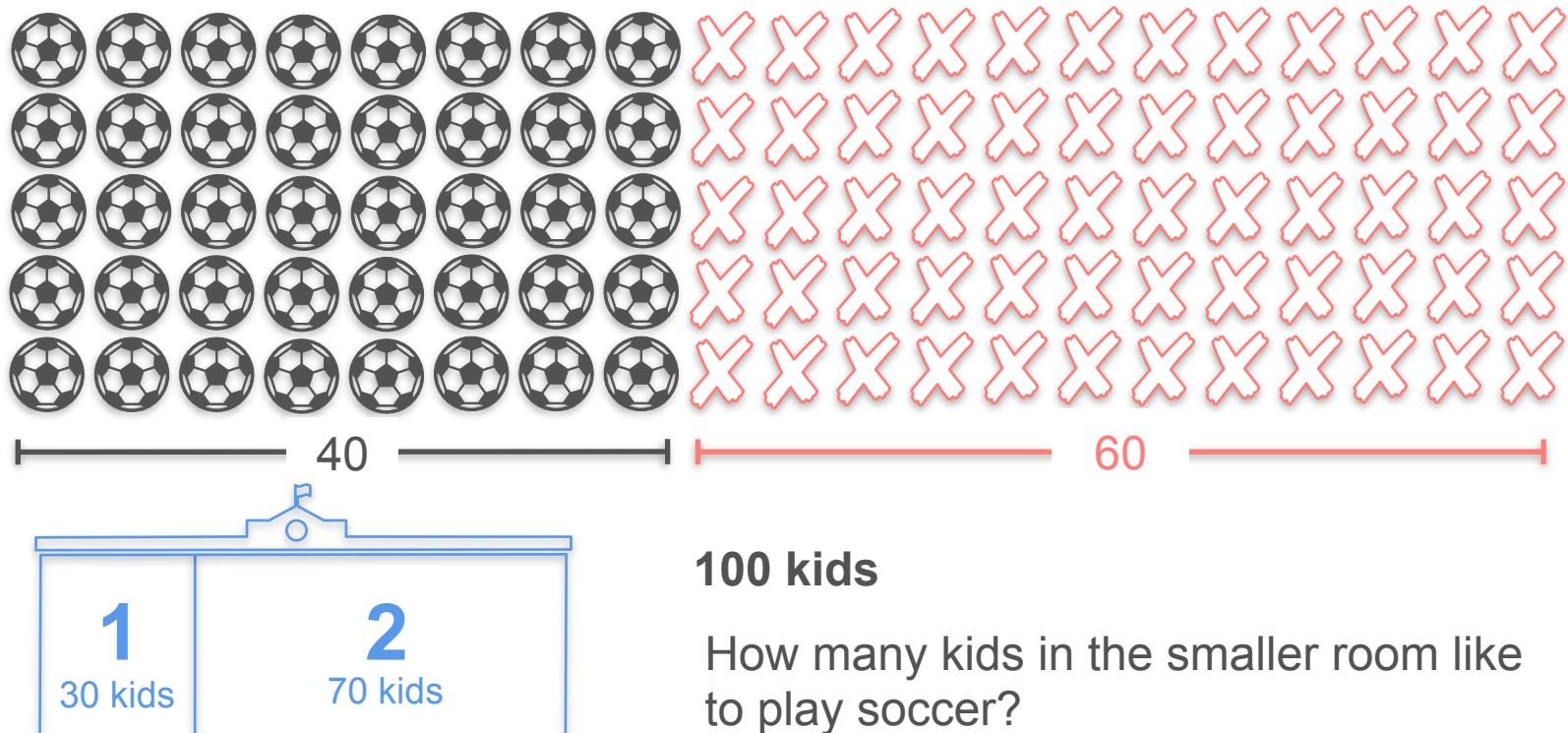
Independence: Quiz 2



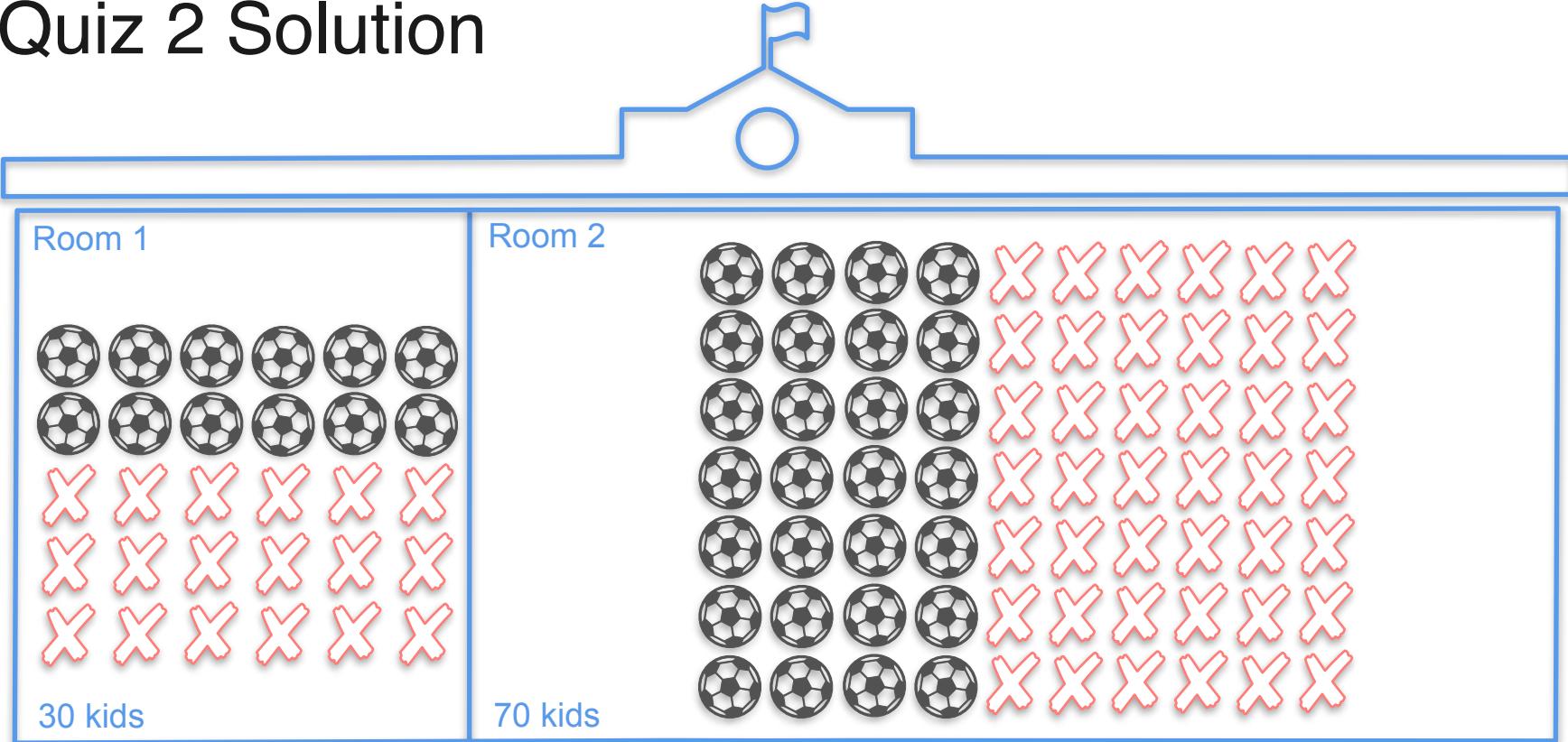
Independence: Quiz 2



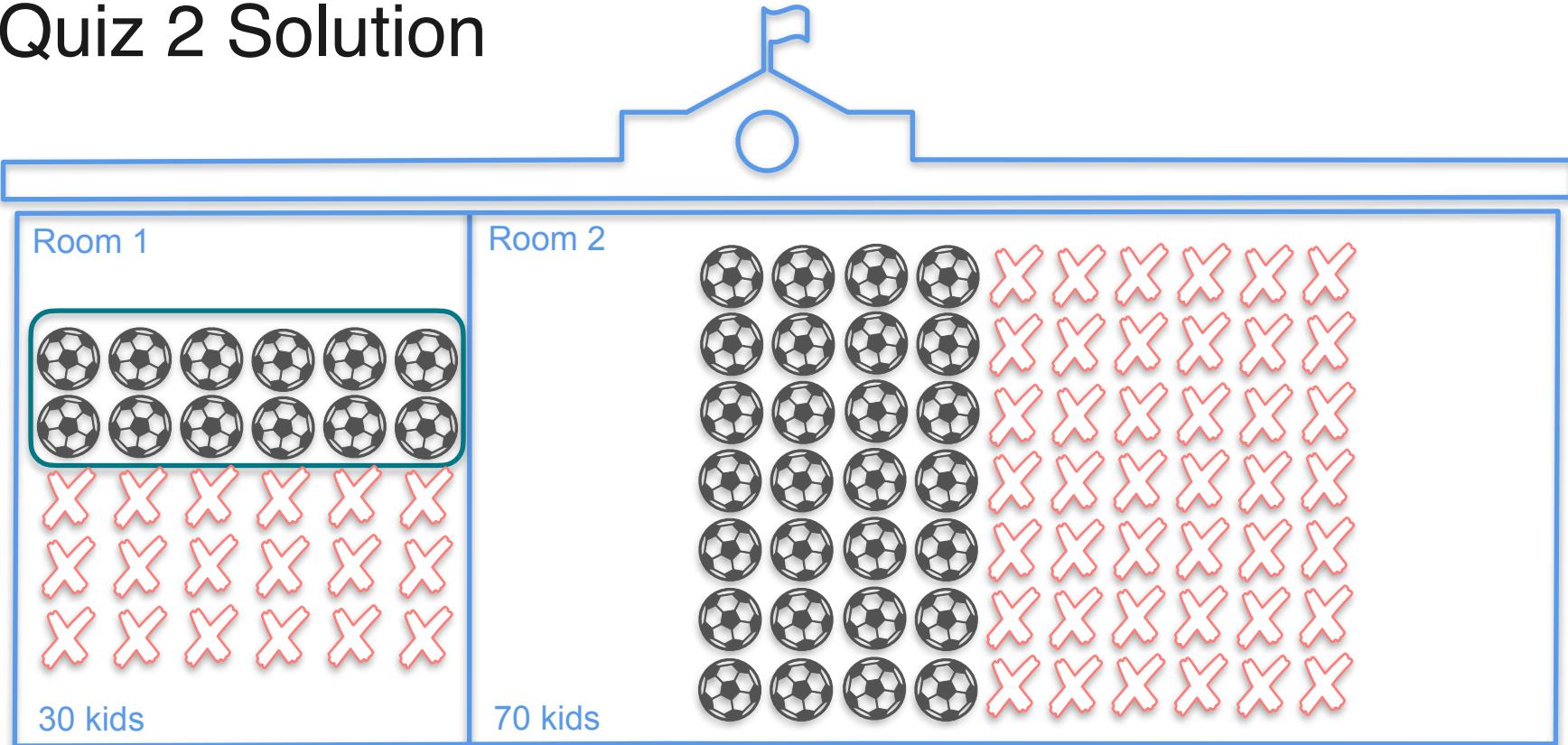
Independence: Quiz 2



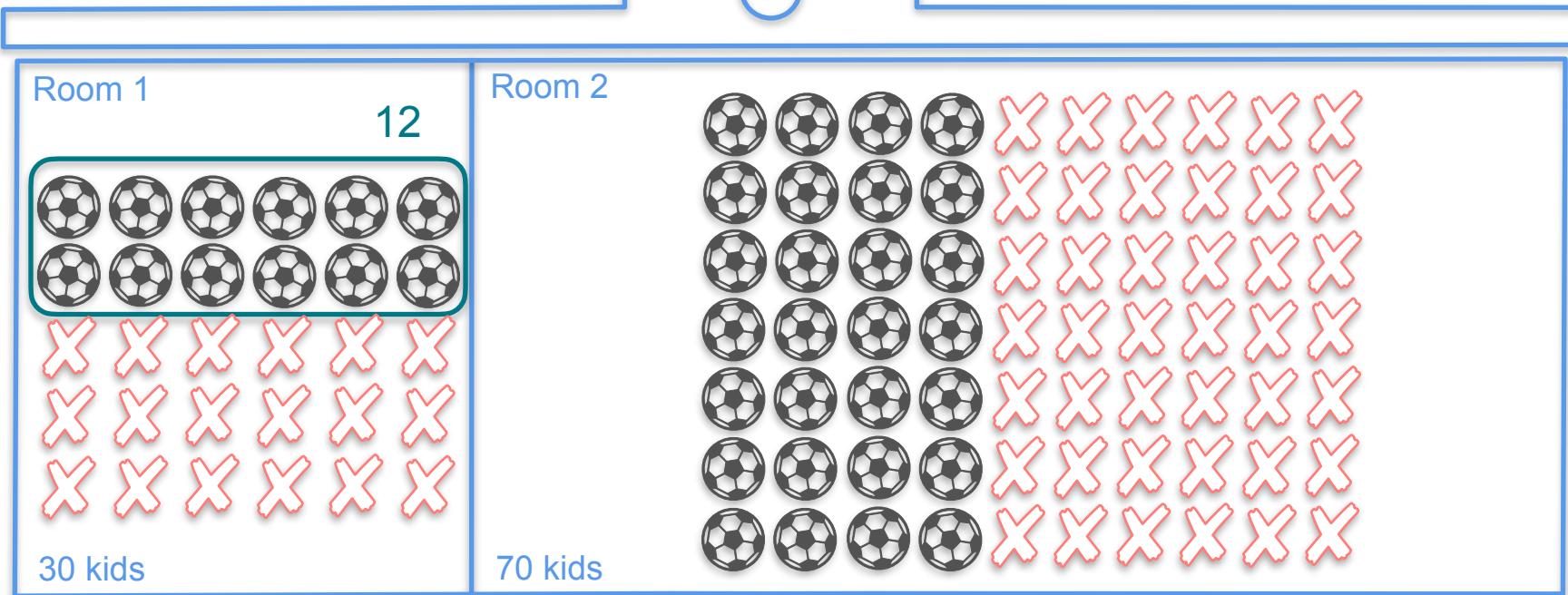
Quiz 2 Solution



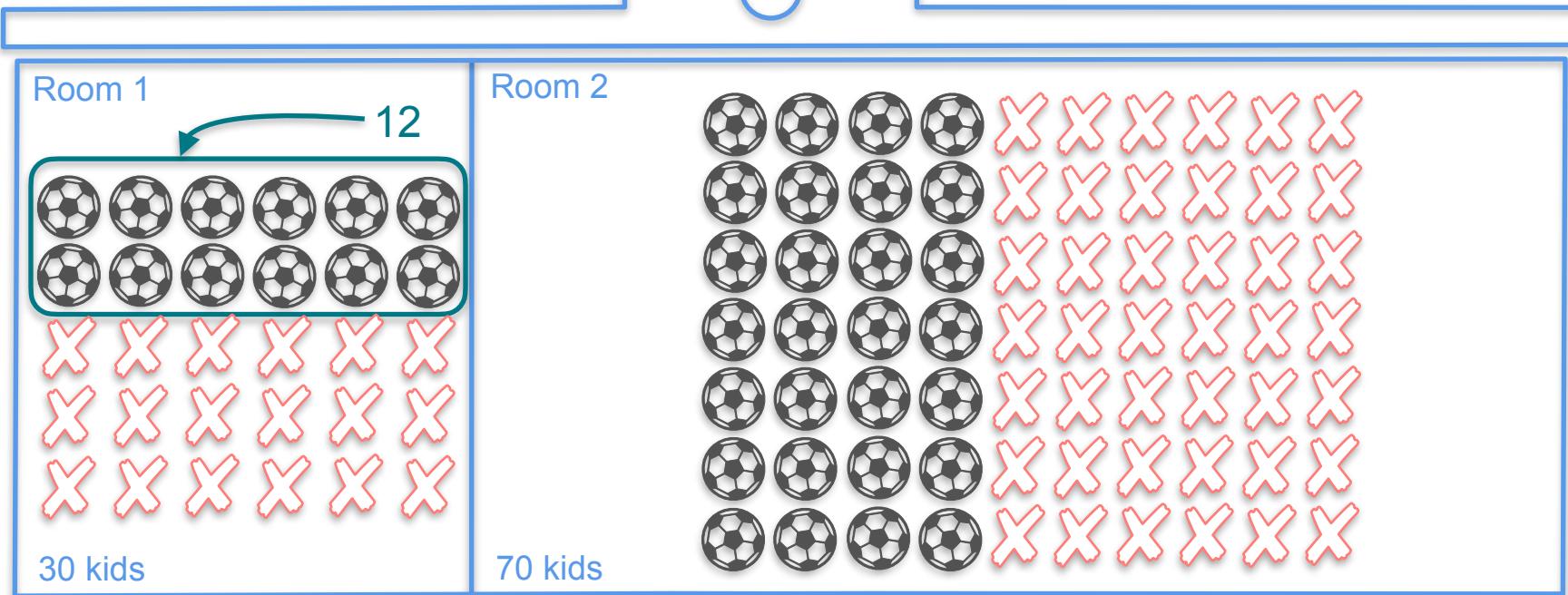
Quiz 2 Solution



Quiz 2 Solution

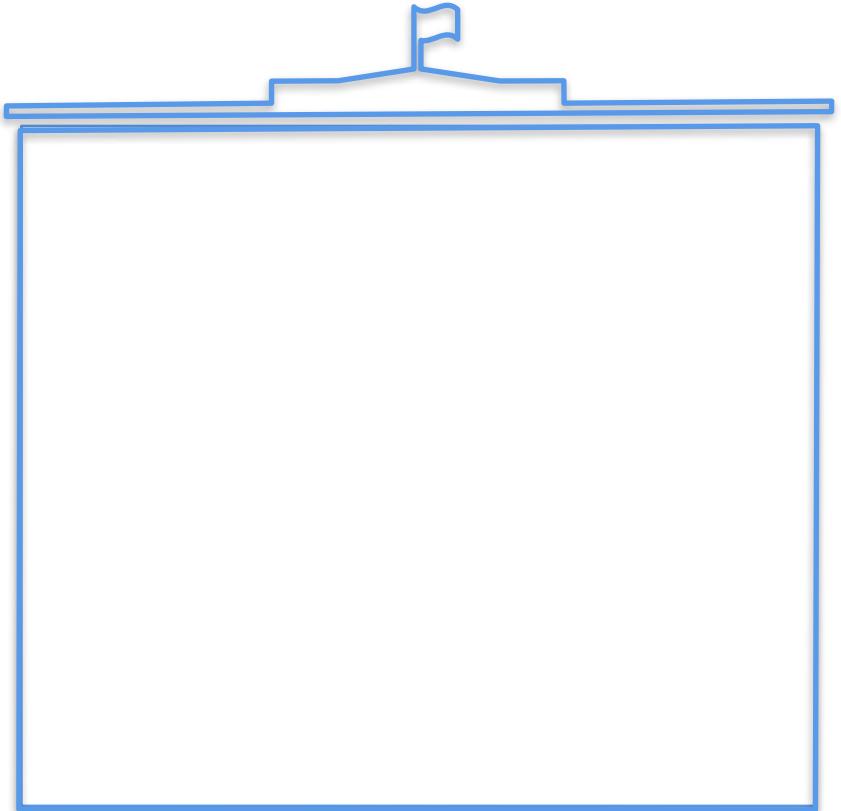


Quiz 2 Solution



Independent Events

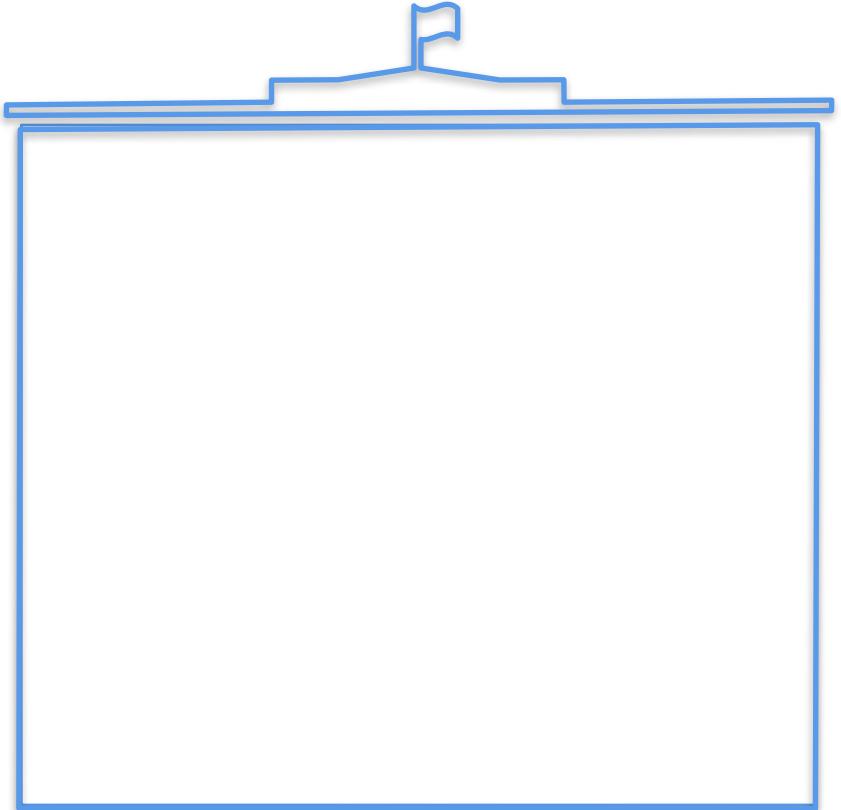
Independent Events



Independent Events



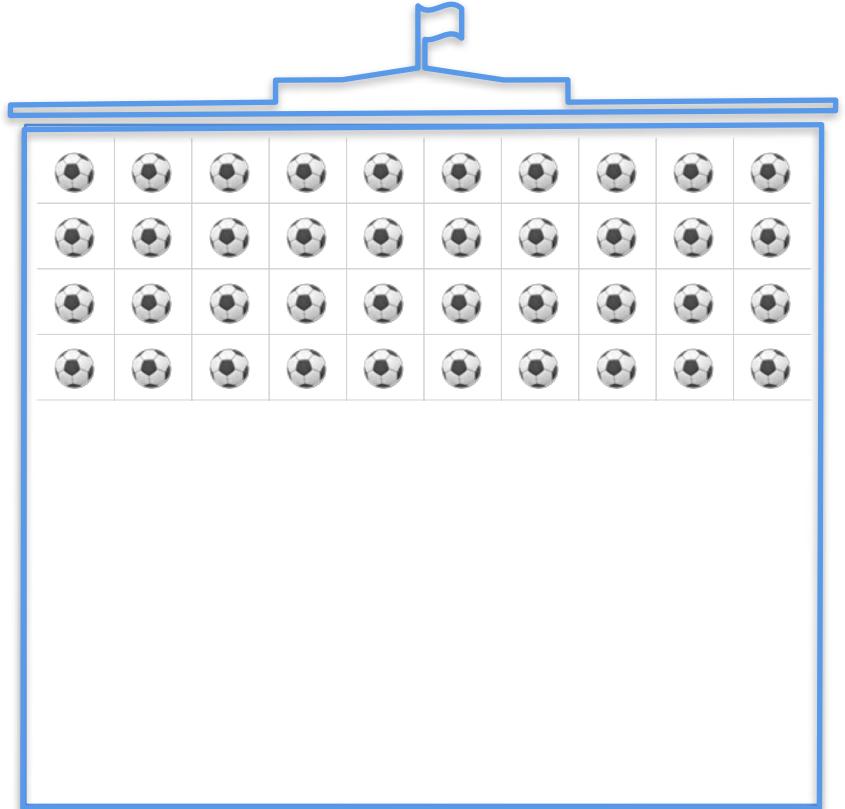
$$\mathbf{P}(S) = 0.4$$



Independent Events



$$\mathbf{P}(S) = 0.4$$



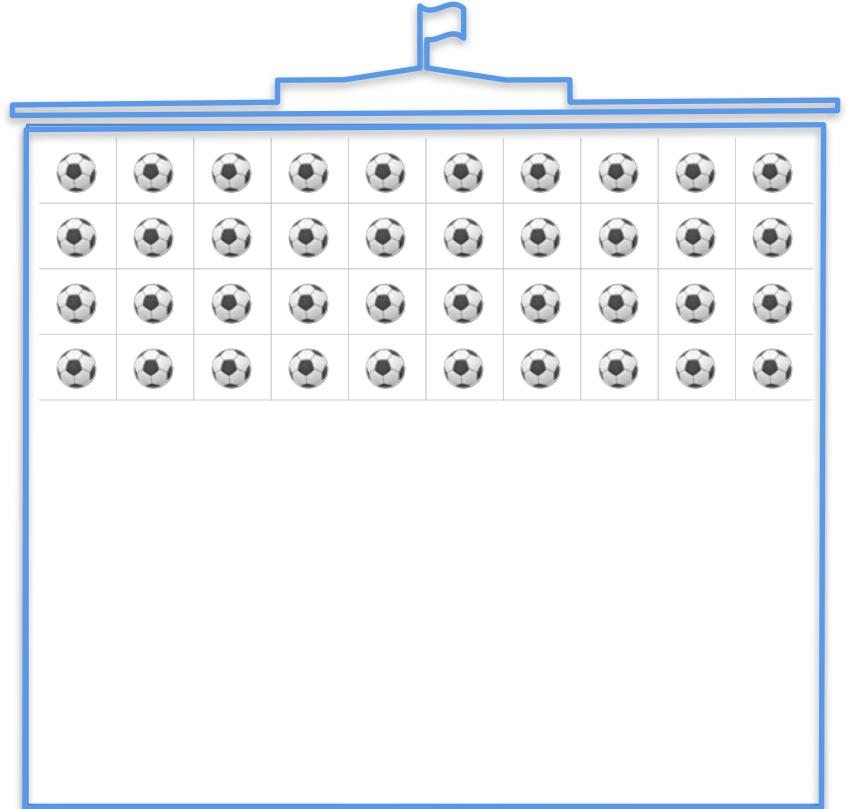
Independent Events



$$P(S) = 0.4$$



$$P(\text{not } S) = 0.6$$



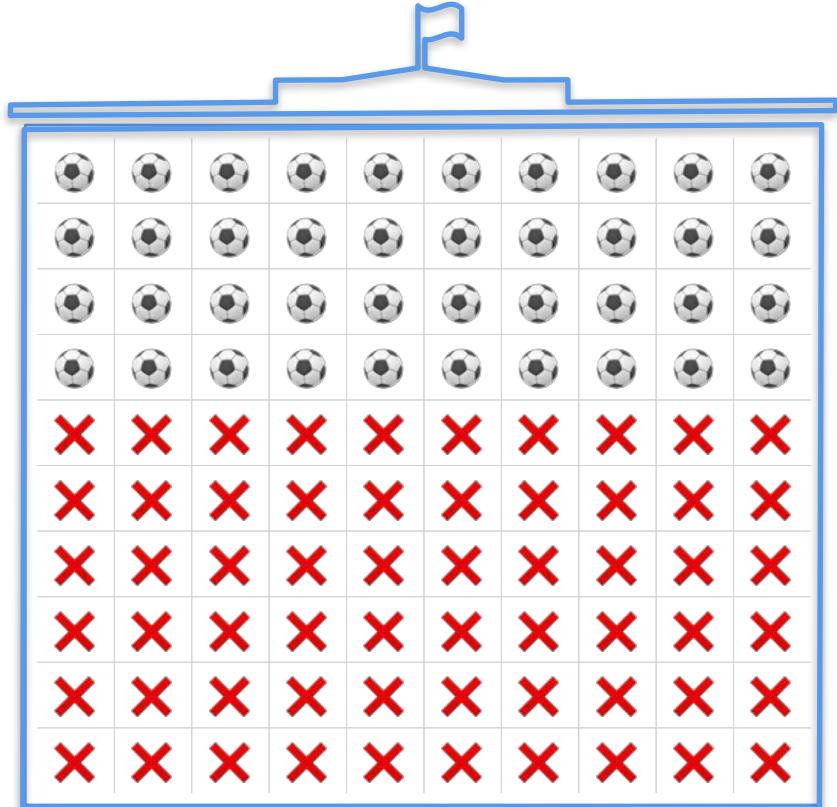
Independent Events



$$P(S) = 0.4$$



$$P(\text{not } S) = 0.6$$



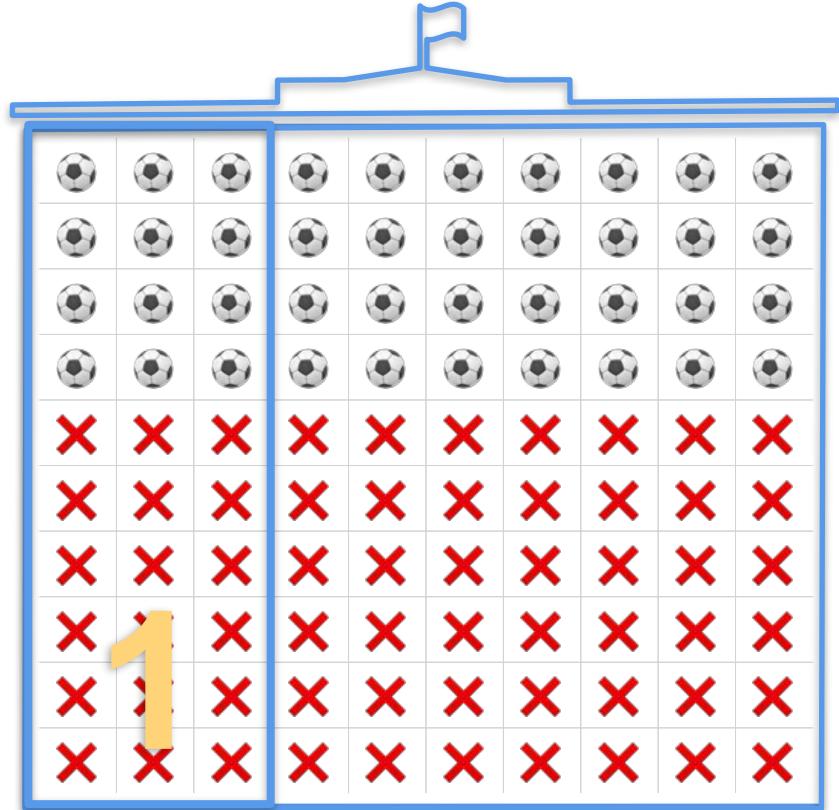
Independent Events



$$P(S) = 0.4$$



$$P(\text{not } S) = 0.6$$



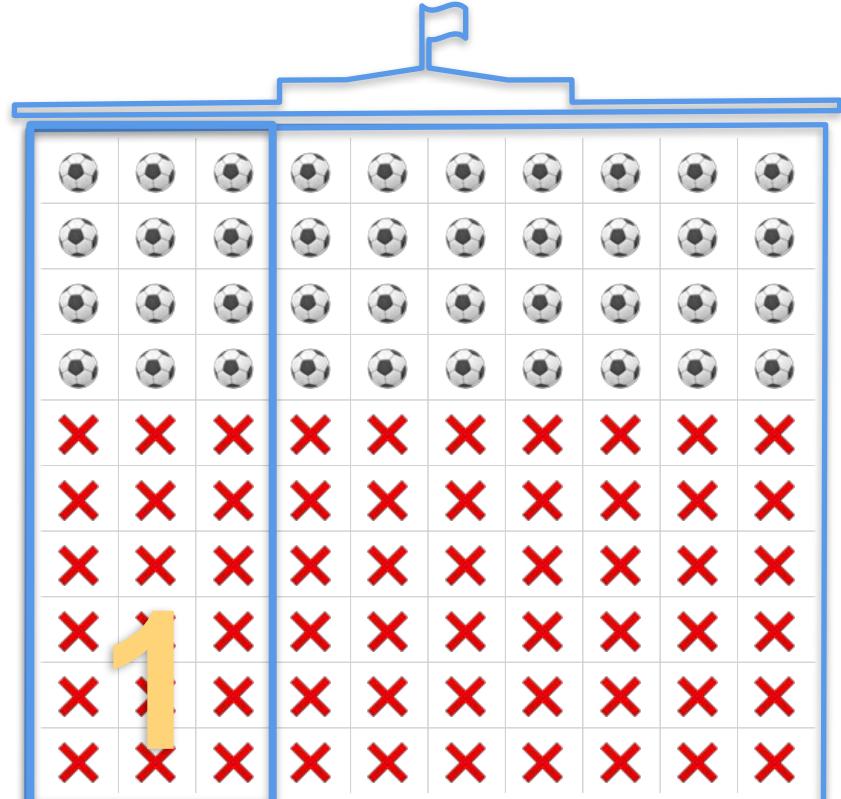
Independent Events



$$P(S) = 0.4$$



$$P(\text{not } S) = 0.6$$



$$P(R_1) = 0.3$$

Independent Events

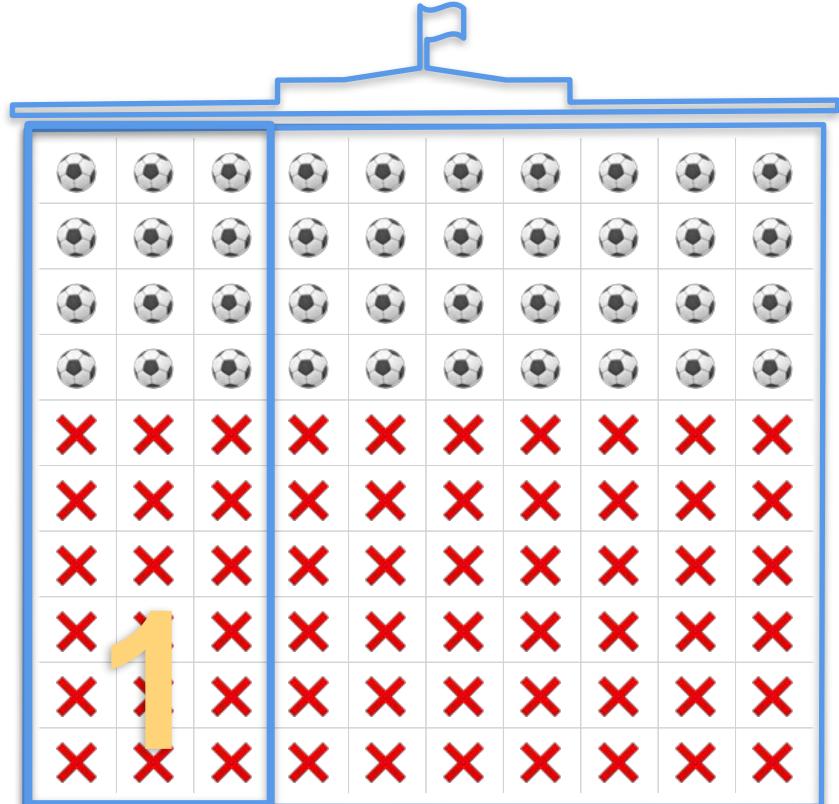
$P(\text{Soccer and Room 1})$



$$P(S) = 0.4$$



$$P(\text{not } S) = 0.6$$



$$P(R_1) = 0.3$$

Independent Events

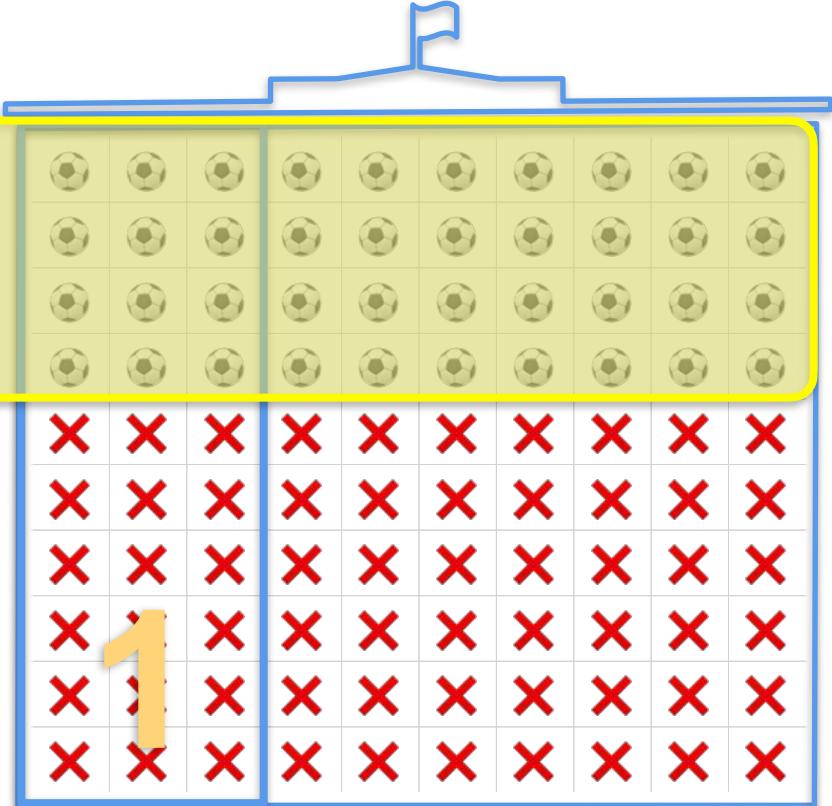
$P(\text{Soccer and Room 1})$



$$P(S) = 0.4$$



$$P(\text{not } S) = 0.6$$



$$P(R_1) = 0.3$$

Independent Events

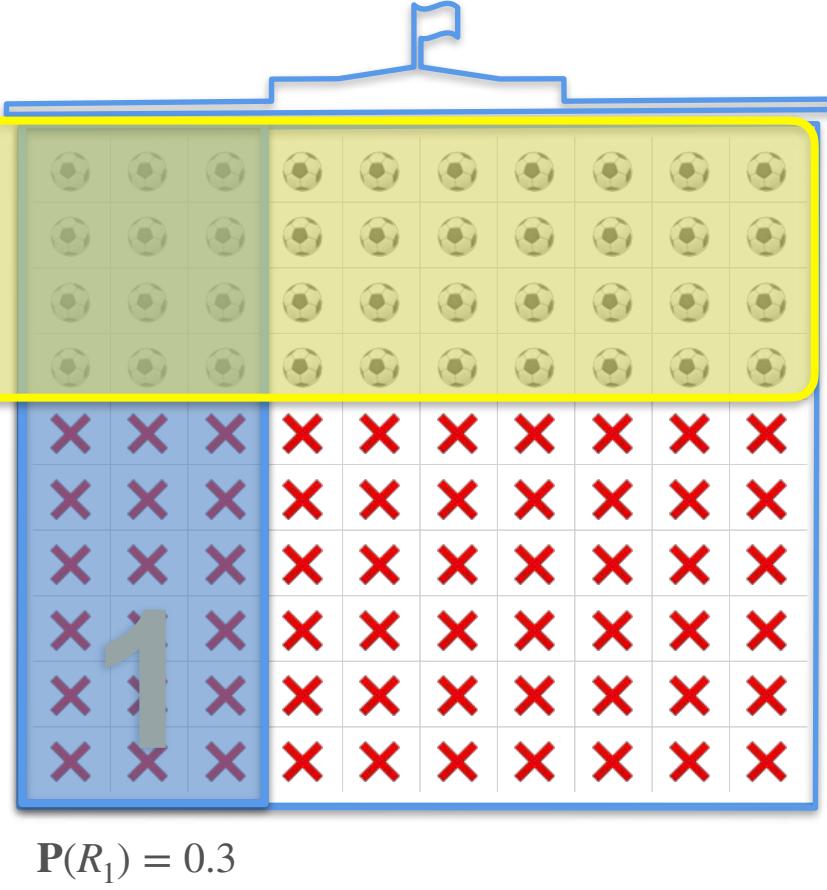
$P(\text{Soccer and Room 1})$



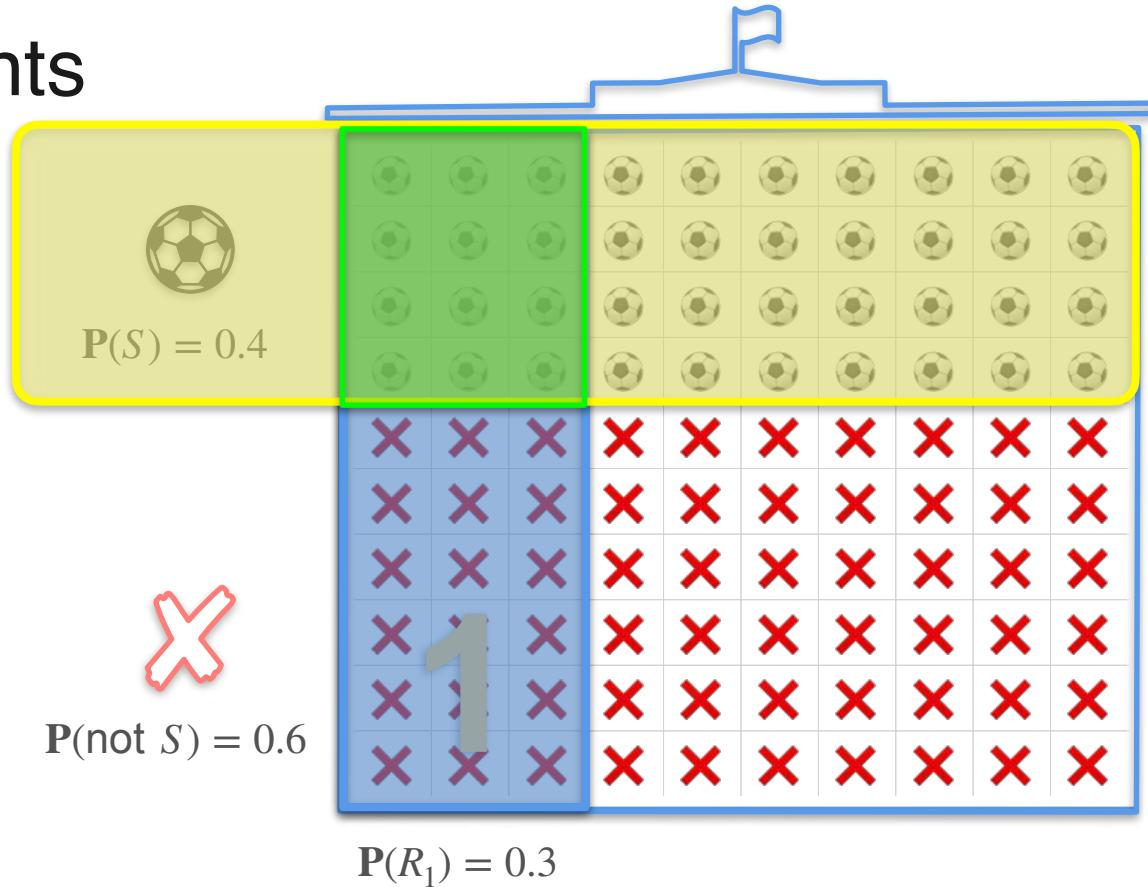
$$P(S) = 0.4$$



$$P(\text{not } S) = 0.6$$



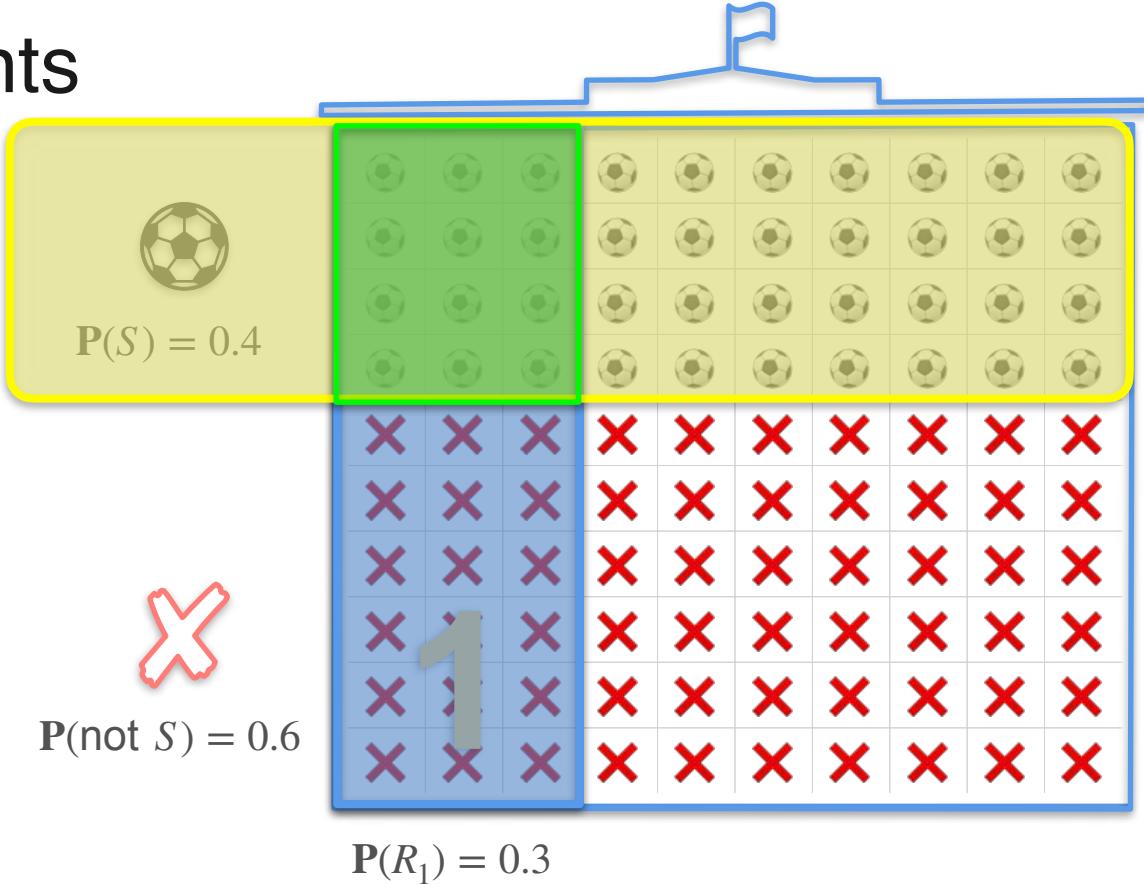
Independent Events



Independent Events

$P(\text{Soccer and Room 1})$

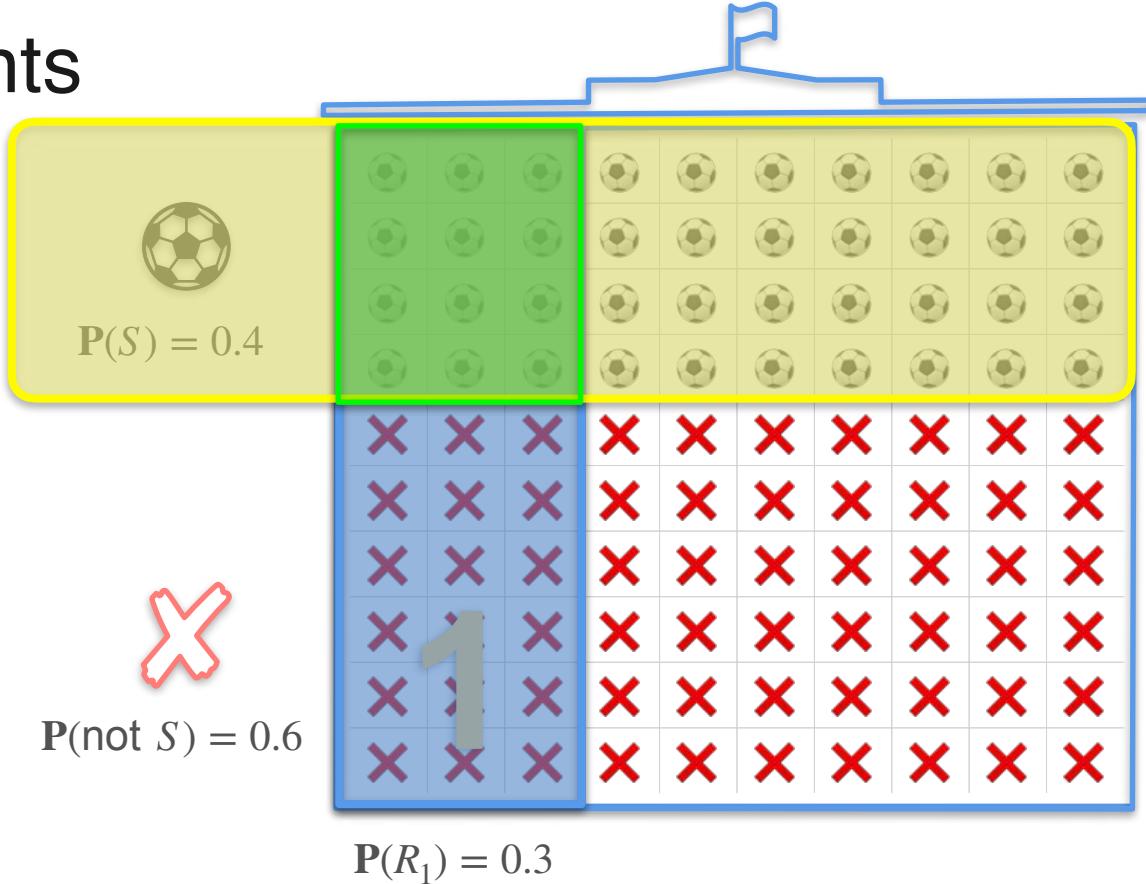
$P(S \cap R_1) =$



Independent Events

$P(\text{Soccer and Room 1})$

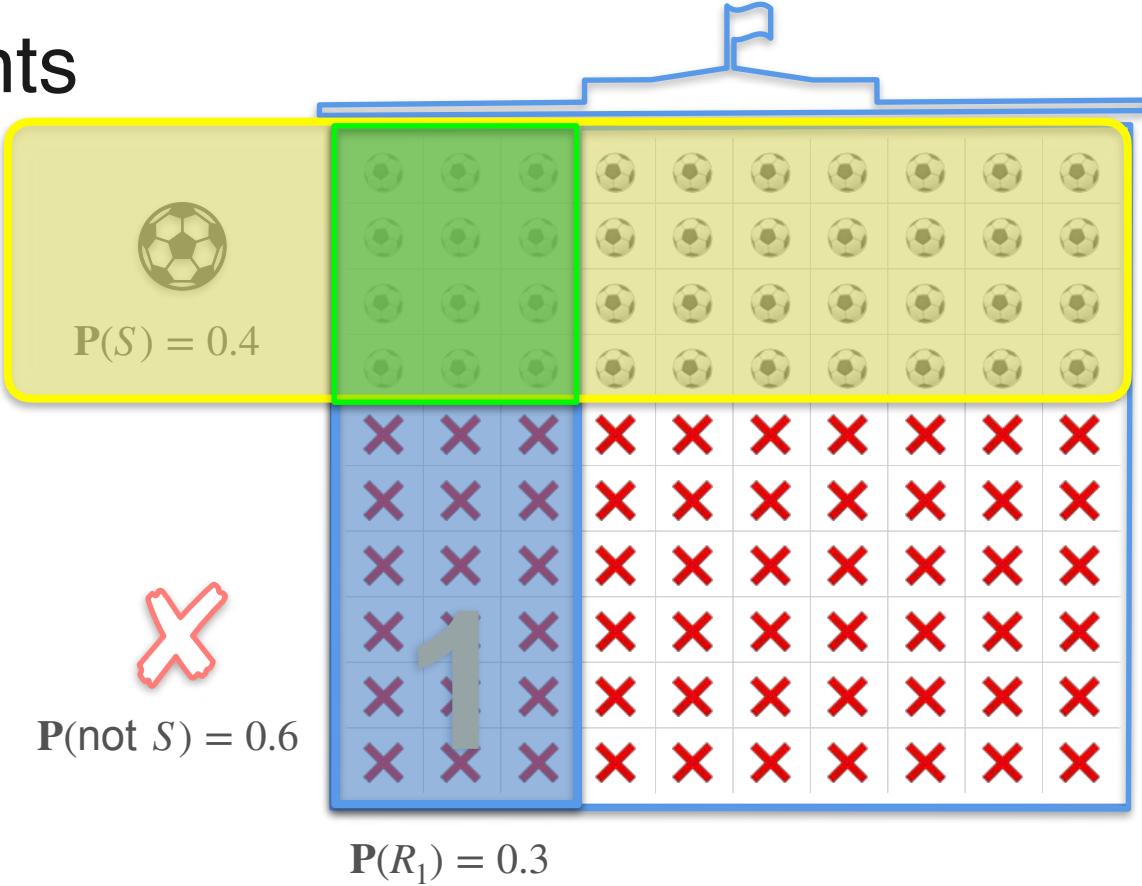
$$P(S \cap R_1) = P(S)$$



Independent Events

$P(\text{Soccer and Room 1})$

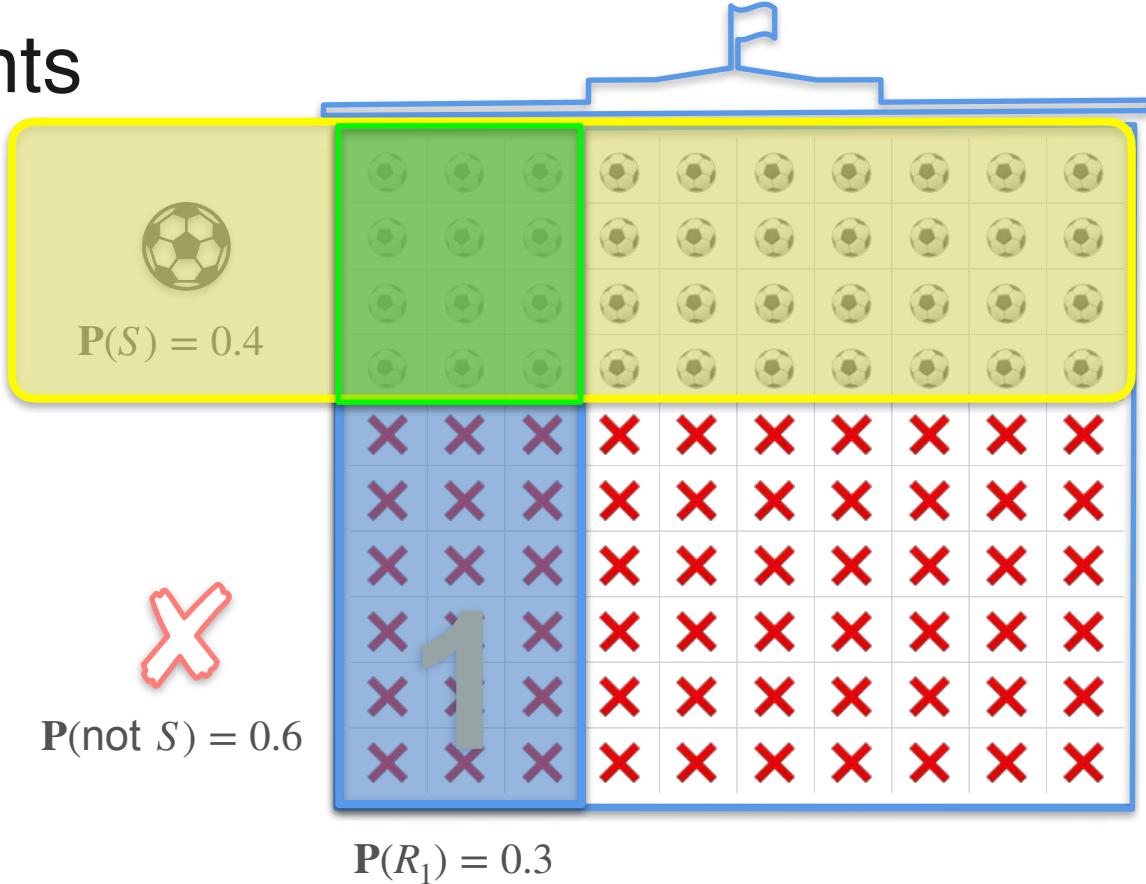
$$P(S \cap R_1) = P(S)$$



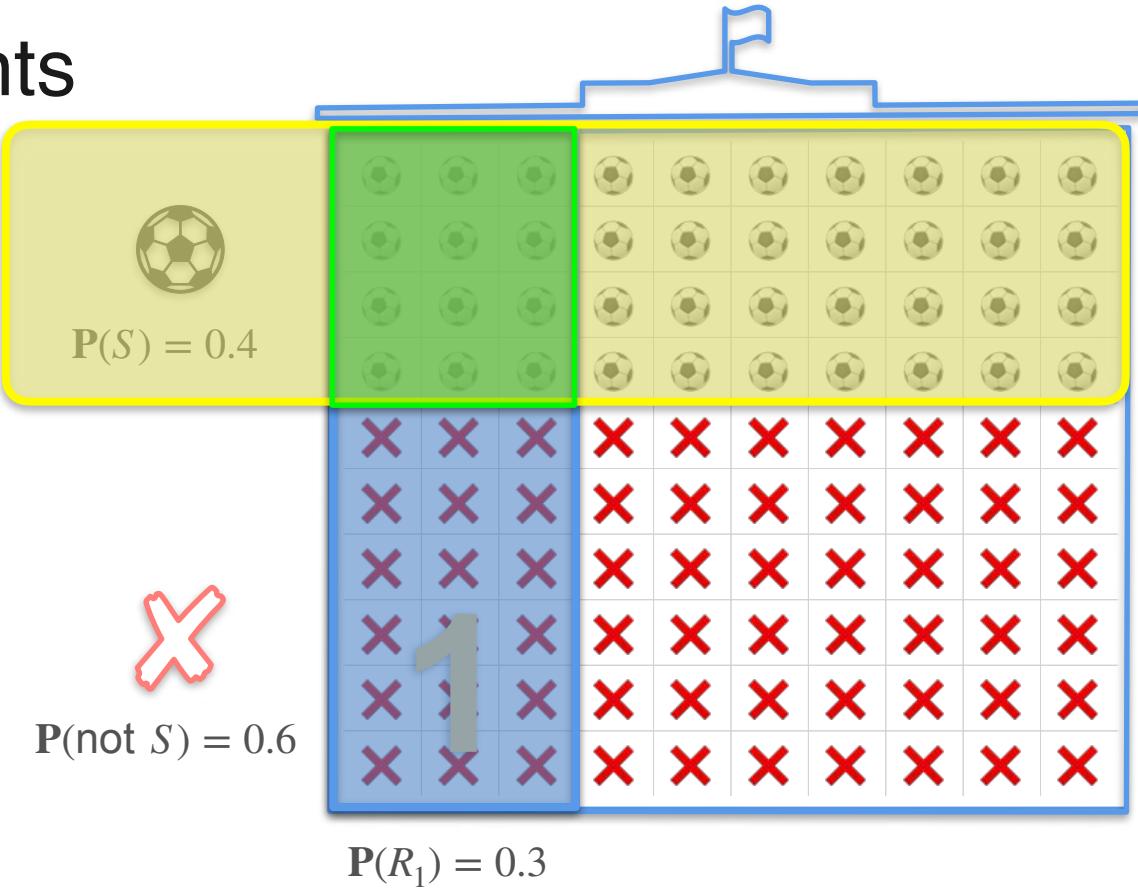
Independent Events

$P(\text{Soccer and Room 1})$

$P(S \cap R_1) = P(S) \bullet$



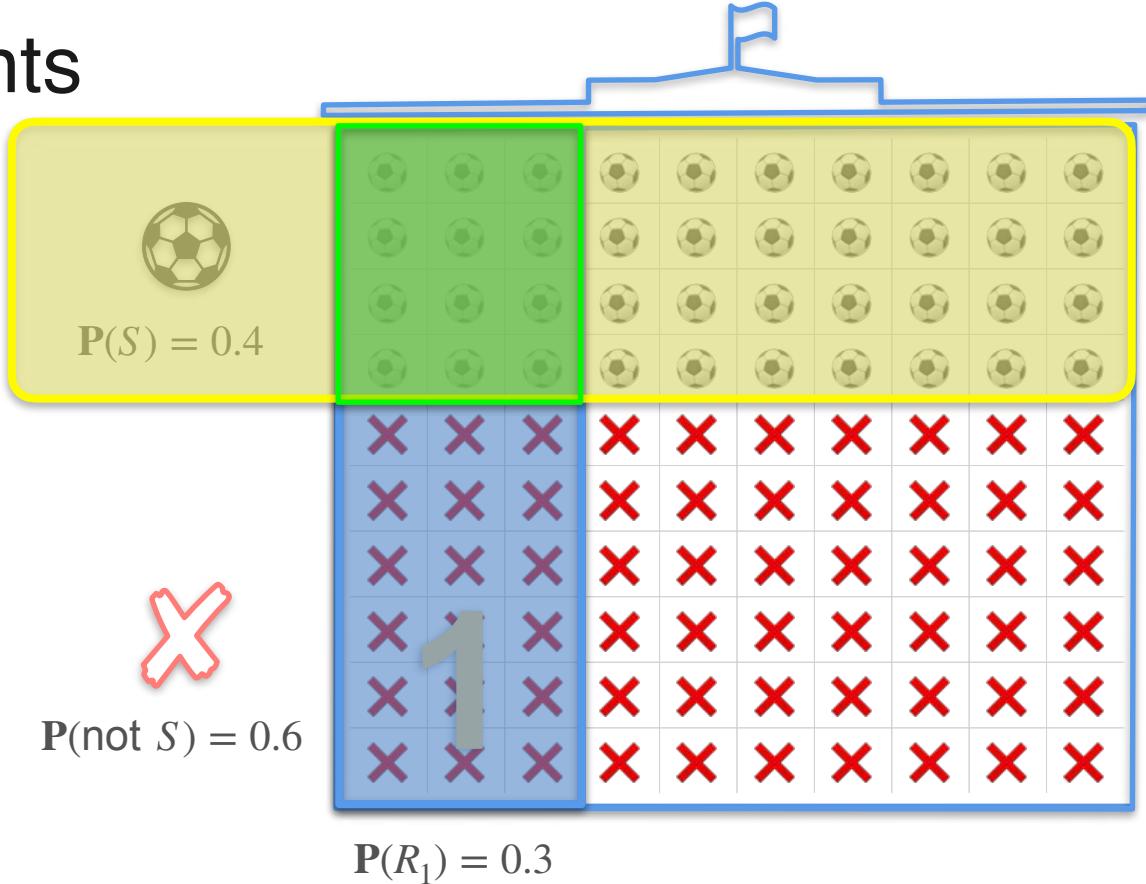
Independent Events



Independent Events

$P(\text{Soccer and Room 1})$

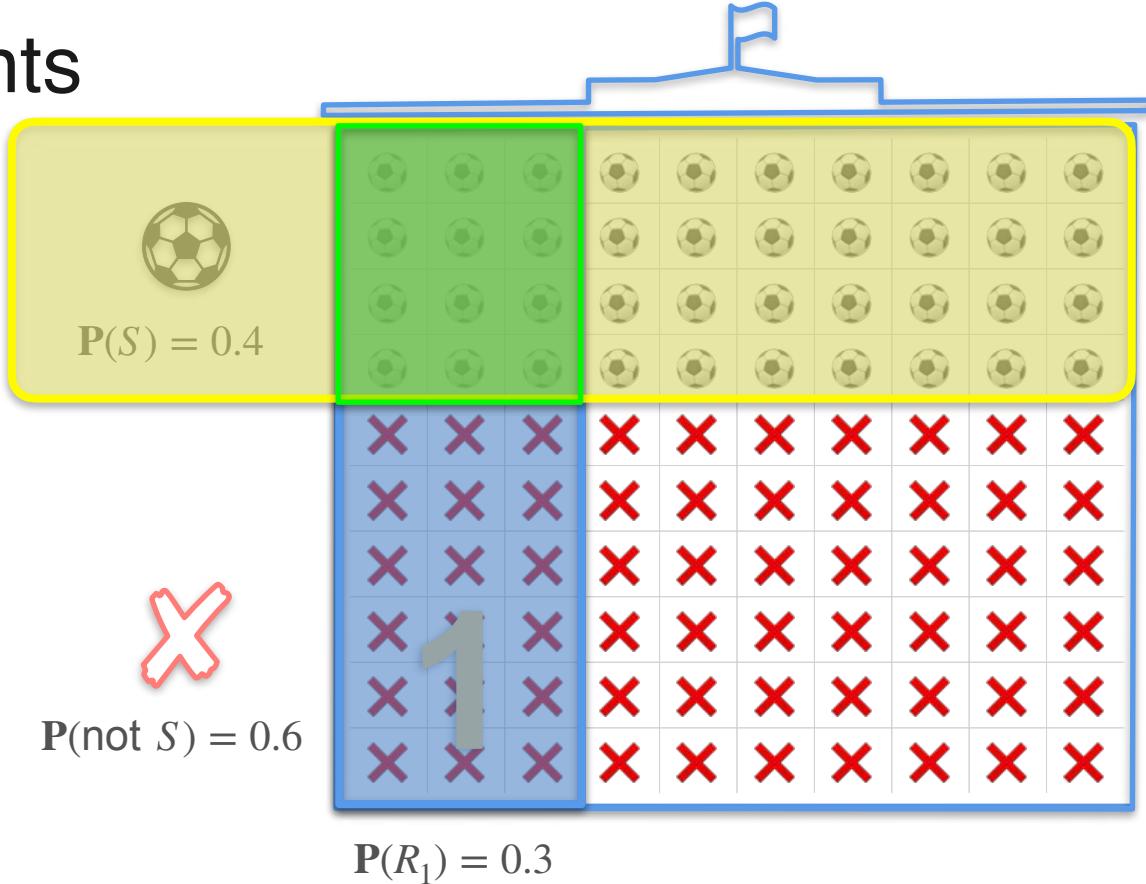
$$P(S \cap R_1) = P(S) \bullet P(R_1)$$



Independent Events

$P(\text{Soccer and Room 1})$

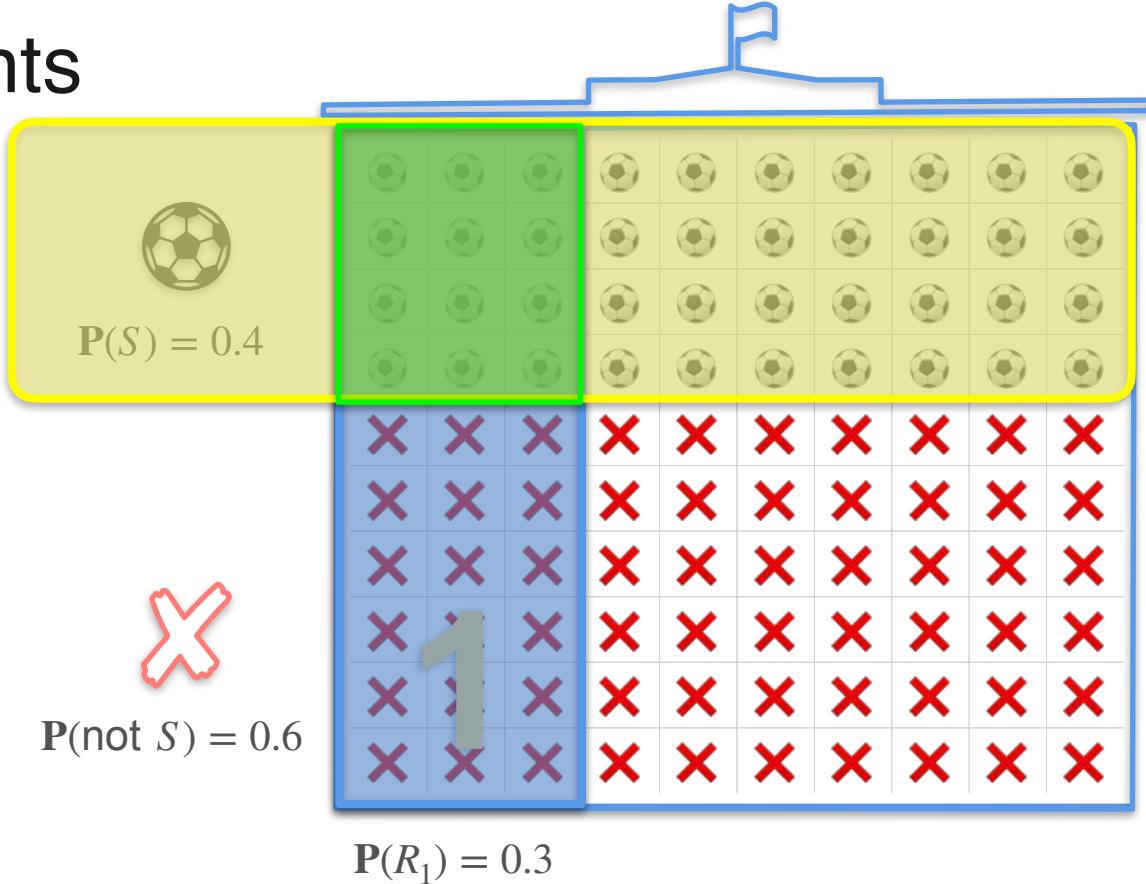
$$\begin{aligned} P(S \cap R_1) &= P(S) \bullet P(R_1) \\ &= 0.4 \end{aligned}$$



Independent Events

$P(\text{Soccer and Room 1})$

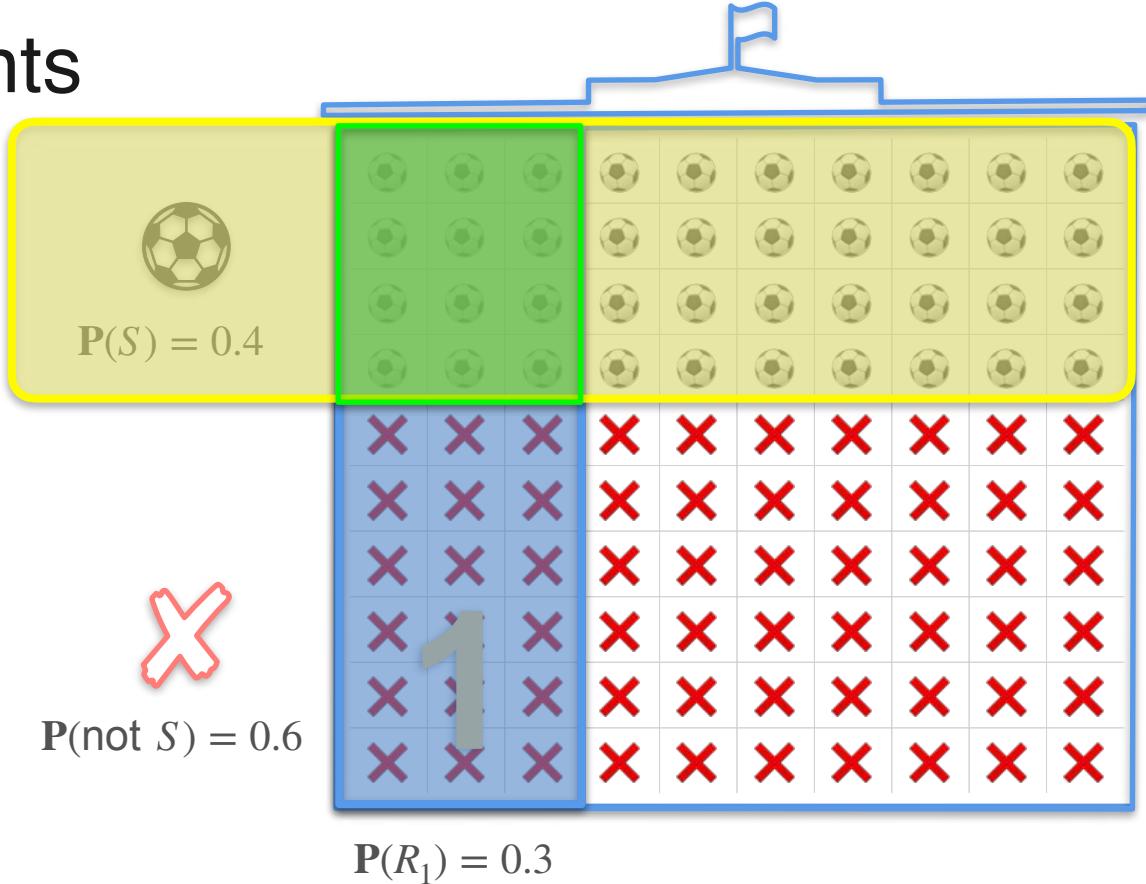
$$\begin{aligned} P(S \cap R_1) &= P(S) \bullet P(R_1) \\ &= 0.4 \bullet \end{aligned}$$



Independent Events

$P(\text{Soccer and Room 1})$

$$\begin{aligned} P(S \cap R_1) &= P(S) \bullet P(R_1) \\ &= 0.4 \bullet 0.3 \end{aligned}$$



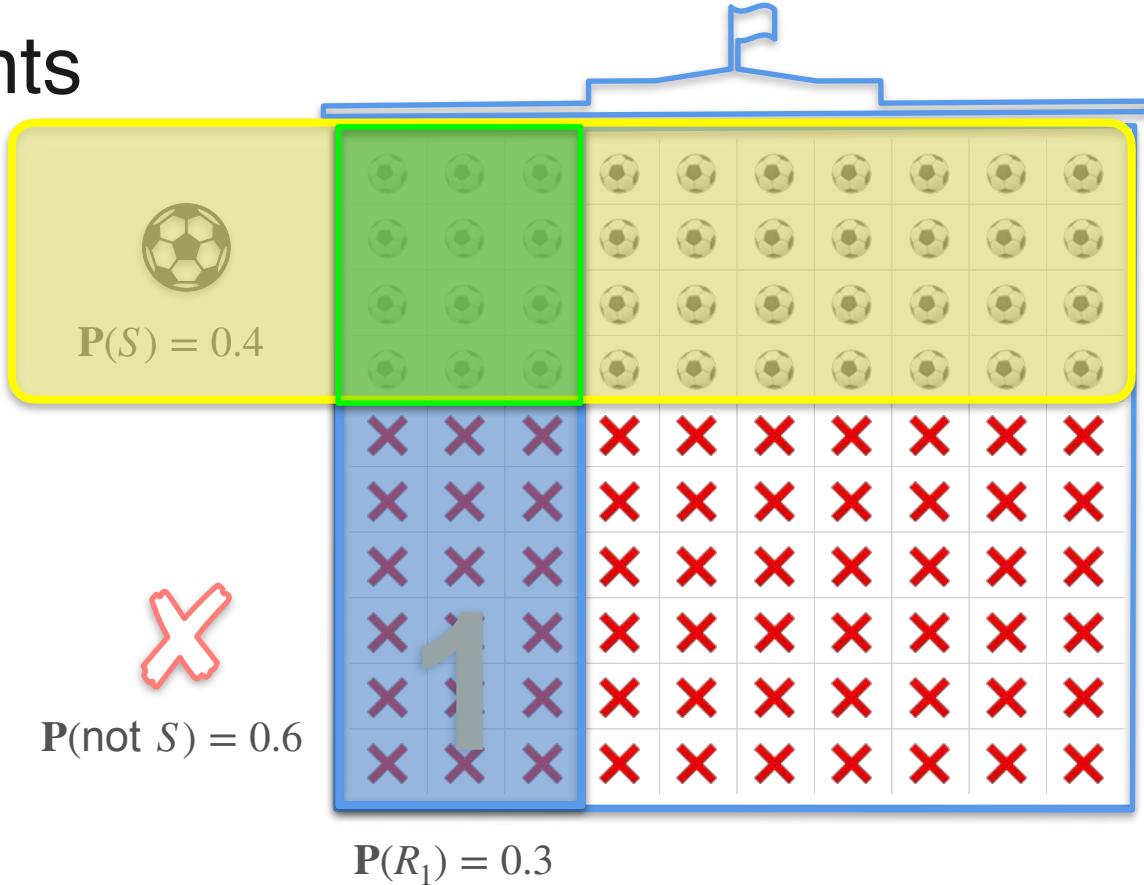
Independent Events

$P(\text{Soccer and Room 1})$

$$P(S \cap R_1) = P(S) \bullet P(R_1)$$

$$= 0.4 \bullet 0.3$$

$$= 0.12$$



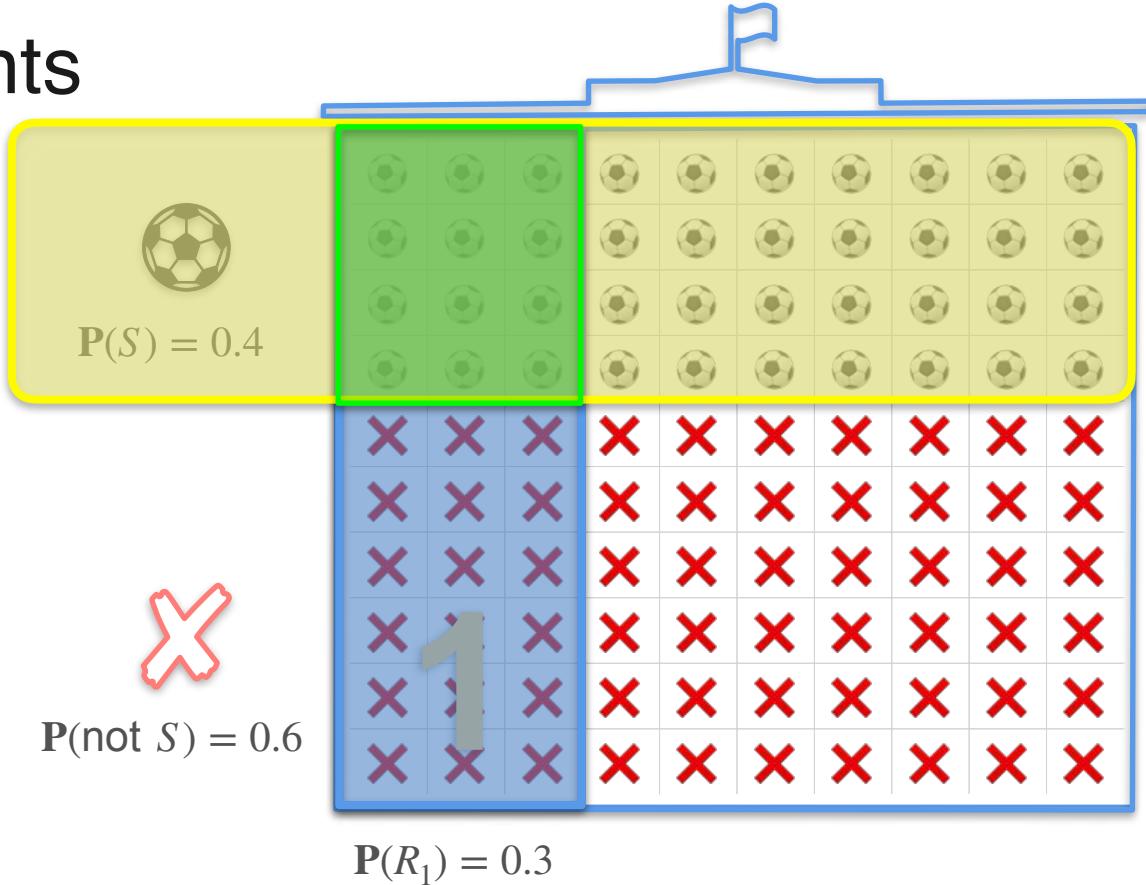
Independent Events

$P(\text{Soccer and Room 1})$

$$P(S \cap R_1) = P(S) \bullet P(R_1)$$

$$= 0.4 \bullet 0.3$$

$$= 0.12$$



Product Rule (for Independent Events)

Product Rule (for Independent Events)

$$\mathbf{P}(A \cap B) = \mathbf{P}(A) \cdot \mathbf{P}(B)$$

Independent Events: Coin Example 1



50% 50%

Independent Events: Coin Example 1



50% 50%



Independent Events: Coin Example 1



50% 50%

What is the probability of landing on heads five times?



Independent Events: Coin Example 1



What is the probability of landing on heads five times?



Independent Events: Coin Example 1



What is the probability of landing on heads five times?



Independent Events: Coin Example 1



50% 50%

What is the probability of landing on heads five times?

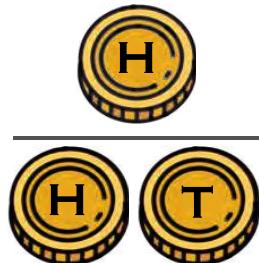


Independent Events: Coin Example 1



50% 50%

What is the probability of landing on heads five times?



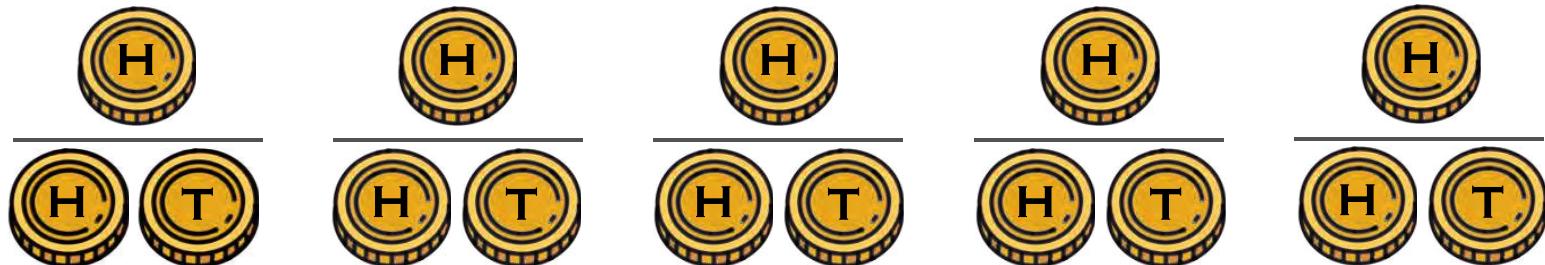
$$\frac{1}{2}$$

Independent Events: Coin Example 1



50% 50%

What is the probability of landing on heads five times?



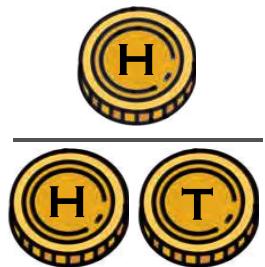
$$\frac{1}{2}$$

Independent Events: Coin Example 1

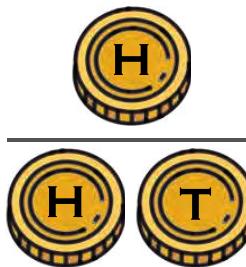


50% 50%

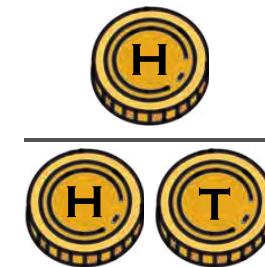
What is the probability of landing on heads five times?



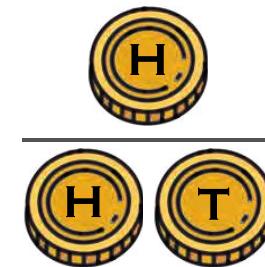
$$\frac{1}{2}$$



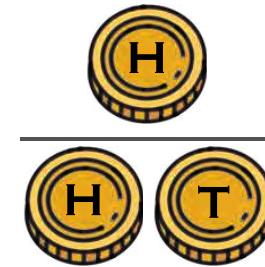
$$\frac{1}{2}$$



$$\frac{1}{2}$$



$$\frac{1}{2}$$



$$\frac{1}{2}$$

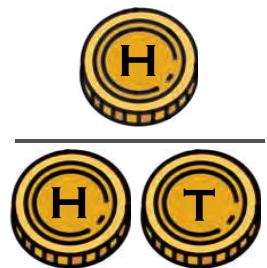
Independent Events: Coin Example 1



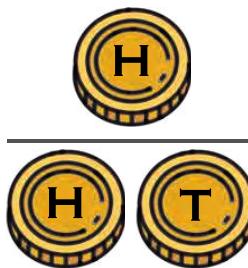
50% 50%

What is the probability of landing on heads five times?

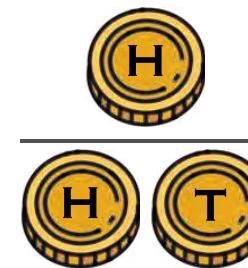
$$P(5 \text{ heads}) =$$



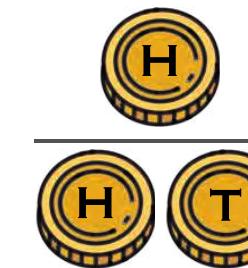
$$\frac{1}{2}$$



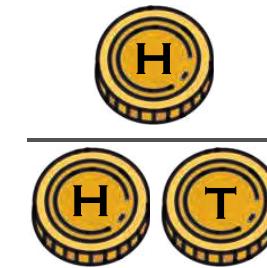
$$\frac{1}{2}$$



$$\frac{1}{2}$$



$$\frac{1}{2}$$



$$\frac{1}{2}$$

Independent Events: Coin Example 1



50% 50%

What is the probability of landing on heads five times?

$P(5 \text{ heads}) =$

$$\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$$

The equation shows the probability of getting 5 heads in a row as the product of the probability of getting heads on each individual flip. There are five fractions, each with a value of $\frac{1}{2}$ above a horizontal line and a gold coin with 'H' facing up above it. Between the first fraction and the second, there is a black dot. Between the second and third, there is a black dot. Between the third and fourth, there is a black dot. Between the fourth and fifth, there is a black dot.

Independent Events: Coin Example 1



50% 50%

What is the probability of landing on heads five times?

$P(5 \text{ heads}) =$

$$\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$$
A sequence of five coin tosses is shown. Each toss is represented by a horizontal line with two circles. The top circle contains 'H' (heads) and the bottom circle contains 'T' (tails). The first toss shows H-T. Subsequent tosses show H-H, H-T, H-T, and H-T. Between each pair of circles is a small black dot, and between each pair of lines is a larger black dot, representing the multiplication of probabilities.

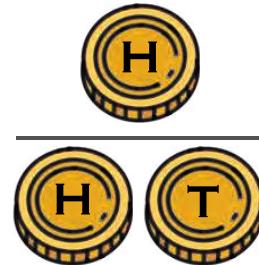
Independent Events: Coin Example 2



50% 50%

What is the probability of landing on heads five times?

$$P(5 \text{ heads}) =$$



$$\frac{1}{2}$$

Independent Events: Coin Example 2



50% 50%

What is the probability of landing on heads five times?

$$P(5 \text{ heads}) = \left(\frac{1}{2} \right)^5$$
A diagram showing three gold-colored coins. Two coins are at the bottom, both showing heads (H). A third coin is positioned above them, also showing heads (H).

$$\frac{1}{2}$$

Independent Events: Coin Example 2



50% 50%

What is the probability of landing 5n heads five times?

$$P(5 \text{ heads}) = \left(\frac{\text{Diagram of 5 coins}}{\text{Diagram of 3 coins}} \right)$$
A fraction is shown where the numerator is a stack of five coins, with the top coin showing heads (H). The denominator is a stack of three coins, with the top two coins showing heads (H) and the bottom coin showing tails (T).

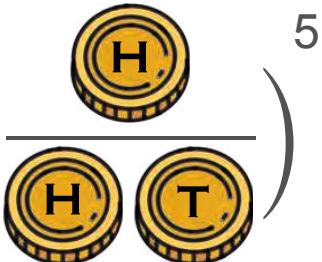
$$\frac{1}{2}$$

Independent Events: Coin Example 2



50% 50%

What is the probability of landing on heads five times?

$$P(5 \text{ heads}) = \left(\frac{\text{Heads}}{\text{Heads} + \text{Tails}} \right)^5$$
A diagram showing three gold-colored coins. The top coin shows 'H' (heads). The bottom-left coin shows 'H' (heads). The bottom-right coin shows 'T' (tails).

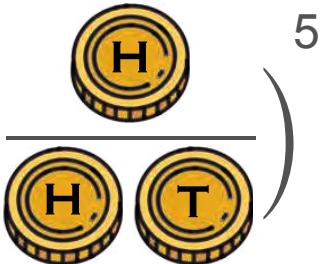
$$\frac{1}{2}$$

Independent Events: Coin Example 2



50% 50%

What is the probability of landing on heads five times?

$$P(5 \text{ heads}) = \left(\frac{\text{Heads}}{\text{Heads} + \text{Tails}} \right)^5$$
A diagram showing three gold-colored coins. The top coin shows 'H' (heads). The bottom-left coin shows 'H' (heads). The bottom-right coin shows 'T' (tails).

$$\left(\frac{1}{2} \right)^5$$

Independent Events: Coin Example 2



50% 50%

What is the probability of landing on heads five times?

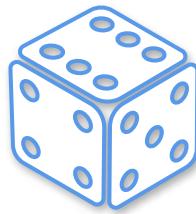
$$P(5 \text{ heads}) = \left(\frac{\text{Diagram of 5 heads}}{\text{Diagram of 2 heads and 1 tail}} \right)^5$$

The fraction in the equation compares two scenarios. The numerator is a stack of five coins, all showing heads ('H'). The denominator is a stack of three coins: the top one shows heads ('H'), the bottom-left shows heads ('H'), and the bottom-right shows tails ('T').

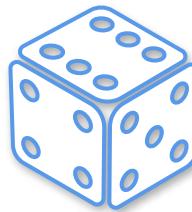
$$\left(\frac{1}{2} \right)^5 = \frac{1}{32}$$

Independent Events: Dice Example 1

Independent Events: Dice Example 1

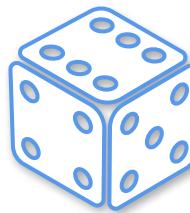


Independent Events: Dice Example 1



$$P(6) = \underline{\hspace{2cm}}$$

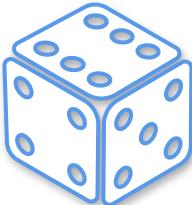
Independent Events: Dice Example 1



$$P(6) = \underline{\hspace{2cm}}$$

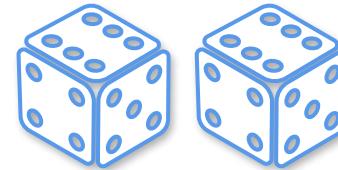


Independent Events: Dice Example 1

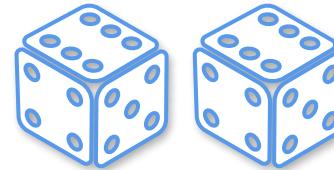
$$P(6) = \frac{1}{6}$$


Independent Events: Dice Example 1

Independent Events: Dice Example 1



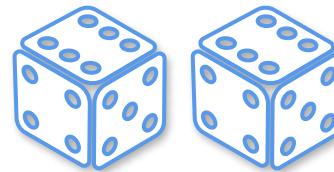
Independent Events: Dice Example 1



2 dice

Independent Events: Dice Example 1

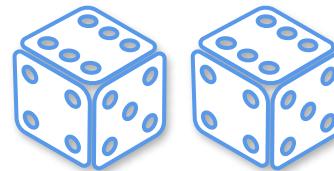
	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
1,2	3,1	3,2	3,3	3,4	3,5	3,6
1,3	4,1	4,2	4,3	4,4	4,5	4,6
1,4	5,1	5,2	5,3	5,4	5,5	5,6
1,5	6,1	6,2	6,3	6,4	6,5	6,6



2 dice

Independent Events: Dice Example 1

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
1,2	3,1	3,2	3,3	3,4	3,5	3,6
1,3	4,1	4,2	4,3	4,4	4,5	4,6
1,4	5,1	5,2	5,3	5,4	5,5	5,6
1,5	6,1	6,2	6,3	6,4	6,5	6,6
1,6						

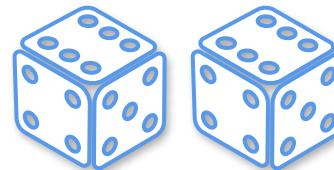


2 dice

$$P(6,6) = \underline{\hspace{2cm}}$$

Independent Events: Dice Example 1

						
	1,1	1,2	1,3	1,4	1,5	1,6
	2,1	2,2	2,3	2,4	2,5	2,6
	3,1	3,2	3,3	3,4	3,5	3,6
	4,1	4,2	4,3	4,4	4,5	4,6
	5,1	5,2	5,3	5,4	5,5	5,6
	6,1	6,2	6,3	6,4	6,5	6,6

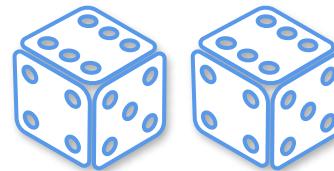


2 dice

$$P(6,6) = \underline{\hspace{2cm}}$$

Independent Events: Dice Example 1

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
1,2	3,1	3,2	3,3	3,4	3,5	3,6
1,3	4,1	4,2	4,3	4,4	4,5	4,6
1,4	5,1	5,2	5,3	5,4	5,5	5,6
1,5	6,1	6,2	6,3	6,4	6,5	6,6



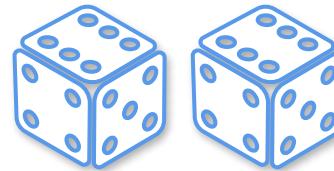
2 dice

6,6

$$P(6,6) = \underline{\hspace{2cm}}$$

Independent Events: Dice Example 1

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6



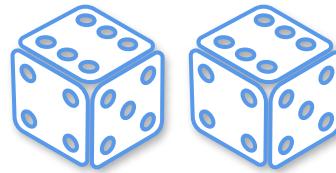
2 dice

$$P(6,6) = \frac{1}{36}$$

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

Independent Events: Dice Example 1

	1,1	1,2	1,3	1,4	1,5	1,6
	2,1	2,2	2,3	2,4	2,5	2,6
	3,1	3,2	3,3	3,4	3,5	3,6
	4,1	4,2	4,3	4,4	4,5	4,6
	5,1	5,2	5,3	5,4	5,5	5,6
	6,1	6,2	6,3	6,4	6,5	6,6



2 dice

$$P(6,6) = \frac{1}{36}$$

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

Independent Events: Dice Example 1

2 dice

						
	1,1	1,2	1,3	1,4	1,5	1,6
	2,1	2,2	2,3	2,4	2,5	2,6
	3,1	3,2	3,3	3,4	3,5	3,6
	4,1	4,2	4,3	4,4	4,5	4,6
	5,1	5,2	5,3	5,4	5,5	5,6
	6,1	6,2	6,3	6,4	6,5	6,6



Independent Events: Dice Example 1

						
1,1	1,2	1,3	1,4	1,5	1,6	
2,1	2,2	2,3	2,4	2,5	2,6	
3,1	3,2	3,3	3,4	3,5	3,6	
4,1	4,2	4,3	4,4	4,5	4,6	
5,1	5,2	5,3	5,4	5,5	5,6	
6,1	6,2	6,3	6,4	6,5		

$$P(6,6) =$$

2 dice



Independent Events: Dice Example 1

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
1,2	3,1	3,2	3,3	3,4	3,5	3,6
1,3	4,1	4,2	4,3	4,4	4,5	4,6
1,4	5,1	5,2	5,3	5,4	5,5	5,6
1,5	6,1	6,2	6,3	6,4	6,5	6,6
$\frac{1}{6}$						

$$P(6,6) =$$



2 dice

Independent Events: Dice Example 1

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
1,2	3,1	3,2	3,3	3,4	3,5	3,6
1,3	4,1	4,2	4,3	4,4	4,5	4,6
1,4	5,1	5,2	5,3	5,4	5,5	5,6
1,5	6,1	6,2	6,3	6,4	6,5	6,6
1,6						

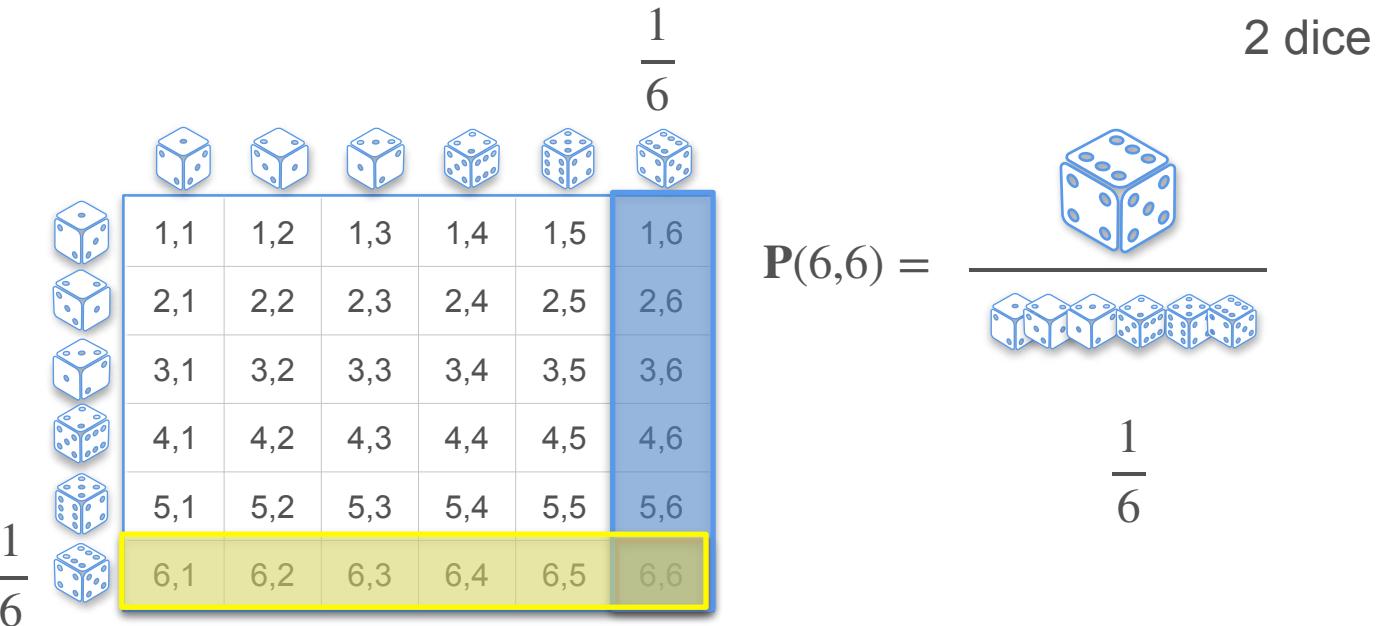
2 dice

$$P(6,6) =$$

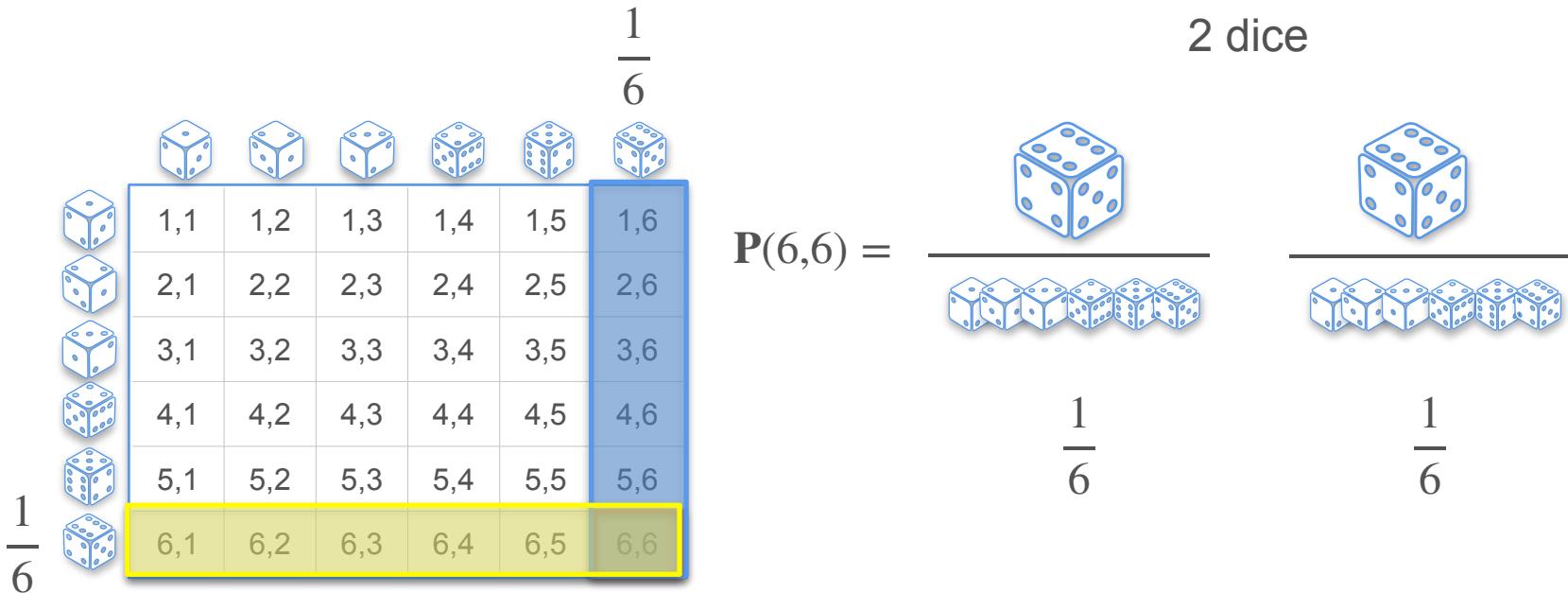


$$\frac{1}{6}$$

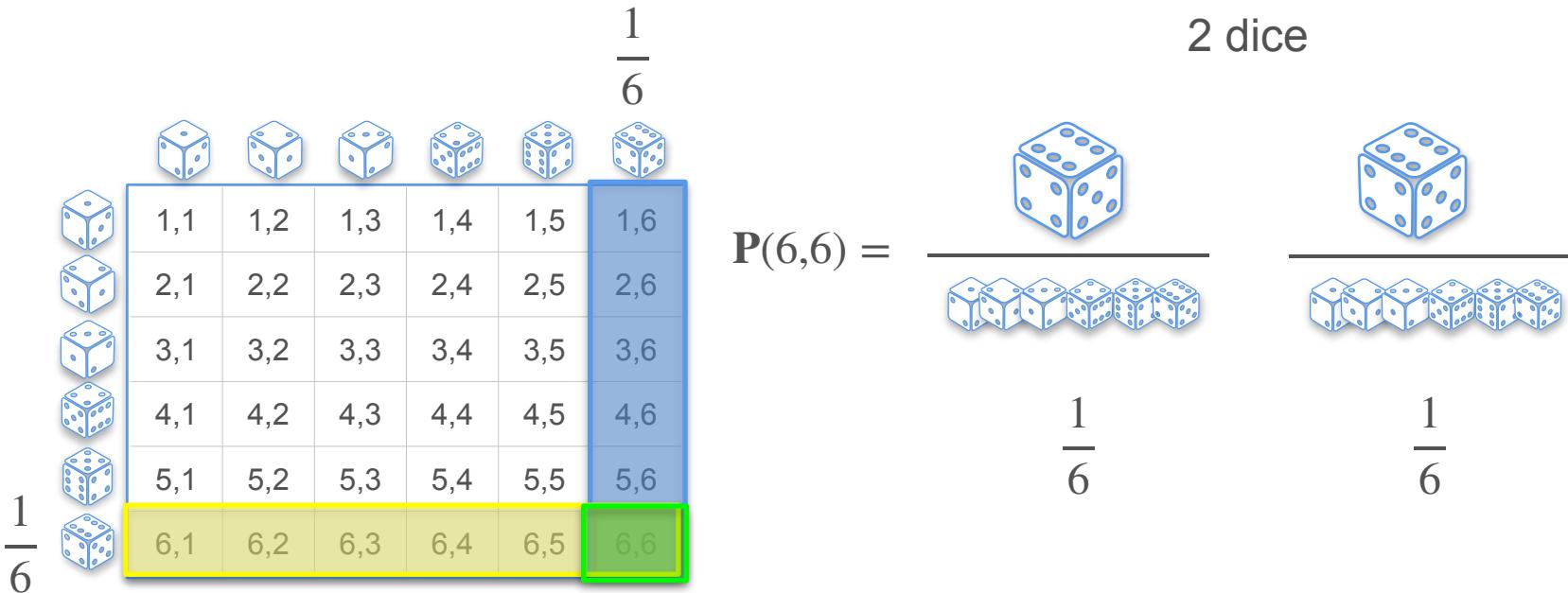
Independent Events: Dice Example 1



Independent Events: Dice Example 1



Independent Events: Dice Example 1



Independent Events: Dice Example 1

						$\frac{1}{6}$
	1,1	1,2	1,3	1,4	1,5	1,6
	2,1	2,2	2,3	2,4	2,5	2,6
	3,1	3,2	3,3	3,4	3,5	3,6
	4,1	4,2	4,3	4,4	4,5	4,6
	5,1	5,2	5,3	5,4	5,5	5,6
$\frac{1}{6}$	6,1	6,2	6,3	6,4	6,5	6,6

2 dice

$$P(6,6) = \frac{\text{one outcome}}{\text{all outcomes}} = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$$

Independent Events: Dice Example 1

					$\frac{1}{6}$	
	1,1	1,2	1,3	1,4	1,5	1,6
	2,1	2,2	2,3	2,4	2,5	2,6
	3,1	3,2	3,3	3,4	3,5	3,6
	4,1	4,2	4,3	4,4	4,5	4,6
	5,1	5,2	5,3	5,4	5,5	5,6
$\frac{1}{6}$	6,1	6,2	6,3	6,4	6,5	6,6

2 dice

$$P(6,6) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}} = \frac{1}{6} \cdot \frac{1}{6} = \left(\frac{1}{6}\right)$$

The diagram illustrates the probability calculation for rolling two dice. It shows a 6x6 grid of outcomes. The outcome (6,6) is highlighted with a yellow background and a green border. The total number of outcomes is 36, represented by 36 dice icons in a 6x6 grid. The number of favorable outcomes is 1, represented by a single die showing a 6.

Independent Events: Dice Example 1

						$\frac{1}{6}$
	1,1	1,2	1,3	1,4	1,5	1,6
	2,1	2,2	2,3	2,4	2,5	2,6
	3,1	3,2	3,3	3,4	3,5	3,6
	4,1	4,2	4,3	4,4	4,5	4,6
	5,1	5,2	5,3	5,4	5,5	5,6
$\frac{1}{6}$	6,1	6,2	6,3	6,4	6,5	6,6

2 dice

$$P(6,6) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}} = \frac{1}{6} \cdot \frac{1}{6} = \left(\frac{1}{6}\right)$$

The diagram illustrates the probability calculation for rolling two dice. It shows a 6x6 grid of outcomes. The outcome (6,6) is highlighted with a yellow background and a green border. The total number of outcomes is 36, represented by 36 dice icons in a 6x6 grid. The number of favorable outcomes is 1, represented by a single die showing a 6.

Independent Events: Dice Example 1

					$\frac{1}{6}$
	1,1	1,2	1,3	1,4	1,5
1,1	2,1	2,2	2,3	2,4	2,5
1,2	3,1	3,2	3,3	3,4	3,5
1,3	4,1	4,2	4,3	4,4	4,5
1,4	5,1	5,2	5,3	5,4	5,5
1,5	6,1	6,2	6,3	6,4	6,5
1,6	6,6				

2 dice

$$P(6,6) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}} = \frac{1}{6} \cdot \frac{1}{6} = \left(\frac{1}{6}\right)^2$$

The diagram illustrates the probability calculation for rolling two dice. It shows a 6x6 grid of outcomes. The top row and left column are labeled with dice icons. The bottom-right cell, representing the outcome (6,6), is highlighted with a green border. The entire grid is divided into two main sections by a diagonal line from the top-left to the bottom-right. The section above the diagonal is shaded yellow, and the section below it is shaded blue. The total number of outcomes is represented by the product of the number of dice (2) and the number of faces per die (6). The number of favorable outcomes is 1, corresponding to the single cell (6,6).

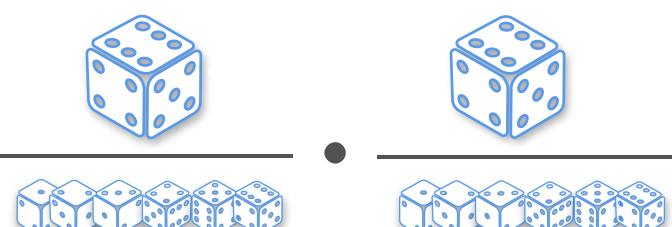
Independent Events: Dice Example 1

						$\frac{1}{6}$
	1,1	1,2	1,3	1,4	1,5	1,6
	2,1	2,2	2,3	2,4	2,5	2,6
	3,1	3,2	3,3	3,4	3,5	3,6
	4,1	4,2	4,3	4,4	4,5	4,6
	5,1	5,2	5,3	5,4	5,5	5,6
$\frac{1}{6}$	6,1	6,2	6,3	6,4	6,5	6,6

2 dice

$$P(6,6) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}} = \frac{1}{6} \cdot \frac{1}{6} = \left(\frac{1}{6}\right)^2 = \frac{1}{36}$$

$P(6,6) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}} = \frac{1}{6} \cdot \frac{1}{6} = \left(\frac{1}{6}\right)^2 = \frac{1}{36}$



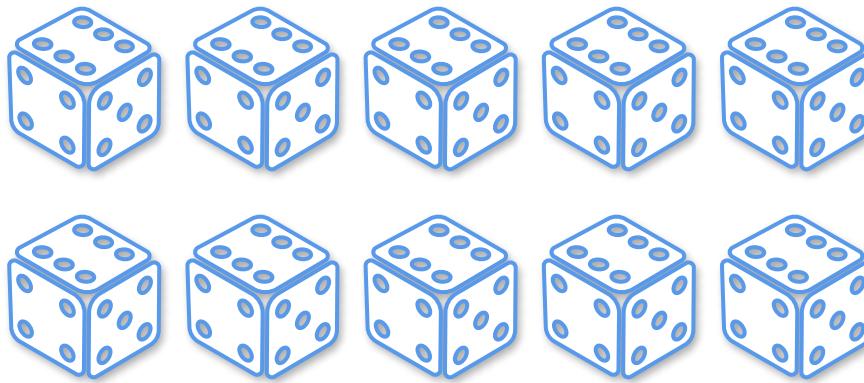
Independent Events: Dice Example 2

Independent Events: Dice Example 2

What is the probability of getting 10 sixes?

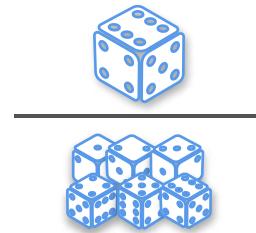
Independent Events: Dice Example 2

What is the probability of getting 10 sixes?



Independent Events: Dice Example 2

What is the probability of getting 10 sixes?



Independent Events: Dice Example 2

What is the probability of getting 10 sixes?

$$P(10 \text{ sixes}) = \frac{\text{one outcome}}{\text{all outcomes}}$$


Independent Events: Dice Example 2

What is the probability of getting 10 sixes?

$$P(10 \text{ sixes}) = \left(\frac{\text{one die}}{\text{ten dice}} \right)$$

Independent Events: Dice Example 2

What is the probability of getting 10 sixes?

$$P(10 \text{ sixes}) = \left(\frac{\text{one die showing 6}}{\text{one row of 6 dice}} \right)^{10}$$

Independent Events: Dice Example 2

What is the probability of getting 10 sixes?

$$\begin{aligned} P(10 \text{ sixes}) &= \left(\frac{\text{one die showing 6}}{\text{one die showing 6}} \right)^{10} \\ &= \left(\frac{1}{6} \right)^{10} \end{aligned}$$



DeepLearning.AI

Introduction to probability

Birthday problem

6. The Birthday Problem

- Quiz: You have a party with 30 friends. What do you think is more likely, that there are two with the same birthday, or not?
 - Answer: Same birthday
- Calculate the probability that two people have the same birthday. Show that it's very close to 1.
- Question: How many people do you think there should be for the probability that 2 have the same birthday is 50?
 - Answer: 23
 - Show calculation

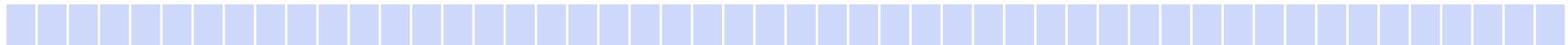
Quiz

- You have 30 friends at a party. What do you think is more likely:
 - That there exist two people with the same birthday
 - That no two of them have the same birthday
- (Assume the year has 365 days, nobody has a birthday on Feb 29).

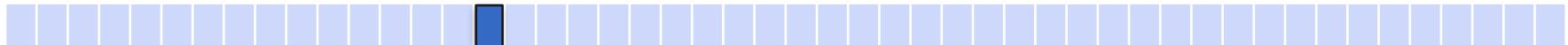
Quiz

- Answer: It's more likely that 2 people have the same birthday.
- In fact, the probability of no two people having the same birthday is around 0.3.

Probability That Everyone Has a Different Birthday

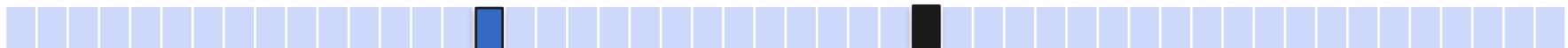


Probability That Everyone Has a Different Birthday



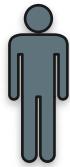
$$\frac{365}{365}$$

Probability That Everyone Has a Different Birthday



$$\frac{365}{365} \quad \frac{364}{365}$$

Probability That Everyone Has a Different Birthday

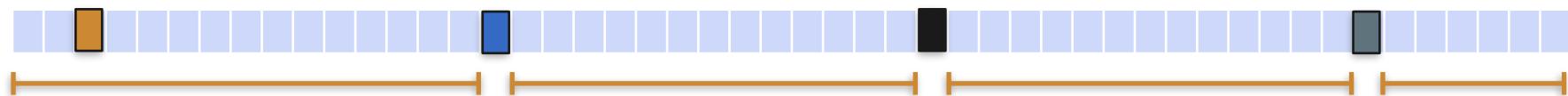


$$\frac{365}{365}$$

$$\frac{364}{365}$$

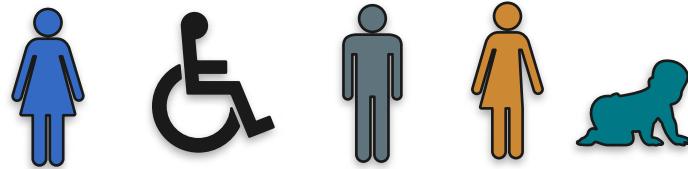
$$\frac{363}{365}$$

Probability That Everyone Has a Different Birthday



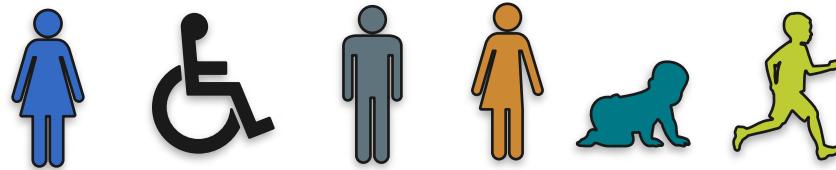
$$\frac{365}{365} \quad \frac{364}{365} \quad \frac{363}{365} \quad \frac{362}{365}$$

Probability That Everyone Has a Different Birthday



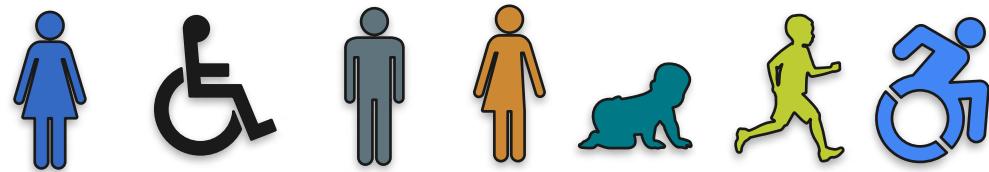
$$\frac{365}{365} \quad \frac{364}{365} \quad \frac{363}{365} \quad \frac{362}{365} \quad \frac{361}{365}$$

Probability That Everyone Has a Different Birthday



$$\frac{365}{365} \quad \frac{364}{365} \quad \frac{363}{365} \quad \frac{362}{365} \quad \frac{361}{365} \quad \frac{360}{365}$$

Probability That Everyone Has a Different Birthday



$$\frac{365}{365}$$

$$\frac{364}{365}$$

$$\frac{363}{365}$$

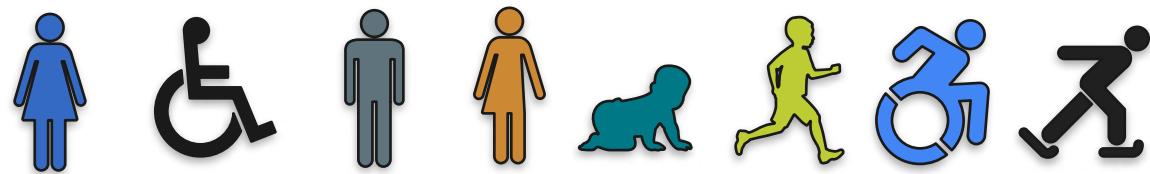
$$\frac{362}{365}$$

$$\frac{361}{365}$$

$$\frac{360}{365}$$

$$\frac{359}{365}$$

Probability That Everyone Has a Different Birthday



$$\frac{365}{365}$$

$$\frac{364}{365}$$

$$\frac{363}{365}$$

$$\frac{362}{365}$$

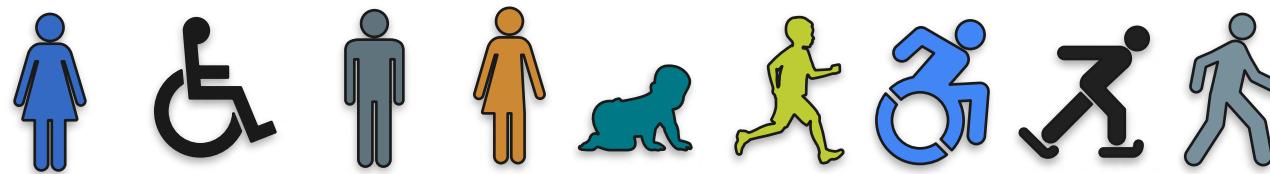
$$\frac{361}{365}$$

$$\frac{360}{365}$$

$$\frac{359}{365}$$

$$\frac{358}{365}$$

Probability That Everyone Has a Different Birthday



$$\frac{365}{365}$$

$$\frac{364}{365}$$

$$\frac{363}{365}$$

$$\frac{362}{365}$$

$$\frac{361}{365}$$

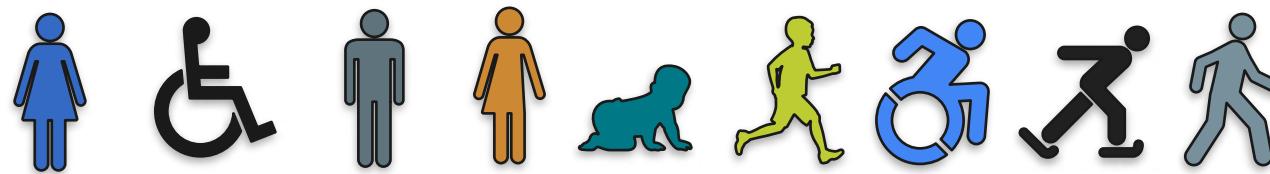
$$\frac{360}{365}$$

$$\frac{359}{365}$$

$$\frac{358}{365}$$

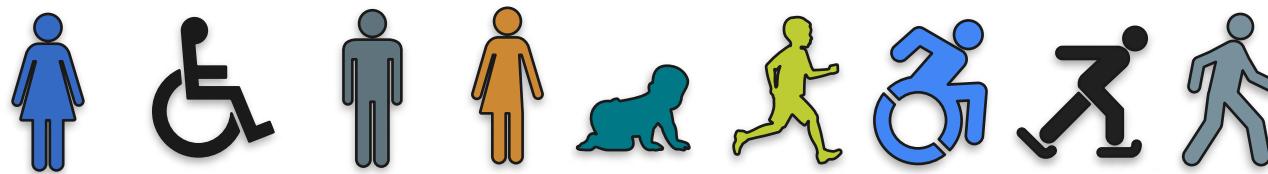
$$\frac{357}{365}$$

Probability That Everyone Has a Different Birthday



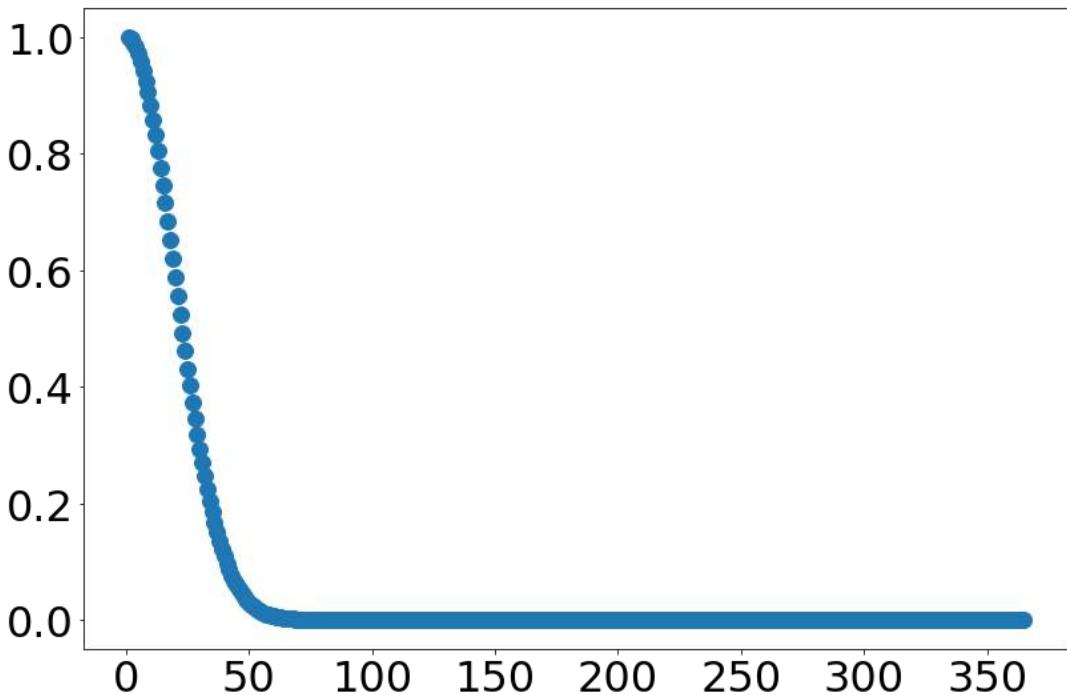
$$\frac{365}{365} \cdot \frac{364}{365} \cdot \frac{363}{365} \cdot \frac{362}{365} \cdot \frac{361}{365} \cdot \frac{360}{365} \cdot \frac{359}{365} \cdot \frac{358}{365} \cdot \frac{357}{365} =$$

Probability That Everyone Has a Different Birthday



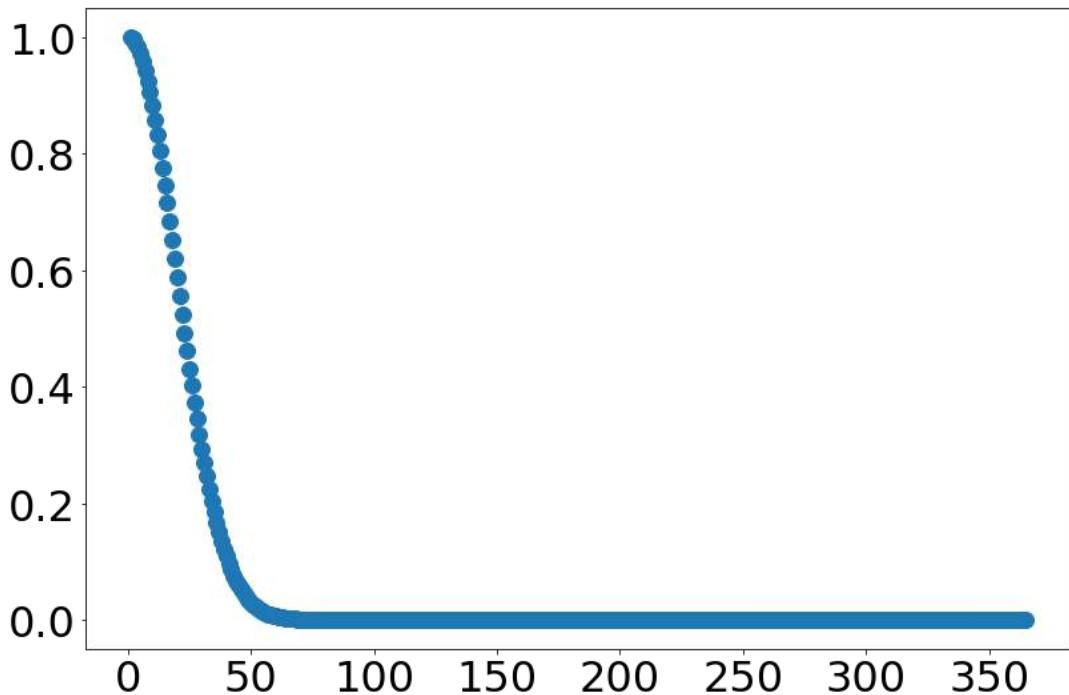
$$\frac{365}{365} \cdot \frac{364}{365} \cdot \frac{363}{365} \cdot \frac{362}{365} \cdot \frac{361}{365} \cdot \frac{360}{365} \cdot \frac{359}{365} \cdot \frac{358}{365} \cdot \frac{357}{365} = 0.905$$

Probability That no Two People Have the Same Birthday



Probability That no Two People Have the Same Birthday

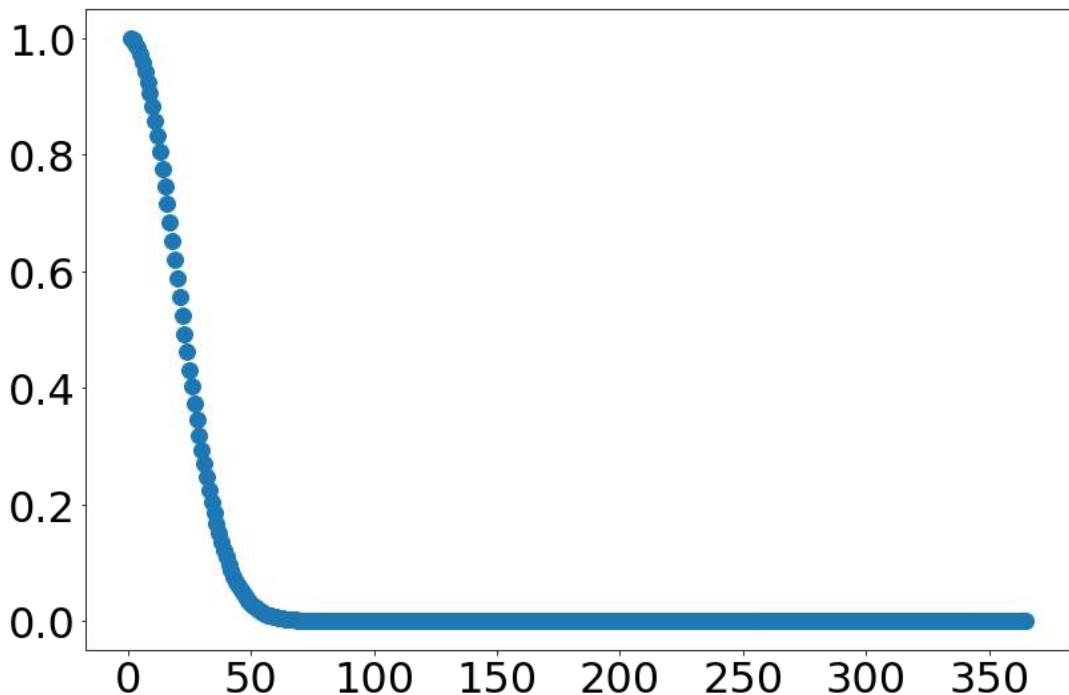
1 person: 1



Probability That no Two People Have the Same Birthday

1 person: 1

2 people: 0.997

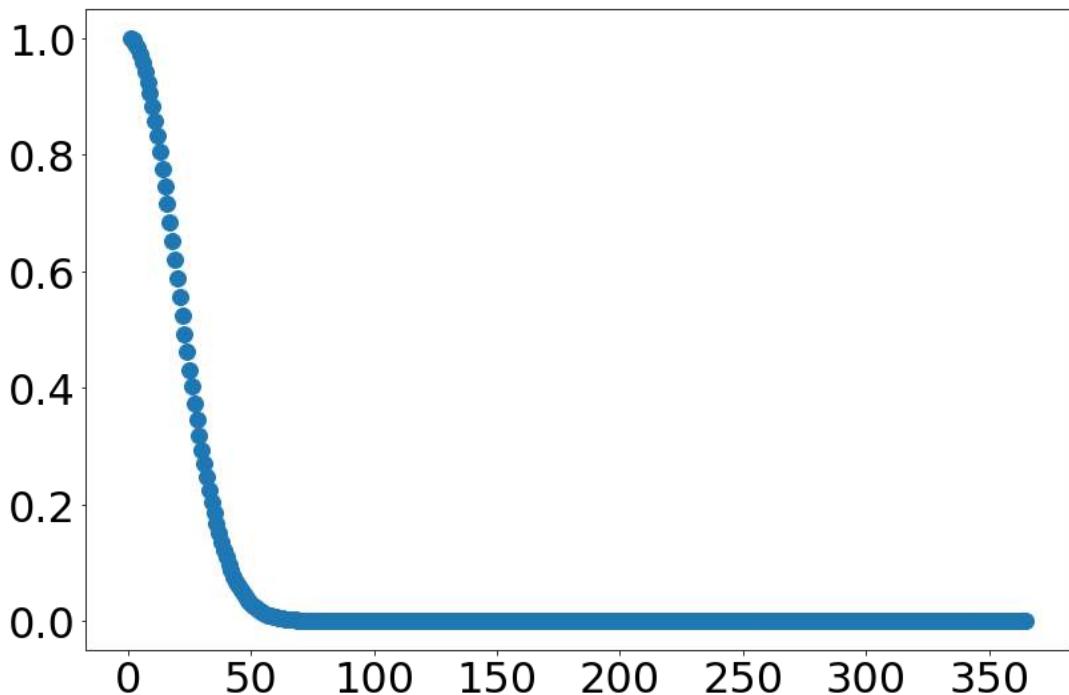


Probability That no Two People Have the Same Birthday

1 person: 1

2 people: 0.997

3 people: 0.992



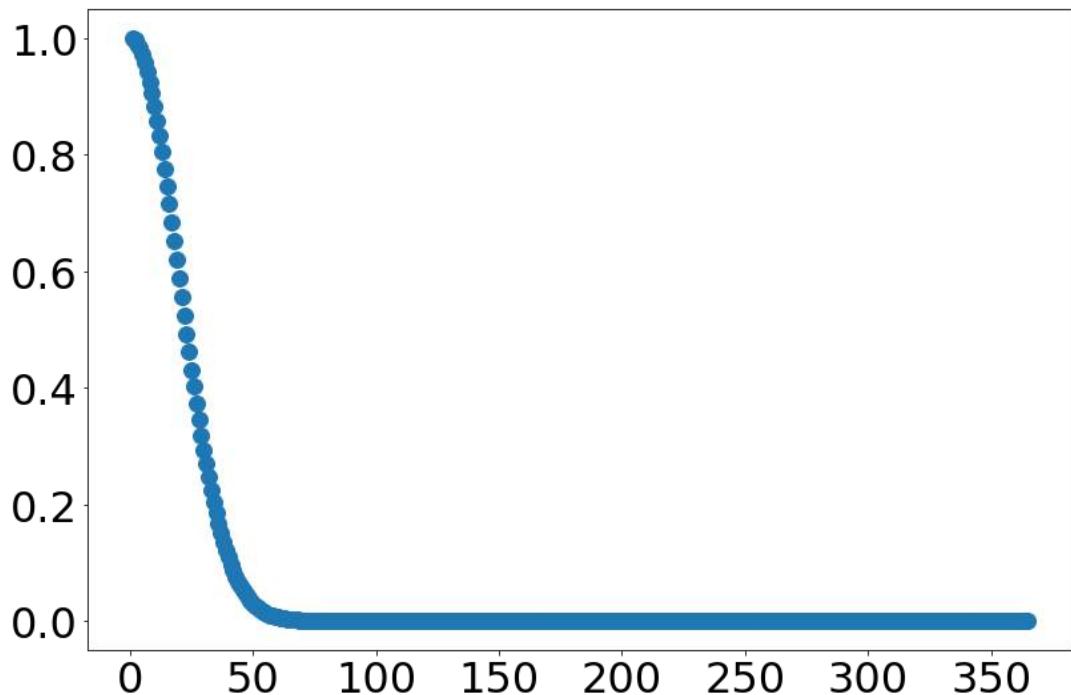
Probability That no Two People Have the Same Birthday

1 person: 1

2 people: 0.997

3 people: 0.992

4 people: 0.984



Probability That no Two People Have the Same Birthday

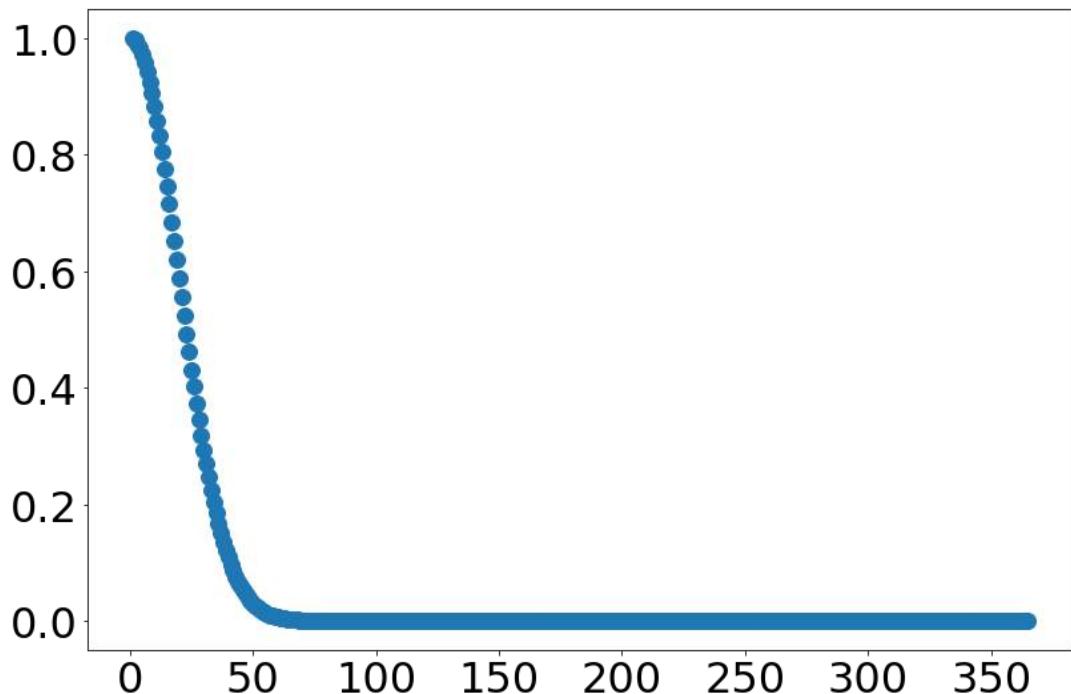
1 person: 1

2 people: 0.997

3 people: 0.992

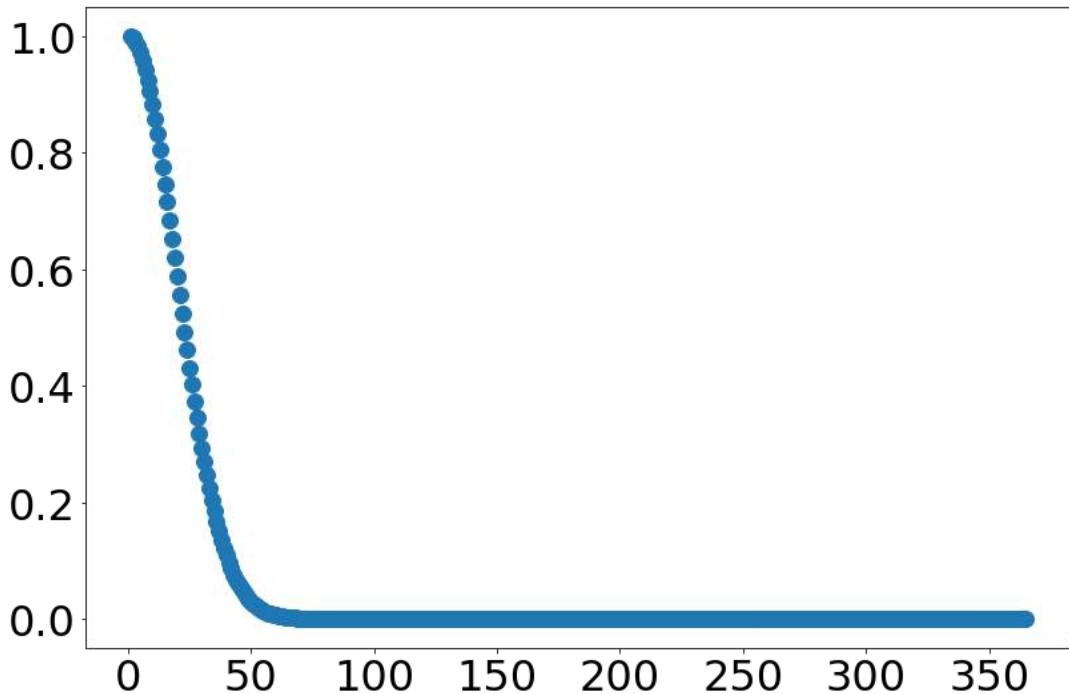
4 people: 0.984

5 people: 0.973



Probability That no Two People Have the Same Birthday

1 person: 1
2 people: 0.997
3 people: 0.992
4 people: 0.984
5 people: 0.973
10 people: 0.883



Probability That no Two People Have the Same Birthday

1 person: 1

2 people: 0.997

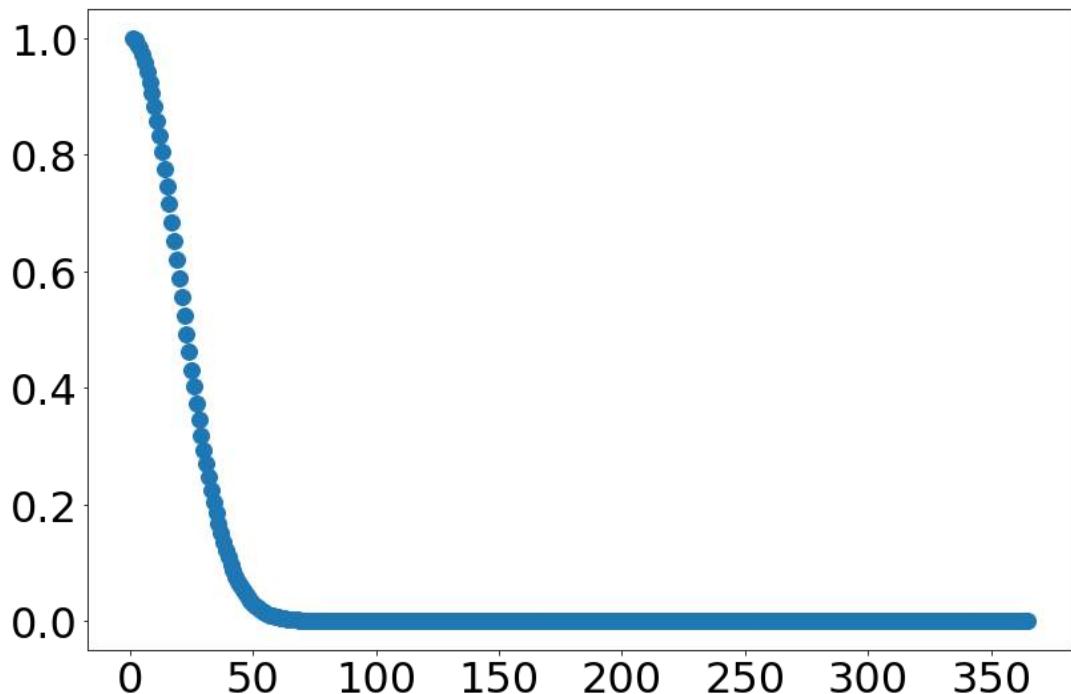
3 people: 0.992

4 people: 0.984

5 people: 0.973

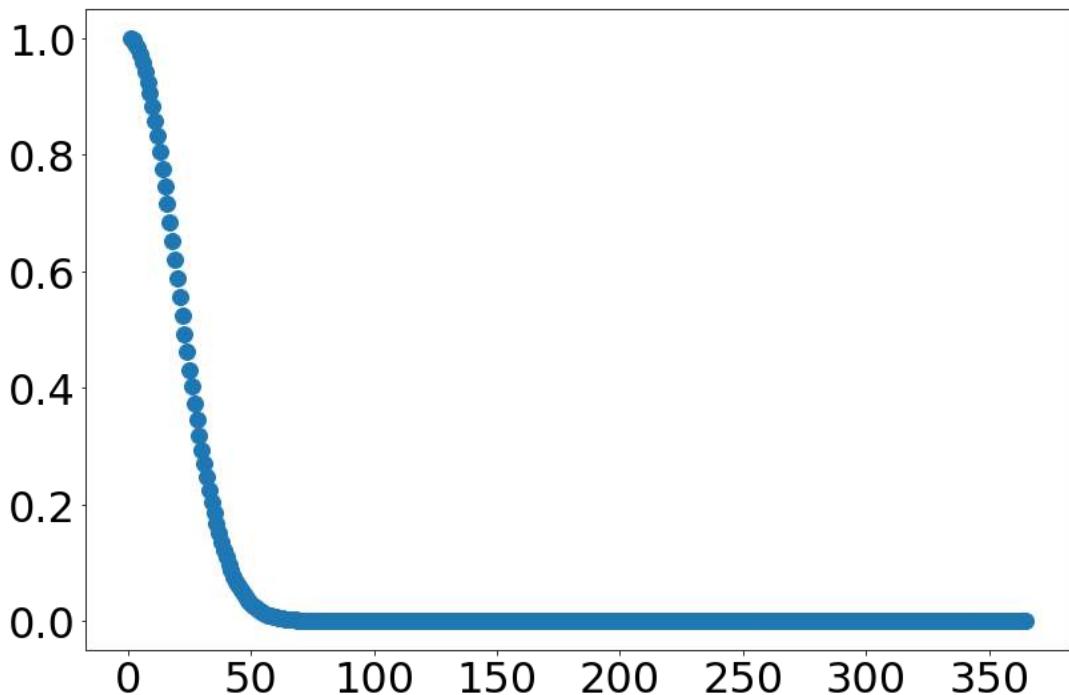
10 people: 0.883

20 people: 0.589



Probability That no Two People Have the Same Birthday

1 person: 1
2 people: 0.997
3 people: 0.992
4 people: 0.984
5 people: 0.973
10 people: 0.883
20 people: 0.589
23 people: 0.493



Probability That no Two People Have the Same Birthday

1 person: 1

2 people: 0.997

3 people: 0.992

4 people: 0.984

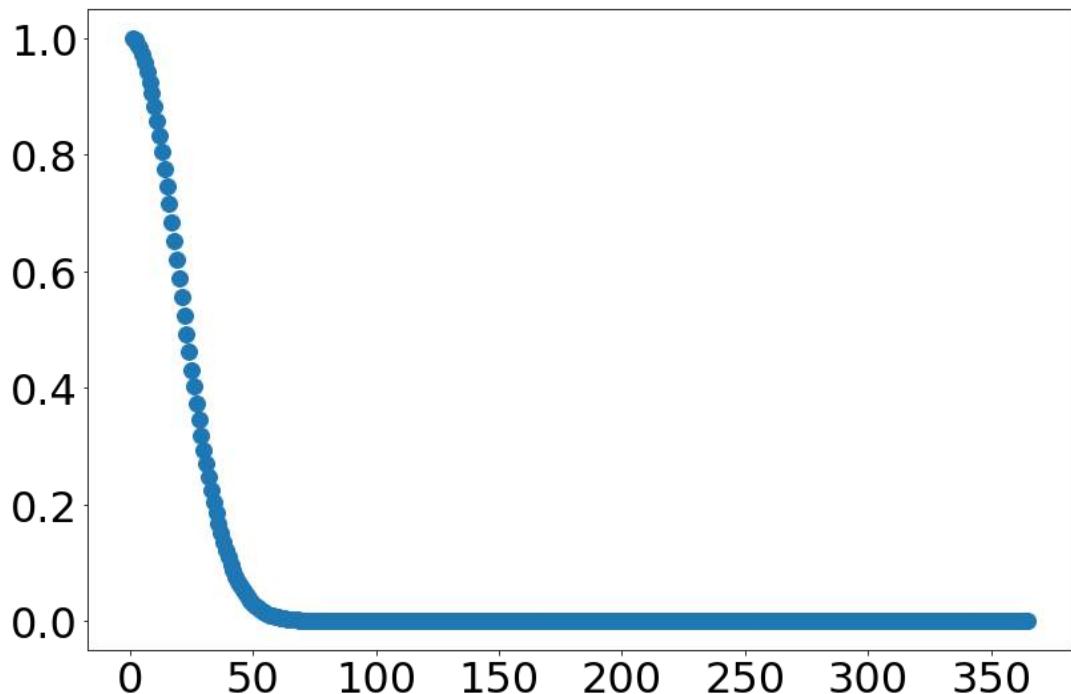
5 people: 0.973

10 people: 0.883

20 people: 0.589

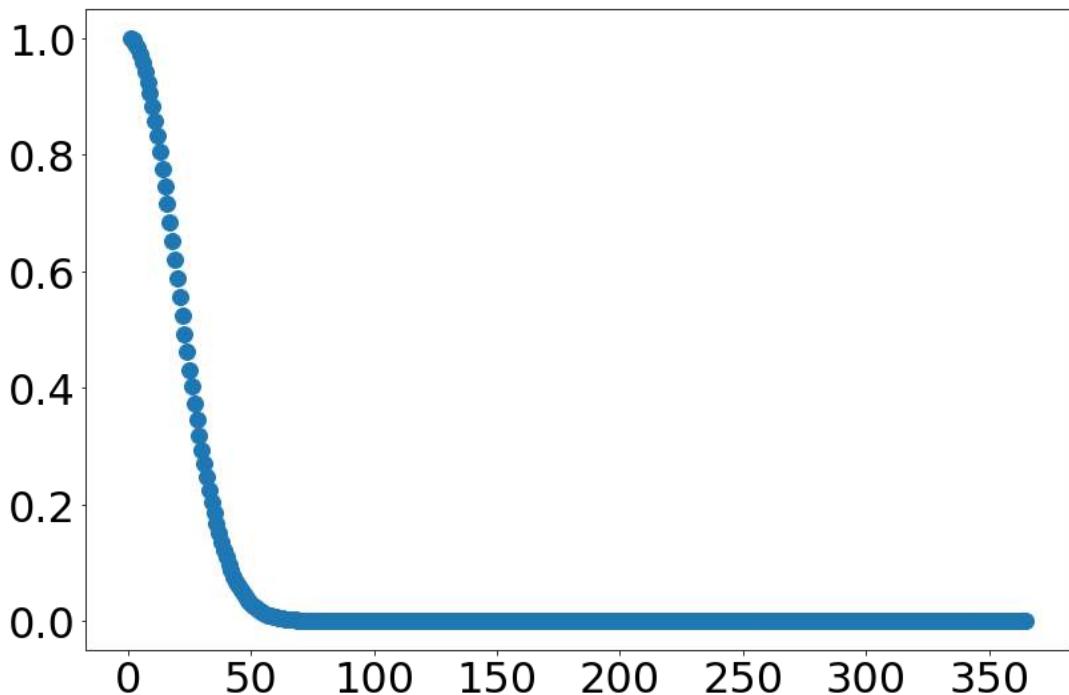
23 people: 0.493

30 people: 0.294



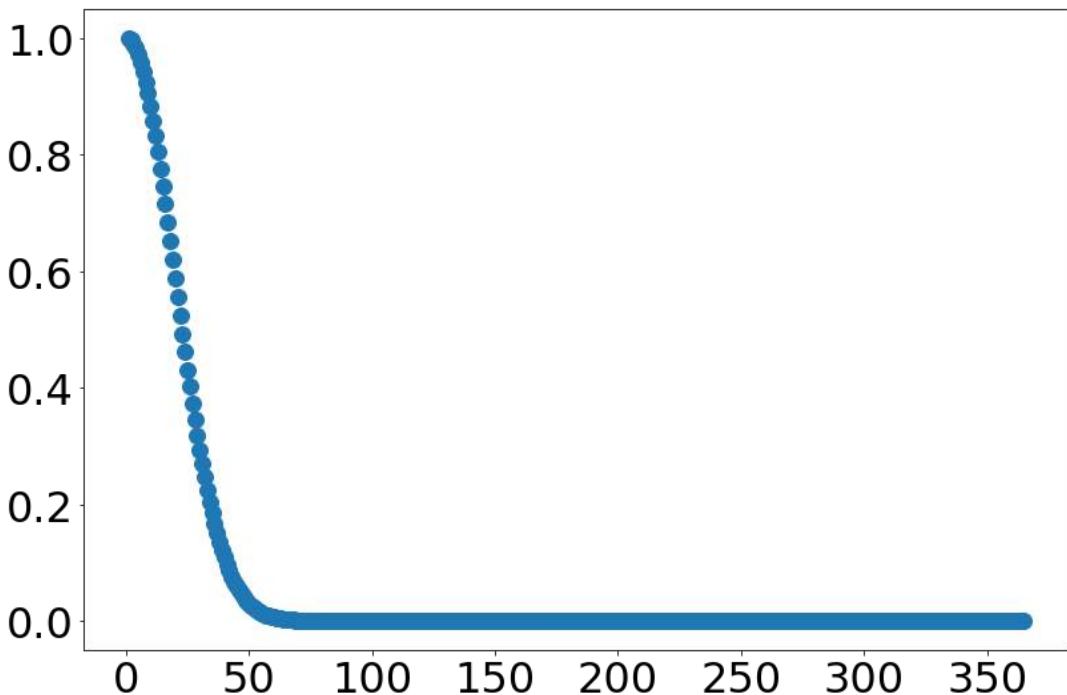
Probability That no Two People Have the Same Birthday

1 person: 1
2 people: 0.997
3 people: 0.992
4 people: 0.984
5 people: 0.973
10 people: 0.883
20 people: 0.589
23 people: 0.493
30 people: 0.294
50 people: 0.030



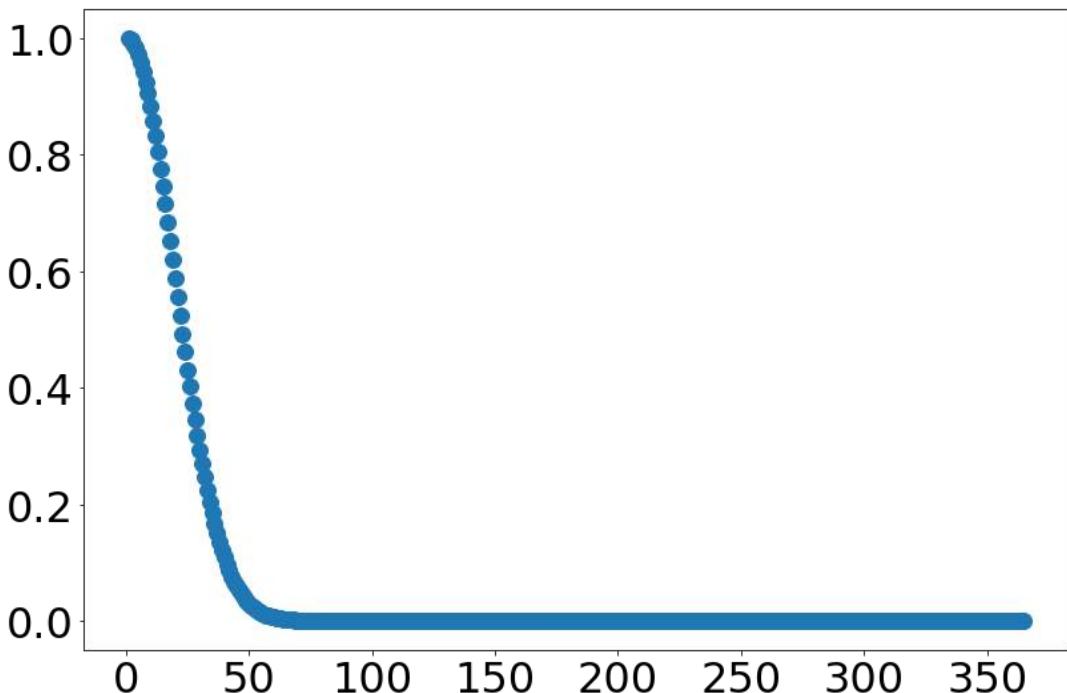
Probability That no Two People Have the Same Birthday

1 person: 1
2 people: 0.997
3 people: 0.992
4 people: 0.984
5 people: 0.973
10 people: 0.883
20 people: 0.589
23 people: 0.493
30 people: 0.294
50 people: 0.030
100 people: 0.0000003



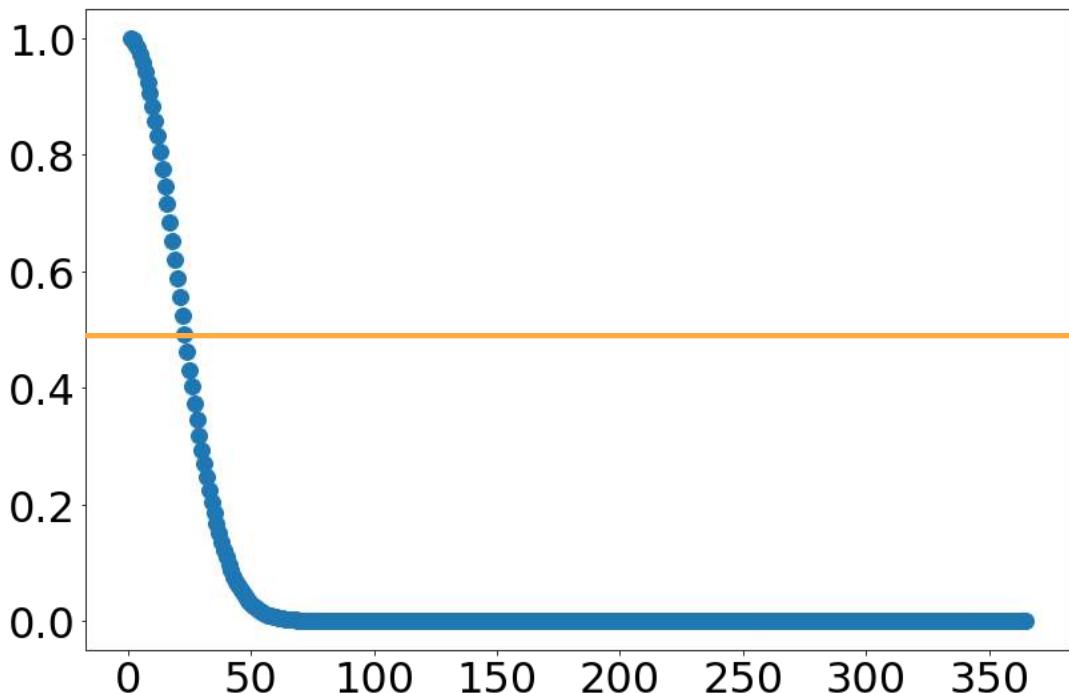
Probability That no Two People Have the Same Birthday

1 person: 1
2 people: 0.997
3 people: 0.992
4 people: 0.984
5 people: 0.973
10 people: 0.883
20 people: 0.589
23 people: 0.493
30 people: 0.294
50 people: 0.030
100 people: 0.0000003
365 people: 0



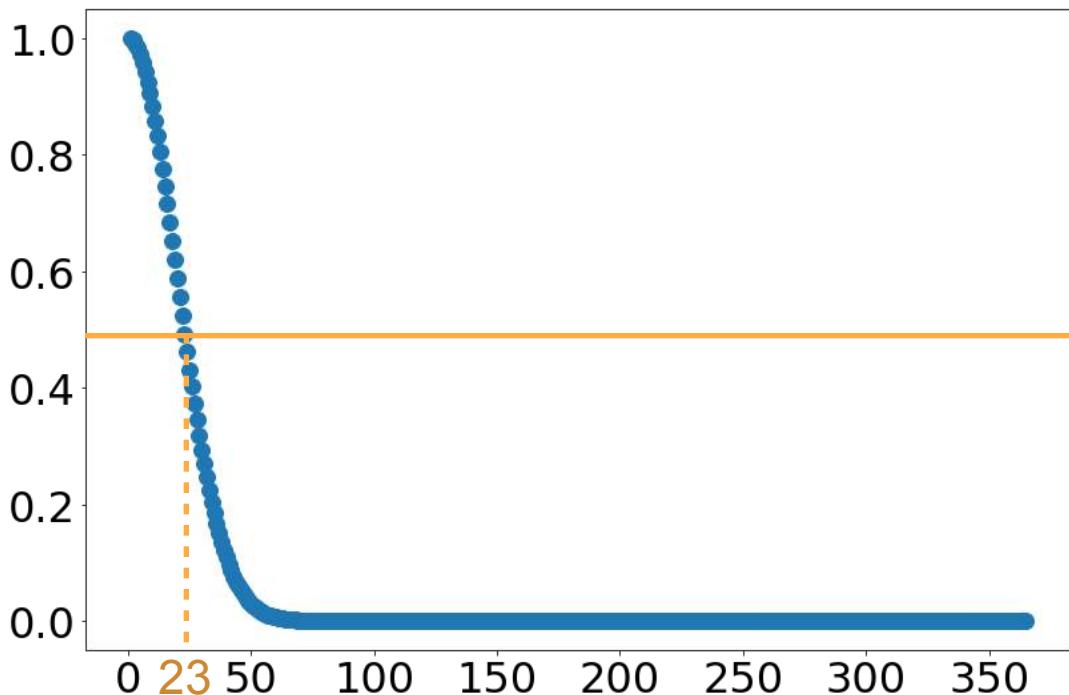
Probability That no Two People Have the Same Birthday

1 person: 1
2 people: 0.997
3 people: 0.992
4 people: 0.984
5 people: 0.973
10 people: 0.883
20 people: 0.589
23 people: 0.493
30 people: 0.294
50 people: 0.030
100 people: 0.0000003
365 people: 0



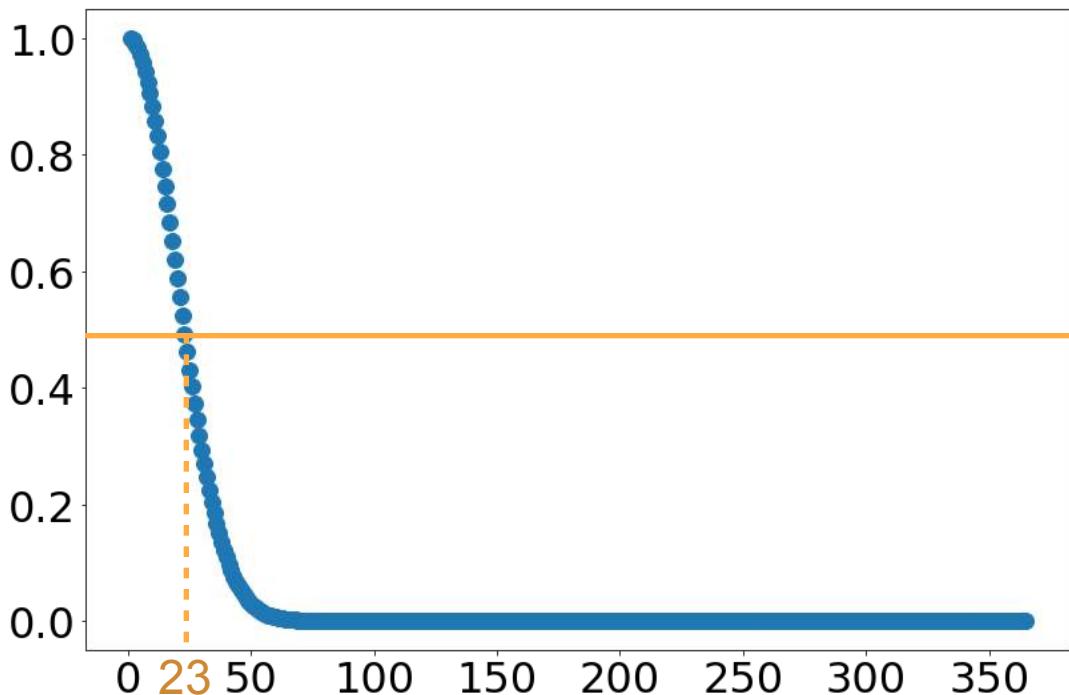
Probability That no Two People Have the Same Birthday

1 person: 1
2 people: 0.997
3 people: 0.992
4 people: 0.984
5 people: 0.973
10 people: 0.883
20 people: 0.589
23 people: 0.493
30 people: 0.294
50 people: 0.030
100 people: 0.0000003
365 people: 0



Probability That no Two People Have the Same Birthday

1 person: 1
2 people: 0.997
3 people: 0.992
4 people: 0.984
5 people: 0.973
10 people: 0.883
20 people: 0.589
23 people: 0.493
30 people: 0.294
50 people: 0.030
100 people: 0.0000003
365 people: 0





DeepLearning.AI

Introduction to probability

Conditional probability

Conditional Probability: Coin Example 1



50% 50%

Conditional Probability: Coin Example 1



50% 50%

What is the probability of landing on heads twice?

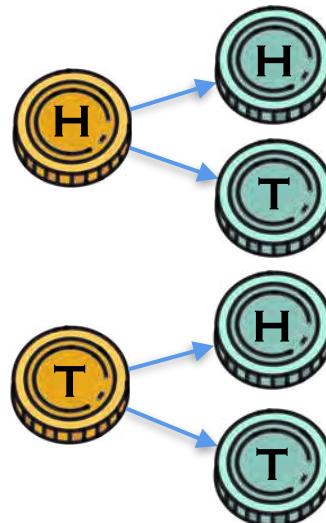
Conditional Probability: Coin Example 1



50% 50%

What is the probability of landing on heads twice?

1st 2nd



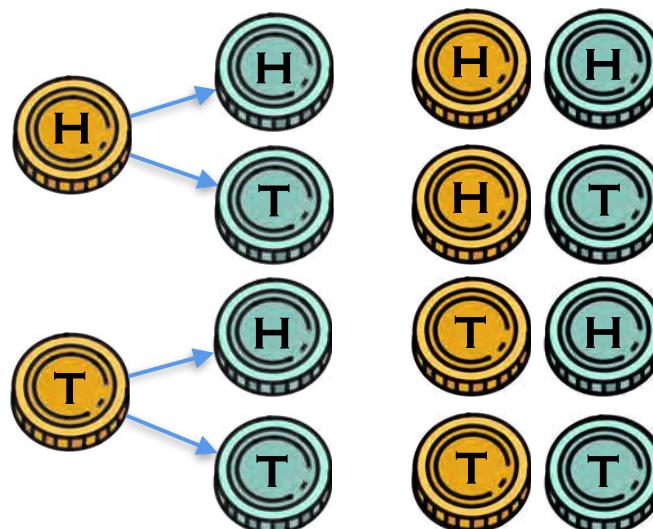
Conditional Probability: Coin Example 1



50% 50%

What is the probability of landing on heads twice?

1st 2nd



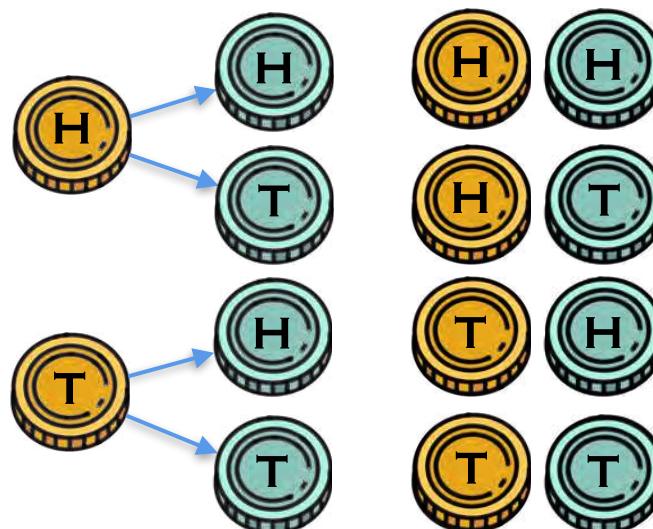
Conditional Probability: Coin Example 1



50% 50%

What is the probability of landing on heads twice?

1st 2nd



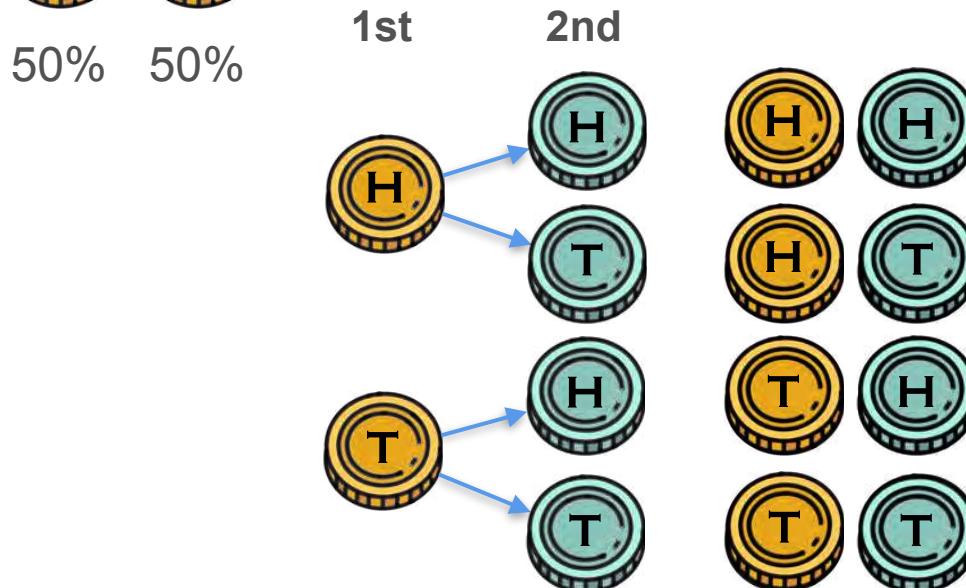
$$P(HH) = \frac{\text{Number of HH outcomes}}{\text{Total number of outcomes}}$$



Conditional Probability: Coin Example 1



What is the probability of landing on heads twice?



$$P(HH) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$

The numerator shows two coins both showing heads (H). The denominator shows all four possible outcomes: (H, H), (H, T), (T, H), and (T, T).

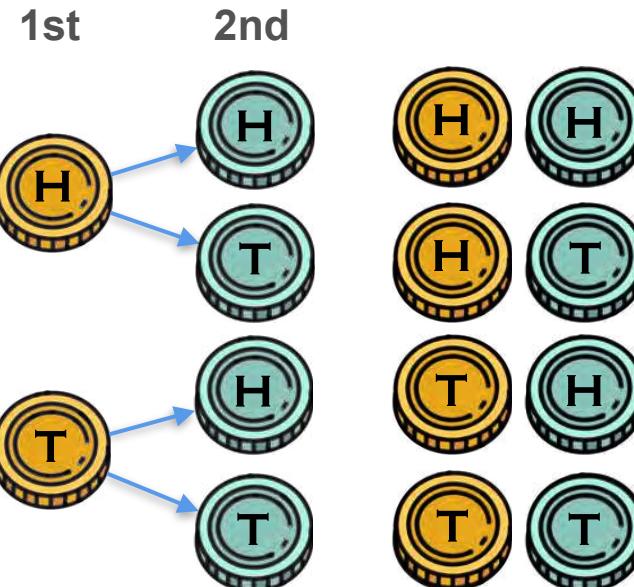
Two yellow coins, each with 'H' in blue, followed by two teal coins, each with 'H' in blue.

Conditional Probability: Coin Example 1



50% 50%

What is the probability of landing on heads twice?



$$P(HH) = \frac{1}{4} = \frac{1}{4}$$

A vertical stack of four pairs of coins, each pair showing heads (H) and tails (T). The pairs are: (H, H), (H, T), (T, H), and (T, T). This represents the sample space of all possible outcomes of two coin flips.

Conditional Probability: Coin Example 1

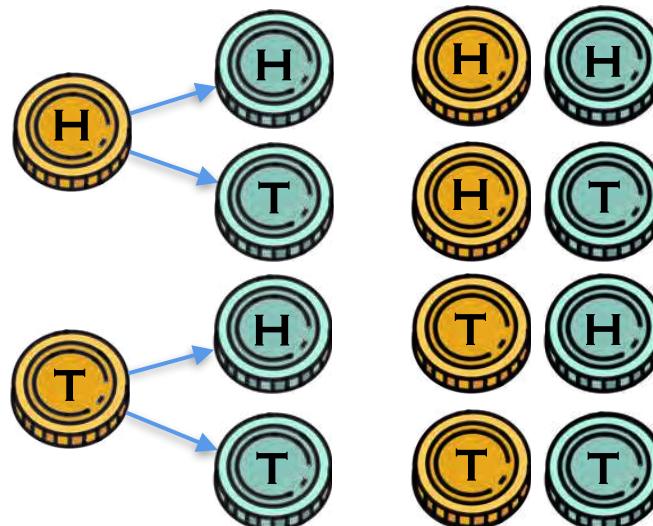


50% 50%

What is the probability of landing on heads twice?

1st 2nd

GIVEN that the first one is heads



$$P(HH) = \frac{1}{4} = \frac{1}{4}$$



Conditional Probability: Coin Example 1

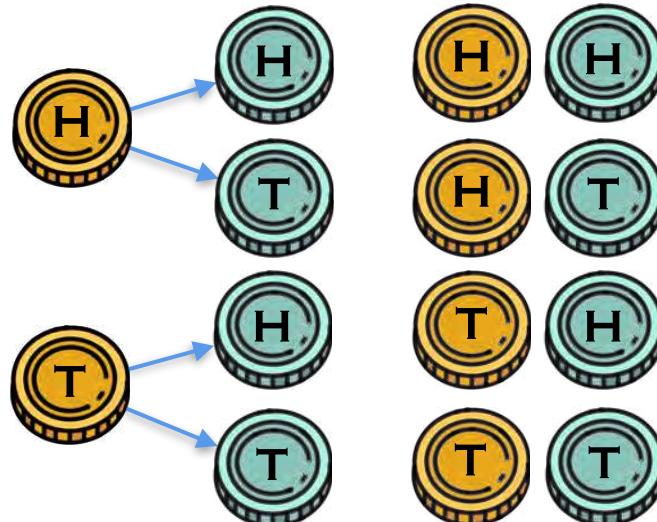


50% 50%

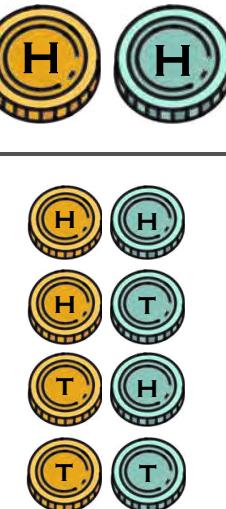
What is the probability of landing on heads twice?

1st 2nd

GIVEN that the first one is heads



$$P(HH) = \frac{1}{4} = \frac{1}{4}$$



Conditional Probability: Coin Example 1

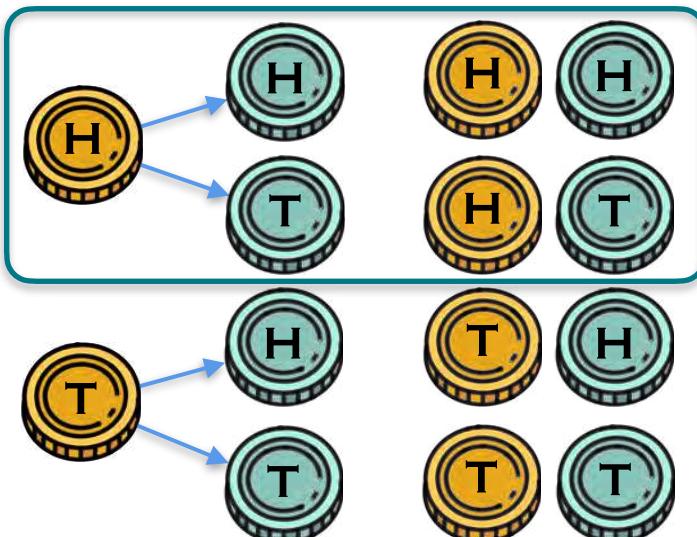


50% 50%

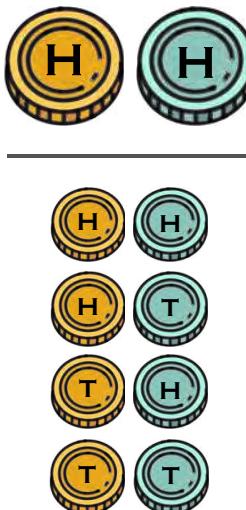
What is the probability of landing on heads twice?

1st 2nd

GIVEN that the first one is heads



$$P(HH) = \frac{1}{4} = \frac{1}{4}$$



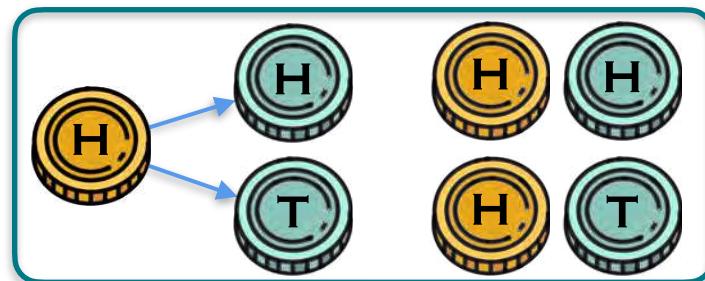
Conditional Probability: Coin Example 1



What is the probability of landing on heads twice?

1st 2nd

GIVEN that the first one is heads



$$P(HH) = \frac{1}{4} = \frac{1}{4}$$



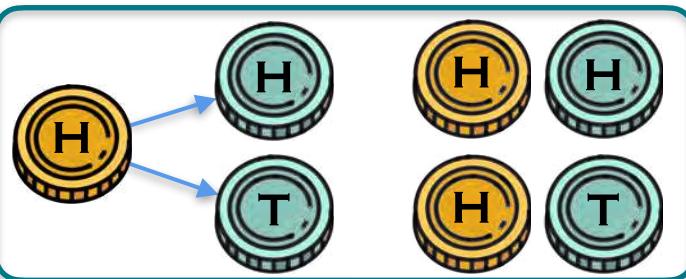
Conditional Probability: Coin Example 1



What is the probability of landing on heads twice?

1st 2nd

GIVEN that the first one is heads



$$P(HH) = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcomes}} = \frac{1}{4}$$
A diagram illustrating the calculation of the probability P(HH). It shows two rows of four coins each. The top row represents the outcome of two coin flips: the first coin is heads (H) and the second is heads (H). The bottom row represents all possible outcomes for the second flip, given that the first flip was heads (H). The bottom row shows four outcomes: H, H; H, T; H, H; and T, T. This illustrates that there is only one favorable outcome (HH) out of four possible outcomes (HH, HT, HH, TT).

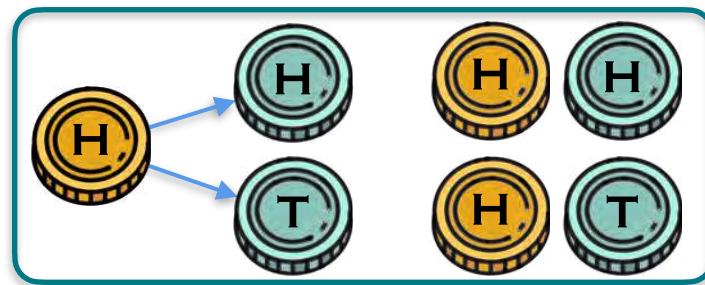
Conditional Probability: Coin Example 1



What is the probability of landing on heads twice?

1st 2nd

GIVEN that the first one is heads



$$P(HH \mid \text{1st is } H) = \frac{\text{Number of HH outcomes}}{\text{Total number of outcomes}} = \frac{1}{2}$$



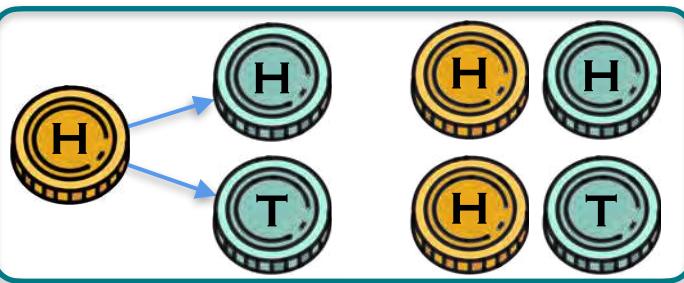
Conditional Probability: Coin Example 1



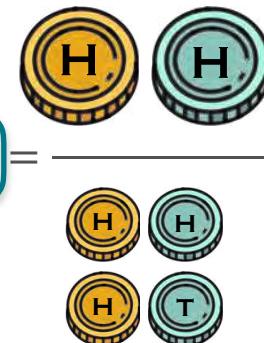
What is the probability of landing on heads twice?

1st 2nd

GIVEN that the first one is heads



$$P(HH | \text{1st is } H) = \frac{\text{Number of HH outcomes}}{\text{Total number of outcomes}} = \frac{1}{2}$$



Conditional Probability: Coin Example 1

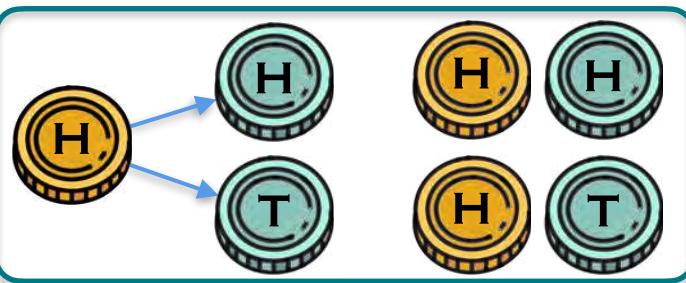


50% 50%

What is the probability of landing on heads twice?

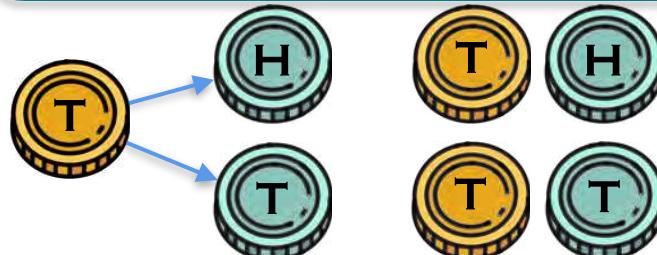
1st 2nd

GIVEN that the first one is heads



$$P(HH \mid 1\text{st is } H) =$$

$$= \frac{1}{2}$$



Conditional Probability: Coin Example 1

What is the probability of landing on heads twice?

GIVEN that the first one is heads

$$P(HH | \text{1st is } H)$$

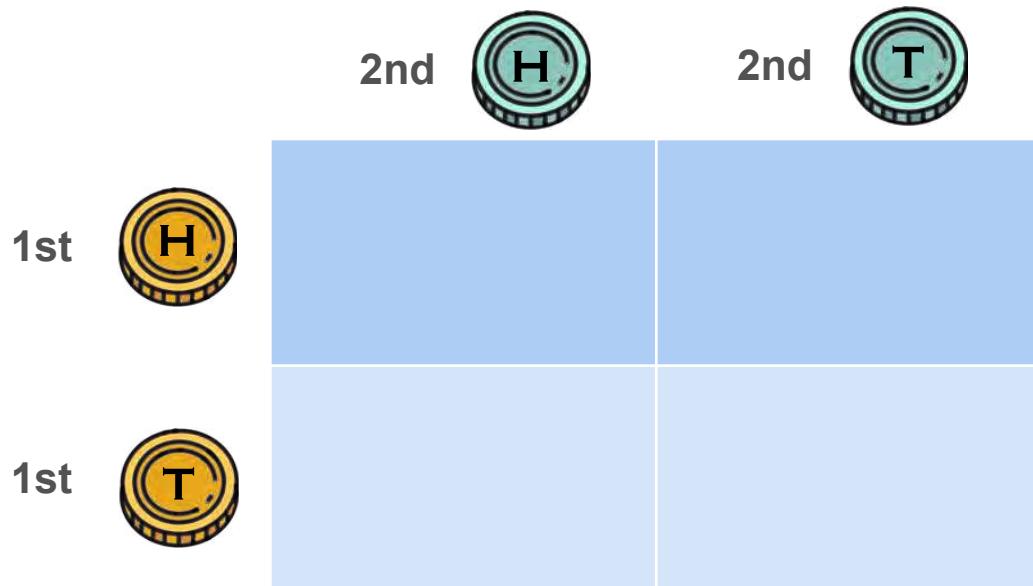


Conditional Probability: Coin Example 1

Conditional Probability: Coin Example 1



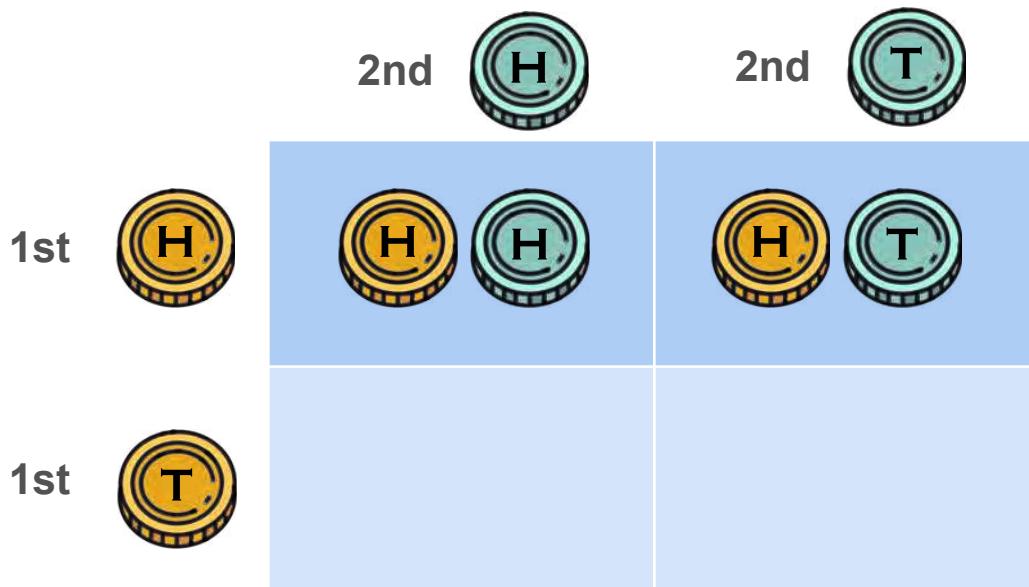
Conditional Probability: Coin Example 1



Conditional Probability: Coin Example 1



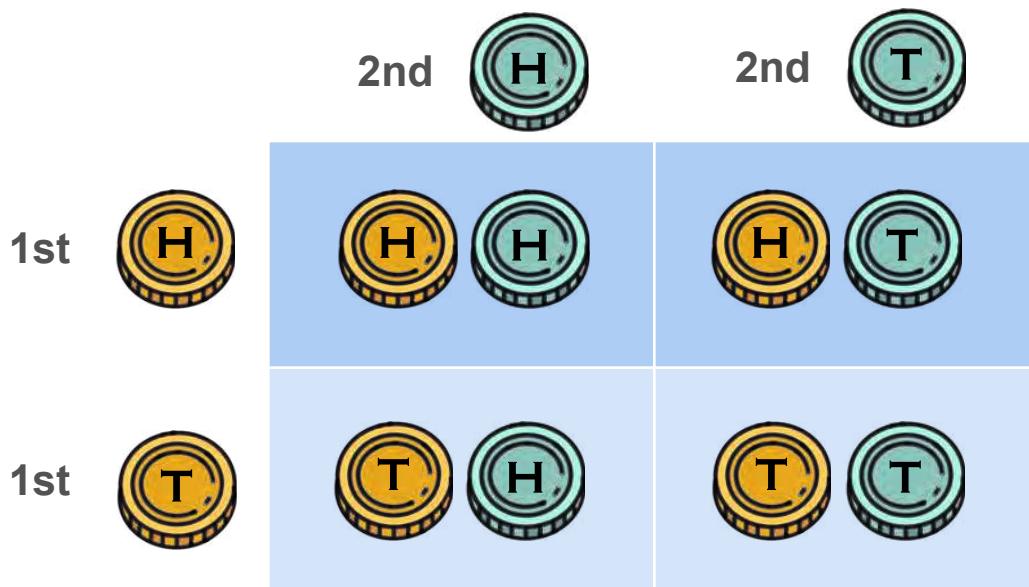
Conditional Probability: Coin Example 1



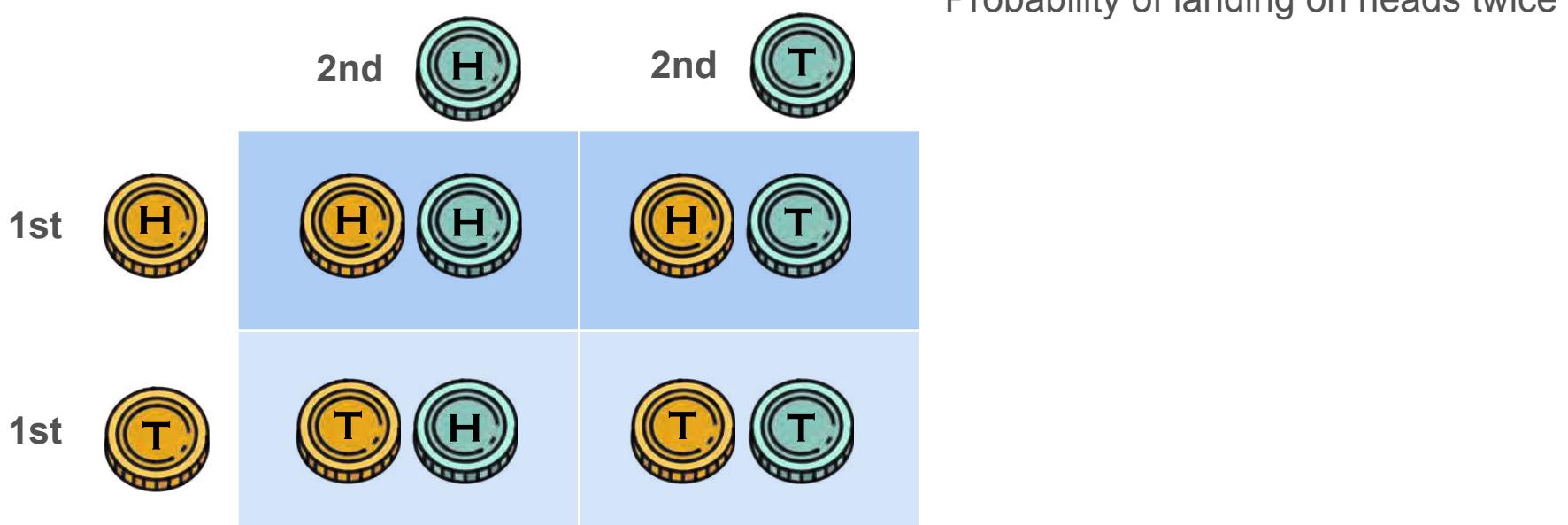
Conditional Probability: Coin Example 1



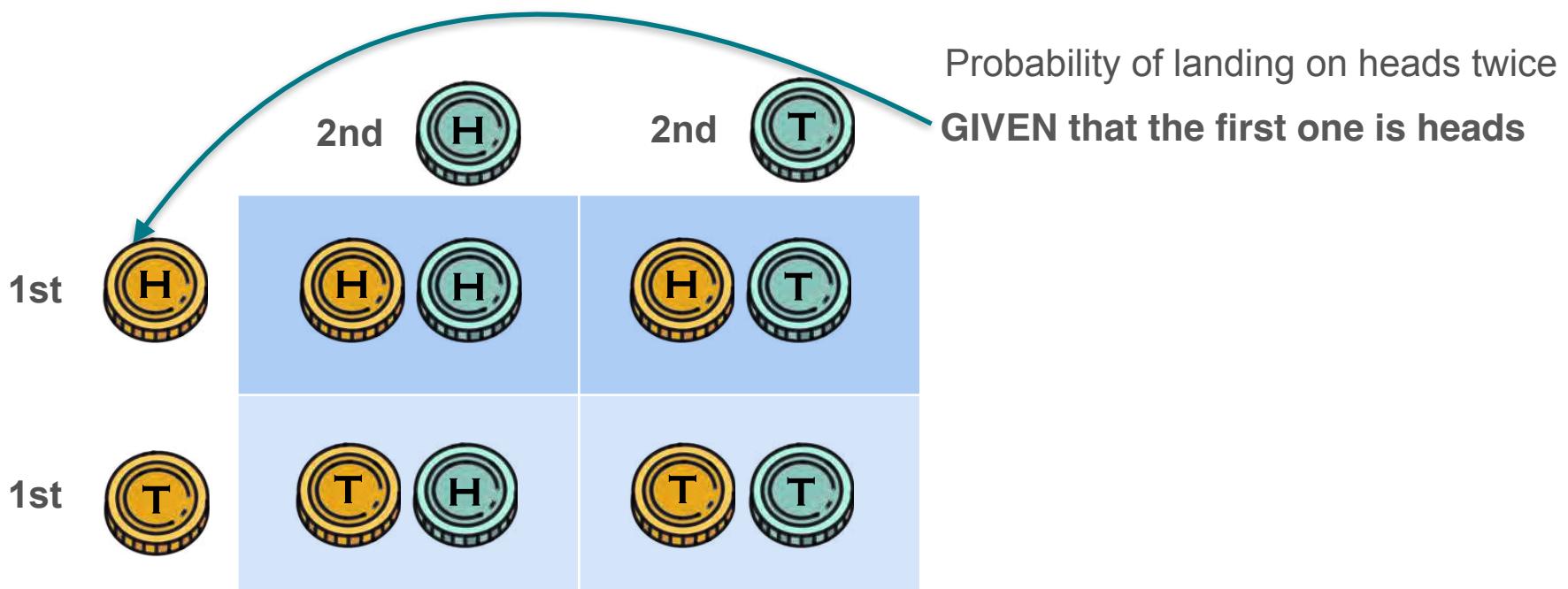
Conditional Probability: Coin Example 1



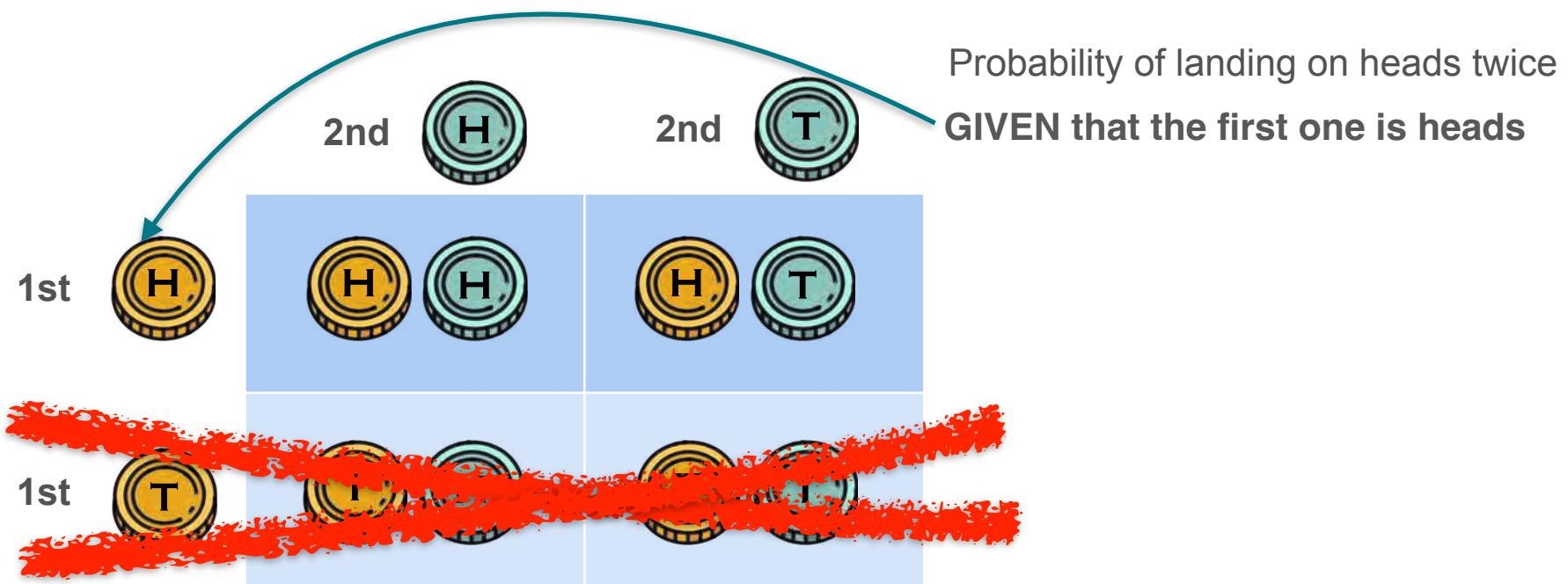
Conditional Probability: Coin Example 1



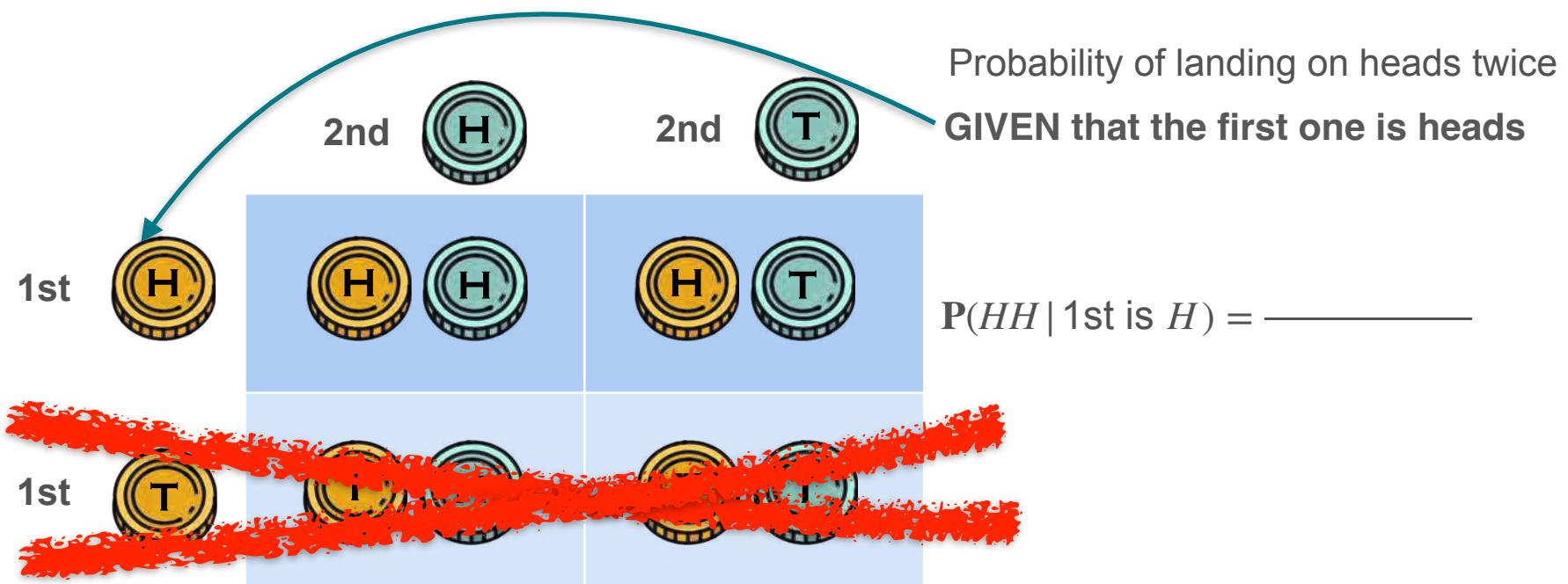
Conditional Probability: Coin Example 1



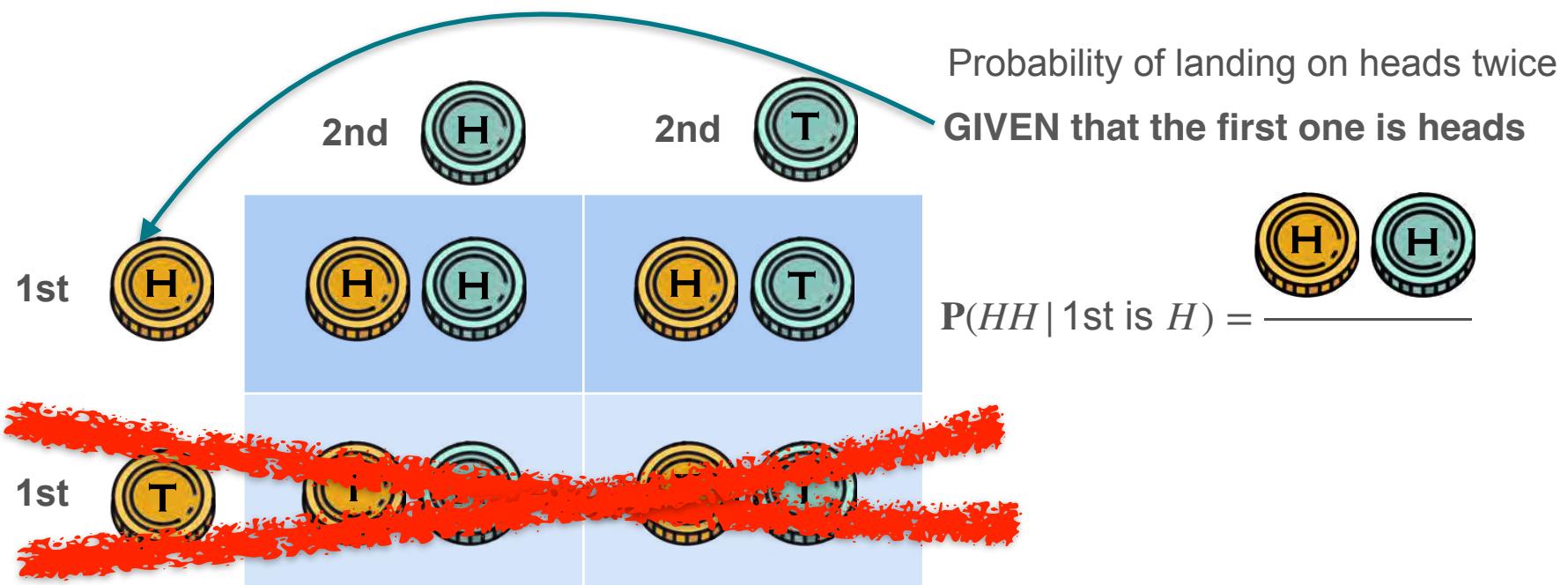
Conditional Probability: Coin Example 1



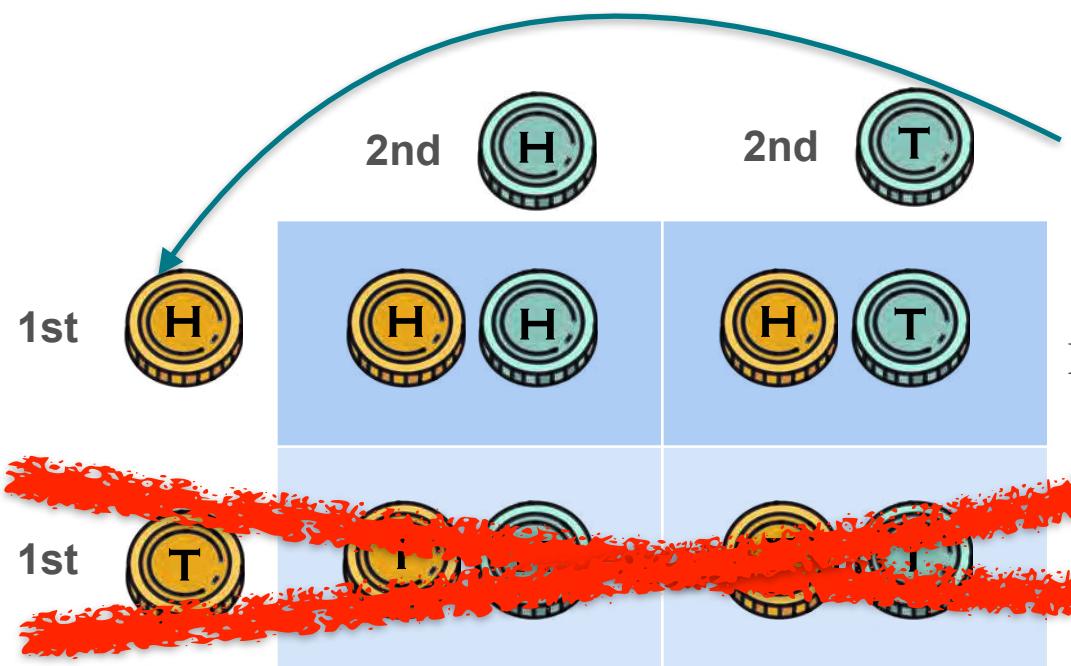
Conditional Probability: Coin Example 1



Conditional Probability: Coin Example 1



Conditional Probability: Coin Example 1



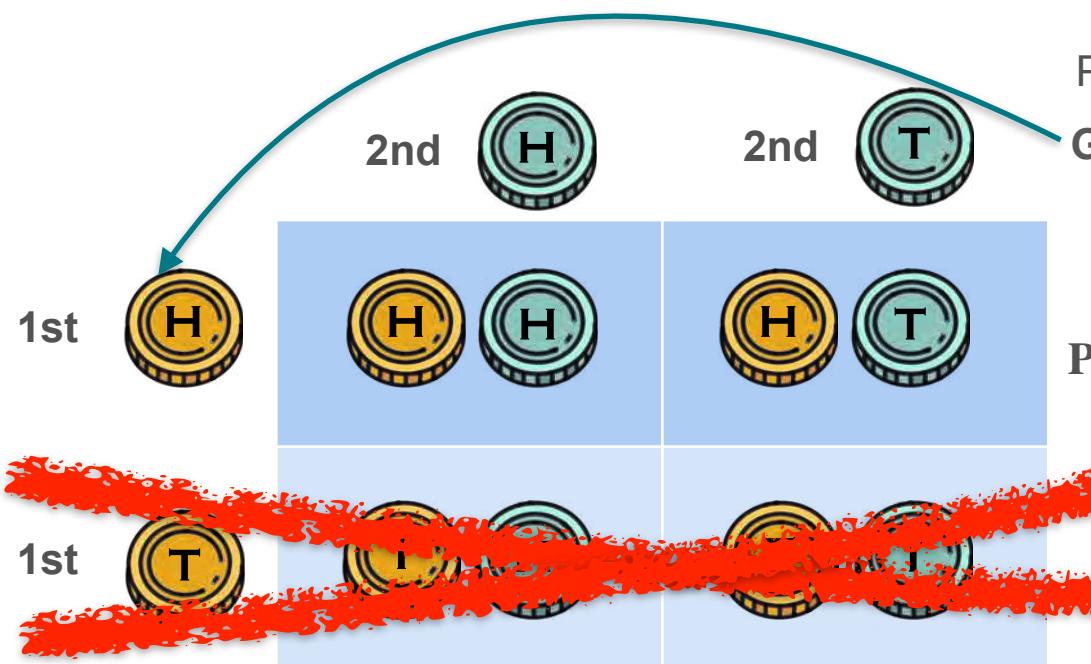
Probability of landing on heads twice
GIVEN that the first one is heads

$$P(HH \mid \text{1st is } H) = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcomes}}$$

Number of favorable outcomes: 1 (HH)

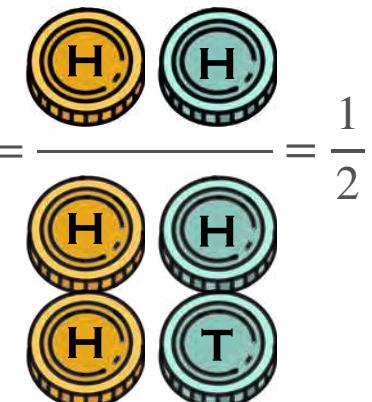
Total number of possible outcomes: 2 (HH, HT)

Conditional Probability: Coin Example 1



Probability of landing on heads twice
GIVEN that the first one is heads

$$P(HH \mid \text{1st is } H) = \frac{1}{2}$$



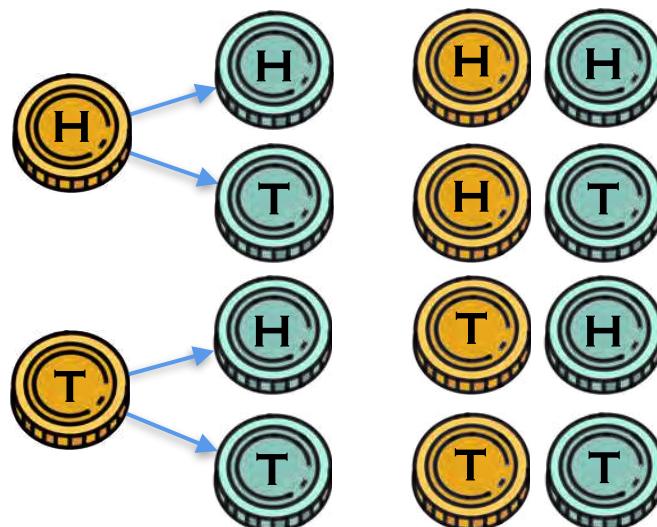
Conditional Probability: Coin Example 2



50% 50%

What is the probability of landing on heads twice?

1st 2nd



$$P(HH) = \frac{1}{4}$$

The equation shows the probability of getting heads on both the first and second coin flips as 1/4. To the right of the equation is a diagram of the four possible outcomes: HH, HT, TH, and TT. HH is highlighted in yellow and light blue, while the other three outcomes are shown in grey.

Conditional Probability: Coin Example 2

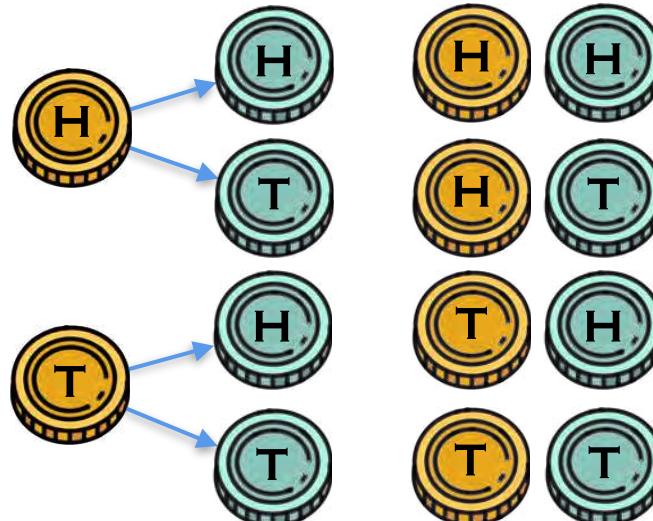


50% 50%

What is the probability of landing on heads twice?

1st 2nd

GIVEN that the first one is tails



$$P(HH) = \frac{1}{4} = \frac{1}{4}$$



Conditional Probability: Coin Example 2

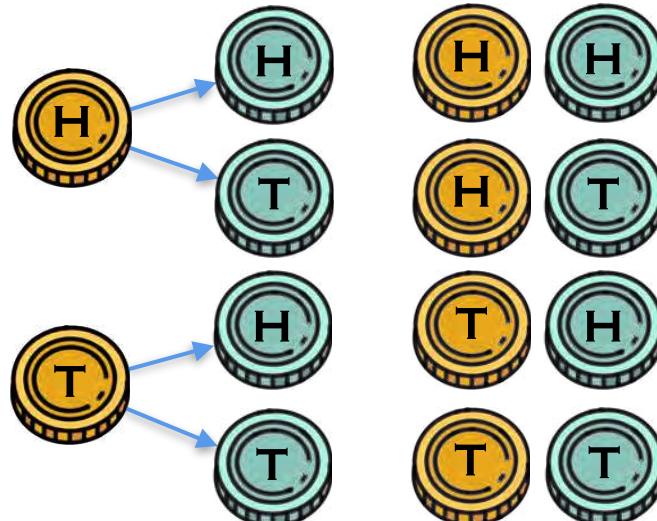


50% 50%

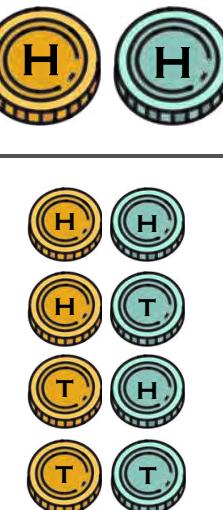
What is the probability of landing on heads twice?

1st 2nd

GIVEN that the first one is tails



$$P(HH) = \frac{1}{4} = \frac{1}{4}$$



Conditional Probability: Coin Example 2

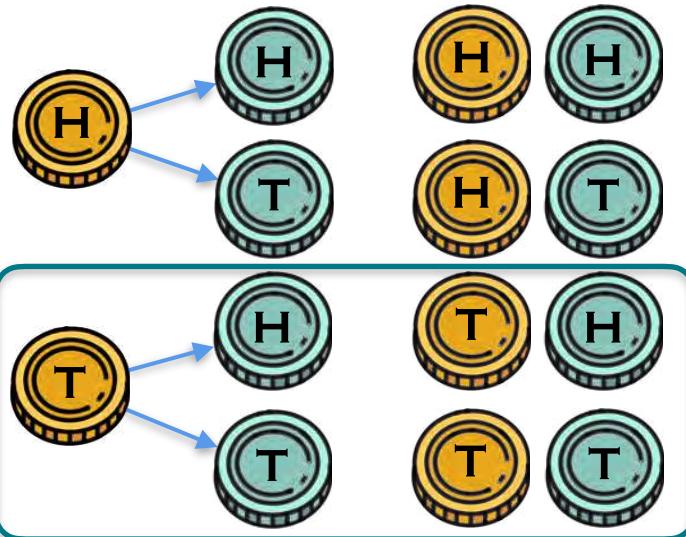


50% 50%

What is the probability of landing on heads twice?

1st 2nd

GIVEN that the first one is tails



$$P(HH) = \frac{1}{4} = \frac{1}{4}$$



Conditional Probability: Coin Example 2

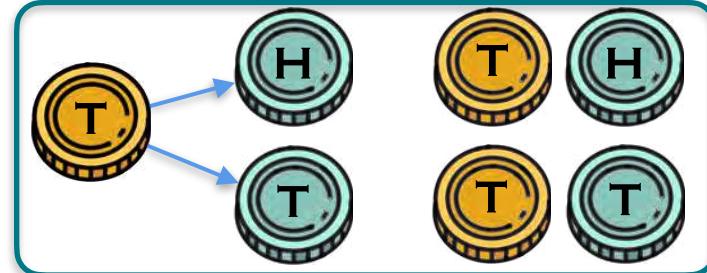


50% 50%

What is the probability of landing on heads twice?

1st 2nd

GIVEN that the first one is tails



$$P(HH) = \frac{1}{4} = \frac{1}{4}$$



Conditional Probability: Coin Example 2



50% 50%

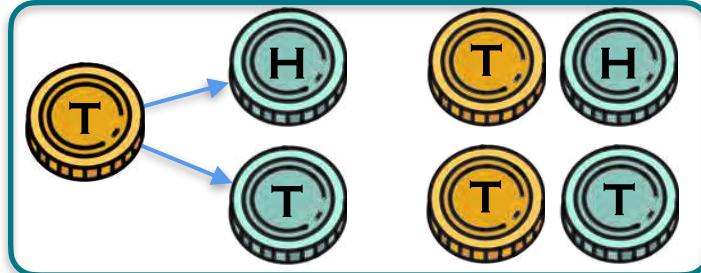
What is the probability of landing on heads twice?

1st 2nd

GIVEN that the first one is tails



$$P(HH) = \frac{1}{4} = \frac{1}{4}$$



Conditional Probability: Coin Example 2



50% 50%

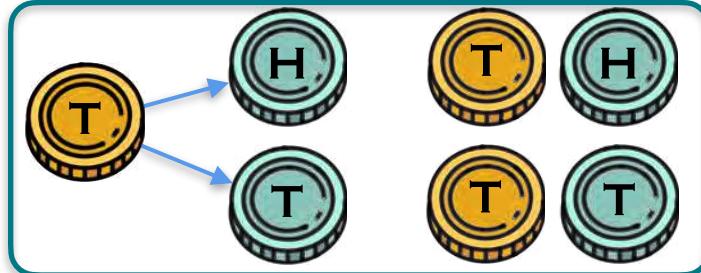
What is the probability of landing on heads twice?

1st 2nd

GIVEN that the first one is tails



$$P(HH) = \text{_____} = \frac{1}{4}$$



Conditional Probability: Coin Example 2

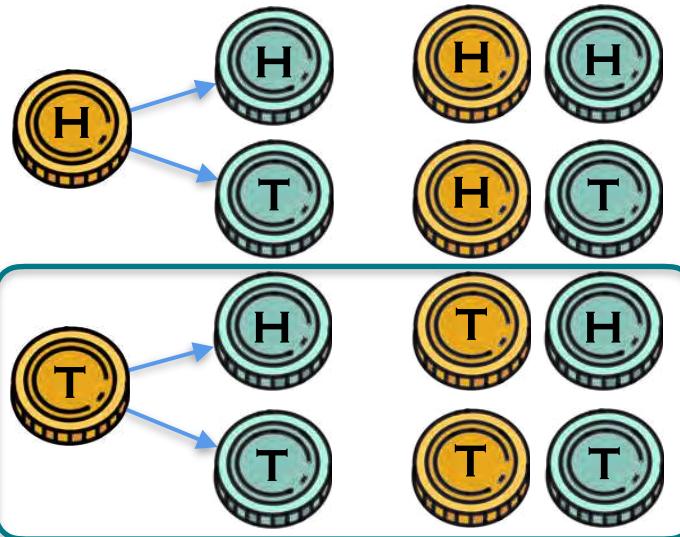


50% 50%

What is the probability of landing on heads twice?

1st 2nd

GIVEN that the first one is tails



$$P(HH \mid \text{1st is } T) = \frac{0}{4} = 0$$

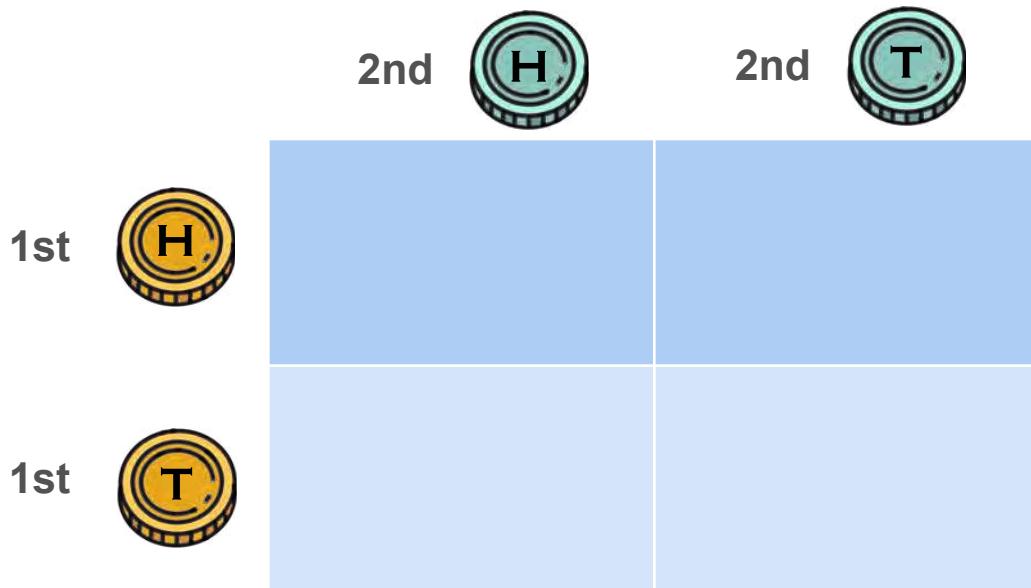


Conditional Probability: Coin Example 2

Conditional Probability: Coin Example 2



Conditional Probability: Coin Example 2



Conditional Probability: Coin Example 2



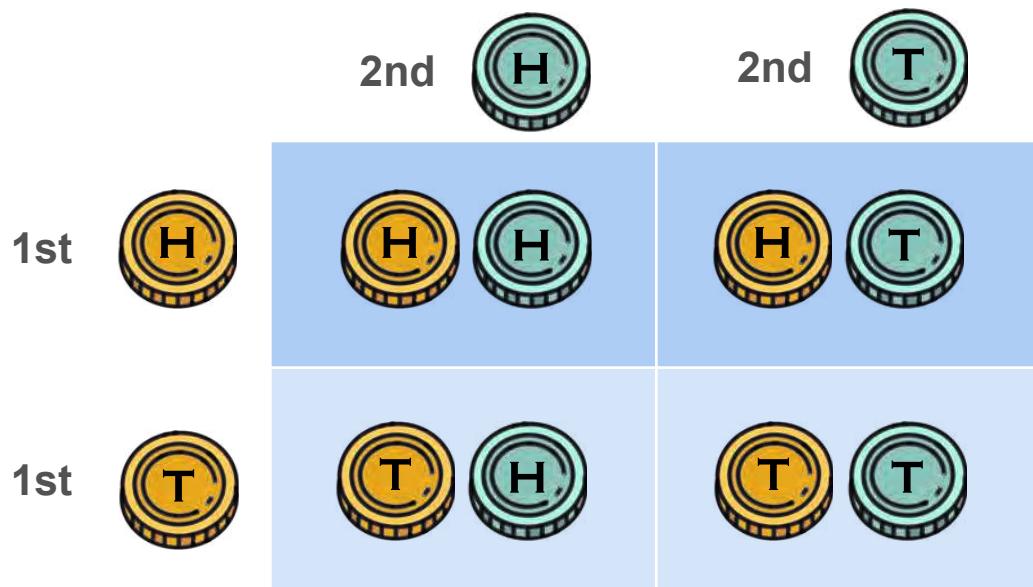
Conditional Probability: Coin Example 2



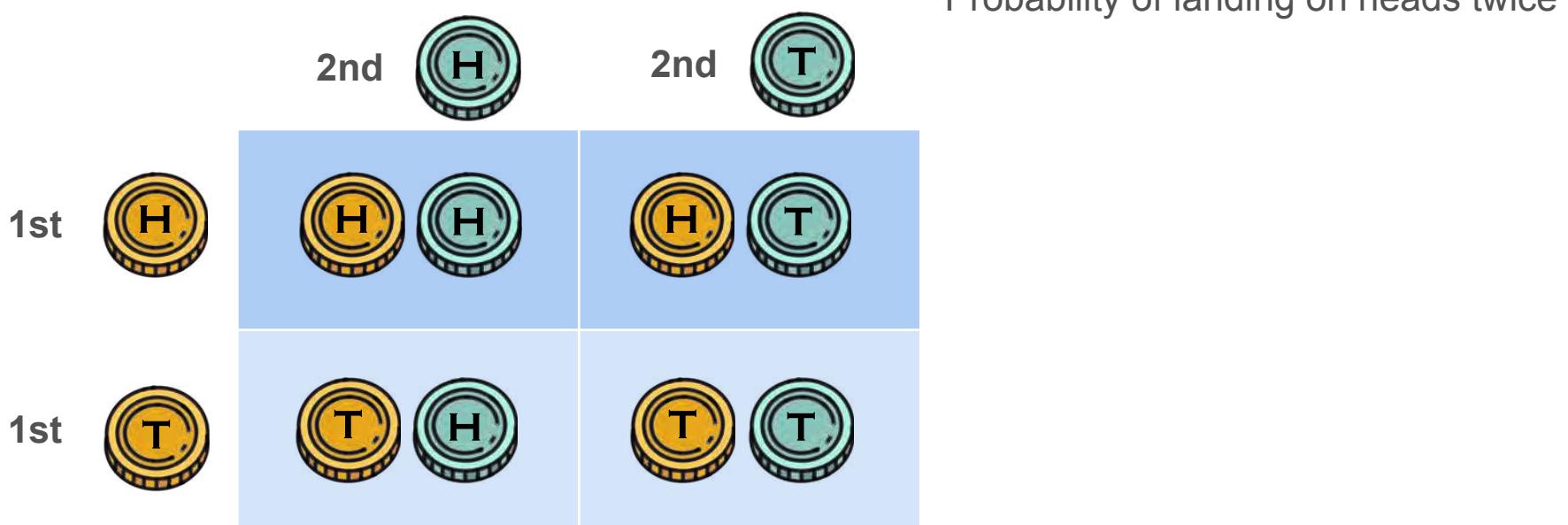
Conditional Probability: Coin Example 2



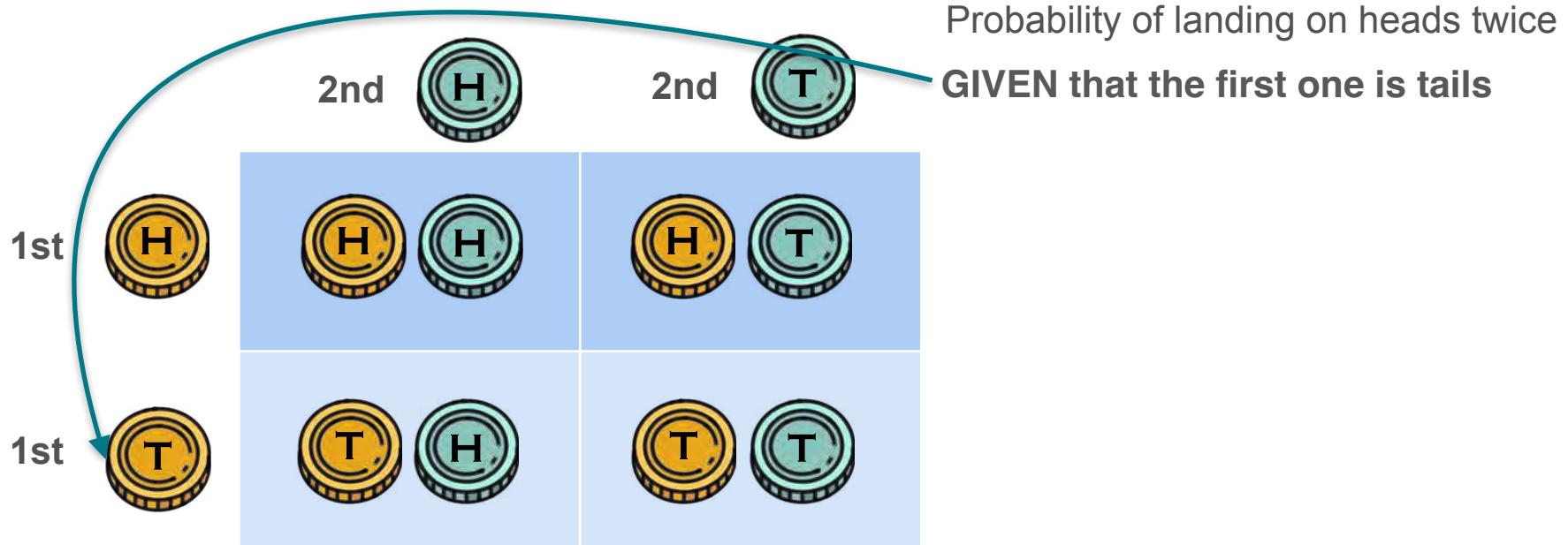
Conditional Probability: Coin Example 2



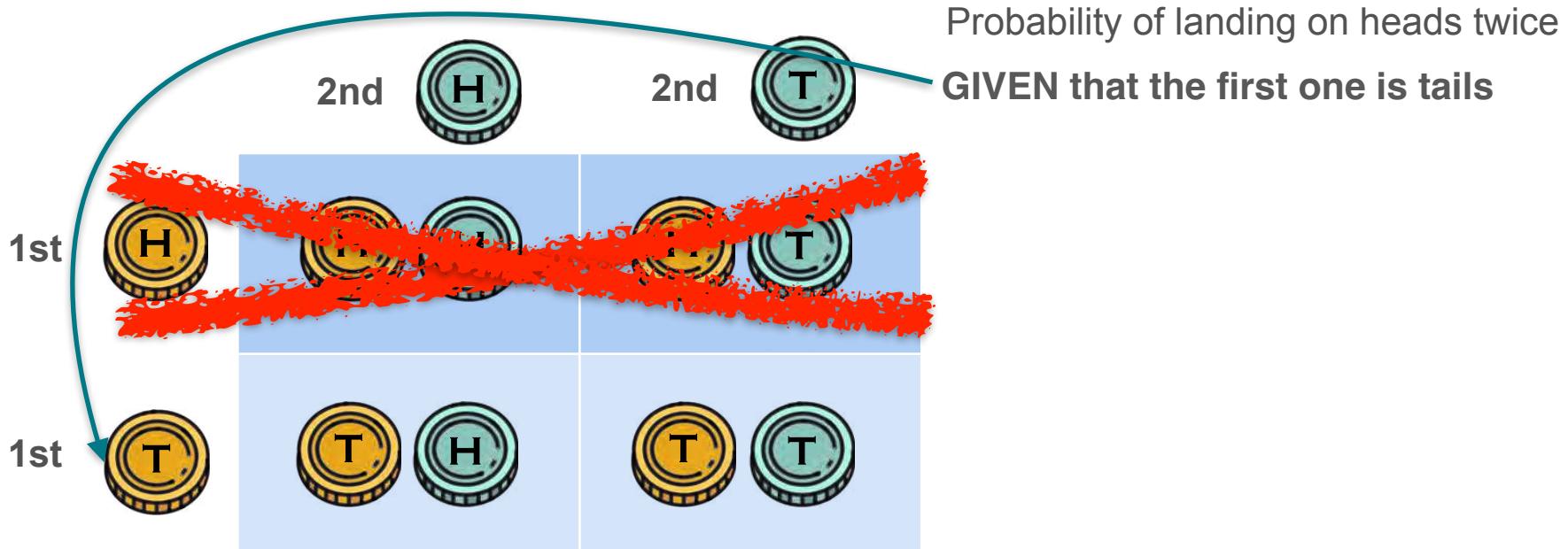
Conditional Probability: Coin Example 2



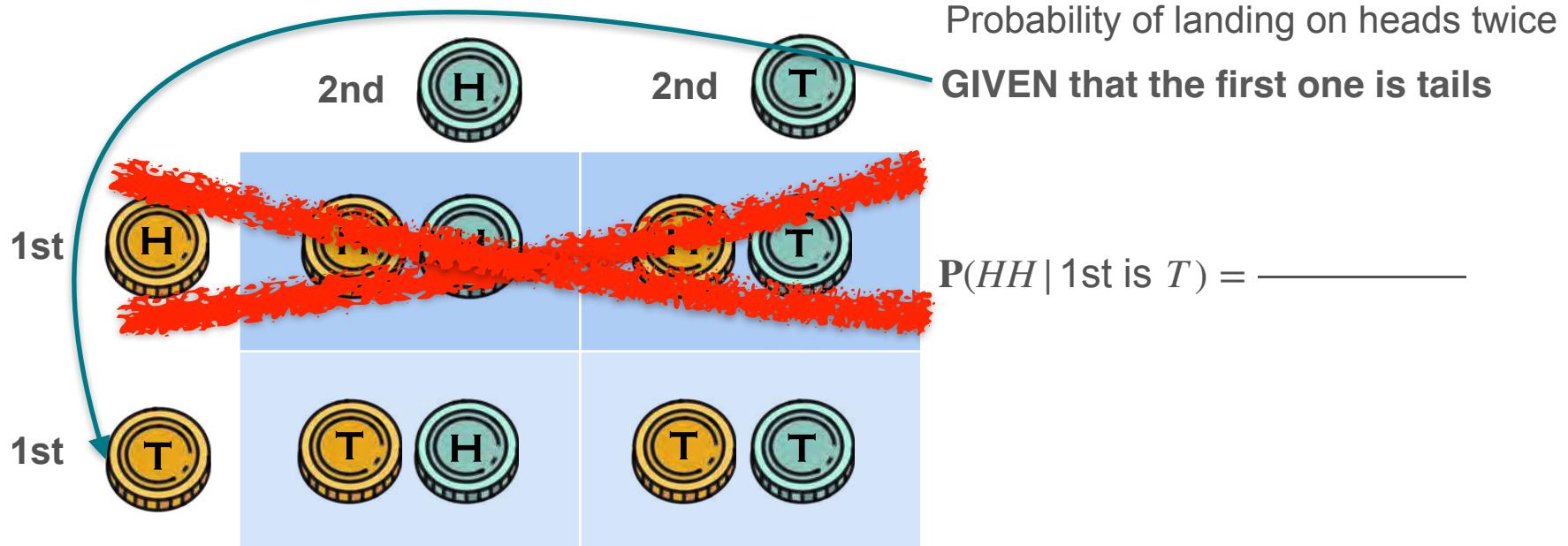
Conditional Probability: Coin Example 2



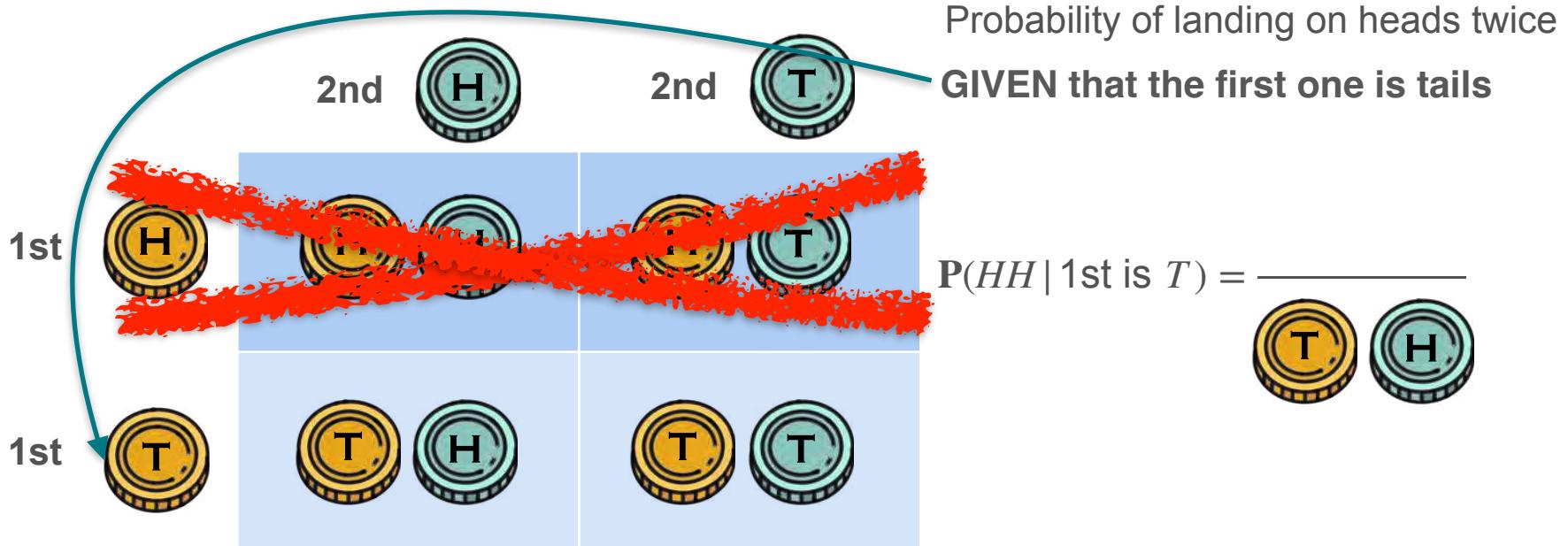
Conditional Probability: Coin Example 2



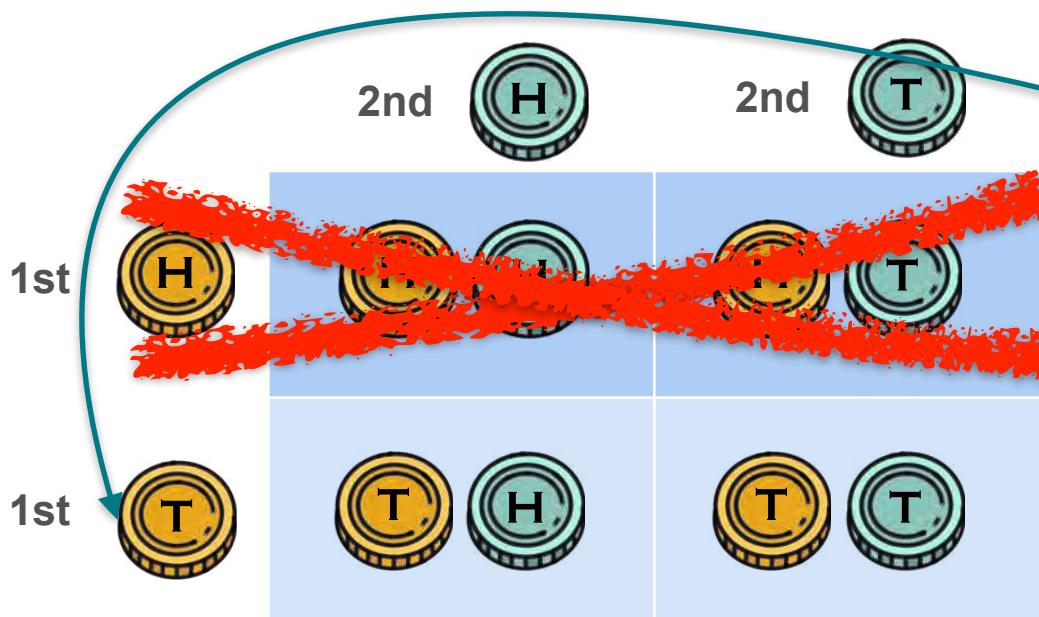
Conditional Probability: Coin Example 2



Conditional Probability: Coin Example 2

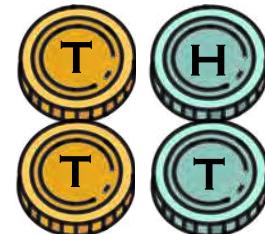


Conditional Probability: Coin Example 2

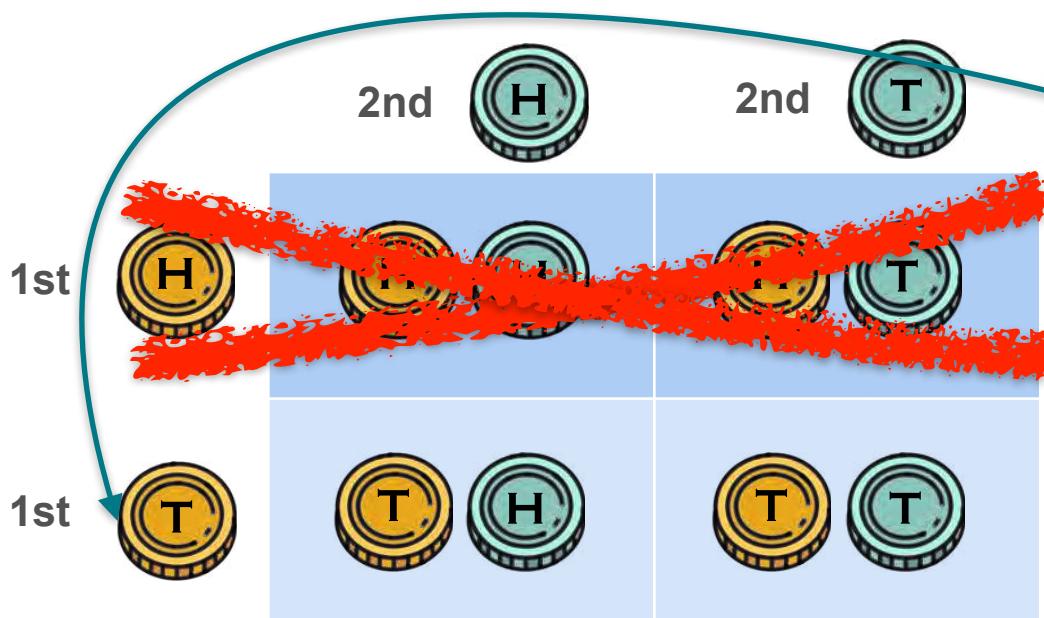


Probability of landing on heads twice
GIVEN that the first one is tails

$$P(HH | \text{1st is } T) = \frac{\text{Number of HH outcomes}}{\text{Number of T outcomes}}$$



Conditional Probability: Coin Example 2

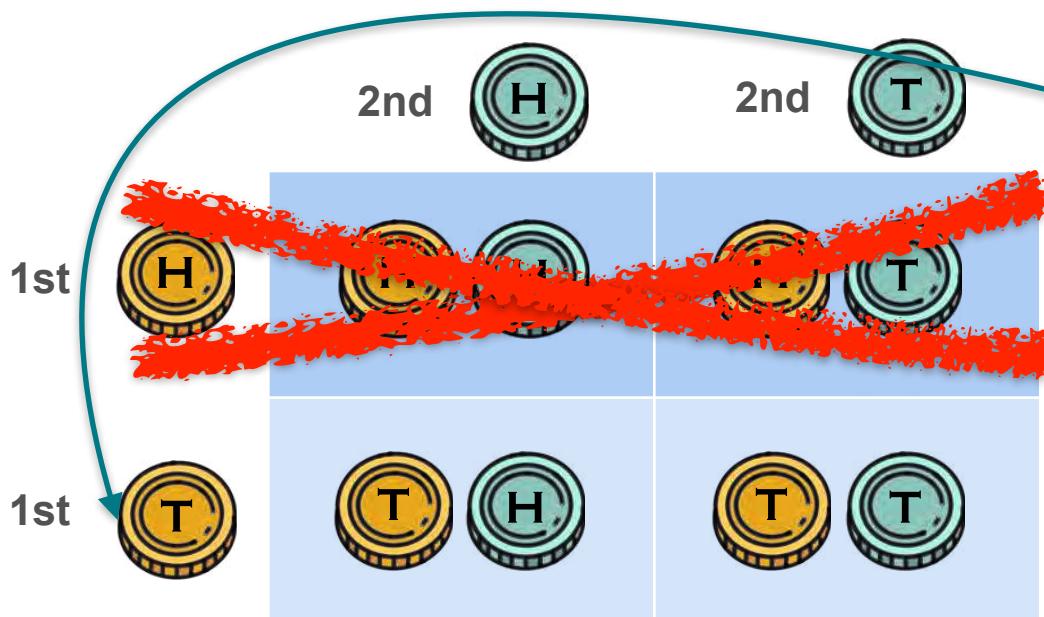


Probability of landing on heads twice
GIVEN that the first one is tails

$$P(HH | \text{1st is } T) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$



Conditional Probability: Coin Example 2

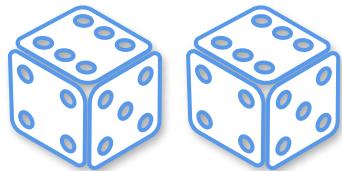


Probability of landing on heads twice
GIVEN that the first one is tails

$$P(HH | \text{1st is } T) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}} = 0$$

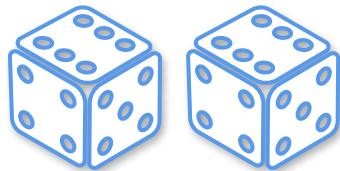


Conditional Probability: Dice Example 1



What is the probability that the sum is 10?

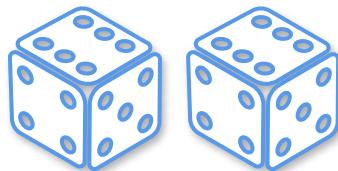
Conditional Probability: Dice Example 1



What is the probability that the sum is 10?

	1,1	1,2	1,3	1,4	1,5	1,6
1,1						
1,2						
1,3						
1,4						
1,5						
1,6						
2,1						
2,2						
2,3						
2,4						
2,5						
2,6						
3,1						
3,2						
3,3						
3,4						
3,5						
3,6						
4,1						
4,2						
4,3						
4,4						
4,5						
4,6						
5,1						
5,2						
5,3						
5,4						
5,5						
5,6						
6,1						
6,2						
6,3						
6,4						
6,5						
6,6						

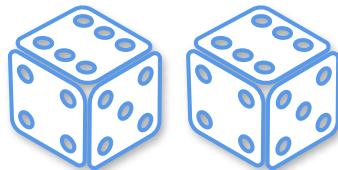
Conditional Probability: Dice Example 1



What is the probability that the sum is 10?

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6

Conditional Probability: Dice Example 1

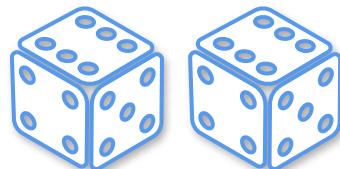


What is the probability that the sum is 10?

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{sum} = 10) = \underline{\hspace{10em}}$$

Conditional Probability: Dice Example 1



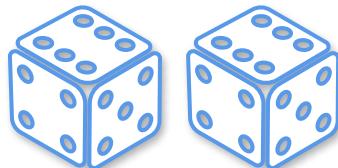
What is the probability that the sum is 10?

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{sum} = 10) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$

The diagram shows a 6x6 grid of outcomes from two dice rolls. Outcomes where the sum is 10 are highlighted with red boxes: (4,6), (5,5), (6,4), and (6,5). The outcome (5,5) is also highlighted with a red box.

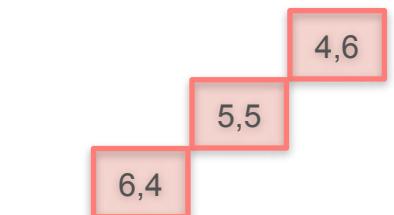
Conditional Probability: Dice Example 1



What is the probability that the sum is 10?

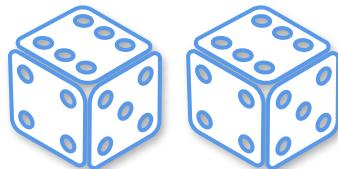
1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{sum} = 10) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$



1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

Conditional Probability: Dice Example 1



What is the probability that the sum is 10?

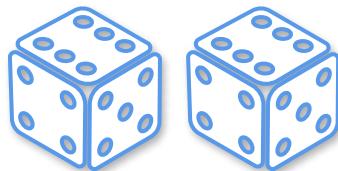
1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{sum} = 10) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}} = \frac{3}{36}$$

The diagram shows a 6x6 grid of outcomes from two dice rolls. The outcomes are represented by small dice icons above each cell. The cells are colored based on their sum: light blue for sums 2 through 9, pink for sum 10, and light green for sums 11 and 12. The three pink cells representing a sum of 10 are highlighted with red boxes. The total number of outcomes is 36.

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

Conditional Probability: Dice Example 1



What is the probability that the sum is 10?

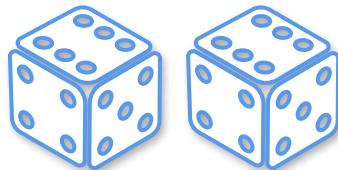
1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{sum} = 10) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}} = \frac{3}{12} = \frac{1}{4}$$

The diagram shows a 6x6 grid of outcomes from two dice rolls. The outcomes are represented by small dice icons above each cell. The cells are colored based on their sum: light blue for sums 2-5, pink for sum 6, red for sum 7, and light green for sums 8-12. The three outcomes where the sum is 10 (4,6), (5,5), and (6,4) are highlighted with red boxes.

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

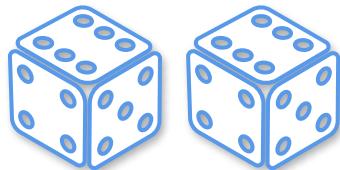
Conditional Probability: Dice Example 1



What is the probability that the sum is 10?

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6

Conditional Probability: Dice Example 1

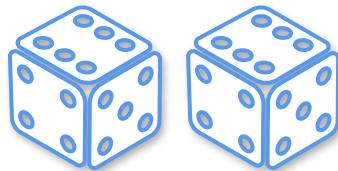


What is the probability that the sum is 10?

GIVEN that the first one is 6

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

Conditional Probability: Dice Example 1



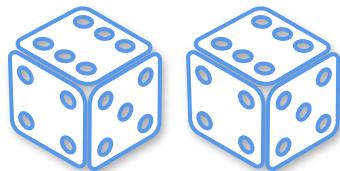
What is the probability that the sum is 10?

GIVEN that the first one is 6

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{sum} = 10 \mid \text{1st is } 6) = \underline{\hspace{10cm}}$$

Conditional Probability: Dice Example 1



What is the probability that the sum is 10?

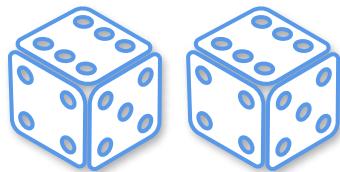
GIVEN that the first one is 6

A 6x6 grid representing all possible outcomes of two dice rolls. The columns are labeled with the outcome of the first die (1, 2, 3, 4, 5, 6) and the rows are labeled with the outcome of the second die (1, 2, 3, 4, 5, 6). The cell at the intersection of the 6th column and 4th row is highlighted with a red border. The entire grid is enclosed in a red border.

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{sum} = 10 \mid \text{1st is } 6) = \underline{\hspace{10cm}}$$

Conditional Probability: Dice Example 1



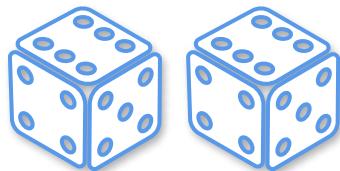
What is the probability that the sum is 10?

GIVEN that the first one is 6

	1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6	
3,1	3,2	3,3	3,4	3,5	3,6	
4,1	4,2	4,3	4,4	4,5	4,6	
5,1	5,2	5,3	5,4	5,5	5,6	
6,1	6,2	6,3	6,4	6,5	6,6	

$$P(\text{sum} = 10 \mid \text{1st is } 6) = \underline{\hspace{10cm}}$$

Conditional Probability: Dice Example 1



What is the probability that the sum is 10?

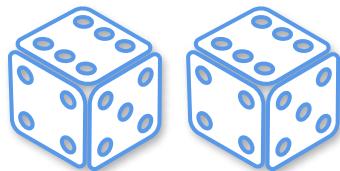
GIVEN that the first one is 6

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6
6,1	6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{sum} = 10 \mid \text{1st is } 6) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$

6,1	6,2	6,3	6,4	6,5	6,6
-----	-----	-----	-----	-----	-----

Conditional Probability: Dice Example 1



What is the probability that the sum is 10?

GIVEN that the first one is 6

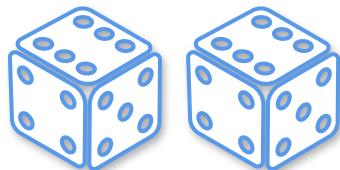
	1,1	1,2	1,3	1,4	1,5	1,6
	2,1	2,2	2,3	2,4	2,5	2,6
	3,1	3,2	3,3	3,4	3,5	3,6
	4,1	4,2	4,3	4,4	4,5	4,6
	5,1	5,2	5,3	5,4	5,5	5,6
	6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{sum} = 10 \mid \text{1st is } 6) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$

6,4

6,1	6,2	6,3	6,4	6,5	6,6
-----	-----	-----	-----	-----	-----

Conditional Probability: Dice Example 1



What is the probability that the sum is 10?

GIVEN that the first one is 6

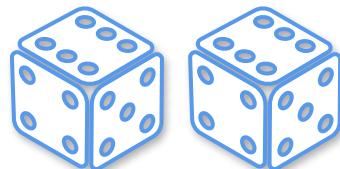
	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6
6,1	6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{sum} = 10 \mid \text{1st is } 6) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$

6,4 6,1 6,2 6,3 6,4 6,5 6,6

$$= \frac{1}{6}$$

Conditional Probability: Dice Example 2



What is the probability that the sum is 10?

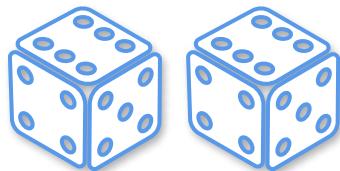
	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{sum} = 10) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}} = \frac{3}{36}$$

A diagram showing a staircase path through a 6x6 grid of dice rolls. The steps are red-bordered boxes. The path starts at (1,1), goes up-right to (2,2), up to (3,3), right to (4,4), up to (5,5), and right to (6,6). The final outcome (6,6) is also highlighted with a red border.

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

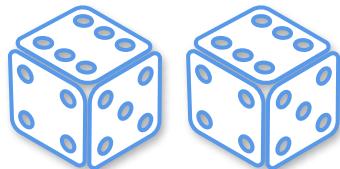
Conditional Probability: Dice Example 2



What is the probability that the sum is 10?

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6

Conditional Probability: Dice Example 2

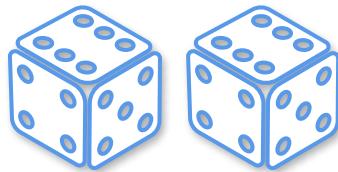


What is the probability that the sum is 10?

GIVEN that the first one is 1

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6

Conditional Probability: Dice Example 2



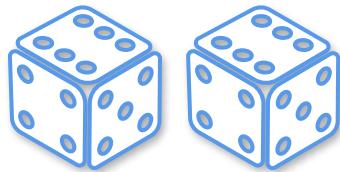
What is the probability that the sum is 10?

GIVEN that the first one is 1

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{sum} = 10 \mid \text{1st is } 1) = \underline{\hspace{10cm}}$$

Conditional Probability: Dice Example 2



What is the probability that the sum is 10?

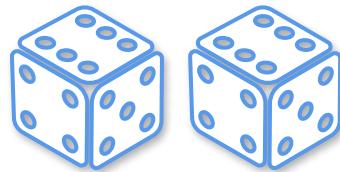
GIVEN that the first one is 1

A 6x6 grid representing all possible outcomes of two dice rolls. The columns are labeled by the outcome of the first die (1, 2, 3, 4, 5, 6) and the rows by the outcome of the second die (1, 2, 3, 4, 5, 6). The first column is highlighted in blue, and the last row is highlighted in red. The cell at the intersection of the first column and the last row (1,6) is also highlighted in red.

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{sum} = 10 \mid \text{1st is } 1) = \underline{\hspace{10cm}}$$

Conditional Probability: Dice Example 2



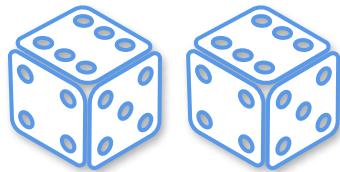
What is the probability that the sum is 10?

GIVEN that the first one is 1

	1,1	1,2	1,3	1,4	1,5	1,6
	2,1	2,2	2,3	2,4	2,5	2,6
	3,1	3,2	3,3	3,4	3,5	3,6
	4,1	4,2	4,3	4,4	4,5	4,6
	5,1	5,2	5,3	5,4	5,5	5,6
	6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{sum} = 10 \mid \text{1st is } 1) = \underline{\hspace{10cm}}$$

Conditional Probability: Dice Example 2



What is the probability that the sum is 10?

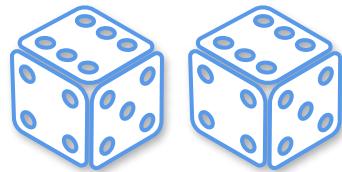
GIVEN that the first one is 1

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{sum} = 10 \mid \text{1st is } 1) = \underline{\hspace{10cm}}$$

1,1	1,2	1,3	1,4	1,5	1,6
-----	-----	-----	-----	-----	-----

Conditional Probability: Dice Example 2



What is the probability that the sum is 10?

GIVEN that the first one is 1

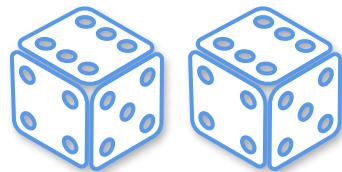
1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{sum} = 10 \mid \text{1st is } 1) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$

1,1	1,2	1,3	1,4	1,5	1,6
1,1	1,2	1,3	1,4	1,5	1,6

1,1	1,2	1,3	1,4	1,5	1,6
1,1	1,2	1,3	1,4	1,5	1,6

Conditional Probability: Dice Example 2



What is the probability that the sum is 10?

GIVEN that the first one is 1

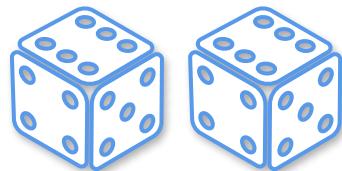
1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{sum} = 10 \mid \text{1st is } 1) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$

1,1	1,2	1,3	1,4	1,5	1,6
0	1,2	1,3	1,4	1,5	1,6

1,1	1,2	1,3	1,4	1,5	1,6
0	1,2	1,3	1,4	1,5	1,6

Conditional Probability: Dice Example 2



What is the probability that the sum is 10?

GIVEN that the first one is 1

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{sum} = 10 \mid \text{1st is } 1) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$

The total number of outcomes is 6 (the first die) times 6 (the second die), which is 36.

The number of favorable outcomes where the sum is 10 given the first die is 1 is 0 (1,9), (2,8), (3,7), (4,6), (5,5), and (6,4).

Therefore, the probability is $\frac{0}{36} = 0$.

Product Rule (for Independent Events)

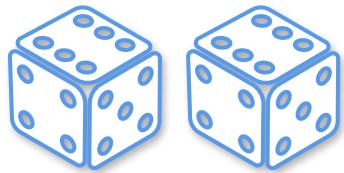
$$\mathbf{P}(A \cap B) = \mathbf{P}(A) \cdot \mathbf{P}(B)$$

Product Rule (for Independent Events)

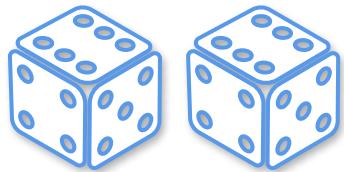
When A and B independent

$$\mathbf{P}(A \cap B) = \mathbf{P}(A) \cdot \mathbf{P}(B)$$

Conditional Probability: Dice Example 3

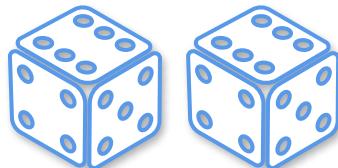


Conditional Probability: Dice Example 3



What is the probability that
the first is 6 **AND** the sum = 10?

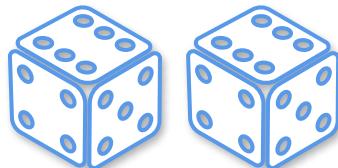
Conditional Probability: Dice Example 3



What is the probability that
the first is 6 **AND** the sum = 10?

	1,1	1,2	1,3	1,4	1,5	1,6
1,1						
1,2						
1,3						
1,4						
1,5						
1,6						
2,1	2,1	2,2	2,3	2,4	2,5	2,6
2,2						
2,3						
2,4						
2,5						
2,6						
3,1	3,1	3,2	3,3	3,4	3,5	3,6
3,2						
3,3						
3,4						
3,5						
3,6						
4,1	4,1	4,2	4,3	4,4	4,5	4,6
4,2						
4,3						
4,4						
4,5						
4,6						
5,1	5,1	5,2	5,3	5,4	5,5	5,6
5,2						
5,3						
5,4						
5,5						
5,6						
6,1	6,1	6,2	6,3	6,4	6,5	6,6
6,2						
6,3						
6,4						
6,5						
6,6						

Conditional Probability: Dice Example 3

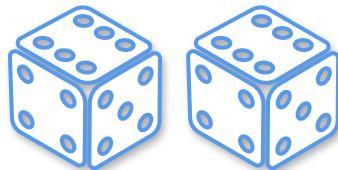


What is the probability that
the first is 6 **AND** the sum = 10?

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6
6,1						

$$P(\text{1st is 6} \cap \text{sum} = 10) = \underline{\hspace{10cm}}$$

Conditional Probability: Dice Example 3



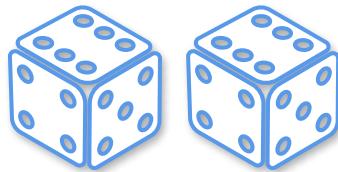
What is the probability that
the first is 6 **AND** the sum = 10?

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{1st is 6} \cap \text{sum} = 10) =$$

6,4

Conditional Probability: Dice Example 3



What is the probability that the first is 6 **AND** the sum = 10?

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{1st is 6} \cap \text{sum} = 10) =$$

6,4

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$= \frac{1}{36}$$

Conditional Probability: Dice Example 3

	1,1	1,2	1,3	1,4	1,5	1,6
1,1						
1,2						
1,3						
1,4						
1,5						
1,6						
2,1	2,1	2,2	2,3	2,4	2,5	2,6
2,2						
2,3						
2,4						
2,5						
2,6						
3,1	3,1	3,2	3,3	3,4	3,5	3,6
3,2						
3,3						
3,4						
3,5						
3,6						
4,1	4,1	4,2	4,3	4,4	4,5	4,6
4,2						
4,3						
4,4						
4,5						
4,6						
5,1	5,1	5,2	5,3	5,4	5,5	5,6
5,2						
5,3						
5,4						
5,5						
5,6						
6,1	6,1	6,2	6,3	6,4	6,5	6,6
6,2						
6,3						
6,4						
6,5						
6,6						

Conditional Probability: Dice Example 3

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{1st is } 6 \cap \text{sum} = 10) =$$

Conditional Probability: Dice Example 3

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{1st is } 6 \cap \text{sum} = 10) =$$

$$P(\text{1st is } 6)$$

Conditional Probability: Dice Example 3

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6
6,1						

$$P(\text{1st is } 6 \cap \text{sum} = 10) =$$

$$P(\text{1st is } 6)$$

Conditional Probability: Dice Example 3

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6
6,1	6,2	6,3	6,4	6,5	6,6	

$$P(\text{1st is } 6 \cap \text{sum} = 10) =$$

$$P(\text{1st is } 6)$$

6,1	6,2	6,3	6,4	6,5	6,6
-----	-----	-----	-----	-----	-----

Conditional Probability: Dice Example 3

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6
6,1	6,2	6,3	6,4	6,5	6,6	

$$P(\text{1st is } 6 \cap \text{sum} = 10) =$$

$$P(\text{1st is } 6)$$

6,1	6,2	6,3	6,4	6,5	6,6
-----	-----	-----	-----	-----	-----

Conditional Probability: Dice Example 3

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6
6,1	6,2	6,3	6,4	6,5	6,6	

$$P(\text{1st is } 6 \cap \text{sum} = 10) =$$

$$P(\text{1st is } 6)$$

6,1	6,2	6,3	6,4	6,5	6,6
<hr/>					
1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

Conditional Probability: Dice Example 3

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6
6,1	6,2	6,3	6,4	6,5	6,6	

$$P(\text{1st is } 6 \cap \text{sum} = 10) =$$

$$P(\text{1st is } 6)$$

6,1	6,2	6,3	6,4	6,5	6,6
<hr/>					
1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$\frac{6}{36}$$

Conditional Probability: Dice Example 3

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6
6,1	6,2	6,3	6,4	6,5	6,6	

$$P(\text{1st is } 6 \cap \text{sum} = 10) =$$

$$P(\text{1st is } 6)$$

6,1	6,2	6,3	6,4	6,5	6,6
<hr/>					
1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$\frac{1}{6}$$

Conditional Probability: Dice Example 3

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6
6,1	6,2	6,3	6,4	6,5	6,6	

$$P(\text{1st is } 6 \cap \text{sum} = 10) =$$

$$P(\text{1st is } 6)$$

$$P(\text{sum} = 10 | \text{1st } 6)$$

6,1	6,2	6,3	6,4	6,5	6,6
<hr/>					
1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$\frac{1}{6}$$

Conditional Probability: Dice Example 3

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6
6,1	6,2	6,3	6,4	6,5	6,6	

$$P(\text{1st is } 6 \cap \text{sum} = 10) =$$

$$P(\text{1st is } 6)$$

$$P(\text{sum} = 10 | \text{1st } 6)$$

6,1	6,2	6,3	6,4	6,5	6,6
1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

6,4

$$\frac{1}{6}$$

Conditional Probability: Dice Example 3

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6
6,1	6,2	6,3	6,4	6,5	6,6	

$$P(\text{1st is } 6 \cap \text{sum} = 10) =$$

$$P(\text{1st is } 6)$$

$$P(\text{sum} = 10 | \text{1st } 6)$$

6,1	6,2	6,3	6,4	6,5	6,6
1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$\frac{1}{6}$$

Conditional Probability: Dice Example 3

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6
6,1	6,2	6,3	6,4	6,5	6,6	

$$P(\text{1st is } 6 \cap \text{sum} = 10) =$$

$$P(\text{1st is } 6)$$

$$P(\text{sum} = 10 | \text{1st } 6)$$

6,1	6,2	6,3	6,4	6,5	6,6
1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

6,4
6,1
6,2
6,3
6,4
6,5
6,6

$$\frac{1}{6}$$

$$\frac{1}{6}$$

Conditional Probability: Dice Example 3

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6
6,1	6,2	6,3	6,4	6,5	6,6	

$$P(1\text{st is } 6 \cap \text{sum} = 10) =$$

$$P(1\text{st is } 6)$$

$$\bullet P(\text{sum} = 10 | 1\text{st } 6)$$

6,1	6,2	6,3	6,4	6,5	6,6
-----	-----	-----	-----	-----	-----

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

6,4
6,1 6,2 6,3 6,4 6,5 6,6

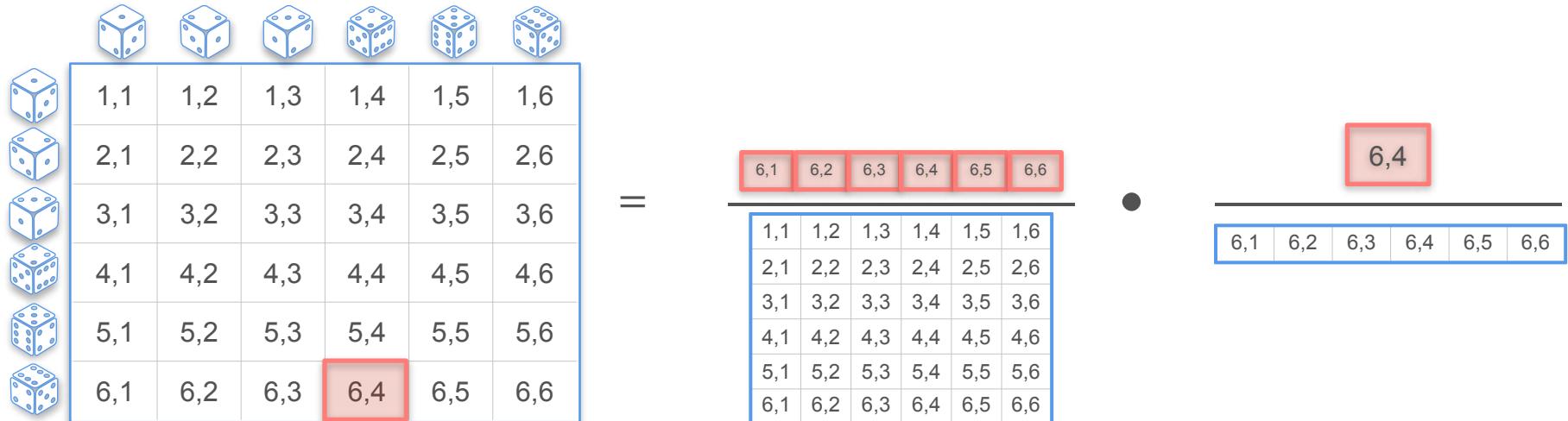
$$\frac{1}{6}$$

•

$$\frac{1}{6}$$

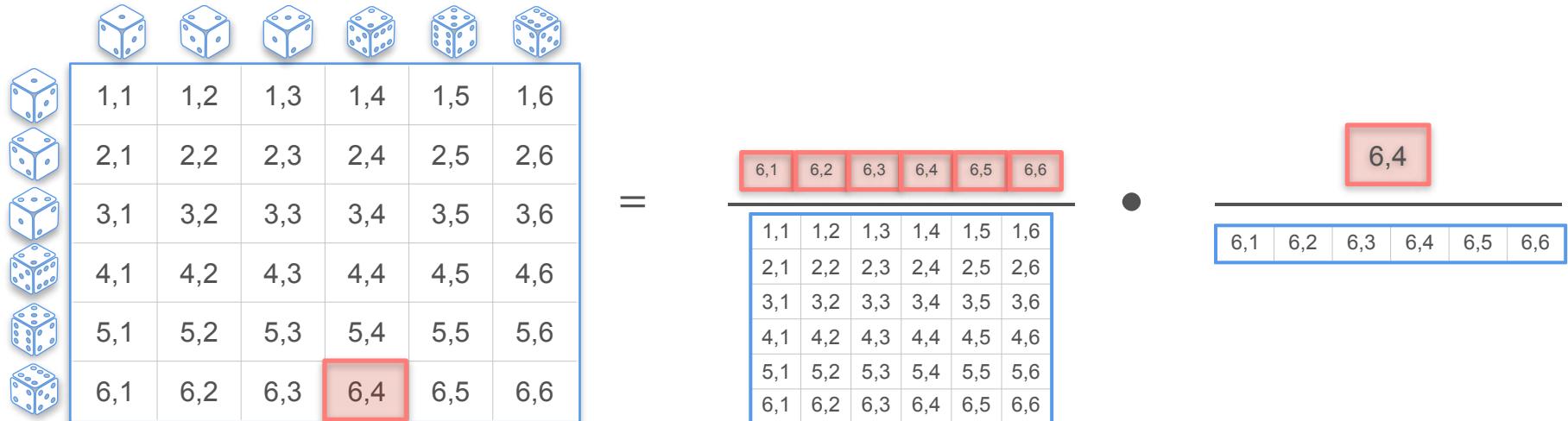
$$= \frac{1}{36}$$

Conditional Probability: Dice Example 3



$$P(1\text{st is } 6 \cap \text{sum} = 10) = P(1\text{st is } 6) \bullet P(\text{sum} = 10 | 1\text{st } 6)$$

Conditional Probability: Dice Example 3



$$P(A \cap B) = P(A) \cdot P(B | A)$$

The General Product Rule

$$\mathbf{P}(A \cap B) = \mathbf{P}(A) \cdot \mathbf{P}(B | A)$$

The General Product Rule

$$\mathbf{P}(A \cap B) = \mathbf{P}(A) \cdot \mathbf{P}(B | A)$$



When independent

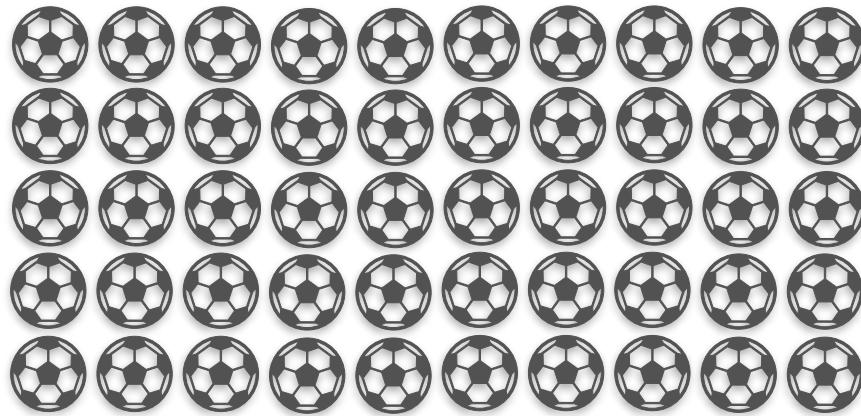
$$\mathbf{P}(B | A) = \mathbf{P}(B)$$

Quiz 1

Quiz 1

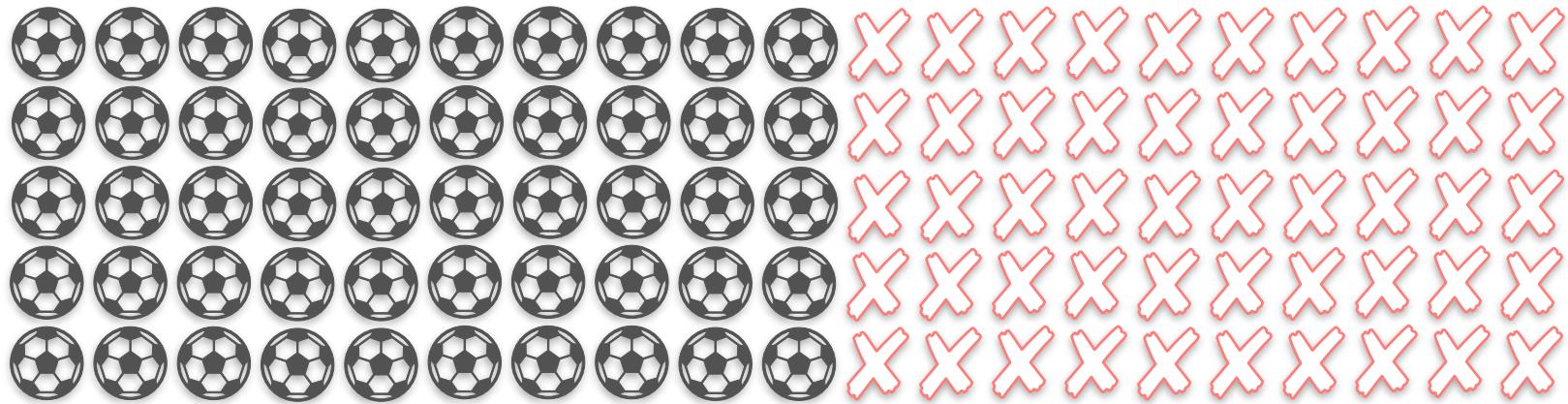
100 kids

Quiz 1



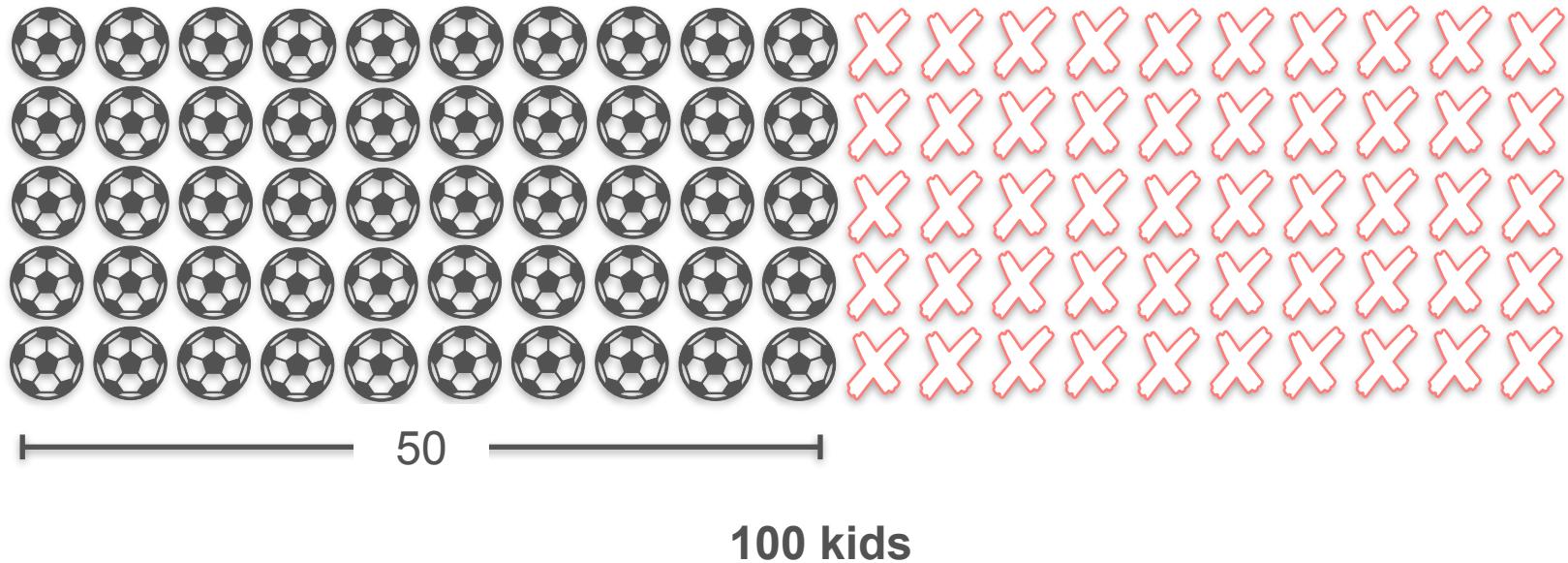
100 kids

Quiz 1

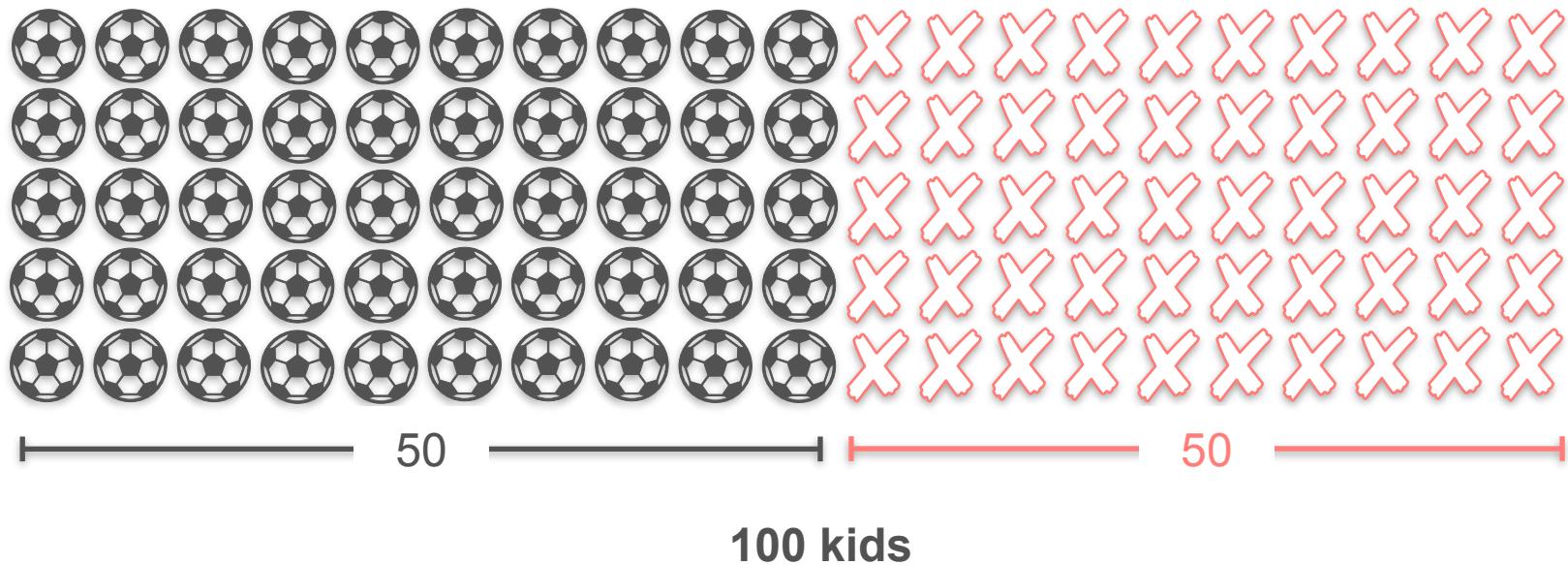


100 kids

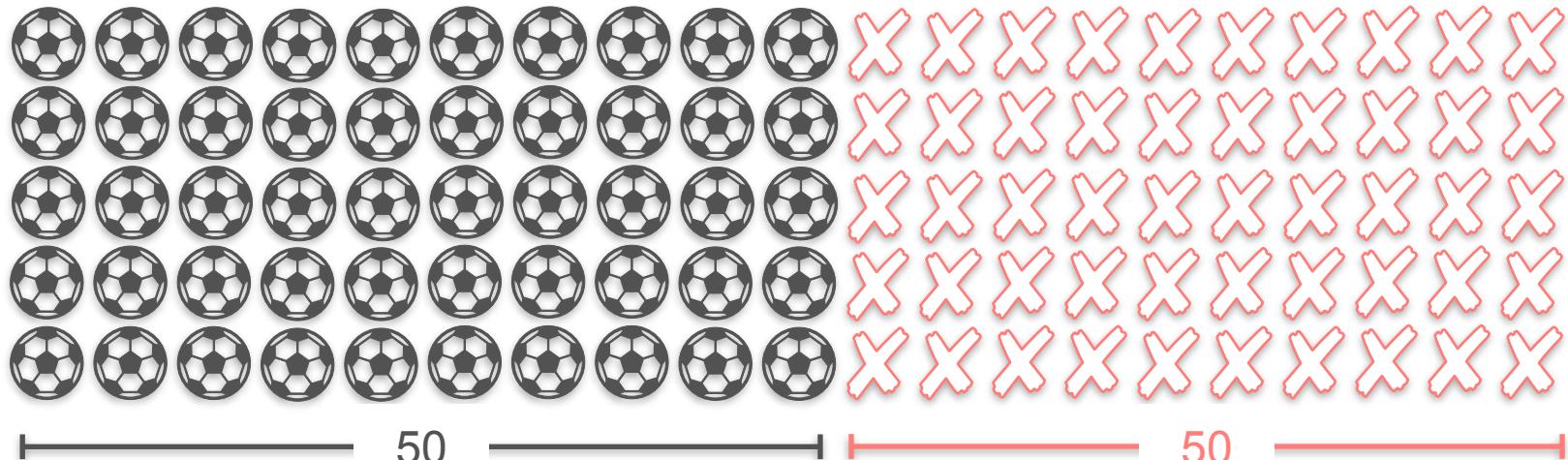
Quiz 1



Quiz 1

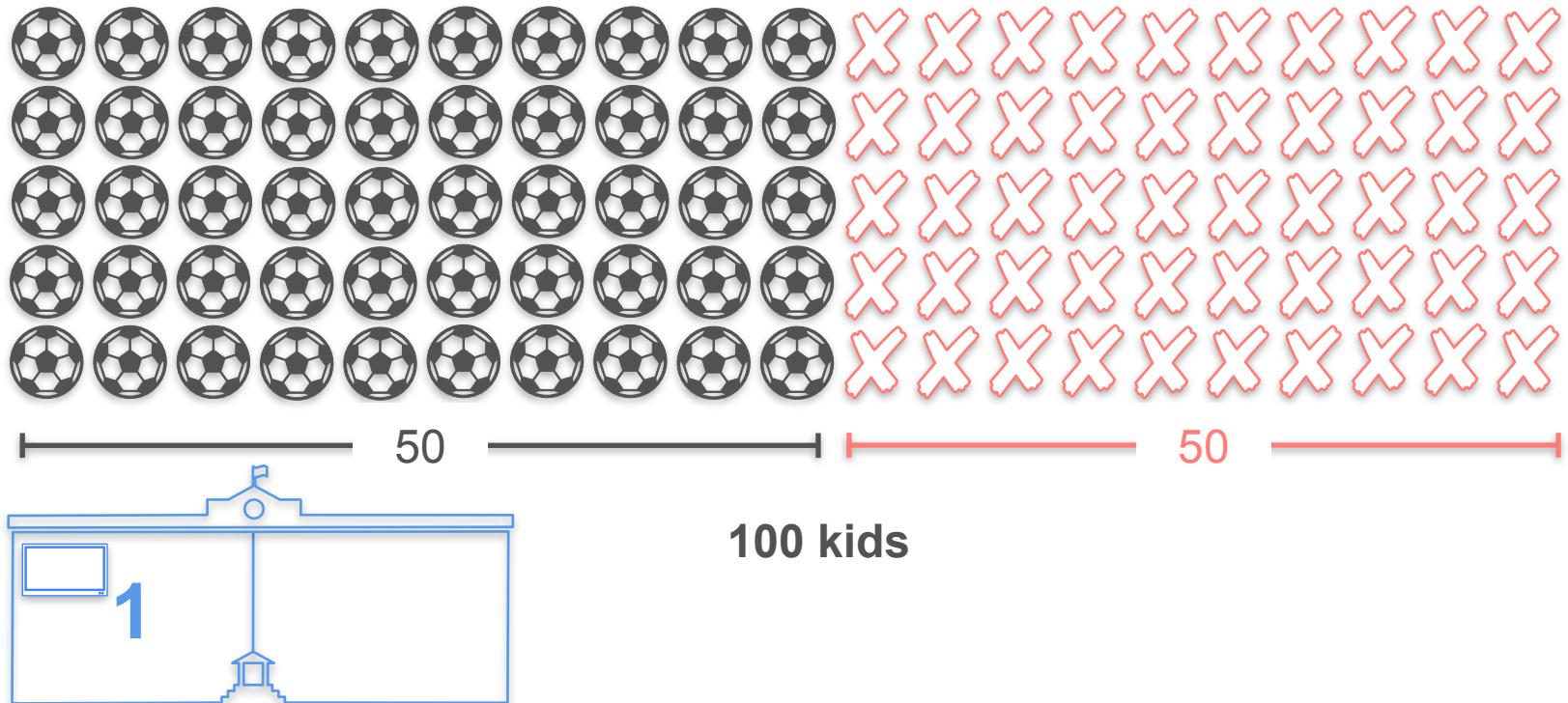


Quiz 1

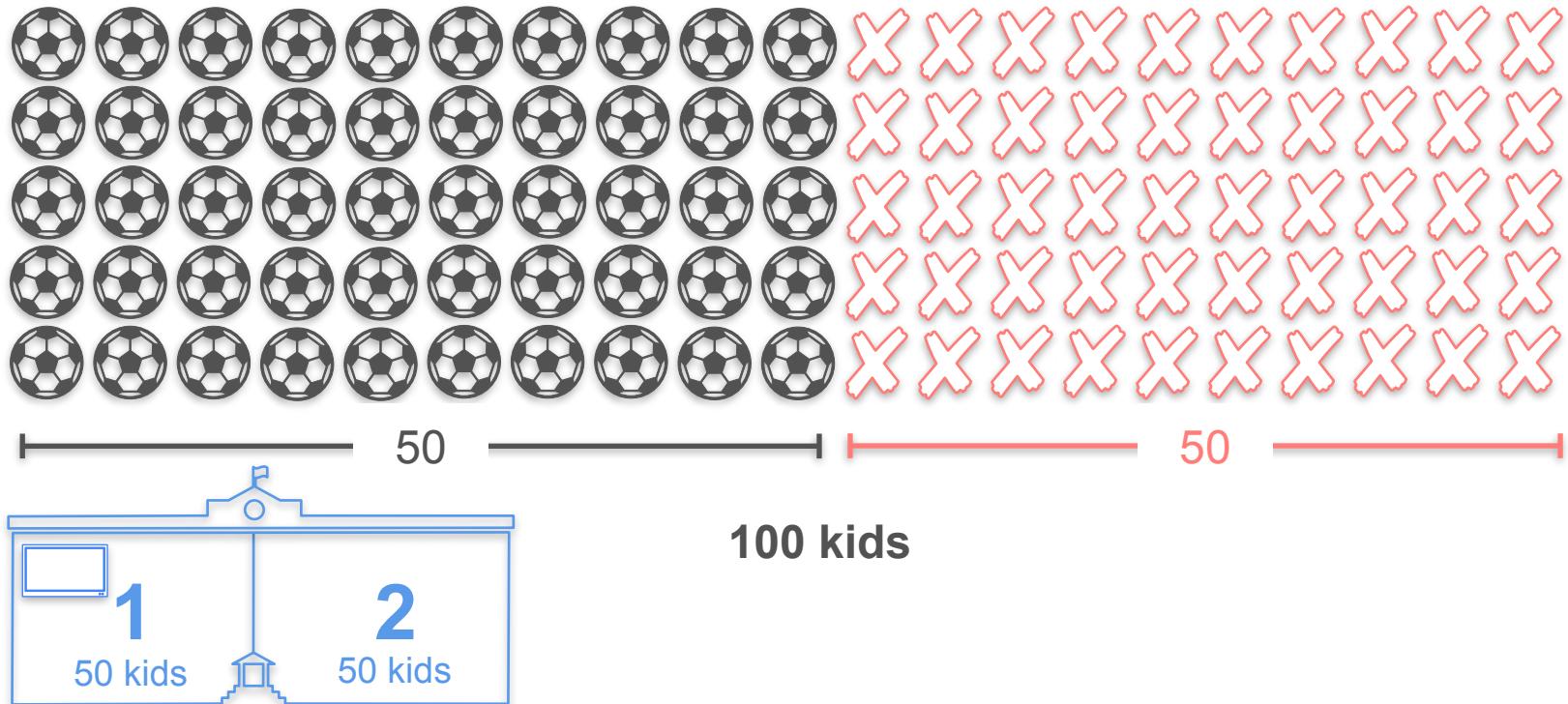


100 kids

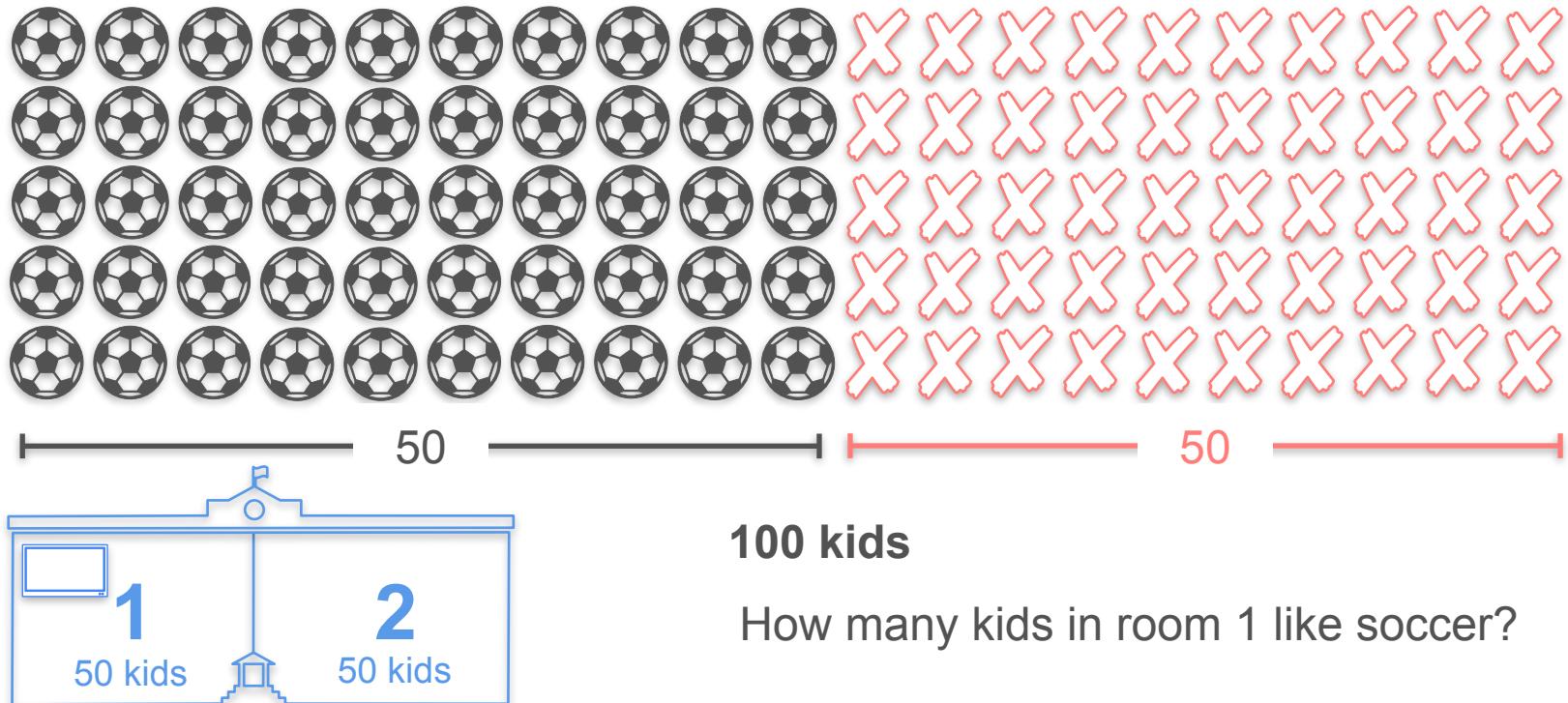
Quiz 1



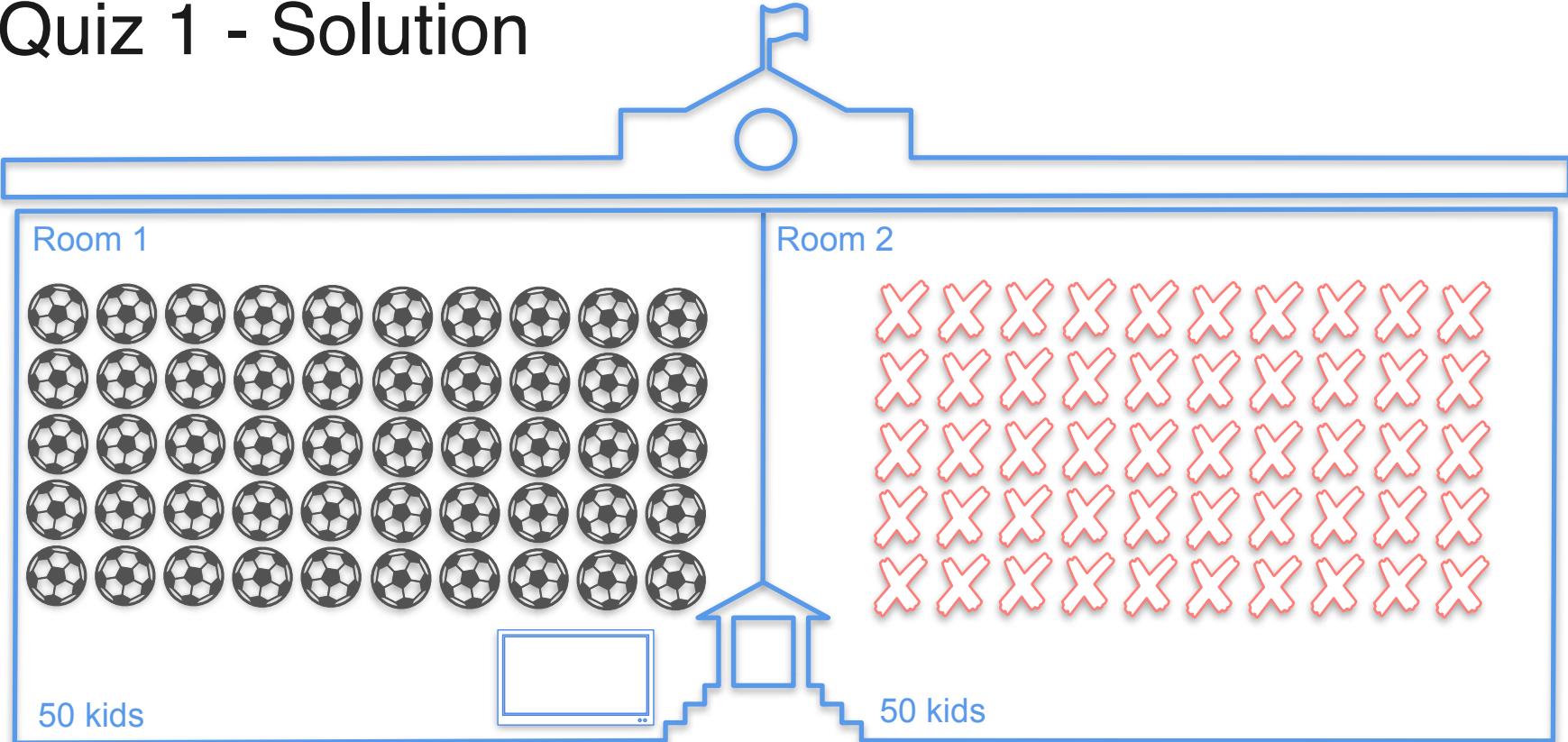
Quiz 1



Quiz 1

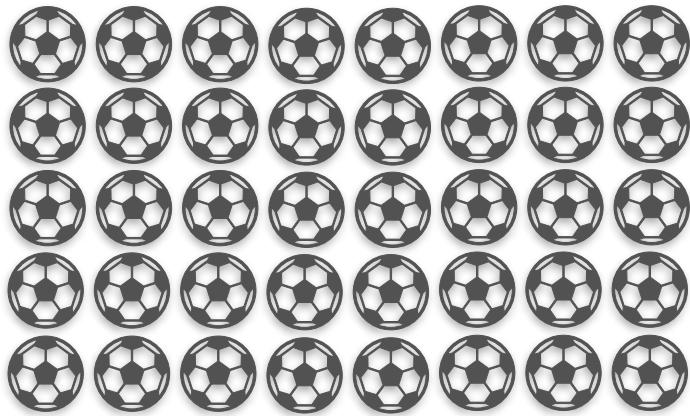


Quiz 1 - Solution

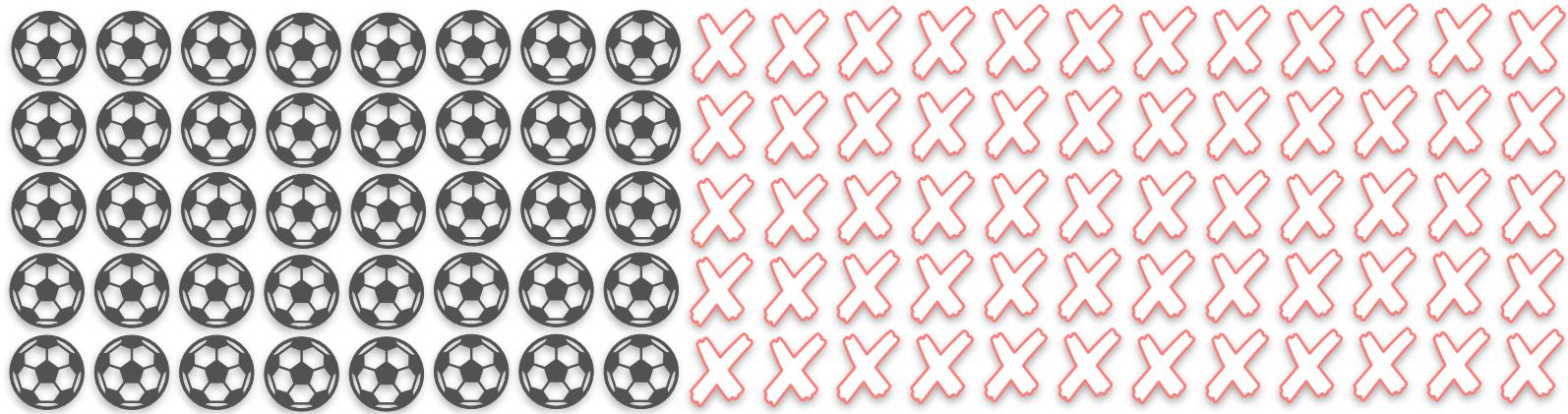


Quiz 2

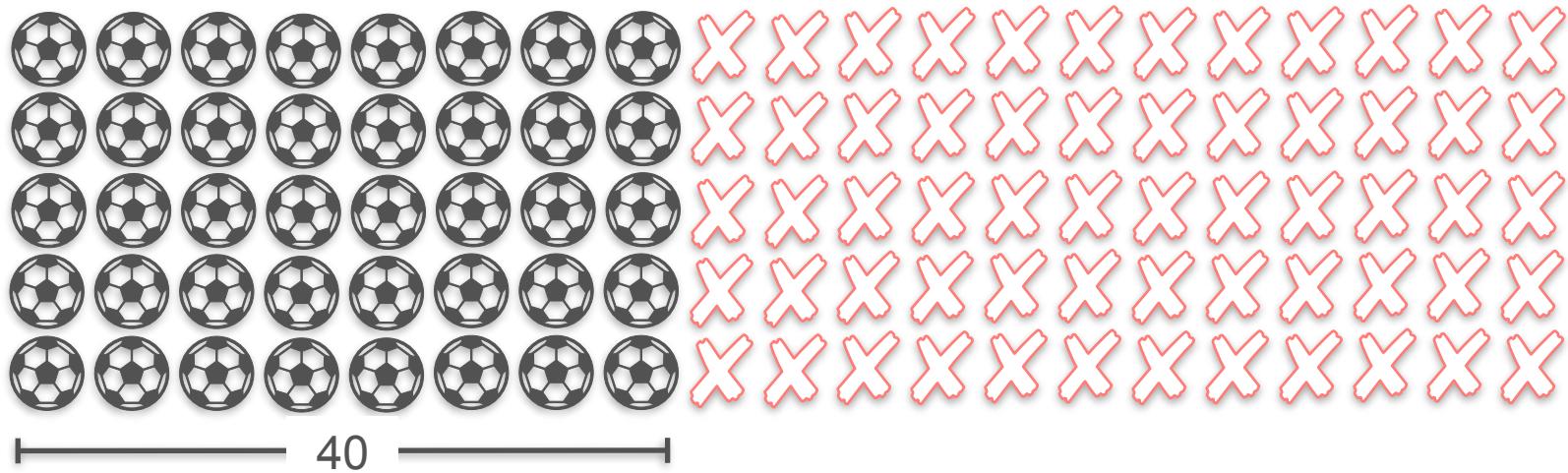
Quiz 2



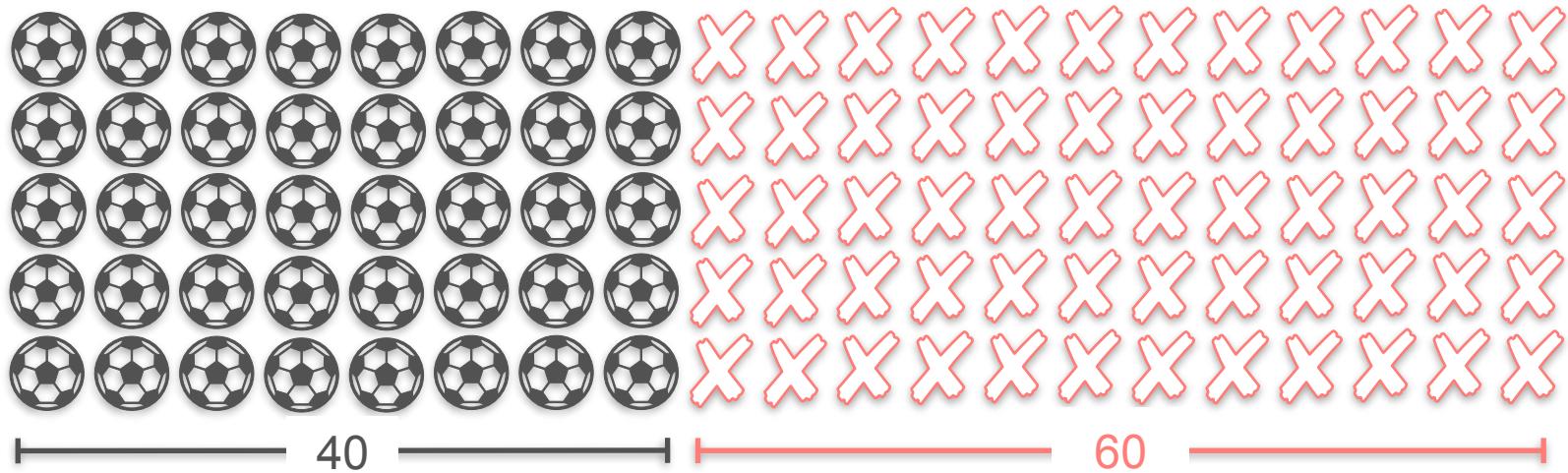
Quiz 2



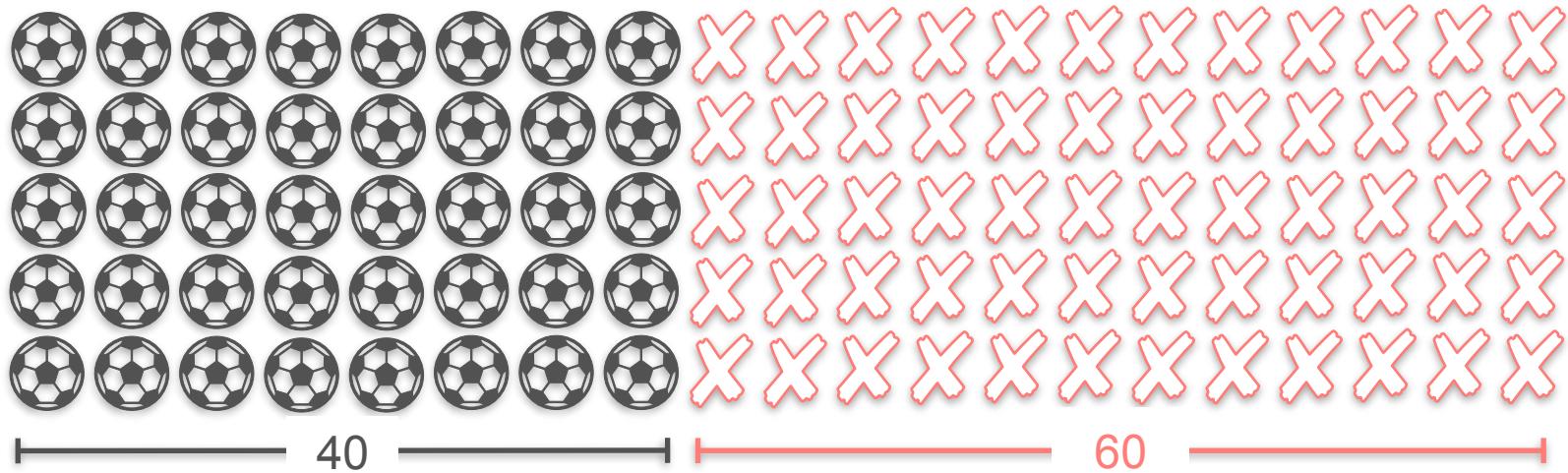
Quiz 2



Quiz 2

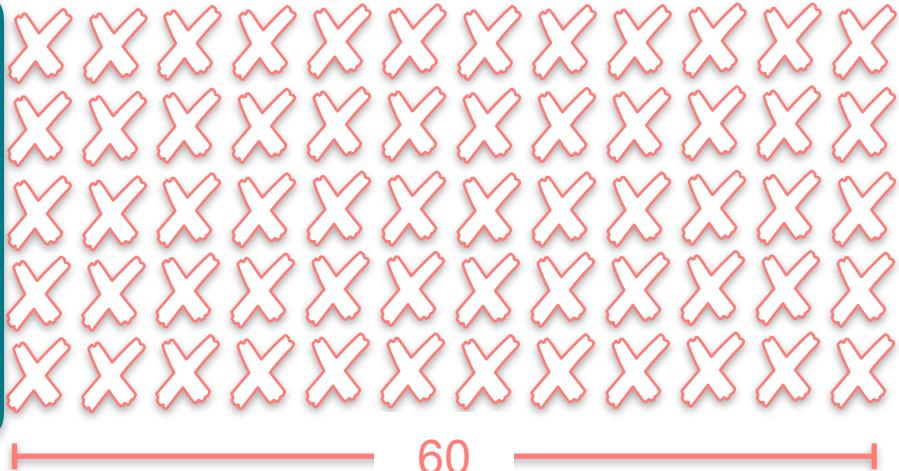
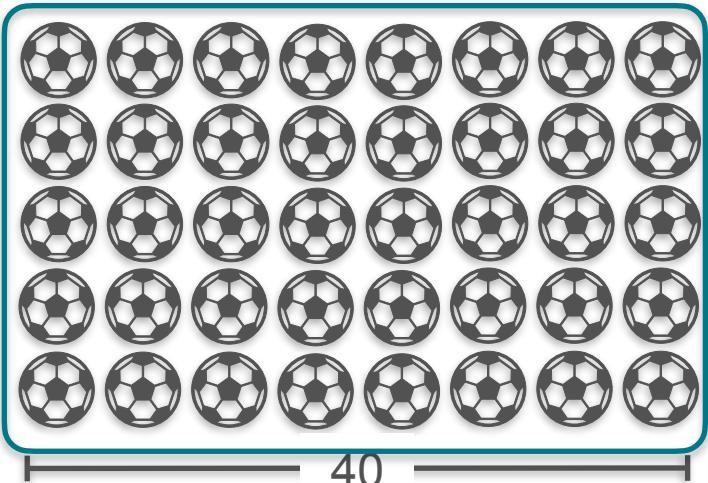


Quiz 2



100 kids

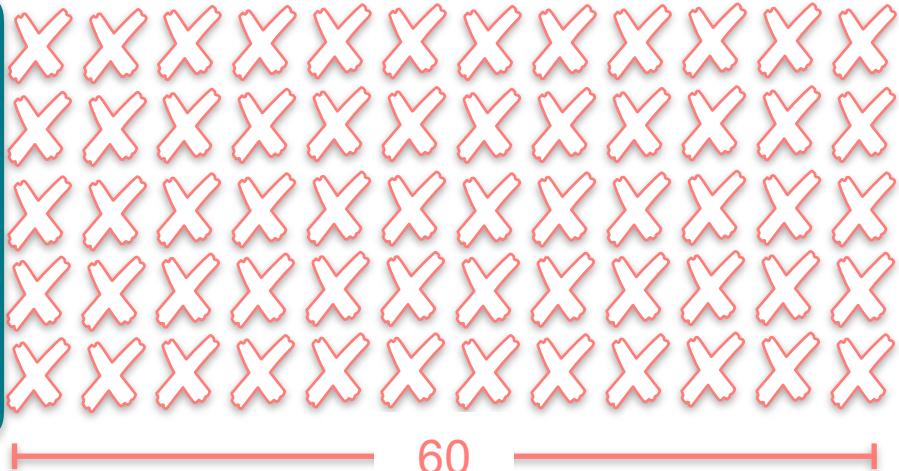
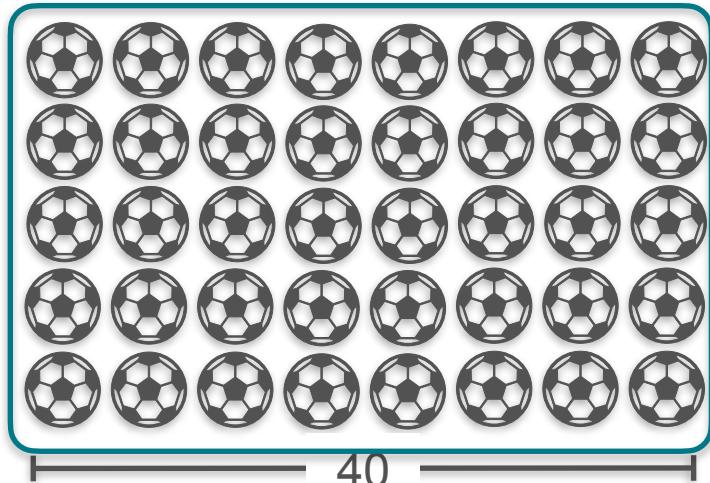
Quiz 2



80%

100 kids

Quiz 2

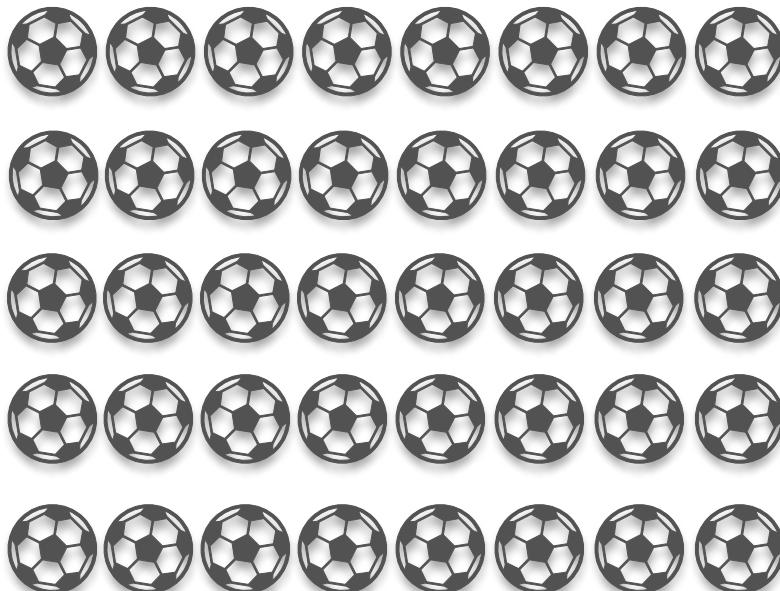


80%

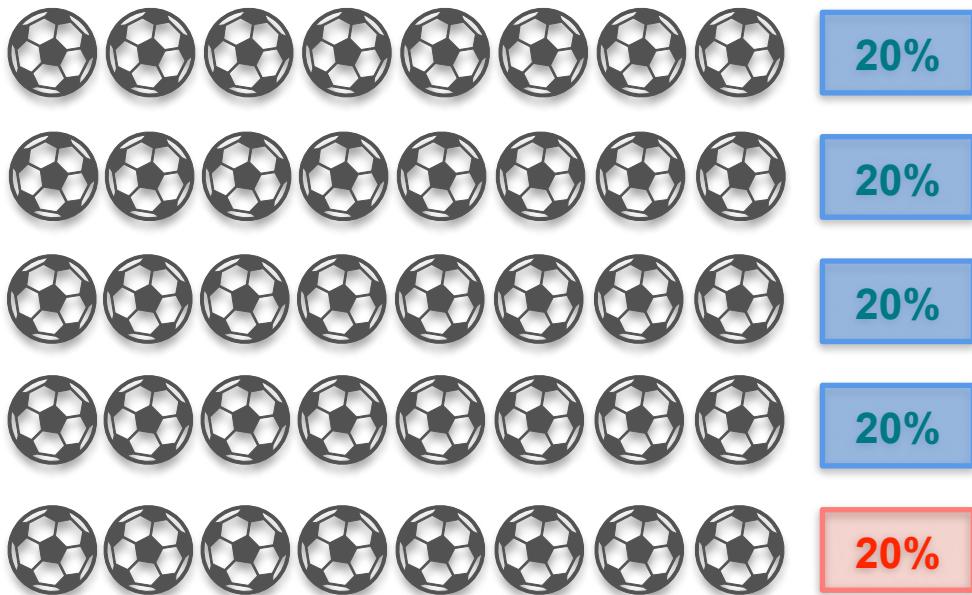
100 kids

How many kids play soccer and wear running shoes

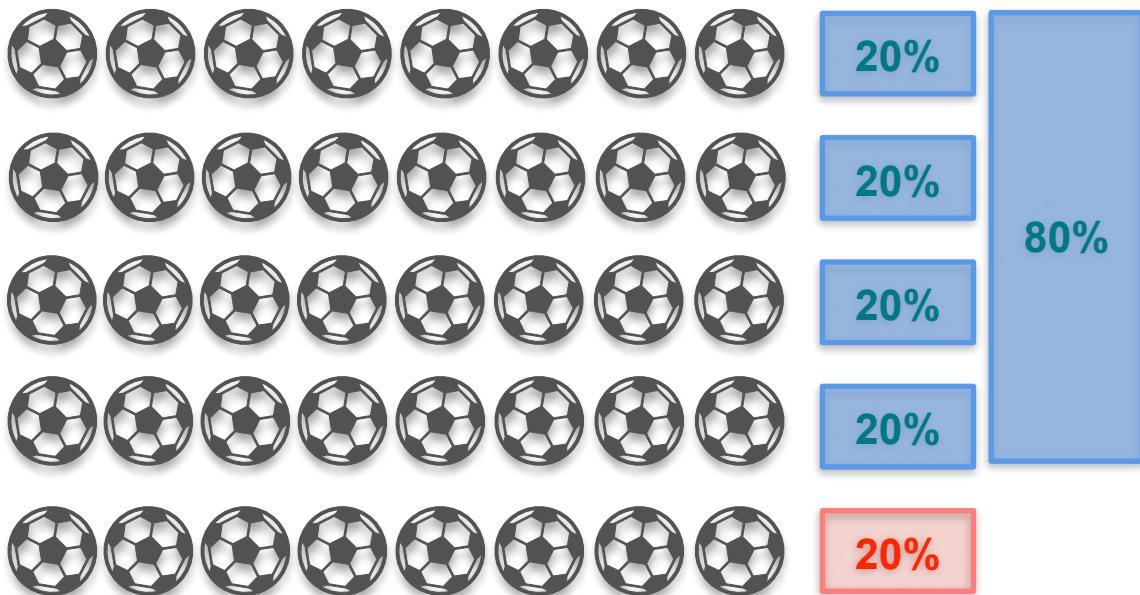
Quiz 2 - Solution



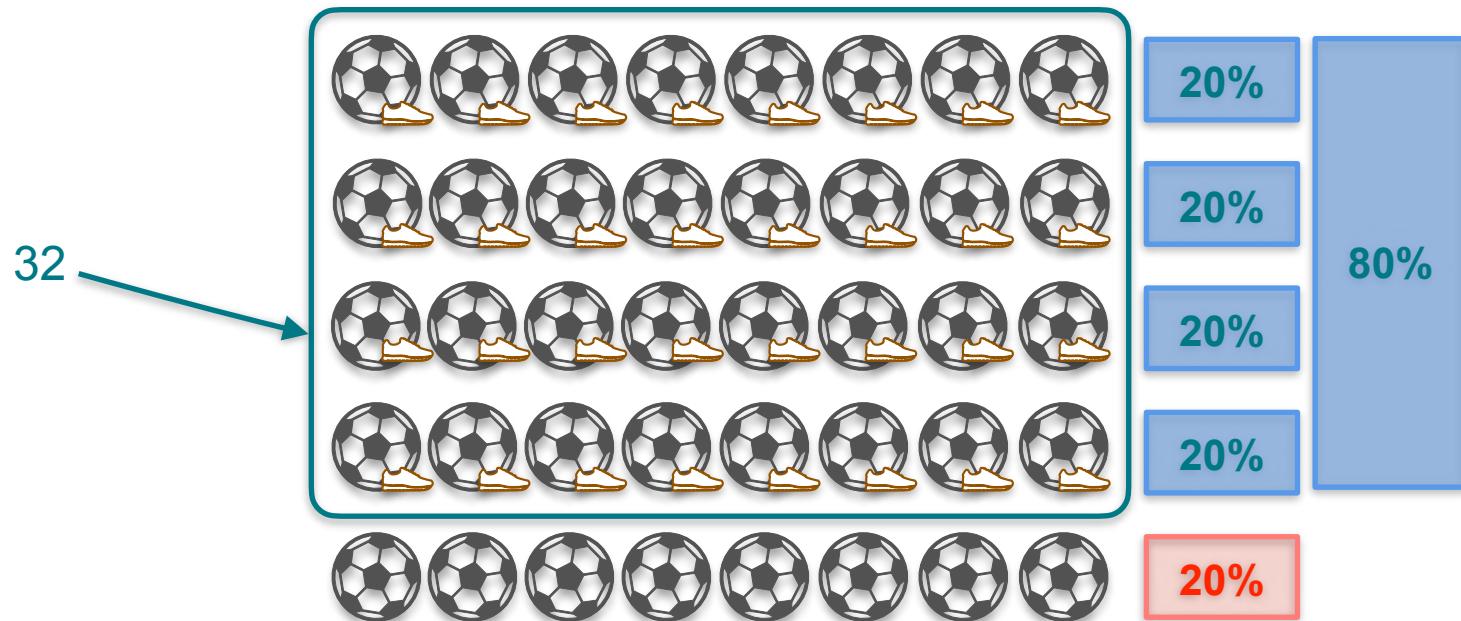
Quiz 2 - Solution



Quiz 2 - Solution

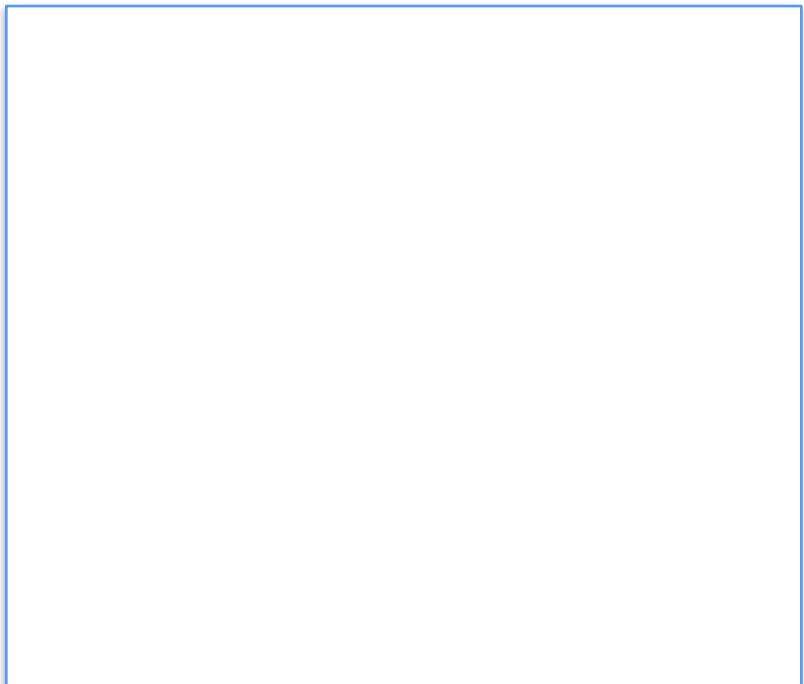


Quiz 2 - Solution



Conditional Probability

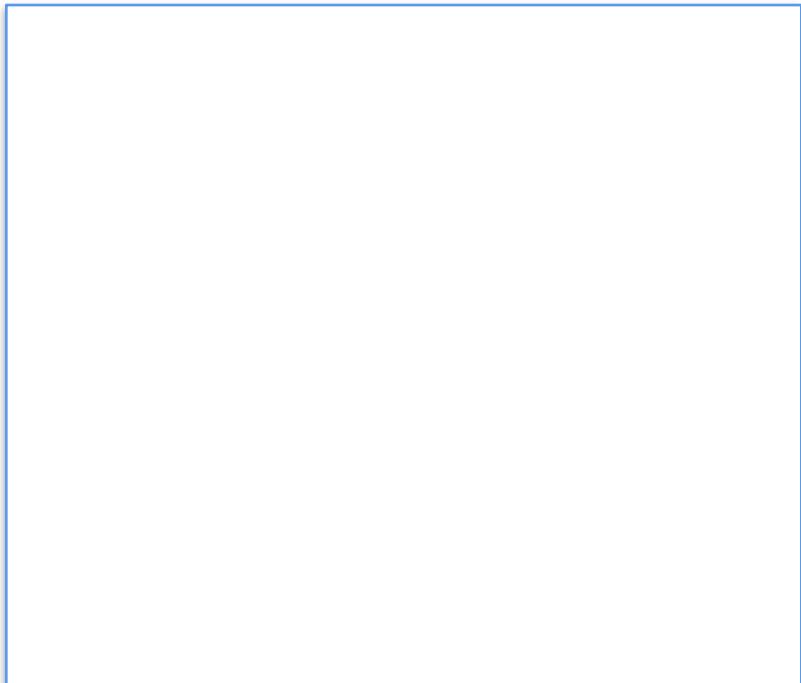
Conditional Probability



Conditional Probability



$$P(S) = 0.4$$



Conditional Probability



$$P(S) = 0.4$$

○	○	○	○	○	○	○	○	○	○
○	○	○	○	○	○	○	○	○	○
○	○	○	○	○	○	○	○	○	○
○	○	○	○	○	○	○	○	○	○
○	○	○	○	○	○	○	○	○	○

Conditional Probability



$P(S) = 0.4$



$P(\text{not } S) = 0.6$

○	○	○	○	○	○	○	○	○
○	○	○	○	○	○	○	○	○
○	○	○	○	○	○	○	○	○
○	○	○	○	○	○	○	○	○
○	○	○	○	○	○	○	○	○

Conditional Probability



$$P(S) = 0.4$$



$$P(\text{not } S) = 0.6$$

○	○	○	○	○	○	○	○	○	○
○	○	○	○	○	○	○	○	○	○
○	○	○	○	○	○	○	○	○	○
○	○	○	○	○	○	○	○	○	○
○	○	○	○	○	○	○	○	○	○
✗	✗	✗	✗	✗	✗	✗	✗	✗	✗
✗	✗	✗	✗	✗	✗	✗	✗	✗	✗
✗	✗	✗	✗	✗	✗	✗	✗	✗	✗
✗	✗	✗	✗	✗	✗	✗	✗	✗	✗
✗	✗	✗	✗	✗	✗	✗	✗	✗	✗

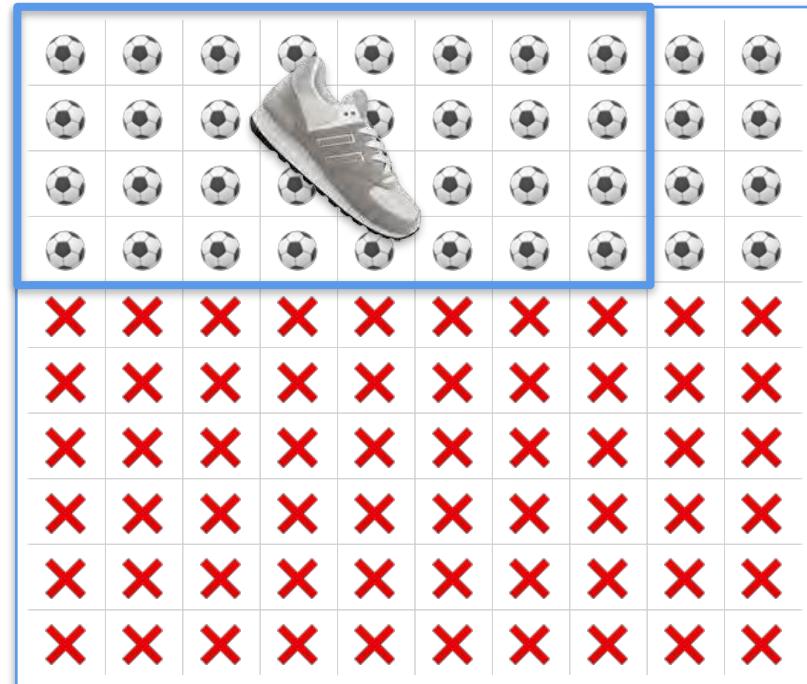
Conditional Probability



$$P(S) = 0.4$$



$$P(\text{not } S) = 0.6$$



Conditional Probability

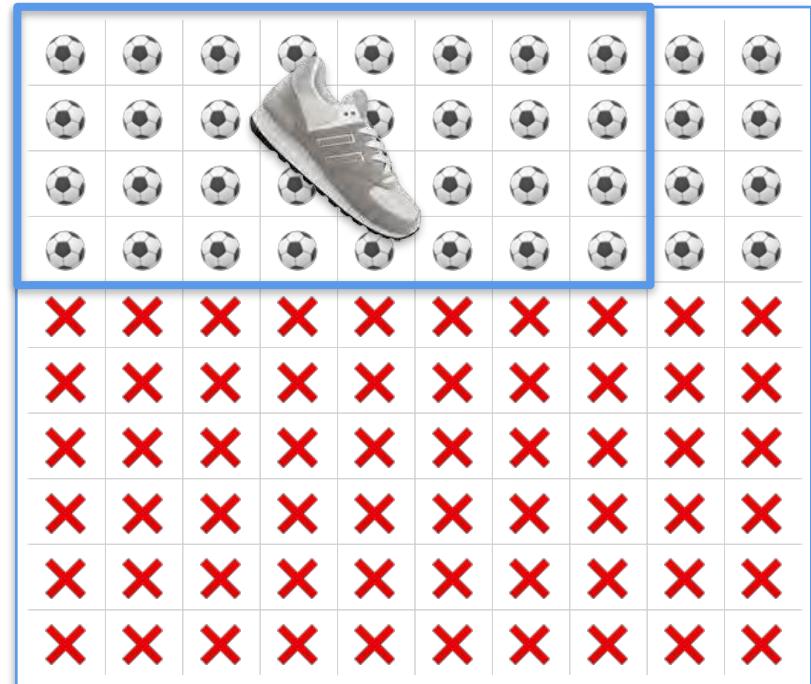


$$P(S) = 0.4$$



$$P(\text{not } S) = 0.6$$

$$P(R | S) = 0.8$$



Conditional Probability

$P(\text{Soccer and Running shoes})$

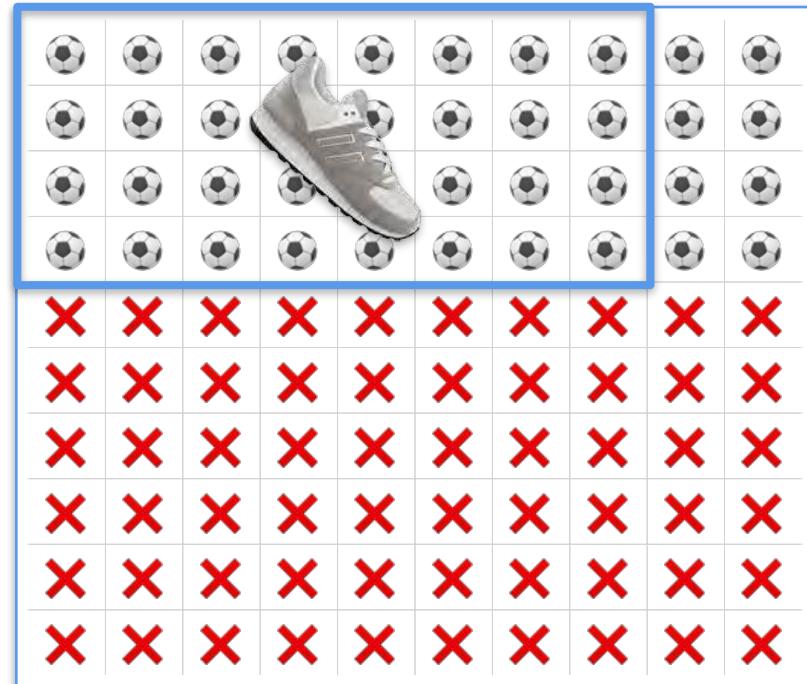


$P(S) = 0.4$



$P(\text{not } S) = 0.6$

$$P(R | S) = 0.8$$



Conditional Probability

$$P(R | S) = 0.8$$

$P(\text{Soccer and Running shoes})$



$$P(S) = 0.4$$



$$P(\text{not } S) = 0.6$$



Conditional Probability

$$P(R | S) = 0.8$$

$P(\text{Soccer and Running shoes})$



$$P(S) = 0.4$$



$$P(\text{not } S) = 0.6$$



Conditional Probability

$$P(R | S) = 0.8$$

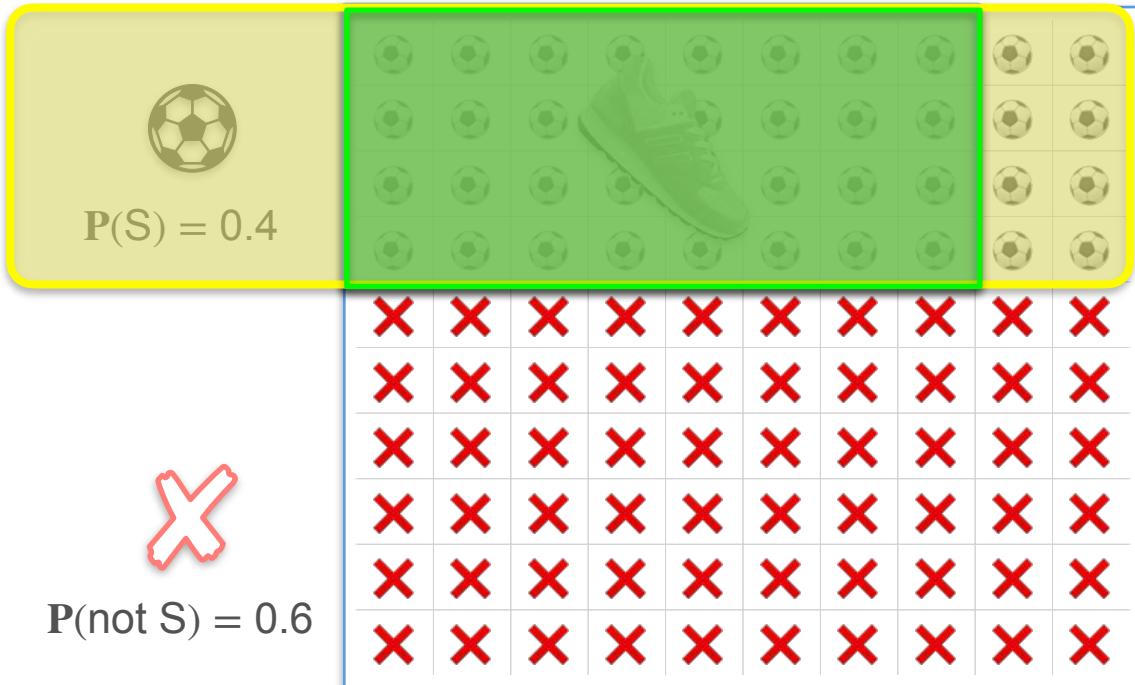
$P(\text{Soccer and Running shoes})$



$$P(S) = 0.4$$



$$P(\text{not } S) = 0.6$$



Conditional Probability

$$P(R | S) = 0.8$$

$P(\text{Soccer and Running shoes})$

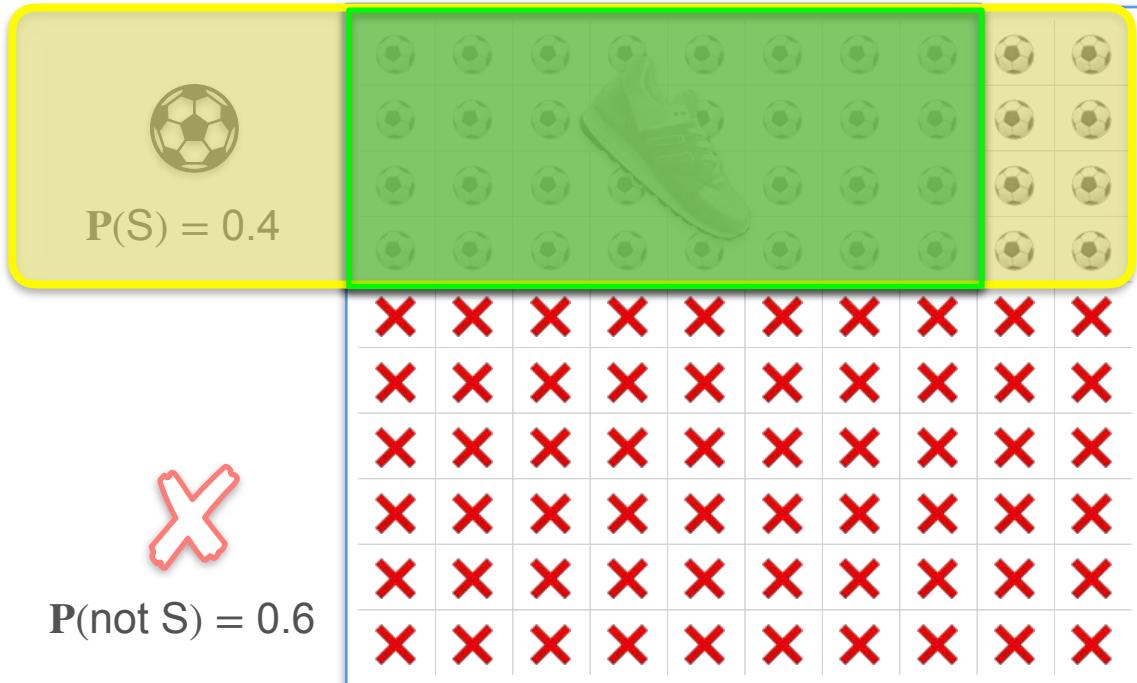
$$P(S \cap R) =$$



$$P(S) = 0.4$$



$$P(\text{not } S) = 0.6$$



Conditional Probability

$$P(R | S) = 0.8$$

$P(\text{Soccer and Running shoes})$

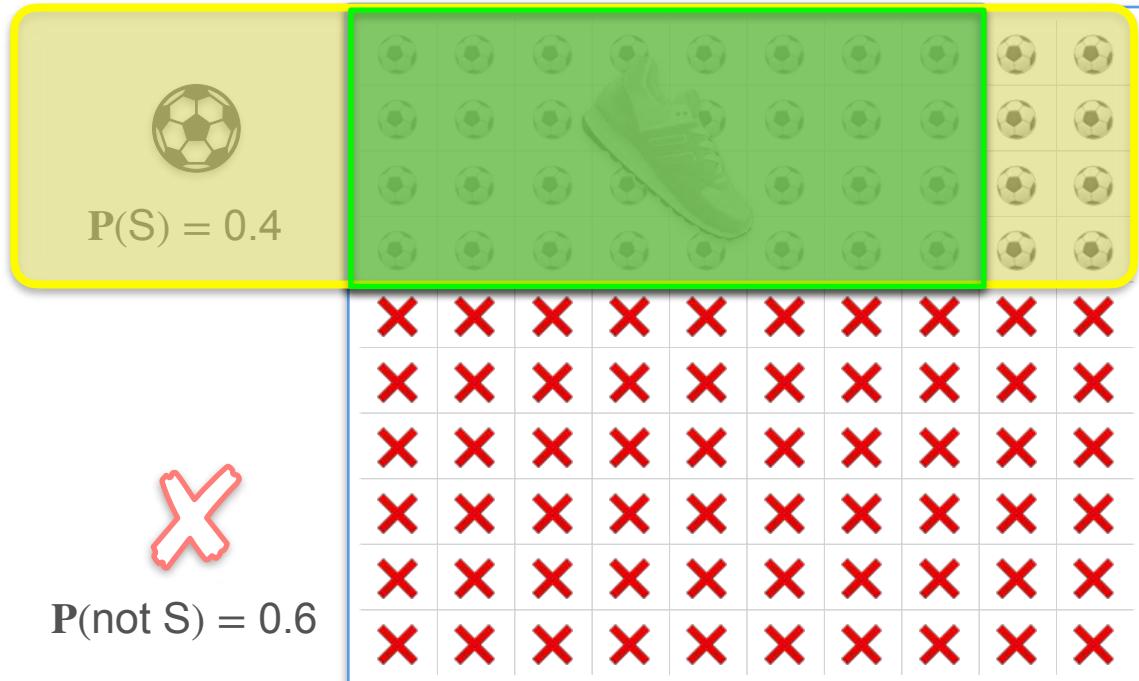
$$P(S \cap R) = P(S)$$



$$P(S) = 0.4$$



$$P(\text{not } S) = 0.6$$



Conditional Probability

$$P(R | S) = 0.8$$

$P(\text{Soccer and Running shoes})$

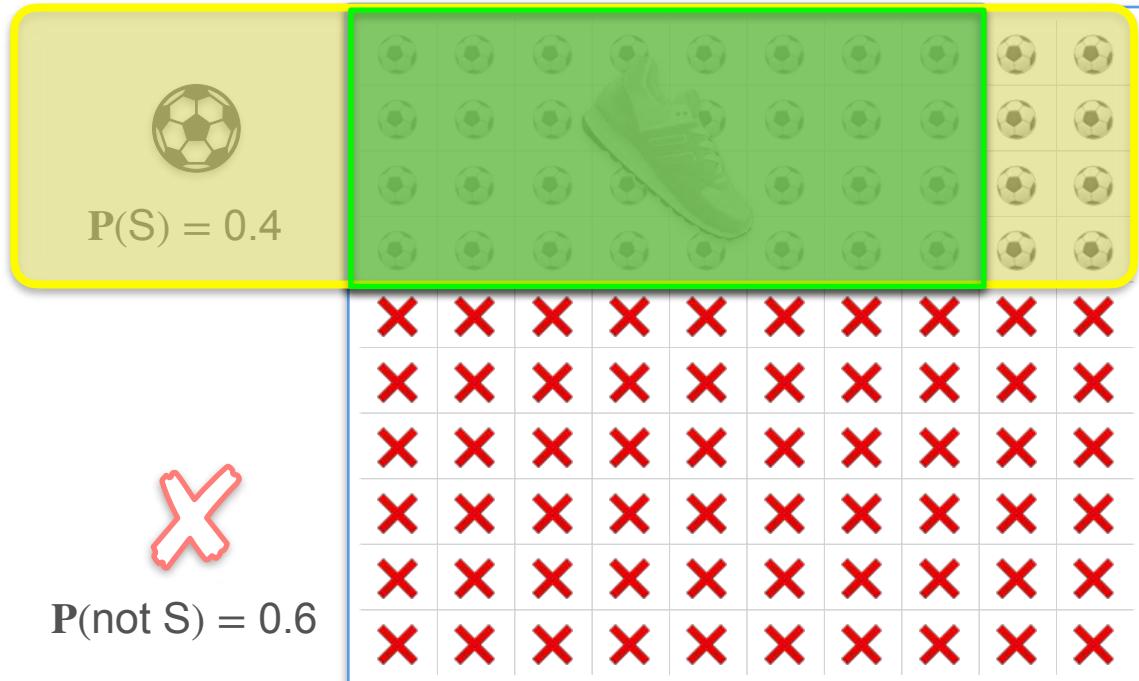
$$P(S \cap R) = P(S)$$



$$P(S) = 0.4$$



$$P(\text{not } S) = 0.6$$



Conditional Probability

$$P(R | S) = 0.8$$

$P(\text{Soccer and Running shoes})$

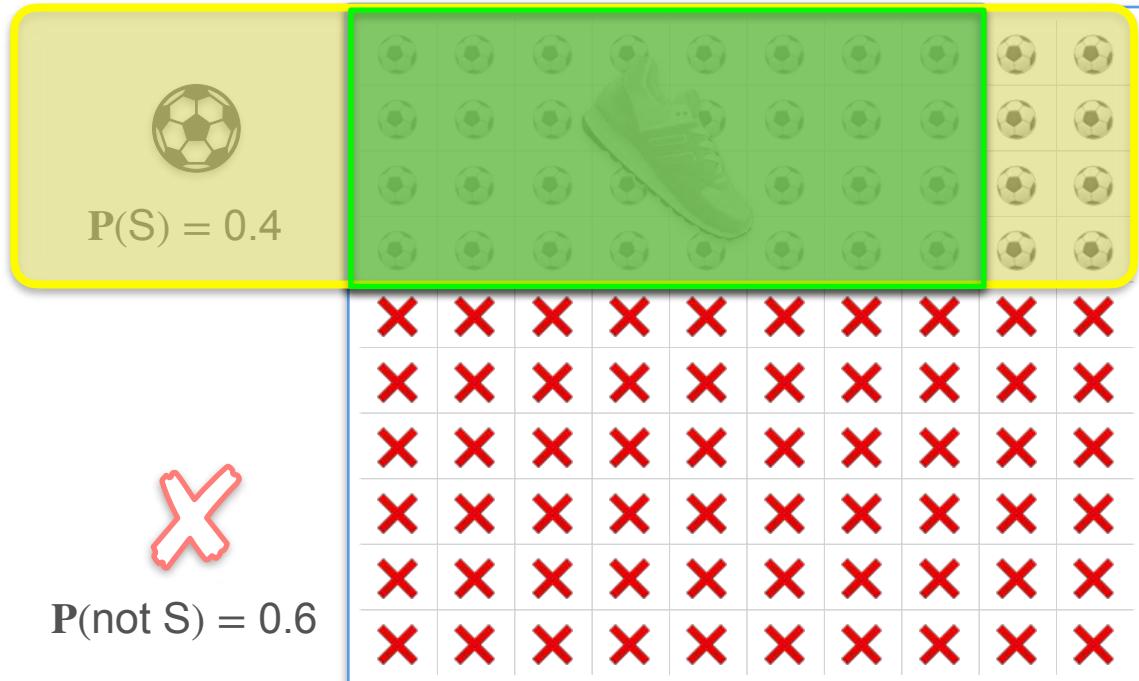
$$P(S \cap R) = P(S) \cdot$$



$$P(S) = 0.4$$



$$P(\text{not } S) = 0.6$$



Conditional Probability

$$P(R | S) = 0.8$$

$P(\text{Soccer and Running shoes})$

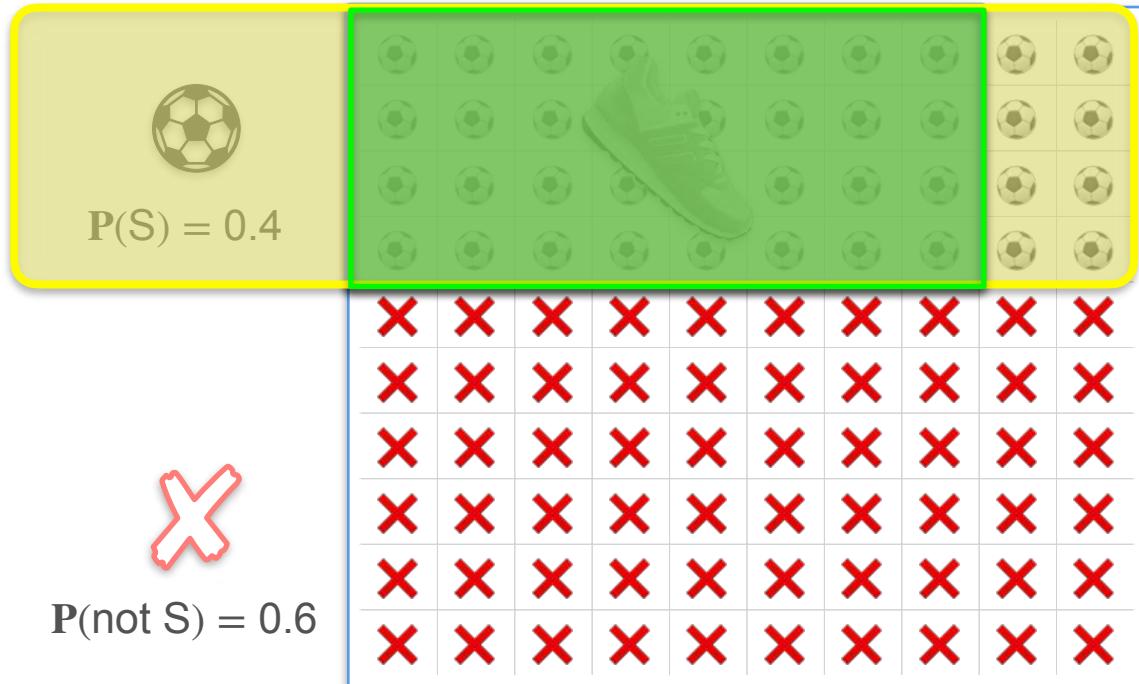
$$P(S \cap R) = P(S) \bullet P(R|S)$$



$$P(S) = 0.4$$



$$P(\text{not } S) = 0.6$$



Conditional Probability

$$P(R | S) = 0.8$$

$P(\text{Soccer and Running shoes})$

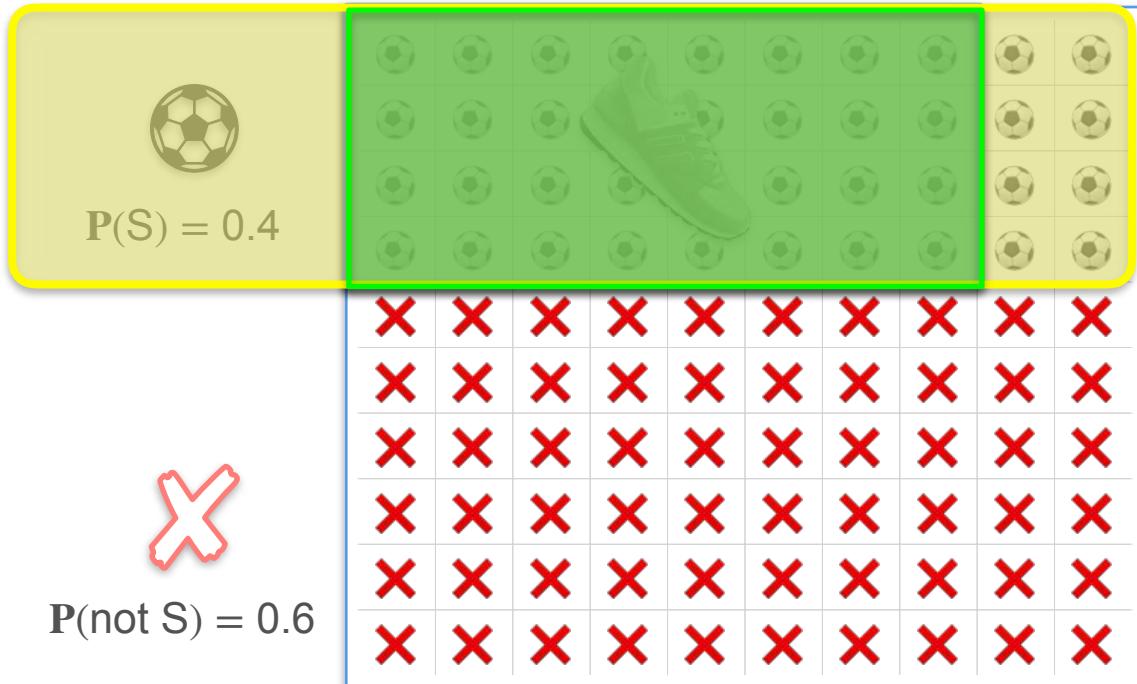
$$P(S \cap R) = P(S) \bullet P(R|S)$$



$$P(S) = 0.4$$



$$P(\text{not } S) = 0.6$$

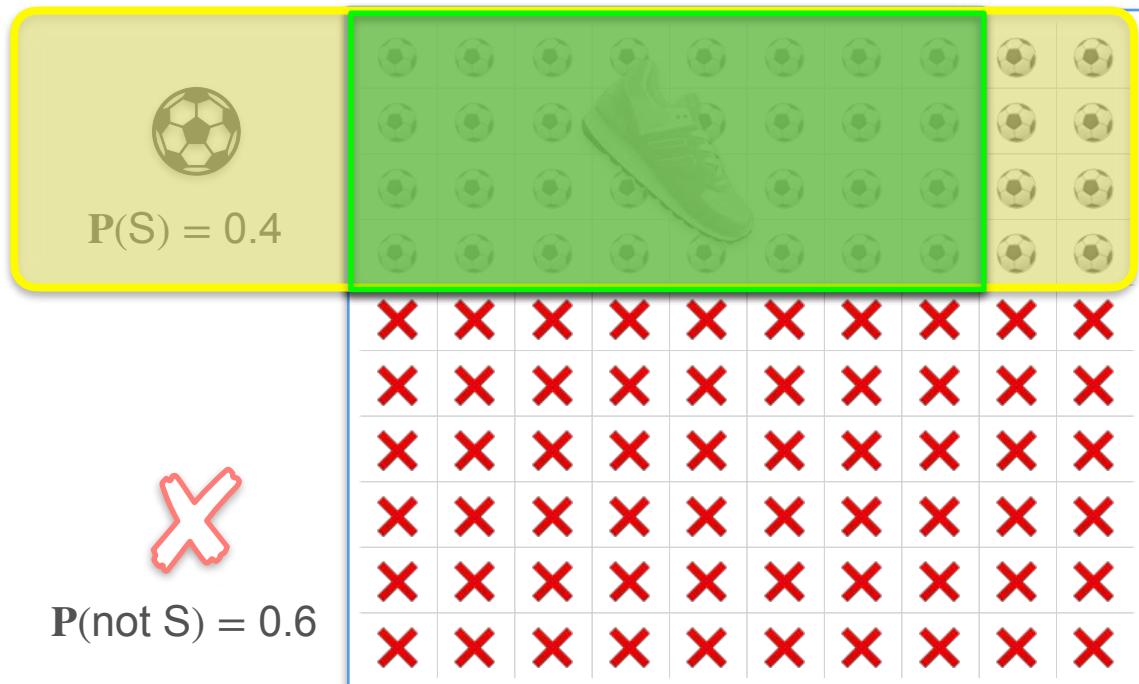


Conditional Probability

$P(\text{Soccer and Running shoes})$

$$\begin{aligned} P(S \cap R) &= P(S) \bullet P(R|S) \\ &= 0.4 \end{aligned}$$

$$P(R | S) = 0.8$$



Conditional Probability

$$P(R | S) = 0.8$$

$P(\text{Soccer and Running shoes})$

$$P(S \cap R) = P(S) \bullet P(R|S)$$

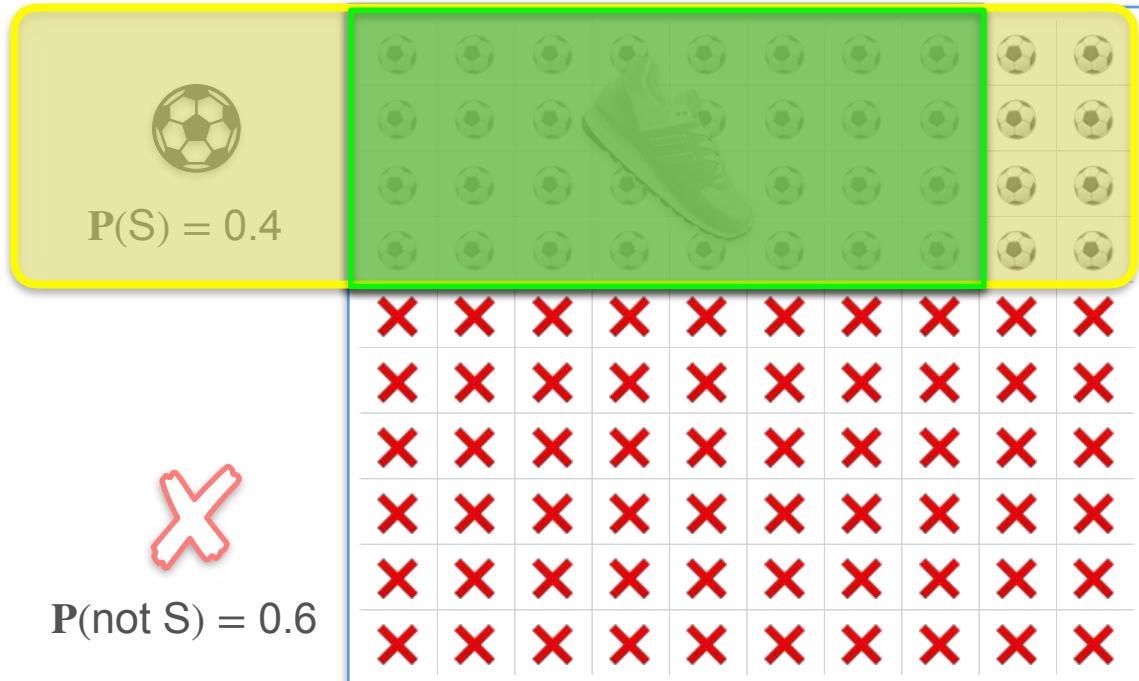
$$= 0.4 \bullet$$



$$P(S) = 0.4$$



$$P(\text{not } S) = 0.6$$

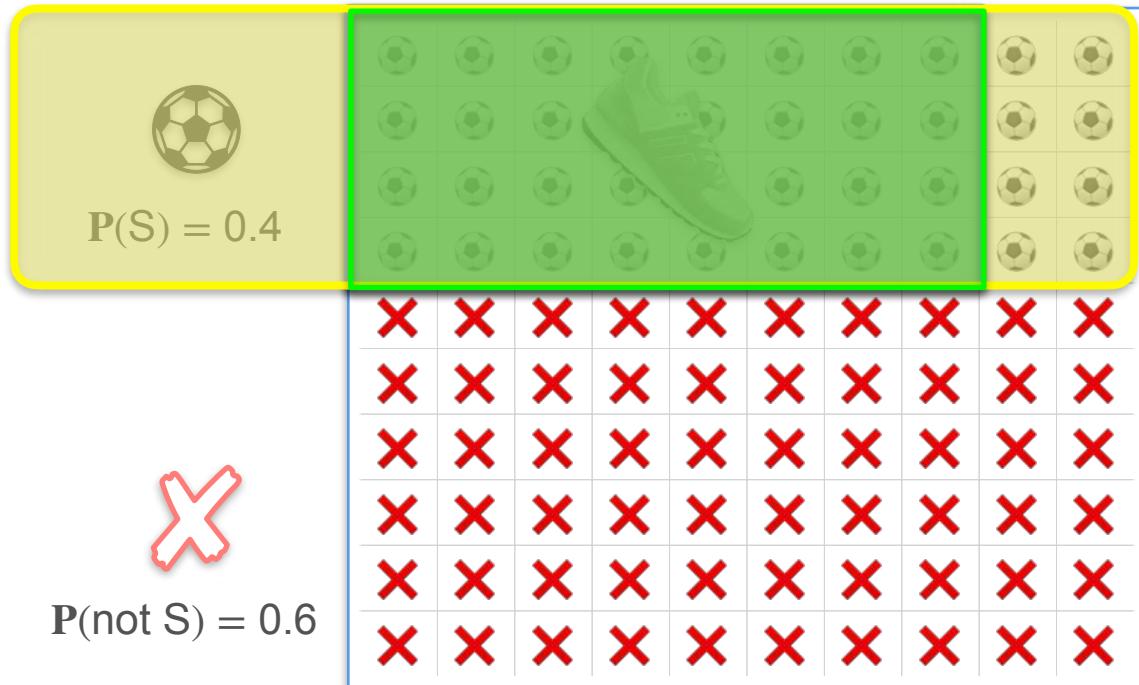


Conditional Probability

$P(\text{Soccer and Running shoes})$

$$\begin{aligned} P(S \cap R) &= P(S) \bullet P(R|S) \\ &= 0.4 \bullet 0.8 \end{aligned}$$

$$P(R | S) = 0.8$$



Conditional Probability

$P(\text{Soccer and Running shoes})$

$$P(S \cap R) = P(S) \bullet P(R|S)$$

$$= 0.4 \bullet 0.8$$

$$= 0.32$$



$$P(S) = 0.4$$

$$P(R | S) = 0.8$$



$$P(\text{not } S) = 0.6$$



Conditional Probability

$P(\text{Soccer and Running shoes})$

$$P(S \cap R) = P(S) \bullet P(R|S)$$

$$= 0.4 \bullet 0.8$$

$$= 0.32$$



$$P(S) = 0.4$$



$$P(\text{not } S) = 0.6$$

$$P(R | S) = 0.8$$



Conditional Probability

$P(\text{Soccer and Running shoes})$

$P(S \cap R) = 0.32$



$P(S) = 0.4$



$P(\text{not } S) = 0.6$

$$P(R | S) = 0.8$$

○	○	○	○	○	○	○	○	○	○
○	○	○	○	○	○	○	○	○	○
○	○	○	○	○	○	○	○	○	○
○	○	○	○	○	○	○	○	○	○
○	○	○	○	○	○	○	○	○	○
✗	✗	✗	✗	✗	✗	✗	✗	✗	✗
✗	✗	✗	✗	✗	✗	✗	✗	✗	✗
✗	✗	✗	✗	✗	✗	✗	✗	✗	✗
✗	✗	✗	✗	✗	✗	✗	✗	✗	✗
✗	✗	✗	✗	✗	✗	✗	✗	✗	✗

Conditional Probability

$P(\text{Soccer and Running shoes})$

$P(S \cap R) = 0.32$



$P(S) = 0.4$



$P(\text{not } S) = 0.6$

$$P(R | S) = 0.8$$

○	○	○	○	○	○	○	○	○	○
○	○	○	○	○	○	○	○	○	○
○	○	○	○	○	○	○	○	○	○
○	○	○	○	○	○	○	○	○	○
○	○	○	○	○	○	○	○	○	○
✗	✗	✗	✗	✗	✗	✗	✗	✗	✗
✗	✗	✗	✗	✗	✗	✗	✗	✗	✗
✗	✗	✗	✗	✗	✗	✗	✗	✗	✗
✗	✗	✗	✗	✗	✗	✗	✗	✗	✗
✗	✗	✗	✗	✗	✗	✗	✗	✗	✗

$$P(R | \text{not } S) = 0.5$$

Conditional Probability

$P(\text{Soccer and Running shoes})$



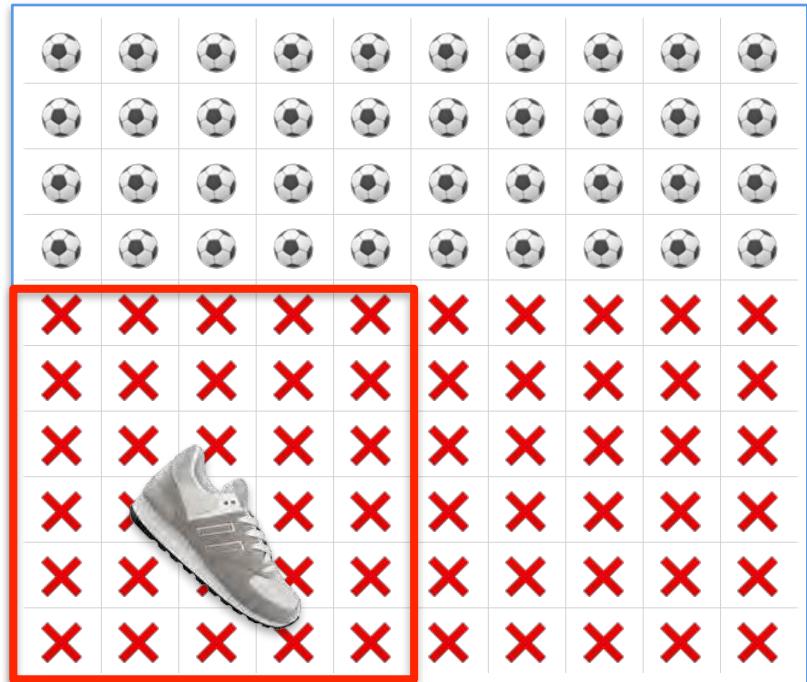
$P(S \cap R) = 0.32$

$P(S) = 0.4$



$P(\text{not } S) = 0.6$

$P(R | S) = 0.8$



$P(R | \text{not } S) = 0.5$

Conditional Probability

$P(\text{Soccer and Running shoes})$



$P(S \cap R) = 0.32$

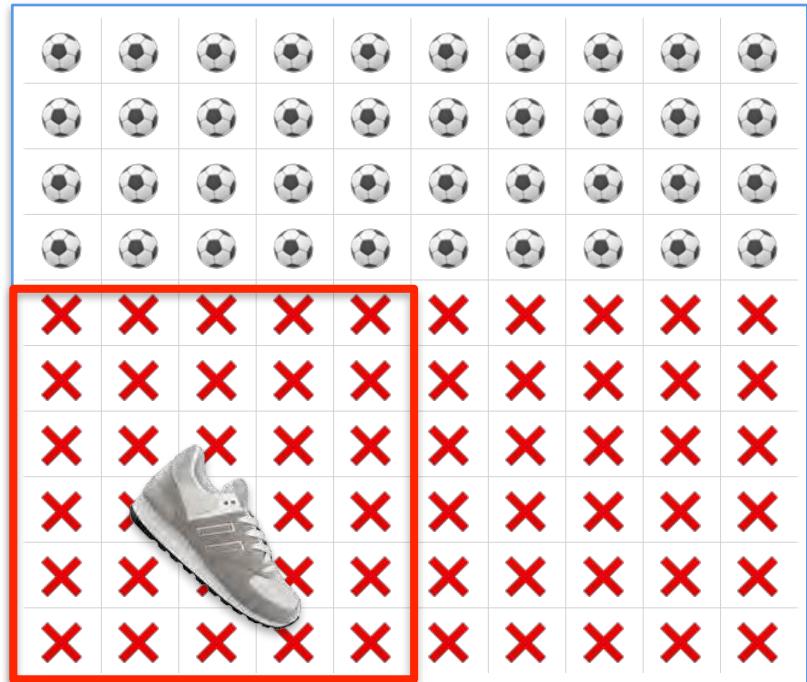
$P(S) = 0.4$

$P(\text{not Soccer and Running shoes})$



$P(\text{not } S) = 0.6$

$$P(R | S) = 0.8$$



$P(R | \text{not } S) = 0.5$

Conditional Probability

$P(\text{Soccer and Running shoes})$



$$P(S \cap R) = 0.32$$

$$P(S) = 0.4$$

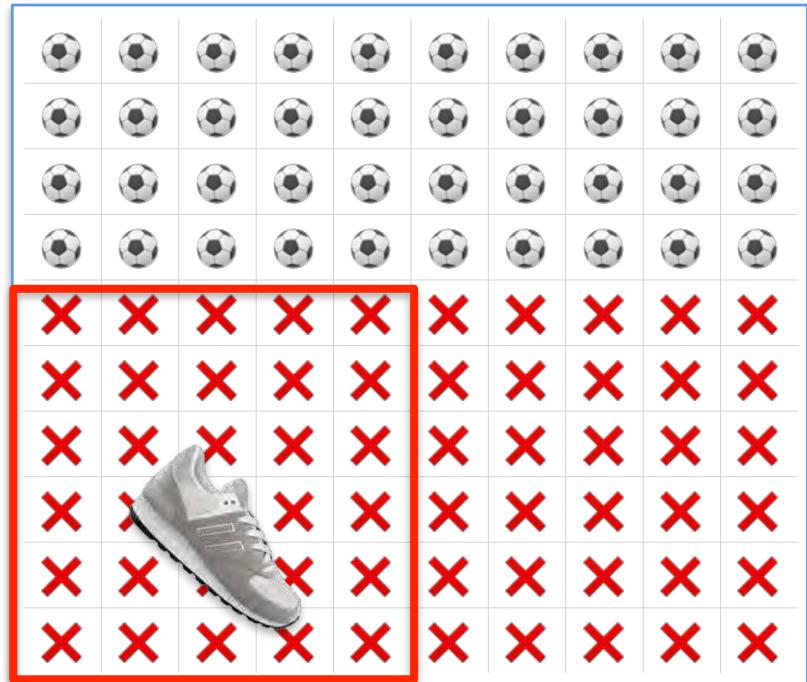
$P(\text{not Soccer and Running shoes})$

$$P(\text{not } S \cap R) =$$



$$P(\text{not } S) = 0.6$$

$$P(R | S) = 0.8$$



Conditional Probability

$P(\text{Soccer and Running shoes})$



$$P(S \cap R) = 0.32$$

$$P(S) = 0.4$$

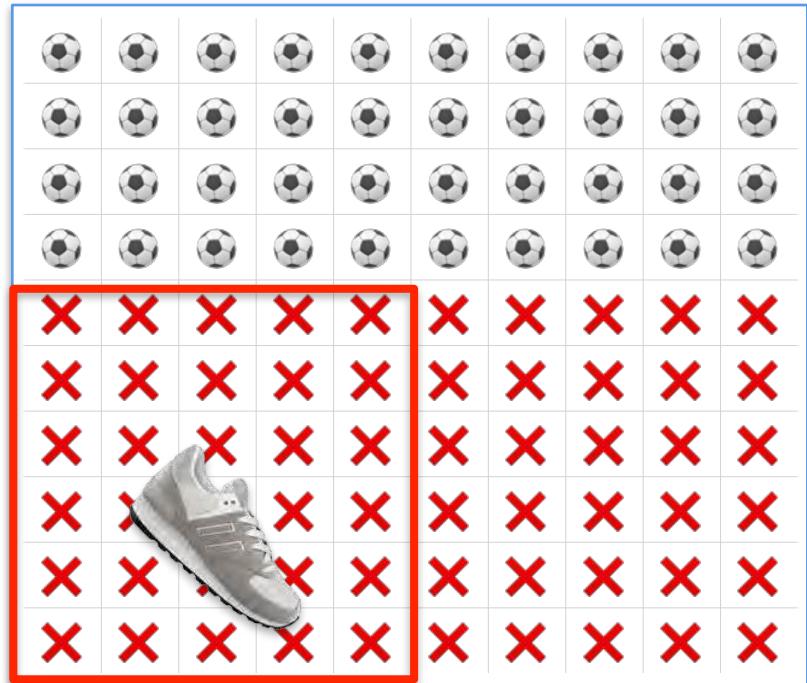
$P(\text{not Soccer and Running shoes})$

$$P(\text{not } S \cap R) = P(\text{not } S)$$



$$P(\text{not } S) = 0.6$$

$$P(R | S) = 0.8$$



$$P(R | \text{not } S) = 0.5$$

Conditional Probability

$P(\text{Soccer and Running shoes})$



$$P(S \cap R) = 0.32$$

$$P(S) = 0.4$$

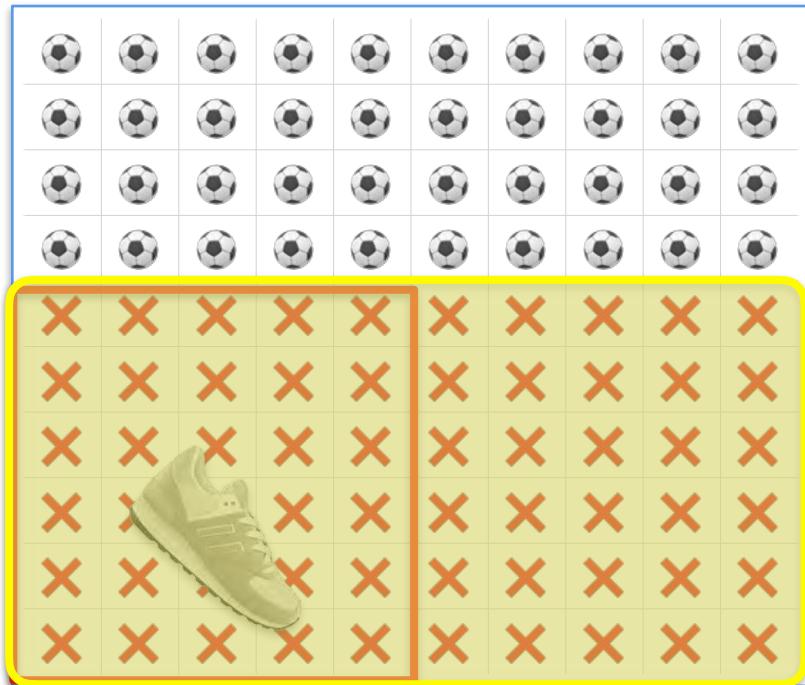
$P(\text{not Soccer and Running shoes})$

$$P(\text{not } S \cap R) = P(\text{not } S)$$



$$P(\text{not } S) = 0.6$$

$$P(R | S) = 0.8$$



$$P(R | \text{not } S) = 0.5$$

Conditional Probability

$P(\text{Soccer and Running shoes})$



$$P(S \cap R) = 0.32$$

$$P(S) = 0.4$$

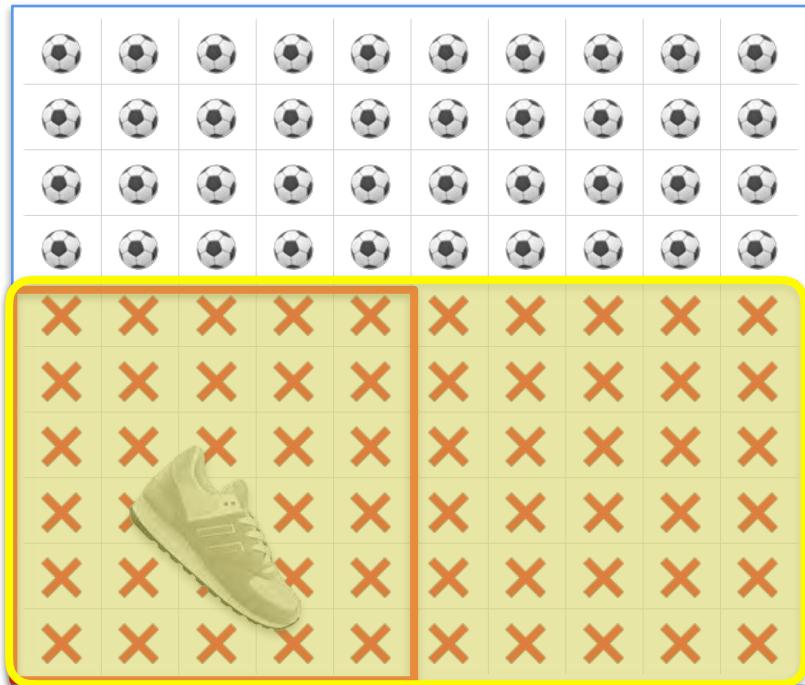
$P(\text{not Soccer and Running shoes})$

$$P(\text{not } S \cap R) = P(\text{not } S) \bullet$$



$$P(\text{not } S) = 0.6$$

$$P(R | S) = 0.8$$



$$P(R | \text{not } S) = 0.5$$

Conditional Probability

$P(\text{Soccer and Running shoes})$



$$P(S \cap R) = 0.32$$

$$P(S) = 0.4$$

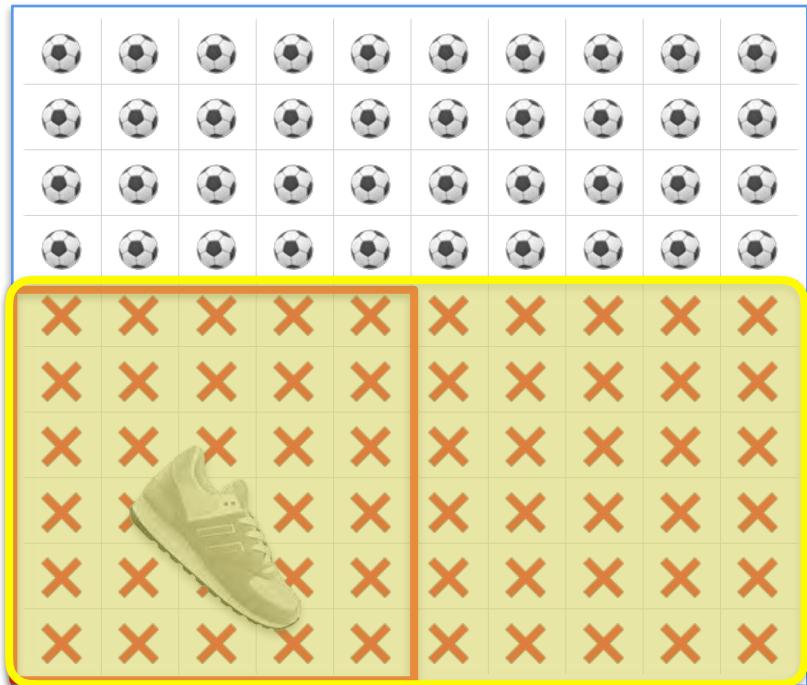
$P(\text{not Soccer and Running shoes})$

$$P(\text{not } S \cap R) = P(\text{not } S) \bullet P(R | \text{not } S)$$



$$P(\text{not } S) = 0.6$$

$$P(R | S) = 0.8$$



$$P(R | \text{not } S) = 0.5$$

Conditional Probability

$P(\text{Soccer and Running shoes})$



$$P(S \cap R) = 0.32$$

$$P(S) = 0.4$$

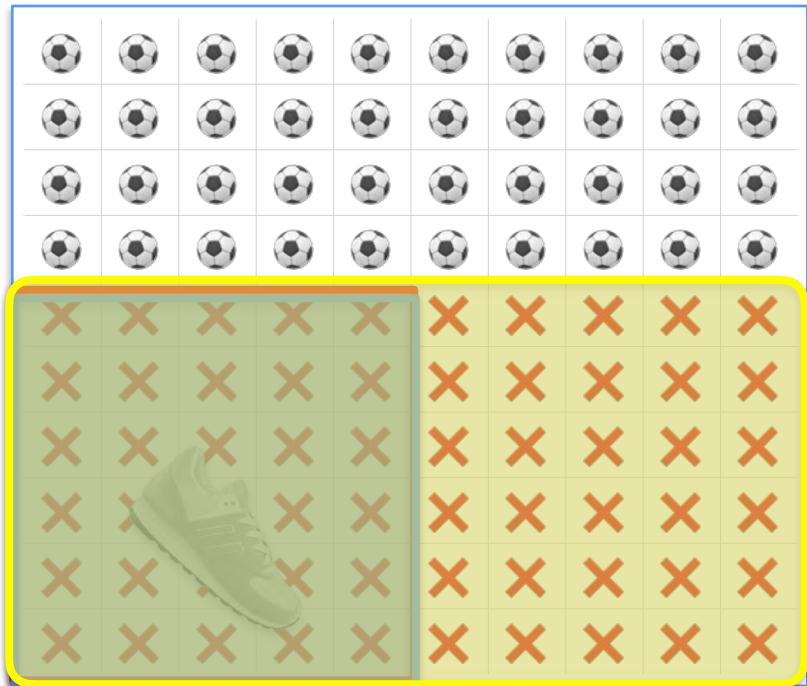
$P(\text{not Soccer and Running shoes})$

$$P(\text{not } S \cap R) = P(\text{not } S) \bullet P(R | \text{not } S)$$



$$P(\text{not } S) = 0.6$$

$$P(R | S) = 0.8$$



$$P(R | \text{not } S) = 0.5$$

Conditional Probability

$P(\text{Soccer and Running shoes})$



$$P(S \cap R) = 0.32$$

$$P(S) = 0.4$$

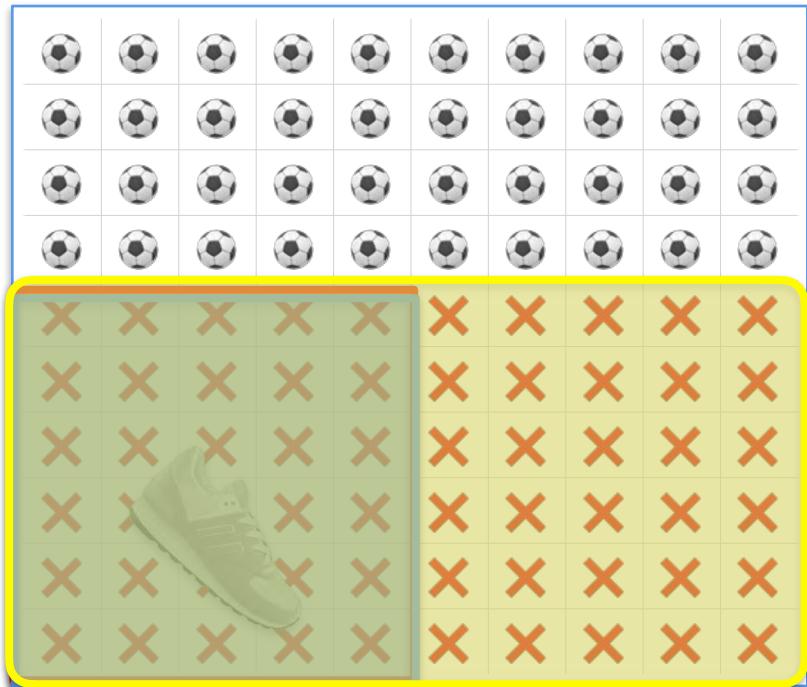
$P(\text{not Soccer and Running shoes})$

$$\begin{aligned} P(\text{not } S \cap R) &= P(\text{not } S) \bullet P(R | \text{not } S) \\ &= 0.6 \end{aligned}$$



$$P(\text{not } S) = 0.6$$

$$P(R | S) = 0.8$$



Conditional Probability

$P(\text{Soccer and Running shoes})$



$$P(S \cap R) = 0.32$$

$$P(S) = 0.4$$

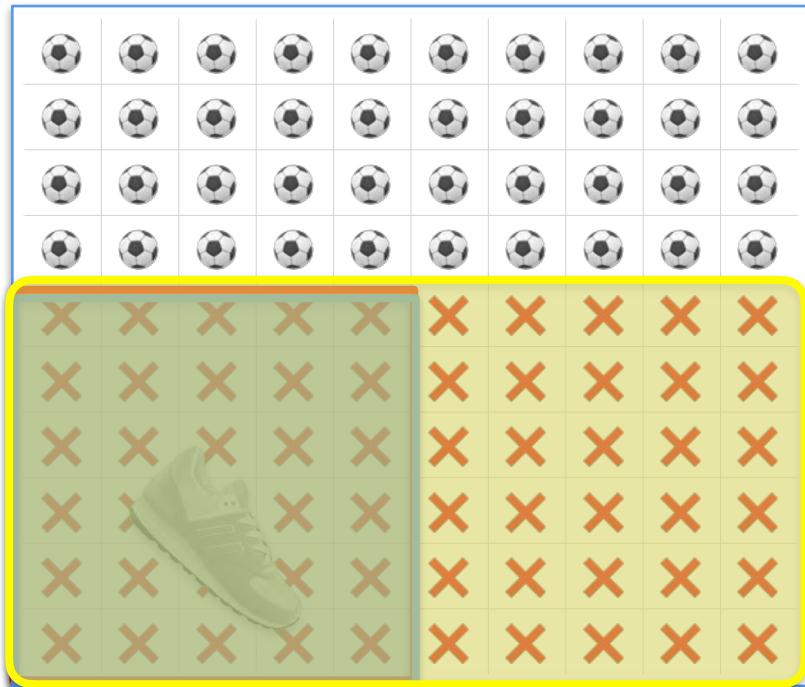
$P(\text{not Soccer and Running shoes})$

$$\begin{aligned} P(\text{not } S \cap R) &= P(\text{not } S) \bullet P(R | \text{not } S) \\ &= 0.6 \bullet \end{aligned}$$



$$P(\text{not } S) = 0.6$$

$$P(R | S) = 0.8$$



$$P(R | \text{not } S) = 0.5$$

Conditional Probability

$P(\text{Soccer and Running shoes})$



$$P(S \cap R) = 0.32$$

$$P(S) = 0.4$$

$P(\text{not Soccer and Running shoes})$

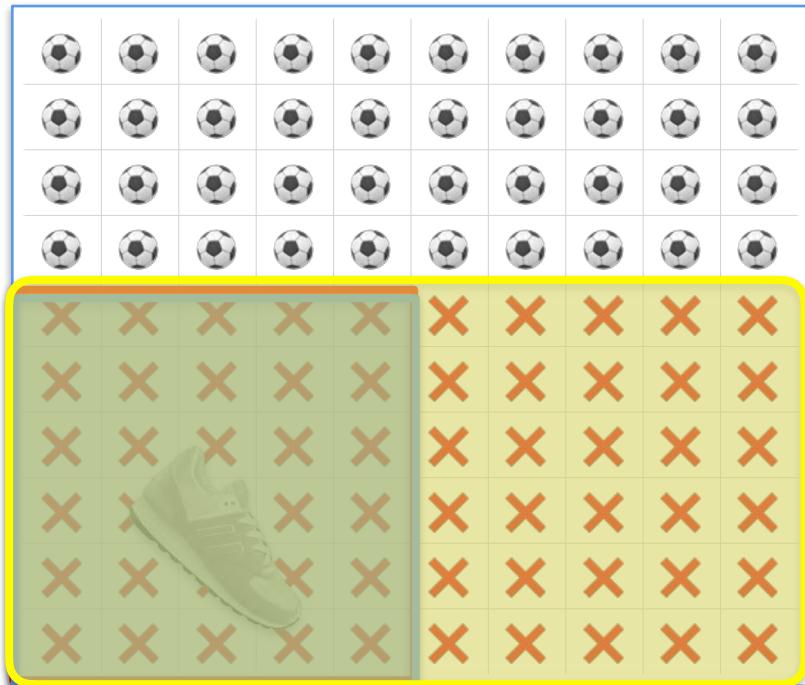
$$P(\text{not } S \cap R) = P(\text{not } S) \bullet P(R | \text{not } S)$$

$$= 0.6 \bullet 0.5$$



$$P(\text{not } S) = 0.6$$

$$P(R | S) = 0.8$$



Conditional Probability

$P(\text{Soccer and Running shoes})$



$$P(S \cap R) = 0.32$$

$$P(S) = 0.4$$

$P(\text{not Soccer and Running shoes})$

$$P(\text{not } S \cap R) = P(\text{not } S) \bullet P(R | \text{not } S)$$

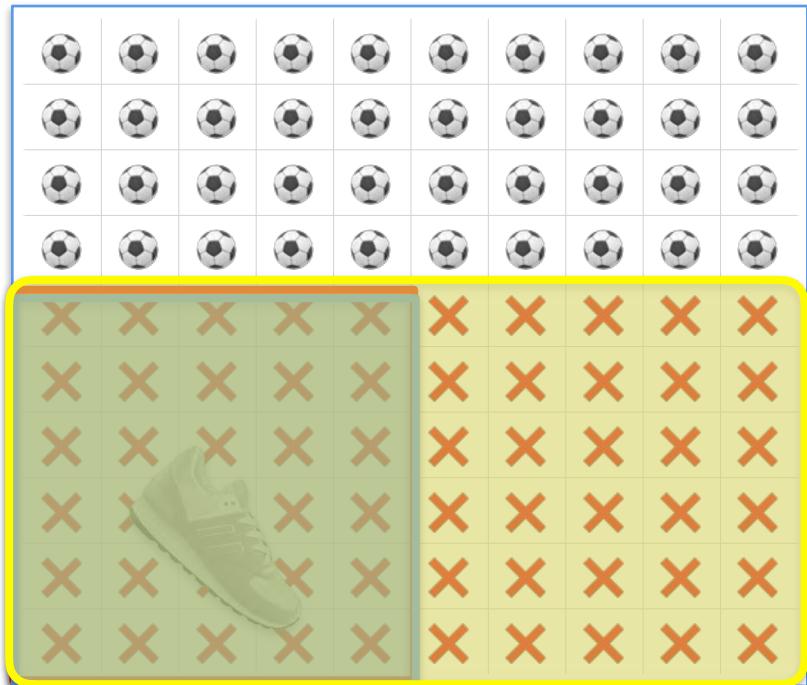
$$= 0.6 \bullet 0.5$$

$$= 0.3$$



$$P(\text{not } S) = 0.6$$

$$P(R | S) = 0.8$$



$$P(R | \text{not } S) = 0.5$$

Conditional Probability

$P(\text{Soccer and Running shoes})$



$$P(S \cap R) = 0.32$$

$$P(S) = 0.4$$

$P(\text{not Soccer and Running shoes})$

$$P(\text{not } S \cap R) = P(\text{not } S) \bullet P(R | \text{not } S)$$

$$= 0.6 \bullet 0.5$$

$$= 0.3$$

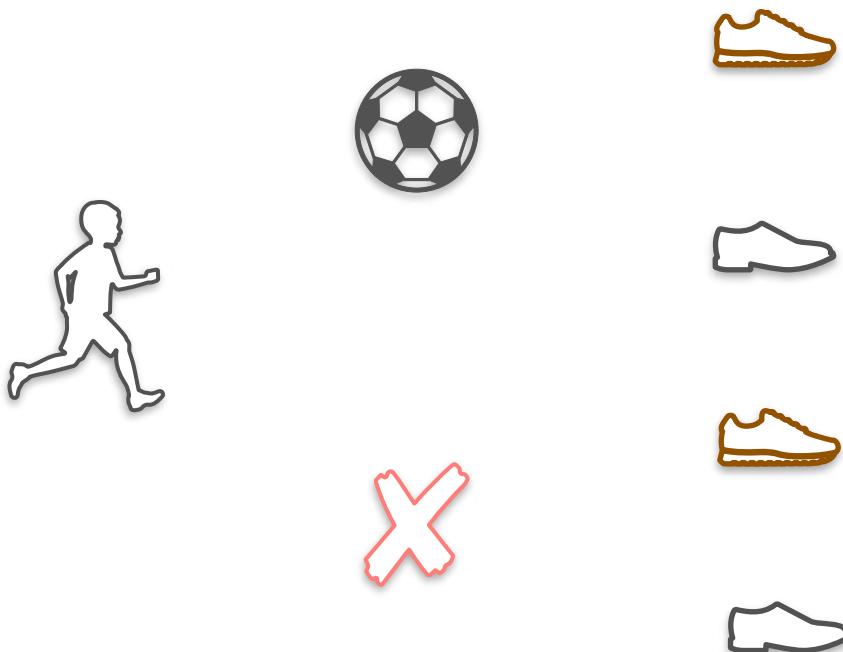


$$P(\text{not } S) = 0.6$$

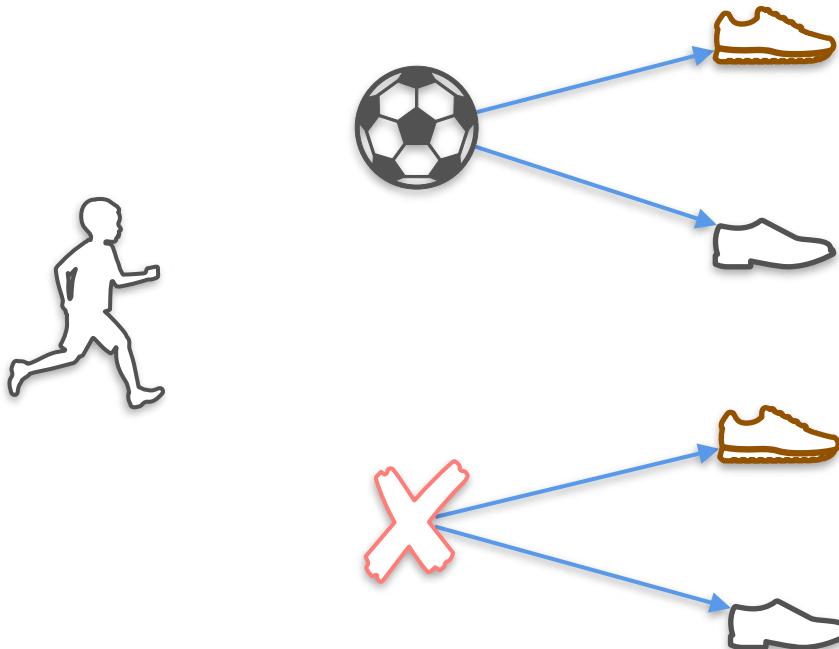
$$P(R | S) = 0.8$$



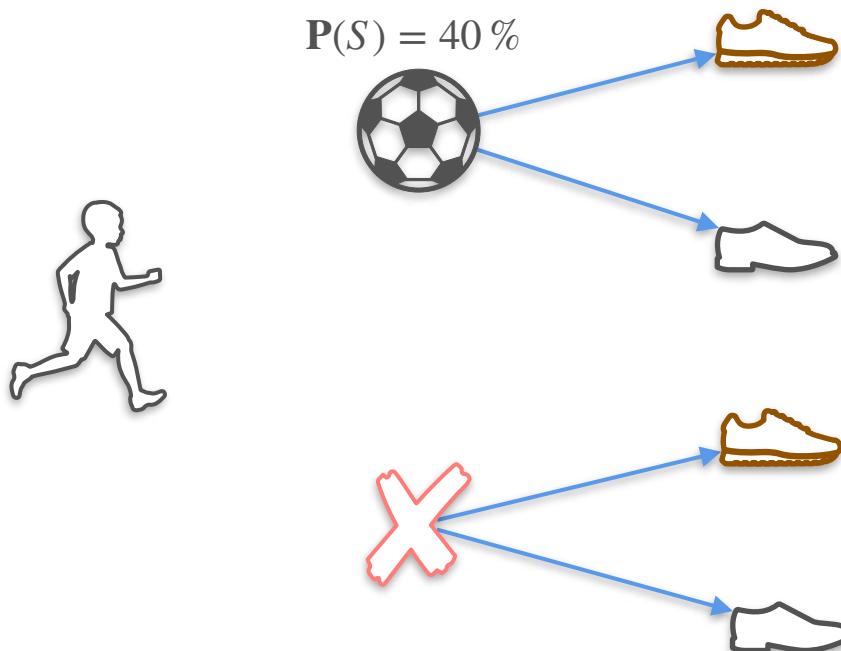
Conditional Probability



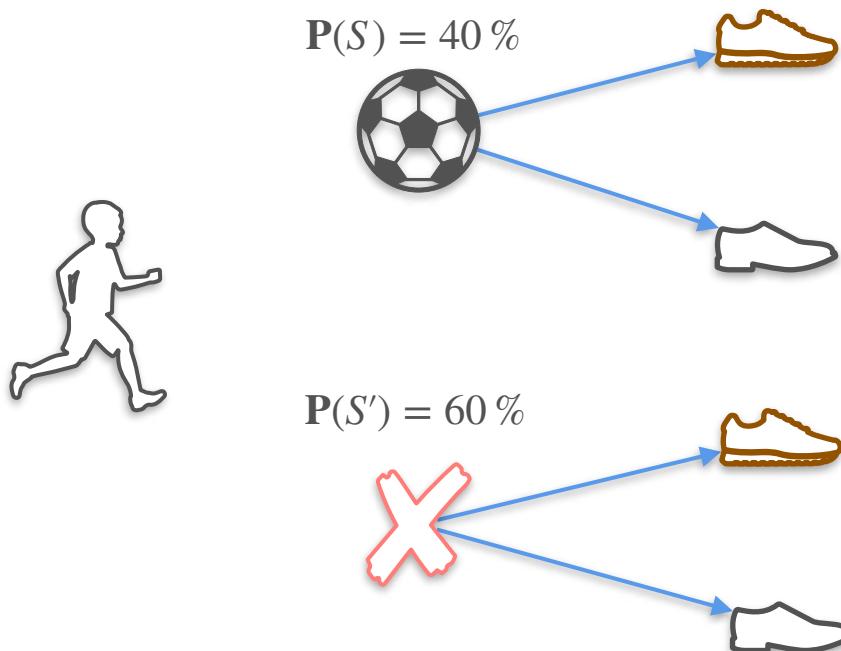
Conditional Probability



Conditional Probability

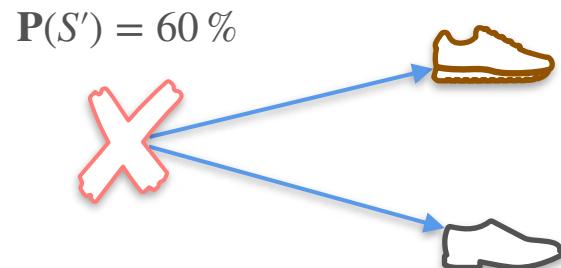
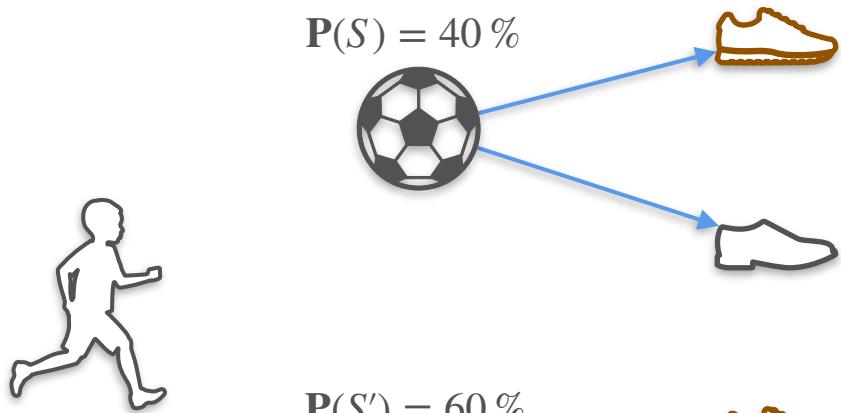


Conditional Probability

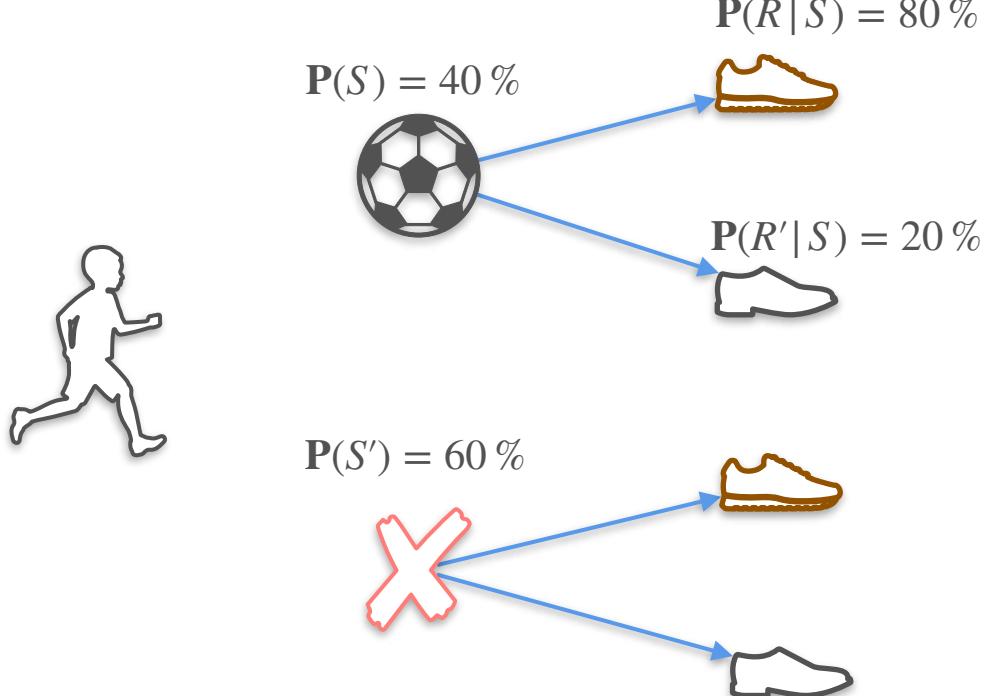


Conditional Probability

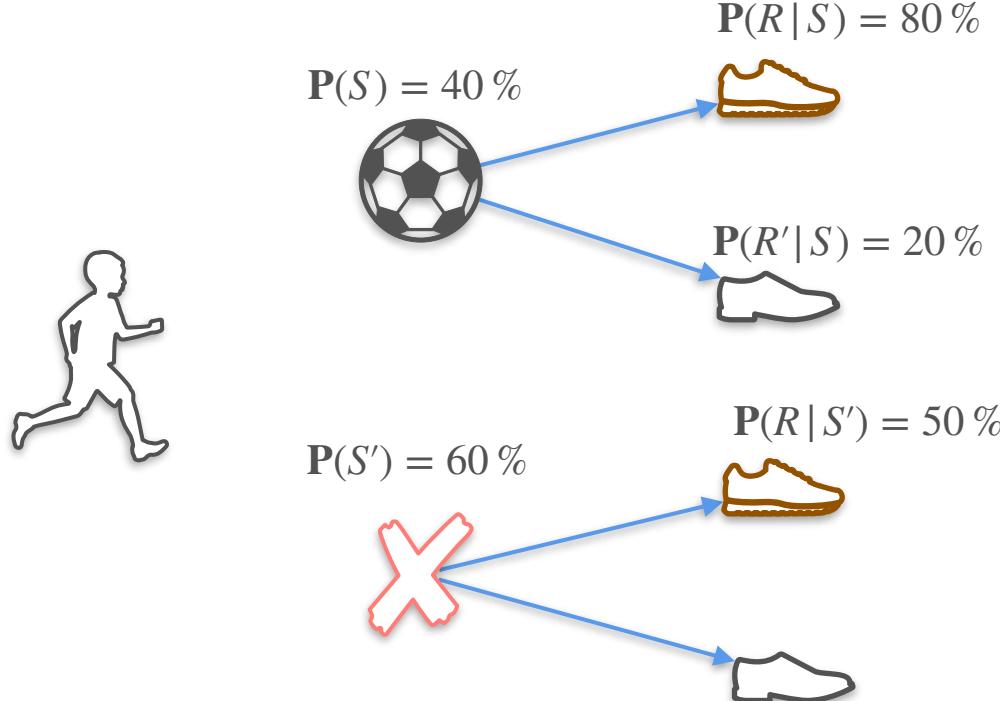
$$P(R | S) = 80 \%$$



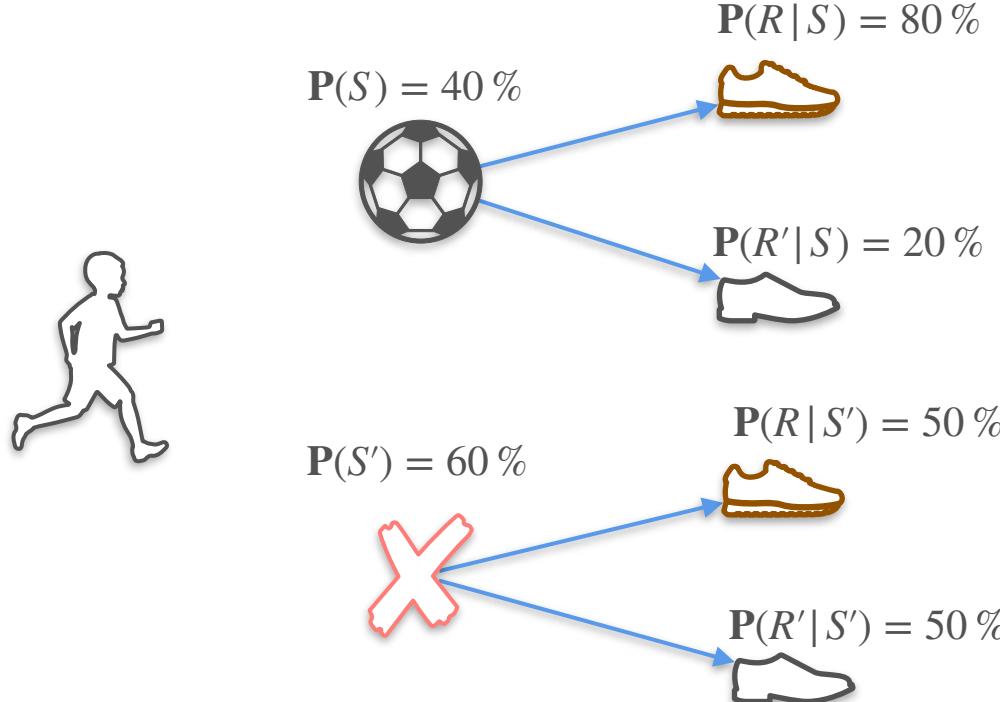
Conditional Probability



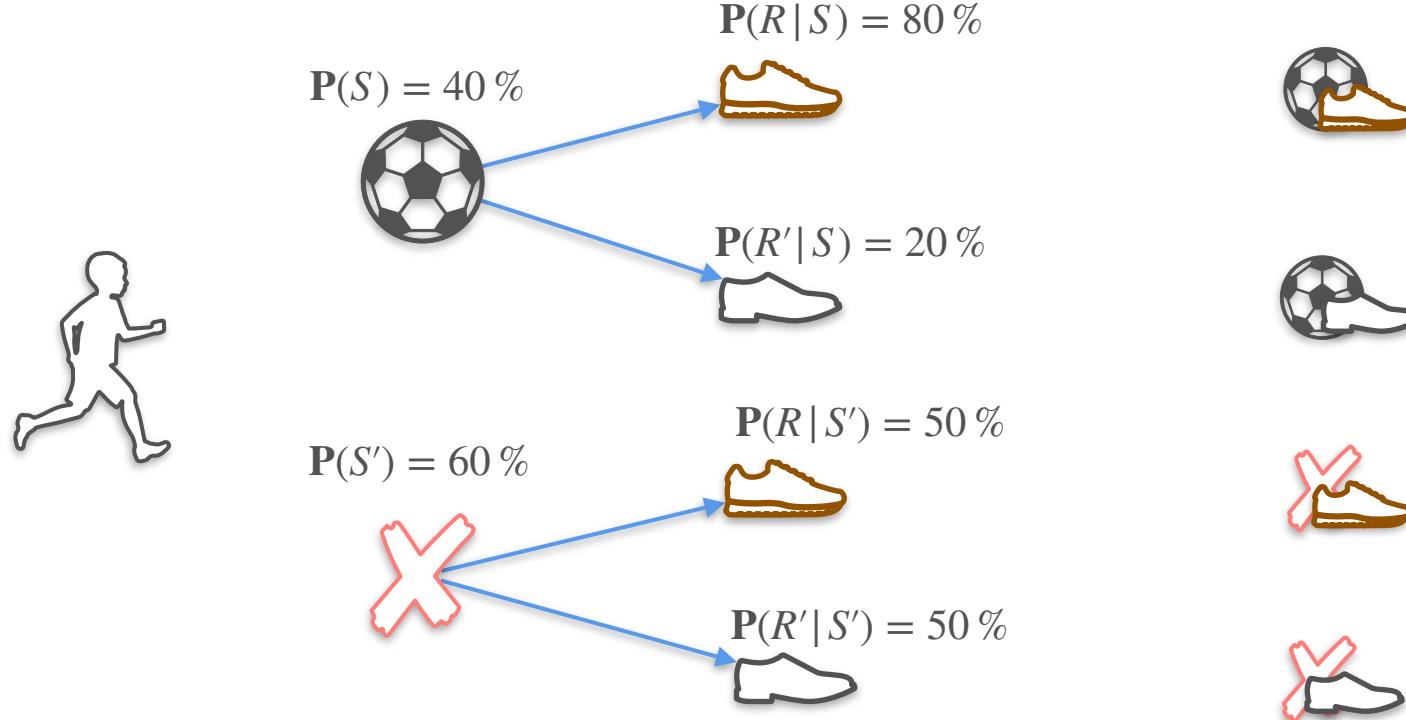
Conditional Probability



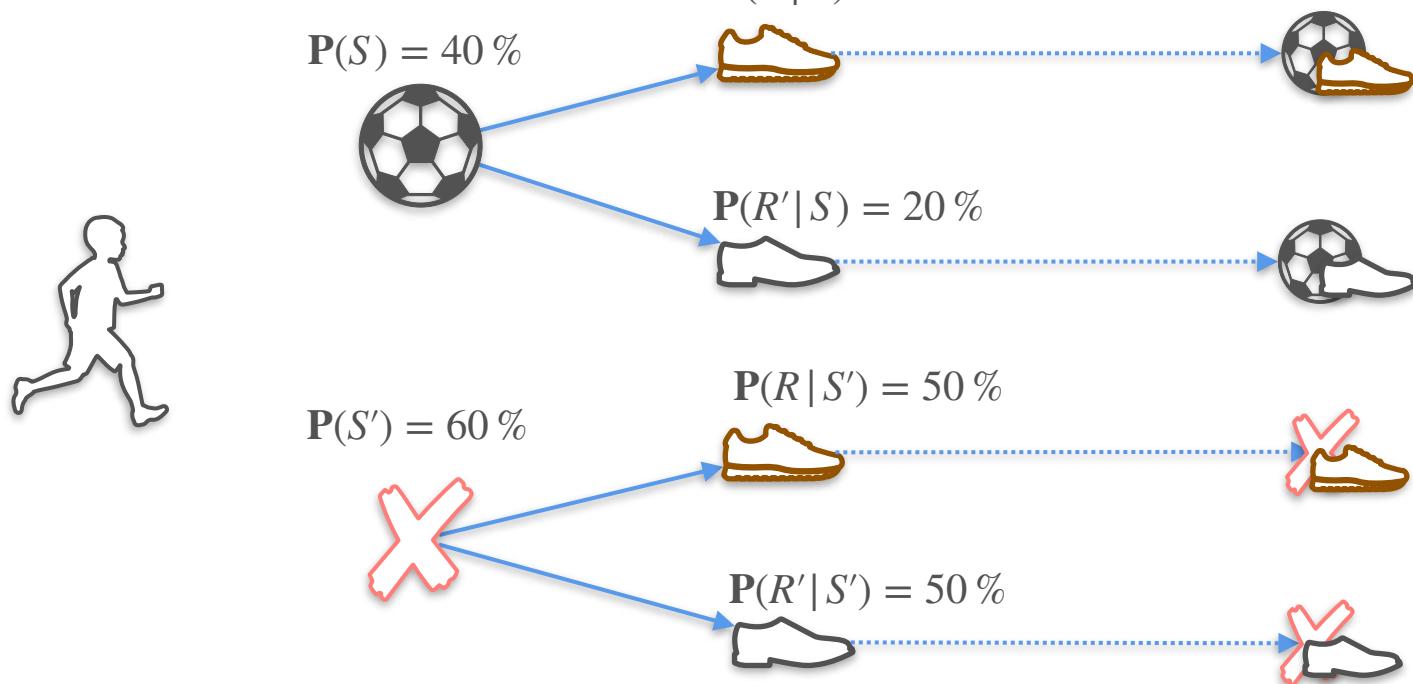
Conditional Probability



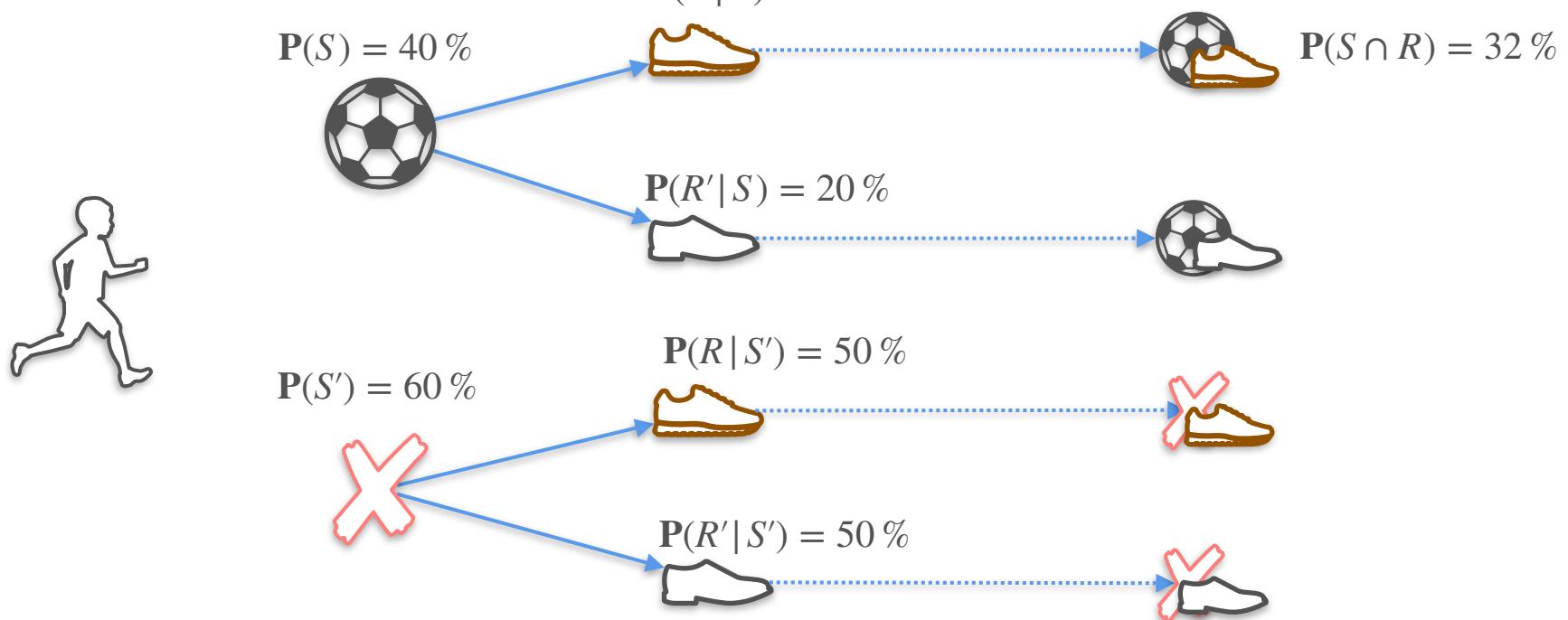
Conditional Probability



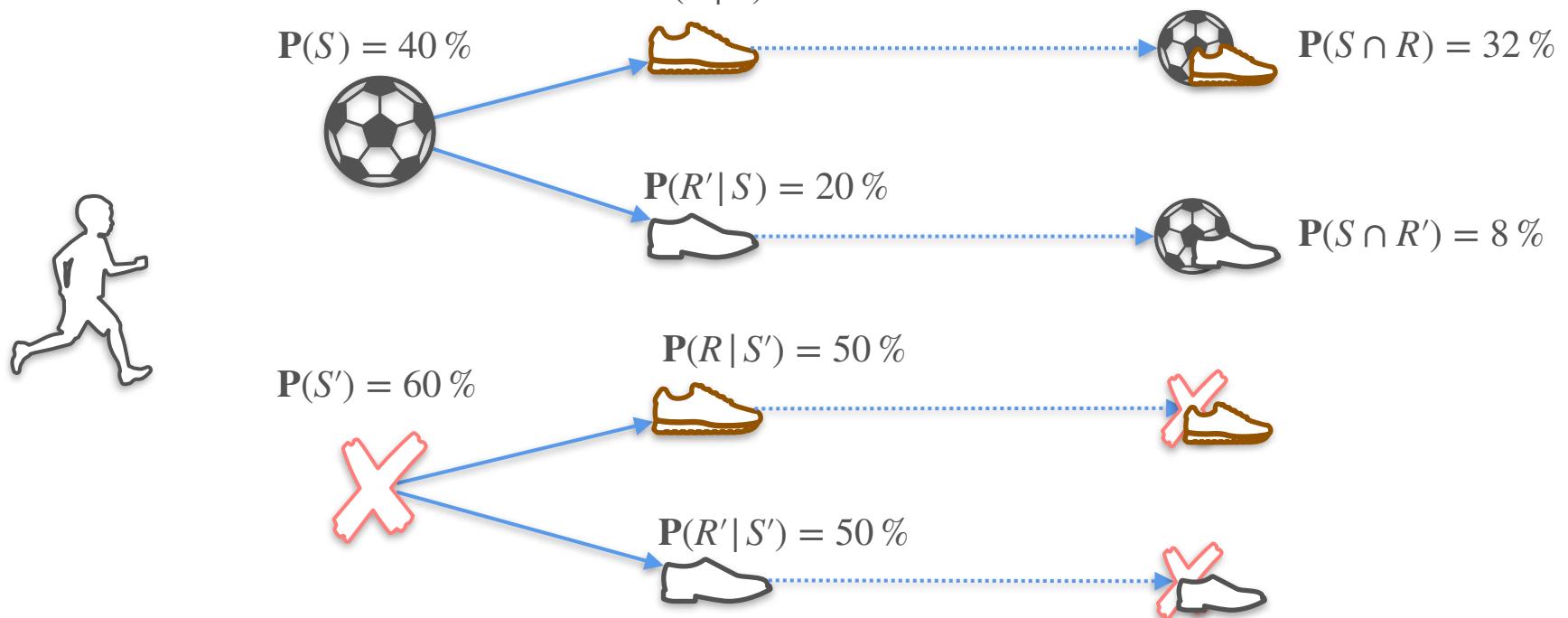
Conditional Probability



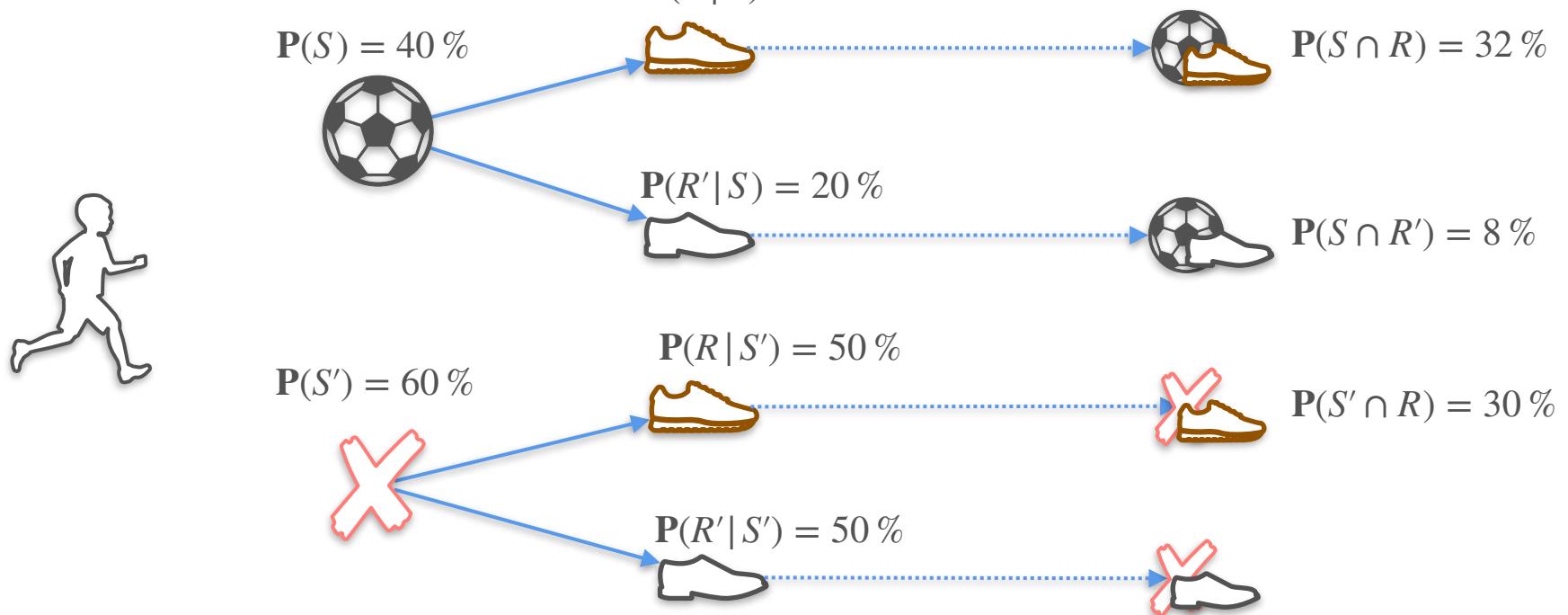
Conditional Probability



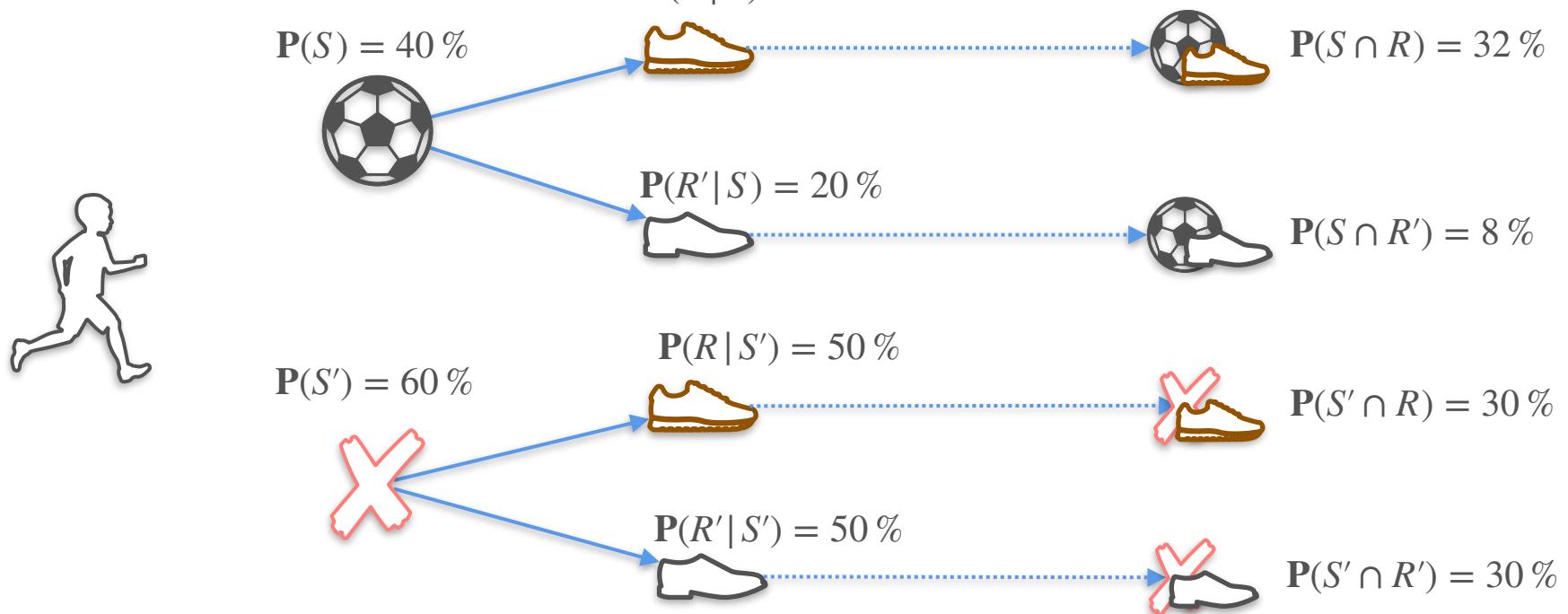
Conditional Probability



Conditional Probability

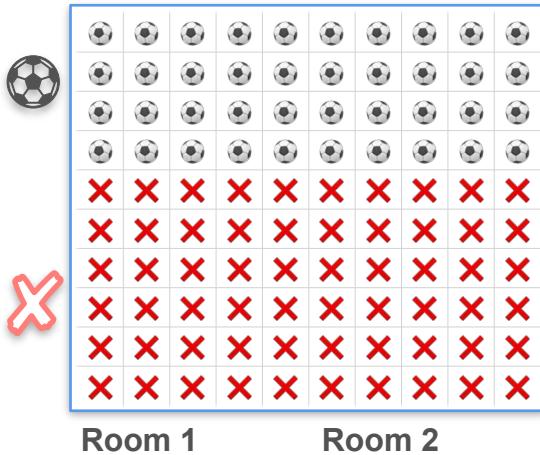


Conditional Probability

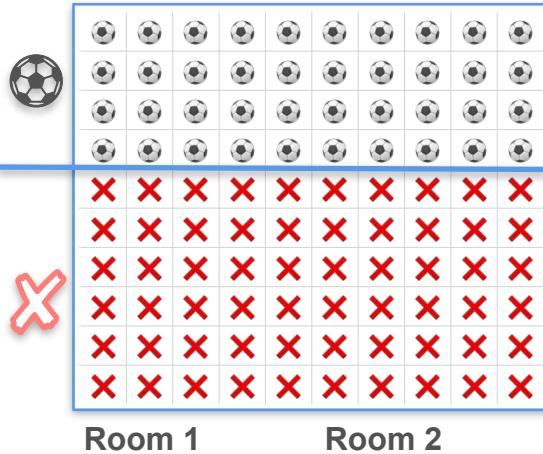


Independent vs Dependent Events

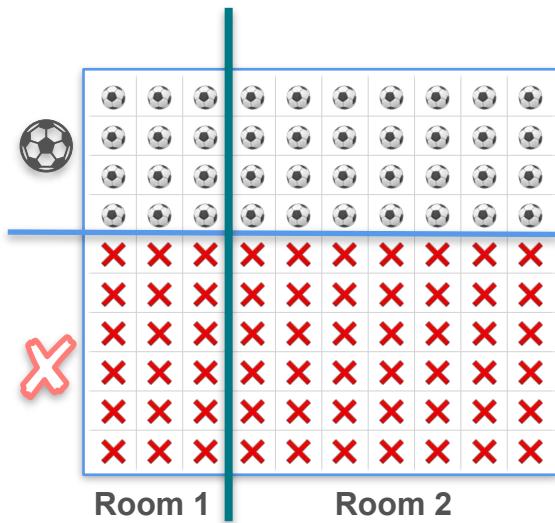
Independent vs Dependent Events



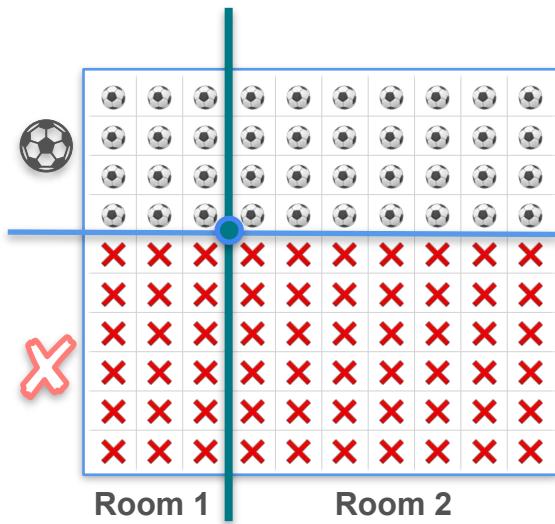
Independent vs Dependent Events



Independent vs Dependent Events



Independent vs Dependent Events



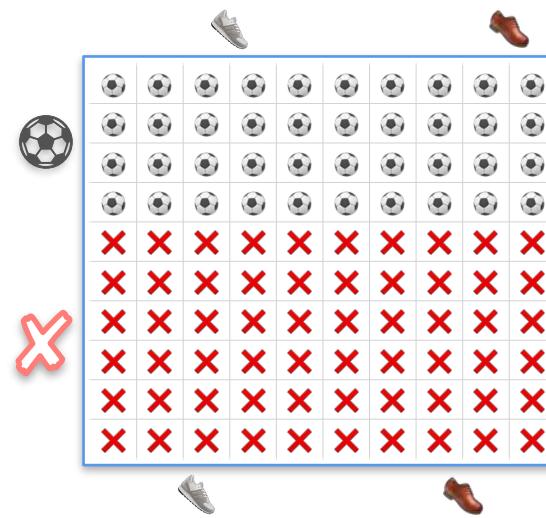
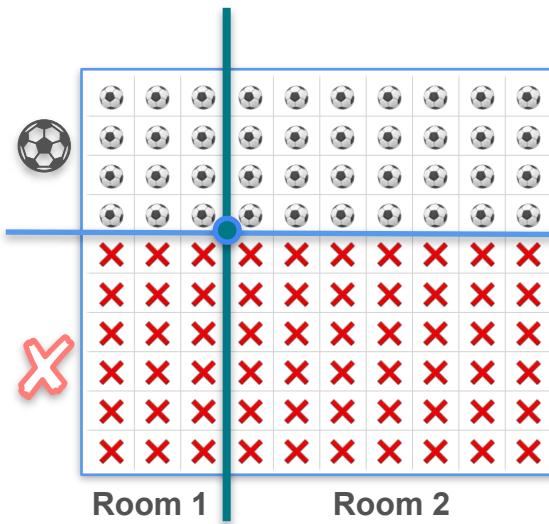
Independent vs Dependent Events

Independent



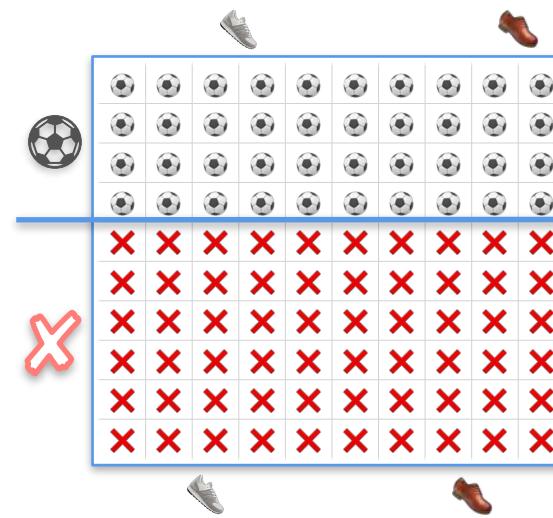
Independent vs Dependent Events

Independent



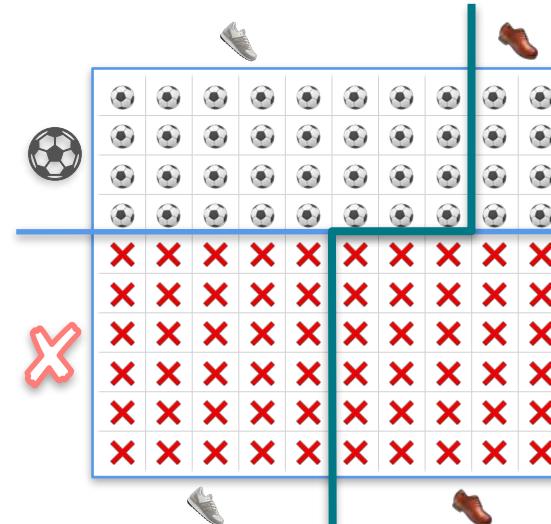
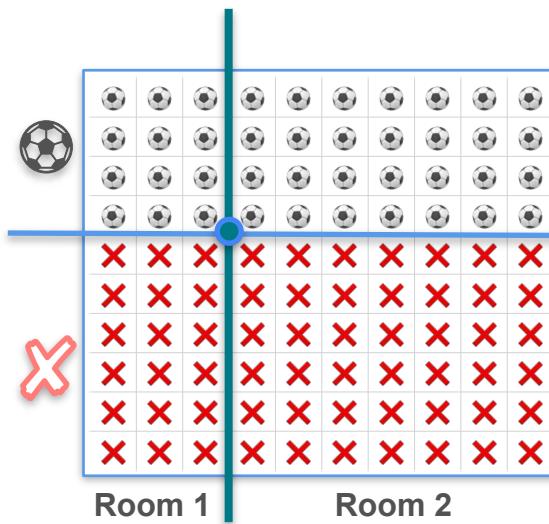
Independent vs Dependent Events

Independent



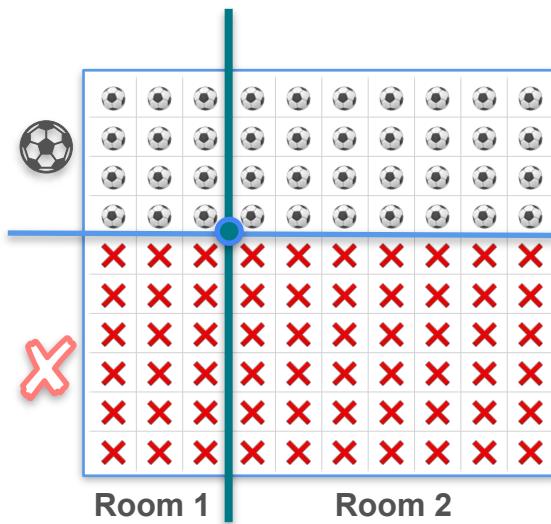
Independent vs Dependent Events

Independent

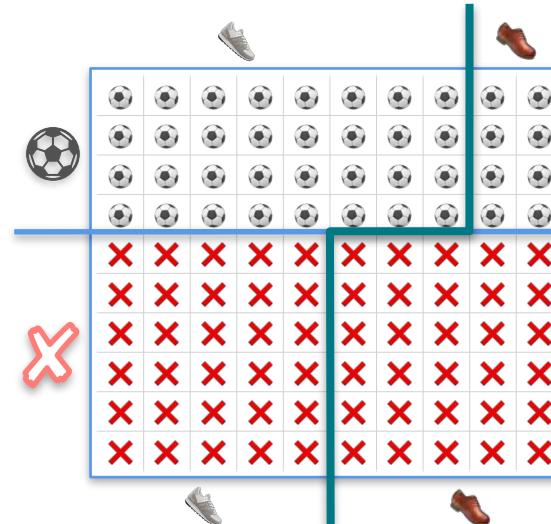


Independent vs Dependent Events

Independent

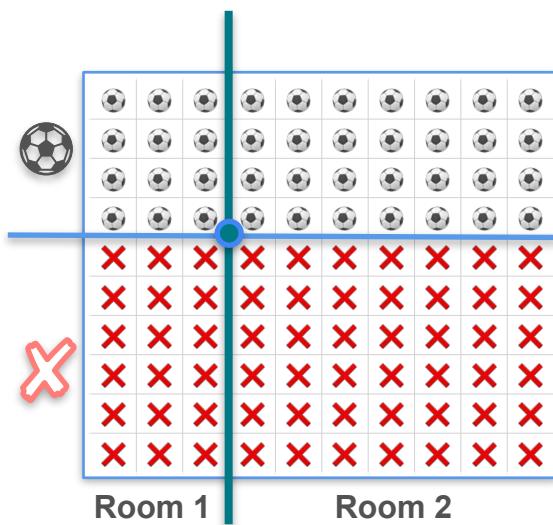


Dependent

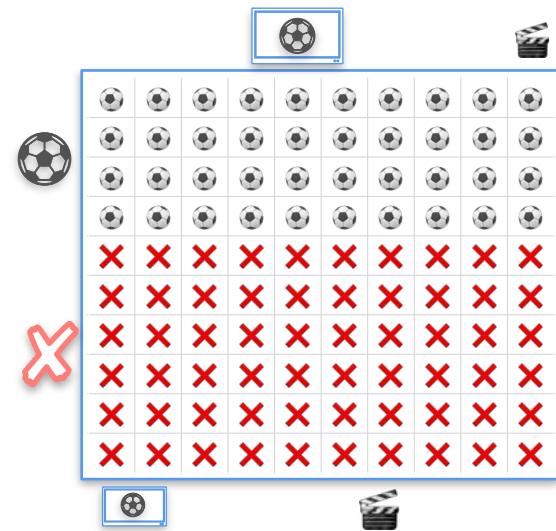
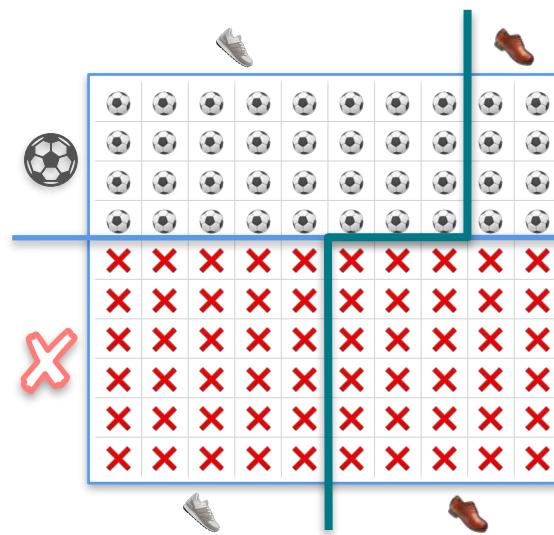


Independent vs Dependent Events

Independent



Dependent

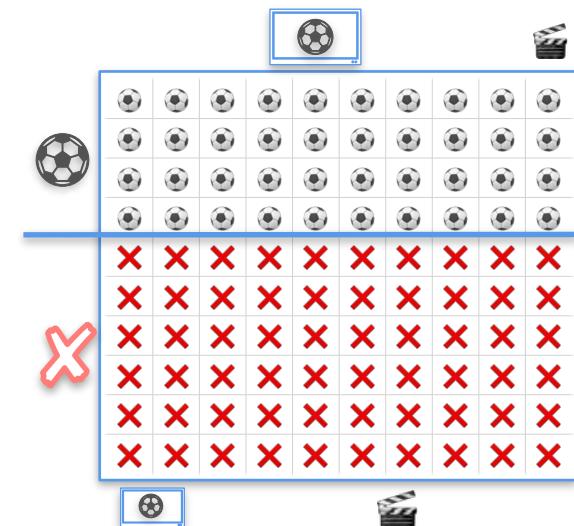
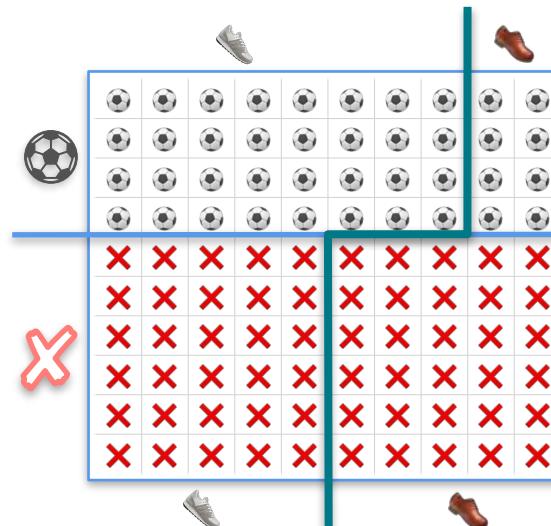


Independent vs Dependent Events

Independent



Dependent

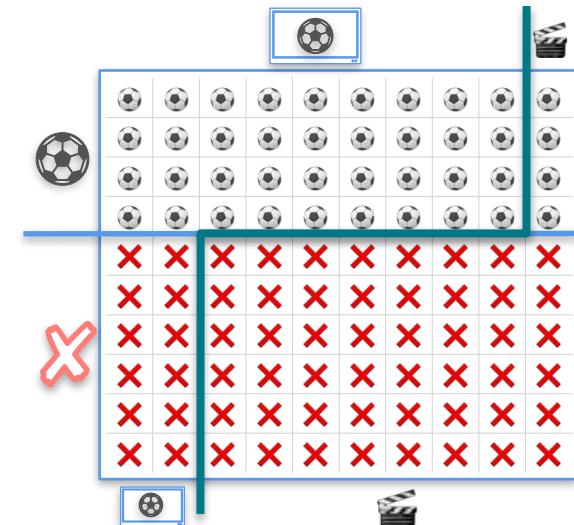
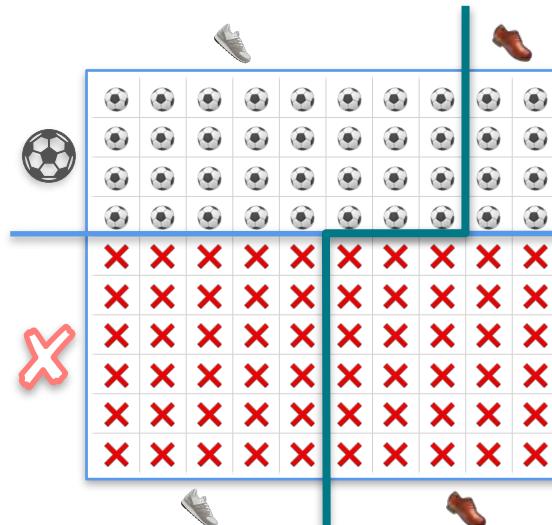


Independent vs Dependent Events

Independent

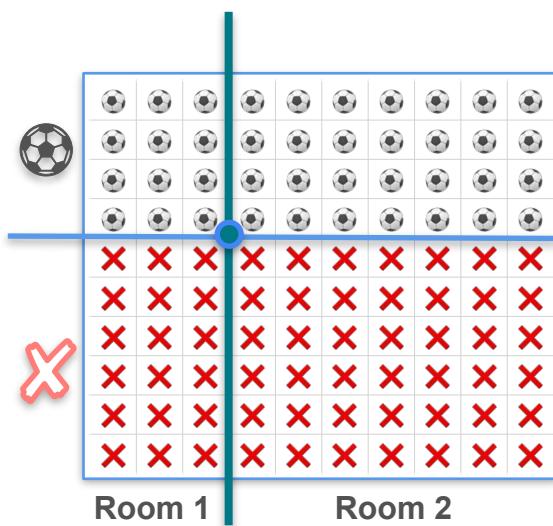


Dependent

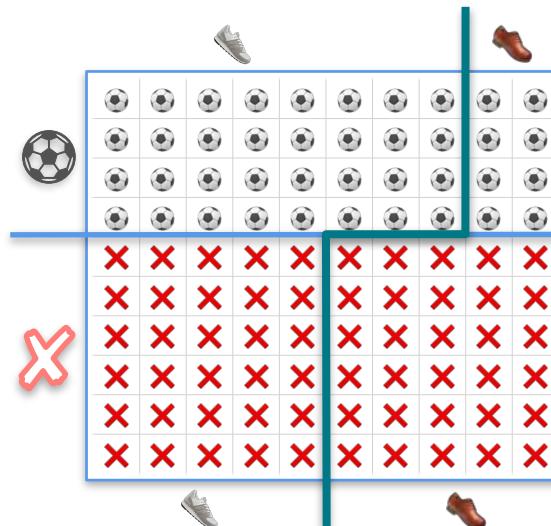


Independent vs Dependent Events

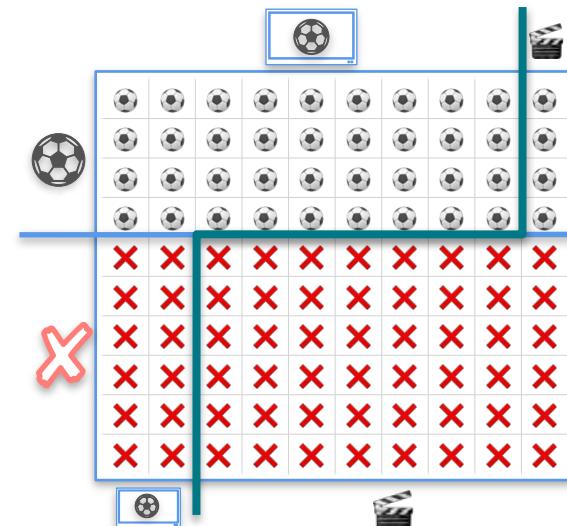
Independent



Dependent



Dependent



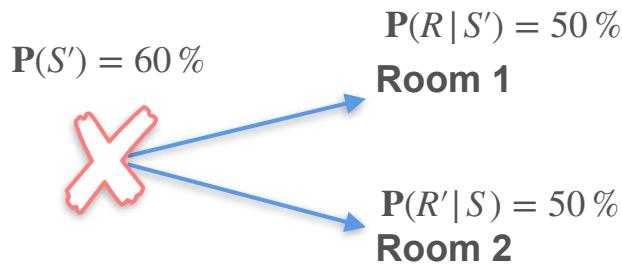
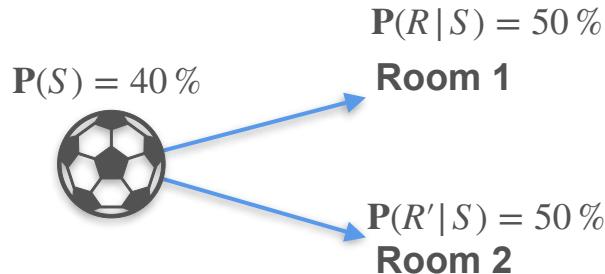
Conditional Probability

Conditional Probability

Independent

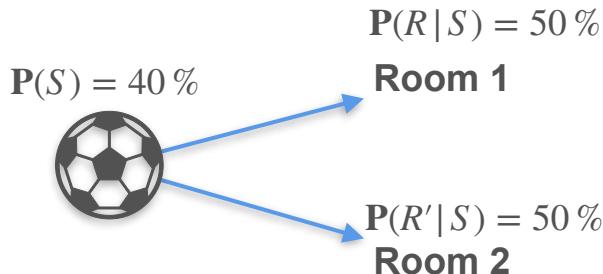
Conditional Probability

Independent

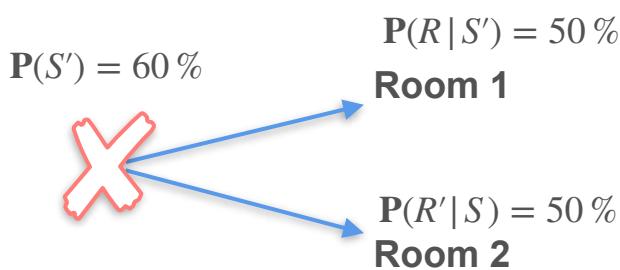


Conditional Probability

Independent

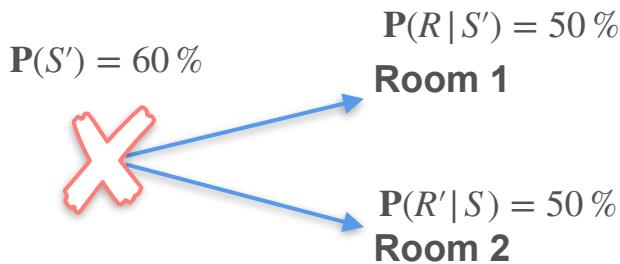
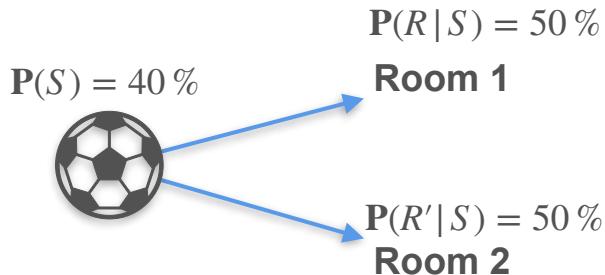


Dependent

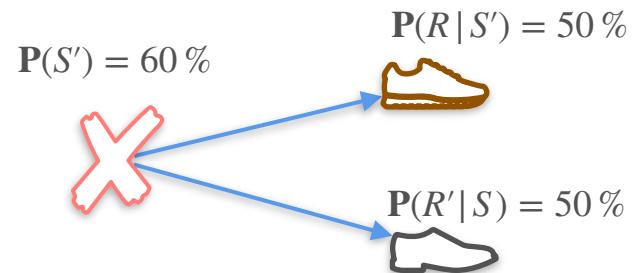
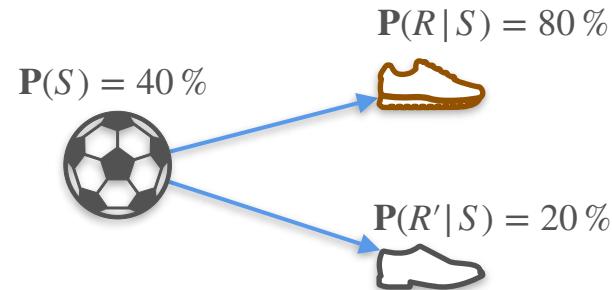


Conditional Probability

Independent

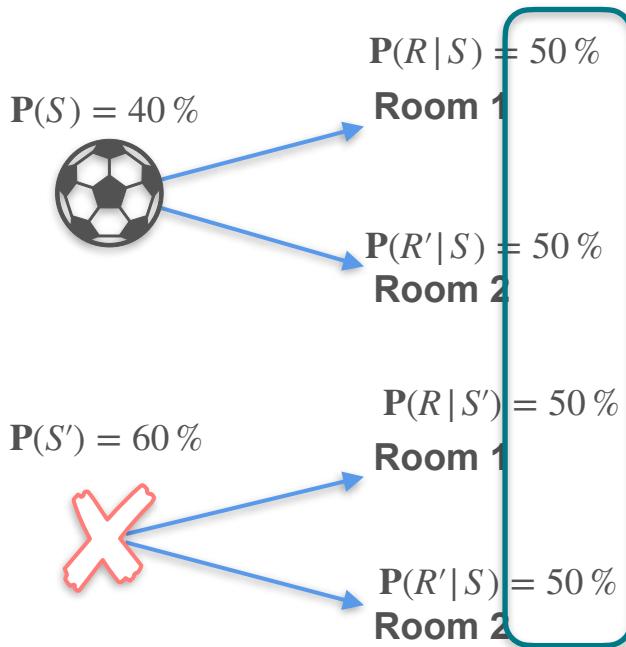


Dependent

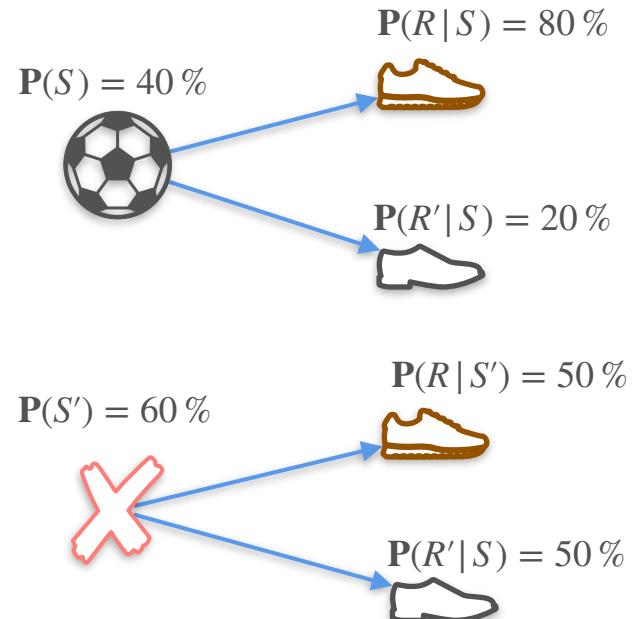


Conditional Probability

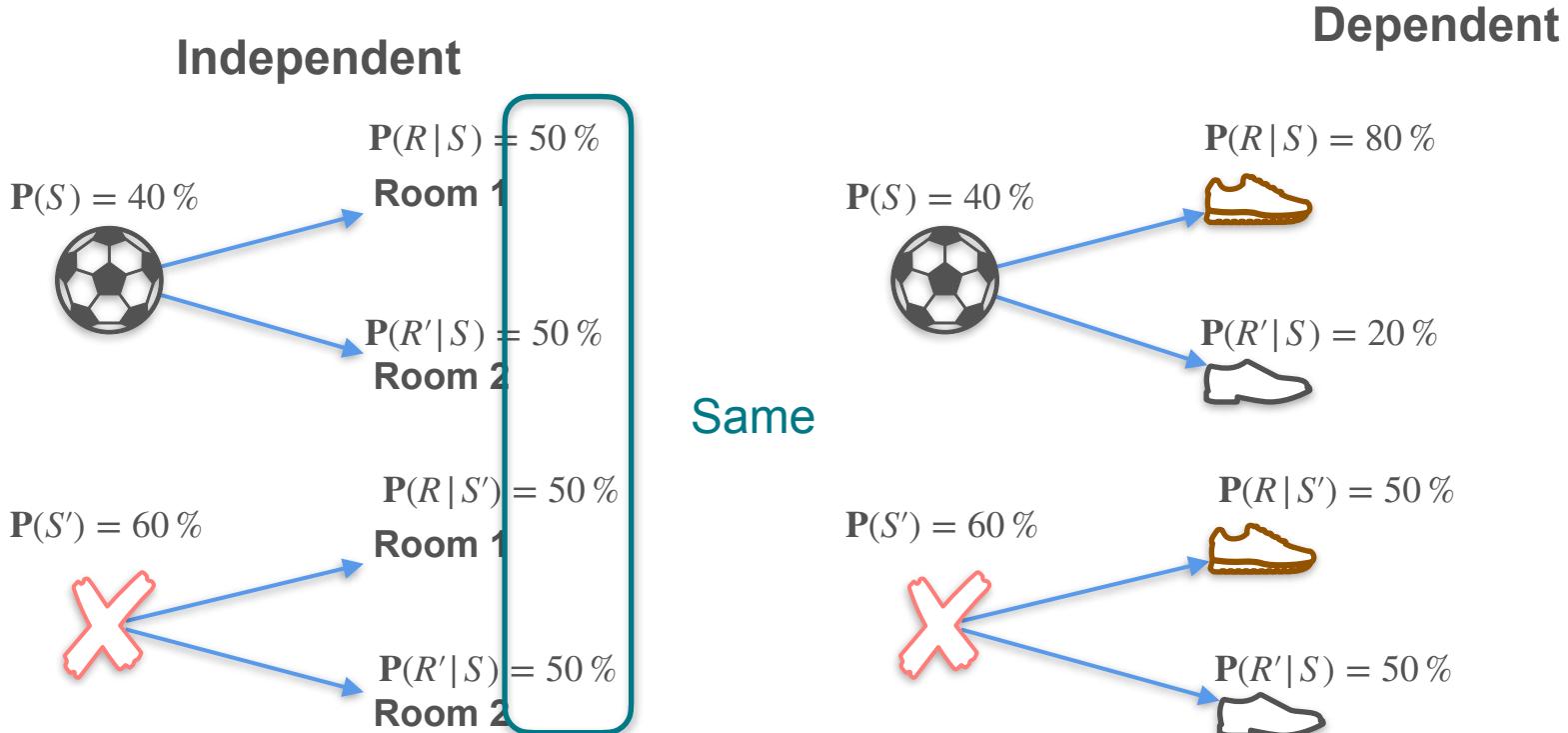
Independent



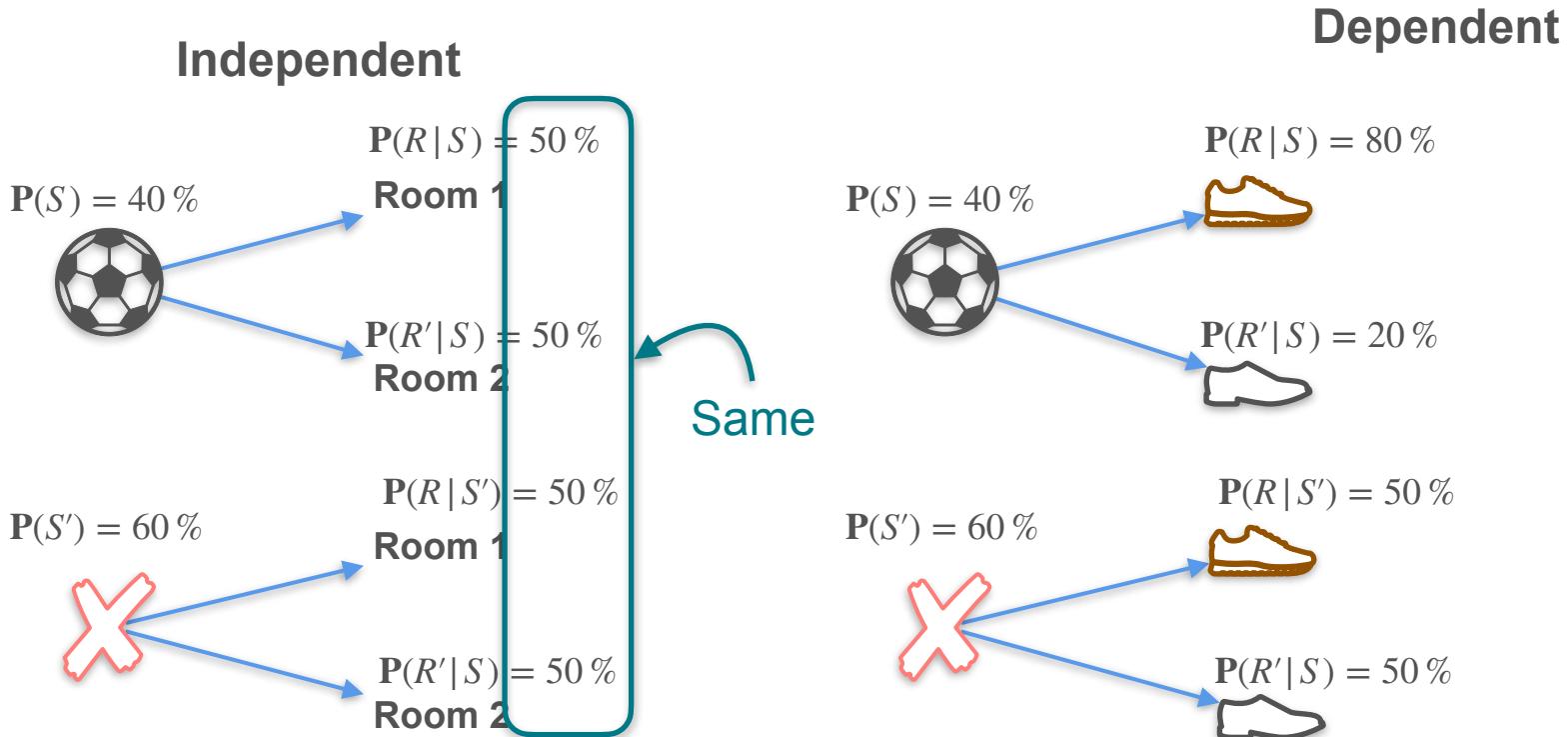
Dependent



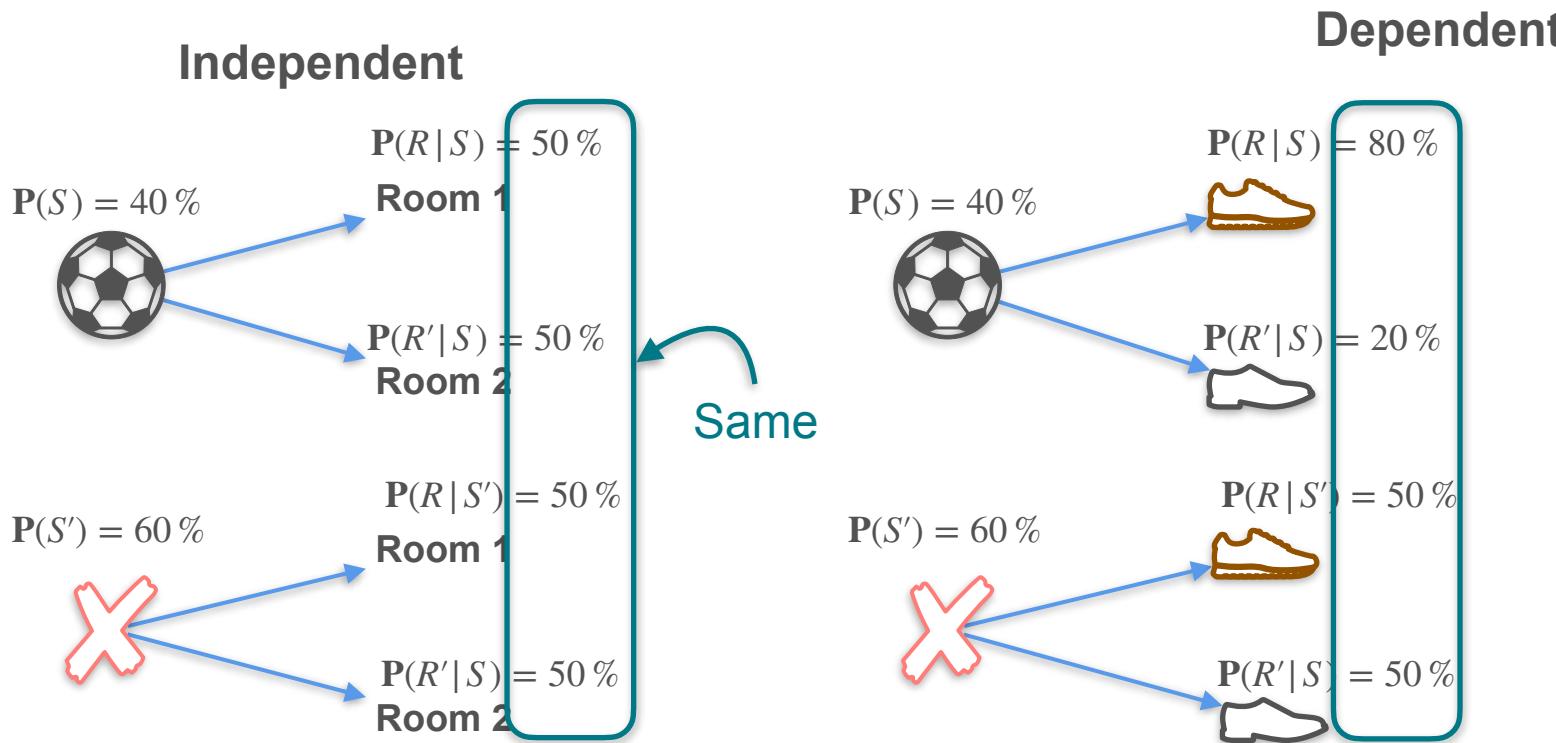
Conditional Probability



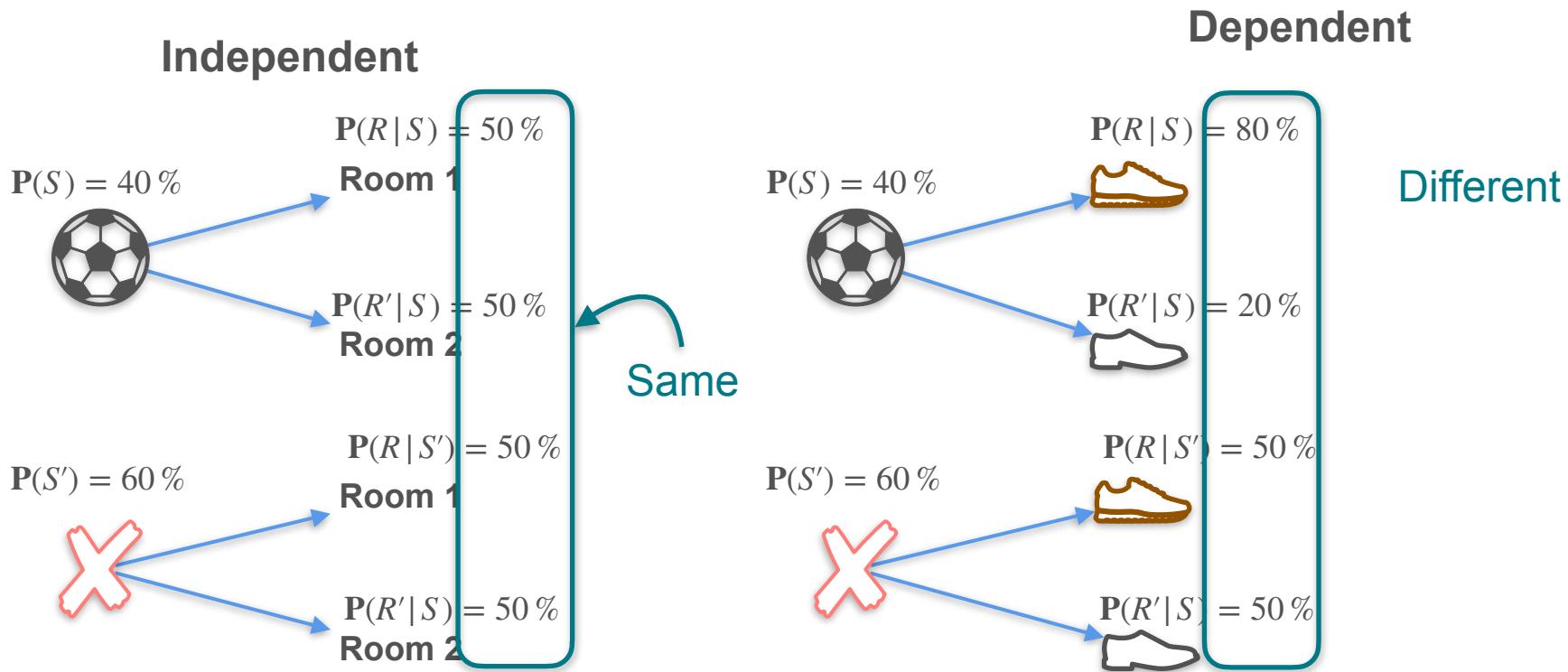
Conditional Probability



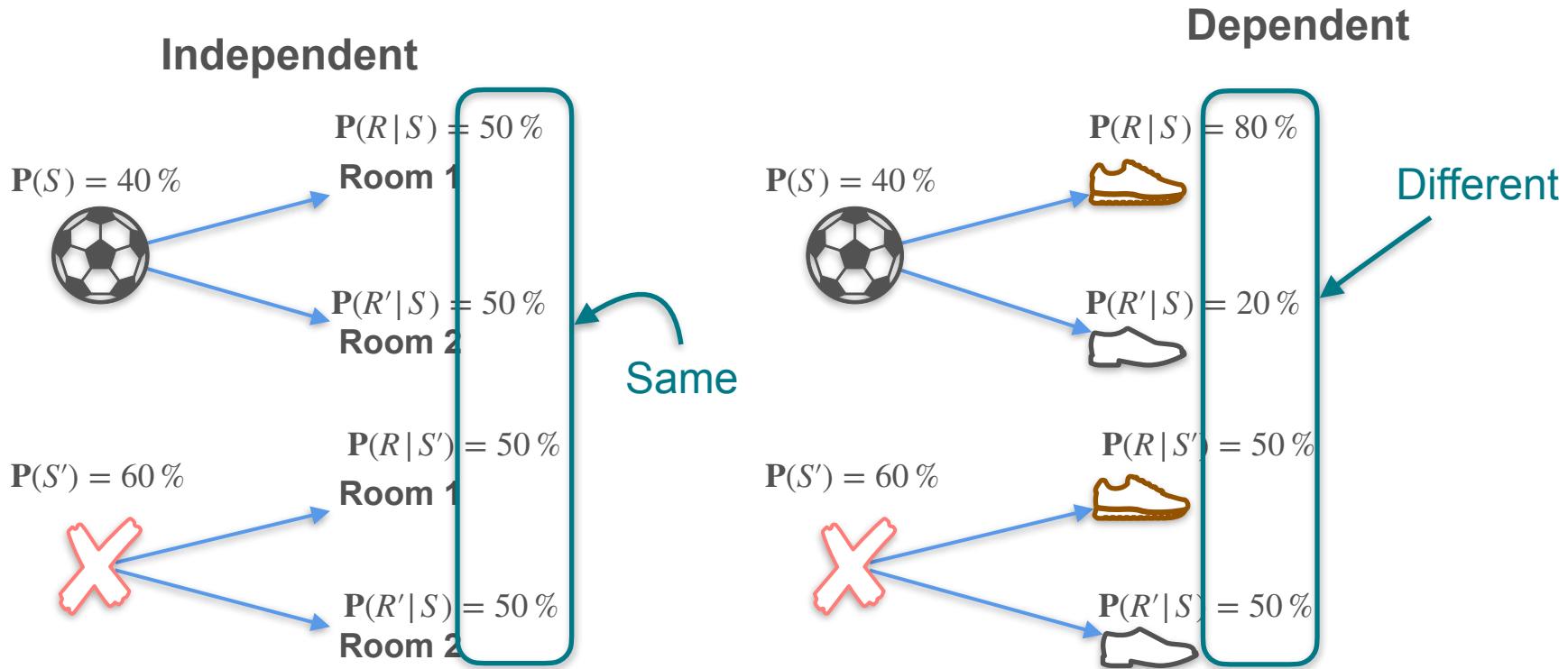
Conditional Probability



Conditional Probability



Conditional Probability





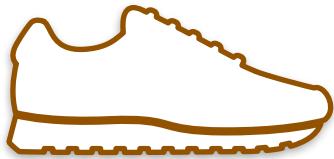
DeepLearning.AI

Introduction to probability

Bayes theorem

Bayes Theorem - Motivation

Bayes Theorem - Motivation



60%

Bayes Theorem - Motivation

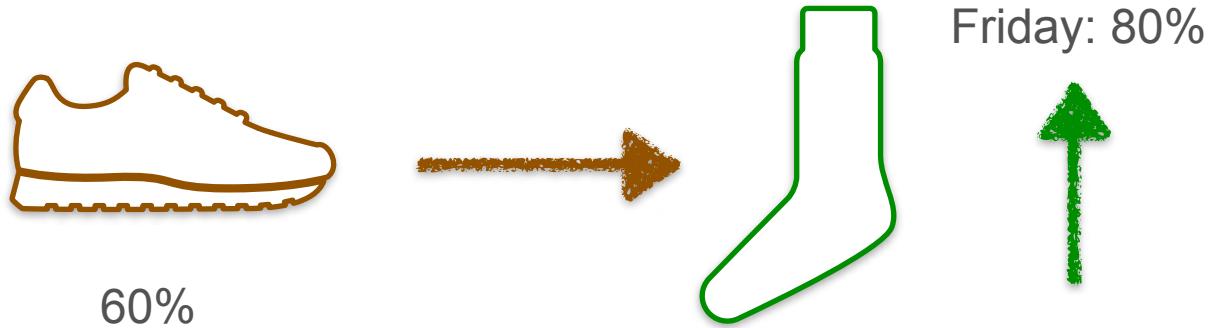


60%

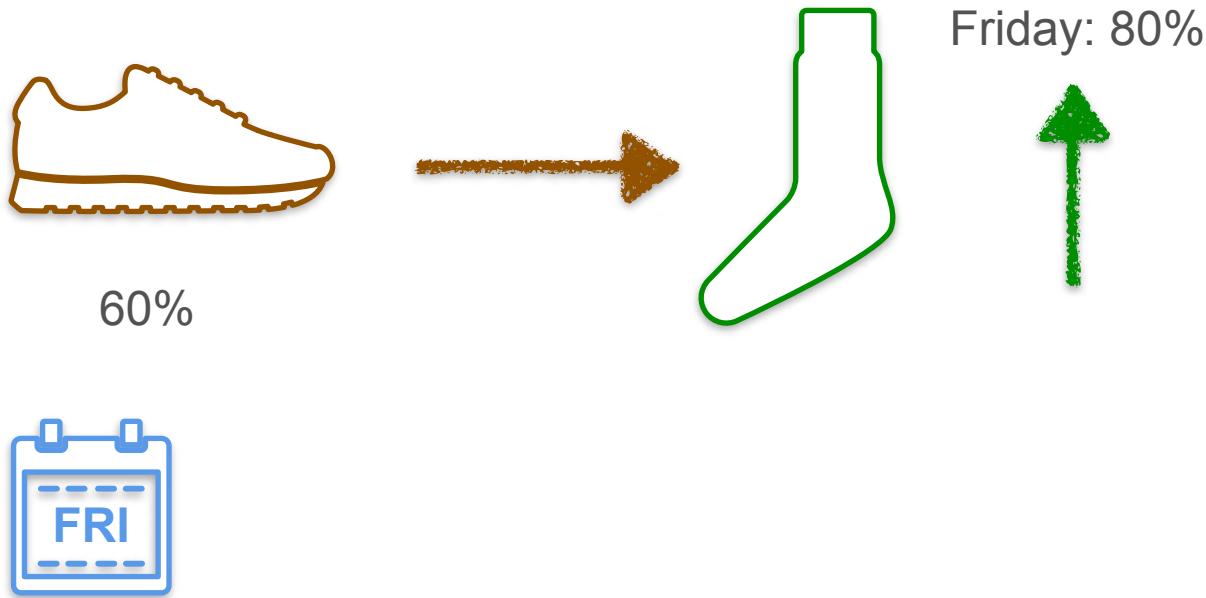
Bayes Theorem - Motivation



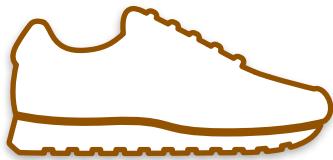
Bayes Theorem - Motivation



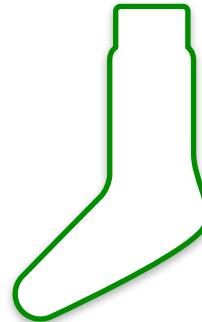
Bayes Theorem - Motivation



Bayes Theorem - Motivation



60%



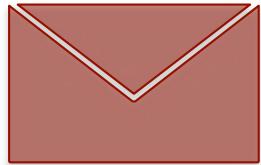
Friday: 80%



What is the probability that a customer will purchase a pair of socks given that they purchased shoes?

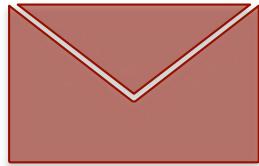
Bayes Theorem: Motivation

Bayes Theorem: Motivation



spam

Bayes Theorem: Motivation

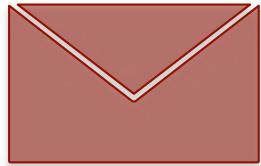


spam



contains
“lottery”

Bayes Theorem: Motivation



spam

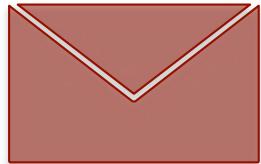


contains
“lottery”

so



Bayes Theorem: Motivation

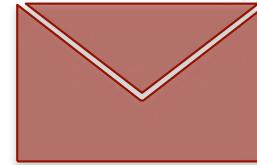


spam



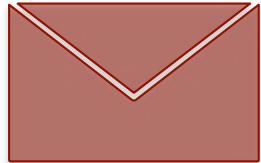
contains
“lottery”

so



spam

Bayes Theorem: Motivation

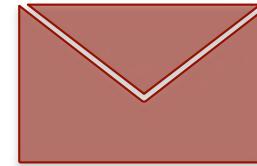


spam

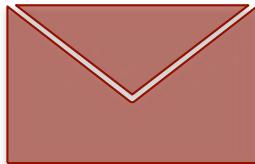


contains
“lottery”

so

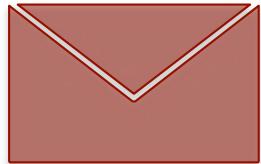


spam



spam

Bayes Theorem: Motivation

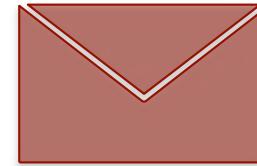


spam

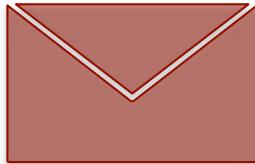


contains
“lottery”

so



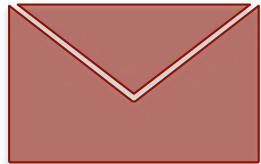
spam



spam

so

Bayes Theorem: Motivation

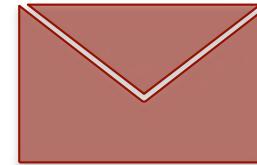


spam



contains
“lottery”

so



spam



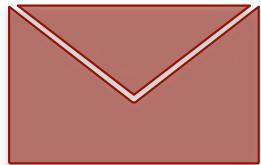
spam

so



contains
lottery

Bayes Theorem: Motivation

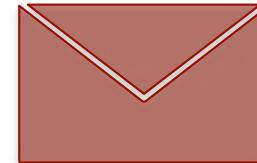


spam

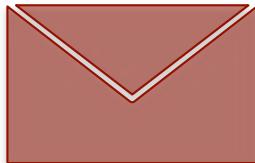


contains
“lottery”

so



spam



spam

?



contains
lottery

so

Bayes Theorem: Intuition

Bayes Theorem: Intuition



1,000,000 people

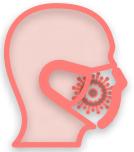


1 / 10,000 people

Bayes Theorem: Intuition



1,000,000 people



1 / 10,000 people



99% Effective

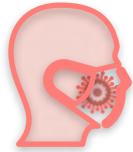
Bayes Theorem: Intuition



1,000,000 people

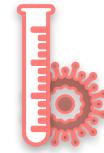


1 / 10,000 people



100 people

Diagnosed Sick



99

Diagnosed Healthy



1

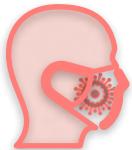


99% Effective

Bayes Theorem: Intuition



1,000,000 people

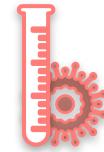


1 / 10,000 people



100 people

Diagnosed Sick



99

Diagnosed Healthy



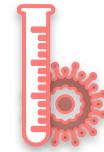
1



99% Effective



100 people



1

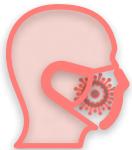


99

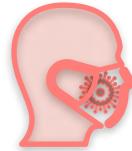
Bayes Theorem: Intuition



1,000,000 people



1 / 10,000 people



100 people



99



1



99% Effective



Tested Sick



100 people



1

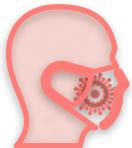


99

Bayes Theorem: Intuition



1,000,000 people



1 / 10,000 people



100 people



99



1



99% Effective



Tested Sick



100 people



1



99



What's the probability that **you are sick**

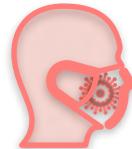
Bayes Theorem: Intuition



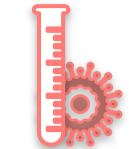
1,000,000 people



1 / 10,000 people



100 people



99



1



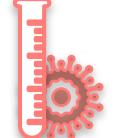
99% Effective



Tested Sick



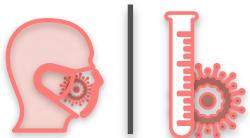
100 people



1



99

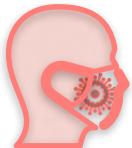


What's the probability that **you are sick**
GIVEN that you tested sick?

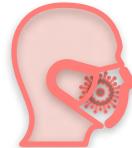
Bayes Theorem: Intuition



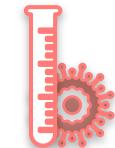
1,000,000 people



1 / 10,000 people



100 people



99



1



99% Effective



Tested Sick



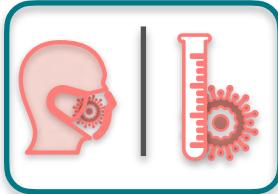
100 people



1

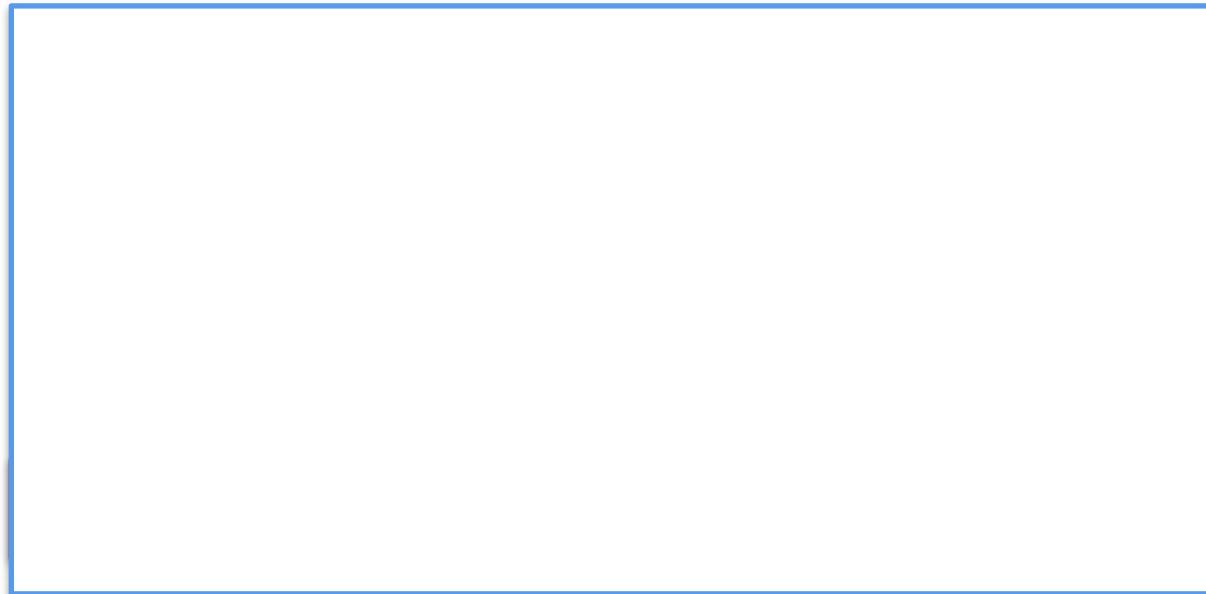
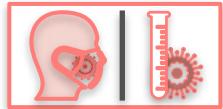


99



What's the probability that **you are sick**
GIVEN that you tested sick?

Bayes Theorem: Intuition



1,000,000 people

Bayes Theorem: Intuition



Healthy

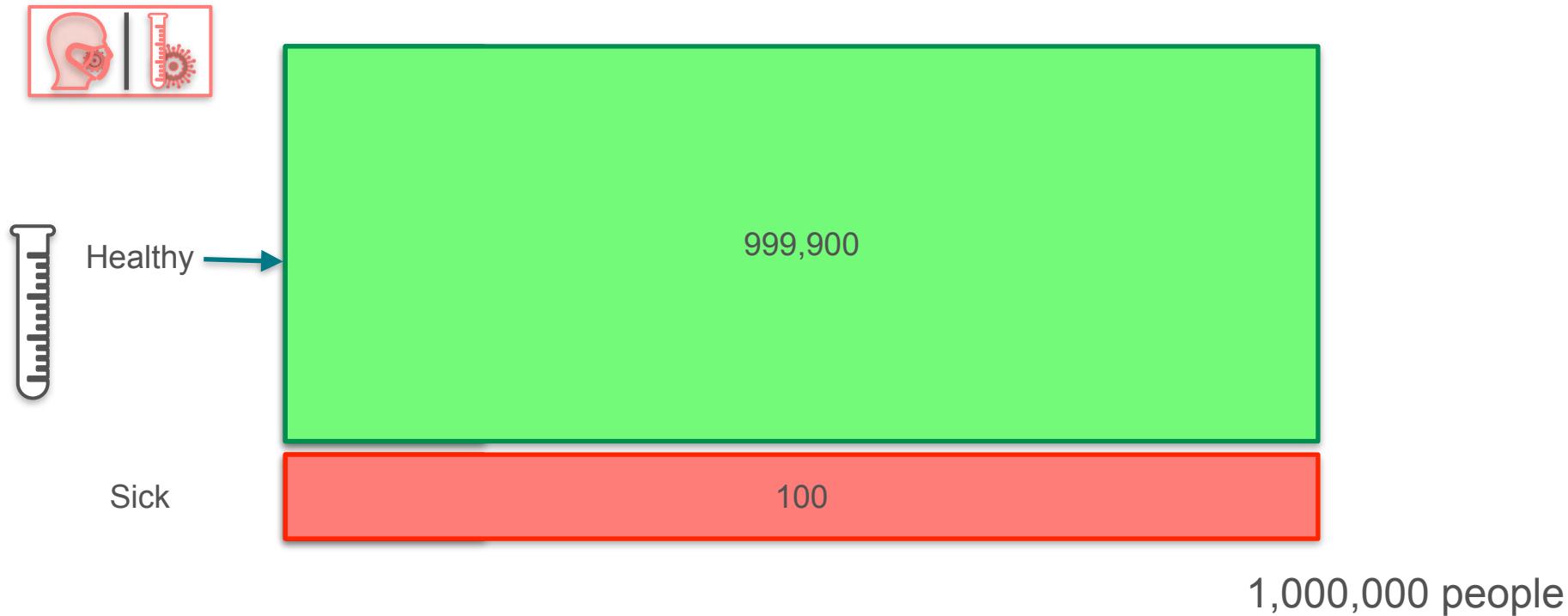
999,900

Sick

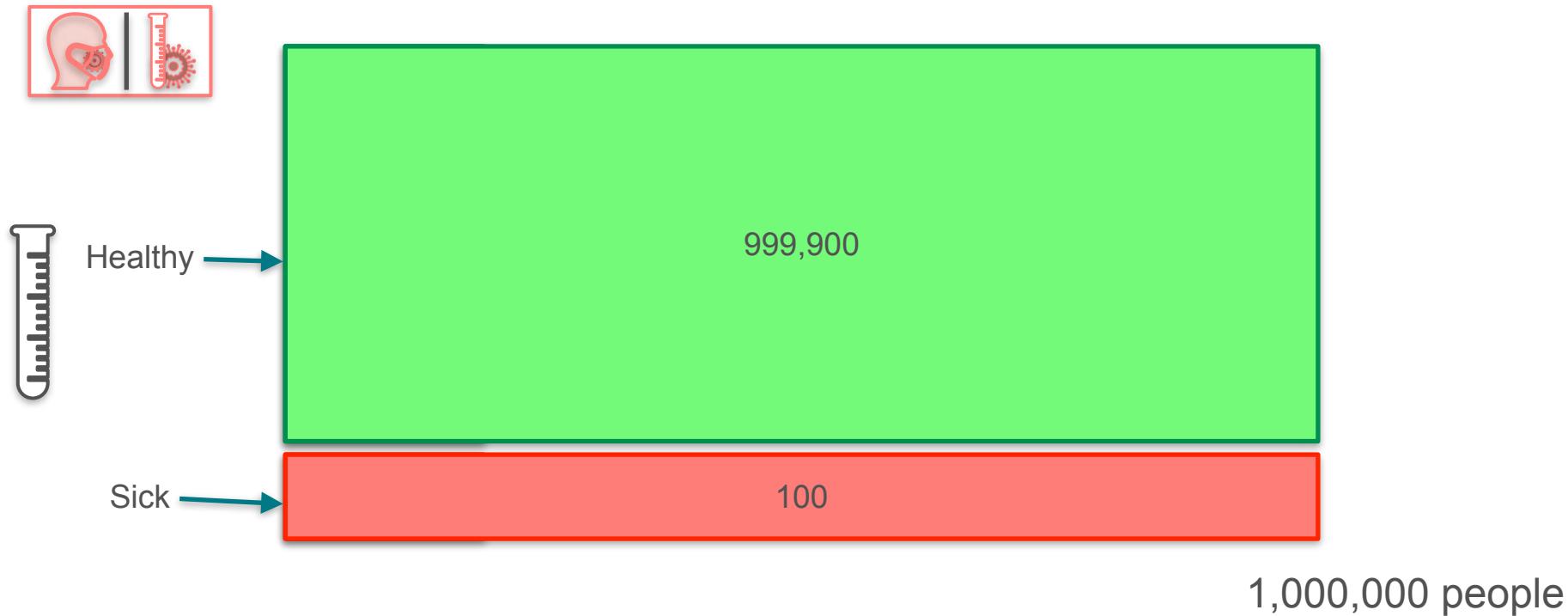
100

1,000,000 people

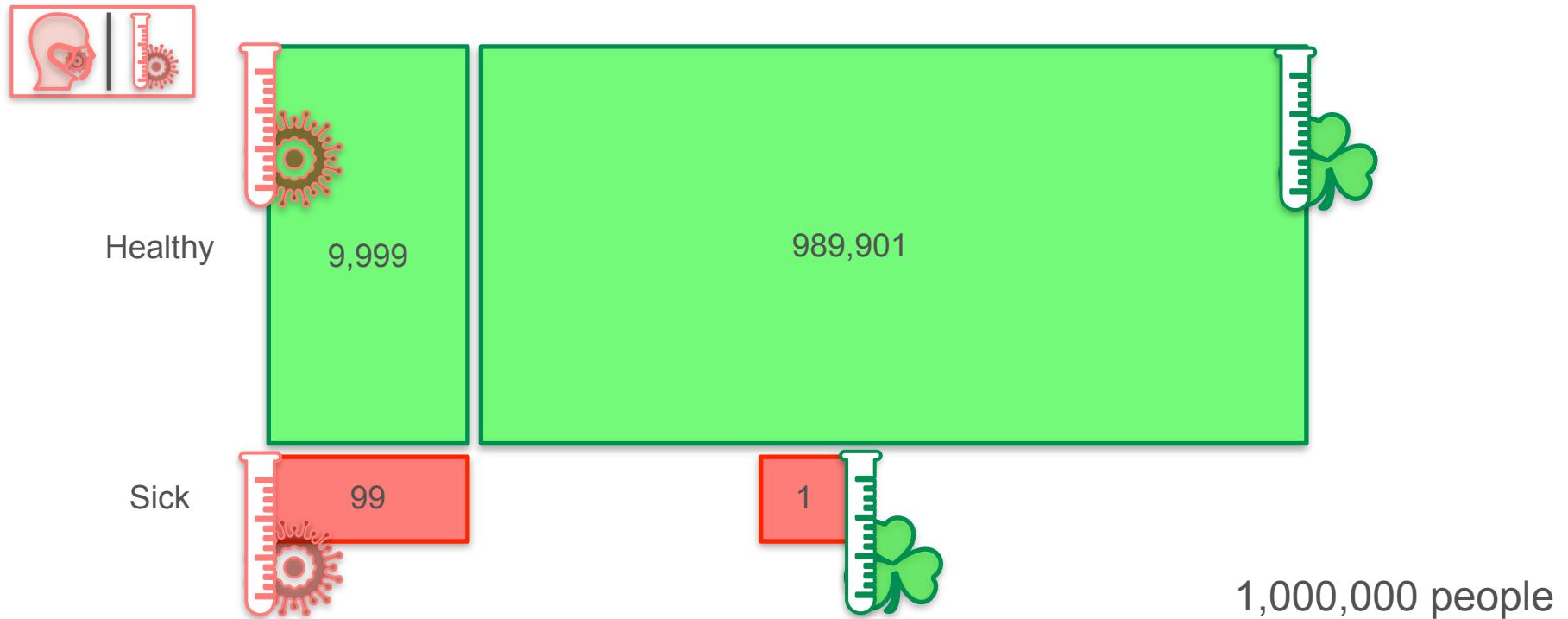
Bayes Theorem: Intuition



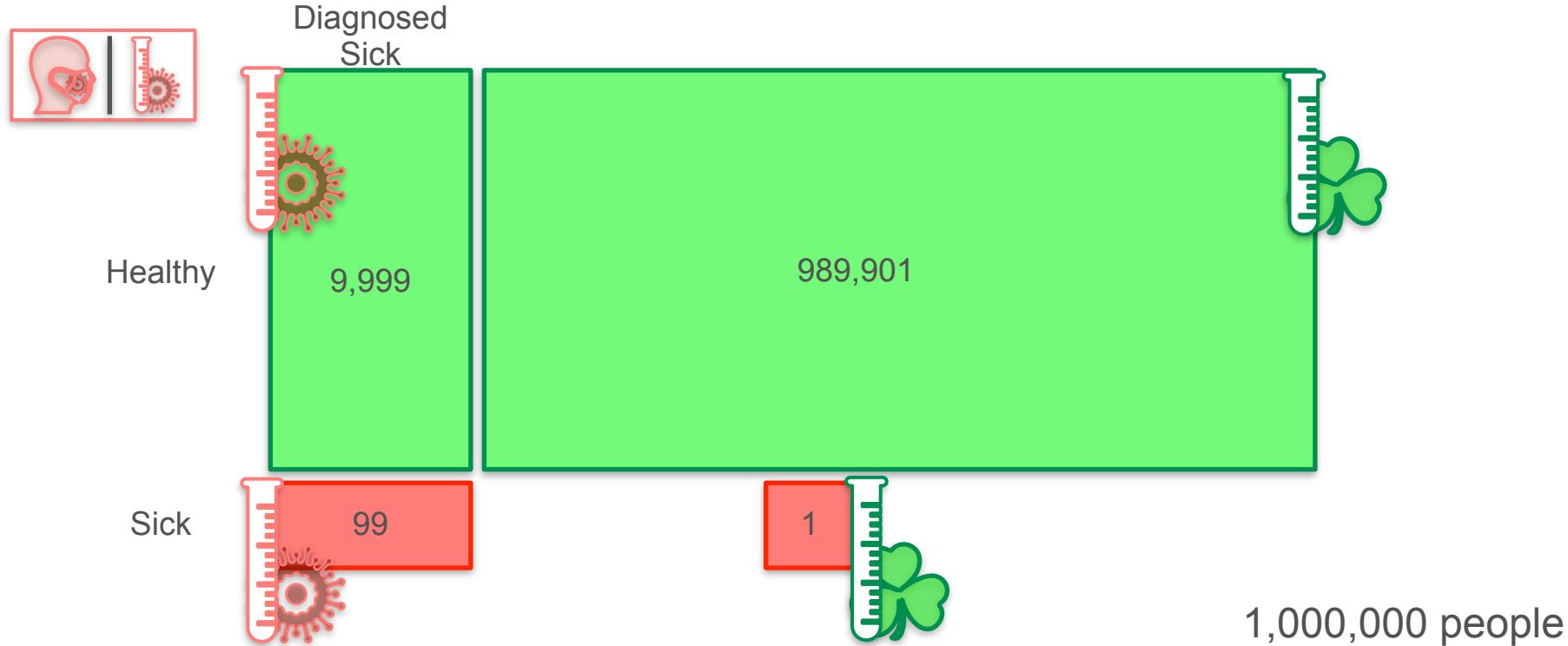
Bayes Theorem: Intuition



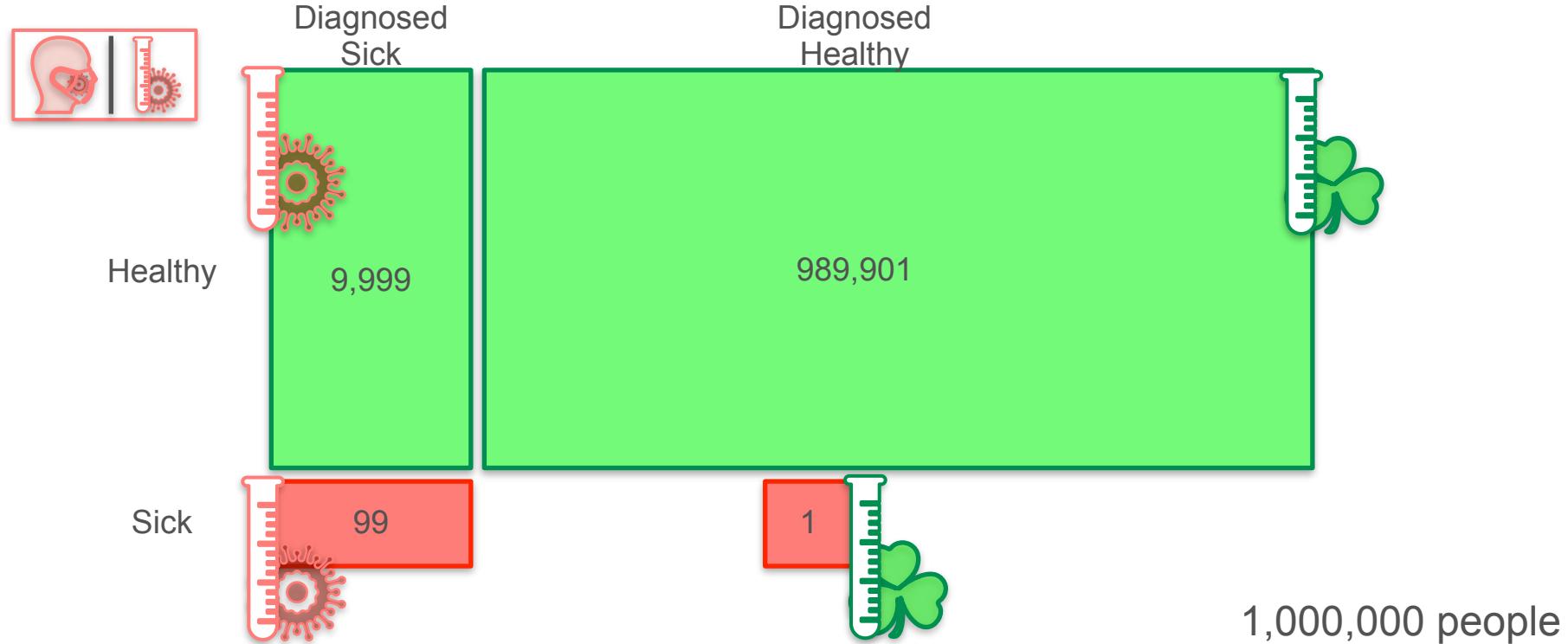
Bayes Theorem: Intuition



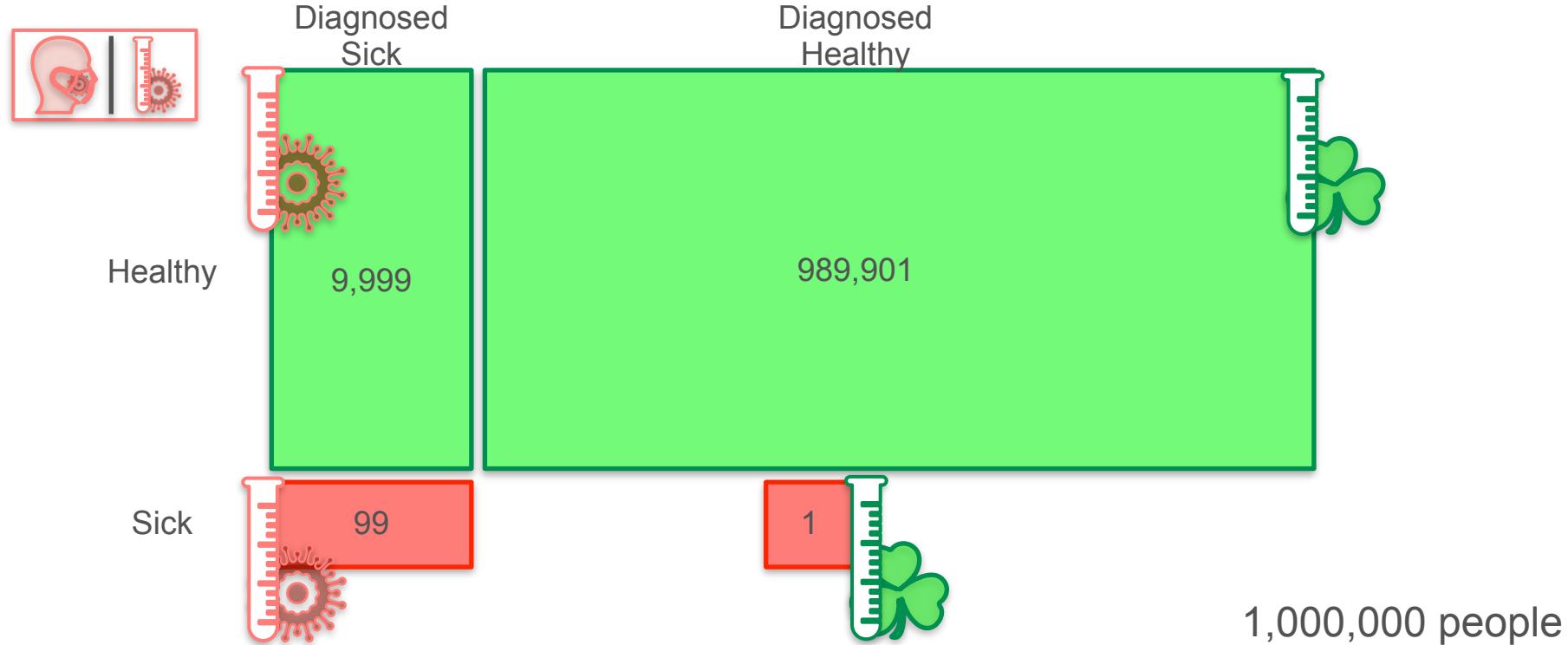
Bayes Theorem: Intuition



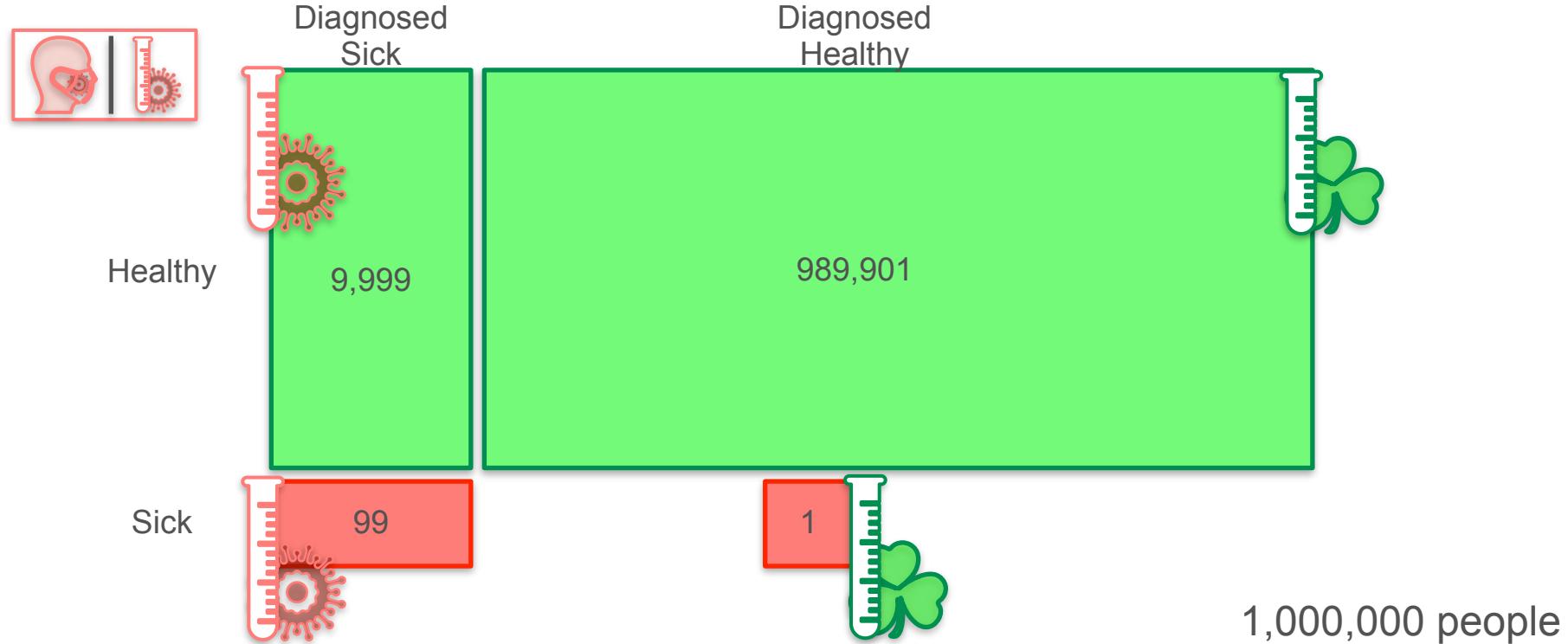
Bayes Theorem: Intuition



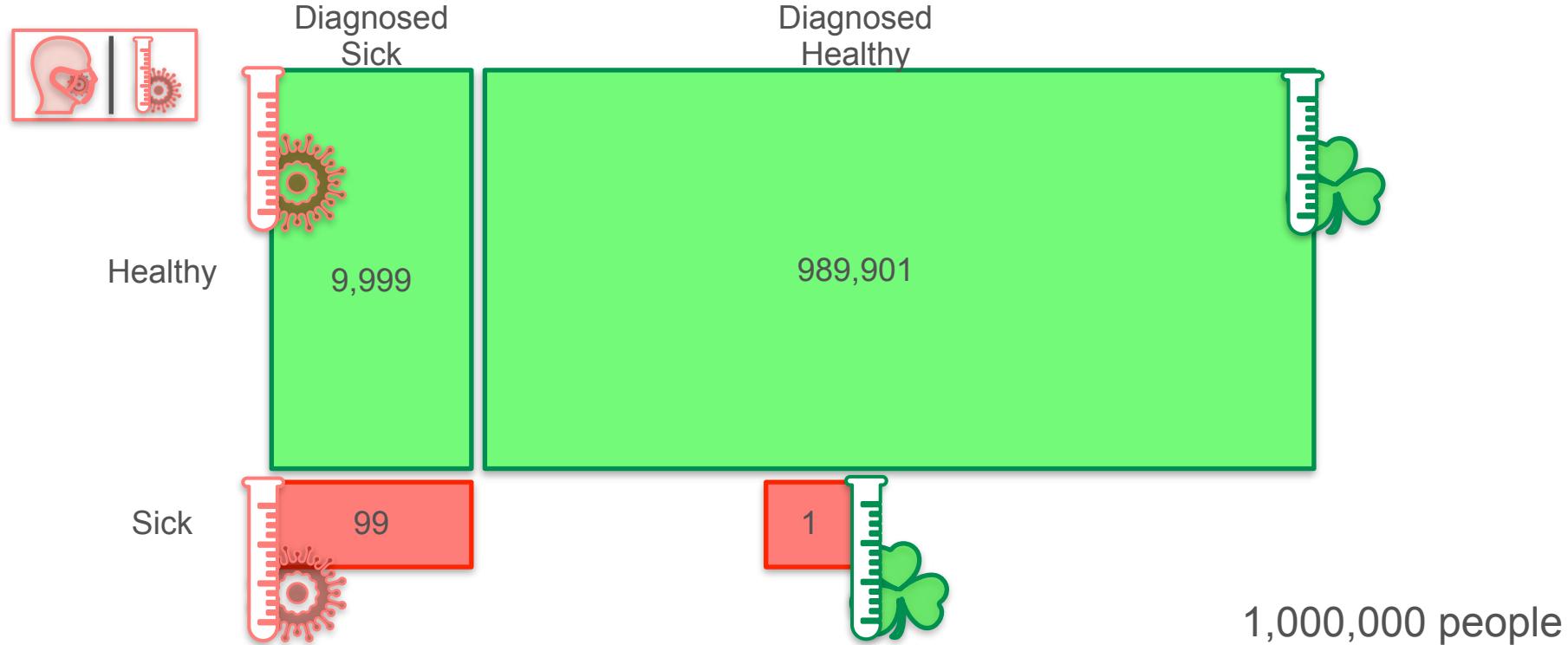
Bayes Theorem: Intuition



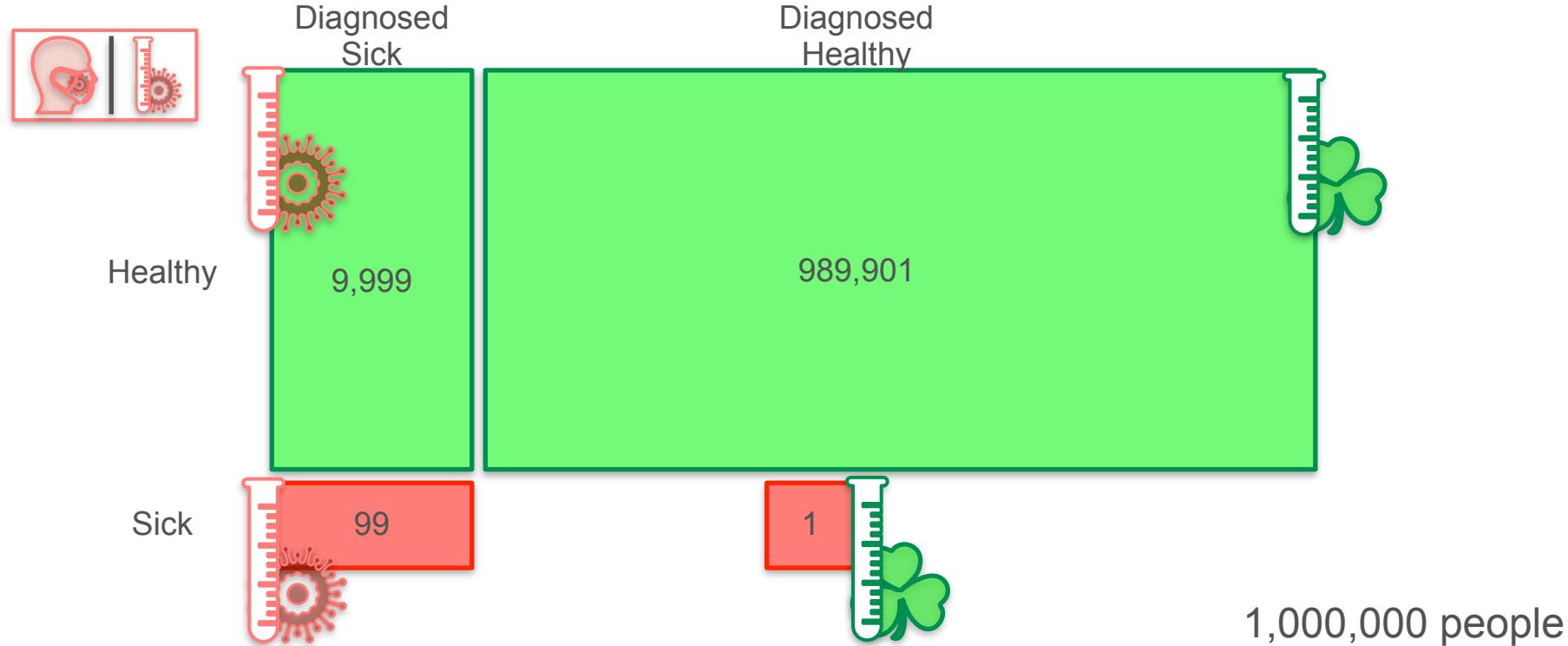
Bayes Theorem: Intuition



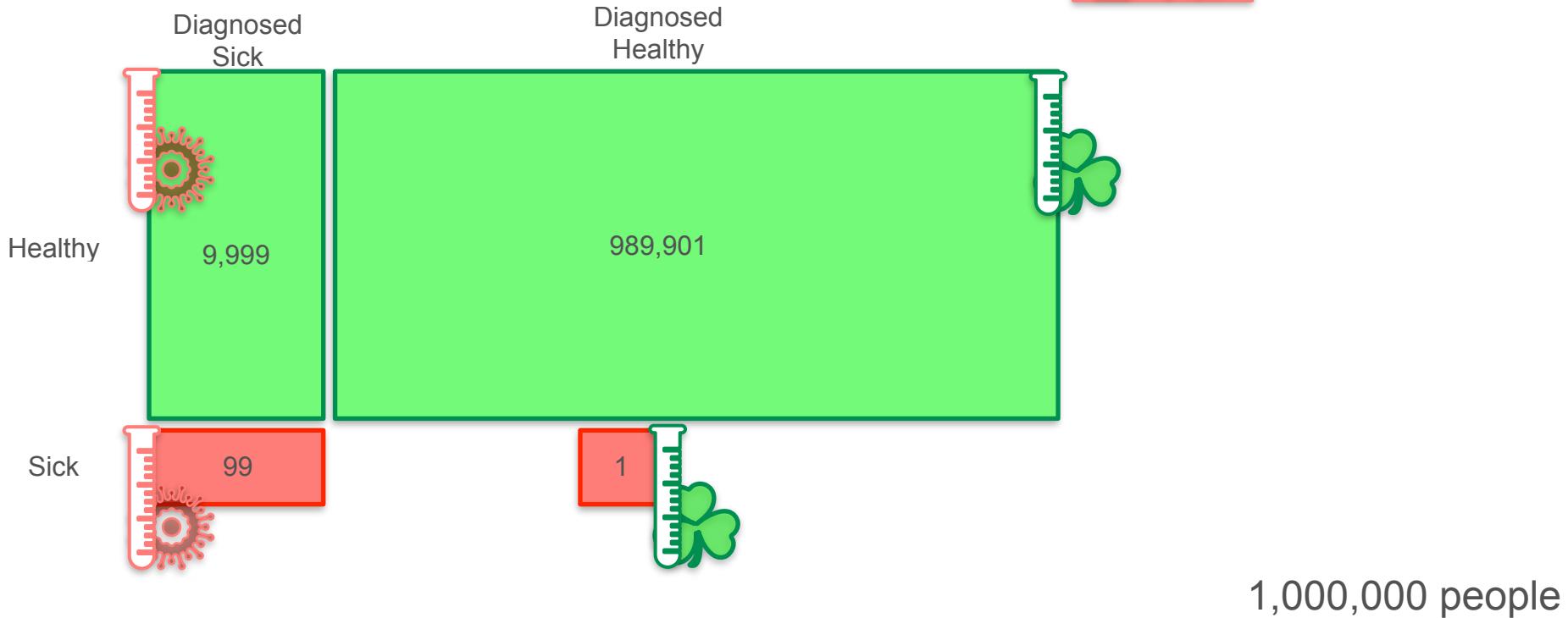
Bayes Theorem: Intuition



Bayes Theorem: Intuition



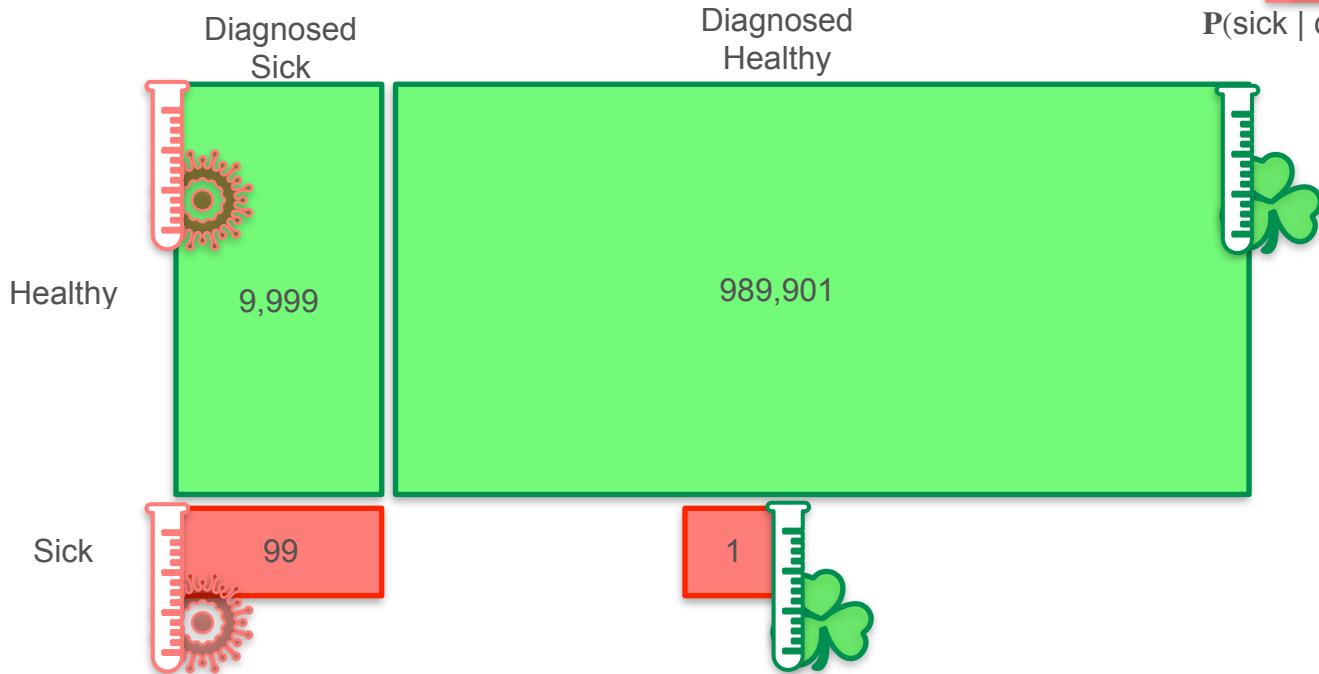
Bayes Theorem: Intuition



Bayes Theorem: Intuition



$P(\text{sick} | \text{diagnosed sick}) = \underline{\hspace{2cm}}$

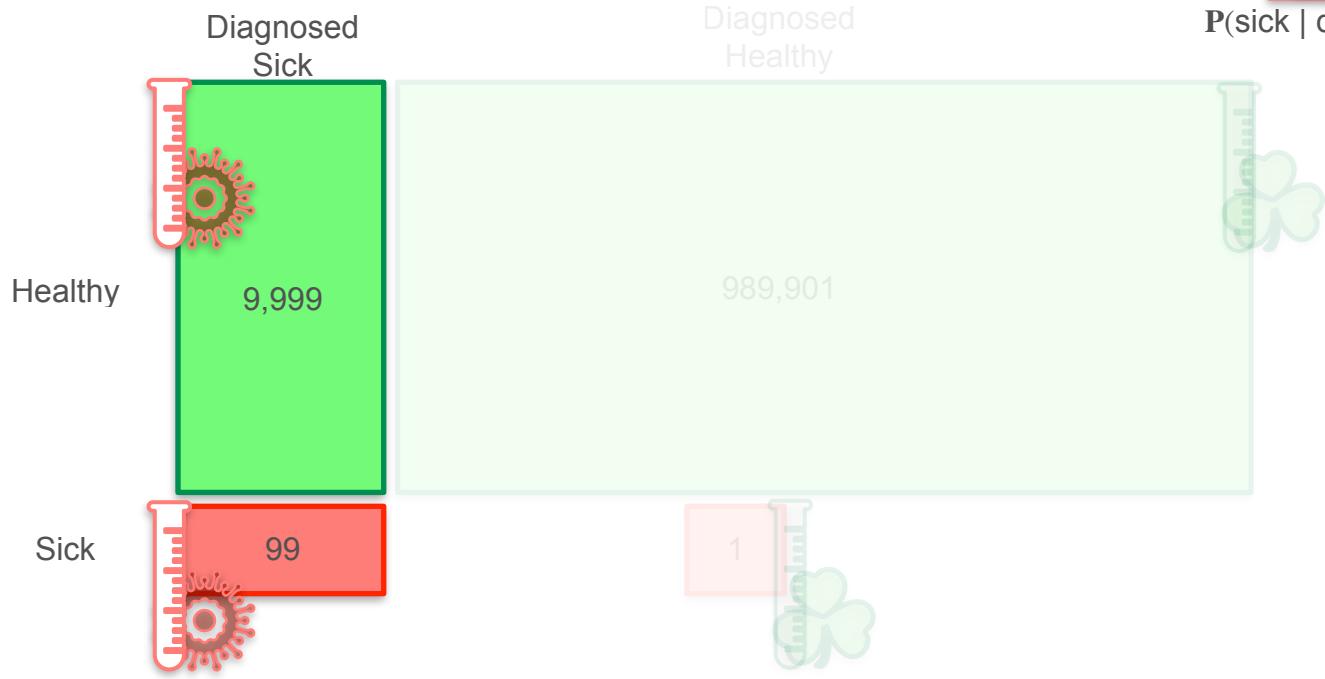


1,000,000 people

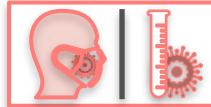
Bayes Theorem: Intuition



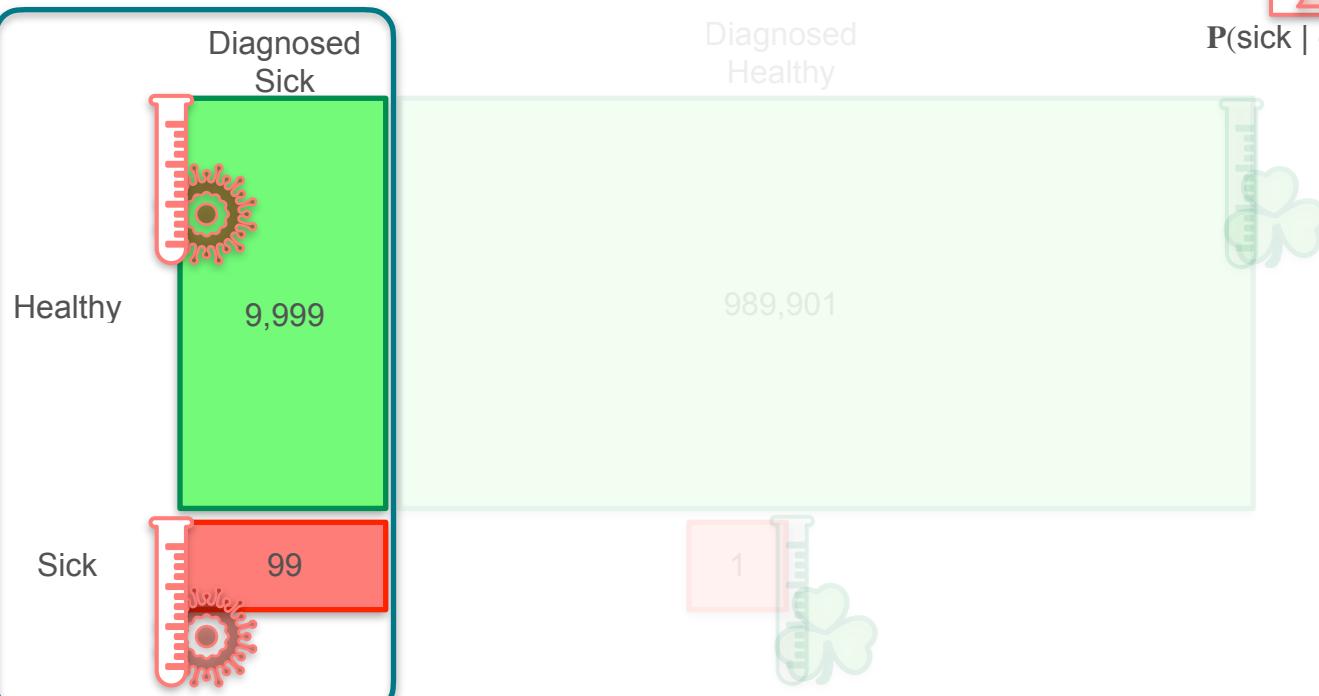
$P(\text{sick} | \text{diagnosed sick}) = \underline{\hspace{2cm}}$



Bayes Theorem: Intuition

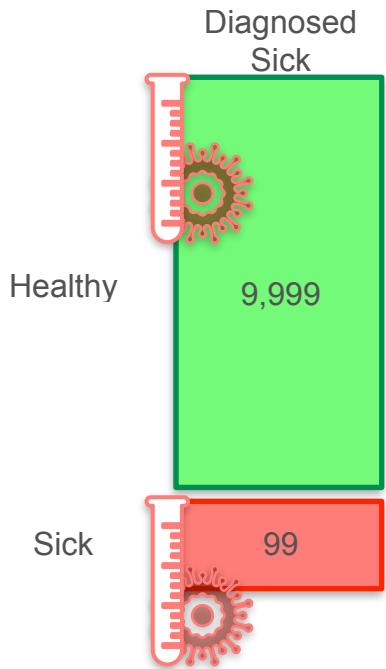


$P(\text{sick} | \text{diagnosed sick}) = \underline{\hspace{2cm}}$



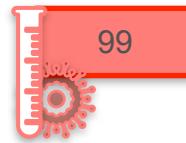
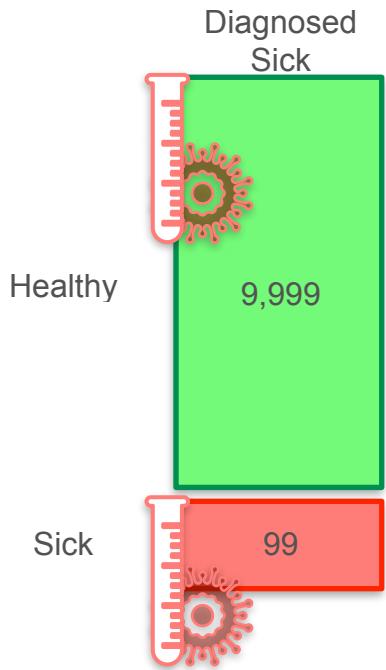
1,000,000 people

Bayes Theorem: Intuition



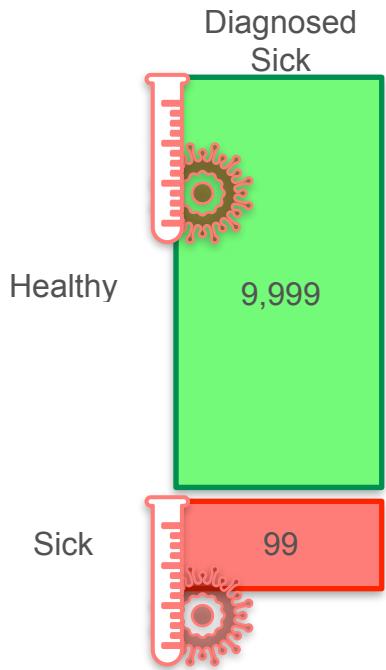
$$P(\text{sick} \mid \text{diagnosed sick}) = \underline{\hspace{2cm}}$$

Bayes Theorem: Intuition

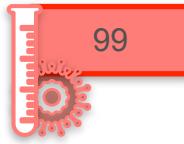
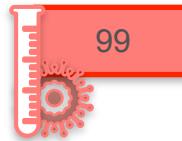


$$P(\text{sick} \mid \text{diagnosed sick}) = \frac{\text{_____}}{\text{_____}}$$

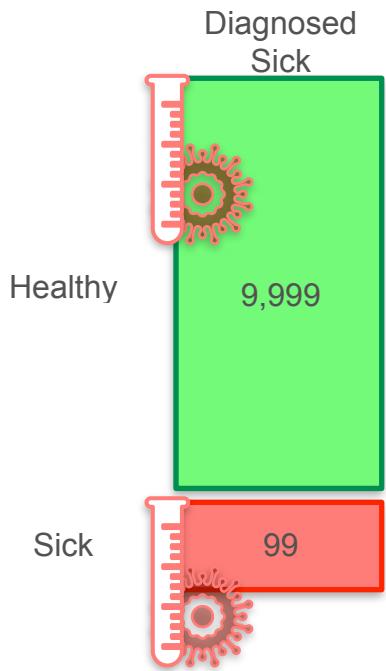
Bayes Theorem: Intuition



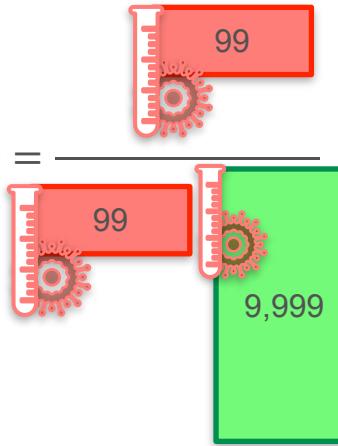
$$P(\text{sick} \mid \text{diagnosed sick}) = \frac{\text{_____}}{\text{_____}}$$



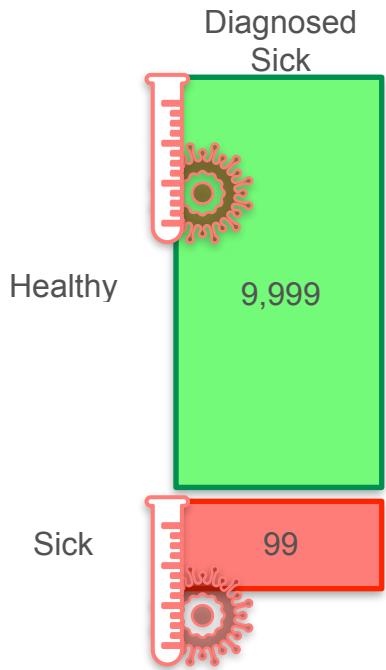
Bayes Theorem: Intuition



$$P(\text{sick} \mid \text{diagnosed sick}) = \frac{\text{Probability of true positive}}{\text{Probability of true positive} + \text{Probability of false positive}}$$

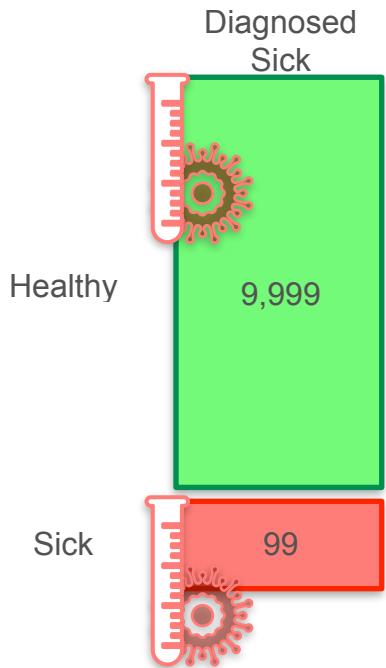


Bayes Theorem: Intuition

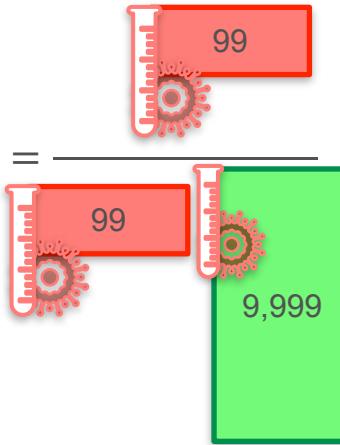


$$\begin{aligned} P(\text{sick} | \text{diagnosed sick}) &= \frac{\text{sick and diagnosed sick}}{\text{sick and diagnosed sick} + \text{healthy and diagnosed sick}} \\ &= \frac{99}{99 + 9,999} \end{aligned}$$

Bayes Theorem: Intuition

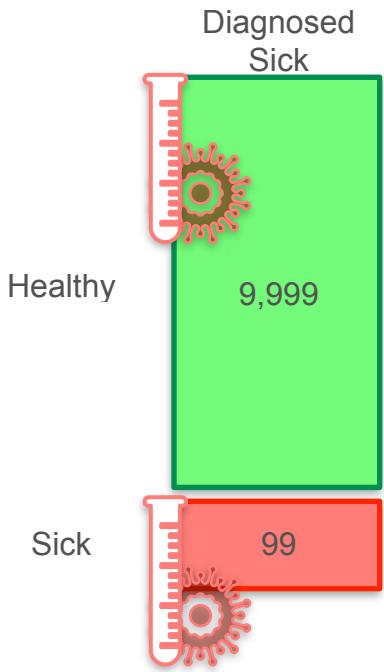


$$P(\text{sick} | \text{diagnosed sick}) = \frac{\text{sick and diagnosed sick}}{\text{sick and diagnosed sick} + \text{healthy and diagnosed sick}}$$



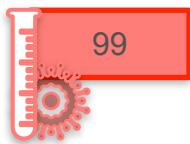
$$= \frac{99}{99 + 9999}$$

Bayes Theorem: Intuition

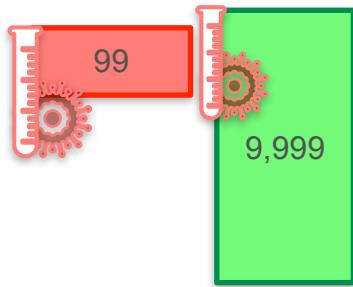


$$\begin{aligned} P(\text{sick} | \text{diagnosed sick}) &= \frac{\text{sick and diagnosed sick}}{\text{sick and diagnosed sick} + \text{healthy and diagnosed sick}} \\ &= \frac{99}{99 + 9999} \\ &= \frac{99}{10098} = 0.0098 \end{aligned}$$

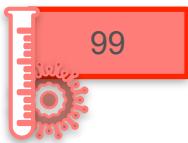
Bayes Theorem: Intuition



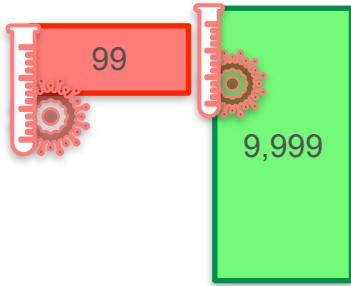
$$P(\text{sick} \mid \text{diagnosed sick}) = \frac{\text{_____}}{\text{_____}}$$



Bayes Theorem: Intuition

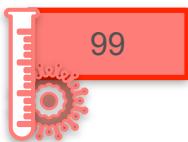


$$P(\text{sick} \mid \text{diagnosed sick}) = \frac{\text{_____}}{\text{_____}}$$

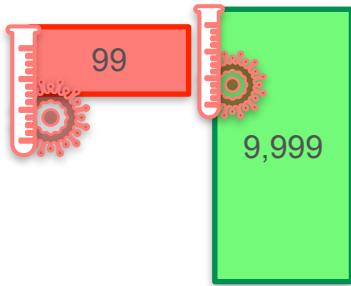


= $\frac{\text{sick and diagnosed sick}}{\text{everyone diagnosed sick}}$

Bayes Theorem: Intuition



$$P(\text{sick} | \text{diagnosed sick}) = \frac{\text{sick and diagnosed sick}}{\text{sick and diagnosed sick} + \text{healthy and diagnosed sick}}$$



$$= \frac{\text{sick and diagnosed sick}}{\text{sick and diagnosed sick} + \text{healthy and diagnosed sick}}$$

$$= \frac{\text{sick and diagnosed sick}}{\text{everyone diagnosed sick}}$$

Bayes Theorem: Intuition



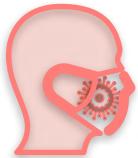
1,000,000
people

Bayes Theorem: Intuition

sick = 100



1,000,000
people

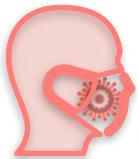


Bayes Theorem: Intuition

sick = 100



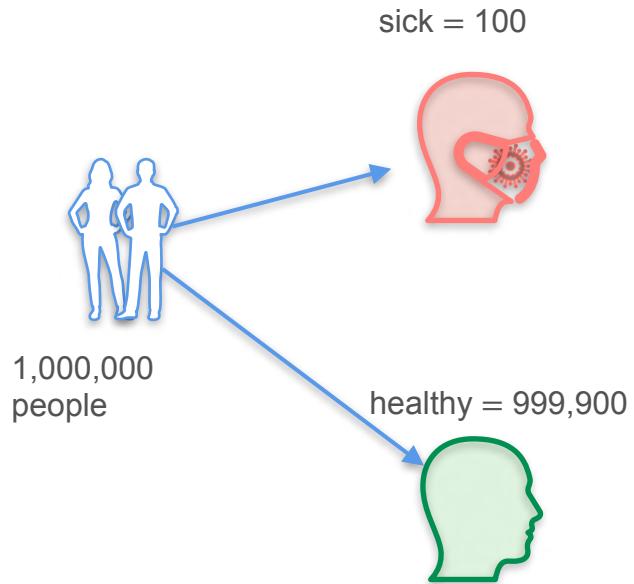
1,000,000
people



healthy = 999,900

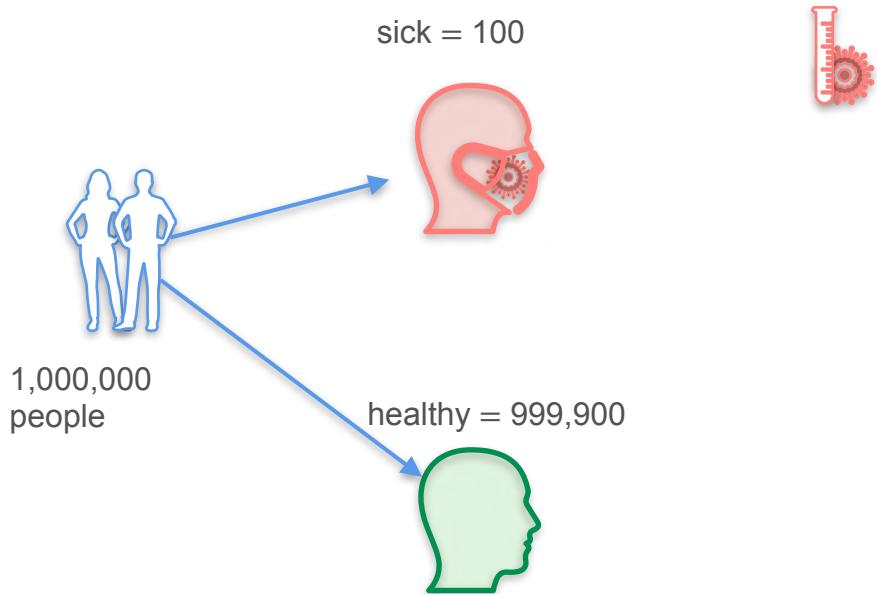


Bayes Theorem: Intuition

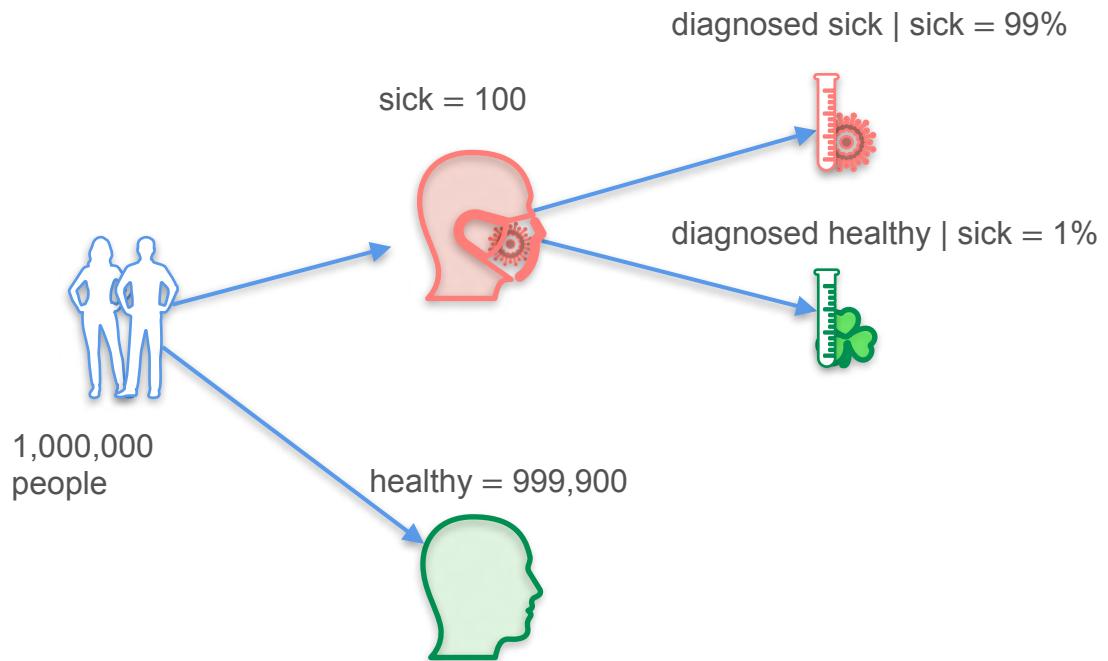


Bayes Theorem: Intuition

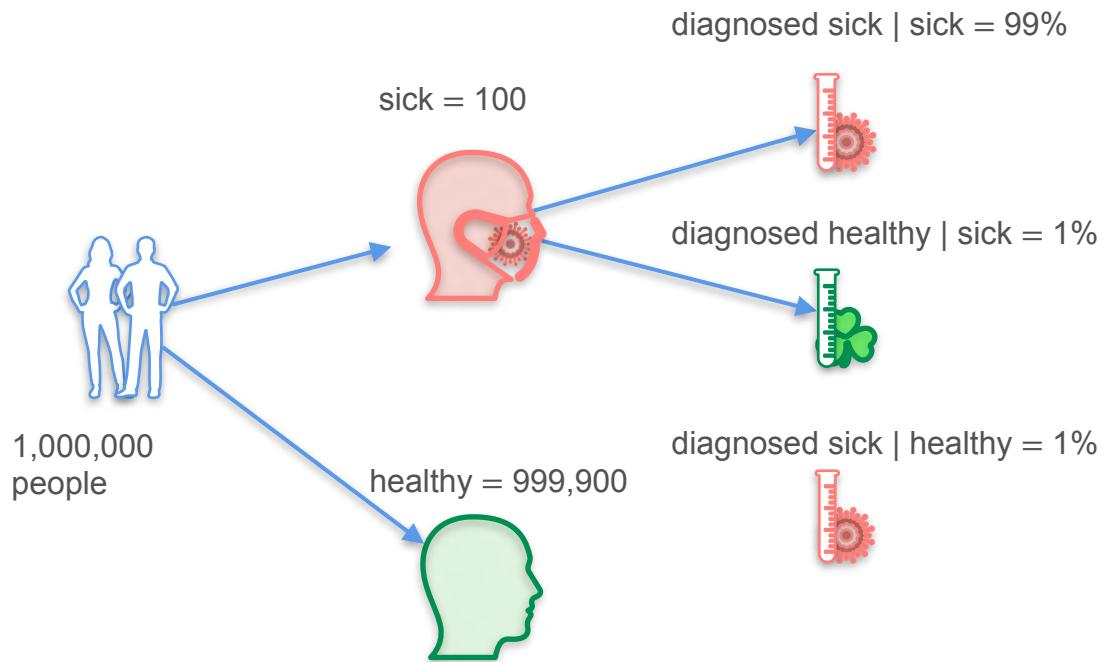
diagnosed sick | sick = 99%



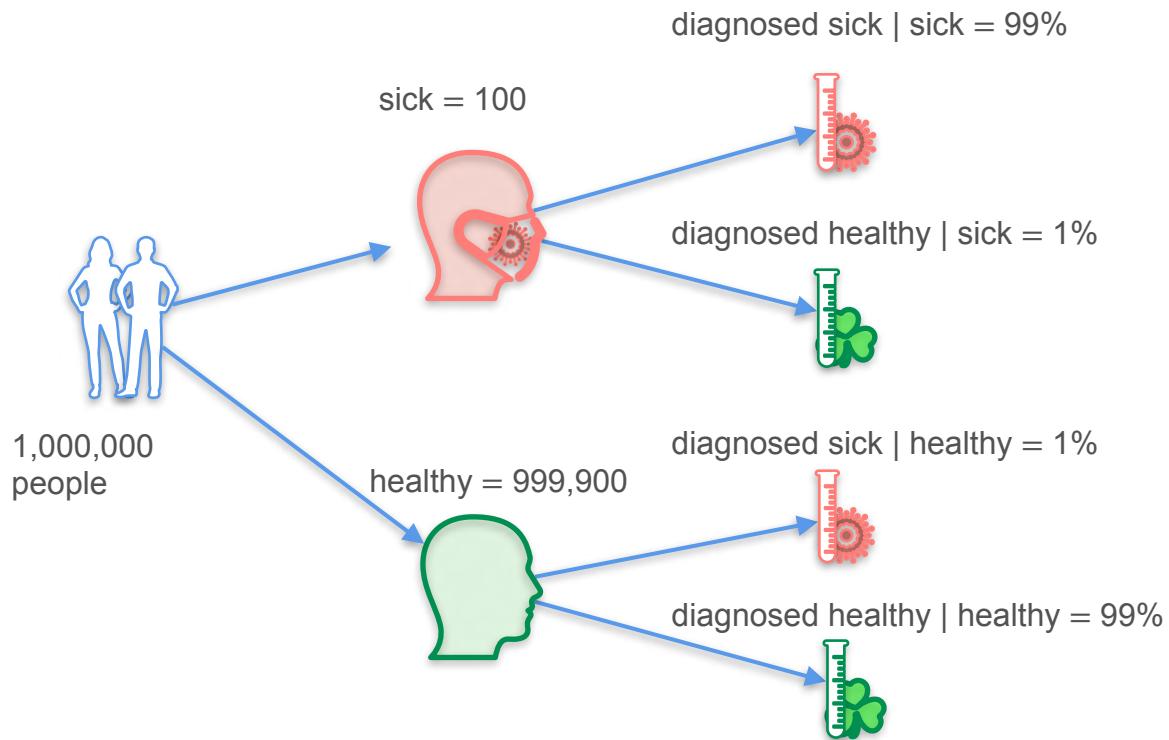
Bayes Theorem: Intuition



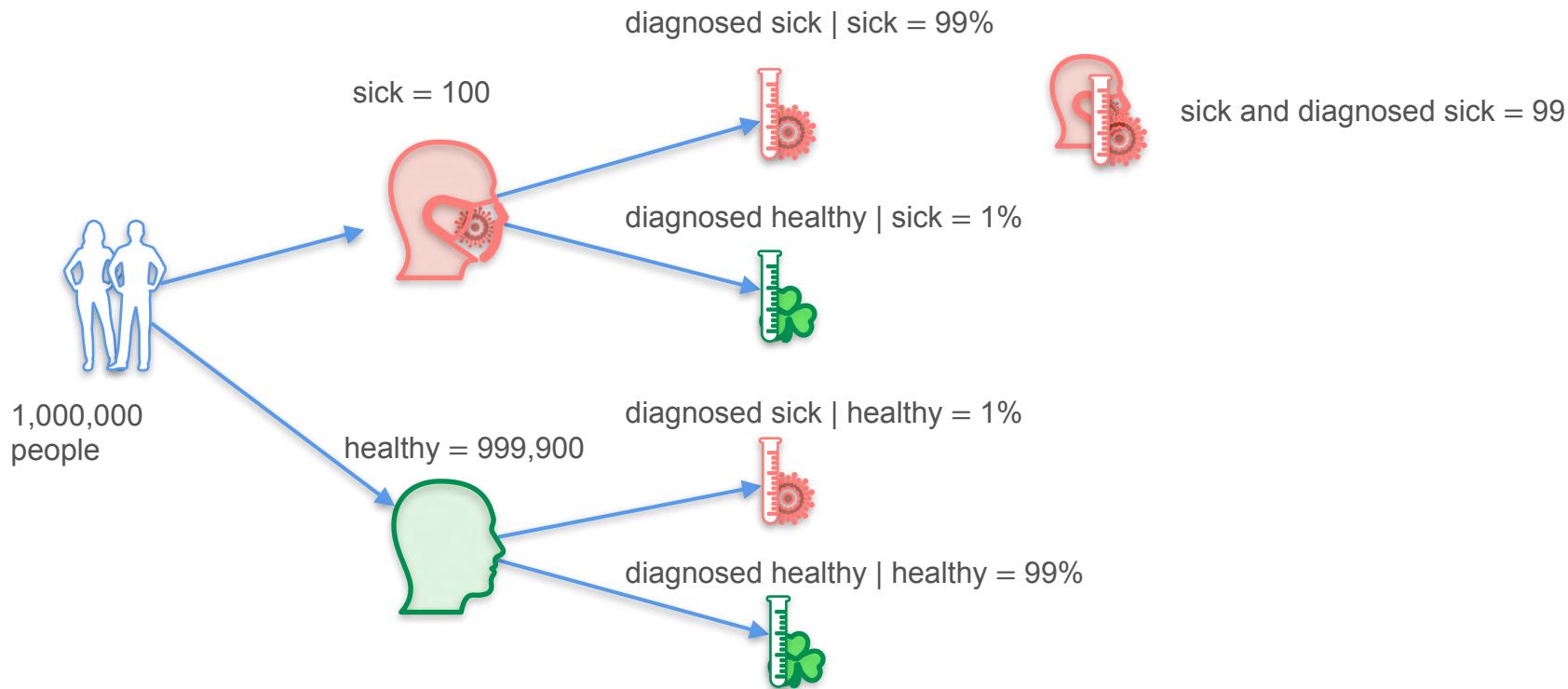
Bayes Theorem: Intuition



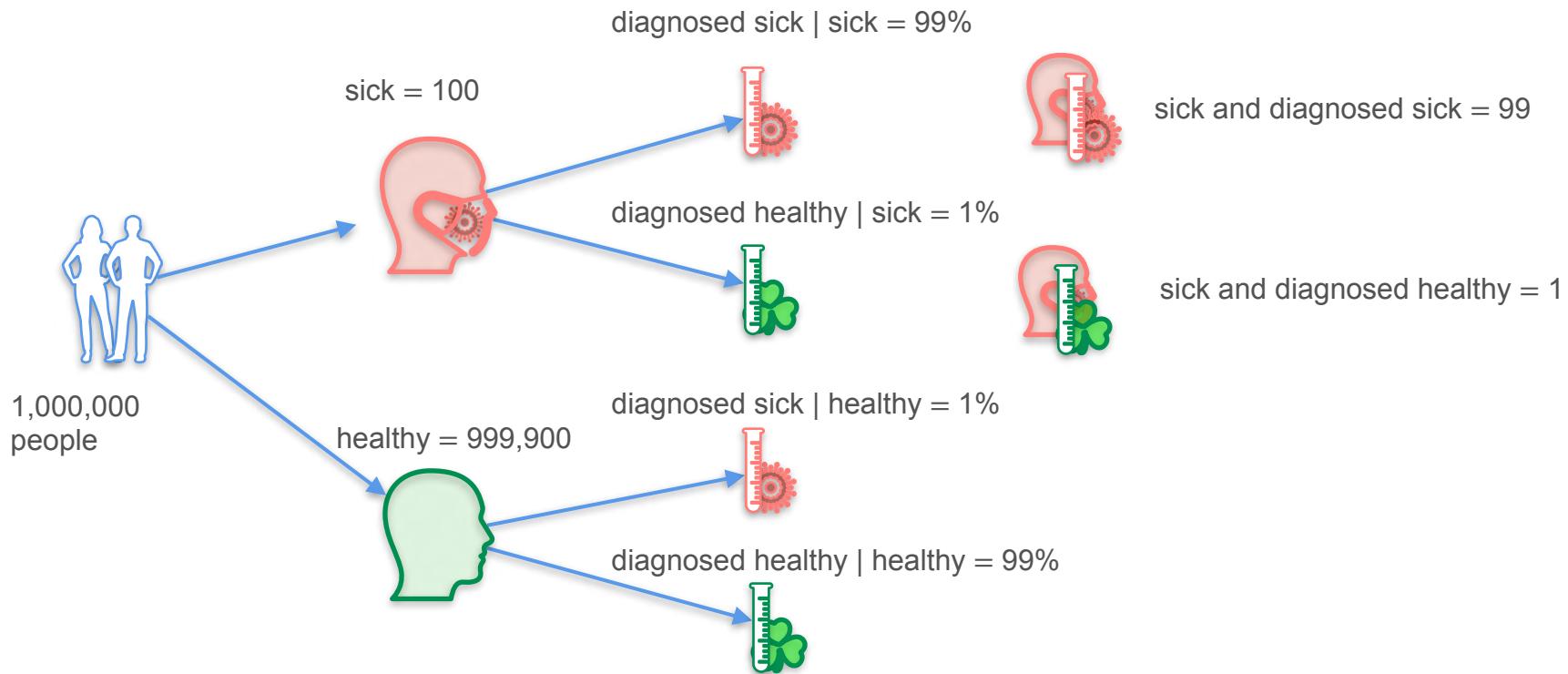
Bayes Theorem: Intuition



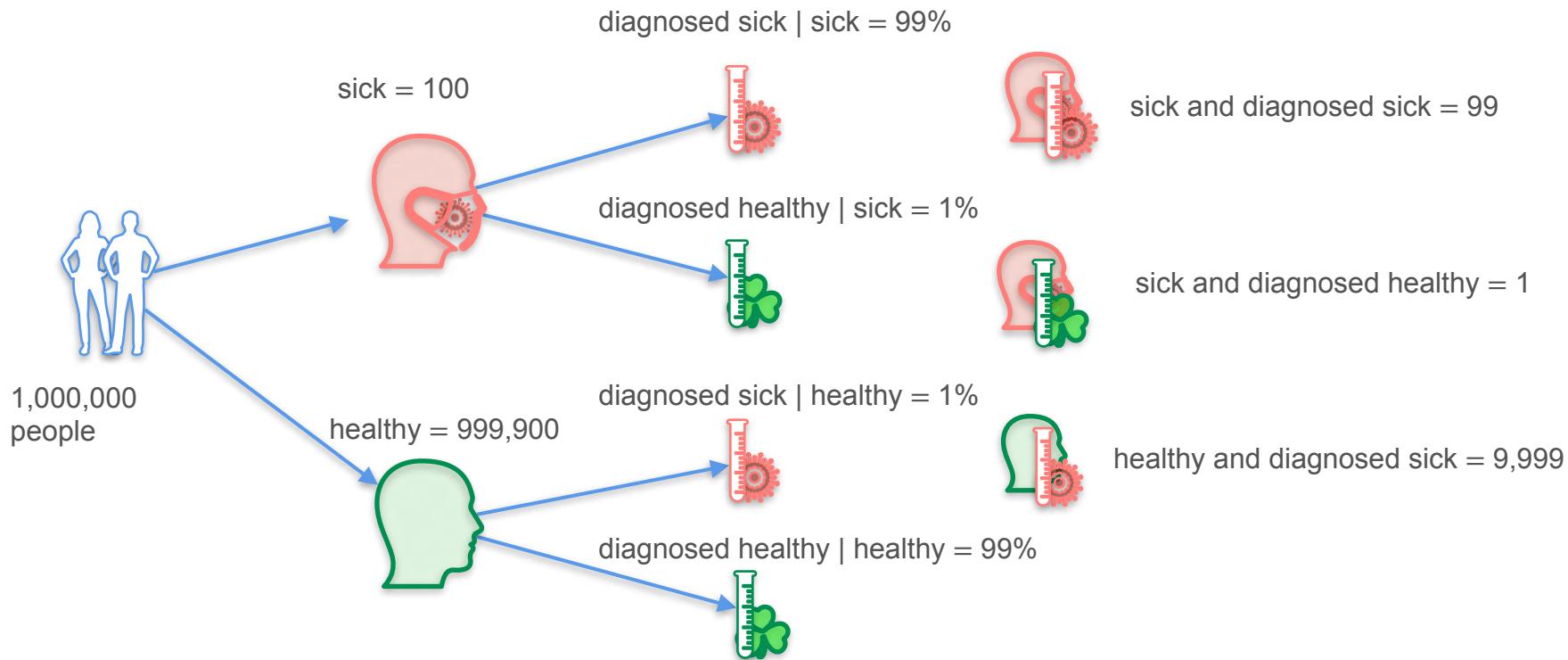
Bayes Theorem: Intuition



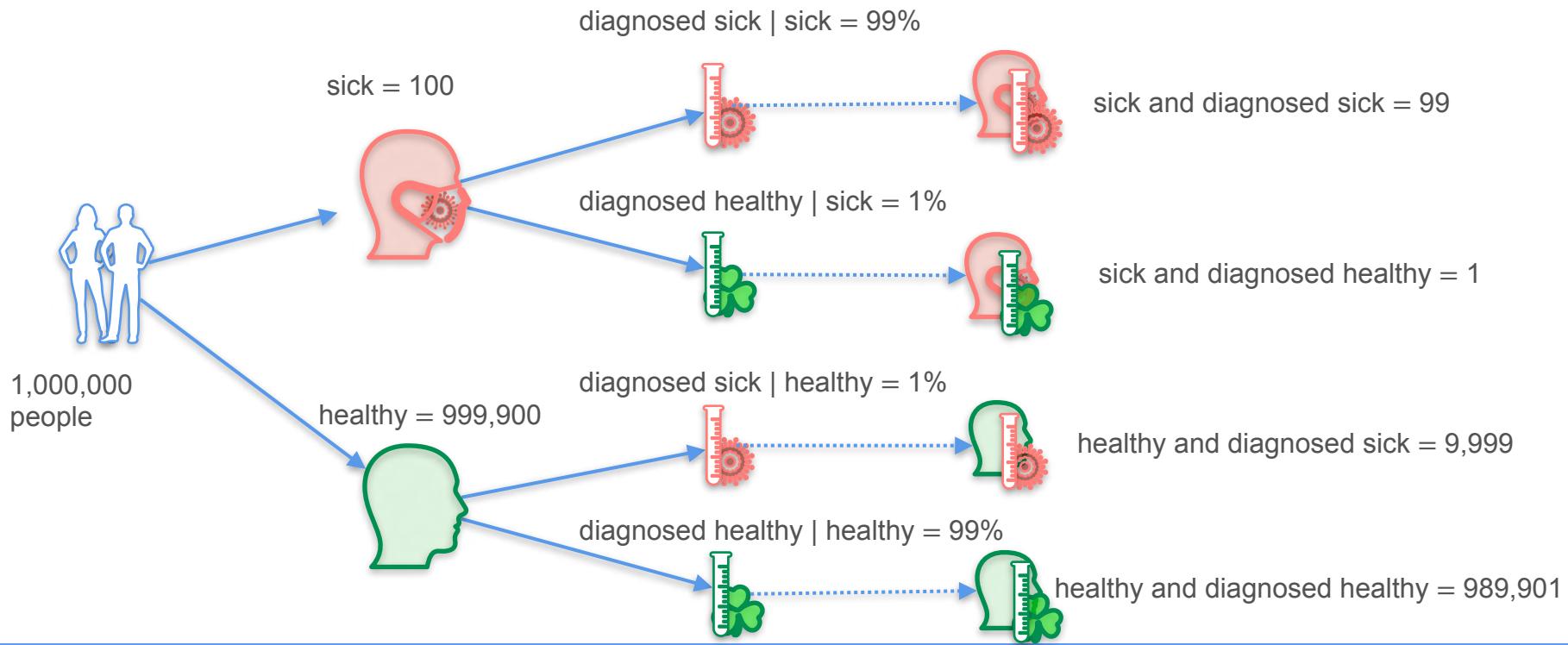
Bayes Theorem: Intuition



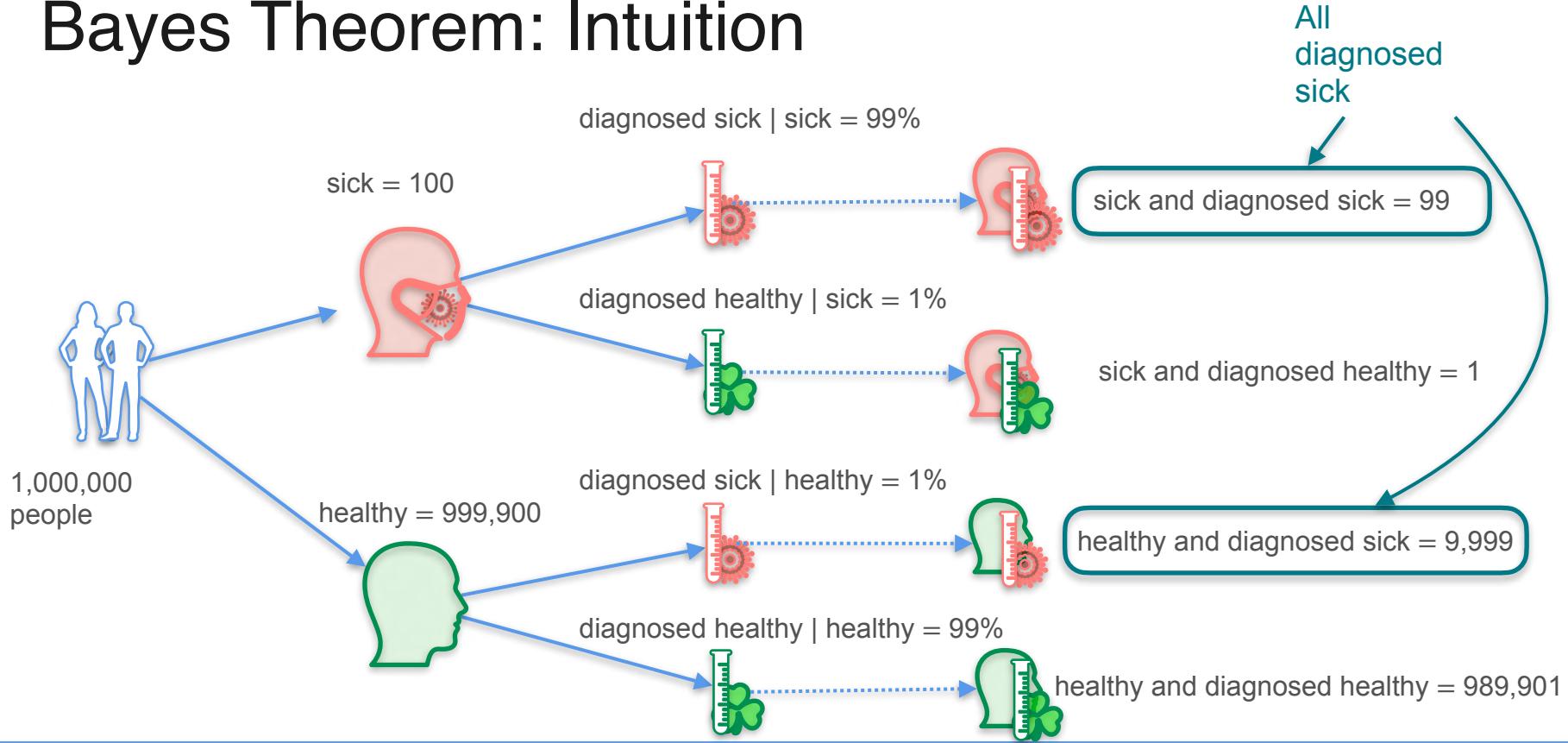
Bayes Theorem: Intuition



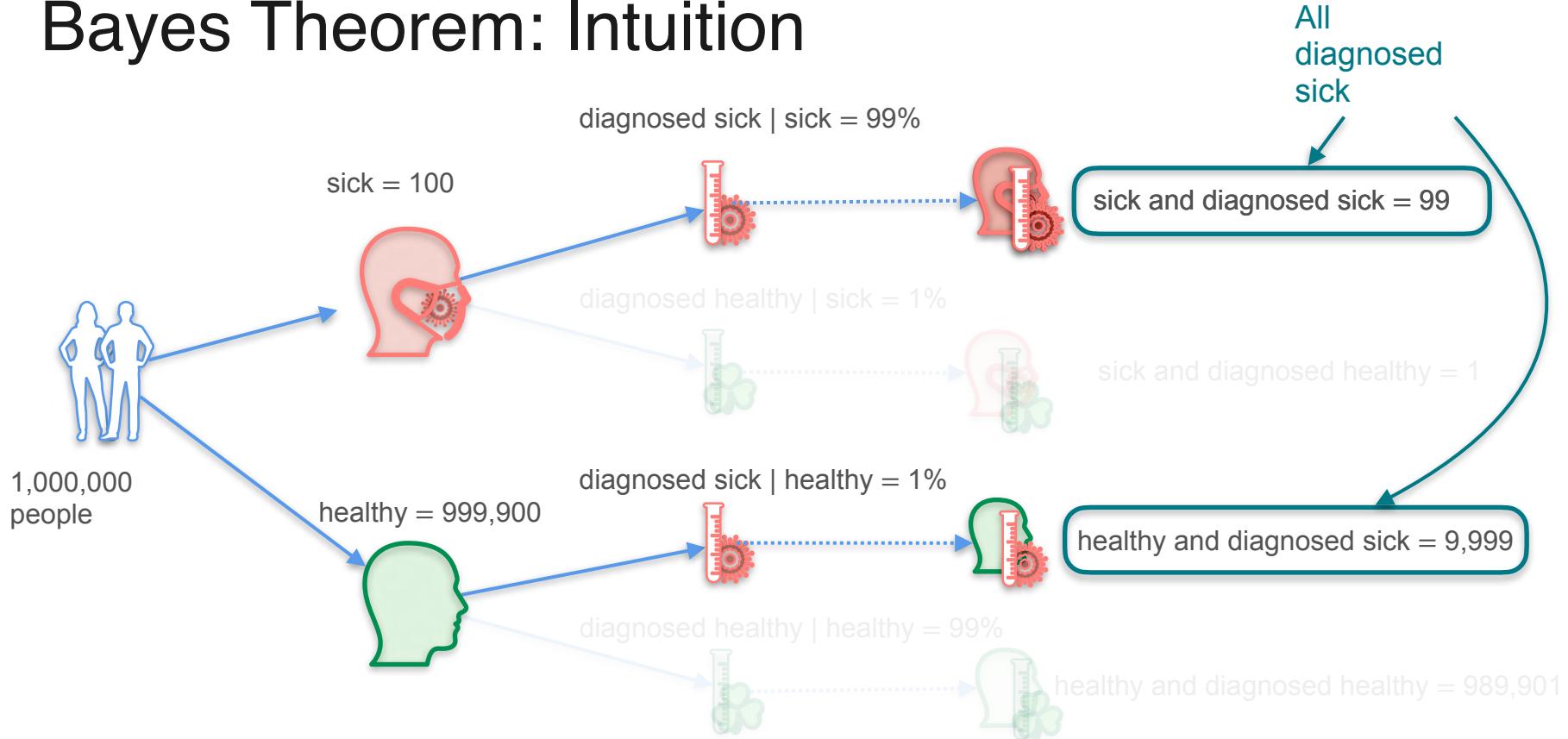
Bayes Theorem: Intuition



Bayes Theorem: Intuition

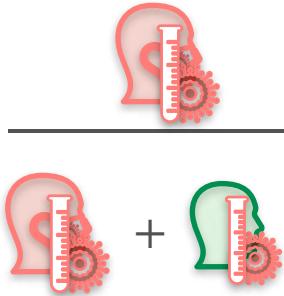


Bayes Theorem: Intuition



Bayes Theorem: Intuition

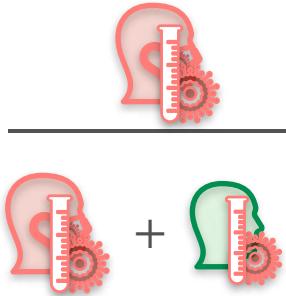
$$P(\text{sick} | \text{diagnosed sick}) = \frac{\text{ }}{\text{ }}$$



$$P(\text{sick} | \text{diagnosed sick}) = \frac{\text{sick and diagnosed sick} = 99}{\text{healthy and diagnosed sick} = 9,999 + \text{sick and diagnosed sick} = 99}$$

Bayes Theorem: Intuition

$$P(\text{sick} \mid \text{diagnosed sick}) = \frac{\text{ }}{\text{ }}$$

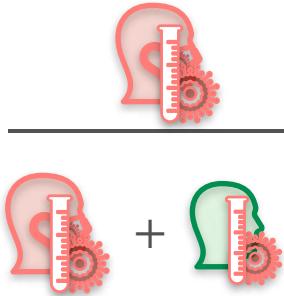


$$P(\text{sick} \mid \text{diagnosed sick}) = \frac{\text{sick and diagnosed sick} = 99}{\text{healthy and diagnosed sick} = 9,999 + \text{sick and diagnosed sick} = 99}$$

$$P(\text{sick} \mid \text{diagnosed sick}) = \frac{99}{10098}$$

Bayes Theorem: Intuition

$$P(\text{sick} \mid \text{diagnosed sick}) = \frac{\text{ }}{\text{ }}$$



$$P(\text{sick} \mid \text{diagnosed sick}) = \frac{\text{sick and diagnosed sick} = 99}{\text{healthy and diagnosed sick} = 9,999 + \text{sick and diagnosed sick} = 99}$$

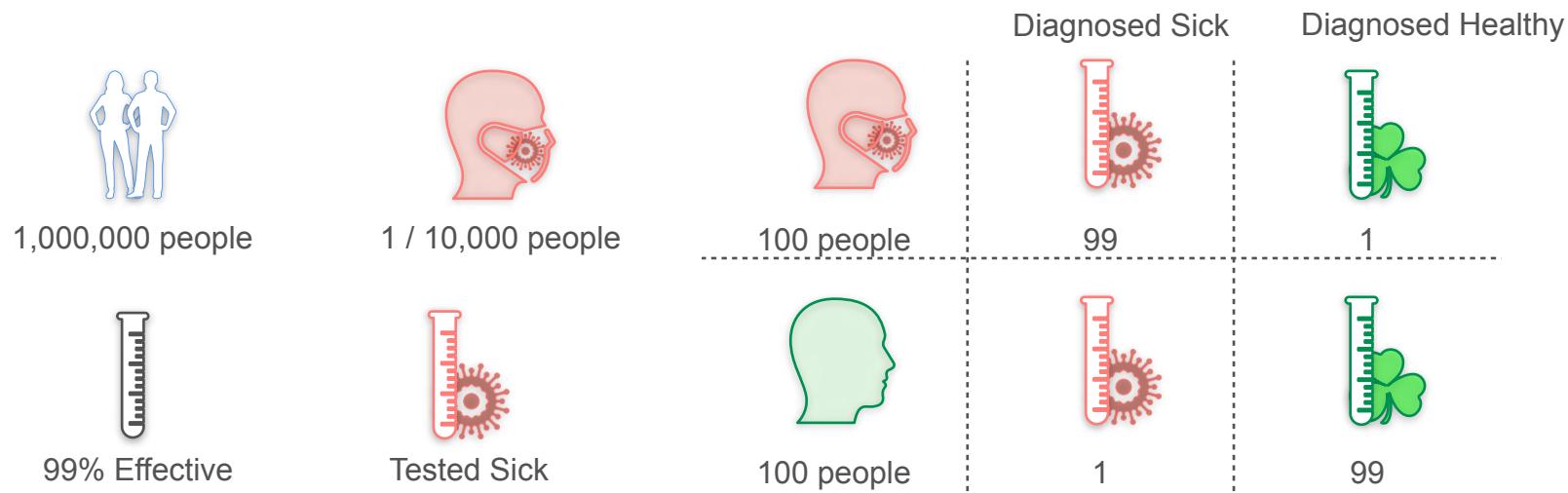
$$P(\text{sick} \mid \text{diagnosed sick}) = \frac{99}{10098} = 0.0098$$

Bayes Theorem: Formula

Bayes Theorem: Formula

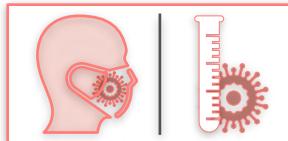
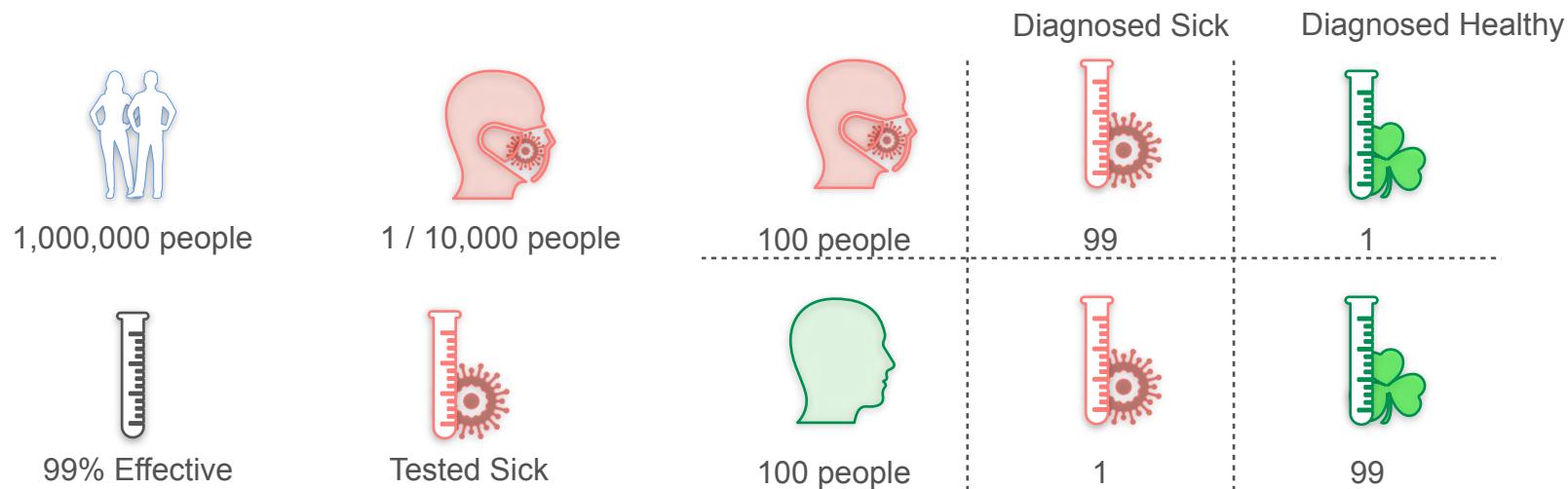
The probability that **you are sick**
GIVEN that you tested sick

Bayes Theorem: Formula



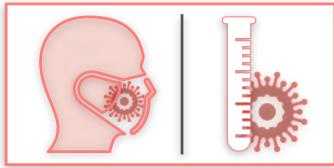
The probability that **you are sick**
GIVEN that you tested sick

Bayes Theorem: Formula



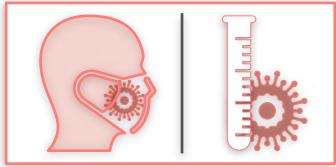
The probability that **you are sick**
GIVEN that you tested sick

Bayes Theorem: Formula



The probability that **you are sick**
GIVEN that you tested sick

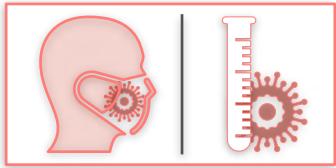
Bayes Theorem: Formula



$P(\text{sick} \mid \text{diagnosed sick}) = ?$

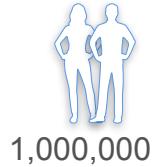
The probability that **you are sick**
GIVEN that you tested sick

Bayes Theorem: Formula

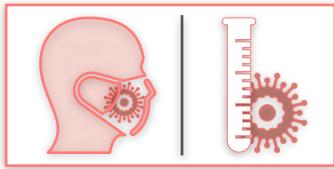


$P(\text{sick} \mid \text{diagnosed sick}) = ?$

The probability that **you are sick**
GIVEN that you tested sick



Bayes Theorem: Formula

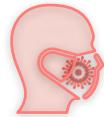


$P(\text{sick} \mid \text{diagnosed sick}) = ?$

The probability that **you are sick**
GIVEN that you tested sick

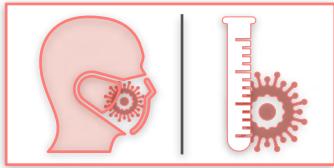


1,000,000



1 / 10,000

Bayes Theorem: Formula



$P(\text{sick} \mid \text{diagnosed sick}) = ?$

$P(\text{sick}) = 0.01\%$

The probability that **you are sick**
GIVEN that you tested sick

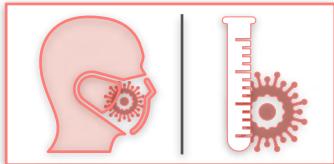


1,000,000



1 / 10,000

Bayes Theorem: Formula



$P(\text{sick} | \text{diagnosed sick}) = ?$

$P(\text{sick}) = 0.01\%$

$P(\text{not sick}) = 99.99\%$

The probability that **you are sick**
GIVEN that you tested sick

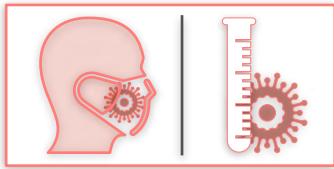


1,000,000



1 / 10,000

Bayes Theorem: Formula



$P(\text{sick} | \text{diagnosed sick}) = ?$

$P(\text{sick}) = 0.01\%$

$P(\text{not sick}) = 99.99\%$

The probability that **you are sick**
GIVEN that you tested sick



1,000,000

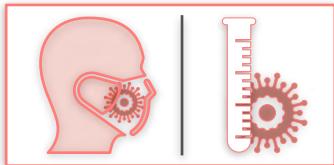


1 / 10,000



99% Effective

Bayes Theorem: Formula



$P(\text{sick} | \text{diagnosed sick}) = ?$

$P(\text{sick}) = 0.01\%$

$P(\text{not sick}) = 99.99\%$

$P(\text{diagnosed sick} | \text{sick}) = 99\%$

The probability that **you are sick**
GIVEN that you tested sick



1,000,000

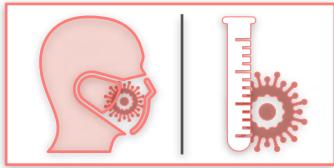


1 / 10,000



99% Effective

Bayes Theorem: Formula



$$P(\text{sick} \mid \text{diagnosed sick}) = ?$$

$$P(\text{sick}) = 0.01\%$$

$$P(\text{not sick}) = 99.99\%$$

$$P(\text{diagnosed sick} \mid \text{sick}) = 99\%$$

$$P(\text{diagnosed sick} \mid \text{not sick}) = 1\%$$

The probability that **you are sick**
GIVEN that you tested sick



1,000,000

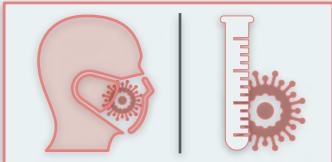


1 / 10,000



99% Effective

Bayes Theorem: Formula



$$P(\text{sick} \mid \text{diagnosed sick}) = ?$$

$$P(\text{sick}) = 0.01\%$$

$$P(\text{not sick}) = 99.99\%$$

$$P(\text{diagnosed sick} \mid \text{sick}) = 99\%$$

$$P(\text{diagnosed sick} \mid \text{not sick}) = 1\%$$

The probability that **you are sick**
GIVEN that you tested sick



1,000,000

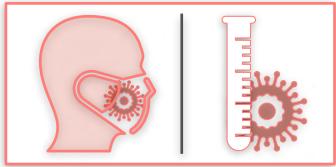


1 / 10,000



99% Effective

Bayes Theorem: Formula



$P(\text{sick} | \text{diagnosed sick}) = ?$

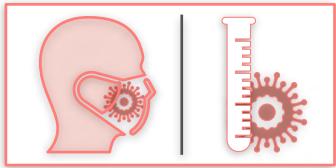
$P(\text{sick}) = 0.01\%$

$P(\text{not sick}) = 99.99\%$

$P(\text{diagnosed sick} | \text{sick}) = 99\%$

$P(\text{diagnosed sick} | \text{not sick}) = 1\%$

Bayes Theorem: Formula



A : sick

B : diagnosed sick

$$P(\text{sick} \mid \text{diagnosed sick}) = ?$$

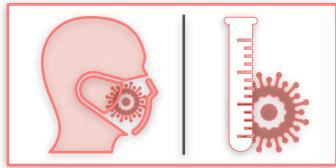
$$P(\text{sick}) = 0.01\%$$

$$P(\text{not sick}) = 99.99\%$$

$$P(\text{diagnosed sick} \mid \text{sick}) = 99\%$$

$$P(\text{diagnosed sick} \mid \text{not sick}) = 1\%$$

Bayes Theorem: Formula



A : sick

B : diagnosed sick

$$P(\text{sick} \mid \text{diagnosed sick}) = ?$$

$$P(A \mid B) = ?$$

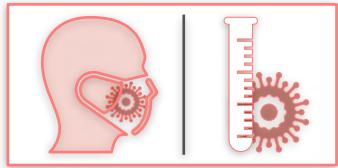
$$P(\text{sick}) = 0.01\%$$

$$P(\text{not sick}) = 99.99\%$$

$$P(\text{diagnosed sick} \mid \text{sick}) = 99\%$$

$$P(\text{diagnosed sick} \mid \text{not sick}) = 1\%$$

Bayes Theorem: Formula



A : sick

B : diagnosed sick

$$P(\text{sick} \mid \text{diagnosed sick}) = ?$$

$$P(A \mid B) = ?$$

$$P(\text{sick}) = 0.01\%$$

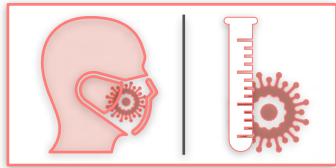
From Conditional Probability

$$P(\text{not sick}) = 99.99\%$$

$$P(\text{diagnosed sick} \mid \text{sick}) = 99\%$$

$$P(\text{diagnosed sick} \mid \text{not sick}) = 1\%$$

Bayes Theorem: Formula



A : sick

B : diagnosed sick

$$P(\text{sick} \mid \text{diagnosed sick}) = ?$$

$$P(A \mid B) = ?$$

$$P(\text{sick}) = 0.01\%$$

From Conditional Probability

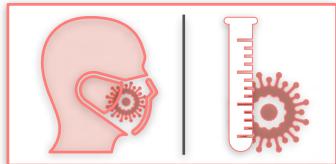
$$P(\text{not sick}) = 99.99\%$$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$P(\text{diagnosed sick} \mid \text{sick}) = 99\%$$

$$P(\text{diagnosed sick} \mid \text{not sick}) = 1\%$$

Bayes Theorem: Formula



A : sick

B : diagnosed sick

$$P(\text{sick} \mid \text{diagnosed sick}) = ?$$

$$P(A \mid B) = ?$$

$$P(\text{sick}) = 0.01\%$$

From Conditional Probability

$$P(\text{not sick}) = 99.99\%$$

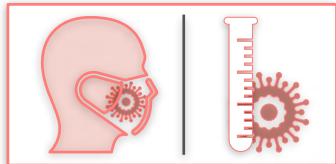
$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$P(\text{diagnosed sick} \mid \text{sick}) = 99\%$$

$$P(\text{diagnosed sick} \mid \text{not sick}) = 1\%$$

$$P(A \mid B) = \underline{\hspace{10em}}$$

Bayes Theorem: Formula



A : sick

B : diagnosed sick

$$P(\text{sick} \mid \text{diagnosed sick}) = ?$$

$$P(A \mid B) = ?$$

$$P(\text{sick}) = 0.01\%$$

From Conditional Probability

$$P(\text{not sick}) = 99.99\%$$

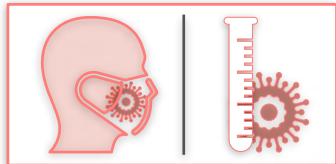
$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$P(\text{diagnosed sick} \mid \text{sick}) = 99\%$$

$$P(A \mid B) = \frac{P(\text{sick and diagnosed sick})}{}$$

$$P(\text{diagnosed sick} \mid \text{not sick}) = 1\%$$

Bayes Theorem: Formula



A : sick

B : diagnosed sick

$$P(\text{sick} \mid \text{diagnosed sick}) = ?$$

$$P(A \mid B) = ?$$

$$P(\text{sick}) = 0.01\%$$

From Conditional Probability

$$P(\text{not sick}) = 99.99\%$$

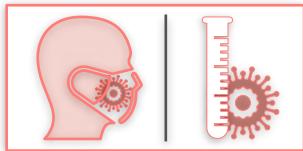
$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$P(\text{diagnosed sick} \mid \text{sick}) = 99\%$$

$$P(A \mid B) = \frac{P(\text{sick and diagnosed sick})}{P(\text{diagnosed sick})}$$

$$P(\text{diagnosed sick} \mid \text{not sick}) = 1\%$$

Bayes Theorem: Formula



A : sick

B : diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(\text{sick and diagnosed sick})}{P(\text{diagnosed sick})}$$

$$P(\text{sick} | \text{diagnosed sick}) = ?$$

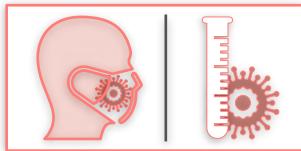
$$P(\text{sick}) = 0.01\%$$

$$P(\text{not sick}) = 99.99\%$$

$$P(\text{diagnosed sick} | \text{sick}) = 99\%$$

$$P(\text{diagnosed sick} | \text{not sick}) = 1\%$$

Bayes Theorem: Formula



A: sick

B: diagnosed sick

$$P(\text{sick} \mid \text{diagnosed sick}) = ?$$

$$P(\text{sick}) = 0.01\%$$

$$P(\text{not sick}) = 99.99\%$$

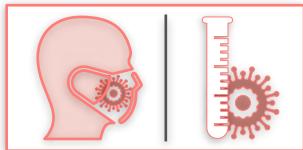
$$P(\text{diagnosed sick} \mid \text{sick}) = 99\%$$

$$P(\text{diagnosed sick} \mid \text{not sick}) = 1\%$$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A \mid B) = \frac{P(\text{sick and diagnosed sick})}{P(\text{diagnosed sick})}$$

Bayes Theorem: Formula



A: sick

B: diagnosed sick

$$P(\text{sick} \mid \text{diagnosed sick}) = ?$$

$$P(\text{sick}) = 0.01\%$$

$$P(\text{not sick}) = 99.99\%$$

$$P(\text{diagnosed sick} \mid \text{sick}) = 99\%$$

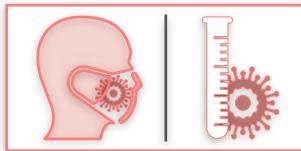
$$P(\text{diagnosed sick} \mid \text{not sick}) = 1\%$$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A \mid B) = \frac{P(\text{sick and diagnosed sick})}{P(\text{diagnosed sick})}$$

$$P(\text{sick and diagnosed sick}) = ?$$

Bayes Theorem: Formula



A: sick

B: diagnosed sick

$$P(\text{sick} \mid \text{diagnosed sick}) = ?$$

$$P(\text{sick}) = 0.01\%$$

$$P(\text{not sick}) = 99.99\%$$

$$P(\text{diagnosed sick} \mid \text{sick}) = 99\%$$

$$P(\text{diagnosed sick} \mid \text{not sick}) = 1\%$$

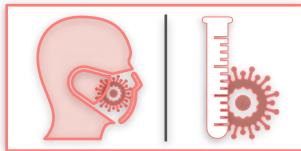
$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A \mid B) = \frac{P(\text{sick and diagnosed sick})}{P(\text{diagnosed sick})}$$

$$P(\text{sick and diagnosed sick}) = ?$$

$$P(\text{diagnosed sick}) = ?$$

Bayes Theorem: Formula



A: sick

B: diagnosed sick

$P(\text{sick} \mid \text{diagnosed sick}) = ?$

$P(\text{sick}) = 0.01\%$

$P(\text{not sick}) = 99.99\%$

$P(\text{diagnosed sick} \mid \text{sick}) = 99\%$

$P(\text{diagnosed sick} \mid \text{not sick}) = 1\%$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

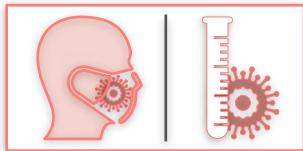
$$P(A \mid B) = \frac{P(\text{sick and diagnosed sick})}{P(\text{diagnosed sick})}$$

$P(\text{sick and diagnosed sick}) = ?$

$P(\text{diagnosed sick}) = ?$

BAYES THEOREM FORMULA CAN HELP

Bayes Theorem: Formula



A: sick

B: diagnosed sick

$P(\text{sick} \mid \text{diagnosed sick}) = ?$

$P(\text{sick}) = 0.01\%$

$P(\text{not sick}) = 99.99\%$

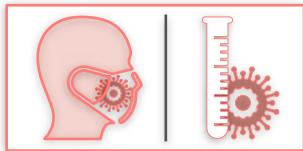
$P(\text{diagnosed sick} \mid \text{sick}) = 99\%$

$P(\text{diagnosed sick} \mid \text{not sick}) = 1\%$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A \mid B) = \frac{P(\text{sick and diagnosed sick}) = ?}{P(\text{diagnosed sick}) = ?}$$

Bayes Theorem: Formula



A: sick

B: diagnosed sick

$P(\text{sick} \mid \text{diagnosed sick}) = ?$

$P(\text{sick}) = 0.01\%$

$P(\text{not sick}) = 99.99\%$

$P(\text{diagnosed sick} \mid \text{sick}) = 99\%$

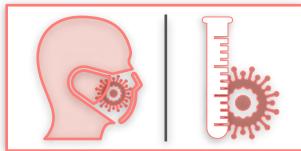
$P(\text{diagnosed sick} \mid \text{not sick}) = 1\%$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$P(\text{sick and diagnosed sick}) = ?$$
$$P(A \mid B) = \frac{P(\text{sick and diagnosed sick})}{P(\text{diagnosed sick})} = ?$$

From Conditional Probability

Bayes Theorem: Formula



A: sick

B: diagnosed sick

$P(\text{sick} \mid \text{diagnosed sick}) = ?$

$P(\text{sick}) = 0.01\%$

$P(\text{not sick}) = 99.99\%$

$P(\text{diagnosed sick} \mid \text{sick}) = 99\%$

$P(\text{diagnosed sick} \mid \text{not sick}) = 1\%$

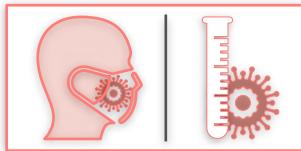
$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$P(\text{sick and diagnosed sick}) = ?$$
$$P(A \mid B) = \frac{P(\text{sick and diagnosed sick})}{P(\text{diagnosed sick})} = ?$$

From Conditional Probability

$$P(A \cap B) = P(A) \cdot P(B \mid A)$$

Bayes Theorem: Formula



A: sick

B: diagnosed sick

$P(\text{sick} \mid \text{diagnosed sick}) = ?$

$P(\text{sick}) = 0.01\%$

$P(\text{not sick}) = 99.99\%$

$P(\text{diagnosed sick} \mid \text{sick}) = 99\%$

$P(\text{diagnosed sick} \mid \text{not sick}) = 1\%$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

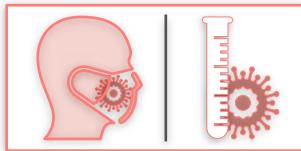
$$P(\text{sick and diagnosed sick}) = ?$$
$$P(A \mid B) = \frac{P(\text{sick and diagnosed sick})}{P(\text{diagnosed sick})} = ?$$

From Conditional Probability

$$P(A \cap B) = P(A) \cdot P(B \mid A)$$

$P(\text{sick and diagnosed sick}) =$

Bayes Theorem: Formula



A: sick

B: diagnosed sick

$P(\text{sick} \mid \text{diagnosed sick}) = ?$

$P(\text{sick}) = 0.01\%$

$P(\text{not sick}) = 99.99\%$

$P(\text{diagnosed sick} \mid \text{sick}) = 99\%$

$P(\text{diagnosed sick} \mid \text{not sick}) = 1\%$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

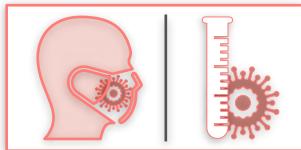
$$P(A \mid B) = \frac{P(\text{sick and diagnosed sick}) = ?}{P(\text{diagnosed sick}) = ?}$$

From Conditional Probability

$$P(A \cap B) = P(A) \cdot P(B \mid A)$$

$$P(\text{sick and diagnosed sick}) = P(\text{sick})$$

Bayes Theorem: Formula



A: sick

B: diagnosed sick

$P(\text{sick} \mid \text{diagnosed sick}) = ?$

$P(\text{sick}) = 0.01\%$

$P(\text{not sick}) = 99.99\%$

$P(\text{diagnosed sick} \mid \text{sick}) = 99\%$

$P(\text{diagnosed sick} \mid \text{not sick}) = 1\%$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

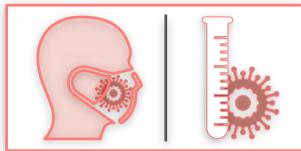
$$P(\text{sick and diagnosed sick}) = ?$$
$$P(A \mid B) = \frac{P(\text{sick and diagnosed sick})}{P(\text{diagnosed sick})} = ?$$

From Conditional Probability

$$P(A \cap B) = P(A) \cdot P(B \mid A)$$

$$P(\text{sick and diagnosed sick}) = P(\text{sick}) \cdot$$

Bayes Theorem: Formula



A: sick

B: diagnosed sick

$P(\text{sick} \mid \text{diagnosed sick}) = ?$

$P(\text{sick}) = 0.01\%$

$P(\text{not sick}) = 99.99\%$

$P(\text{diagnosed sick} \mid \text{sick}) = 99\%$

$P(\text{diagnosed sick} \mid \text{not sick}) = 1\%$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

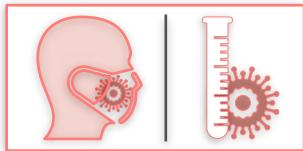
$$P(A \mid B) = \frac{P(\text{sick and diagnosed sick}) = ?}{P(\text{diagnosed sick}) = ?}$$

From Conditional Probability

$$P(A \cap B) = P(A) \cdot P(B \mid A)$$

$$P(\text{sick and diagnosed sick}) = P(\text{sick}) \cdot P(\text{diagnosed sick} \mid \text{sick})$$

Bayes Theorem: Formula



A: sick

B: diagnosed sick

$$P(\text{sick} \mid \text{diagnosed sick}) = ?$$

$$P(\text{sick}) = 0.01\%$$

$$P(\text{not sick}) = 99.99\%$$

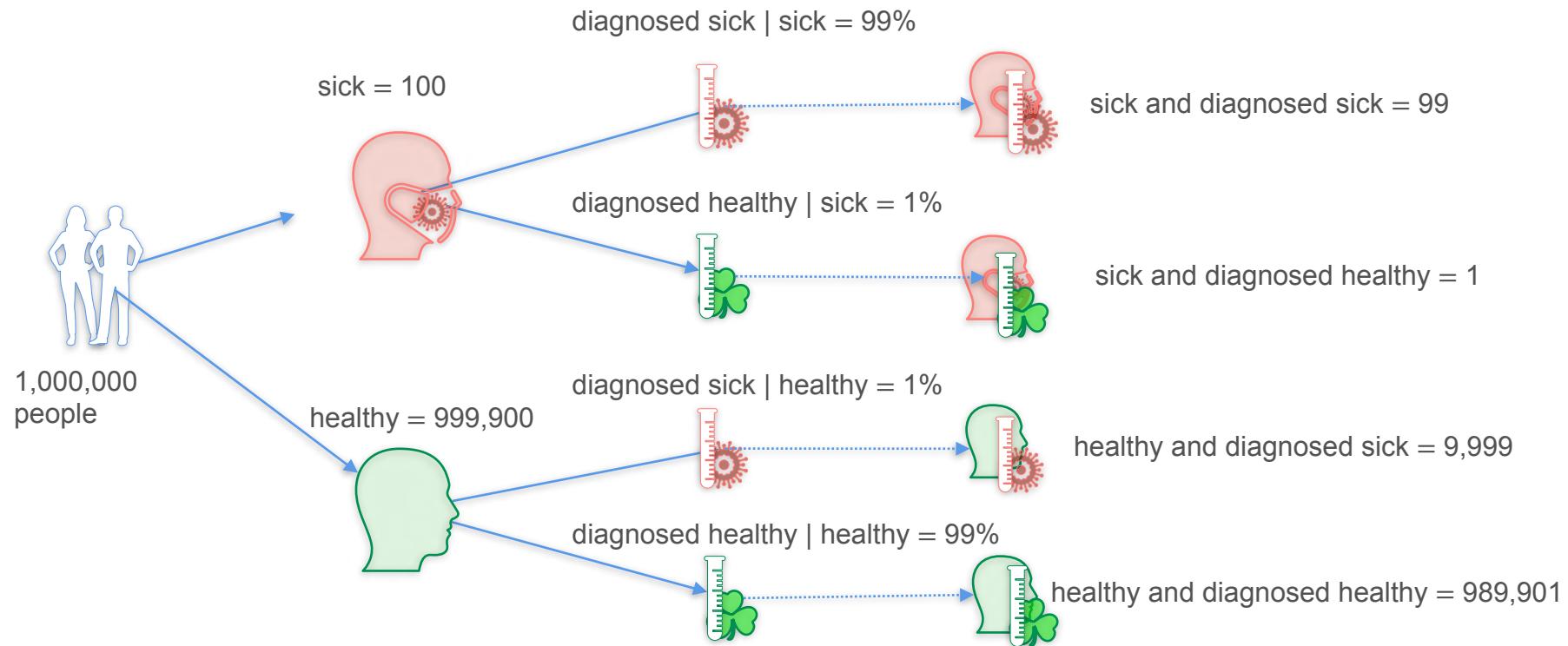
$$P(\text{diagnosed sick} \mid \text{sick}) = 99\%$$

$$P(\text{diagnosed sick} \mid \text{not sick}) = 1\%$$

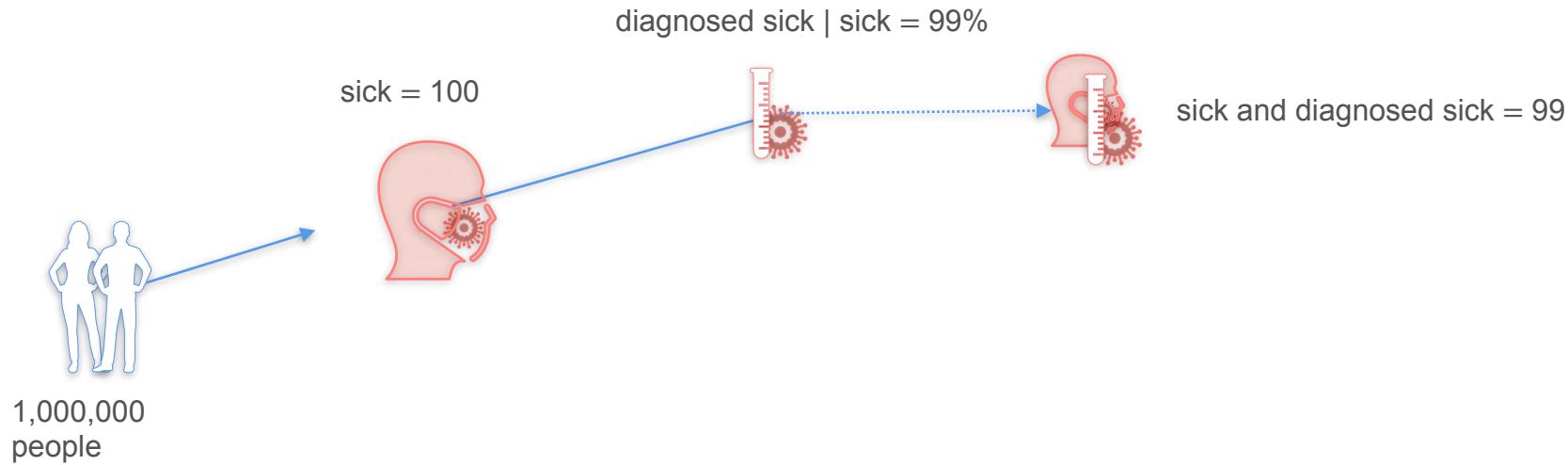
$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A \mid B) = \frac{P(\text{sick}) \cdot P(\text{diagnosed sick} \mid \text{sick})}{P(\text{diagnosed sick})} = ?$$

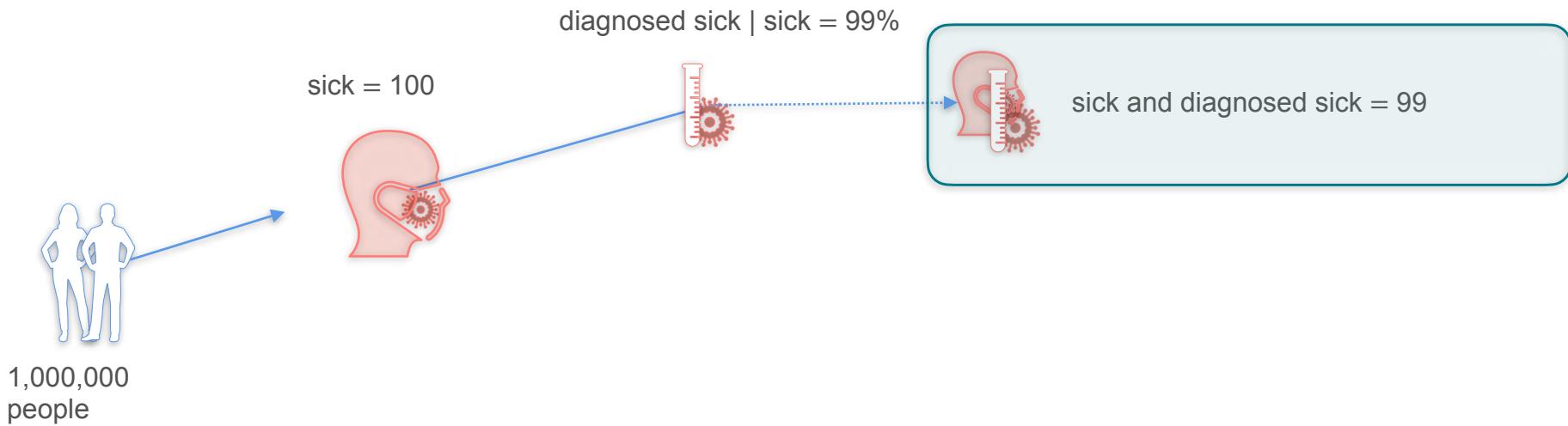
Bayes Theorem: Formula



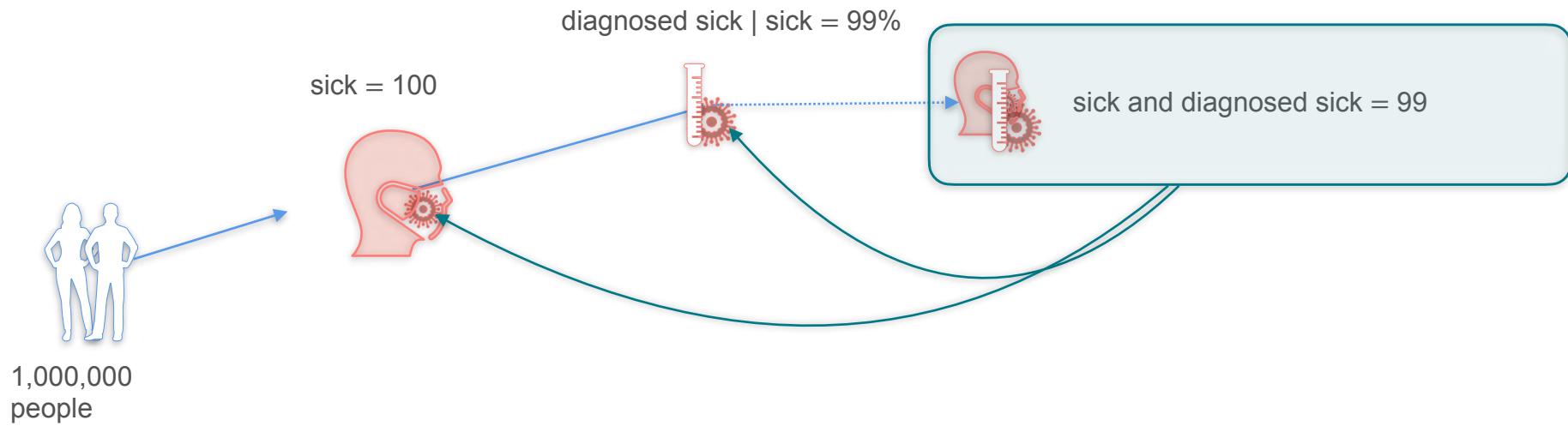
Bayes Theorem: Formula



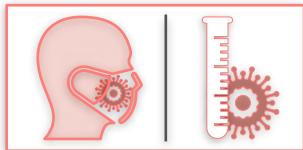
Bayes Theorem: Formula



Bayes Theorem: Formula



Bayes Theorem: Formula



A: sick

B: diagnosed sick

$$P(\text{sick} \mid \text{diagnosed sick}) = ?$$

$$P(\text{sick}) = 0.01\%$$

$$P(\text{not sick}) = 99.99\%$$

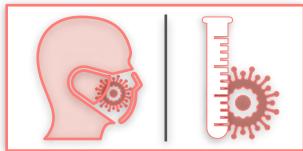
$$P(\text{diagnosed sick} \mid \text{sick}) = 99\%$$

$$P(\text{diagnosed sick} \mid \text{not sick}) = 1\%$$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A \mid B) = \frac{P(\text{sick}) \cdot P(\text{diagnosed sick} \mid \text{sick})}{P(\text{diagnosed sick})} = ?$$

Bayes Theorem: Formula



A: sick

B: diagnosed sick

$$P(\text{sick} \mid \text{diagnosed sick}) = ?$$

$$P(\text{sick}) = 0.01\%$$

$$P(\text{not sick}) = 99.99\%$$

$$P(\text{diagnosed sick} \mid \text{sick}) = 99\%$$

$$P(\text{diagnosed sick} \mid \text{not sick}) = 1\%$$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A \mid B) = \frac{P(\text{sick}) \cdot P(\text{diagnosed sick} \mid \text{sick})}{P(\text{diagnosed sick})} = ?$$

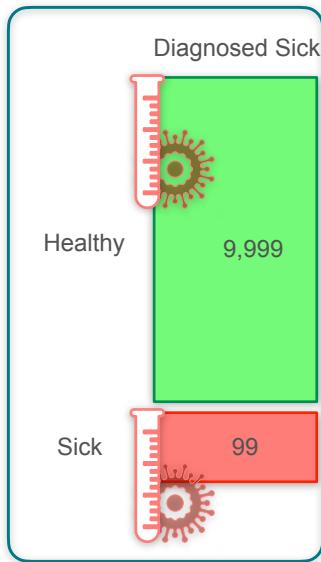
$$P(\text{diagnosed sick}) =$$

Bayes Theorem: Formula

$P(\text{diagnosed sick}) =$

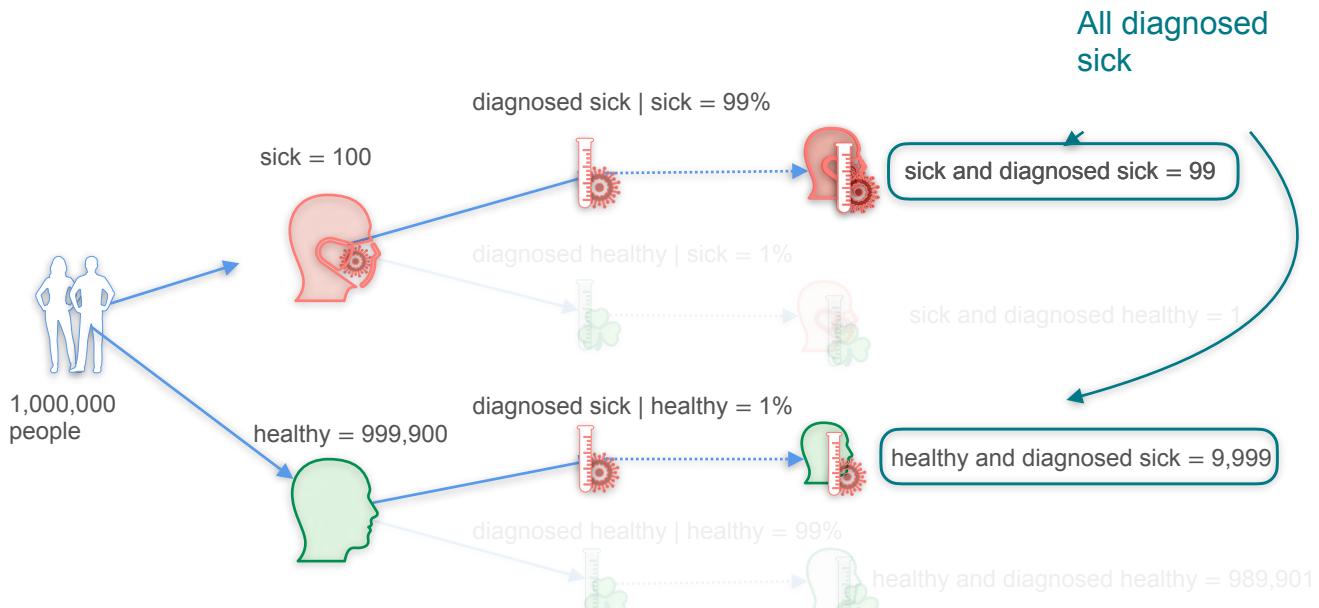
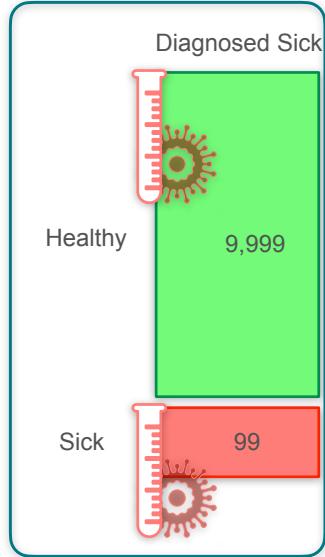
Bayes Theorem: Formula

$P(\text{diagnosed sick}) =$



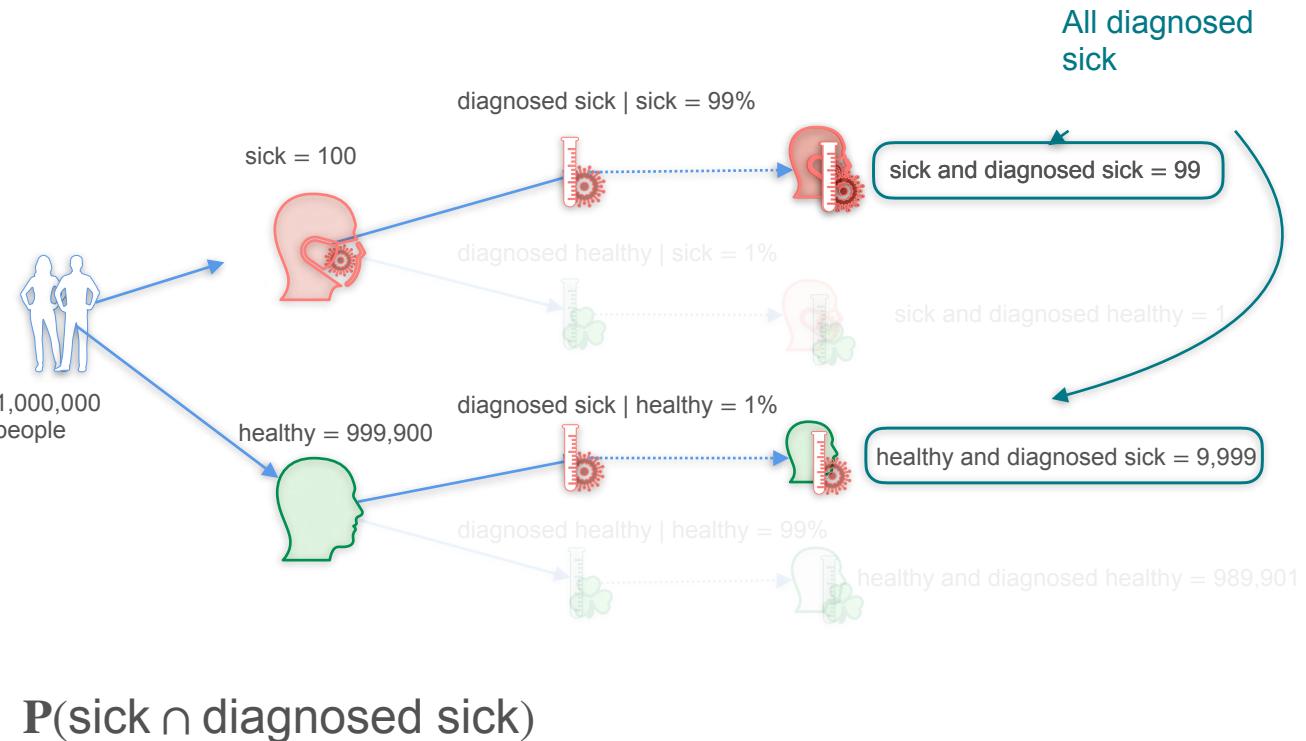
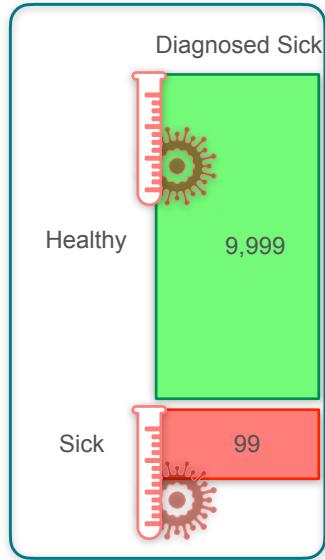
Bayes Theorem: Formula

$$P(\text{diagnosed sick}) =$$



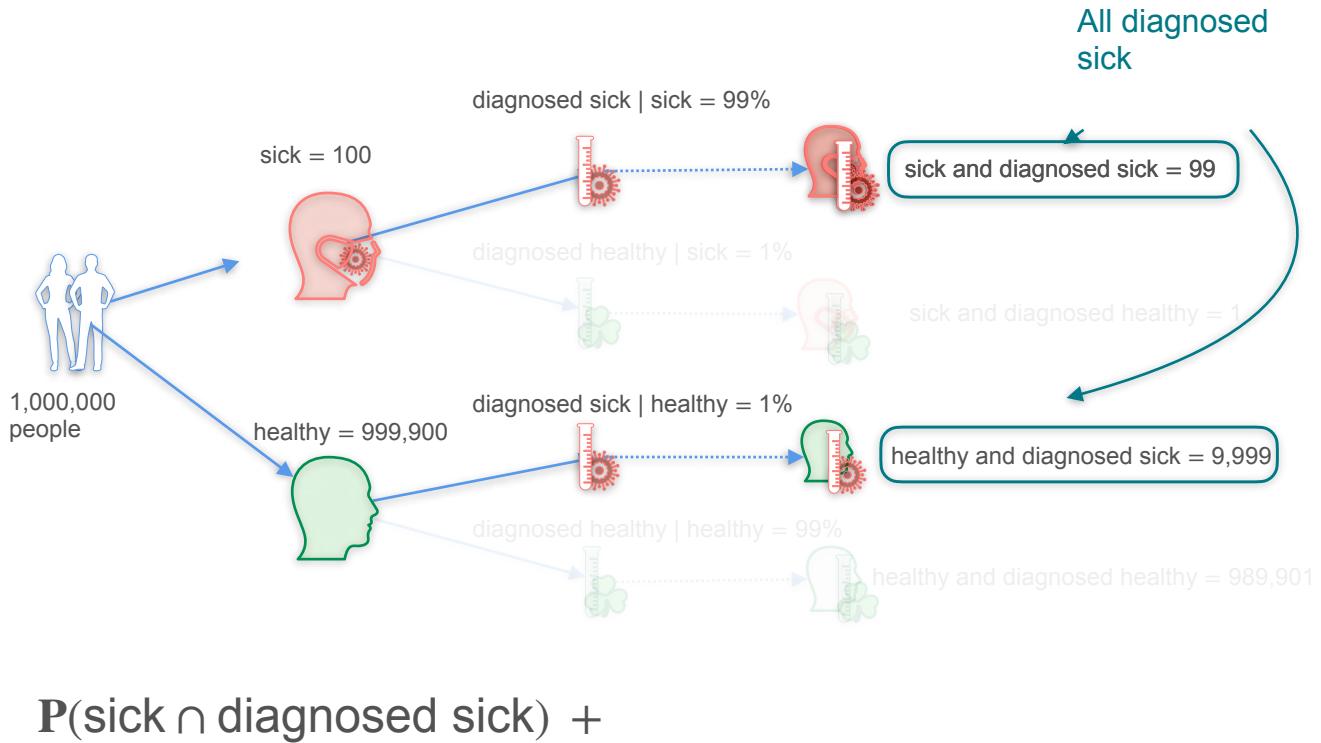
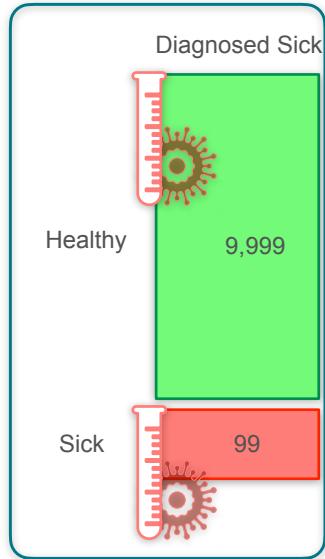
Bayes Theorem: Formula

$$P(\text{diagnosed sick}) =$$



Bayes Theorem: Formula

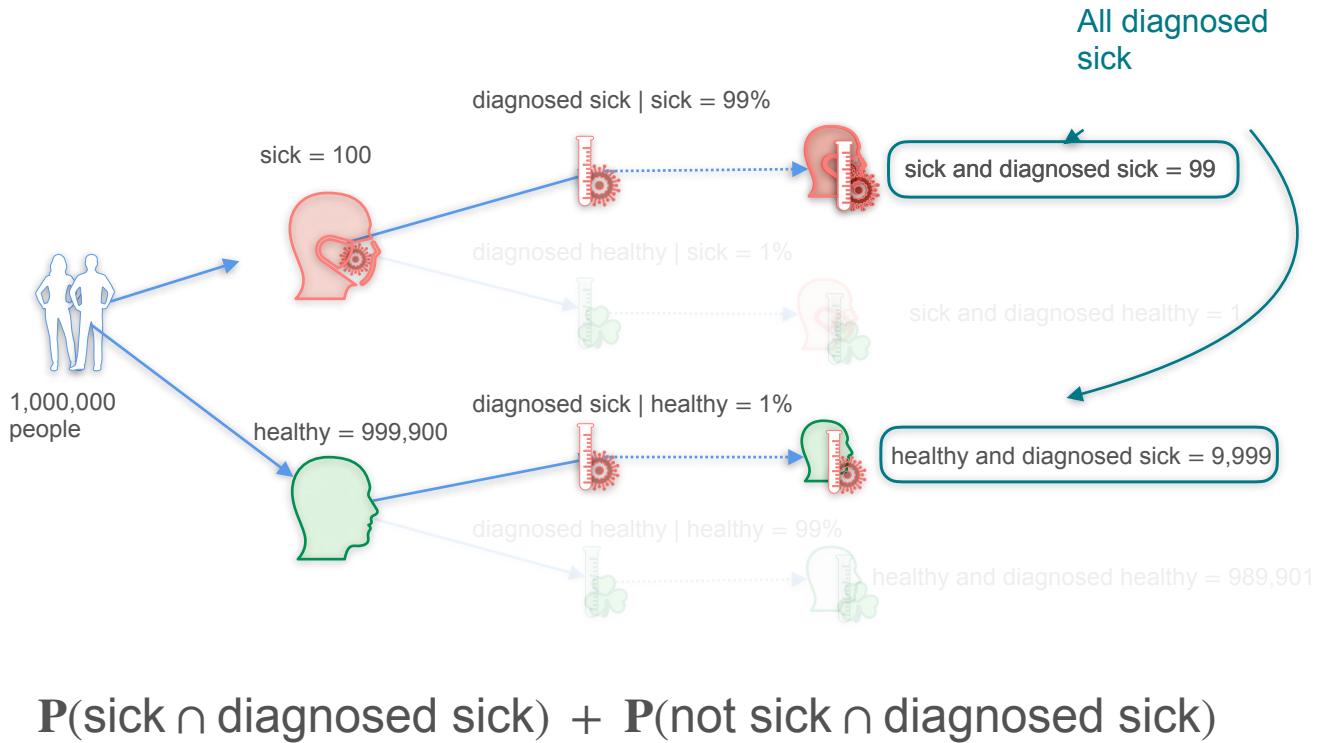
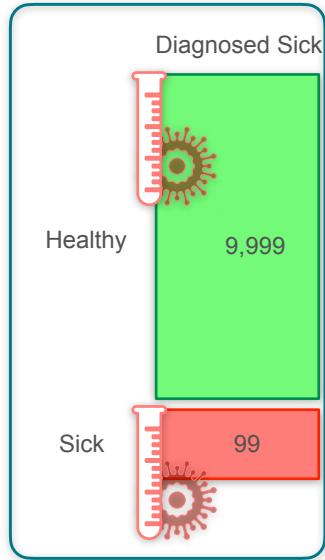
$P(\text{diagnosed sick}) =$



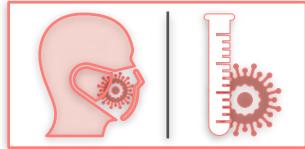
$P(\text{sick} \cap \text{diagnosed sick}) +$

Bayes Theorem: Formula

$P(\text{diagnosed sick}) =$



Bayes Theorem: Formula



A : sick

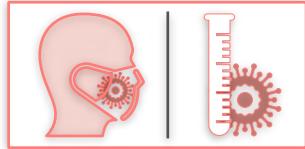
B : diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick})}{P(\text{diagnosed sick}) = ?}$$

$$P(\text{diagnosed sick}) = P(\text{sick} \cap \text{diagnosed sick}) + P(\text{not sick} \cap \text{diagnosed sick})$$

Bayes Theorem: Formula



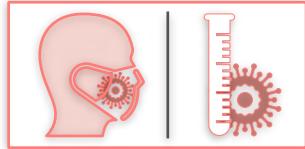
A : sick

B : diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick})}{P(\text{sick} \cap \text{diagnosed sick}) + P(\text{not sick} \cap \text{diagnosed sick})}$$

Bayes Theorem: Formula



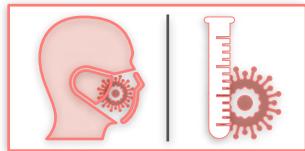
A: sick

B: diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick})}{P(\text{sick} \cap \text{diagnosed sick}) + P(\text{not sick} \cap \text{diagnosed sick})}$$

Bayes Theorem: Formula



A : sick

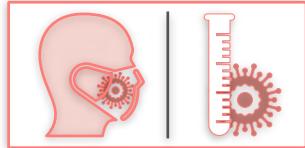
B : diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick})}{P(\text{sick} \cap \text{diagnosed sick}) + P(\text{not sick} \cap \text{diagnosed sick})}$$

$$P(\text{sick} \cap \text{diagnosed sick})$$

Bayes Theorem: Formula



A : sick

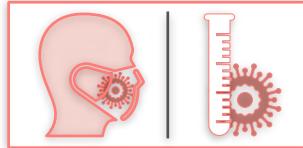
B : diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick})}{P(\text{sick} \cap \text{diagnosed sick}) + P(\text{not sick} \cap \text{diagnosed sick})}$$

$$P(\text{sick} \cap \text{diagnosed sick}) = P(A \cap B)$$

Bayes Theorem: Formula



A: sick

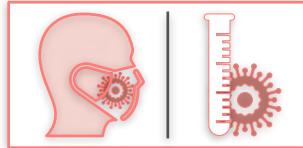
B: diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick})}{P(\text{sick} \cap \text{diagnosed sick}) + P(\text{not sick} \cap \text{diagnosed sick})}$$

$$P(\text{sick} \cap \text{diagnosed sick}) = P(A \cap B)$$

Bayes Theorem: Formula



A: sick

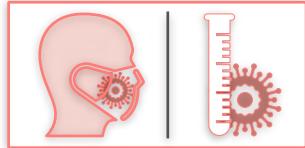
B: diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick})}{P(\text{sick} \cap \text{diagnosed sick}) + P(\text{not sick} \cap \text{diagnosed sick})}$$

$$P(\text{sick} \cap \text{diagnosed sick}) = P(A \cap B)$$

Bayes Theorem: Formula



A: sick

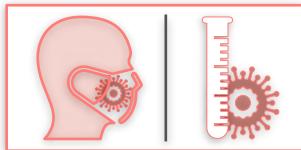
B: diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick})}{P(\text{sick} \cap \text{diagnosed sick}) + P(\text{not sick} \cap \text{diagnosed sick})}$$

$$\begin{aligned} P(\text{sick} \cap \text{diagnosed sick}) &= P(A \cap B) \\ &= P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick}) \end{aligned}$$

Bayes Theorem: Formula



A: sick

B: diagnosed sick

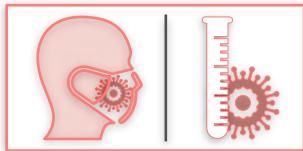
$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick})}{P(\text{sick} \cap \text{diagnosed sick}) + P(\text{not sick} \cap \text{diagnosed sick})}$$

$$\begin{aligned} P(\text{sick} \cap \text{diagnosed sick}) &= P(A \cap B) \\ &= P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick}) \end{aligned}$$

$$P(\text{not sick} \cap \text{diagnosed sick})$$

Bayes Theorem: Formula



A: sick

B: diagnosed sick

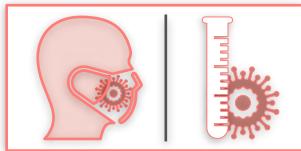
$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick})}{P(\text{sick} \cap \text{diagnosed sick}) + P(\text{not sick} \cap \text{diagnosed sick})}$$

$$\begin{aligned} P(\text{sick} \cap \text{diagnosed sick}) &= P(A \cap B) \\ &= P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick}) \end{aligned}$$

$$P(\text{not sick} \cap \text{diagnosed sick}) = P(A' \cap B)$$

Bayes Theorem: Formula



A: sick

B: diagnosed sick

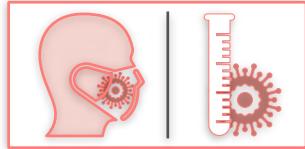
$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick})}{P(\text{sick} \cap \text{diagnosed sick}) + P(\text{not sick} \cap \text{diagnosed sick})}$$

$$\begin{aligned} P(\text{sick} \cap \text{diagnosed sick}) &= P(A \cap B) \\ &= P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick}) \end{aligned}$$

$$\begin{aligned} P(\text{not sick} \cap \text{diagnosed sick}) &= P(A' \cap B) \\ &= P(\text{not sick}) \cdot P(\text{diagnosed sick} | \text{not sick}) \end{aligned}$$

Bayes Theorem: Formula



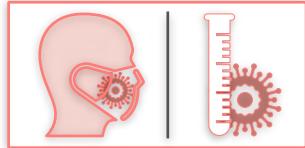
A : sick

B : diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick})}{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick}) + P(\text{not sick}) \cdot P(\text{diagnosed sick} | \text{not sick})}$$

Bayes Theorem: Formula



A: sick

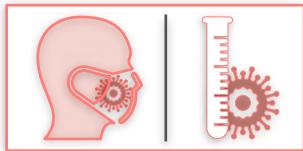
B: diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick})}{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick}) + P(\text{not sick}) \cdot P(\text{diagnosed sick} | \text{not sick})}$$

**BAYES THEOREM
FORMULA**

Bayes Theorem: Formula



A: sick

B: diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick})}{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick}) + P(\text{not sick}) \cdot P(\text{diagnosed sick} | \text{not sick})}$$

$$P(\text{sick}) = 0.01\%$$

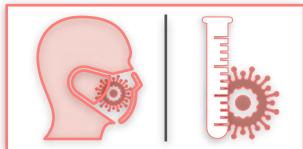
$$P(\text{not sick}) = 99.99\%$$

$$P(\text{diagnosed sick} | \text{sick}) = 99\%$$

$$P(\text{diagnosed sick} | \text{not sick}) = 1\%$$

**BAYES THEOREM
FORMULA**

Bayes Theorem: Formula



A: sick

B: diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick})}{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick}) + P(\text{not sick}) \cdot P(\text{diagnosed sick} | \text{not sick})}$$

$$P(\text{sick}) = 0.01\%$$

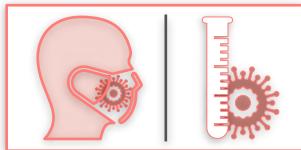
$$P(\text{not sick}) = 99.99\%$$

$$P(\text{diagnosed sick} | \text{sick}) = 99\%$$

$$P(\text{diagnosed sick} | \text{not sick}) = 1\%$$

**BAYES THEOREM
FORMULA**

Bayes Theorem: Formula



A: sick

B: diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick})}{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick}) + P(\text{not sick}) \cdot P(\text{diagnosed sick} | \text{not sick})}$$

$$P(\text{sick}) = 0.01\%$$

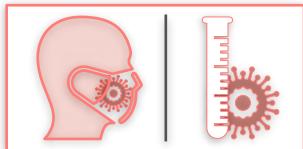
$$P(\text{not sick}) = 99.99\%$$

$$P(\text{diagnosed sick} | \text{sick}) = 99\%$$

$$P(\text{diagnosed sick} | \text{not sick}) = 1\%$$

**BAYES THEOREM
FORMULA**

Bayes Theorem: Formula



A: sick

B: diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick})}{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick}) + P(\text{not sick}) \cdot P(\text{diagnosed sick} | \text{not sick})}$$

$$P(\text{sick}) = 0.01\%$$

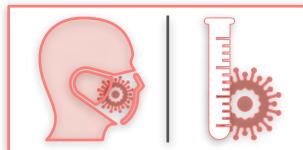
$$P(\text{not sick}) = 99.99\%$$

$$P(\text{diagnosed sick} | \text{sick}) = 99\%$$

$$P(\text{diagnosed sick} | \text{not sick}) = 1\%$$

**BAYES THEOREM
FORMULA**

Bayes Theorem: Formula



A: sick

B: diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick})}{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick}) + P(\text{not sick}) \cdot P(\text{diagnosed sick} | \text{not sick})}$$

$$P(\text{sick}) = 0.01\%$$

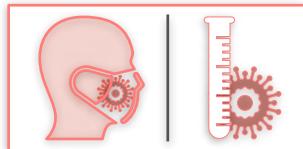
$$P(\text{not sick}) = 99.99\%$$

$$P(\text{diagnosed sick} | \text{sick}) = 99\%$$

$$P(\text{diagnosed sick} | \text{not sick}) = 1\%$$

**BAYES THEOREM
FORMULA**

Bayes Theorem: Formula



A: sick

B: diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick})}{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick}) + P(\text{not sick}) \cdot P(\text{diagnosed sick} | \text{not sick})}$$

$P(\text{sick}) = 0.01\%$

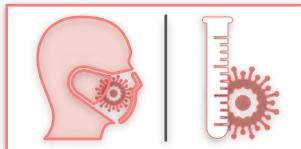
$P(\text{not sick}) = 99.99\%$

$P(\text{diagnosed sick} | \text{sick}) = 99\%$

$P(\text{diagnosed sick} | \text{not sick}) = 1\%$

**BAYES THEOREM
FORMULA**

Bayes Theorem: Formula



A: sick

B: diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

?

$$P(A | B) = \frac{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick})}{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick}) + P(\text{not sick}) \cdot P(\text{diagnosed sick} | \text{not sick})}$$

$P(\text{sick}) = 0.01\%$

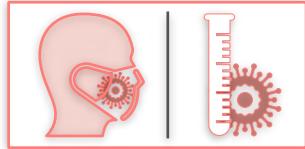
$P(\text{not sick}) = 99.99\%$

$P(\text{diagnosed sick} | \text{sick}) = 99\%$

$P(\text{diagnosed sick} | \text{not sick}) = 1\%$

**BAYES THEOREM
FORMULA**

Bayes Theorem: Formula



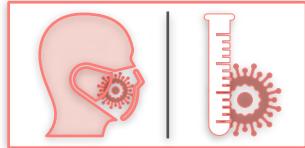
A : sick

B : diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick})}{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick}) + P(\text{not sick}) \cdot P(\text{diagnosed sick} | \text{not sick})}$$

Bayes Theorem: Formula



A: sick

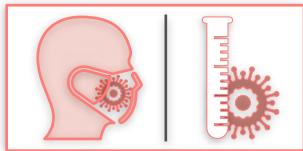
B: diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick})}{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick}) + P(\text{not sick}) \cdot P(\text{diagnosed sick} | \text{not sick})}$$

**BAYES THEOREM
FORMULA**

Bayes Theorem: Formula



A: sick

B: diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick})}{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick}) + P(\text{not sick}) \cdot P(\text{diagnosed sick} | \text{not sick})}$$

$$P(\text{sick}) = 1\%$$

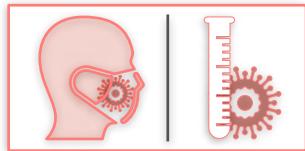
$$P(\text{not sick}) = 99\%$$

$$P(\text{diagnosed sick} | \text{sick}) = 99\%$$

$$P(\text{diagnosed sick} | \text{not sick}) = 1\%$$

**BAYES THEOREM
FORMULA**

Bayes Theorem: Formula



A: sick

B: diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick})}{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick}) + P(\text{not sick}) \cdot P(\text{diagnosed sick} | \text{not sick})}$$

$$P(A) = 0.01\%$$

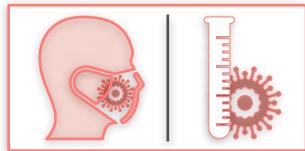
$$P(\text{not sick}) = 99\%$$

$$P(\text{diagnosed sick} | \text{sick}) = 99\%$$

$$P(\text{diagnosed sick} | \text{not sick}) = 1\%$$

**BAYES THEOREM
FORMULA**

Bayes Theorem: Formula



A: sick

B: diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick})}{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick}) + P(\text{not sick}) \cdot P(\text{diagnosed sick} | \text{not sick})}$$

$$P(A) = 0.01\%$$

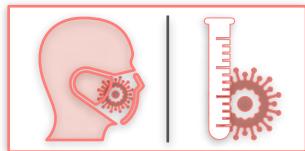
$$P(A') = 99.99\%$$

$$P(\text{diagnosed sick} | \text{sick}) = 99\%$$

$$P(\text{diagnosed sick} | \text{not sick}) = 1\%$$

**BAYES THEOREM
FORMULA**

Bayes Theorem: Formula



A: sick

B: diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick})}{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick}) + P(\text{not sick}) \cdot P(\text{diagnosed sick} | \text{not sick})}$$

$$P(A) = 0.01\%$$

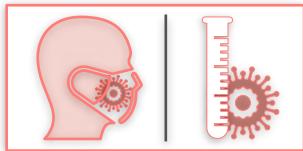
$$P(A') = 99.99\%$$

$$P(B | A) = 99\%$$

$$P(\text{diagnosed sick} | \text{not sick}) = 1\%$$

**BAYES THEOREM
FORMULA**

Bayes Theorem: Formula



A : sick

B : diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick})}{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick}) + P(\text{not sick}) \cdot P(\text{diagnosed sick} | \text{not sick})}$$

$$P(A) = 0.01\%$$

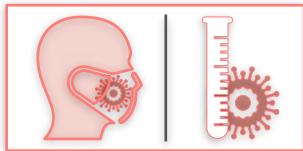
$$P(A') = 99.99\%$$

$$P(B | A) = 99\%$$

$$P(B | A') = 1\%$$

**BAYES THEOREM
FORMULA**

Bayes Theorem: Formula



A : sick

B : diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(A) \cdot P(\text{diagnosed sick} | \text{sick})}{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick}) + P(\text{not sick}) \cdot P(\text{diagnosed sick} | \text{not sick})}$$

$$P(A) = 0.01\%$$

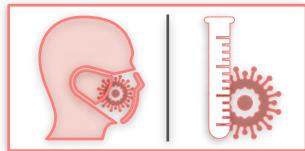
$$P(A') = 99.99\%$$

$$P(B | A) = 99\%$$

$$P(B | A') = 1\%$$

**BAYES THEOREM
FORMULA**

Bayes Theorem: Formula



A : sick

B : diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(A) \cdot P(B | A)}{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick}) + P(\text{not sick}) \cdot P(\text{diagnosed sick} | \text{not sick})}$$

$$P(A) = 0.01\%$$

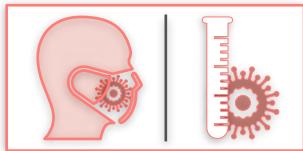
$$P(A') = 99.99\%$$

$$P(B | A) = 99\%$$

$$P(B | A') = 1\%$$

**BAYES THEOREM
FORMULA**

Bayes Theorem: Formula



A : sick

B : diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(A) \cdot P(B | A)}{P(A) \cdot P(\text{diagnosed sick} | \text{sick}) + P(\text{not sick}) \cdot P(\text{diagnosed sick} | \text{not sick})}$$

$$P(A) = 0.01\%$$

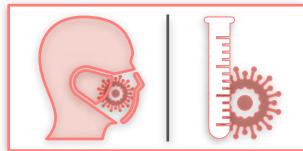
$$P(A') = 99.99\%$$

$$P(B | A) = 99\%$$

$$P(B | A') = 1\%$$

**BAYES THEOREM
FORMULA**

Bayes Theorem: Formula



A : sick

B : diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(A) \cdot P(B | A)}{P(A) \cdot P(B | A) + P(\text{not sick}) \cdot P(\text{diagnosed sick} | \text{not sick})}$$

$$P(A) = 0.01\%$$

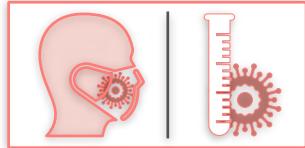
$$P(A') = 99.99\%$$

$$P(B | A) = 99\%$$

$$P(B | A') = 1\%$$

**BAYES THEOREM
FORMULA**

Bayes Theorem: Formula



A: sick

B: diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(A) \cdot P(B | A)}{P(A) \cdot P(B | A) + P(A') \cdot P(\text{diagnosed sick} | \text{not sick})}$$

$$P(A) = 0.01\%$$

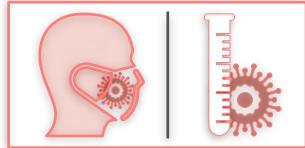
$$P(A') = 99.99\%$$

$$P(B | A) = 99\%$$

$$P(B | A') = 1\%$$

**BAYES THEOREM
FORMULA**

Bayes Theorem: Formula



A : sick

B : diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(A) \cdot P(B | A)}{P(A) \cdot P(B | A) + P(A') \cdot P(B | A')}$$

$$P(A) = 0.01\%$$

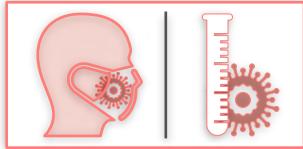
$$P(A') = 99.99\%$$

$$P(B | A) = 99\%$$

$$P(B | A') = 1\%$$

**BAYES THEOREM
FORMULA**

Bayes Theorem: Formula



A : sick

B : diagnosed sick

$$\mathbf{P}(A | B) = \frac{\mathbf{P}(A \cap B)}{\mathbf{P}(B)}$$

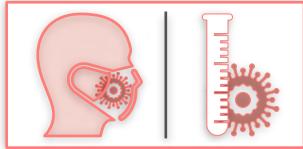
$$\mathbf{P}(A) = 0.01\%$$

$$\mathbf{P}(A') = 99.99\%$$

$$\mathbf{P}(B | A) = 99\%$$

$$\mathbf{P}(B | A') = 1\%$$

Bayes Theorem: Formula



A : sick

B : diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A) = 0.01\%$$

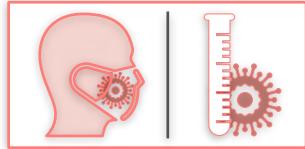
$$P(A') = 99.99\%$$

$$P(B | A) = 99\%$$

$$P(B | A') = 1\%$$

**BAYES THEOREM
FORMULA**

Bayes Theorem: Formula



A : sick

B : diagnosed sick

$$\mathbf{P}(A | B) = \frac{\mathbf{P}(A \cap B)}{\mathbf{P}(B)}$$

$$\mathbf{P}(A | B) = \frac{\mathbf{P}(A) \cdot \mathbf{P}(B | A)}{\mathbf{P}(A) \cdot \mathbf{P}(B | A) + \mathbf{P}(A') \cdot \mathbf{P}(B | A')}$$

$$\mathbf{P}(A) = 0.01\%$$

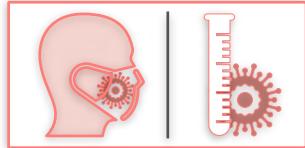
$$\mathbf{P}(A') = 99.99\%$$

$$\mathbf{P}(B | A) = 99\%$$

$$\mathbf{P}(B | A') = 1\%$$

**BAYES THEOREM
FORMULA**

Bayes Theorem: Formula



A : sick

B : diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(A) \cdot P(B | A)}{P(A) \cdot P(B | A) + P(A') \cdot P(B | A')}$$

$$P(A) = 0.01\%$$

$$P(A') = 99.99\%$$

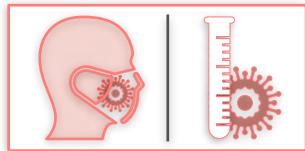
$$P(B | A) = 99\%$$

$$P(B | A') = 1\%$$

$$P(A | B) = \frac{0.0001 \times 0.99}{(0.0001 \times 0.99) + (0.9999 \times 0.01)}$$

**BAYES THEOREM
FORMULA**

Bayes Theorem: Formula



A : sick

B : diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(A) \cdot P(B | A)}{P(A) \cdot P(B | A) + P(A') \cdot P(B | A')}$$

$$P(A) = 0.01\%$$

$$P(A') = 99.99\%$$

$$P(B | A) = 99\%$$

$$P(B | A') = 1\%$$

$$P(A | B) = \frac{0.0001 \times 0.99}{(0.0001 \times 0.99) + (0.9999 \times 0.01)}$$

$$P(A | B) = 0.0098$$

**BAYES THEOREM
FORMULA**

Bayes Theorem: Spam Example

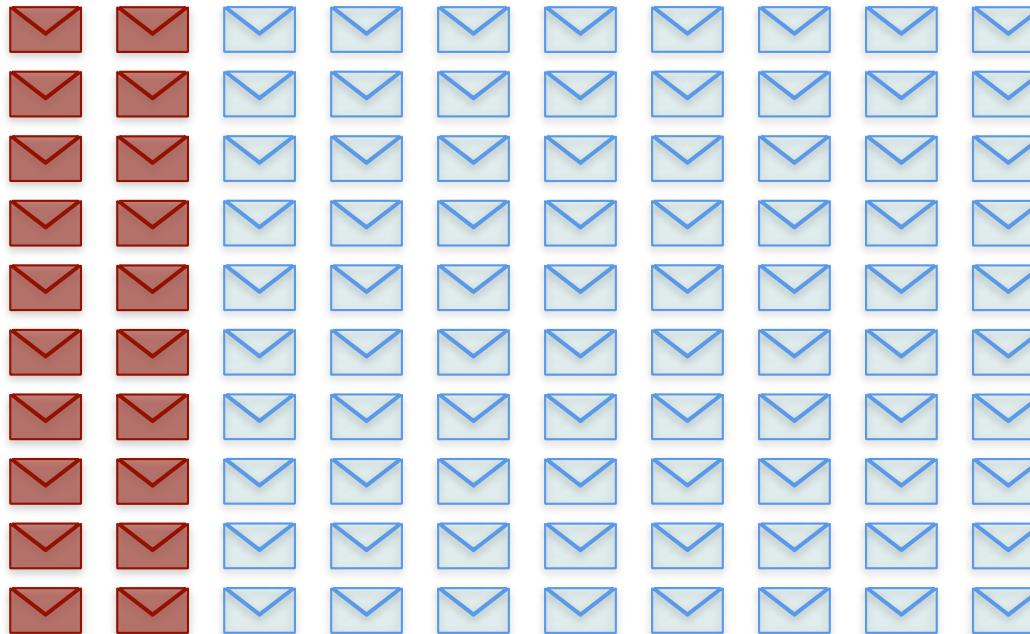
Bayes Theorem: Spam Example



Bayes Theorem: Spam Example



Bayes Theorem: Spam Example



20 spam

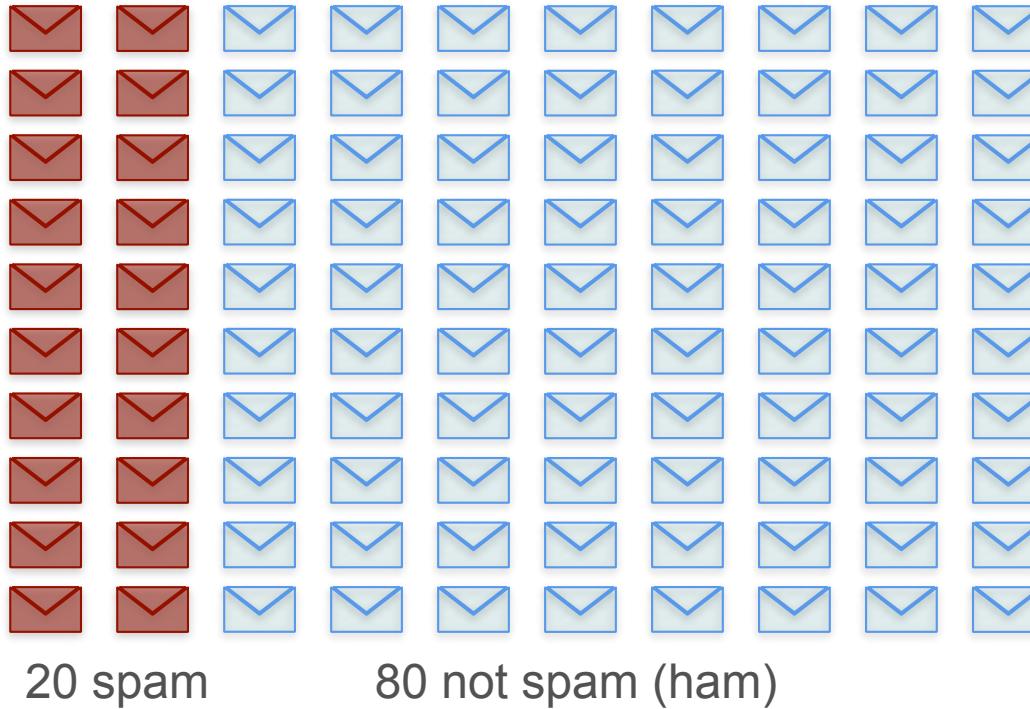
Bayes Theorem: Spam Example



20 spam

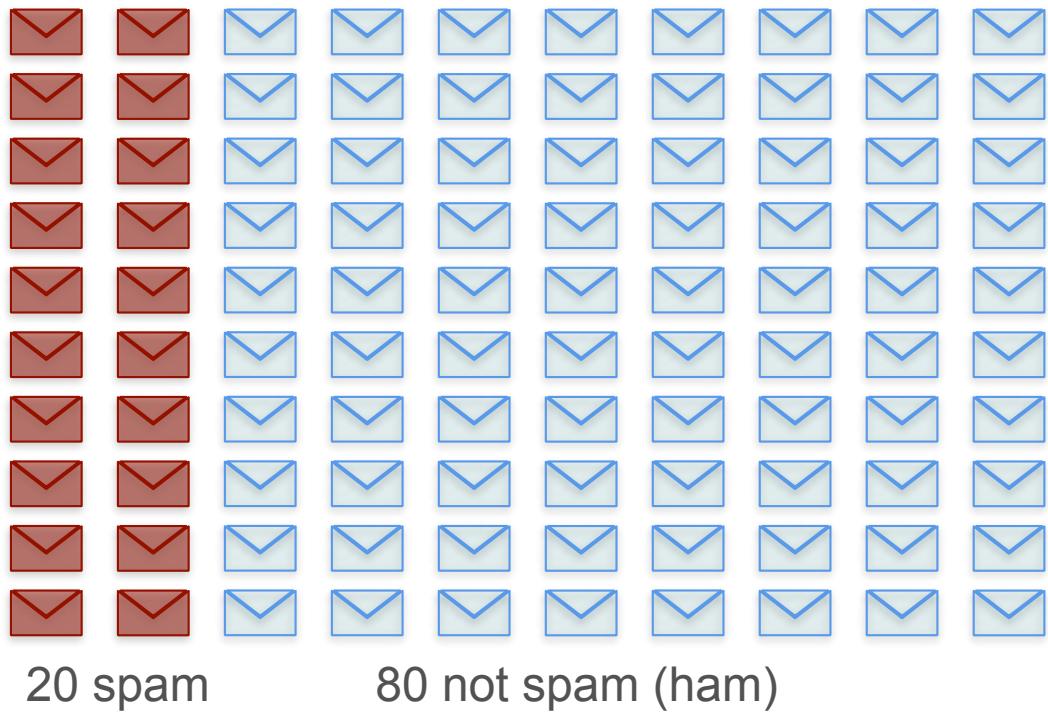
80 not spam (ham)

Bayes Theorem: Spam Example

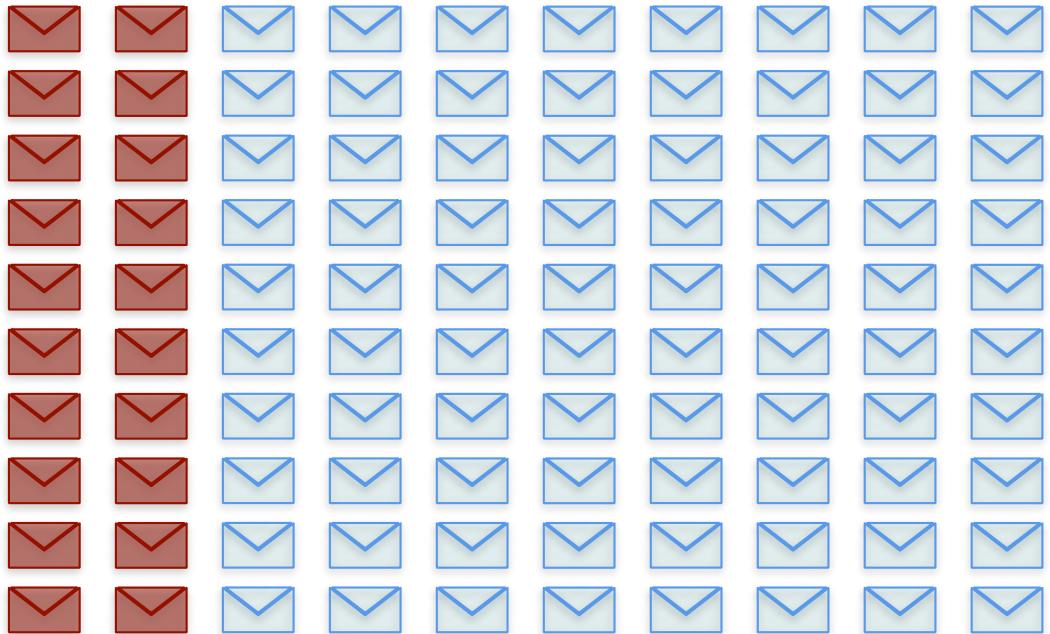


“lottery”

Bayes Theorem: Spam Example



Bayes Theorem: Spam Example



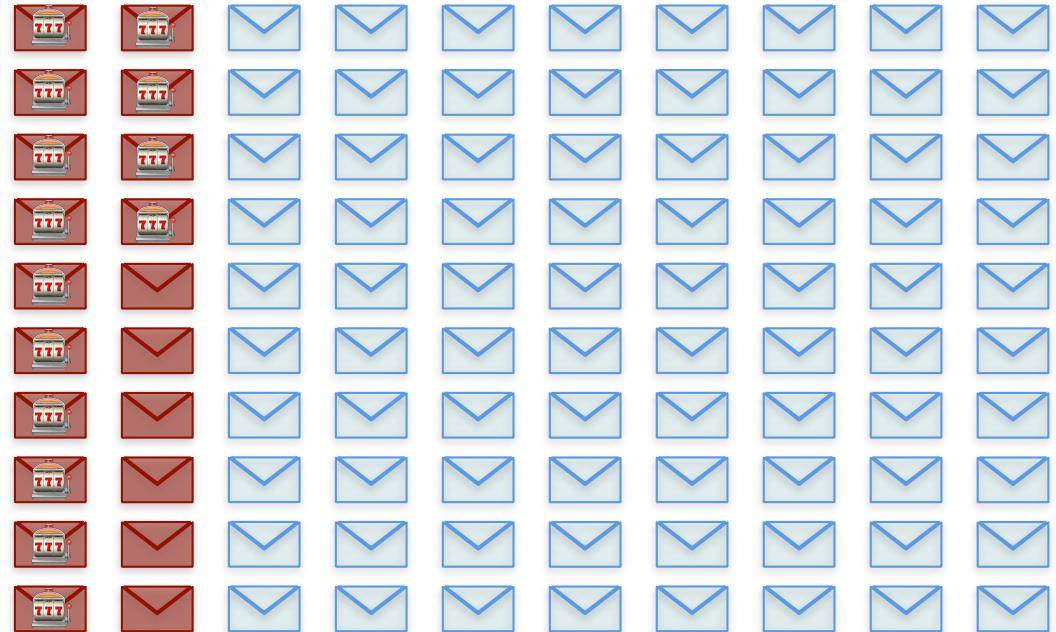
20 spam

80 not spam (ham)

Bayes Theorem: Spam Example



14

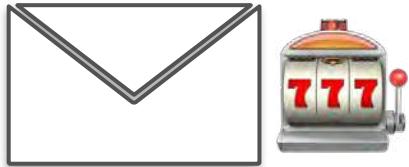


20 spam

80 not spam (ham)

Bayes Theorem: Spam Example

10



14



20 spam

80 not spam (ham)

Bayes Theorem: Spam Example

10



What is the probability that an email containing lottery is a spam?

14

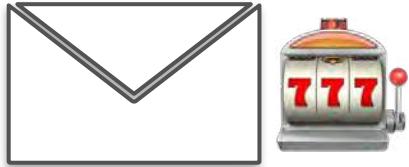


20 spam

80 not spam (ham)

Bayes Theorem: Spam Example

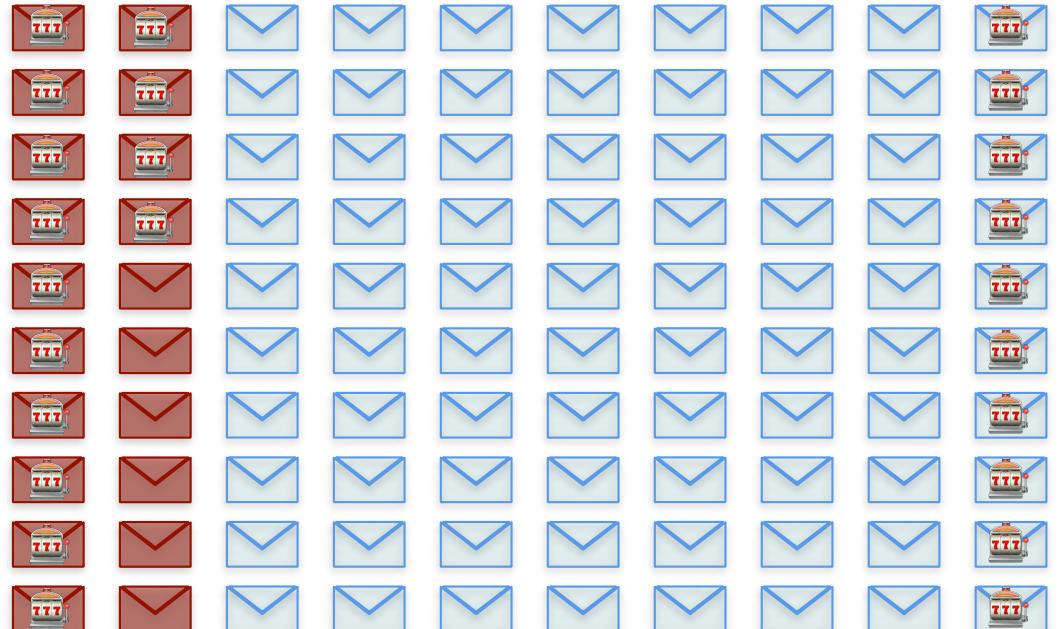
10



What is the probability that an email containing lottery is a spam?

$P(\text{spam} \mid \text{lottery})$

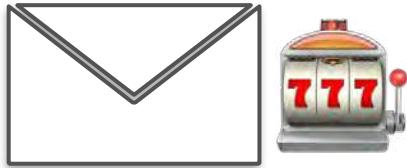
14



20 spam

80 not spam (ham)

Bayes Theorem: Spam Example (Intuition Solution)



$P(\text{spam} \mid \text{lottery})$

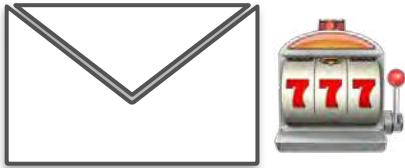


14



10

Bayes Theorem: Spam Example (Intuition Solution)



$P(\text{spam} \mid \text{lottery})$



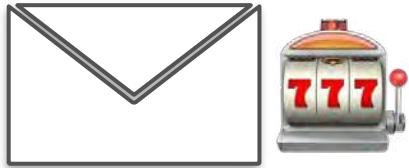
14

24 emails
containing lottery



10

Bayes Theorem: Spam Example (Intuition Solution)



$P(\text{spam} \mid \text{lottery})$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$



14

24 emails
containing lottery



10

Bayes Theorem: Spam Example (Intuition Solution)



$P(\text{spam} \mid \text{lottery})$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$



24 emails
containing lottery

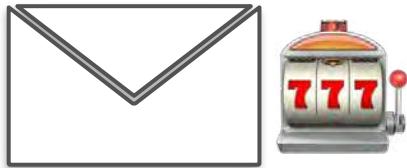
$$P(\text{spam} \mid \text{lottery}) = \frac{\text{spam and lottery}}{\text{all lottery}}$$

14

10

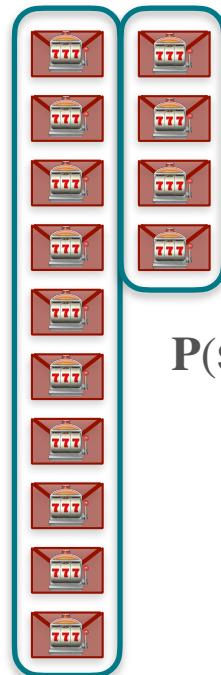


Bayes Theorem: Spam Example (Intuition Solution)



$P(\text{spam} \mid \text{lottery})$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$



14

24 emails
containing lottery

$$P(\text{spam} \mid \text{lottery}) = \frac{\text{spam and lottery}}{\text{all lottery}}$$

10



Bayes Theorem: Spam Example (Intuition Solution)



$P(\text{spam} \mid \text{lottery})$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$



14

24 emails
containing lottery

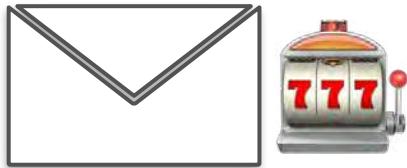
$$P(\text{spam} \mid \text{lottery}) = \frac{\text{spam and lottery}}{\text{all lottery}}$$

$$= \frac{14}{24}$$

10



Bayes Theorem: Spam Example (Intuition Solution)



$P(\text{spam} \mid \text{lottery})$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$



14

$$P(\text{spam} \mid \text{lottery}) = \frac{\text{spam and lottery}}{\text{all lottery}}$$

$$= \frac{14}{24}$$

$$= \frac{7}{12} = 0.583$$

10



Bayes Theorem: Spam Example (Formula Solution)

Bayes Theorem: Spam Example (Formula Solution)

$$P(\text{spam} \mid \text{lottery})$$

Bayes Theorem: Spam Example (Formula Solution)

$P(\text{spam} \mid \text{lottery})$

$$P(A \mid B) = \frac{P(A) \cdot P(B \mid A)}{P(A) \cdot P(B \mid A) + P(A') \cdot P(B \mid A')}$$

Bayes Theorem: Spam Example (Formula Solution)

$P(\text{spam} \mid \text{lottery})$

$$P(A \mid B) = \frac{P(A) \cdot P(B \mid A)}{P(A) \cdot P(B \mid A) + P(A') \cdot P(B \mid A')}$$

A: Email is spam

Bayes Theorem: Spam Example (Formula Solution)

$P(\text{spam} \mid \text{lottery})$

$$P(A \mid B) = \frac{P(A) \cdot P(B \mid A)}{P(A) \cdot P(B \mid A) + P(A') \cdot P(B \mid A')}$$

A : Email is spam B : Email contains lottery

Bayes Theorem: Spam Example (Formula Solution)

$P(\text{spam} \mid \text{lottery})$

$$P(A \mid B) = \frac{P(A) \cdot P(B \mid A)}{P(A) \cdot P(B \mid A) + P(A') \cdot P(B \mid A')}$$

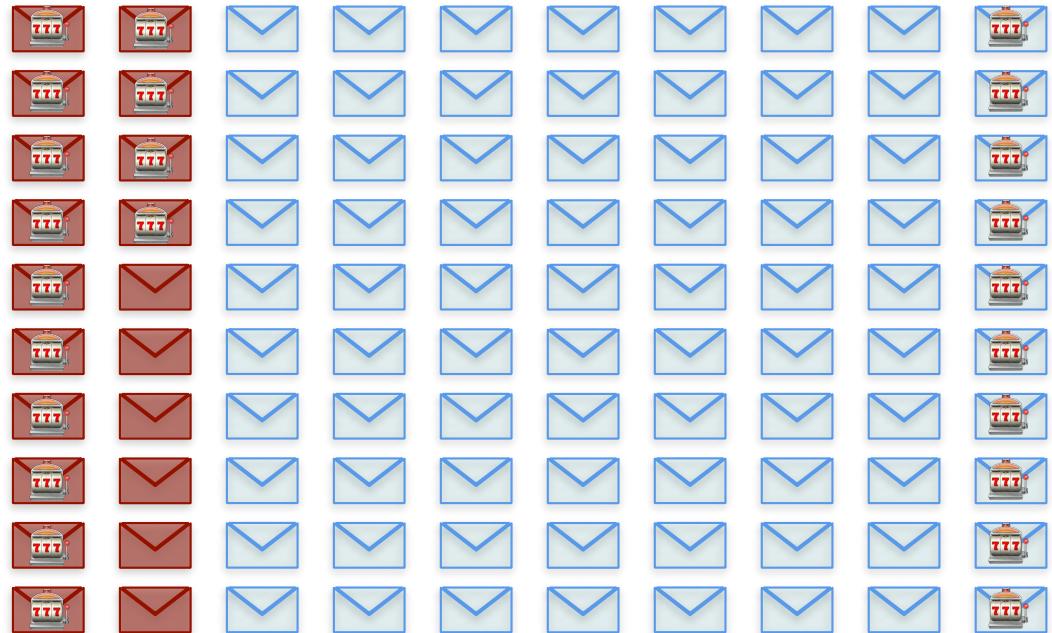
A : Email is spam B : Email contains lottery

$$P(\text{spam} \mid \text{lottery}) = \frac{P(\text{spam}) \cdot P(\text{lottery} \mid \text{spam})}{P(\text{spam}) \cdot P(\text{lottery} \mid \text{spam}) + P(\text{not spam}) \cdot P(\text{lottery} \mid \text{not spam})}$$

Bayes Theorem: Spam Example (Formula Solution)

10

14



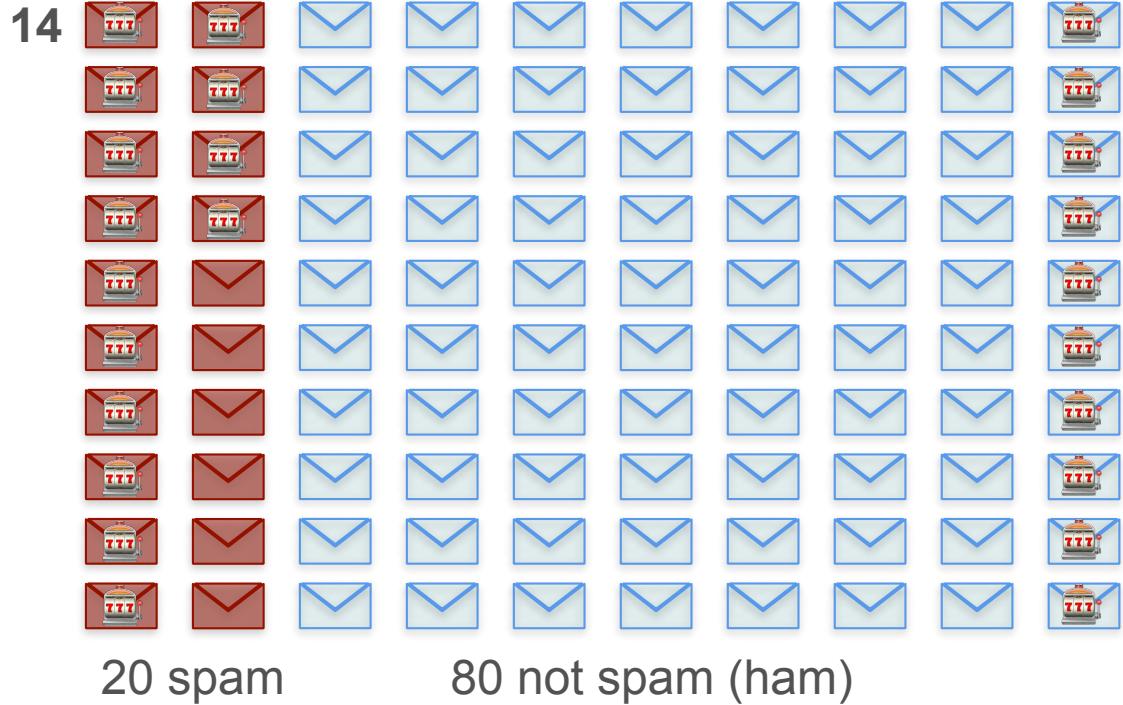
20 spam

80 not spam (ham)

Bayes Theorem: Spam Example (Formula Solution)

10

$$P(\text{spam}) = \frac{20}{100} = 0.2$$

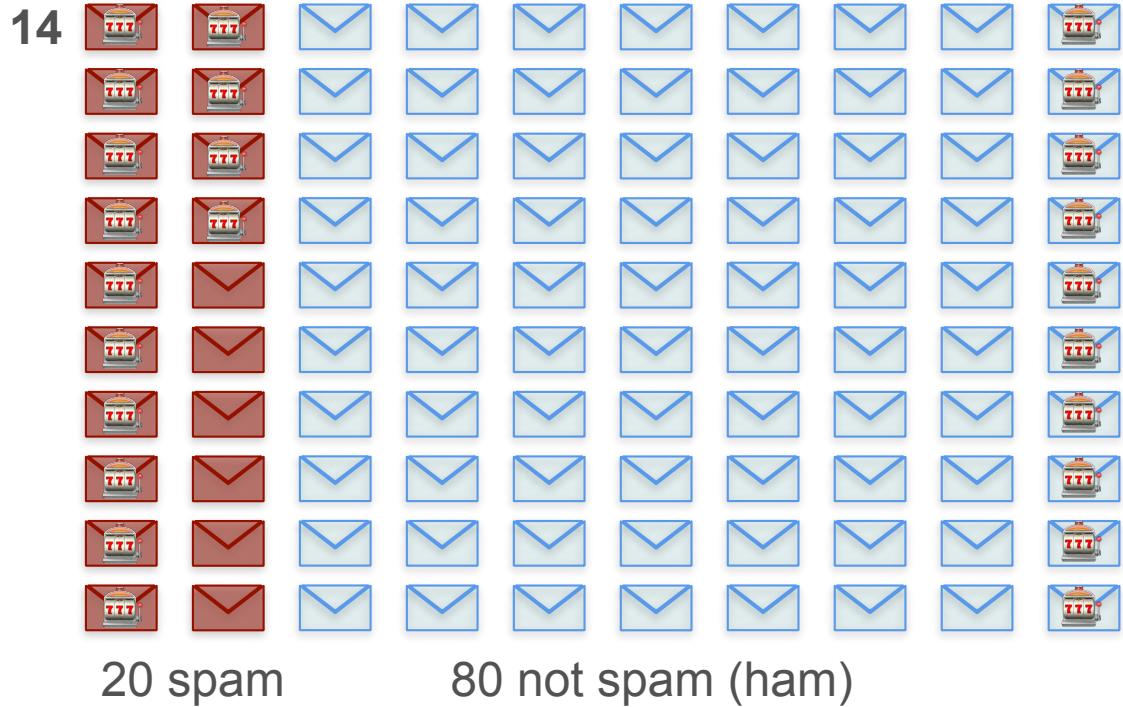


Bayes Theorem: Spam Example (Formula Solution)

10

$$P(\text{spam}) = \frac{20}{100} = 0.2$$

$$P(\text{not spam}) = \frac{80}{100} = 0.8$$



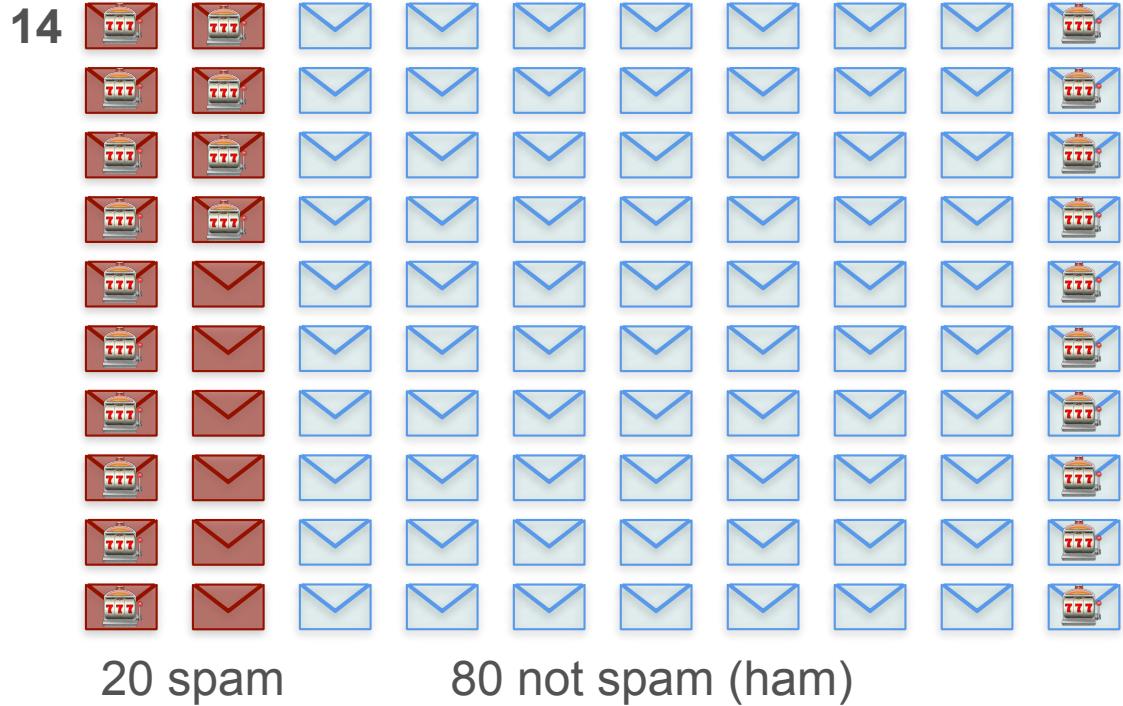
Bayes Theorem: Spam Example (Formula Solution)

10

$$P(\text{spam}) = \frac{20}{100} = 0.2$$

$$P(\text{not spam}) = \frac{80}{100} = 0.8$$

$$P(\text{lottery} \mid \text{spam}) = \frac{14}{20} = 0.7$$



Bayes Theorem: Spam Example (Formula Solution)

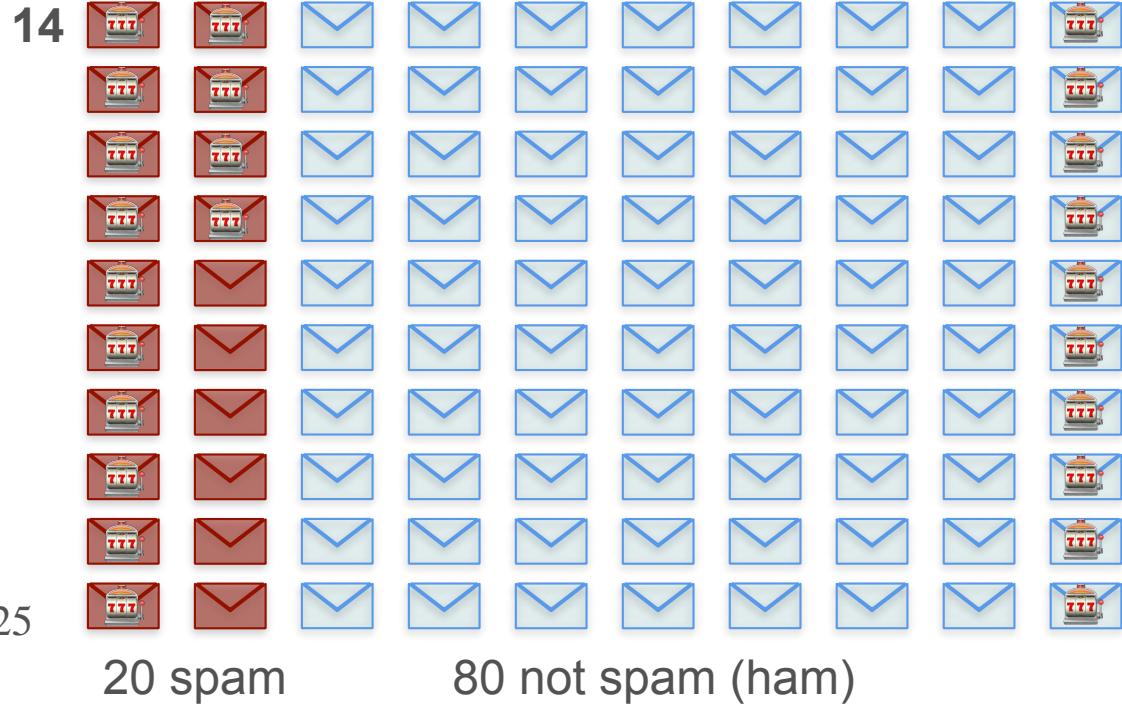
10

$$P(\text{spam}) = \frac{20}{100} = 0.2$$

$$P(\text{not spam}) = \frac{80}{100} = 0.8$$

$$P(\text{lottery} | \text{spam}) = \frac{14}{20} = 0.7$$

$$P(\text{lottery} | \text{not spam}) = \frac{10}{80} = 0.125$$



Bayes Theorem: Spam Example (Formula Solution)

Bayes Theorem: Spam Example (Formula Solution)

$$P(\text{spam}) = 0.2$$

Bayes Theorem: Spam Example (Formula Solution)

$$P(\text{spam}) = 0.2$$

$$P(\text{lottery} \mid \text{spam}) = 0.7$$

Bayes Theorem: Spam Example (Formula Solution)

$$P(\text{spam}) = 0.2$$

$$P(\text{lottery} \mid \text{spam}) = 0.7$$

$$P(\text{not spam}) = 0.8$$

Bayes Theorem: Spam Example (Formula Solution)

$$P(\text{spam}) = 0.2$$

$$P(\text{lottery} \mid \text{spam}) = 0.7$$

$$P(\text{not spam}) = 0.8$$

$$P(\text{lottery} \mid \text{not spam}) = 0.125$$

Bayes Theorem: Spam Example (Formula Solution)

$$P(\text{spam}) = 0.2$$

$$P(\text{lottery} \mid \text{spam}) = 0.7$$

$$P(\text{not spam}) = 0.8$$

$$P(\text{lottery} \mid \text{not spam}) = 0.125$$

$$P(\text{spam} \mid \text{lottery}) = \frac{P(\text{spam}) \cdot P(\text{lottery} \mid \text{spam})}{P(\text{spam}) \cdot P(\text{lottery} \mid \text{spam}) + P(\text{not spam}) \cdot P(\text{lottery} \mid \text{not spam})}$$

Bayes Theorem: Spam Example (Formula Solution)

$$P(\text{spam}) = 0.2$$

$$P(\text{lottery} \mid \text{spam}) = 0.7$$

$$P(\text{not spam}) = 0.8$$

$$P(\text{lottery} \mid \text{not spam}) = 0.125$$

$$P(\text{spam} \mid \text{lottery}) = \frac{P(\text{spam}) \cdot P(\text{lottery} \mid \text{spam})}{P(\text{spam}) \cdot P(\text{lottery} \mid \text{spam}) + P(\text{not spam}) \cdot P(\text{lottery} \mid \text{not spam})}$$

$$P(\text{spam} \mid \text{lottery}) = \frac{0.2 \times 0.7}{(0.2 \times 0.7) + (0.8 \times 0.125)}$$

Bayes Theorem: Spam Example (Formula Solution)

$$P(\text{spam}) = 0.2$$

$$P(\text{lottery} \mid \text{spam}) = 0.7$$

$$P(\text{not spam}) = 0.8$$

$$P(\text{lottery} \mid \text{not spam}) = 0.125$$

$$P(\text{spam} \mid \text{lottery}) = \frac{P(\text{spam}) \cdot P(\text{lottery} \mid \text{spam})}{P(\text{spam}) \cdot P(\text{lottery} \mid \text{spam}) + P(\text{not spam}) \cdot P(\text{lottery} \mid \text{not spam})}$$

$$P(\text{spam} \mid \text{lottery}) = \frac{0.2 \times 0.7}{(0.2 \times 0.7) + (0.8 \times 0.125)} = 0.583$$

Bayes Theorem

Bayes Theorem

PRIOR

Bayes Theorem

PRIOR

EVENT

Bayes Theorem

PRIOR

EVENT

POSTERIOR

Bayes Theorem

PRIOR

EVENT

POSTERIOR

$$\mathbf{P}(A)$$

Bayes Theorem

PRIOR

$\mathbf{P}(A)$

EVENT

E

POSTERIOR

Bayes Theorem

PRIOR

$$\mathbf{P}(A)$$

EVENT

$$E$$

POSTERIOR

$$\mathbf{P}(A | E)$$

Prior and Posterior

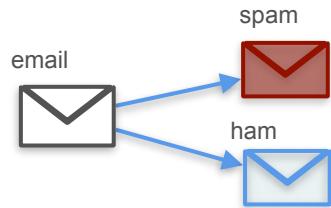
PRIOR

EVENT

POSTERIOR

Prior and Posterior

PRIOR

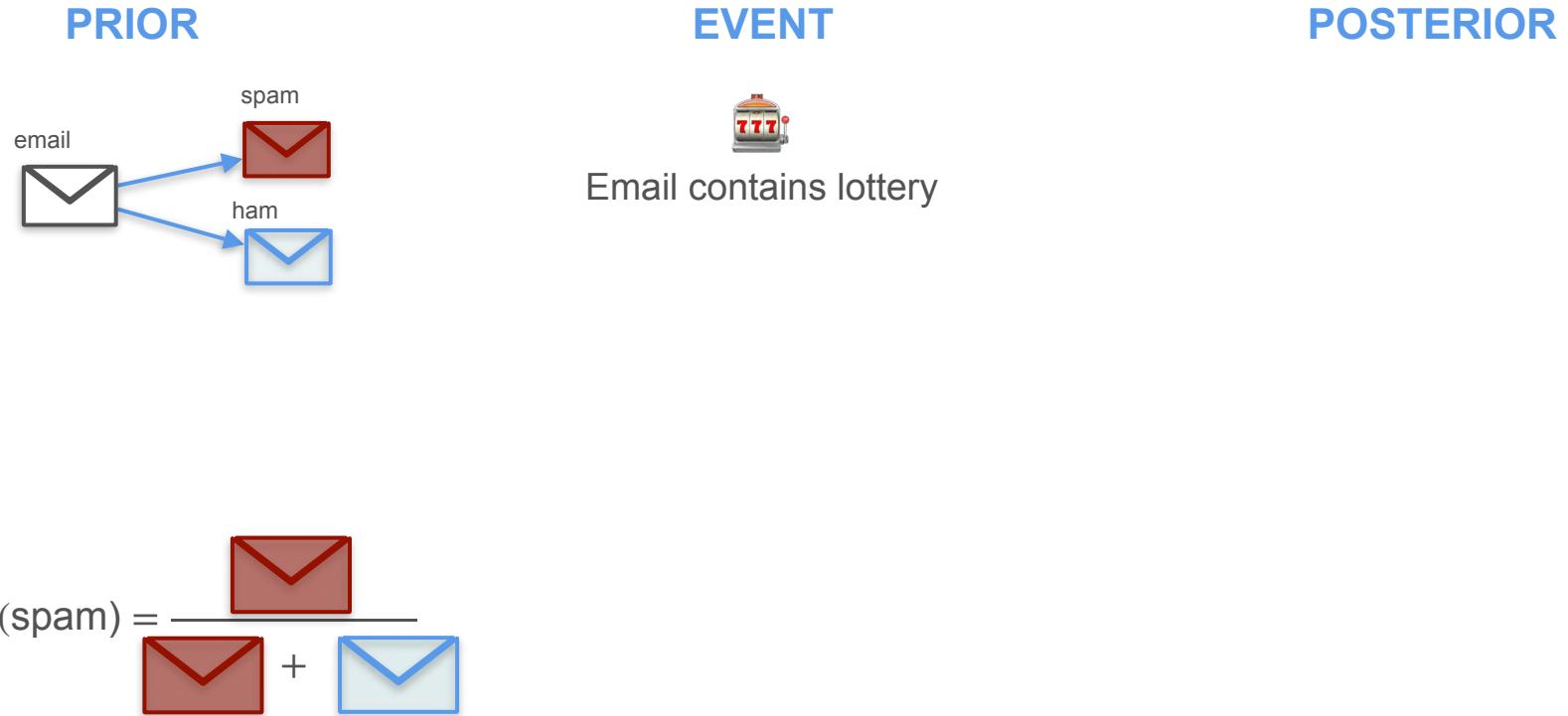


EVENT

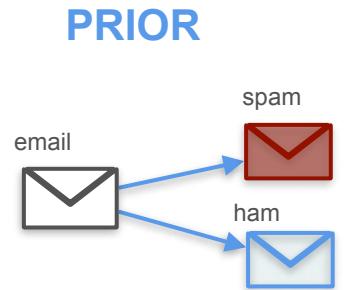
POSTERIOR

$$P(\text{spam}) = \frac{\text{red envelope}}{\text{red envelope} + \text{blue envelope}}$$

Prior and Posterior



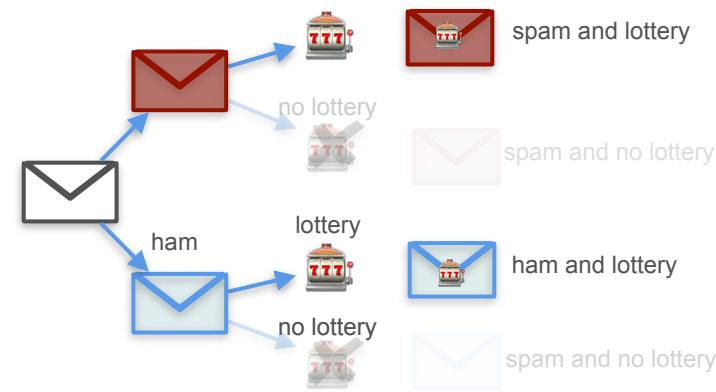
Prior and Posterior



EVENT



Email contains lottery



$$P(\text{spam}) = \frac{\text{spam}}{\text{spam} + \text{ham}}$$

$$P(\text{spam} | \text{lottery}) = \frac{\text{spam and lottery}}{\text{spam and lottery} + \text{ham and lottery}}$$

Prior and Posterior

PRIOR

EVENT

POSTERIOR

Prior and Posterior

PRIOR

	1,1	1,2	1,3	1,4	1,5	1,6
	2,1	2,2	2,3	2,4	2,5	2,6
	3,1	3,2	3,3	3,4	3,5	3,6
	4,1	4,2	4,3	4,4	4,5	4,6
	5,1	5,2	5,3	5,4	5,5	5,6
	6,1	6,2	6,3	6,4	6,5	6,6

EVENT

POSTERIOR

Prior and Posterior

PRIOR

	1,1	1,2	1,3	1,4	1,5	1,6
	2,1	2,2	2,3	2,4	2,5	2,6
	3,1	3,2	3,3	3,4	3,5	3,6
	4,1	4,2	4,3	4,4	4,5	4,6
	5,1	5,2	5,3	5,4	5,5	5,6
	6,1	6,2	6,3	6,4	6,5	6,6

EVENT

POSTERIOR

Prior and Posterior

PRIOR

	1,1	1,2	1,3	1,4	1,5	1,6
	2,1	2,2	2,3	2,4	2,5	2,6
	3,1	3,2	3,3	3,4	3,5	3,6
	4,1	4,2	4,3	4,4	4,5	4,6
	5,1	5,2	5,3	5,4	5,5	5,6
	6,1	6,2	6,3	6,4	6,5	6,6

EVENT

POSTERIOR

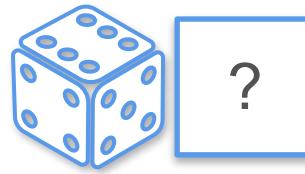
$$P(\text{sum} = 10) = \frac{3}{36}$$

Prior and Posterior

PRIOR

	1,1	1,2	1,3	1,4	1,5	1,6
	2,1	2,2	2,3	2,4	2,5	2,6
	3,1	3,2	3,3	3,4	3,5	3,6
	4,1	4,2	4,3	4,4	4,5	4,6
	5,1	5,2	5,3	5,4	5,5	5,6
	6,1	6,2	6,3	6,4	6,5	6,6

EVENT



POSTERIOR

1st dice is 6

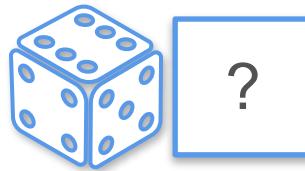
$$P(\text{sum} = 10) = \frac{3}{36}$$

Prior and Posterior

PRIOR

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

EVENT



1st dice is 6

POSTERIOR

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

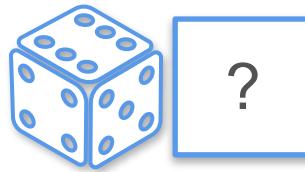
$$P(\text{sum} = 10) = \frac{3}{36}$$

Prior and Posterior

PRIOR

	1,1	1,2	1,3	1,4	1,5	1,6
	2,1	2,2	2,3	2,4	2,5	2,6
	3,1	3,2	3,3	3,4	3,5	3,6
	4,1	4,2	4,3	4,4	4,5	4,6
	5,1	5,2	5,3	5,4	5,5	5,6
	6,1	6,2	6,3	6,4	6,5	6,6

EVENT



1st dice is 6

POSTERIOR

	1,1	1,2	1,3	1,4	1,5	1,6
	2,1	2,2	2,3	2,4	2,5	2,6
	3,1	3,2	3,3	3,4	3,5	3,6
	4,1	4,2	4,3	4,4	4,5	4,6
	5,1	5,2	5,3	5,4	5,5	5,6
	6,1	6,2	6,3	6,4	6,5	6,6

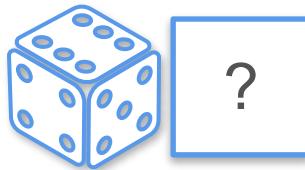
$$P(\text{sum} = 10) = \frac{3}{36}$$

Prior and Posterior

PRIOR

	1,1	1,2	1,3	1,4	1,5	1,6
	2,1	2,2	2,3	2,4	2,5	2,6
	3,1	3,2	3,3	3,4	3,5	3,6
	4,1	4,2	4,3	4,4	4,5	4,6
	5,1	5,2	5,3	5,4	5,5	5,6
	6,1	6,2	6,3	6,4	6,5	6,6

EVENT



1st dice is 6

POSTERIOR

	1,1	1,2	1,3	1,4	1,5	1,6
	2,1	2,2	2,3	2,4	2,5	2,6
	3,1	3,2	3,3	3,4	3,5	3,6
	4,1	4,2	4,3	4,4	4,5	4,6
	5,1	5,2	5,3	5,4	5,5	5,6
	6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{sum} = 10) = \frac{3}{36}$$

$$P(\text{sum} = 10 | \text{1st is } 6) = \frac{1}{6}$$

Prior and Posterior

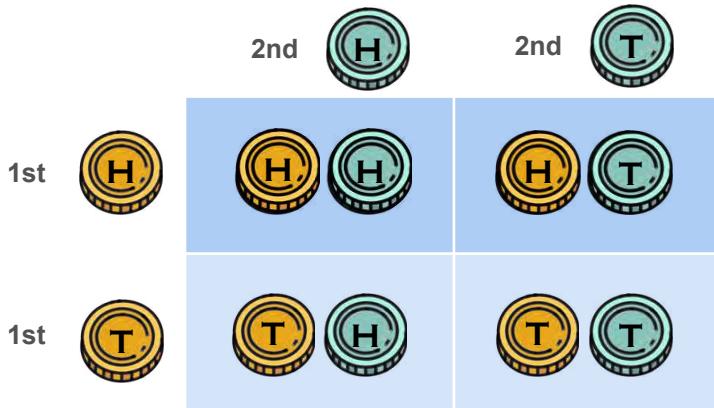
PRIOR

EVENT

POSTERIOR

Prior and Posterior

PRIOR

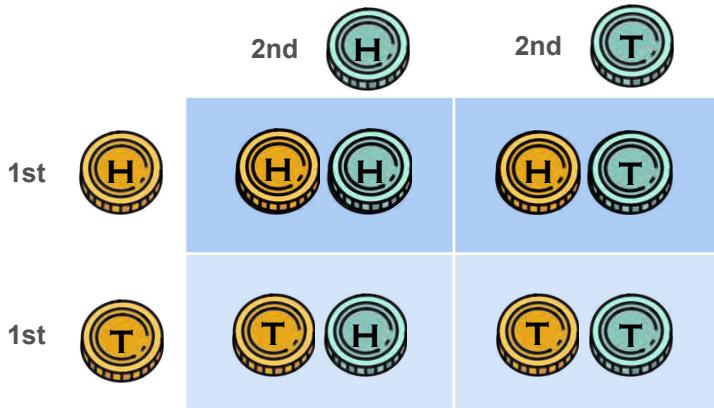


EVENT

POSTERIOR

Prior and Posterior

PRIOR



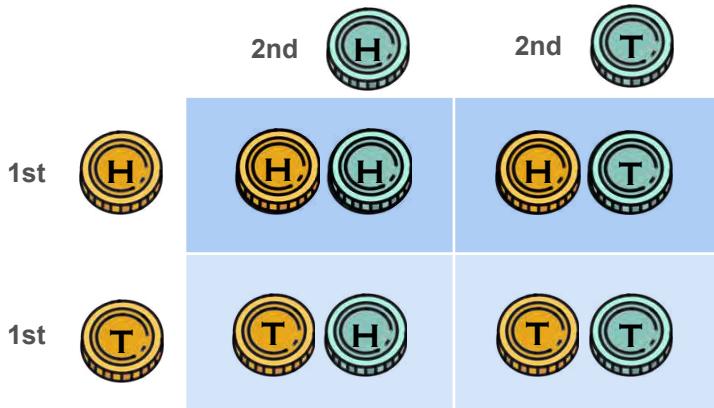
EVENT

POSTERIOR

$$P(HH) = \frac{1}{4}$$

Prior and Posterior

PRIOR



EVENT



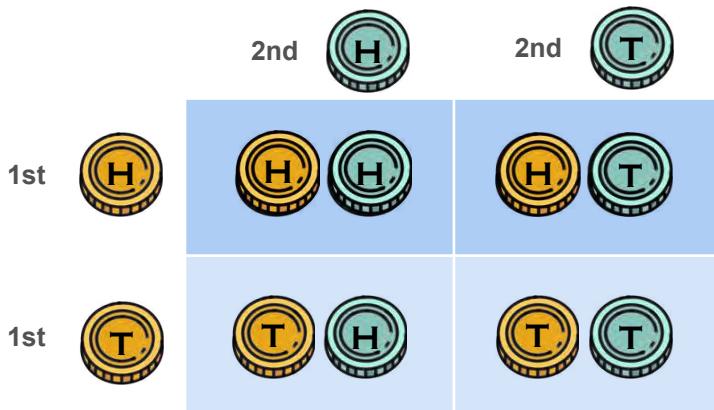
1st coin is H

POSTERIOR

$$P(HH) = \frac{1}{4}$$

Prior and Posterior

PRIOR



EVENT



1st coin is H

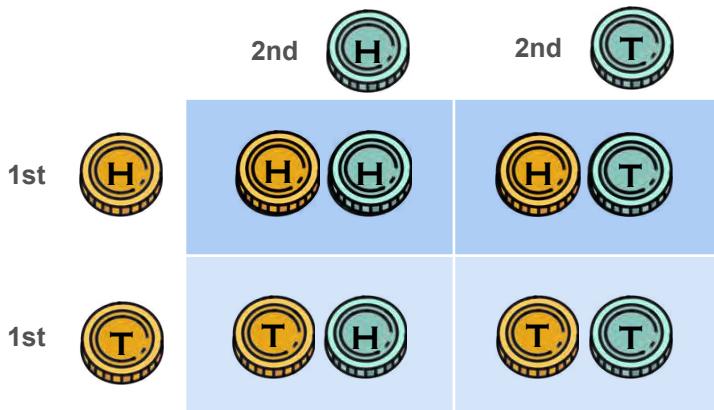
POSTERIOR



$$P(HH) = \frac{1}{4}$$

Prior and Posterior

PRIOR



$$P(HH) = \frac{1}{4}$$

EVENT



POSTERIOR



$$P(HH | \text{1st is } H) = \frac{1}{2}$$

Video 8e: the Naive Bayes Model

What About 2 Events?

PRIOR

EVENT

POSTERIOR

What About 2 Events?

PRIOR

EVENT

POSTERIOR

$$P(\text{spam}) = \frac{\text{Red Envelope}}{\text{Red Envelope} + \text{Blue Envelope}}$$

What About 2 Events?

PRIOR

$$P(\text{spam}) = \frac{\text{Red Envelope}}{\text{Red Envelope} + \text{Blue Envelope}}$$

EVENT



Email contains 'lottery'

POSTERIOR

What About 2 Events?

PRIOR

$$P(\text{spam}) = \frac{\text{Red Envelope}}{\text{Red Envelope} + \text{Blue Envelope}}$$

EVENT



Email contains 'lottery'

POSTERIOR

$$P(\text{spam} | \text{lottery}) = \frac{\text{Red Envelope with lottery icon}}{\text{Red Envelope with lottery icon} + \text{Blue Envelope with lottery icon}}$$

What About 2 Events?

PRIOR

$$P(\text{spam}) = \frac{\text{Red Envelope}}{\text{Red Envelope} + \text{Blue Envelope}}$$

EVENT



Email contains 'lottery'



Email contains 'winning'

POSTERIOR

$$P(\text{spam} | \text{lottery}) = \frac{\text{Red Envelope with lottery icon}}{\text{Red Envelope with lottery icon} + \text{Blue Envelope with lottery icon}}$$

What About 2 Events?

PRIOR

$$P(\text{spam}) = \frac{\text{Red Envelope}}{\text{Red Envelope} + \text{Blue Envelope}}$$

EVENT



Email contains 'lottery'



Email contains 'winning'

POSTERIOR

$$P(\text{spam} | \text{lottery}) = \frac{\text{Red Envelope with lottery icon}}{\text{Red Envelope with lottery icon} + \text{Blue Envelope with lottery icon}}$$

$$P(\text{spam} | \text{winning}) = \frac{\text{Red Envelope with winning icon}}{\text{Red Envelope with winning icon} + \text{Blue Envelope with winning icon}}$$

What About 2 Events?

PRIOR

$$P(\text{spam}) = \frac{\text{Red Envelope}}{\text{Red Envelope} + \text{Blue Envelope}}$$

EVENT



Email contains 'lottery'



Email contains 'winning'

POSTERIOR

$$P(\text{spam} | \text{lottery}) = \frac{\text{Red Envelope with lottery icon}}{\text{Red Envelope with lottery icon} + \text{Blue Envelope with lottery icon}}$$

$$P(\text{spam} | \text{winning}) = \frac{\text{Red Envelope with winning icon}}{\text{Red Envelope with winning icon} + \text{Blue Envelope with winning icon}}$$



Email contains 'lottery' and 'winning'

What About 2 Events?

PRIOR

$$P(\text{spam}) = \frac{\text{Red Envelope}}{\text{Red Envelope} + \text{Blue Envelope}}$$

EVENT



Email contains 'lottery'



Email contains 'winning'



Email contains 'lottery' and 'winning'

POSTERIOR

$$P(\text{spam} | \text{lottery}) = \frac{\text{Red Envelope with lottery icon}}{\text{Red Envelope with lottery icon} + \text{Blue Envelope with lottery icon}}$$

$$P(\text{spam} | \text{winning}) = \frac{\text{Red Envelope with winning icon}}{\text{Red Envelope with winning icon} + \text{Blue Envelope with winning icon}}$$

?

What About 2 Events?

EVENT



POSTERIOR

Email contains 'lottery' and 'winning'

What About 2 Events?

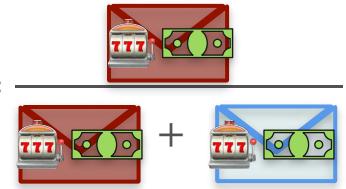
EVENT



Email contains 'lottery' and 'winning'

POSTERIOR

$$P(\text{spam} \mid \text{lottery} \& \text{winning}) = \frac{\dots}{\dots + \dots}$$



What About 2 Events?

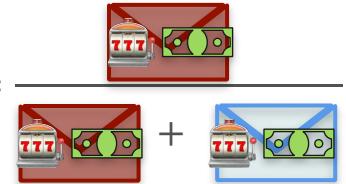
EVENT



Email contains 'lottery' and 'winning'

POSTERIOR

$$P(\text{spam} | \text{lottery} \& \text{winning}) = \frac{\text{Red Envelope}}{\text{Red Envelope} + \text{Blue Envelope}}$$



$$P(\text{spam} | \text{lottery} \& \text{winning}) = \frac{P(\text{spam}) \cdot P(\text{lottery} \& \text{winning} | \text{spam})}{P(\text{spam}) \cdot P(\text{lottery} \& \text{winning} | \text{spam}) + P(\text{ham}) \cdot P(\text{lottery} \& \text{winning} | \text{ham})}$$

What About 2 Events?

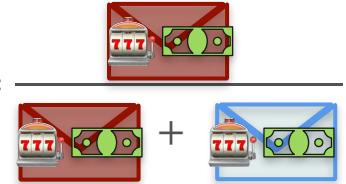
EVENT



Email contains 'lottery' and 'winning'

POSTERIOR

$$P(\text{spam} | \text{lottery} \& \text{winning}) = \frac{\text{?}}{\text{?}}$$



$$P(\text{spam} | \text{lottery} \& \text{winning}) = \frac{P(\text{spam}) \cdot P(\text{lottery} \& \text{winning} | \text{spam})}{P(\text{spam}) \cdot P(\text{lottery} \& \text{winning} | \text{spam}) + P(\text{ham}) \cdot P(\text{lottery} \& \text{winning} | \text{ham})}$$

What About 2 Events?

EVENT



Email contains 'lottery' and 'winning'

POSTERIOR



$$P(\text{spam} | \text{lottery} \& \text{winning}) = \frac{\text{# Spam emails with 'lottery' and 'winning'}}{\text{# Emails with 'lottery' and 'winning'}}$$

$$P(\text{spam} | \text{lottery} \& \text{winning}) = \frac{P(\text{spam}) \cdot P(\text{lottery} \& \text{winning} | \text{spam})}{P(\text{spam}) \cdot P(\text{lottery} \& \text{winning} | \text{spam}) + P(\text{ham}) \cdot P(\text{lottery} \& \text{winning} | \text{ham})}$$

What About More Than 2 Events?

EVENT

POSTERIOR

What About More Than 2 Events?

EVENT

POSTERIOR

Email contains w_1, w_2, \dots, w_{100}

What About More Than 2 Events?

EVENT

POSTERIOR

Email contains w_1, w_2, \dots, w_{100}

$$P(\text{spam} | w_1, \dots, w_{100}) = \frac{P(\text{spam}) \cdot P(w_1, \dots, w_{100} | \text{spam})}{P(\text{spam}) \cdot P(w_1, \dots, w_{100} | \text{spam}) + P(\text{ham}) \cdot P(w_1, \dots, w_{100} | \text{ham})}$$

What About More Than 2 Events?

EVENT

POSTERIOR

Email contains w_1, w_2, \dots, w_{100}

$$P(\text{spam} | w_1, \dots, w_{100}) = \frac{P(\text{spam}) P(w_1, \dots, w_{100} | \text{spam})}{P(\text{spam}) \cdot P(w_1, \dots, w_{100} | \text{spam}) + P(\text{ham}) \cdot P(w_1, \dots, w_{100} | \text{ham})}$$

?

What About More Than 2 Events?

EVENT

POSTERIOR

Email contains w_1, w_2, \dots, w_{100}

$$P(\text{spam} | w_1, \dots, w_{100}) = \frac{\frac{\# \text{ Spam emails with } w_1, \dots, w_{100}}{\# \text{ Emails with } w_1, \dots, w_{100}}}{P(\text{spam}) \cdot P(w_1, \dots, w_{100} | \text{spam}) + P(\text{ham}) \cdot P(w_1, \dots, w_{100} | \text{ham})}$$

A red arrow points from the fraction above to the term $P(w_1, \dots, w_{100} | \text{spam})$, which is highlighted with a red oval and followed by a red question mark.

What About More Than 2 Events?

EVENT

POSTERIOR

Email contains w_1, w_2, \dots, w_{100}

$$P(\text{spam} | w_1, \dots, w_{100}) = \frac{\frac{\# \text{ Spam emails with } w_1, \dots, w_{100}}{\# \text{ Emails with } w_1, \dots, w_{100}}}{P(\text{spam}) \cdot P(w_1, \dots, w_{100} | \text{spam}) + P(\text{ham}) \cdot P(w_1, \dots, w_{100} | \text{ham})}$$

The fraction $\frac{\# \text{ Spam emails with } w_1, \dots, w_{100}}{\# \text{ Emails with } w_1, \dots, w_{100}}$ is highlighted with a red underline and a red arrow points from it to a question mark $?$. The term $P(w_1, \dots, w_{100} | \text{spam})$ is also highlighted with a red circle.

Is There a Quicker Way To Estimate the Probability?

$$P(\text{spam} \mid \text{lottery \& winning}) = \frac{P(\text{spam}) \cdot P(\text{lottery \& winning} \mid \text{spam})}{P(\text{spam}) \cdot P(\text{lottery \& winning} \mid \text{spam}) + P(\text{ham}) \cdot P(\text{lottery \& winning} \mid \text{ham})}$$

Is There a Quicker Way To Estimate the Probability?

$$P(\text{spam} \mid \text{lottery \& winning}) = \frac{P(\text{spam}) \cdot P(\text{lottery \& winning} \mid \text{spam})}{P(\text{spam}) \cdot P(\text{lottery \& winning} \mid \text{spam}) + P(\text{ham}) \cdot P(\text{lottery \& winning} \mid \text{ham})}$$

Is There a Quicker Way To Estimate the Probability?

Naive assumption



The appearances of 'lottery' and 'winning' are independent

$$P(\text{spam} \mid \text{lottery \& winning}) = \frac{P(\text{spam}) \cdot P(\text{lottery \& winning} \mid \text{spam})}{P(\text{spam}) \cdot P(\text{lottery \& winning} \mid \text{spam}) + P(\text{ham}) \cdot P(\text{lottery \& winning} \mid \text{ham})}$$

Is There a Quicker Way To Estimate the Probability?

Naive assumption



The appearances of 'lottery' and 'winning' are independent

$$P(\text{spam} \mid \text{lottery \& winning}) = \frac{P(\text{spam}) \cdot P(\text{lottery \& winning} \mid \text{spam})}{P(\text{spam}) \cdot P(\text{lottery \& winning} \mid \text{spam}) + P(\text{ham}) \cdot P(\text{lottery \& winning} \mid \text{ham})}$$

$P(A \cap B)$

↓

Is There a Quicker Way To Estimate the Probability?

Naive assumption



The appearances of 'lottery' and 'winning' are independent

$$P(\text{spam} \mid \text{lottery \& winning}) = \frac{P(\text{spam}) \cdot P(\text{lottery \& winning} \mid \text{spam})}{P(\text{spam}) \cdot P(\text{lottery \& winning} \mid \text{spam}) + P(\text{ham}) \cdot P(\text{lottery \& winning} \mid \text{ham})}$$

$P(A \cap B) = P(A) \cdot P(B)$

↓

The terms $P(\text{lottery \& winning} \mid \text{spam})$ and $P(\text{lottery \& winning} \mid \text{ham})$ are circled in red.

Is There a Quicker Way To Estimate the Probability?

Naive assumption



The appearances of ‘lottery’ and ‘winning’ are independent

$$\mathbf{P}(A \cap B) = \mathbf{P}(A) \cdot \mathbf{P}(B)$$

$$\mathbf{P}(\text{spam} \mid \text{lottery \& winning}) = \frac{\mathbf{P}(\text{spam}) \cdot \mathbf{P}(\text{lottery \& winning} \mid \text{spam})}{\mathbf{P}(\text{spam}) \cdot \mathbf{P}(\text{lottery \& winning} \mid \text{spam}) + \mathbf{P}(\text{ham}) \cdot \mathbf{P}(\text{lottery \& winning} \mid \text{ham})}$$

Is There a Quicker Way To Estimate the Probability?

Naive assumption



The appearances of 'lottery' and 'winning' are independent

$$\mathbf{P}(A \cap B) = \mathbf{P}(A) \cdot \mathbf{P}(B)$$

$$\mathbf{P}(\text{spam} \mid \text{lottery \& winning}) = \frac{\mathbf{P}(\text{spam}) \cdot \mathbf{P}(\text{lottery} \mid \text{spam}) \cdot \mathbf{P}(\text{winning} \mid \text{spam})}{\mathbf{P}(\text{spam}) \cdot \mathbf{P}(\text{lottery} \mid \text{spam}) \cdot \mathbf{P}(\text{winning} \mid \text{spam}) + \mathbf{P}(\text{ham}) \cdot \mathbf{P}(\text{lottery} \mid \text{ham}) \cdot \mathbf{P}(\text{winning} \mid \text{ham})}$$

Is There a Quicker Way To Estimate the Probability?

Naive assumption

$$P(\text{spam} \mid w_1, \dots, w_n) = \frac{P(\text{spam}) \cdot P(w_1, \dots, w_n \mid \text{spam})}{P(\text{spam}) \cdot P(w_1, \dots, w_n \mid \text{spam}) + P(\text{ham}) \cdot P(w_1, \dots, w_n \mid \text{ham})}$$

Is There a Quicker Way To Estimate the Probability?

Naive assumption

The appearances of the words w_1, w_2, \dots, w_n are independent

$$P(\text{spam} \mid w_1, \dots, w_n) = \frac{P(\text{spam}) \cdot P(w_1, \dots, w_n \mid \text{spam})}{P(\text{spam}) \cdot P(w_1, \dots, w_n \mid \text{spam}) + P(\text{ham}) \cdot P(w_1, \dots, w_n \mid \text{ham})}$$

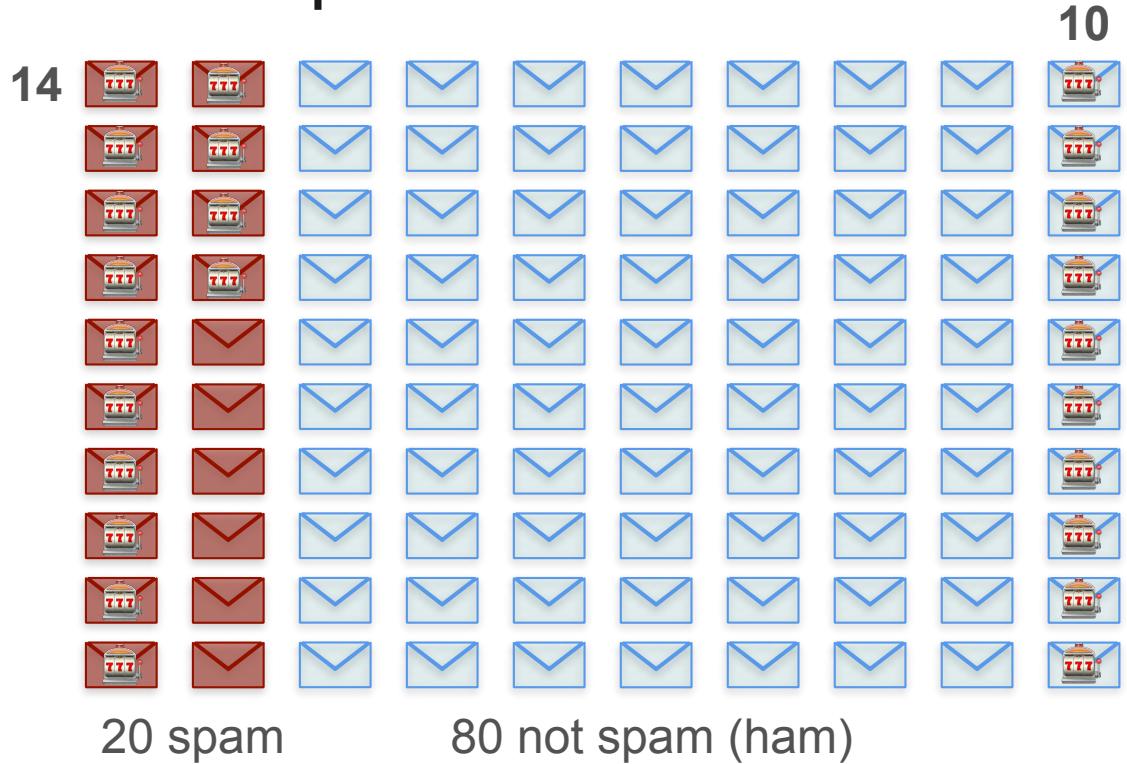
Is There a Quicker Way To Estimate the Probability?

Naive assumption

The appearances of the words w_1, w_2, \dots, w_n are independent

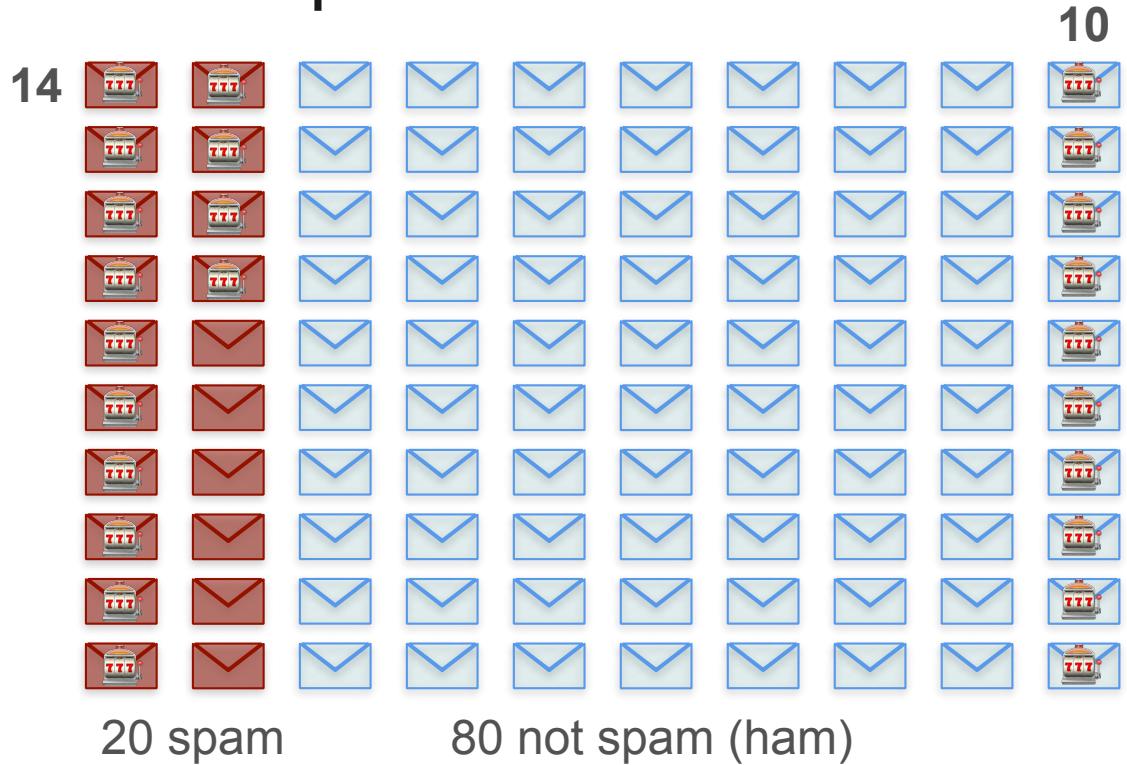
$$P(\text{spam} \mid w_1, \dots, w_n) = \frac{P(\text{spam}) \cdot P(w_1 \mid \text{spam}) \cdots P(w_n \mid \text{spam})}{P(\text{spam}) \cdot P(w_1 \mid \text{spam}) \cdots P(w_n \mid \text{spam}) + P(\text{ham}) \cdot P(w_1 \mid \text{ham}) \cdots P(w_n \mid \text{ham})}$$

Naive Bayes: Spam Example



Naive Bayes: Spam Example

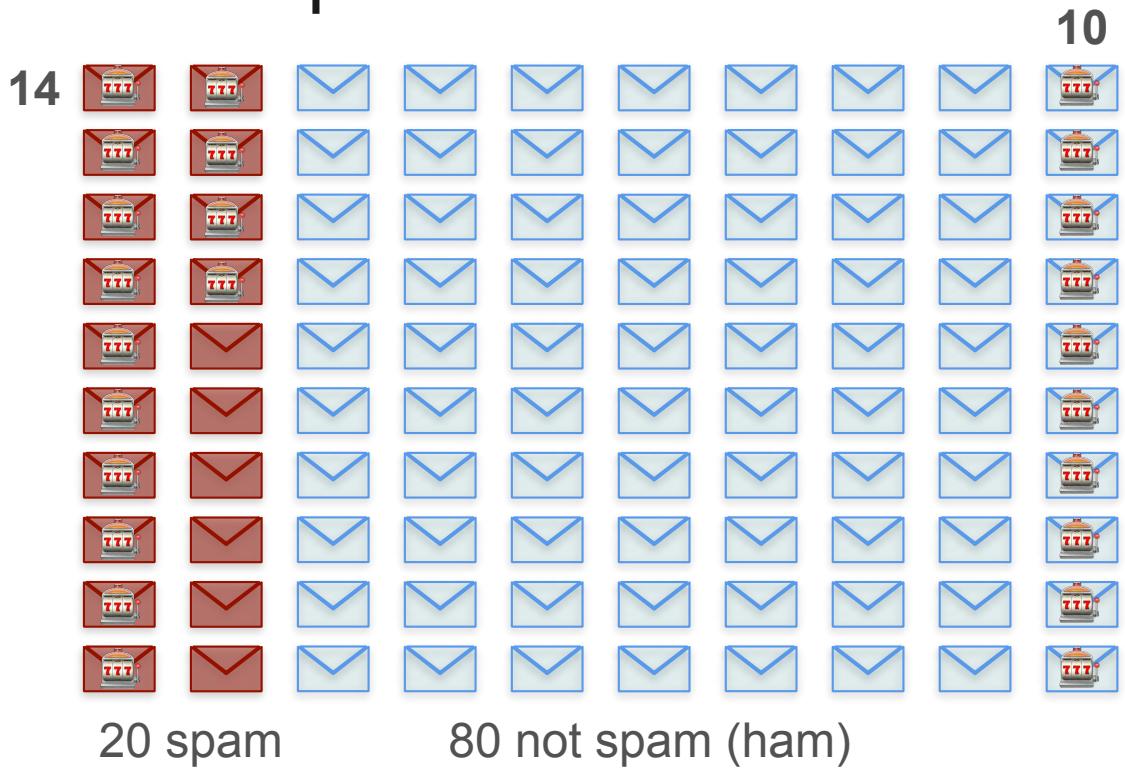
$$P(\text{spam}) = \frac{20}{100} = 0.2$$



Naive Bayes: Spam Example

$$P(\text{spam}) = \frac{20}{100} = 0.2$$

$$P(\text{ham}) = \frac{80}{100} = 0.8$$

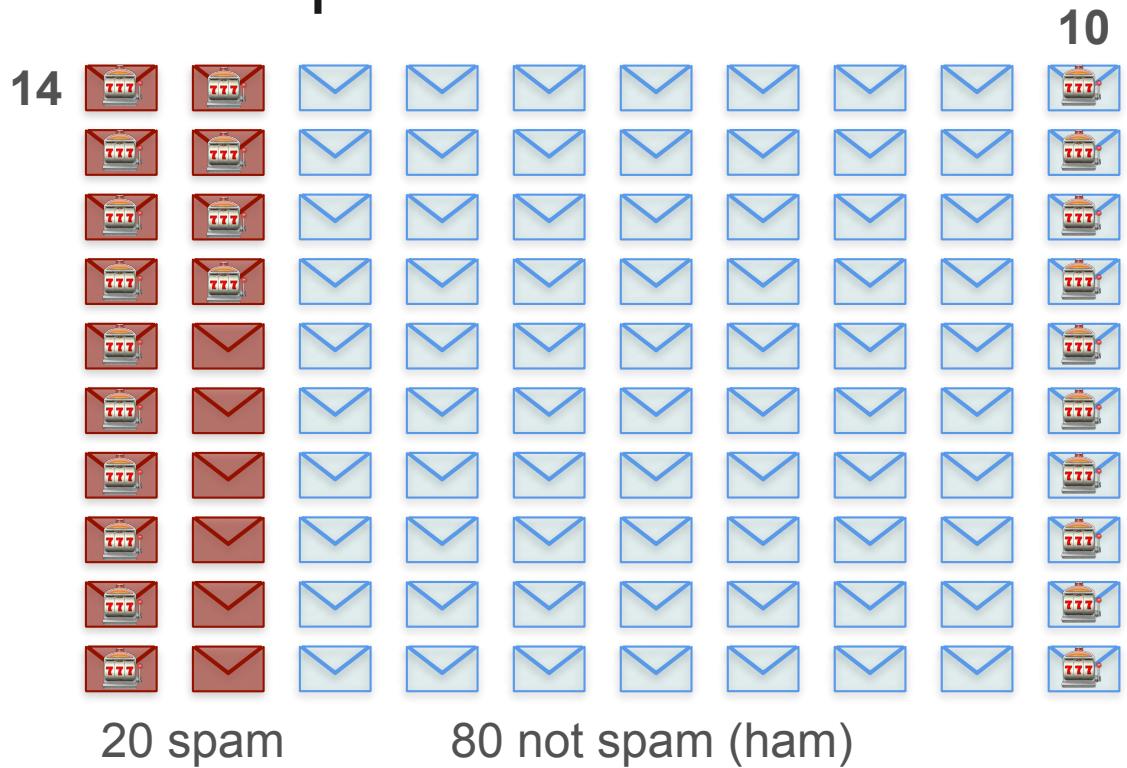


Naive Bayes: Spam Example

$$P(\text{spam}) = \frac{20}{100} = 0.2$$

$$P(\text{ham}) = \frac{80}{100} = 0.8$$

$$P(\text{lottery} \mid \text{spam}) = \frac{14}{20} = 0.7$$



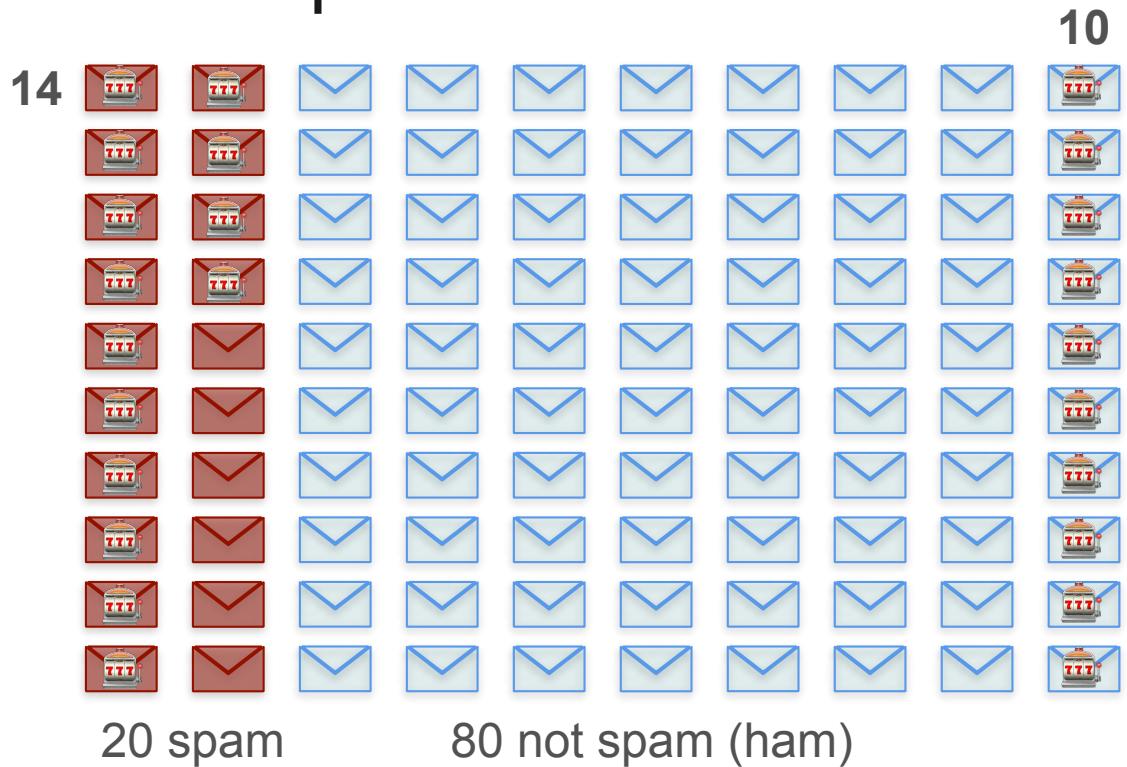
Naive Bayes: Spam Example

$$P(\text{spam}) = \frac{20}{100} = 0.2$$

$$P(\text{ham}) = \frac{80}{100} = 0.8$$

$$P(\text{lottery} | \text{spam}) = \frac{14}{20} = 0.7$$

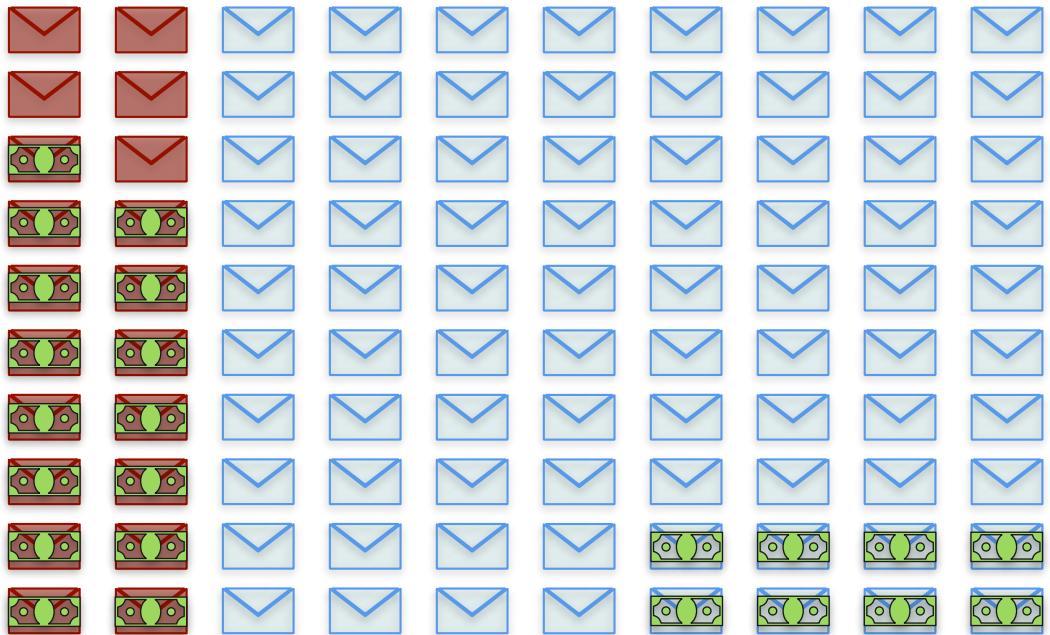
$$P(\text{lottery} | \text{ham}) = \frac{10}{80} = 0.125$$



Naive Bayes: Spam Example

$$P(\text{spam}) = \frac{20}{100} = 0.2$$

$$P(\text{ham}) = \frac{80}{100} = 0.8$$



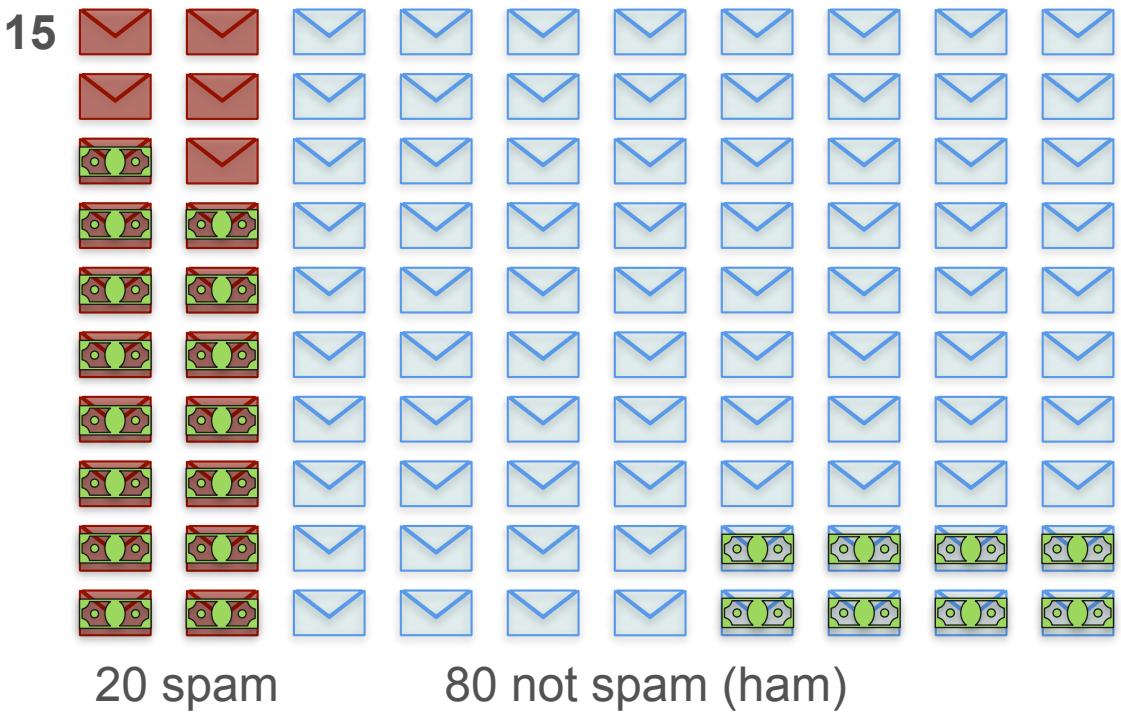
20 spam

80 not spam (ham)

Naive Bayes: Spam Example

$$P(\text{spam}) = \frac{20}{100} = 0.2$$

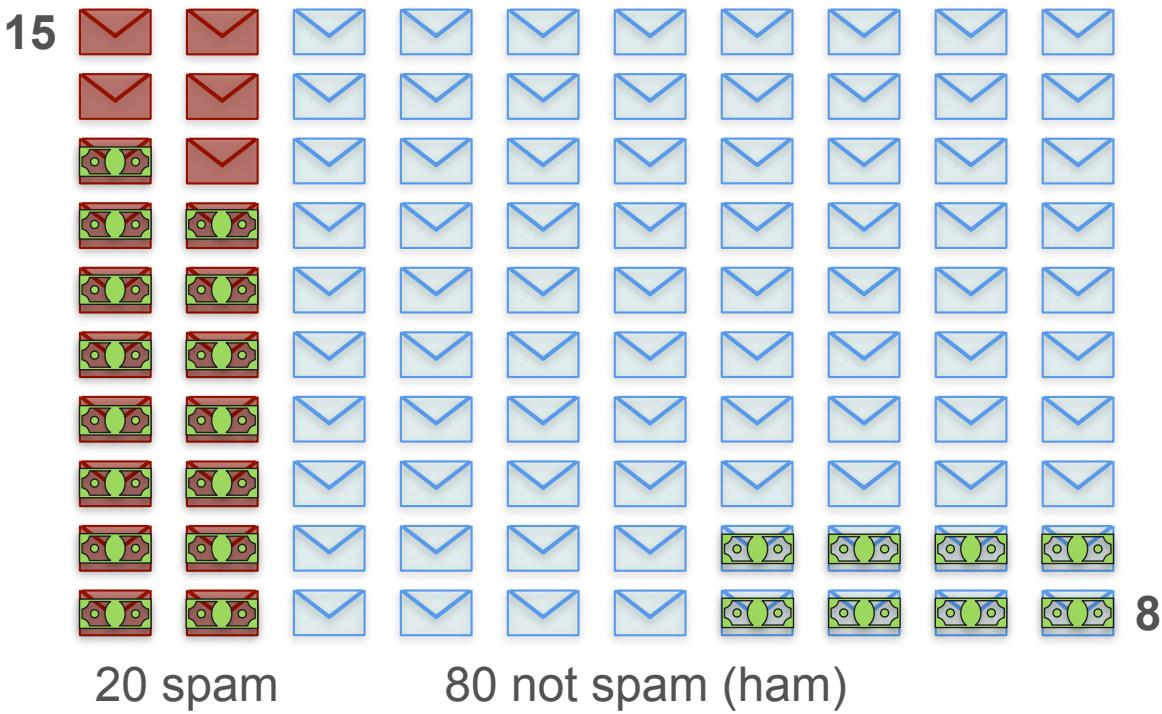
$$P(\text{ham}) = \frac{80}{100} = 0.8$$



Naive Bayes: Spam Example

$$P(\text{spam}) = \frac{20}{100} = 0.2$$

$$P(\text{ham}) = \frac{80}{100} = 0.8$$

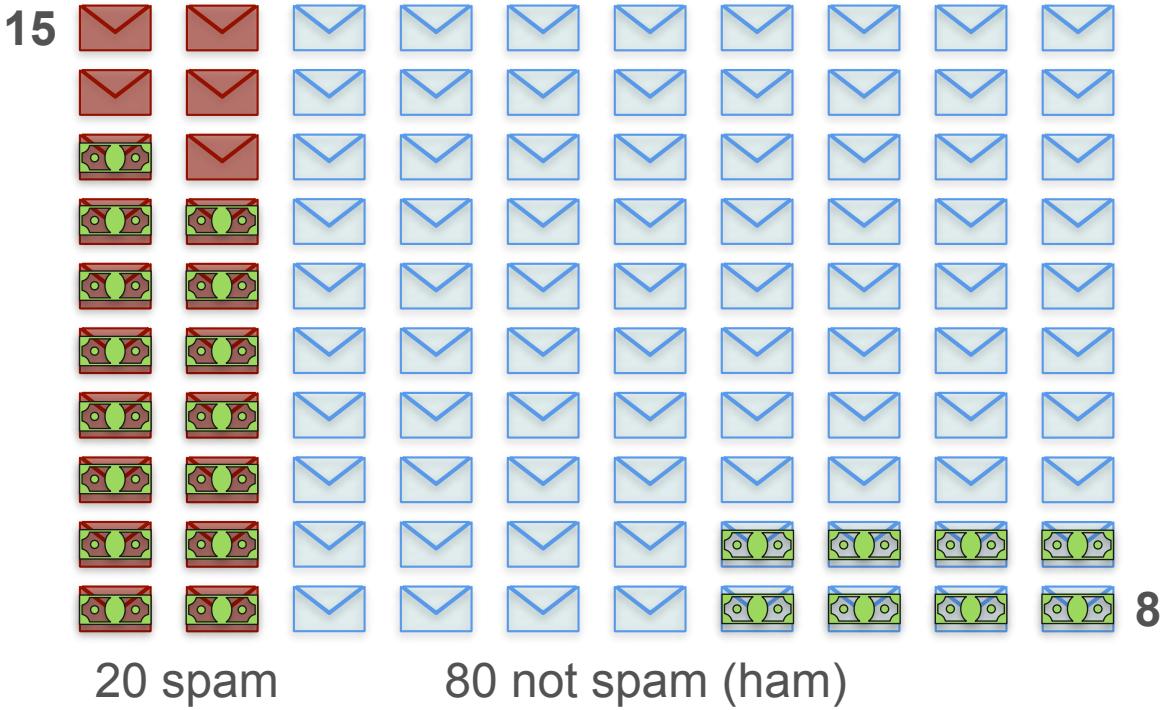


Naive Bayes: Spam Example

$$P(\text{spam}) = \frac{20}{100} = 0.2$$

$$P(\text{ham}) = \frac{80}{100} = 0.8$$

$$P(\text{winning} \mid \text{spam}) = \frac{15}{20} = 0.75$$



Naive Bayes: Spam Example

$$P(\text{spam}) = \frac{20}{100} = 0.2$$

$$P(\text{ham}) = \frac{80}{100} = 0.8$$

$$P(\text{winning} \mid \text{spam}) = \frac{15}{20} = 0.75$$

$$P(\text{winning} \mid \text{ham}) = \frac{8}{80} = 0.1$$



Naive Bayes: Spam Example

$$P(\text{spam}) = 0.2$$

$$P(\text{lottery} \mid \text{spam}) = 0.7$$

$$P(\text{winning} \mid \text{spam}) = 0.75$$

$$P(\text{not spam}) = 0.8$$

$$P(\text{lottery} \mid \text{ham}) = 0.125$$

$$P(\text{winning} \mid \text{ham}) = 0.1$$

Naive Bayes: Spam Example

$$P(\text{spam}) = 0.2$$

$$P(\text{lottery} \mid \text{spam}) = 0.7$$

$$P(\text{winning} \mid \text{spam}) = 0.75$$

$$P(\text{not spam}) = 0.8$$

$$P(\text{lottery} \mid \text{ham}) = 0.125$$

$$P(\text{winning} \mid \text{ham}) = 0.1$$

$$P(\text{spam} \mid \text{lottery}) = \frac{P(\text{spam}) \cdot P(\text{lottery} \mid \text{spam}) \cdot P(\text{winning} \mid \text{spam})}{P(\text{spam}) \cdot P(\text{lottery} \mid \text{spam}) \cdot P(\text{winning} \mid \text{spam}) + P(\text{ham}) \cdot P(\text{lottery} \mid \text{ham}) \cdot P(\text{winning} \mid \text{ham})}$$

Naive Bayes: Spam Example

$$P(\text{spam}) = 0.2$$

$$P(\text{lottery} \mid \text{spam}) = 0.7$$

$$P(\text{winning} \mid \text{spam}) = 0.75$$

$$P(\text{not spam}) = 0.8$$

$$P(\text{lottery} \mid \text{ham}) = 0.125$$

$$P(\text{winning} \mid \text{ham}) = 0.1$$

$$P(\text{spam} \mid \text{lottery}) = \frac{P(\text{spam}) \cdot P(\text{lottery} \mid \text{spam}) \cdot P(\text{winning} \mid \text{spam})}{P(\text{spam}) \cdot P(\text{lottery} \mid \text{spam}) \cdot P(\text{winning} \mid \text{spam}) + P(\text{ham}) \cdot P(\text{lottery} \mid \text{ham}) \cdot P(\text{winning} \mid \text{ham})}$$

$$P(\text{spam} \mid \text{lottery} \& \text{winning}) = \frac{0.2 \times 0.7 \times 0.75}{(0.2 \times 0.7 \times 0.75) + (0.8 \times 0.125 \times 0.1)}$$

Naive Bayes: Spam Example

$$P(\text{spam}) = 0.2$$

$$P(\text{lottery} \mid \text{spam}) = 0.7$$

$$P(\text{winning} \mid \text{spam}) = 0.75$$

$$P(\text{not spam}) = 0.8$$

$$P(\text{lottery} \mid \text{ham}) = 0.125$$

$$P(\text{winning} \mid \text{ham}) = 0.1$$

$$P(\text{spam} \mid \text{lottery}) = \frac{P(\text{spam}) \cdot P(\text{lottery} \mid \text{spam}) \cdot P(\text{winning} \mid \text{spam})}{P(\text{spam}) \cdot P(\text{lottery} \mid \text{spam}) \cdot P(\text{winning} \mid \text{spam}) + P(\text{ham}) \cdot P(\text{lottery} \mid \text{ham}) \cdot P(\text{winning} \mid \text{ham})}$$

$$P(\text{spam} \mid \text{lottery} \& \text{winning}) = \frac{0.2 \times 0.7 \times 0.75}{(0.2 \times 0.7 \times 0.75) + (0.8 \times 0.125 \times 0.1)} = 0.913$$



DeepLearning.AI

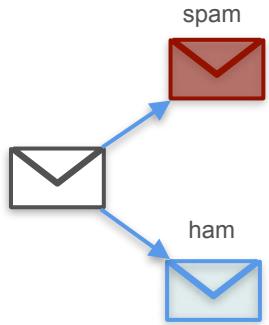
Introduction to probability

Probability in Machine Learning

Bayes Theorem

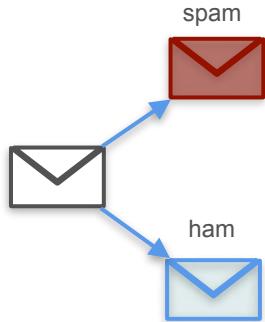
Bayes Theorem

PRIOR



Bayes Theorem

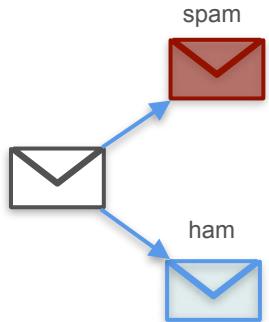
PRIOR



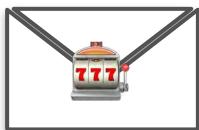
$$P(\text{spam}) = \frac{\text{spam icon}}{\text{spam icon} + \text{ham icon}}$$

Bayes Theorem

PRIOR



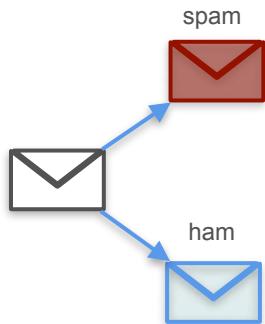
EVENT



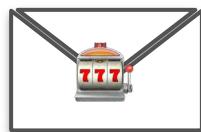
$$P(\text{spam}) = \frac{\text{Red Envelope}}{\text{Red Envelope} + \text{Blue Envelope}}$$

Bayes Theorem

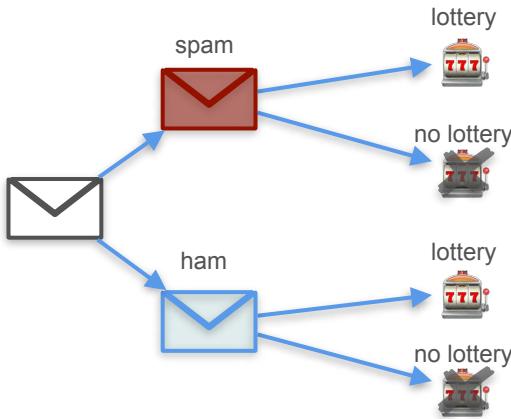
PRIOR



EVENT



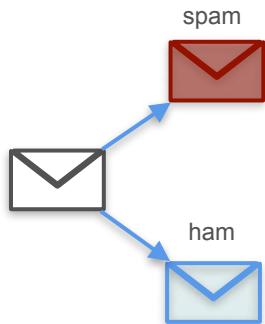
POSTERIOR



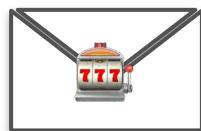
$$P(\text{spam}) = \frac{\text{Red Envelope}}{\text{Red Envelope} + \text{Blue Envelope}}$$

Bayes Theorem

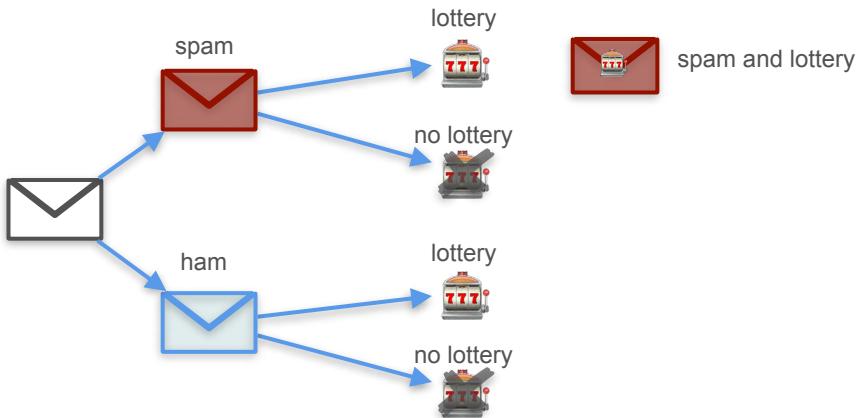
PRIOR



EVENT



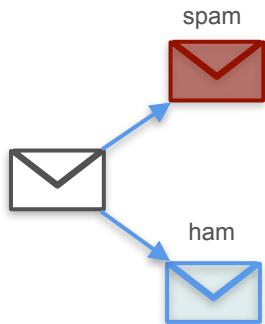
POSTERIOR



$$P(\text{spam}) = \frac{\text{spam}}{\text{spam} + \text{ham}}$$

Bayes Theorem

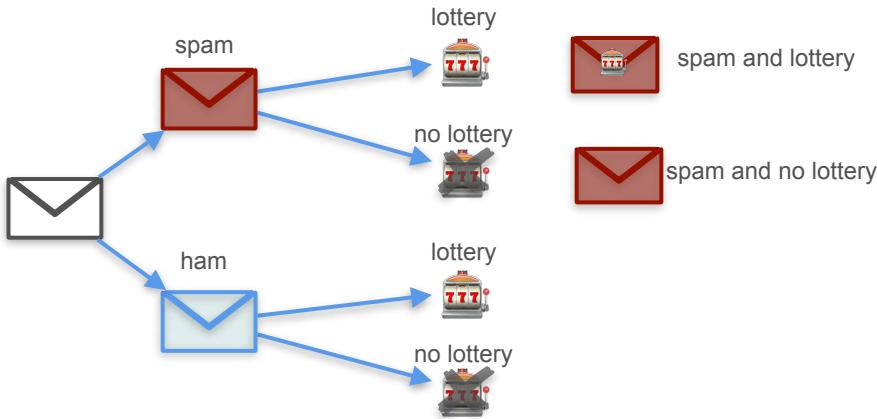
PRIOR



EVENT



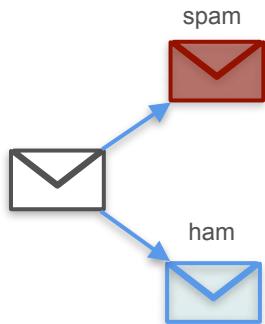
POSTERIOR



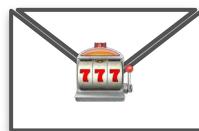
$$P(\text{spam}) = \frac{\text{spam}}{\text{spam} + \text{ham}}$$

Bayes Theorem

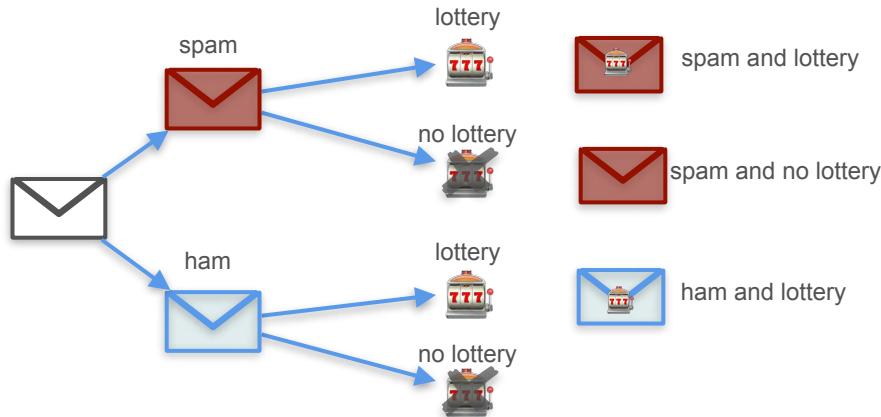
PRIOR



EVENT



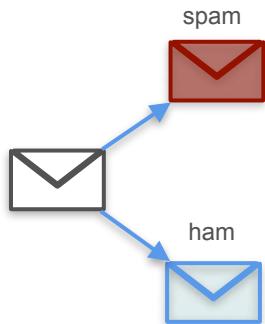
POSTERIOR



$$P(\text{spam}) = \frac{\text{red envelope}}{\text{red envelope} + \text{blue envelope}}$$

Bayes Theorem

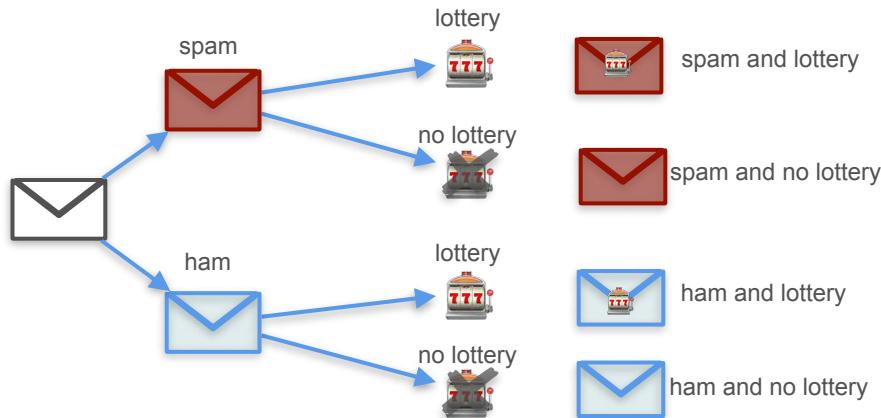
PRIOR



EVENT



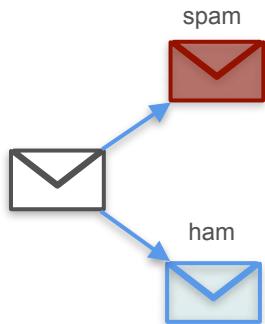
POSTERIOR



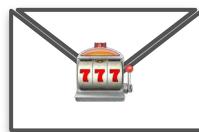
$$P(\text{spam}) = \frac{\text{red envelope}}{\text{red envelope} + \text{blue envelope}}$$

Bayes Theorem

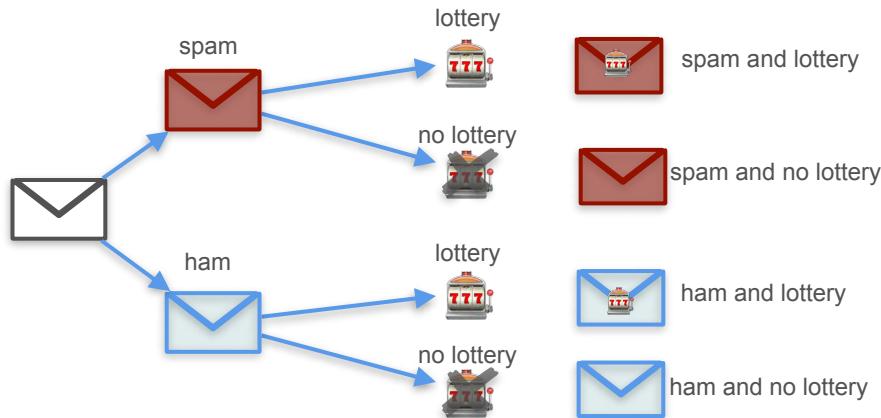
PRIOR



EVENT



POSTERIOR

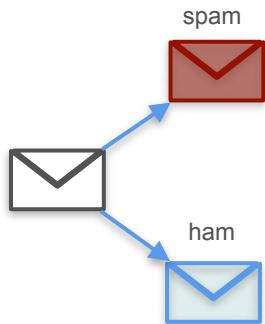


$$P(\text{spam}) = \frac{\text{red envelope}}{\text{red envelope} + \text{blue envelope}}$$

$$P(\text{spam} | \text{lottery}) =$$

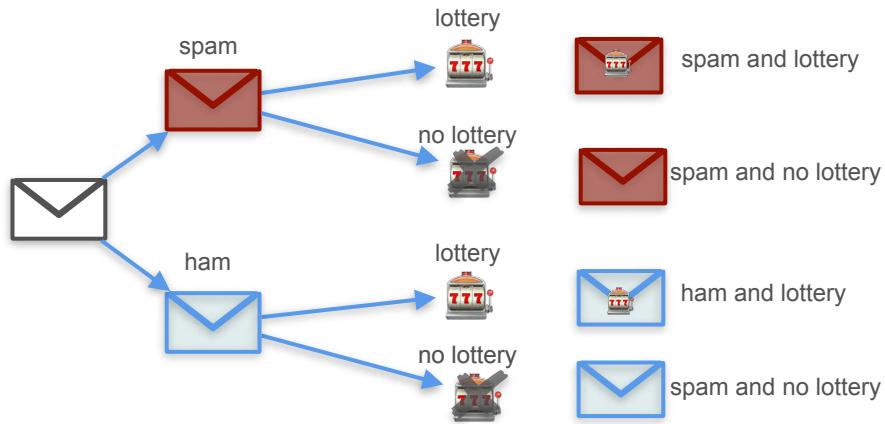
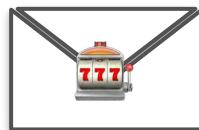
Bayes Theorem

PRIOR



$$P(\text{spam}) = \frac{\text{red envelope}}{\text{red envelope} + \text{blue envelope}}$$

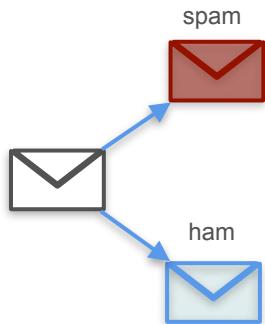
EVENT



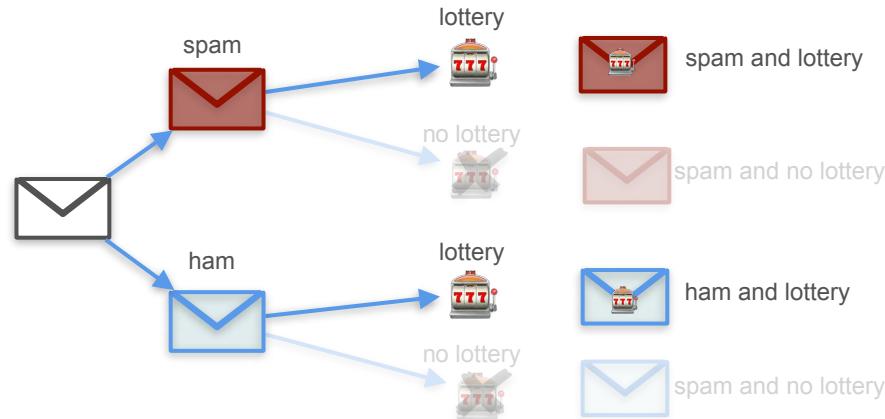
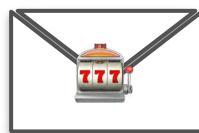
$$P(\text{spam} | \text{lottery}) =$$

Bayes Theorem

PRIOR



EVENT

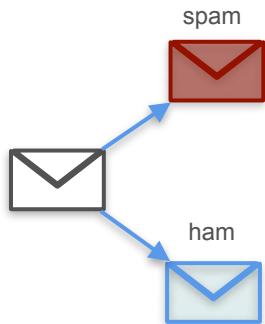


$$P(\text{spam}) = \frac{\text{spam}}{\text{spam} + \text{ham}}$$

$$P(\text{spam} | \text{lottery}) =$$

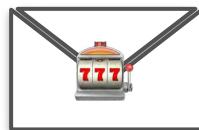
Bayes Theorem

PRIOR

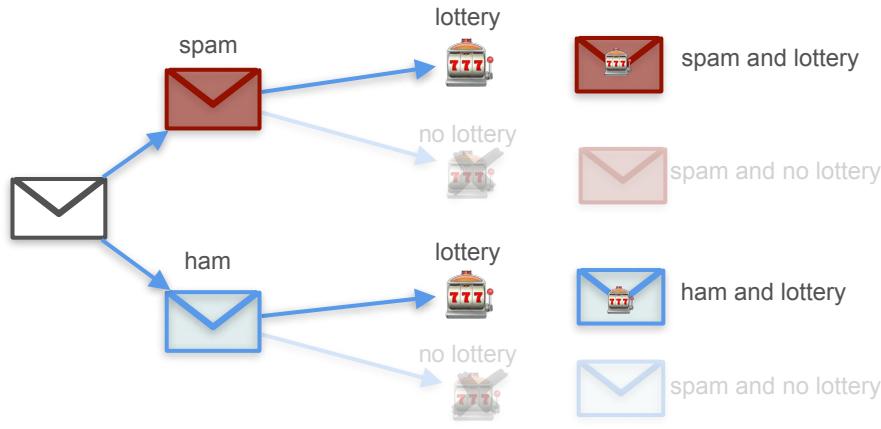


$$P(\text{spam}) = \frac{\text{red envelope}}{\text{red envelope} + \text{blue envelope}}$$

EVENT



POSTERIOR



$$P(\text{spam} | \text{lottery}) = \frac{\text{red envelope}}{\text{red envelope} + \text{blue envelope}}$$

Example Problem

Example Problem

Image recognition

Example Problem

Image recognition

- What is the probability that there is a cat in the image



Example Problem

Image recognition

- What is the probability that there is a cat in the image
- $P(\text{cat} \mid \text{image}) = P(\text{cat} \mid \text{pixel}_1, \text{pixel}_2, \dots, \text{pixel}_n)$



Example Problem

Example Problem

Classification

Example Problem

Classification

Patient 1		
A	Age	29
G	Gender	Female
H	Height	169 cm
W	Weight	62 kg
S	Smoker	No
...
B	Heart rate	63
B	Blood pressure	120 90

- Is this patient healthy?

Example Problem

Classification

Patient 1		
A	Age	29
G	Gender	Female
H	Height	169 cm
W	Weight	62 kg
S	Smoker	No
...
B	Heart rate	63
B	Blood pressure	120 90

- Is this patient healthy?
- Calculate $P(\text{healthy} \mid \text{symptoms and history})$

Example Problem

Example Problem

Sentiment analysis

Example Problem

Sentiment analysis

the first cold shower
even the monkey seems to want
a little coat of straw

Matsuo Bashō

Example Problem

Sentiment analysis

the first cold shower
even the monkey seems to want
a little coat of straw

- Is this a happy sentence?

Matsuo Bashō

Example Problem

Sentiment analysis

the first cold shower
even the monkey seems to want
a little coat of straw

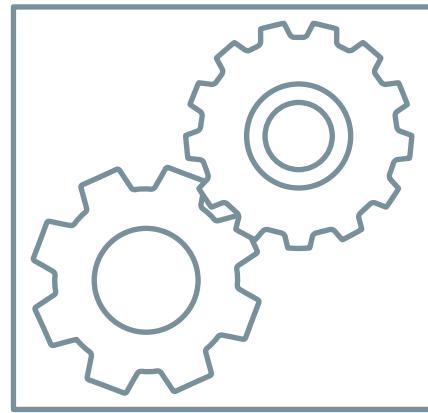
Matsuo Bashō

- Is this a happy sentence?
- Calculate $P(\text{happy} \mid \text{words in the sentence})$

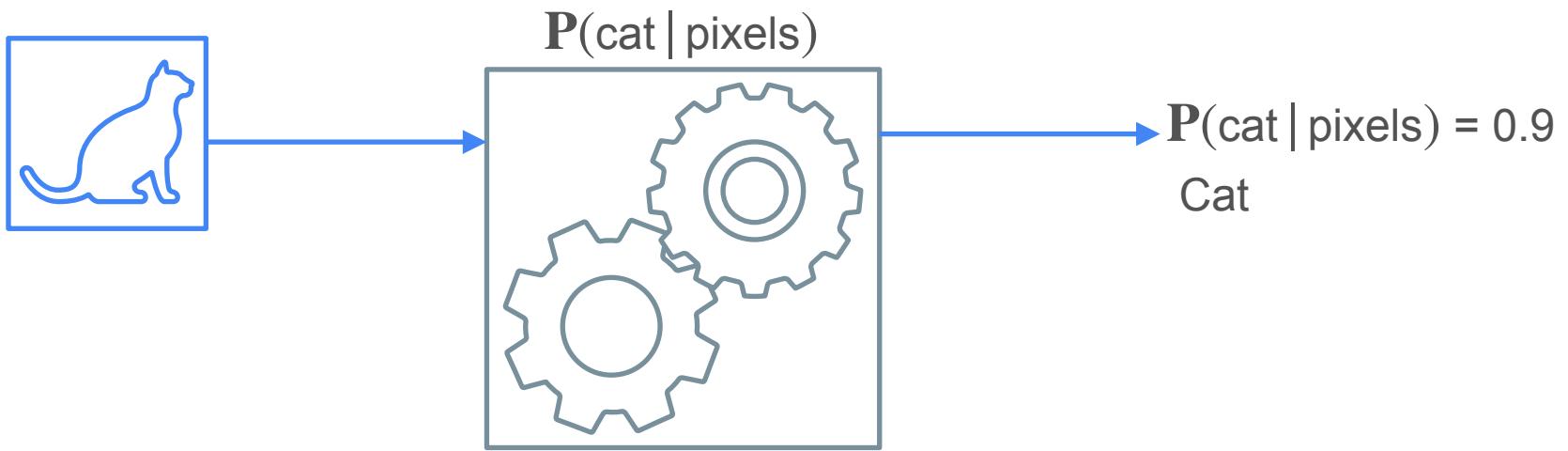
Example Solution

Example Solution

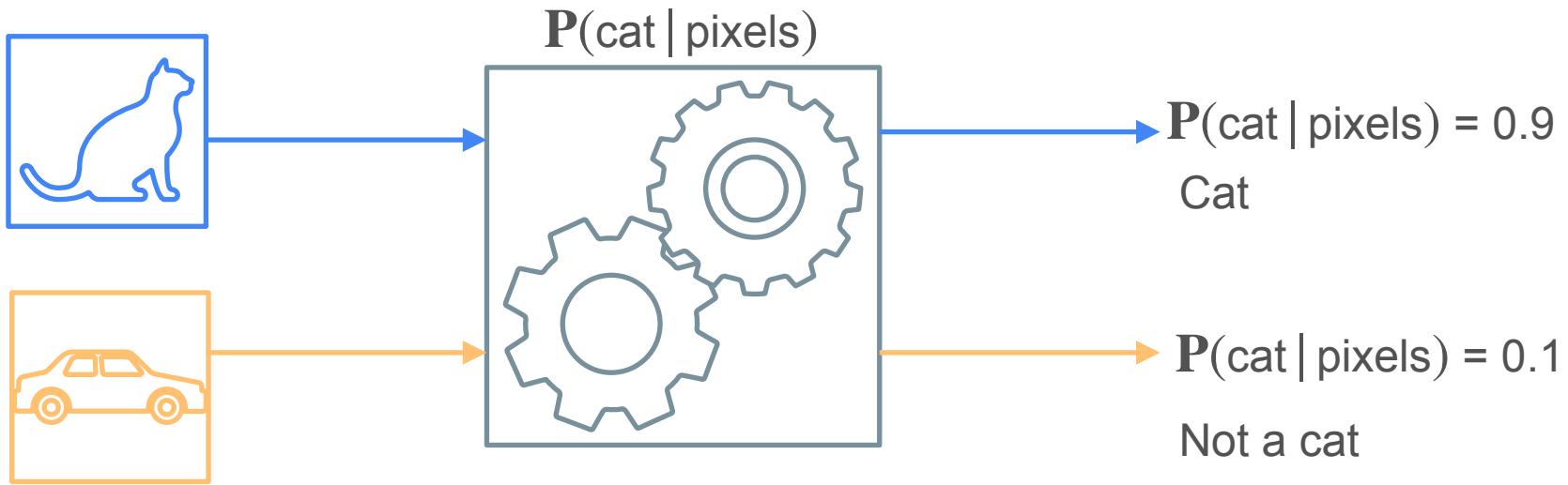
$P(\text{cat} \mid \text{pixels})$



Example Solution



Example Solution



Example Problem: Generative Models

Example Problem: Generative Models

Face generation

Example Problem: Generative Models

Face generation



Image generated by a StyleGAN

Example Problem: Generative Models

Face generation

- Generate a group of pixels such that the resulting image looks like a human face.
- Goal: generate images such that $P(\text{face} \mid \text{pixels})$ is high.

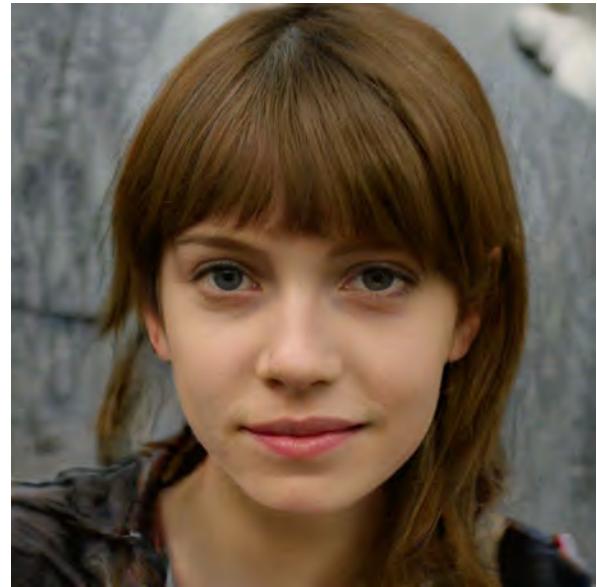


Image generated by a StyleGAN

W1 Lesson 2

Probability Distributions



DeepLearning.AI

Probability Distributions

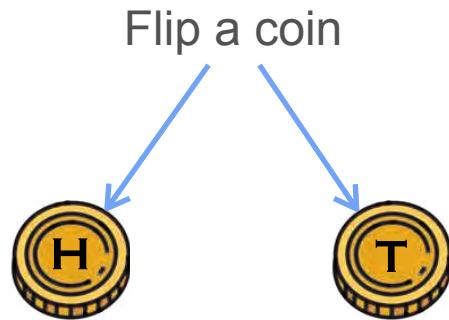
Random Variables

From Events to Random Variables

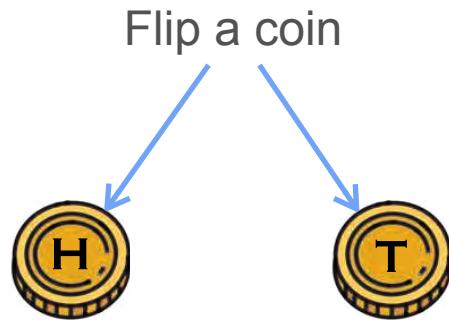
From Events to Random Variables

Flip a coin

From Events to Random Variables

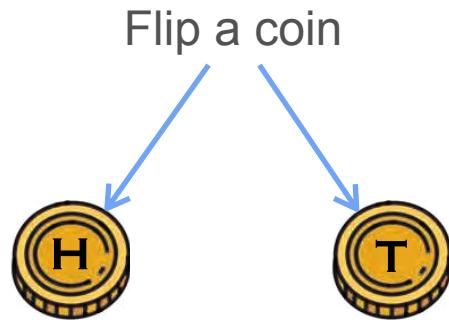


From Events to Random Variables



$$P(H) = 0.5$$

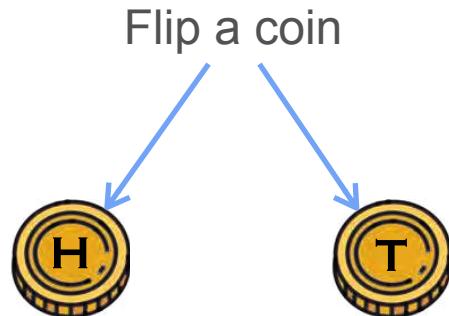
From Events to Random Variables



$$\mathbf{P}(\textcolor{blue}{H}) = 0.5 \quad \mathbf{P}(\textcolor{blue}{T}) = 0.5$$

From Events to Random Variables

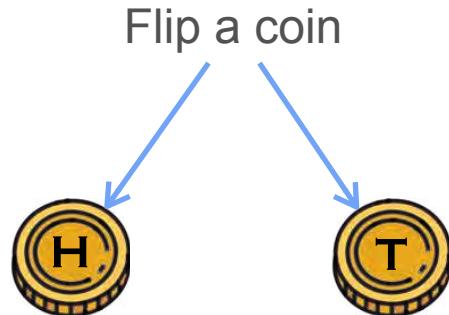
X = Number of heads



$$\mathbf{P}(H) = 0.5 \quad \mathbf{P}(T) = 0.5$$

From Events to Random Variables

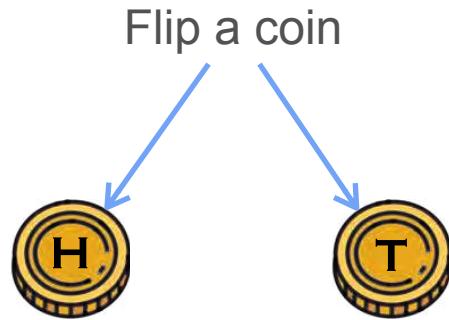
X = Number of heads



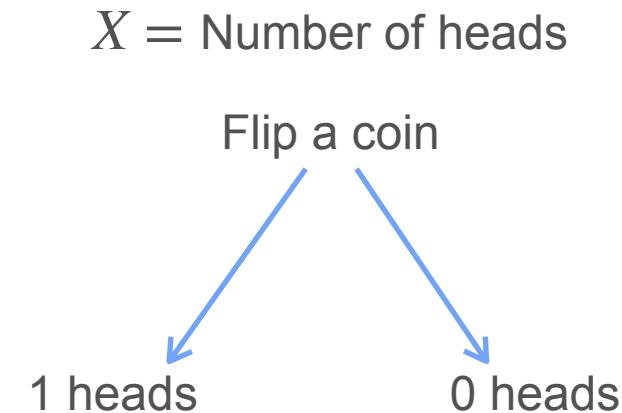
Flip a coin

$$\mathbf{P}(H) = 0.5 \quad \mathbf{P}(T) = 0.5$$

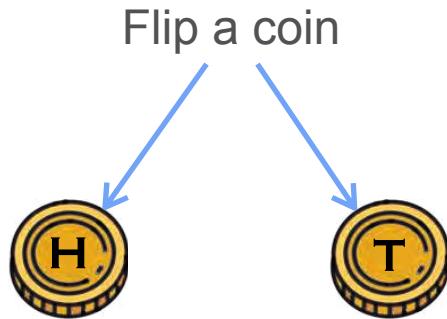
From Events to Random Variables



$$P(H) = 0.5 \quad P(T) = 0.5$$

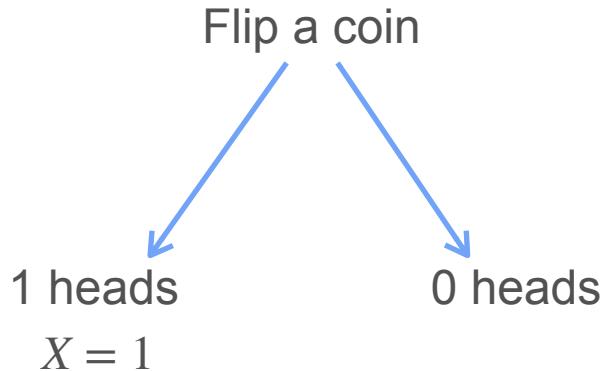


From Events to Random Variables

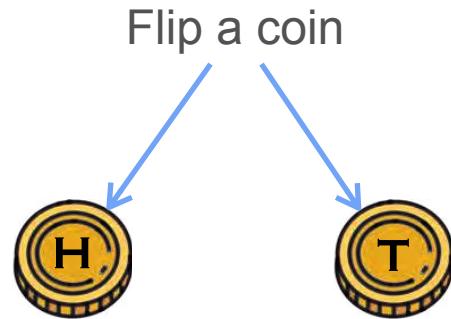


$$P(H) = 0.5 \quad P(T) = 0.5$$

$X = \text{Number of heads}$

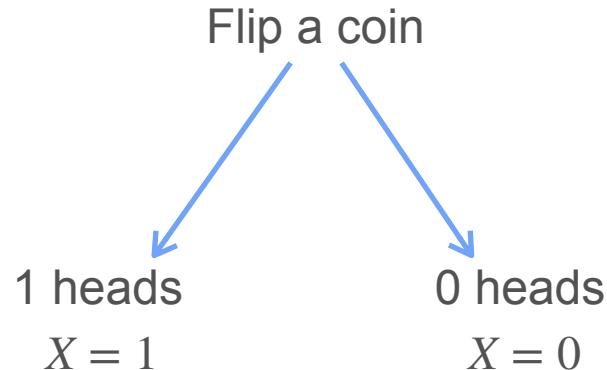


From Events to Random Variables

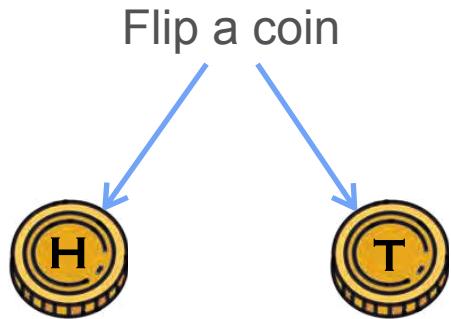


$$\mathbf{P}(H) = 0.5 \quad \mathbf{P}(T) = 0.5$$

$X = \text{Number of heads}$

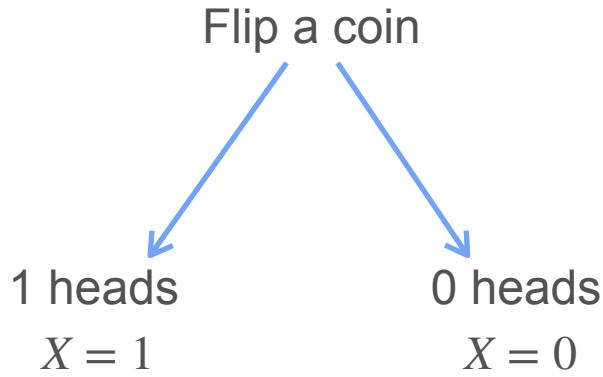


From Events to Random Variables



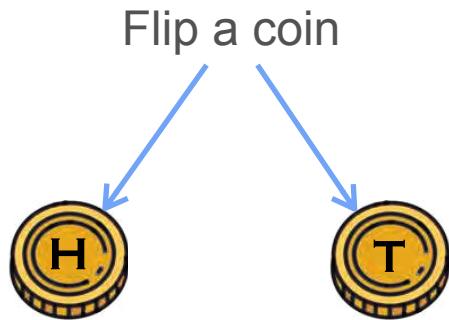
$$\mathbf{P}(H) = 0.5 \quad \mathbf{P}(T) = 0.5$$

$X = \text{Number of heads}$



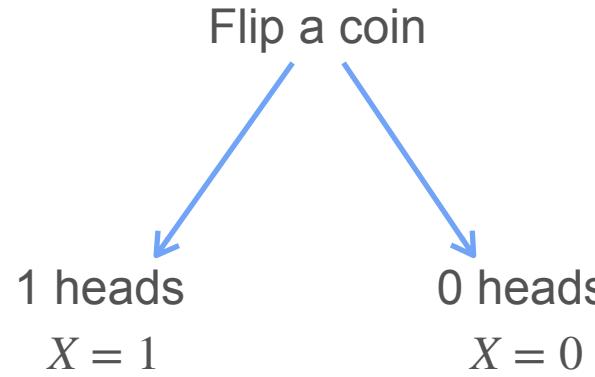
$$\mathbf{P}(X = 1) = 0.5$$

From Events to Random Variables



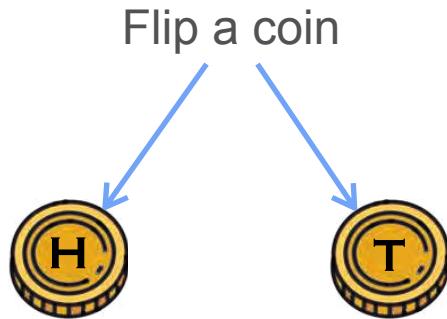
$$\mathbf{P}(H) = 0.5 \quad \mathbf{P}(T) = 0.5$$

$X = \text{Number of heads}$



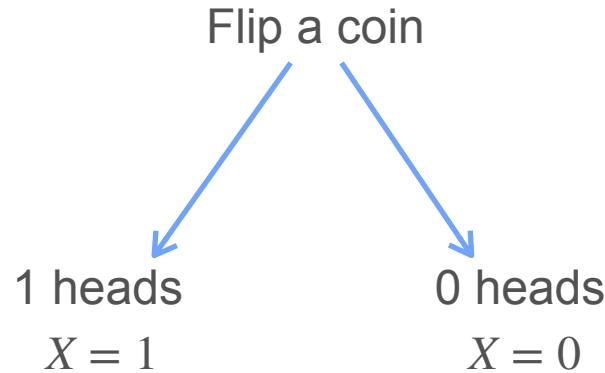
$$\mathbf{P}(X = 1) = 0.5 \quad \mathbf{P}(X = 0) = 0.5$$

From Events to Random Variables



$$\mathbf{P}(H) = 0.5 \quad \mathbf{P}(T) = 0.5$$

X = Number of heads



$$\mathbf{P}(X = 1) = 0.5 \quad \mathbf{P}(X = 0) = 0.5$$

X is a random variable

From Events to Random Variables

From Events to Random Variables

X = Number of heads in 10 coin tosses

From Events to Random Variables

X = Number of heads in 10 coin tosses



$$X = 10$$

From Events to Random Variables

X = Number of heads in 10 coin tosses



$$X = 10$$



$$X = 9$$

From Events to Random Variables

X = Number of heads in 10 coin tosses



$$X = 10$$



$$X = 9$$



$$X = 9$$

From Events to Random Variables

X = Number of heads in 10 coin tosses

$$\mathbf{P}(H) = 0.5$$



$$X = 10$$



$$X = 9$$



$$X = 9$$

From Events to Random Variables

X = Number of heads in 10 coin tosses

$$\mathbf{P}(H) = 0.5$$



$$X = 10$$

$$0.5^{10}$$



$$X = 9$$



$$X = 9$$

From Events to Random Variables

X = Number of heads in 10 coin tosses

$$\mathbf{P}(H) = 0.5$$



$$X = 10$$

$$0.5^{10}$$



$$X = 9$$

$$0.5^9 0.5$$



$$X = 9$$

From Events to Random Variables

X = Number of heads in 10 coin tosses

$$\mathbf{P}(H) = 0.5$$



$$X = 10$$

$$0.5^{10}$$



$$X = 9$$

$$0.5^9 0.5$$



$$X = 9$$

$$0.5^9 0.5$$

From Events to Random Variables

X = Number of heads in 10 coin tosses

$$\mathbf{P}(H) = 0.5$$



$$X = 10$$

$$0.5^{10}$$

$$\mathbf{P}(X = 0)?$$



$$X = 9$$

$$0.5^9 0.5$$



$$X = 9$$

$$0.5^9 0.5$$

From Events to Random Variables

X = Number of heads in 10 coin tosses

$$\mathbf{P}(H) = 0.5$$



$$X = 10$$

$$0.5^{10}$$

$$\mathbf{P}(X = 0)?$$



$$X = 9$$

$$0.5^9 0.5$$

$$\mathbf{P}(X = 1)?$$



$$X = 9$$

$$0.5^9 0.5$$

From Events to Random Variables

X = Number of heads in 10 coin tosses

$$\mathbf{P}(H) = 0.5$$



$$X = 10$$

$$0.5^{10}$$

$$\mathbf{P}(X = 0)?$$



$$X = 9$$

$$0.5^9 0.5$$

$$\mathbf{P}(X = 1)?$$

...



$$X = 9$$

$$0.5^9 0.5$$

From Events to Random Variables

X = Number of heads in 10 coin tosses

$$\mathbf{P}(H) = 0.5$$



$$X = 10$$

$$0.5^{10}$$

$$\mathbf{P}(X = 0)?$$



$$X = 9$$

$$0.5^9 0.5$$

$$\mathbf{P}(X = 1)?$$

...



$$X = 9$$

$$0.5^9 0.5$$

$$\mathbf{P}(X = 10)?$$

From Events to Random Variables

X = Number of heads in 10 coin tosses



From Events to Random Variables

X = Number of heads in 10 coin tosses



From Events to Random Variables

X = Number of heads in 10 coin tosses



From Events to Random Variables

X = Number of heads in 10 coin tosses



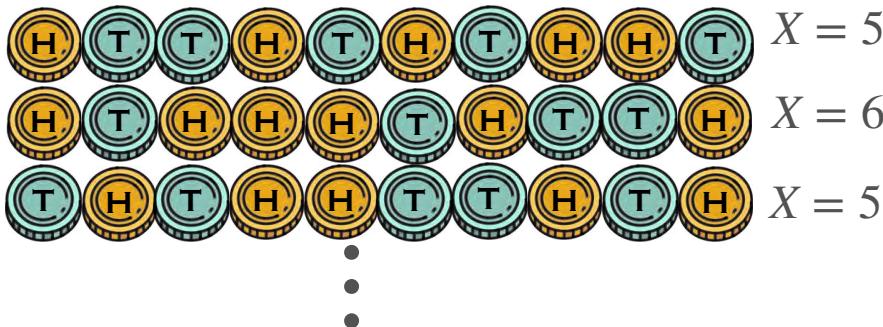
⋮

From Events to Random Variables

X = Number of heads in 10 coin tosses



Possible outcomes:



From Events to Random Variables

X = Number of heads in 10 coin tosses



Possible outcomes:

0	4	8
1	5	9
2	6	10
3	7	



⋮

From Events to Random Variables

X = Number of heads in 10 coin tosses



Possible outcomes:

0	4	8
1	5	9
2	6	10
3	7	



$$X = 5$$



$$X = 6$$



$$X = 5$$

⋮

$$\mathbf{P}(H) = 0.5$$

From Events to Random Variables

X = Number of heads in 10 coin tosses



Possible outcomes:

0	4	8
1	5	9
2	6	10
3	7	



⋮

$$P(H) = 0.5$$

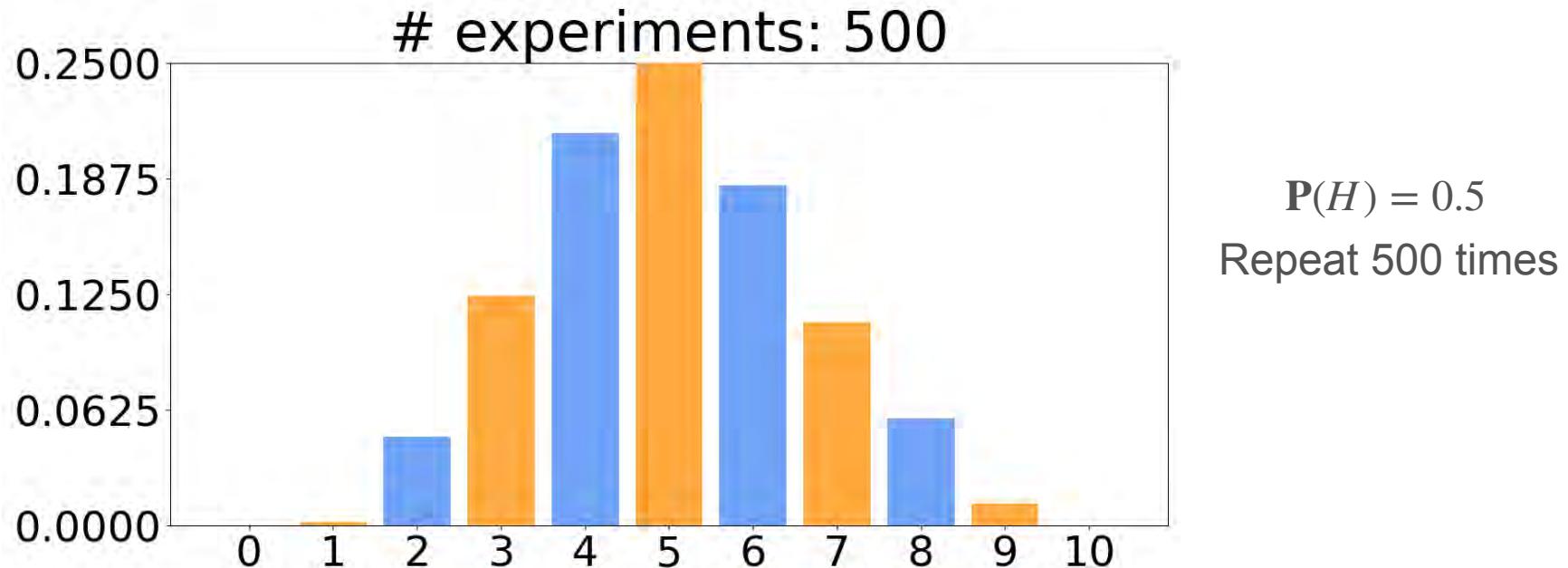
Repeat 500 times

Flipping a Fair Coin 500 Times

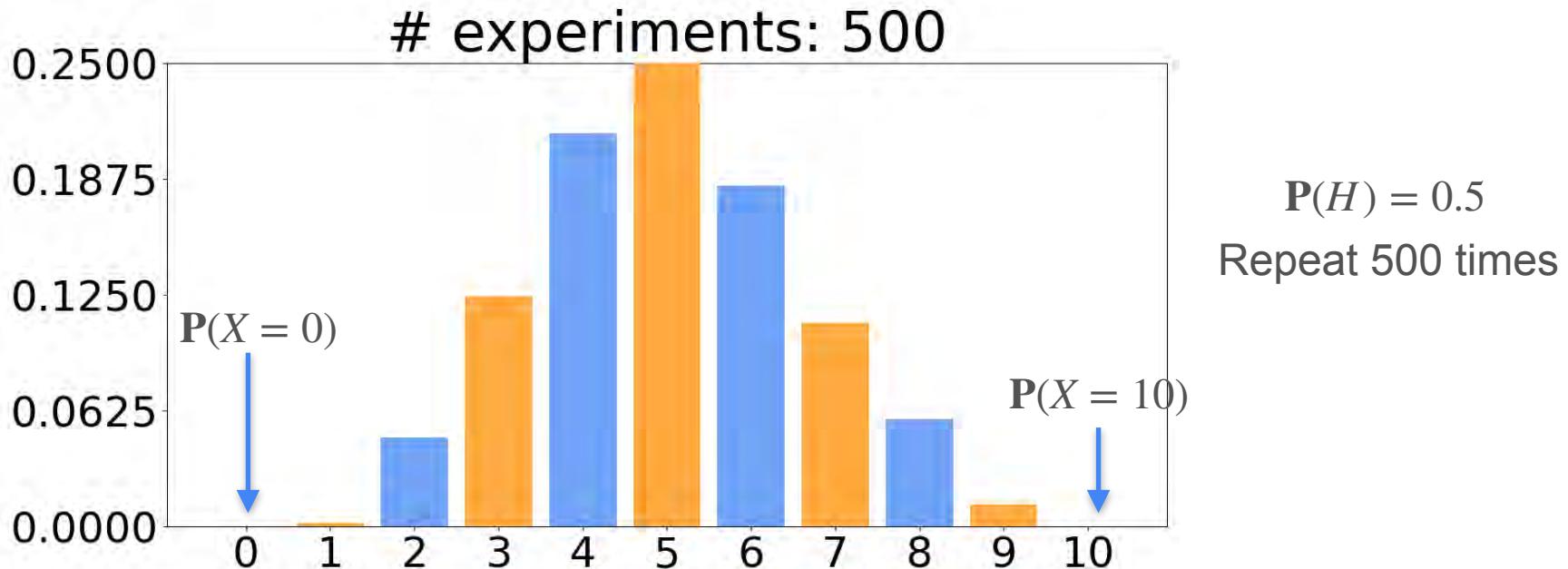
$$\mathbf{P}(H) = 0.5$$

Repeat 500 times

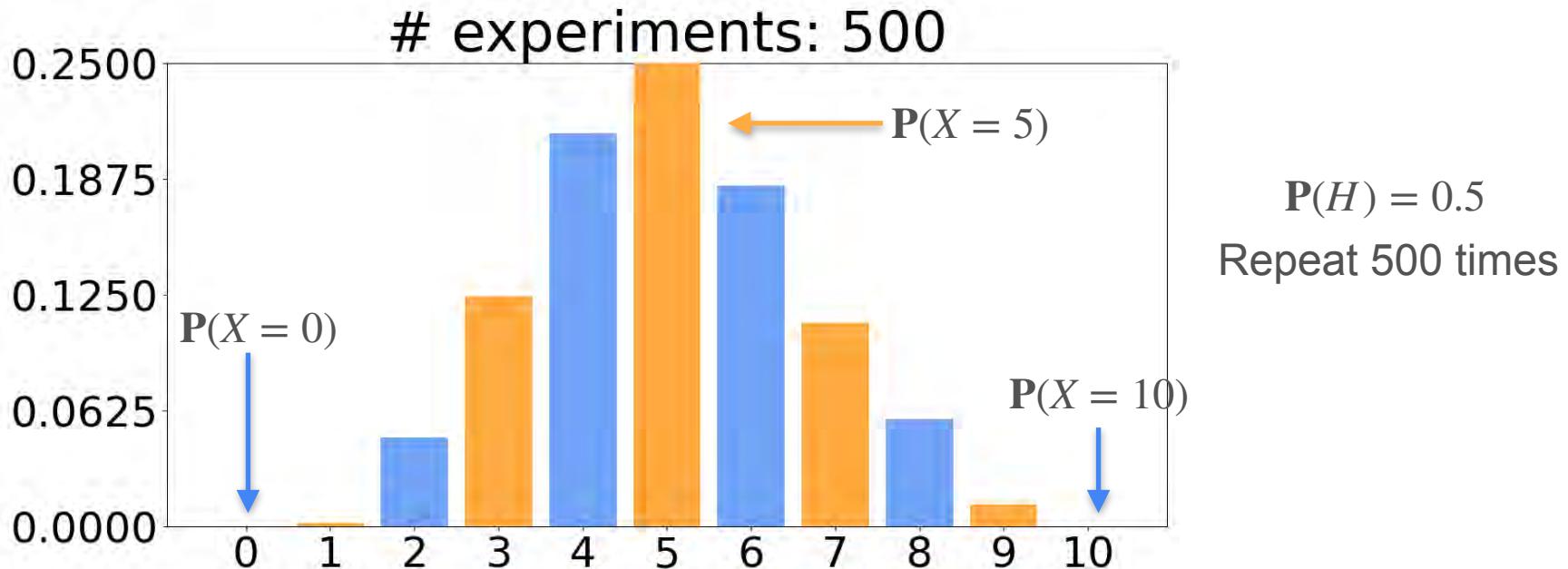
Flipping a Fair Coin 500 Times



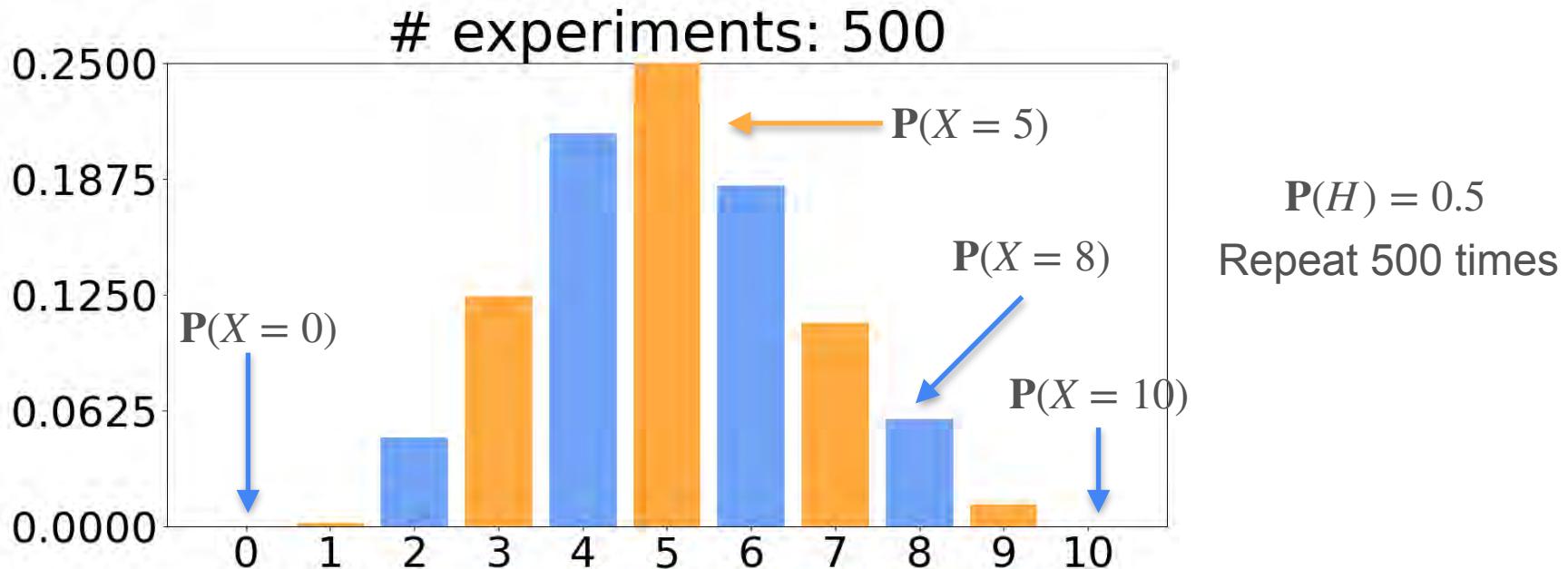
Flipping a Fair Coin 500 Times



Flipping a Fair Coin 500 Times



Flipping a Fair Coin 500 Times



Why Random Variables?

- Random variables allow you to model the whole experiment at once

Why Random Variables?

- Random variables allow you to model the whole experiment at once



X = Number of heads

Why Random Variables?

- Random variables allow you to model the whole experiment at once



X = Number of heads



X = Number of 1's

Why Random Variables?

- Random variables allow you to model the whole experiment at once



X = Number of heads



X = Number of 1's



X = Number of sick patients

Why Random Variables?

- Random variables allow you to model the whole experiment at once



X = Number of heads



X = Number of 1's



X = Number of sick patients

?

Why Random Variables?

- Random variables allow you to model the whole experiment at once



X = Number of heads



X = Number of 1's



X = Number of sick patients

?

$$\mathbf{P}(X = 1) = 0.5$$

Why Random Variables?

- Random variables allow you to model the whole experiment at once



X = Number of heads



X = Number of 1's



X = Number of sick patients



$$P(X = 1) = 0.5$$

$$P(X = -7) = 0.2$$

Why Random Variables?

- Random variables allow you to model the whole experiment at once



X = Number of heads



X = Number of 1's



X = Number of sick patients



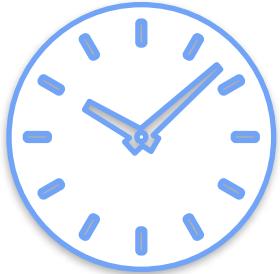
$$\mathbf{P}(X = 1) = 0.5$$

$$\mathbf{P}(X = -7) = 0.2$$

$$\mathbf{P}(X = 3.14159) = 0.3$$

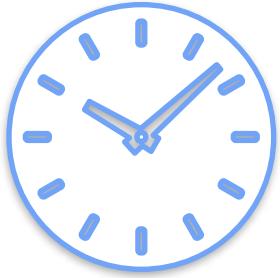
Other Random Variables

Other Random Variables

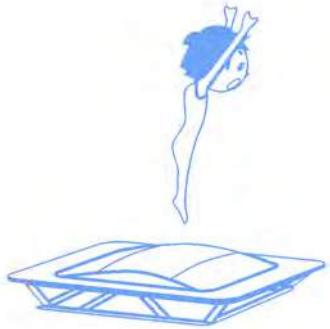


Wait time until the
next bus arrives

Other Random Variables

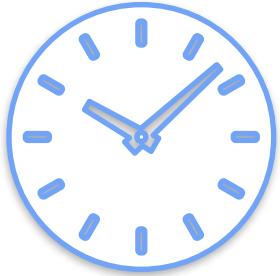


Wait time until the
next bus arrives

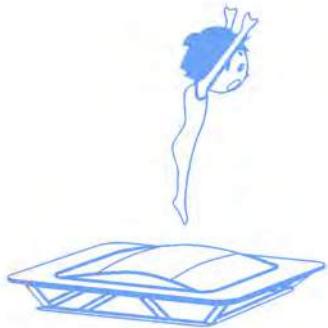


Height of an
gymnast's jump

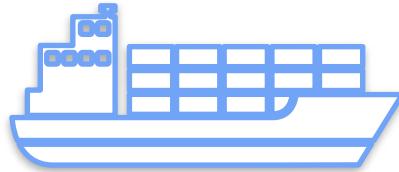
Other Random Variables



Wait time until the
next bus arrives

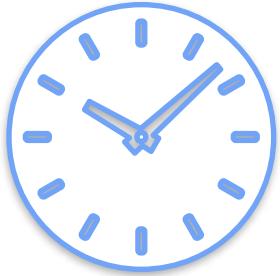


Height of an
gymnast's jump

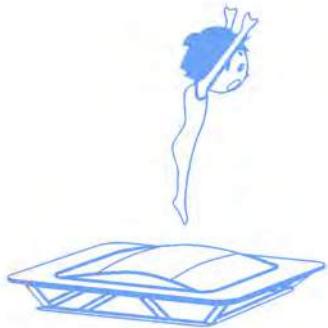


Number of
defective products
in a shipment

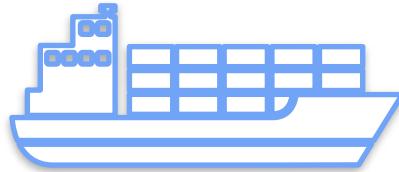
Other Random Variables



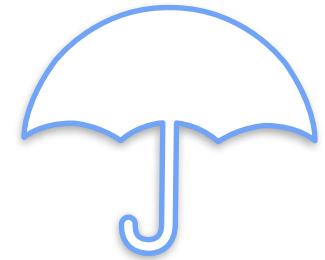
Wait time until the next bus arrives



Height of an gymnast's jump



Number of defective products in a shipment



mm. of rain in November

Discrete and Continuous Random Variables

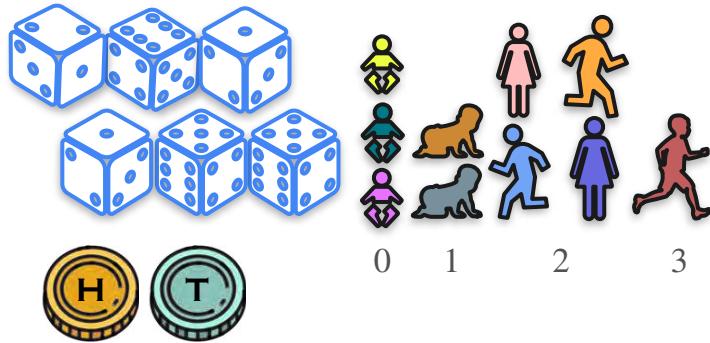
Discrete and Continuous Random Variables

Discrete random variables

Continuous random variables

Discrete and Continuous Random Variables

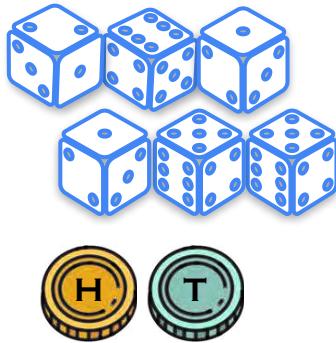
Discrete random variables



Continuous random variables

Discrete and Continuous Random Variables

Discrete random variables

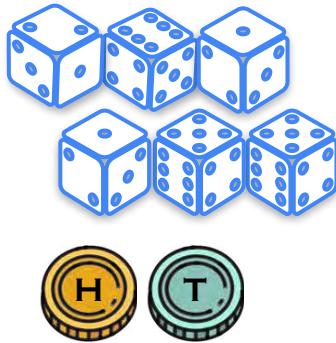


Continuous random variables



Discrete and Continuous Random Variables

Discrete random variables



0 1 2 3

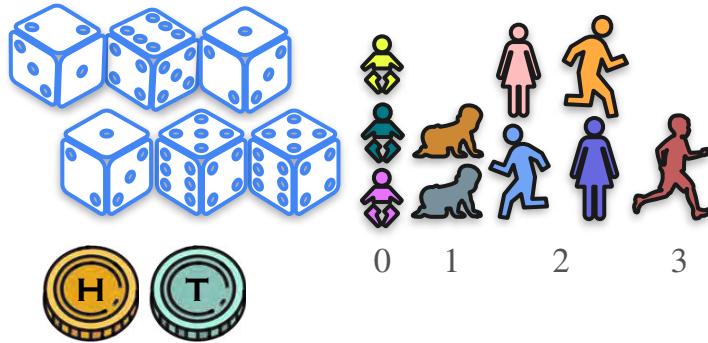
Continuous random variables



Finite number of values

Discrete and Continuous Random Variables

Discrete random variables



Finite number of values

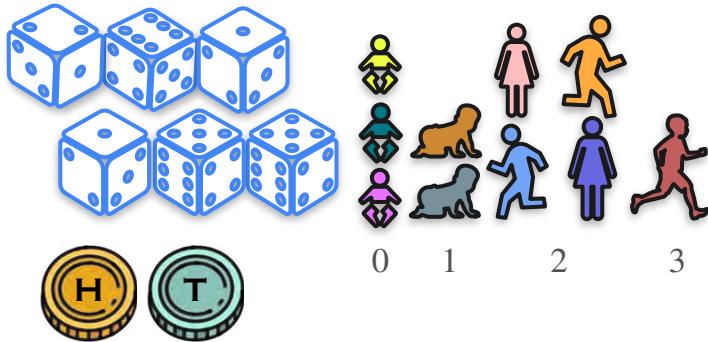
Continuous random variables



Infinite number of values

Discrete and Continuous Random Variables

Discrete random variables



~~Finite number of values~~

(Could be infinite too)

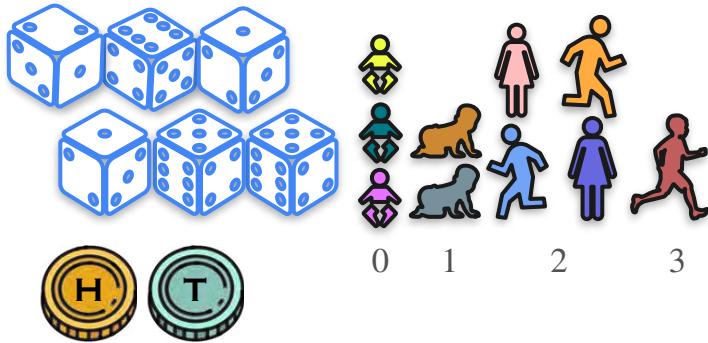
Continuous random variables



Infinite number of values

Discrete and Continuous Random Variables

Discrete random variables



~~Finite number of values~~

(Could be infinite too)

Can take only a **countable**
number of values

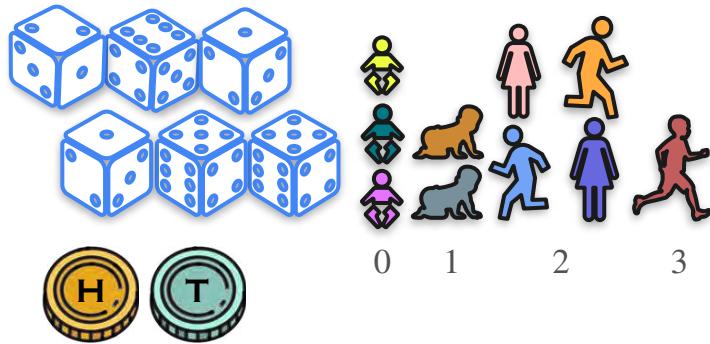
Continuous random variables



Infinite number of values

Discrete and Continuous Random Variables

Discrete random variables



~~Finite number of values~~

(Could be infinite too)

Can take only a **countable** number of values

Continuous random variables



Infinite number of values

Takes values on an interval

Random Variable Vs. Deterministic Variable

Random Variable Vs. Deterministic Variable

Deterministic

Random

Random Variable Vs. Deterministic Variable

Deterministic

$$x = 2, f(x) = x^2$$

Random

Random Variable Vs. Deterministic Variable

Deterministic

$$x = 2, f(x) = x^2$$

Random

X = number of defective items in
a shipment

Random Variable Vs. Deterministic Variable

Deterministic

$$x = 2, f(x) = x^2$$

Fixed outcome

Random

X = number of defective items in
a shipment

Random Variable Vs. Deterministic Variable

Deterministic

$$x = 2, f(x) = x^2$$

Fixed outcome

Random

X = number of defective items in
a shipment

Uncertain outcome



DeepLearning.AI

Probability Distributions

Probability Distributions (Discrete)

Discrete Distributions: Flip Three Coins

Discrete Distributions: Flip Three Coins

X_1 : number of heads in 3
coin tosses

Discrete Distributions: Flip Three Coins

X_1 : number of heads in 3 coin tosses



Coin¹ Coin² Coin³

All three tails ($X = 0$)

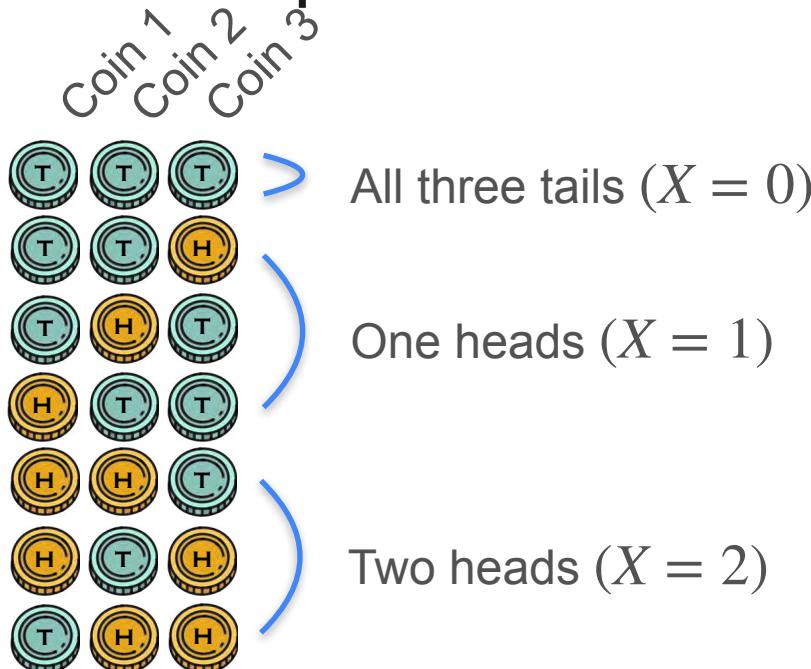
Discrete Distributions: Flip Three Coins

X_1 : number of heads in 3 coin tosses



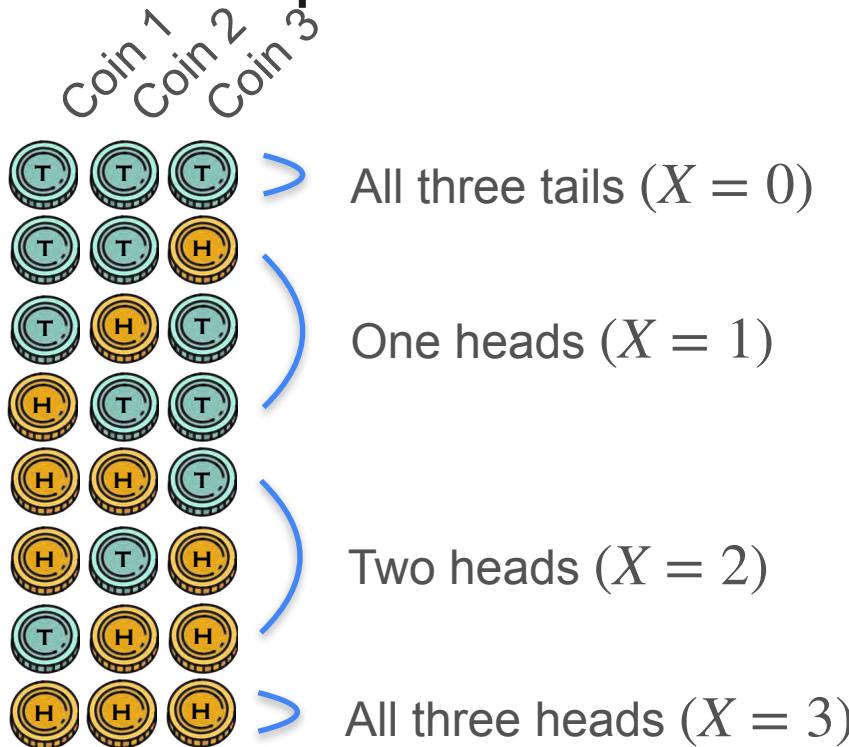
Discrete Distributions: Flip Three Coins

X_1 : number of heads in 3 coin tosses



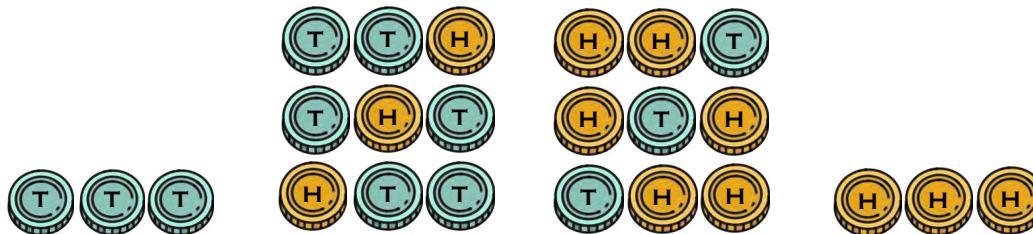
Discrete Distributions: Flip Three Coins

X_1 : number of heads in 3 coin tosses



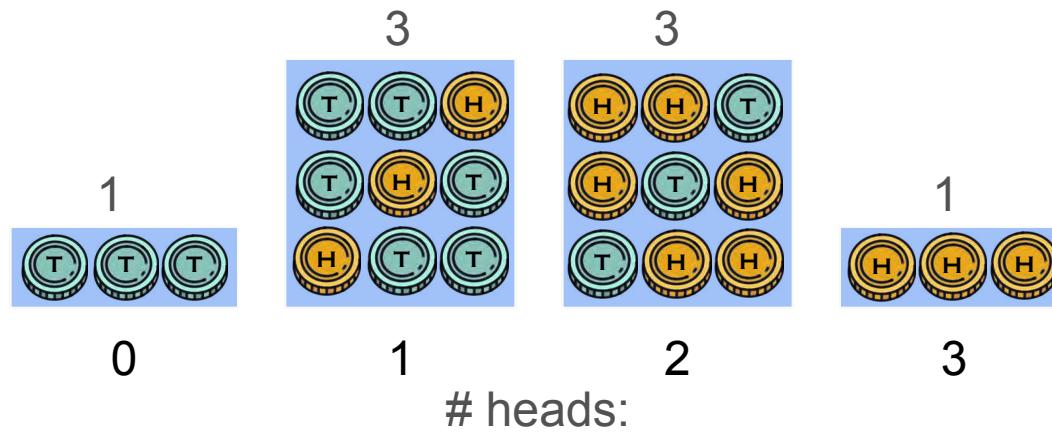
Discrete Distributions: Flip Three Coins

X_1 : number of heads in 3
coin tosses



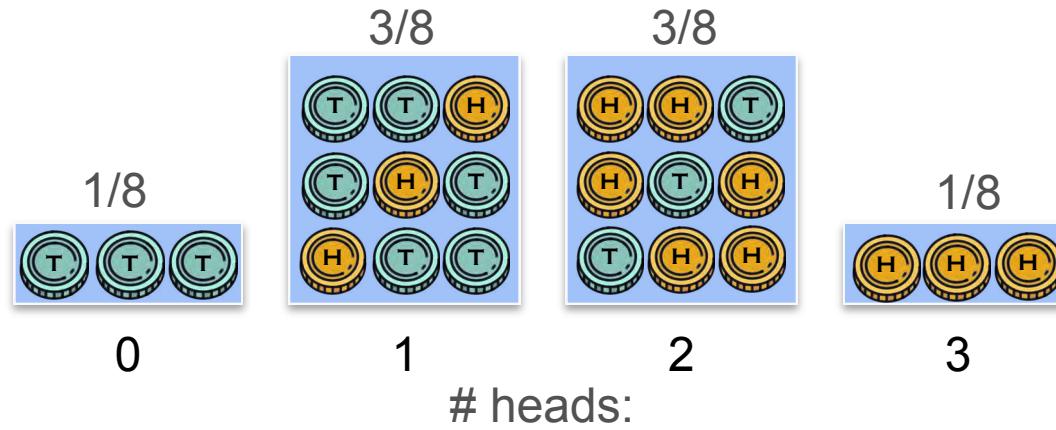
Discrete Distributions: Flip Three Coins

X_1 : number of heads in 3 coin tosses



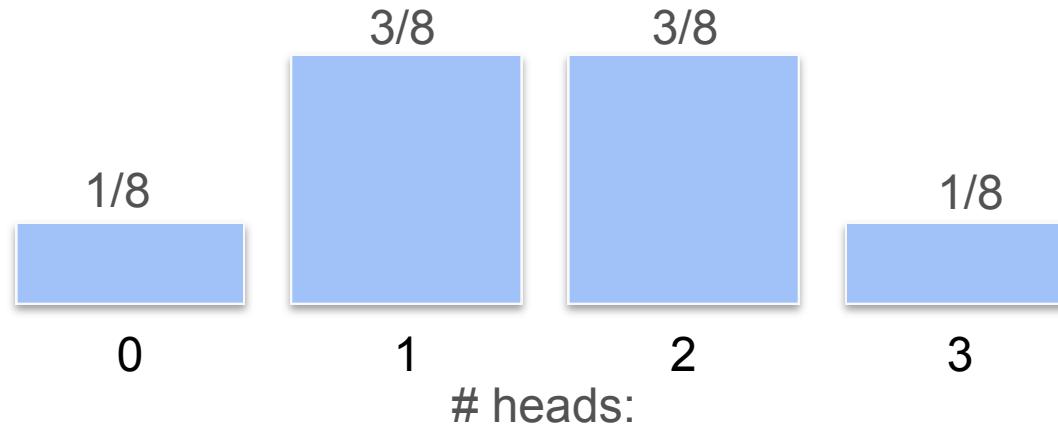
Discrete Distributions: Flip Three Coins

X_1 : number of heads in 3 coin tosses



Discrete Distributions: Flip Three Coins

X_1 : number of heads in 3 coin tosses



Discrete Distributions: Flip Four Coins

Discrete Distributions: Flip Four Coins

X_2 : number of heads in 4
coin tosses

Discrete Distributions: Flip Four Coins

X_2 : number of heads in 4
coin tosses

0 1 2 3 4
heads:

Discrete Distributions: Flip Four Coins

X_2 : number of heads in 4 coin tosses



0

1

2

3

4

heads:



Discrete Distributions: Flip Four Coins

X_2 : number of heads in 4 coin tosses

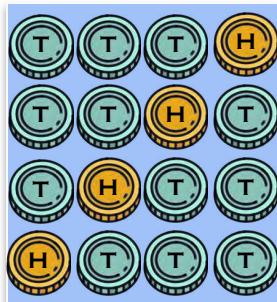


Discrete Distributions: Flip Four Coins

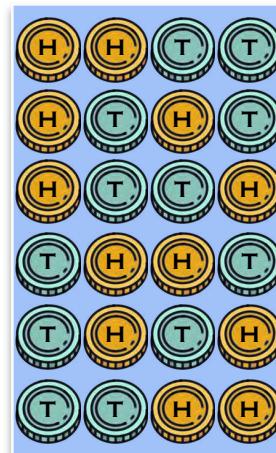
X_2 : number of heads in 4 coin tosses



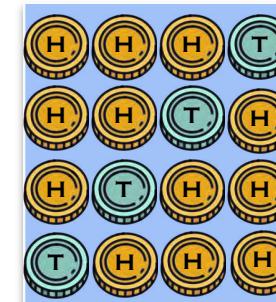
0



1



2



3



4

heads:

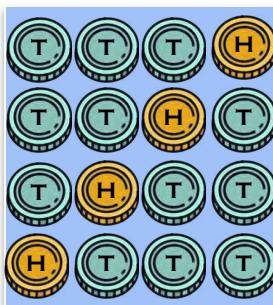
Discrete Distributions: Flip Four Coins

X_2 : number of heads in 4 coin tosses

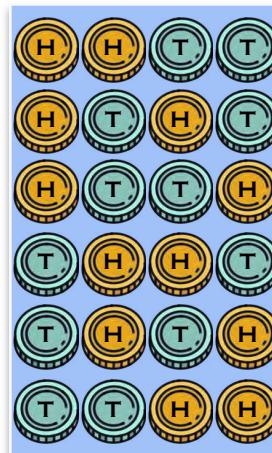


0

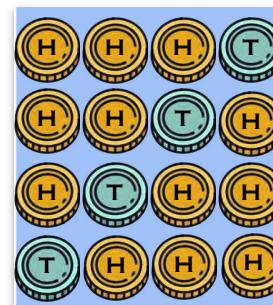
1/16



1



2



3

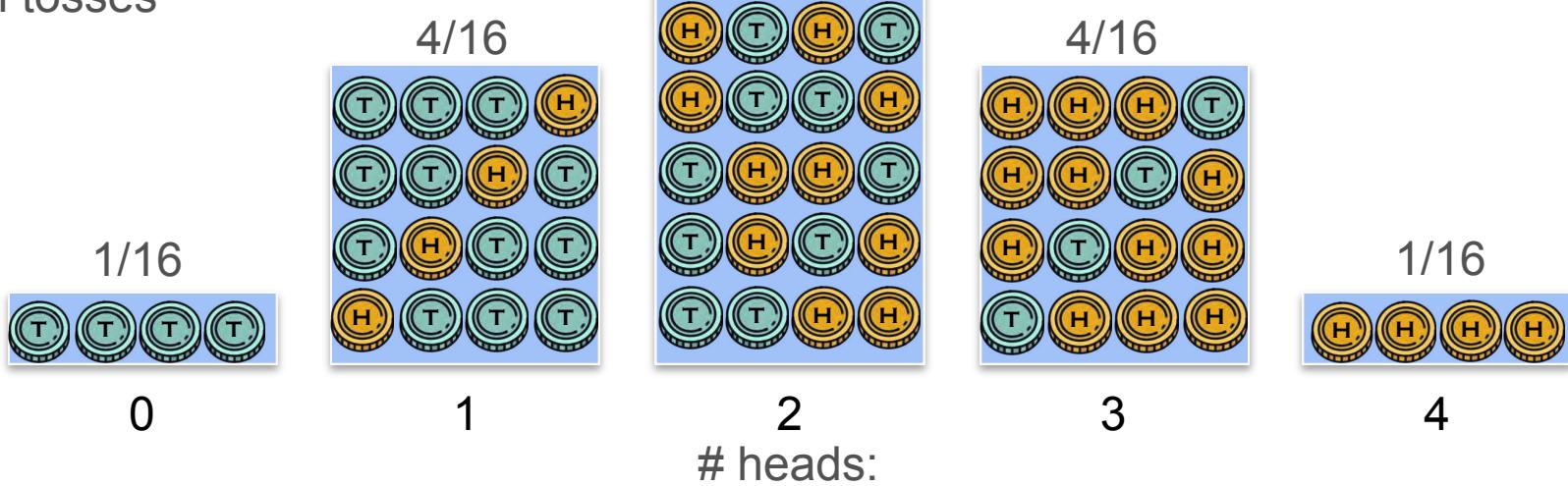


4

heads:

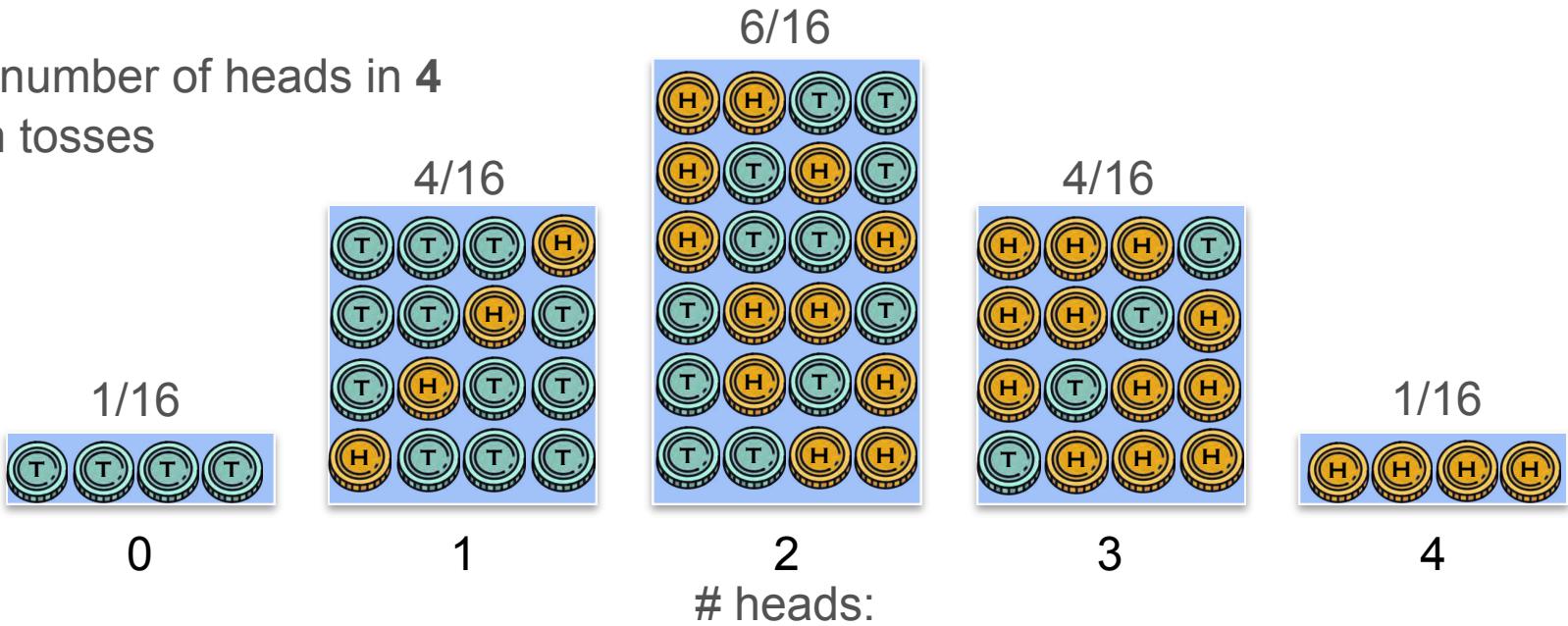
Discrete Distributions: Flip Four Coins

X_2 : number of heads in 4 coin tosses

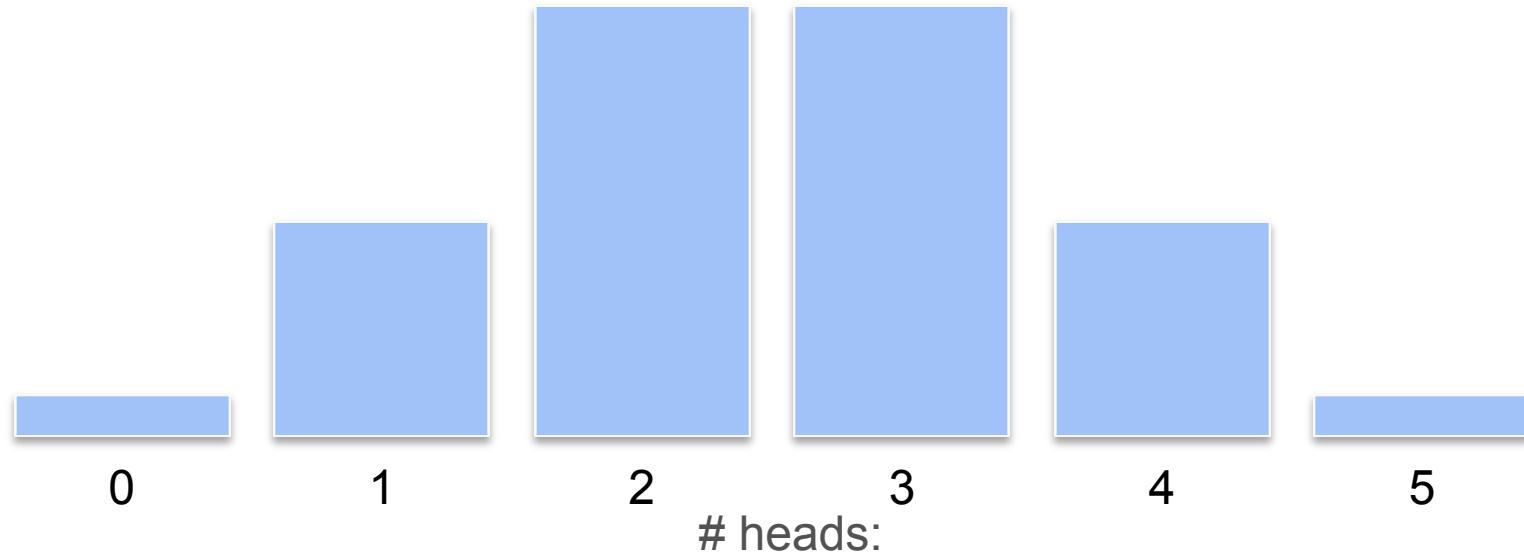


Discrete Distributions: Flip Four Coins

X_2 : number of heads in 4 coin tosses

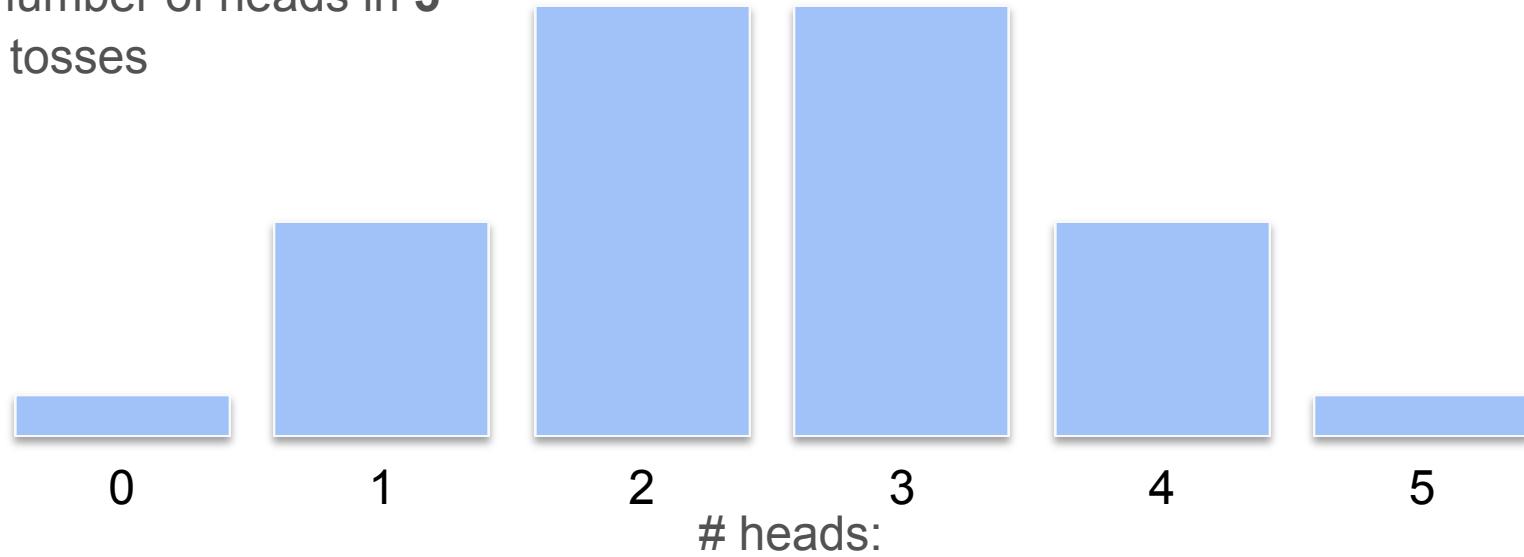


Discrete Distributions: Flip Five Coins



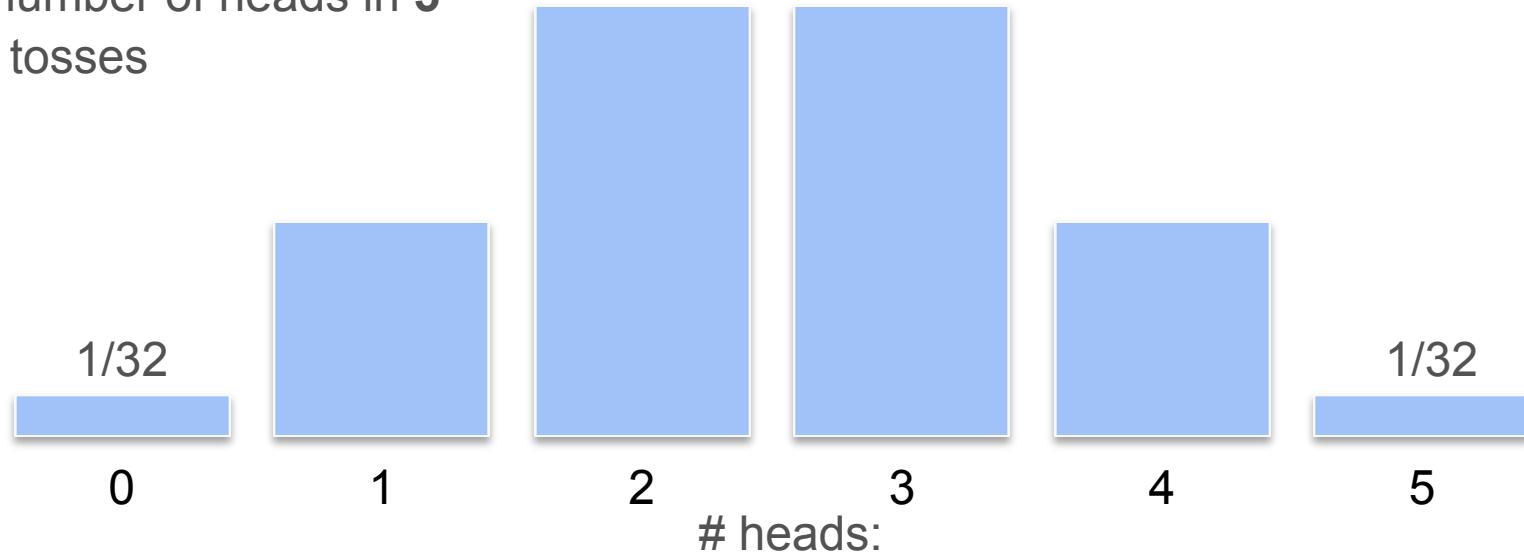
Discrete Distributions: Flip Five Coins

X_3 : number of heads in **5** coin tosses



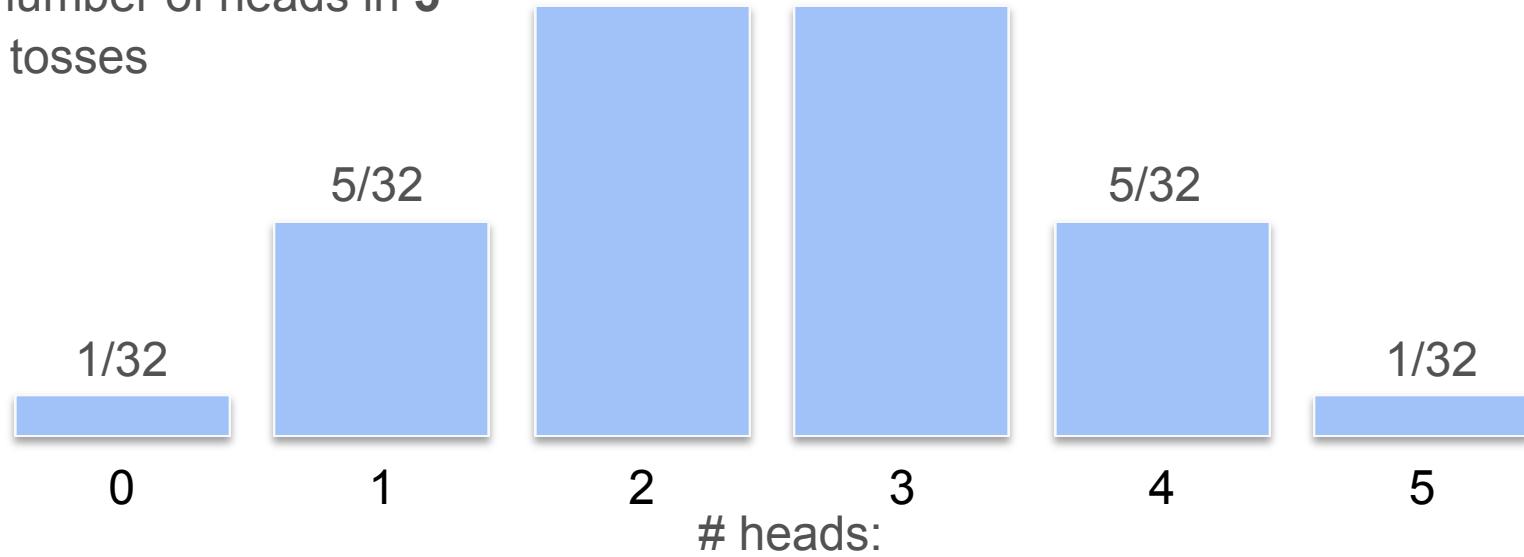
Discrete Distributions: Flip Five Coins

X_3 : number of heads in **5** coin tosses



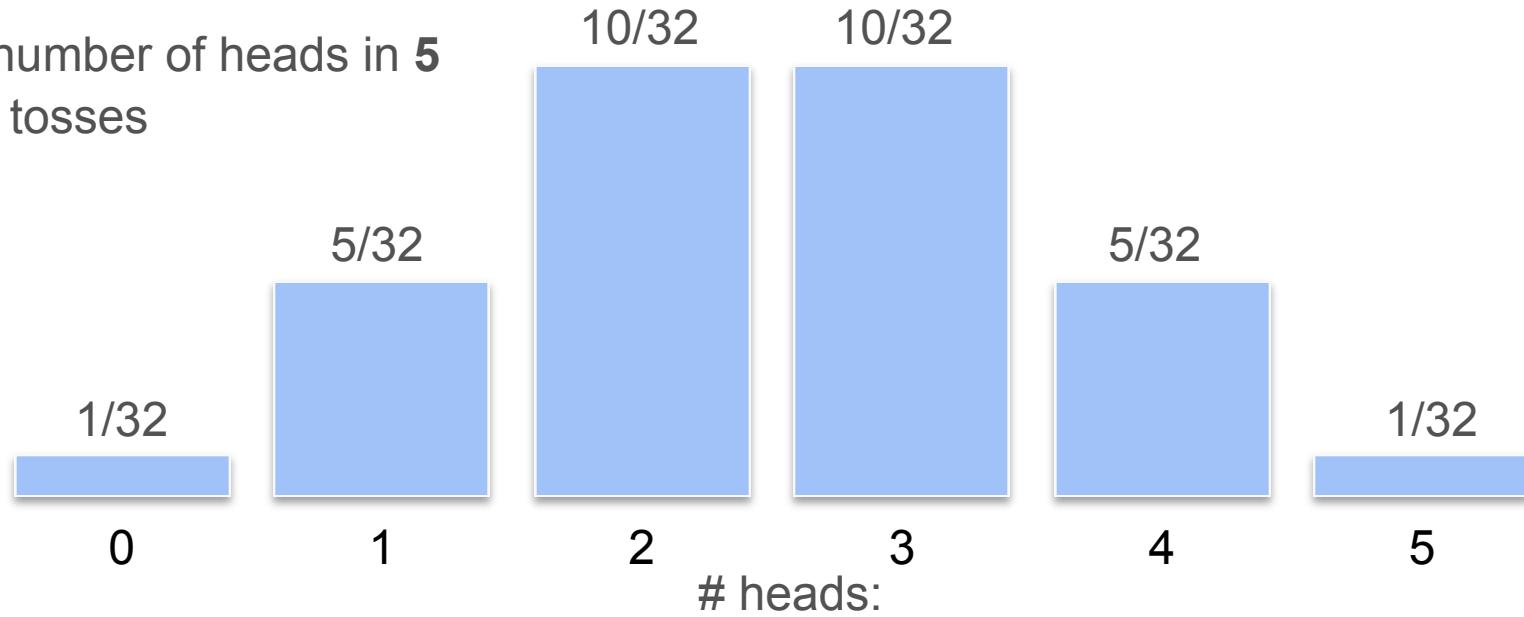
Discrete Distributions: Flip Five Coins

X_3 : number of heads in **5** coin tosses



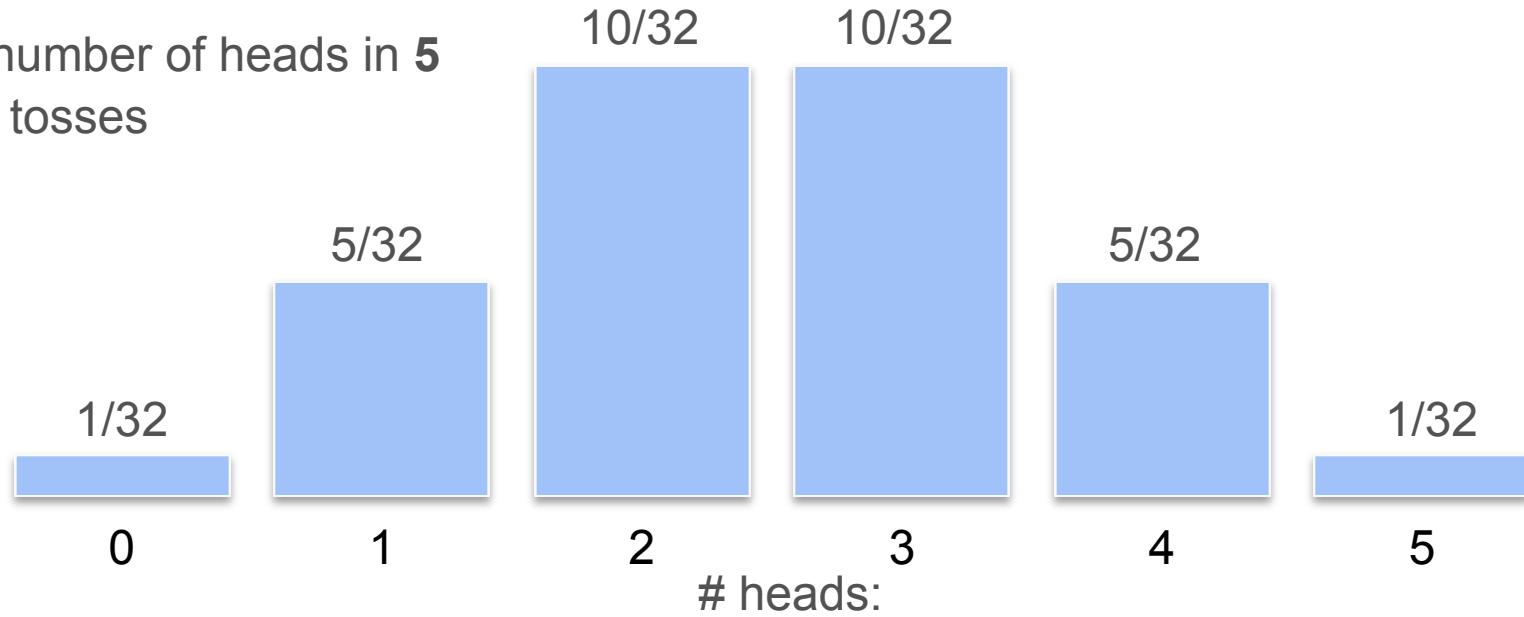
Discrete Distributions: Flip Five Coins

X_3 : number of heads in 5 coin tosses



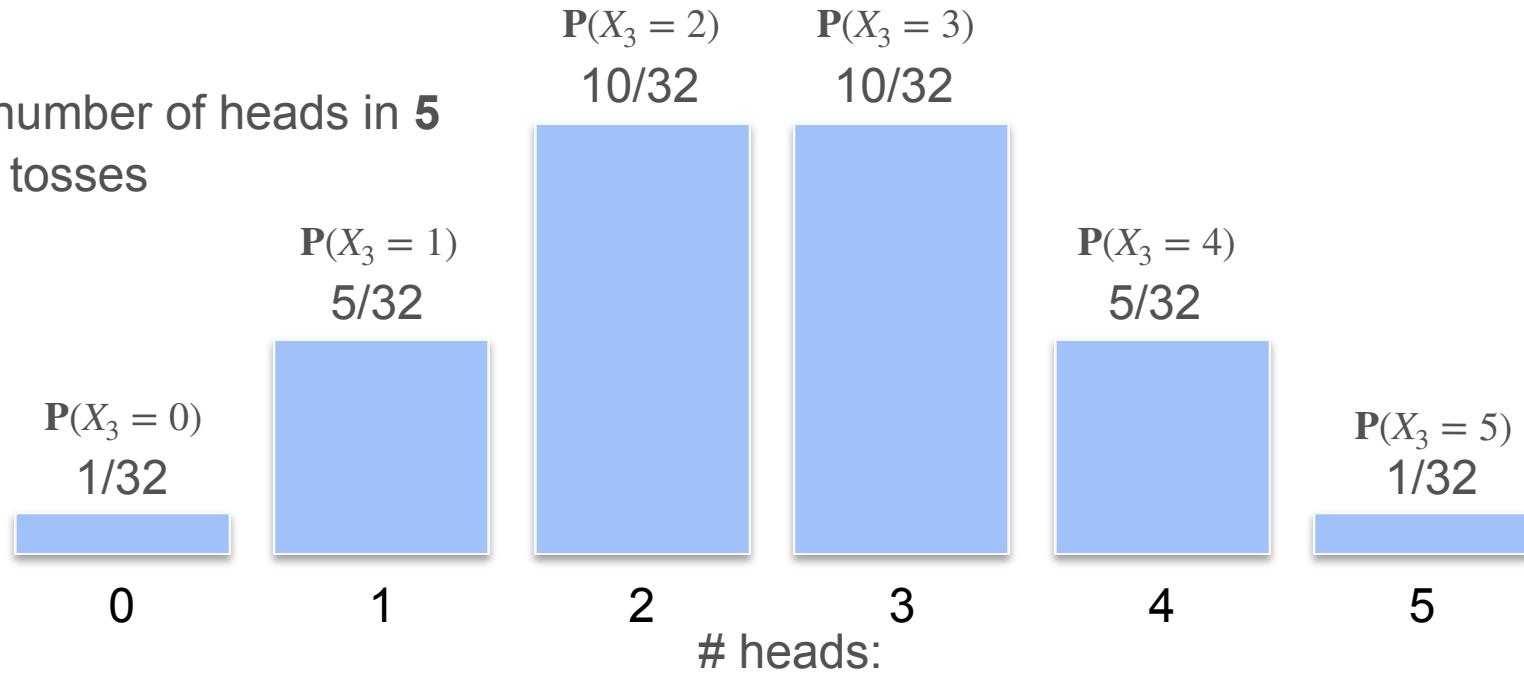
Discrete Distributions: Flip Five Coins

X_3 : number of heads in 5 coin tosses



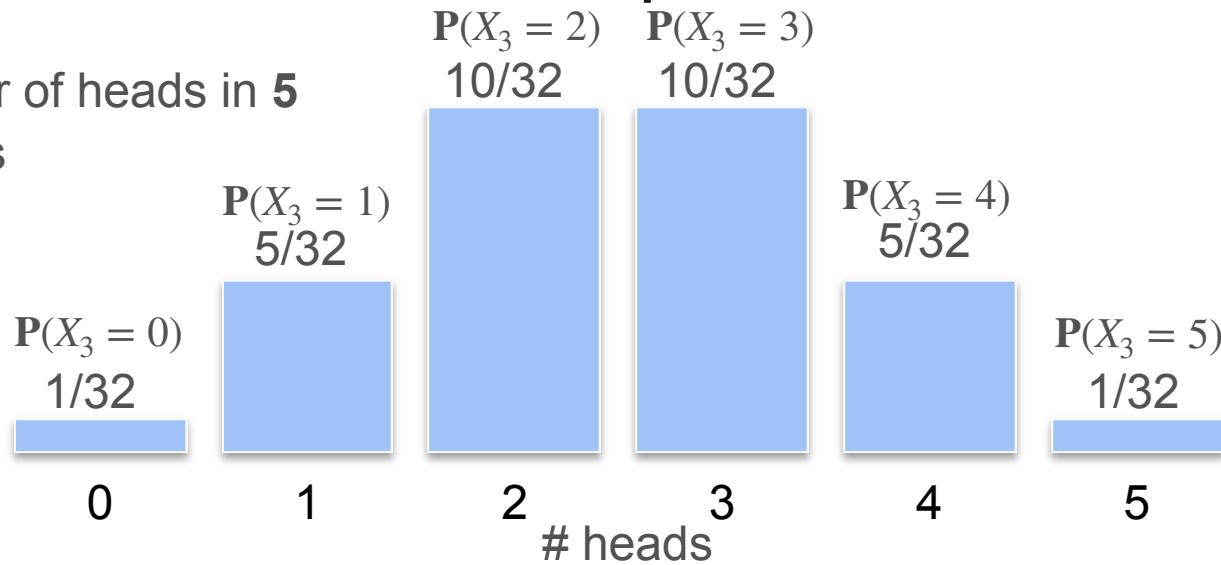
Discrete Distributions: Flip Five Coins

X_3 : number of heads in 5 coin tosses



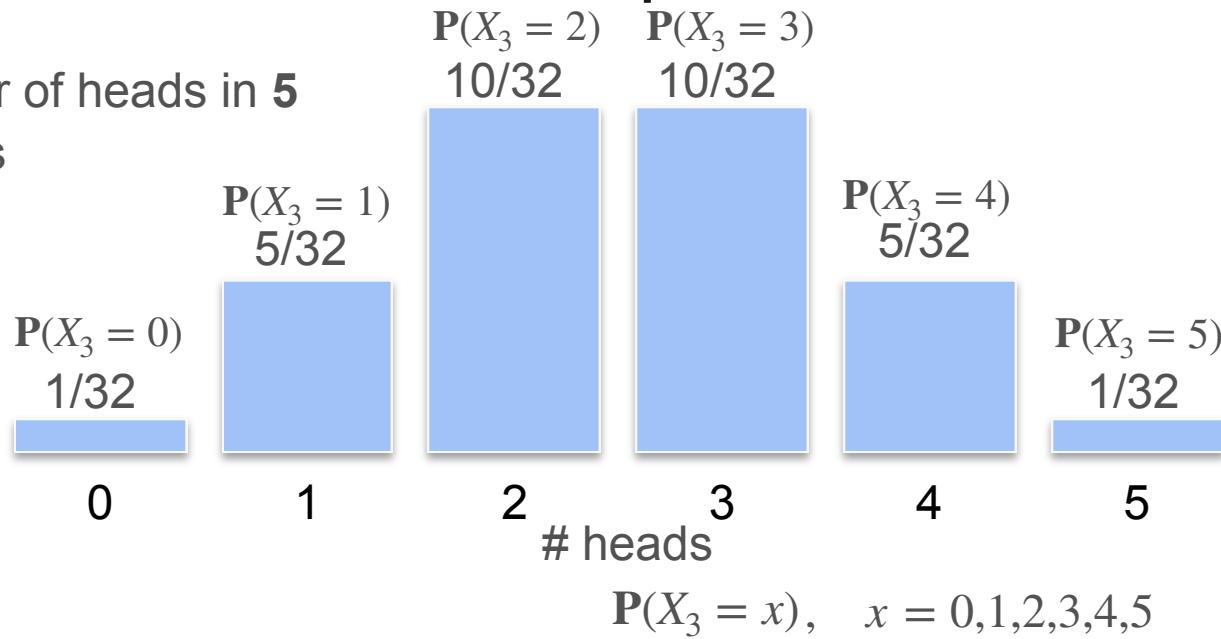
Discrete Distributions: Flip Five Coins

X_3 : number of heads in 5 coin tosses



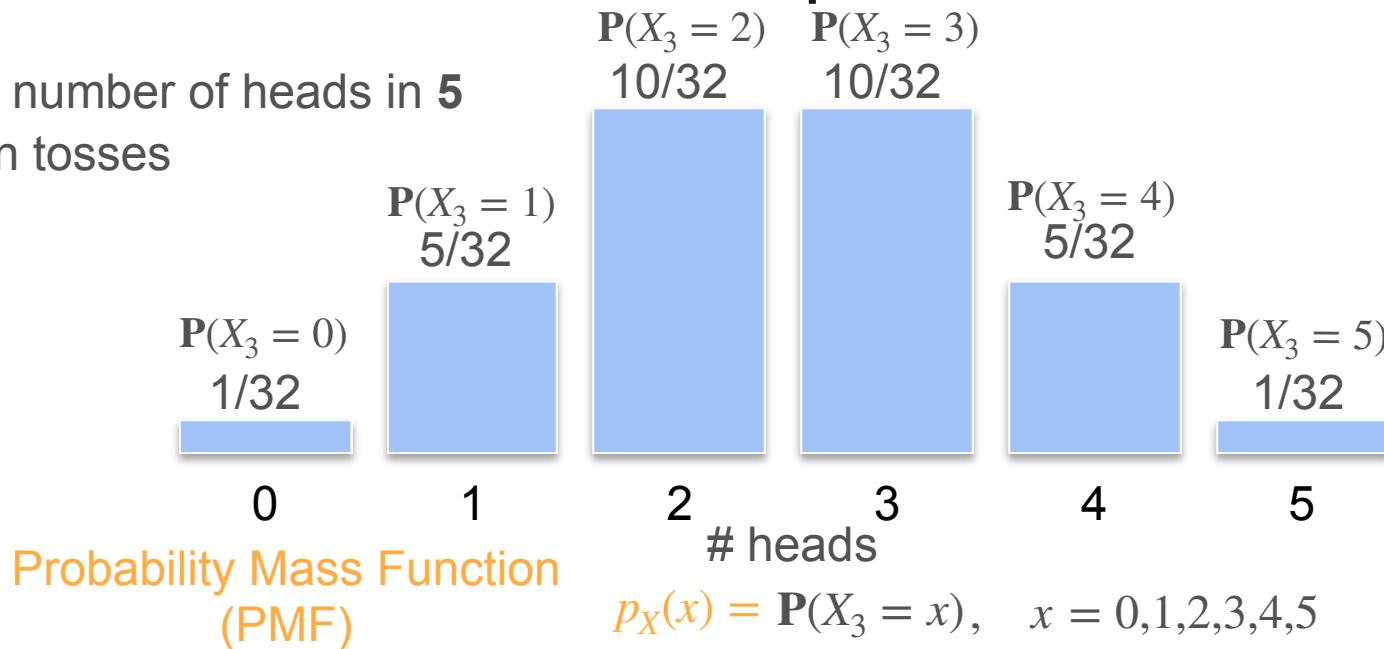
Discrete Distributions: Flip Five Coins

X_3 : number of heads in 5 coin tosses



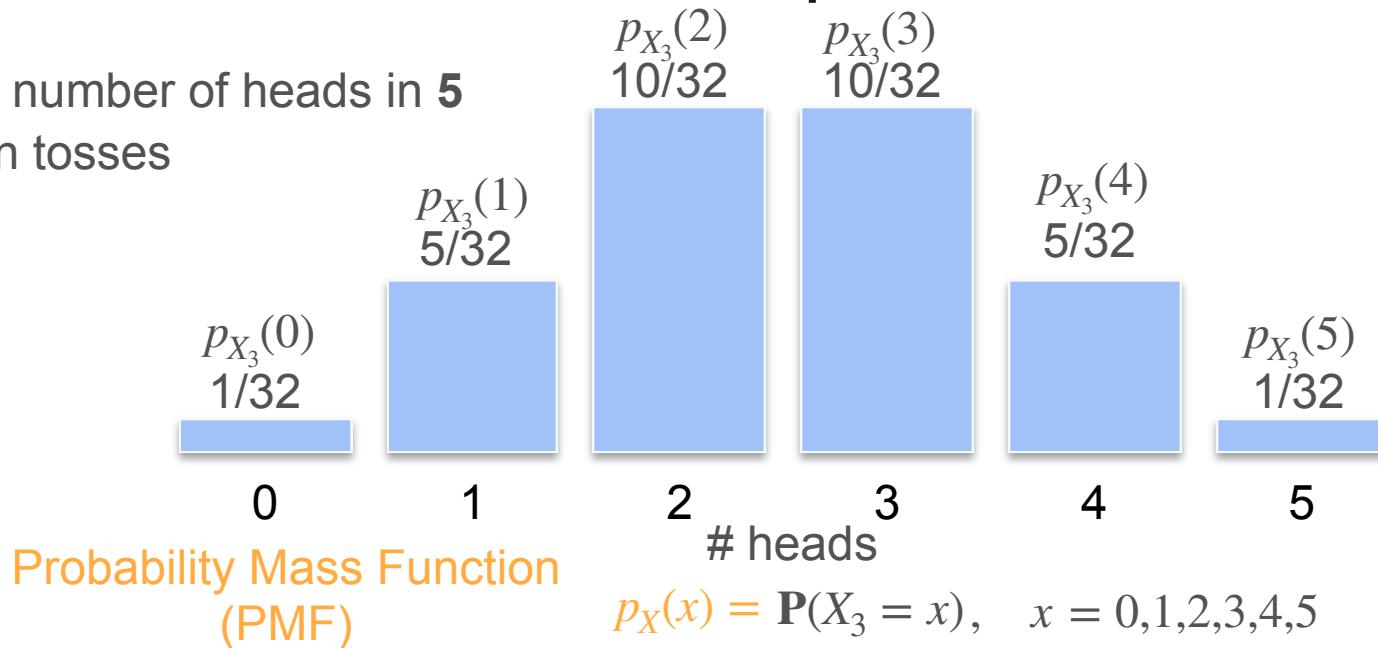
Discrete Distributions: Flip Five Coins

X_3 : number of heads in 5 coin tosses



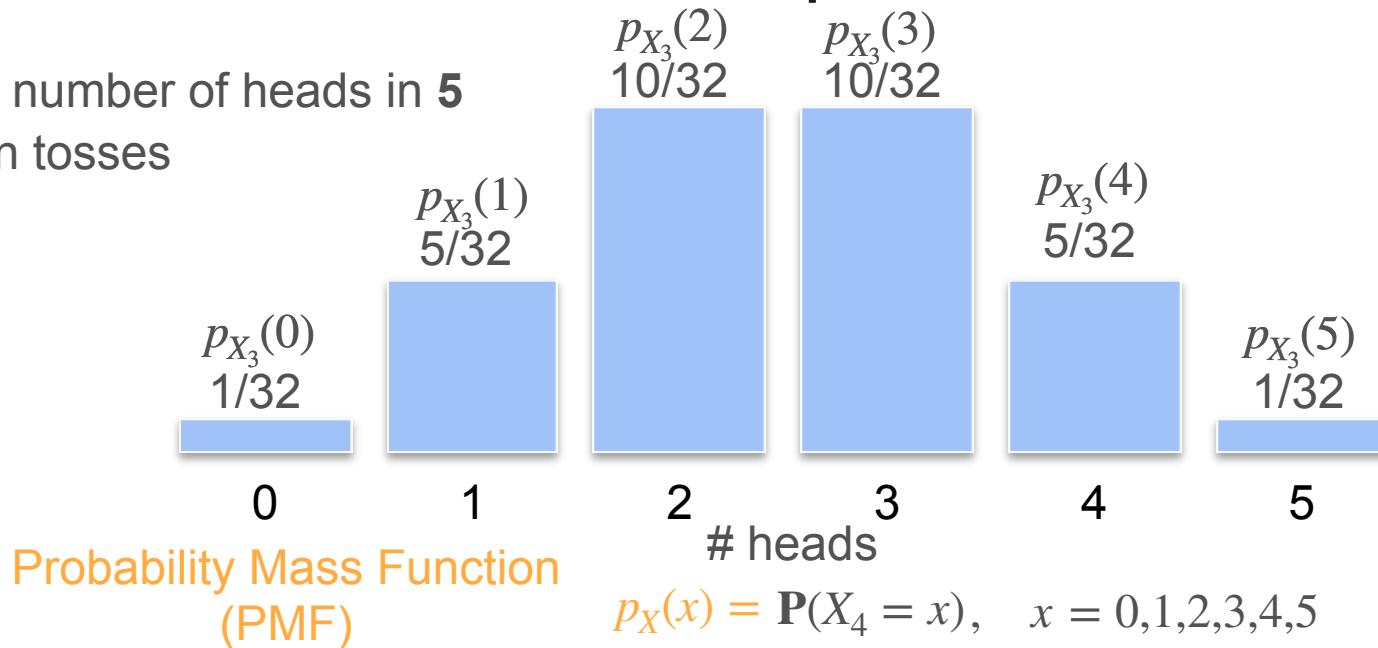
Discrete Distributions: Flip Five Coins

X_3 : number of heads in 5 coin tosses



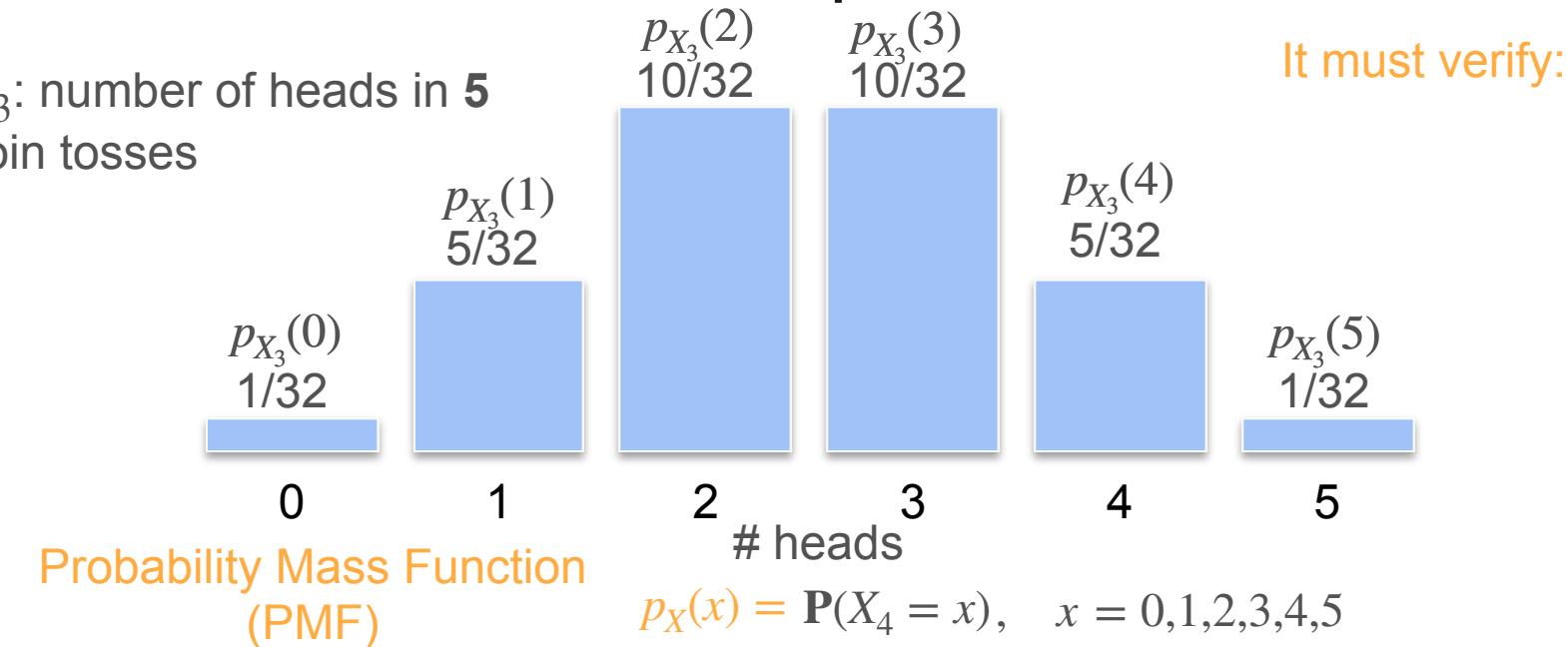
Discrete Distributions: Flip Five Coins

X_3 : number of heads in 5 coin tosses



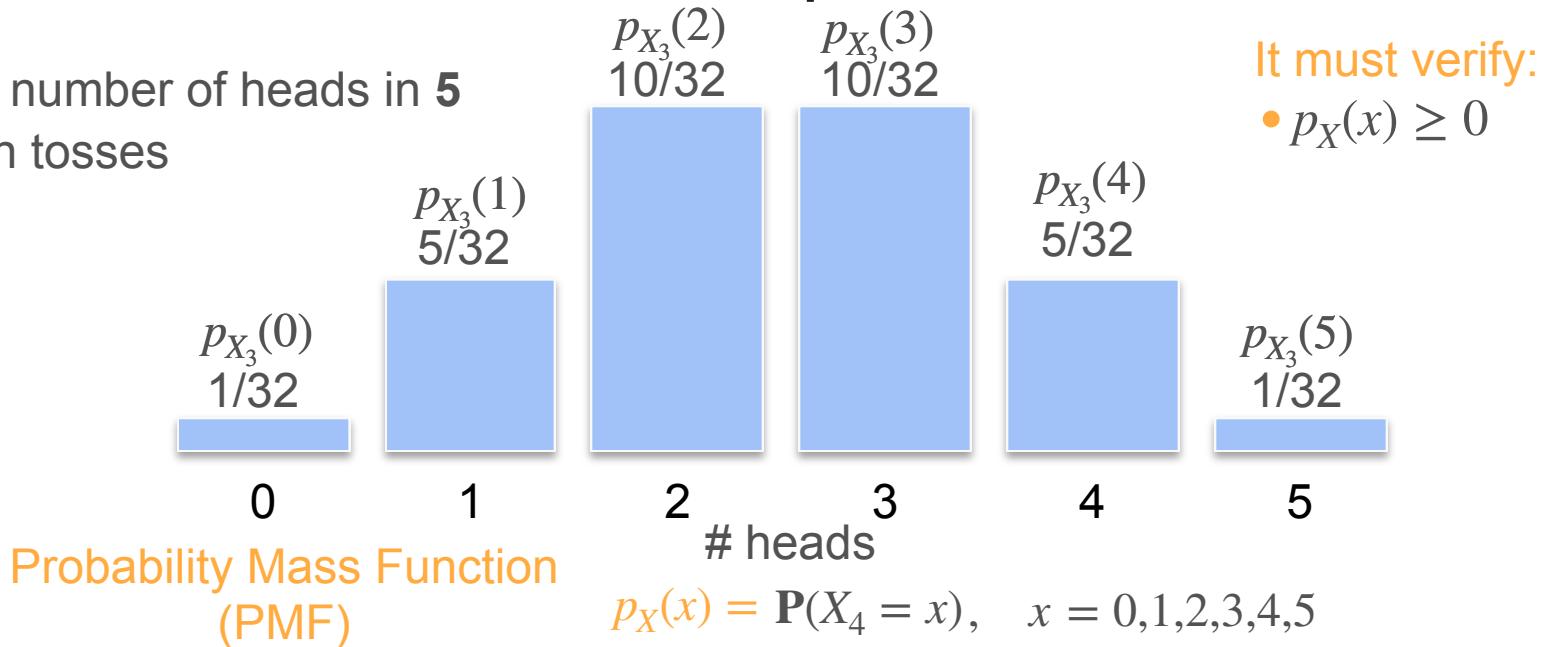
Discrete Distributions: Flip Five Coins

X_3 : number of heads in 5 coin tosses



Discrete Distributions: Flip Five Coins

X_3 : number of heads in 5 coin tosses

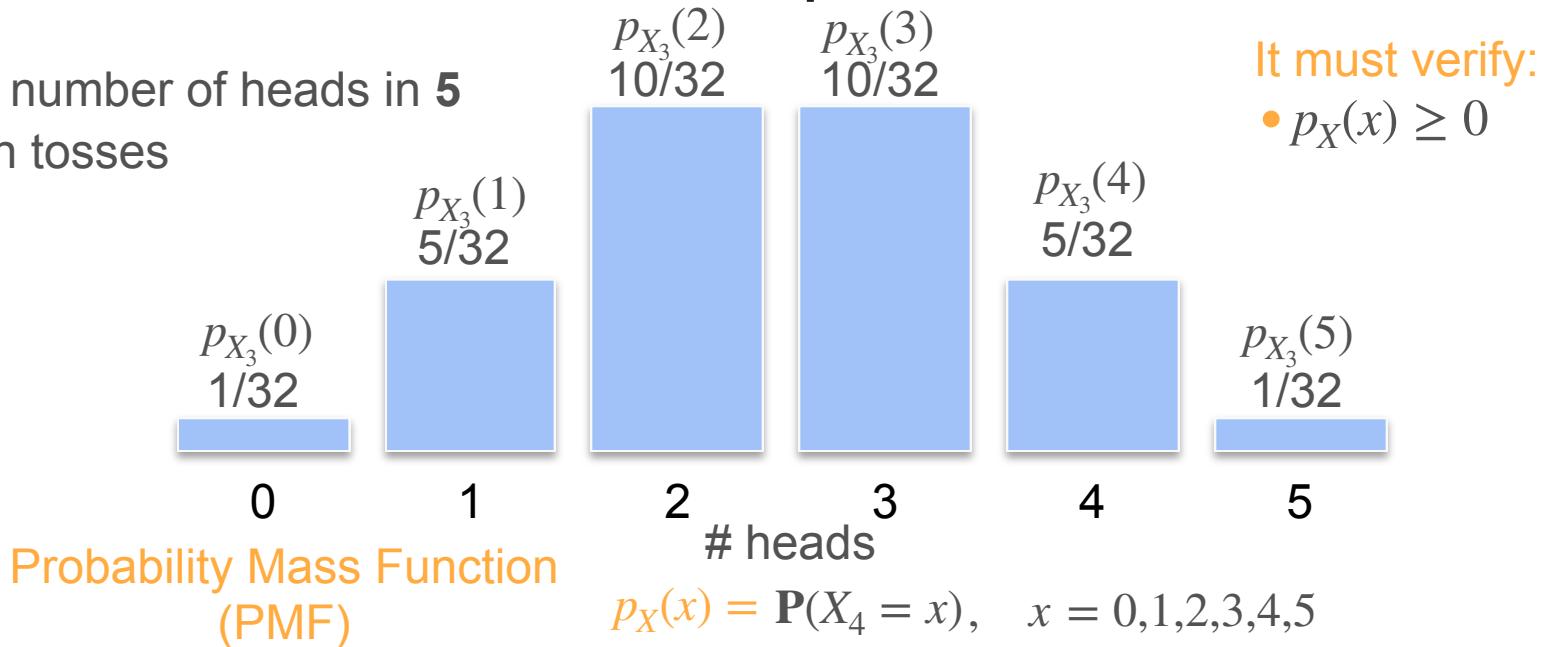


It must verify:

- $p_X(x) \geq 0$

Discrete Distributions: Flip Five Coins

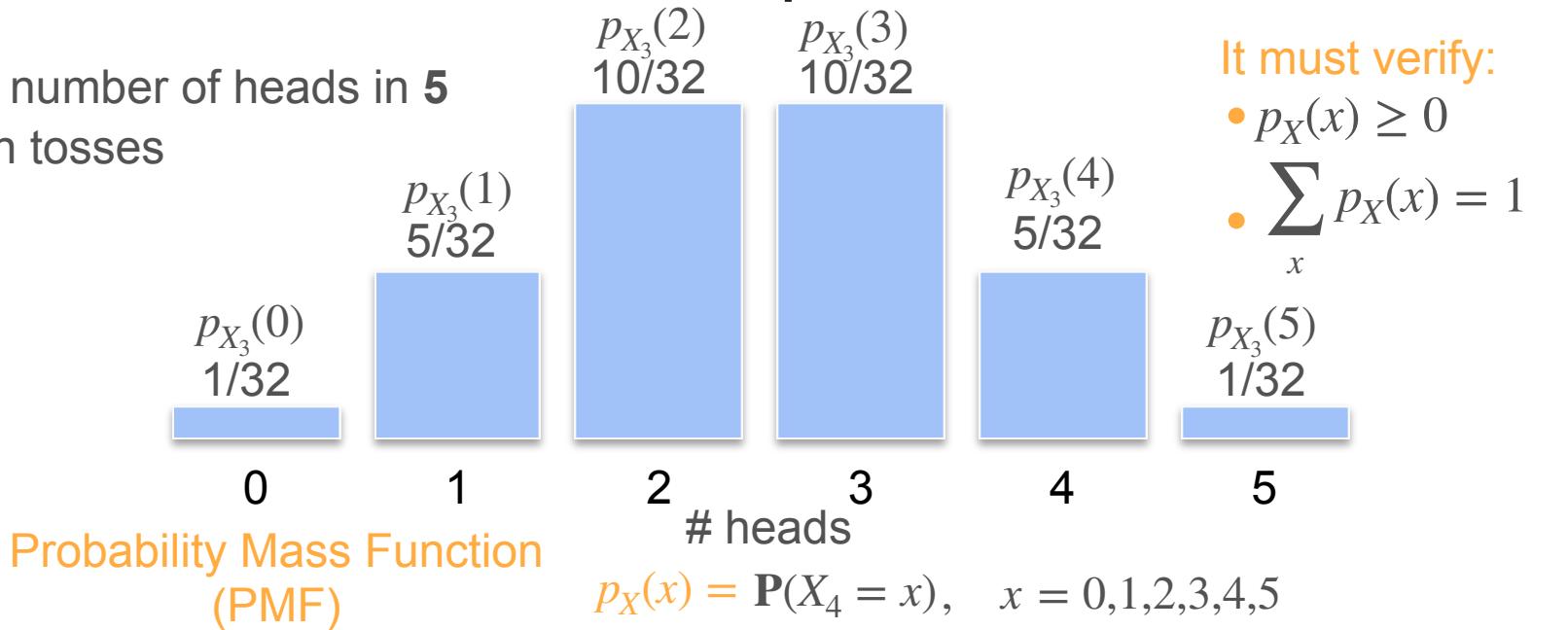
X_3 : number of heads in 5 coin tosses



$$p_{X_3}(0) + p_{X_3}(1) + p_{X_3}(2) + p_{X_3}(3) + p_{X_3}(4) + p_{X_3}(5) = \frac{1}{32} + \frac{5}{32} + \frac{10}{32} + \frac{10}{32} + \frac{5}{32} + \frac{1}{32} = 1$$

Discrete Distributions: Flip Five Coins

X_3 : number of heads in 5 coin tosses



$$p_{X_3}(0) + p_{X_3}(1) + p_{X_3}(2) + p_{X_3}(3) + p_{X_3}(4) + p_{X_3}(5) = \frac{1}{32} + \frac{5}{32} + \frac{10}{32} + \frac{10}{32} + \frac{5}{32} + \frac{1}{32} = 1$$

It must verify:

- $p_X(x) \geq 0$

- $\sum_x p_X(x) = 1$

Binomial Distribution

Binomial Distribution

X_1, X_2, X_3, X_4 are very similar

They all represent **number of heads in n experiments**

Binomial Distribution

X_1, X_2, X_3, X_4 are very similar

They all represent **number of heads in n experiments**

The way the probability distributes along the possible outcomes seems to have a similar pattern

Binomial Distribution

X_1, X_2, X_3, X_4 are very similar

They all represent **number of heads in n experiments**

The way the probability distributes along the possible outcomes seems to have a similar pattern

Could there be a **single model** to represent all this variables?



Binomial distribution



DeepLearning.AI

Probability Distributions

Binomial Distribution

Binomial Distribution: Example

Binomial Distribution: Example

What is the probability that if I flip 5 coins, 2 of them land in heads?

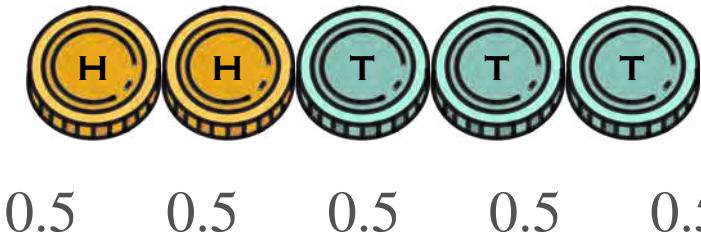
Binomial Distribution: Example

What is the probability that if I flip 5 coins, 2 of them land in heads?



Binomial Distribution: Example

What is the probability that if I flip 5 coins, 2 of them land in heads?



Binomial Distribution: Example

What is the probability that if I flip 5 coins, 2 of them land in heads?



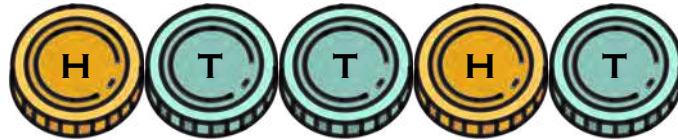
$$0.5 \cdot 0.5 \cdot 0.5 \cdot 0.5 \cdot 0.5 = \frac{1}{32}$$

Binomial Distribution: Example

What is the probability that if I flip 5 coins, 2 of them land in heads?



$$0.5 \cdot 0.5 \cdot 0.5 \cdot 0.5 \cdot 0.5 = \frac{1}{32}$$

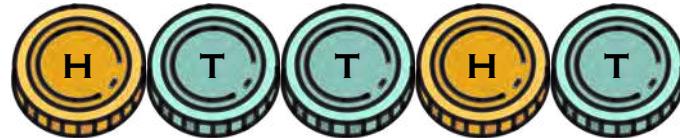


Binomial Distribution: Example

What is the probability that if I flip 5 coins, 2 of them land in heads?



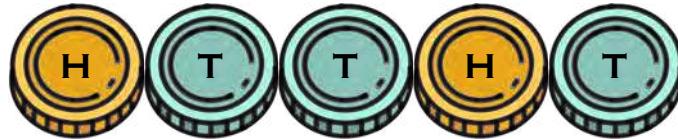
$$0.5 \cdot 0.5 \cdot 0.5 \cdot 0.5 \cdot 0.5 = \frac{1}{32}$$



$$0.5 \cdot 0.5 \cdot 0.5 \cdot 0.5 \cdot 0.5 = \frac{1}{32}$$

Binomial Distribution: Example

What is the probability that if I flip 5 coins, 2 of them land in heads?



Binomial Distribution: Example

What is the probability that if I flip 5 coins, 2 of them land in heads?



10 ways to have 2 heads in 5 coin tosses



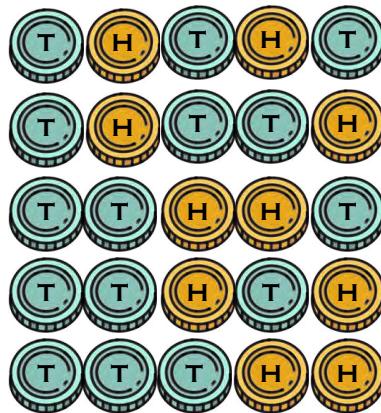
Binomial Distribution: Example

What is the probability that if I flip 5 coins, 2 of them land in heads?



Binomial Distribution: Example

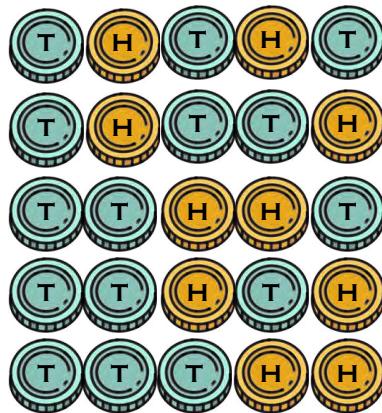
What is the probability that if I flip 5 coins, 2 of them land in heads?



$$10 = \underline{\hspace{2cm}}$$

Binomial Distribution: Example

What is the probability that if I flip 5 coins, 2 of them land in heads?



$$10 = \frac{5!}{\text{Number of ways you can order 5 coins}}$$

Binomial Distribution: Example

What is the probability that if I flip 5 coins, 2 of them land in heads?



$$10 = \frac{5!}{2!}$$

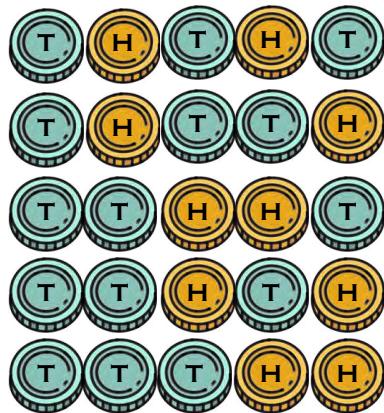
Number of ways you can order 5 coins

Number of H



Binomial Distribution: Example

What is the probability that if I flip 5 coins, 2 of them land in heads?



$$10 = \frac{5!}{2!}$$

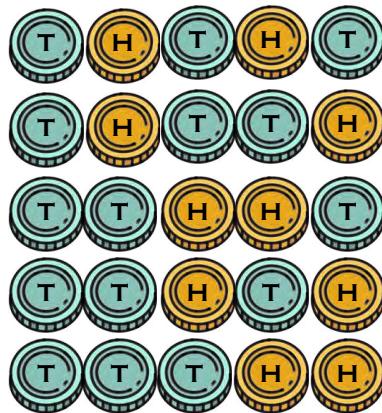
Number of ways you can order 5 coins

Number of H

The equation $10 = \frac{5!}{2!}$ is shown. A blue arrow points from the text "Number of ways you can order 5 coins" to the 5! term in the numerator. Another blue arrow points from the text "Number of H" to the 2! term in the denominator.

Binomial Distribution: Example

What is the probability that if I flip 5 coins, 2 of them land in heads?



$$10 = \frac{5!}{2!(5-2)!}$$

Number of ways you can order 5 coins

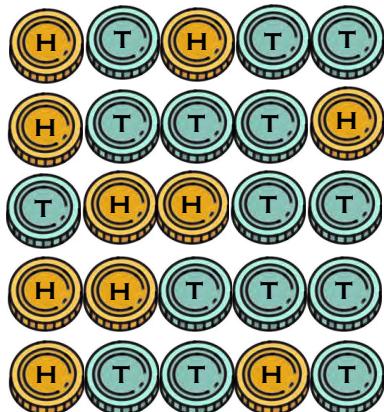
Number of H Number of T

The equation $10 = \frac{5!}{2!(5-2)!}$ represents the binomial coefficient for choosing 2 heads (H) out of 5 coins. The term $5!$ is labeled as "Number of ways you can order 5 coins". The term $2!(5-2)!$ is split into "Number of H" and "Number of T".



Binomial Distribution: Example

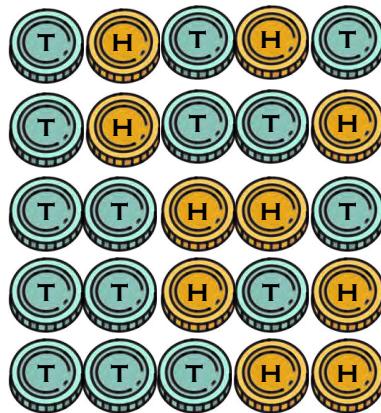
What is the probability that if I flip 5 coins, 2 of them land in heads?



$$10 = \frac{5!}{2!(5-2)!}$$

Binomial Distribution: Example

What is the probability that if I flip 5 coins, 2 of them land in heads?



$$10 = \frac{5!}{2!(5-2)!} = \binom{5}{2}$$

Binomial coefficient

Number of ways you can get 2 heads in 5 coin tosses

Binomial Distribution: Binomial Coefficient

Binomial Distribution: Binomial Coefficient

In general:

$\binom{n}{k}$ counts all the combinations for landing k heads in n coin tosses

Binomial Distribution: Binomial Coefficient

In general:

$\binom{n}{k}$ counts all the combinations for landing k heads in n coin tosses

Property:

$$\binom{n}{k} = \binom{n}{n - k}$$

Binomial Distribution: Binomial Coefficient

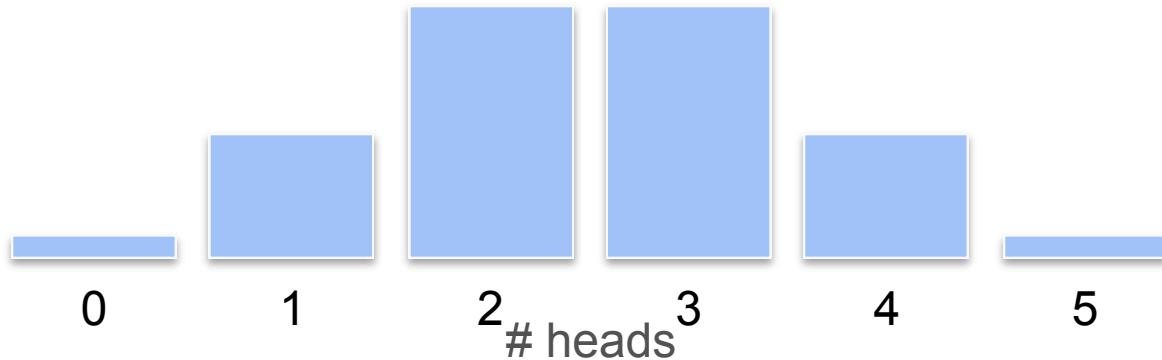
In general:

$\binom{n}{k}$ counts all the combinations for landing k heads in n coin tosses

Property:

The PMF with a fair coin is symmetrical

$$\binom{n}{k} = \binom{n}{n-k}$$



Binomial Distribution: Binomial Coefficient

Binomial Distribution: Binomial Coefficient

General PMF for X : number of heads in 5 coin tosses?

Binomial Distribution: Binomial Coefficient

General PMF for X : number of heads in 5 coin tosses?

Your coin has $\mathbf{P}(H) = p$

Binomial Distribution: Binomial Coefficient

General PMF for X : number of heads in 5 coin tosses?

Your coin has $\mathbf{P}(H) = p$

Event: $X = x$: x heads in 5 tosses

Binomial Distribution: Binomial Coefficient

General PMF for X : number of heads in 5 coin tosses?

Your coin has $\mathbf{P}(H) = p$

Event: $X = x$: x heads in 5 tosses

$$p^x$$

↑
Probability of seeing x heads

Binomial Distribution: Binomial Coefficient

General PMF for X : number of heads in 5 coin tosses?

Your coin has $\mathbf{P}(H) = p$

Event: $X = x$: x heads in 5 tosses

$$p^x (1 - p)^{5-x}$$

Probability of seeing x heads

Probability of seeing $5 - x$ tails

The diagram illustrates the binomial probability formula $p^x (1 - p)^{5-x}$. A blue box encloses the terms p^x and $(1 - p)^{5-x}$. An arrow points from the text "Probability of seeing x heads" to the term p^x . Another arrow points from the text "Probability of seeing $5 - x$ tails" to the term $(1 - p)^{5-x}$.

Binomial Distribution: Binomial Coefficient

General PMF for X : number of heads in 5 coin tosses?

Your coin has $\mathbf{P}(H) = p$

Event: $X = x$: x heads in 5 tosses

$$\binom{5}{x} p^x (1 - p)^{5-x}$$

All the possible orders → $\binom{5}{x}$

Probability of seeing x heads ↑ p^x

Probability of seeing $5 - x$ tails $(1 - p)^{5-x}$

Binomial Distribution

General PMF for X : number of heads in 5 coin tosses?

Your coin has $\mathbf{P}(H) = p$

Event: $X = x$: x heads in 5 tosses

$$\binom{5}{x} p^x (1 - p)^{5-x}$$

Binomial Distribution

General PMF for X : number of heads in 5 coin tosses?

Your coin has $\mathbf{P}(H) = p$

Event: $X = x$: x heads in 5 tosses

$$\binom{5}{x} p^x (1-p)^{5-x}, \quad x = 0,1,2,3,4,5$$

Binomial Distribution

General PMF for X : number of heads in 5 coin tosses?

Your coin has $\mathbf{P}(H) = p$

Event: $X = x$: x heads in 5 tosses

$$p_X(x) = \binom{5}{x} p^x (1-p)^{5-x}, \quad x = 0,1,2,3,4,5$$

Binomial Distribution

General PMF for X : number of heads in 5 coin tosses?

Your coin has $\mathbf{P}(H) = p$

Event: $X = x$: x heads in 5 tosses

$$p_X(x) = \binom{5}{x} p^x (1-p)^{5-x}, \quad x = 0,1,2,3,4,5$$

X follows a binomial distribution

Binomial Distribution

General PMF for X : number of heads in 5 coin tosses?

Your coin has $\mathbf{P}(H) = p$

Event: $X = x$: x heads in 5 tosses

$$p_X(x) = \binom{5}{x} p^x (1-p)^{5-x}, \quad x = 0,1,2,3,4,5$$

X follows a binomial distribution

$X \sim \text{Binomial}(5, p)$

Number of flips

$\mathbf{P}(H)$

Binomial Distribution

General PMF for X : number of heads in 5 coin tosses?

Your coin has $\mathbf{P}(H) = p$

Event: $X = x$: x heads in 5 tosses

$$p_X(x) = \binom{5}{x} p^x (1-p)^{5-x}, \quad x = 0,1,2,3,4,5$$

X follows a binomial distribution

$X \sim \text{Binomial}(5, p)$

Number of flips $\mathbf{P}(H)$

```
graph LR; A["p_X(x) = \binom{5}{x} p^x (1-p)^{5-x}, x = 0,1,2,3,4,5"] --> B["X ~ Binomial(5, p)"]; C["Number of flips"] --> D["5"]
```

Binomial Distribution

Binomial Distribution

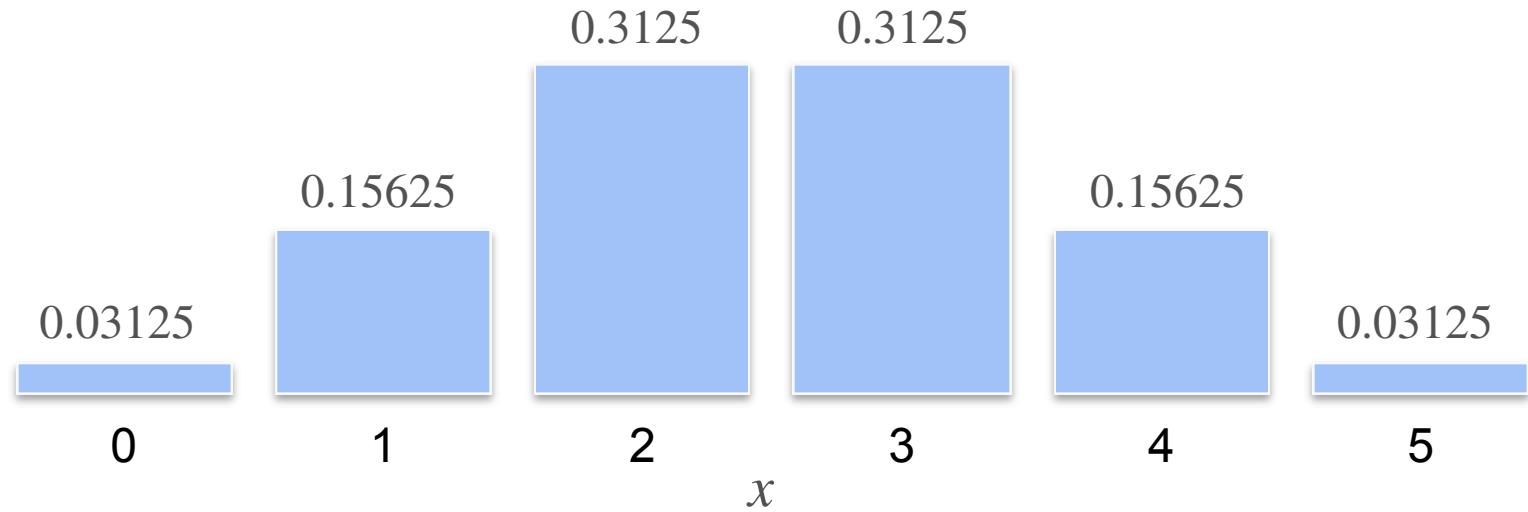
$$\begin{aligned} n &= 5 \\ p &= 0.5 \end{aligned}$$

$$p_X(x) = \mathbf{P}(X = x) = \binom{5}{k} 0.5^k 0.5^{5-k}$$

Binomial Distribution

$$\begin{aligned} n &= 5 \\ p &= 0.5 \end{aligned}$$

$$p_X(x) = \mathbf{P}(X = x) = \binom{5}{k} 0.5^k 0.5^{5-k}$$

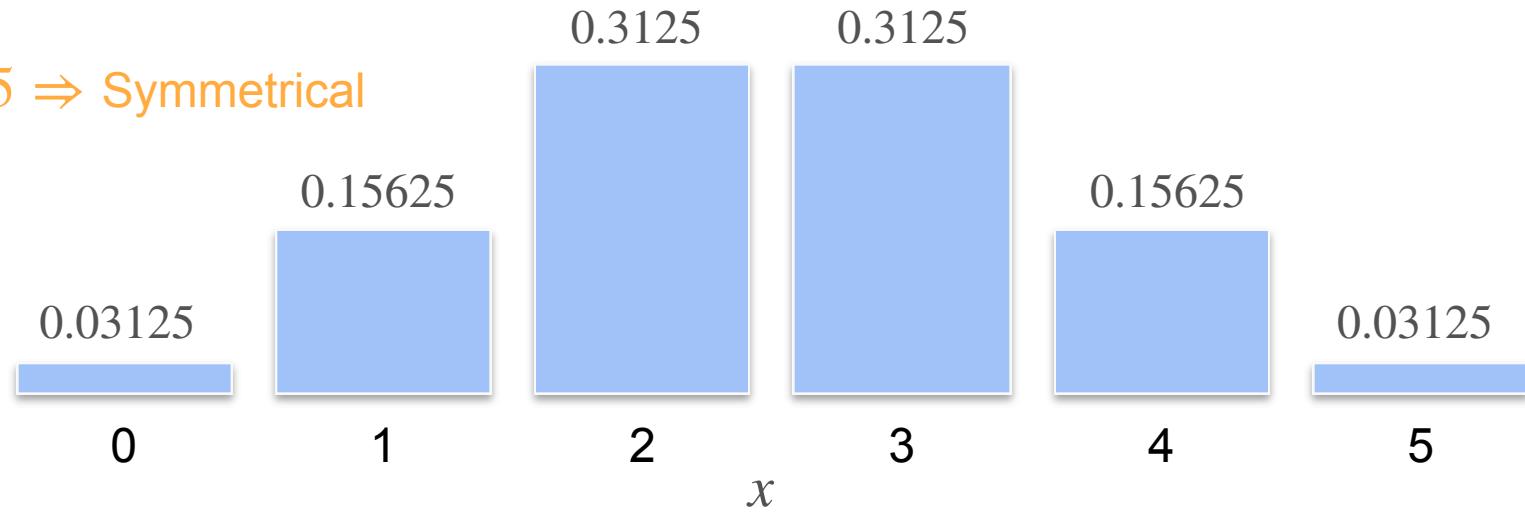


Binomial Distribution

$$\begin{array}{|l|} \hline n = 5 \\ p = 0.5 \\ \hline \end{array}$$

$$p_X(x) = \mathbf{P}(X = x) = \binom{5}{k} 0.5^k 0.5^{5-k}$$

$p = 0.5 \Rightarrow$ Symmetrical



Binomial Distribution

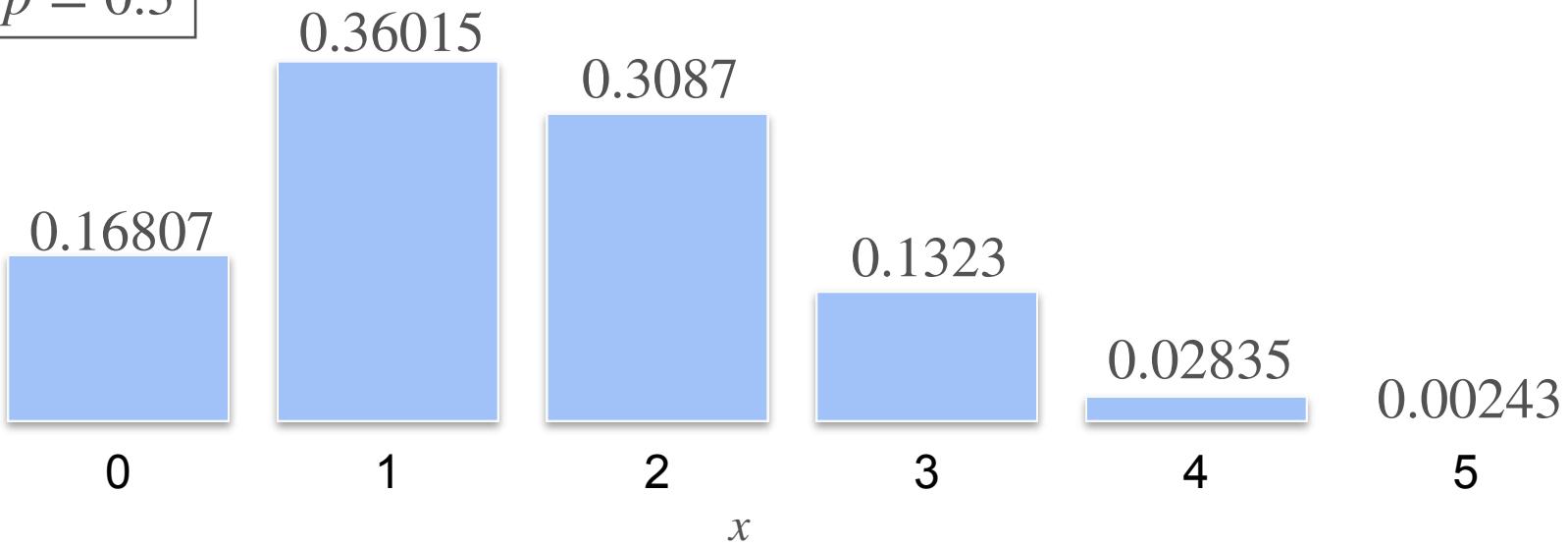
$$\begin{aligned} n &= 5 \\ p &= 0.3 \end{aligned}$$

$$p_X(x) = \mathbf{P}(X = x) = \binom{5}{k} 0.3^k 0.7^{5-k}$$

Binomial Distribution

$$\begin{aligned} n &= 5 \\ p &= 0.3 \end{aligned}$$

$$p_X(x) = \mathbf{P}(X = x) = \binom{5}{k} 0.3^k 0.7^{5-k}$$



Binomial Distribution

Binomial Distribution

General PMF for X : number of heads in 5 coin tosses?

Your coin has $\mathbf{P}(H) = p$

Event: $X = x$: x heads in 5 tosses

$$p_X(x) = \binom{5}{x} p^x (1-p)^{5-x}, \quad x = 0,1,2,3,4,5$$

$$X \sim \text{Binomial}(5,p)$$

Binomial Distribution

General PMF for X : number of heads in n coin tosses?

Your coin has $\mathbf{P}(H) = p$

Event: $X = x$: x heads in n tosses

$$p_X(x) = \binom{n}{x} p^x (1-p)^{n-x}, \quad x = 0, 1, 2, \dots, n$$

$$X \sim \text{Binomial}(n, p)$$

Binomial Distribution

General PMF for X : number of heads in n coin tosses?

Your coin has $\mathbf{P}(H) = p$

Event: $X = x$: x heads in n tosses

$$p_X(x) = \binom{n}{x} p^x (1-p)^{n-x}, \quad x = 0, 1, 2, \dots, n$$

$X \sim \text{Binomial}(n, p)$

n and p are called the **parameters** of the binomial distribution

Binomial Coefficient

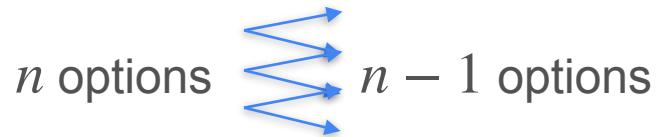
Binomial Coefficient

Pick 1st number

n options

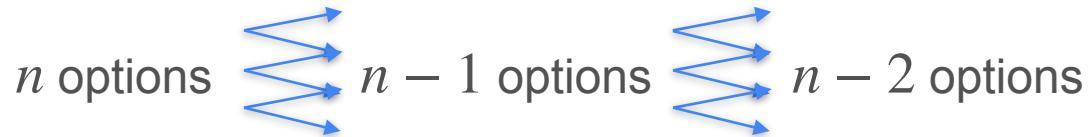
Binomial Coefficient

Pick 1st number Pick 2nd number



Binomial Coefficient

Pick 1st number Pick 2nd number Pick 3rd number



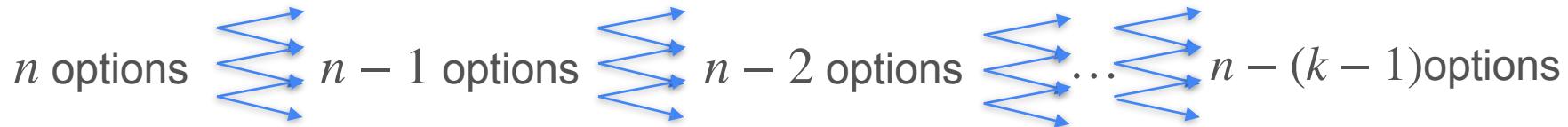
Binomial Coefficient

Pick 1st number Pick 2nd number Pick 3rd number

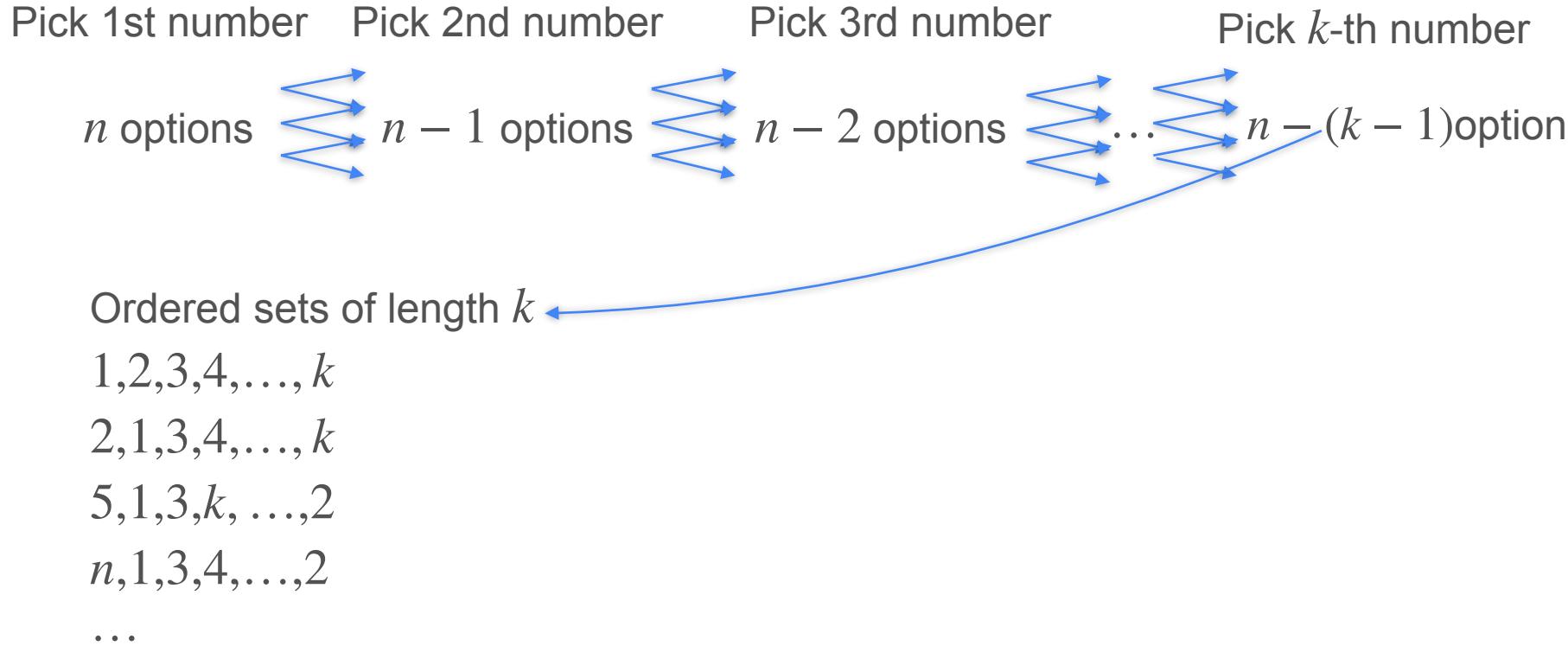


Binomial Coefficient

Pick 1st number Pick 2nd number Pick 3rd number Pick k -th number

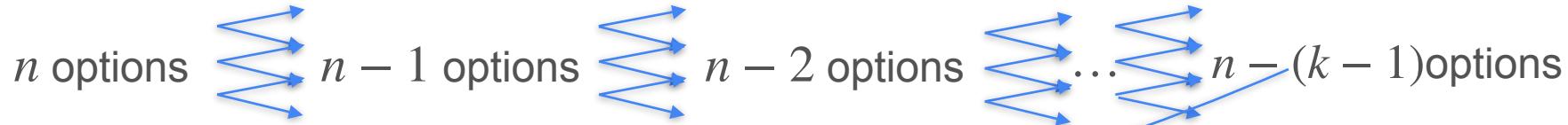


Binomial Coefficient



Binomial Coefficient

Pick 1st number Pick 2nd number Pick 3rd number Pick k -th number



Ordered sets of length k

1,2,3,4,..., k

2,1,3,4,..., k

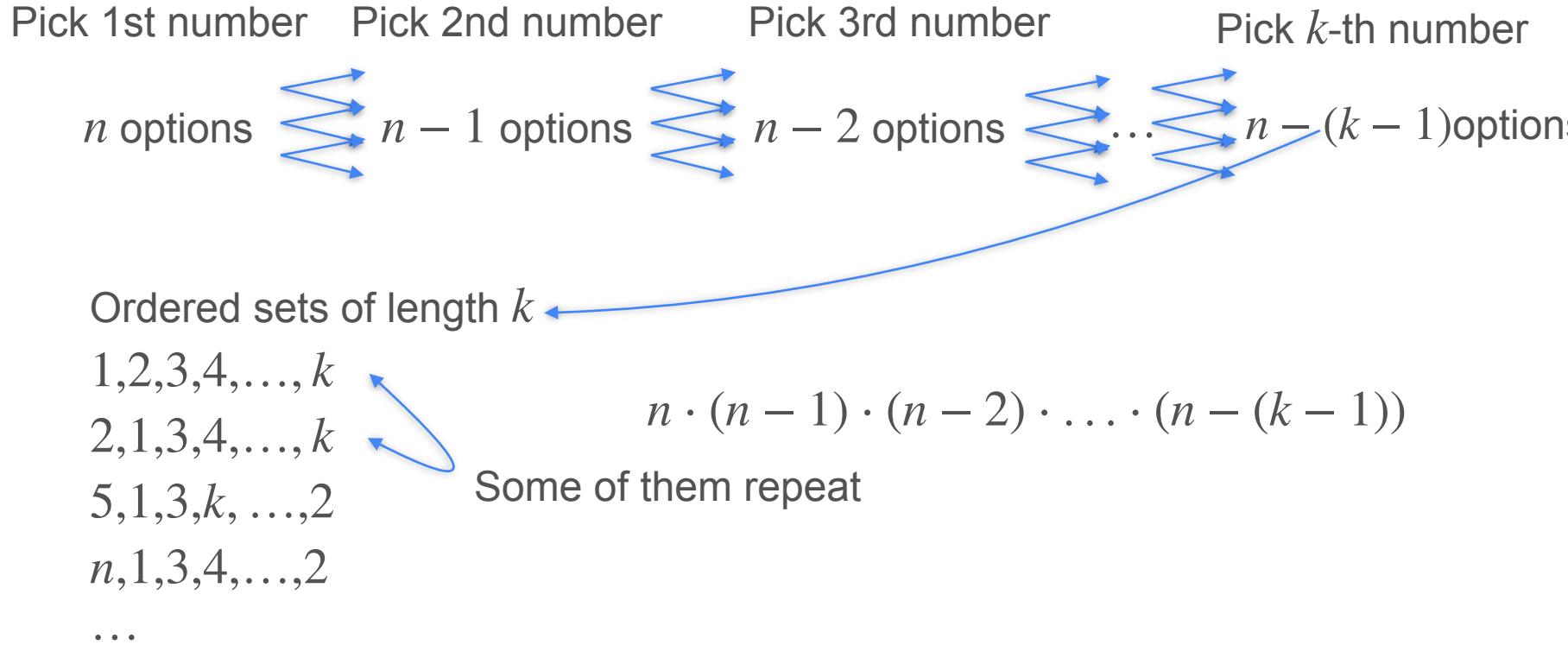
5,1,3, k , ...,2

n ,1,3,4,...,2

...

$$n \cdot (n - 1) \cdot (n - 2) \cdot \dots \cdot (n - (k - 1))$$

Binomial Coefficient



Binomial Coefficient

Binomial Coefficient

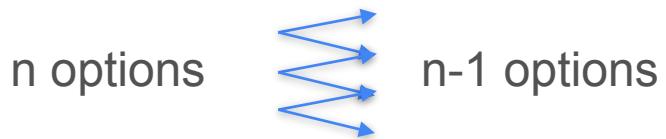
Pick 1st number

n options

Binomial Coefficient

Pick 1st number

Pick 2nd number



Binomial Coefficient

Pick 1st number

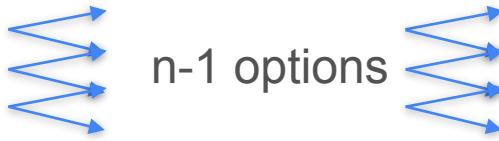
n options

Pick 2nd number

$n-1$ options

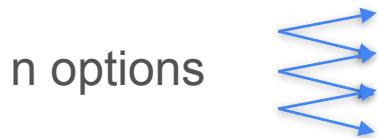
Pick 3rd number

$n-2$ options

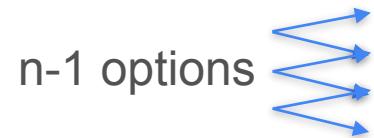


Binomial Coefficient

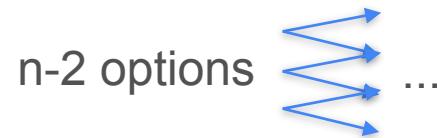
Pick 1st number



Pick 2nd number



Pick 3rd number



...

Binomial Coefficient



Binomial Coefficient

Pick 1st number

n options

Pick 2nd number

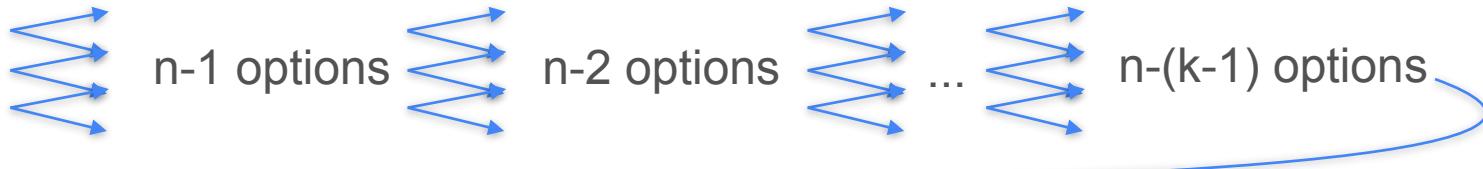
$n-1$ options

Pick 3rd number

$n-2$ options

Pick k -th number

$n-(k-1)$ options



Unordered sets of length k

1, 2, 3, 4, ... k

2, 1, 3, 4, ... k

5, 1, 3, k , ... 2

n , 1, 3, 4, ... 2

...

Binomial Coefficient

Pick 1st number

n options

Pick 2nd number

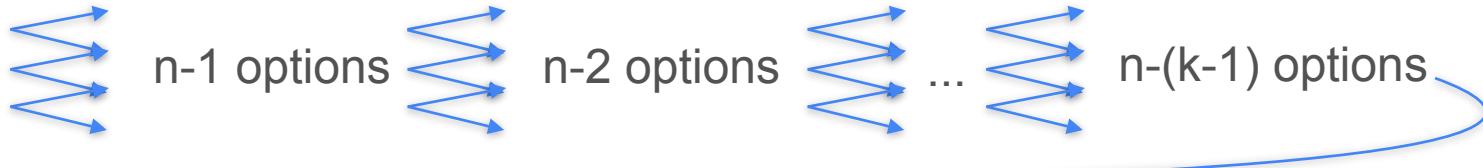
n-1 options

Pick 3rd number

n-2 options

Pick k-th number

n-(k-1) options



Unordered sets of length k

1, 2, 3, 4, ... k

$n * (n-1) * (n-2) * \dots * (n-(k-1))$

2, 1, 3, 4, ... k

5, 1, 3, k, ... 2

n, 1, 3, 4, ... 2

...

Binomial Coefficient

Pick 1st number

n options

Pick 2nd number

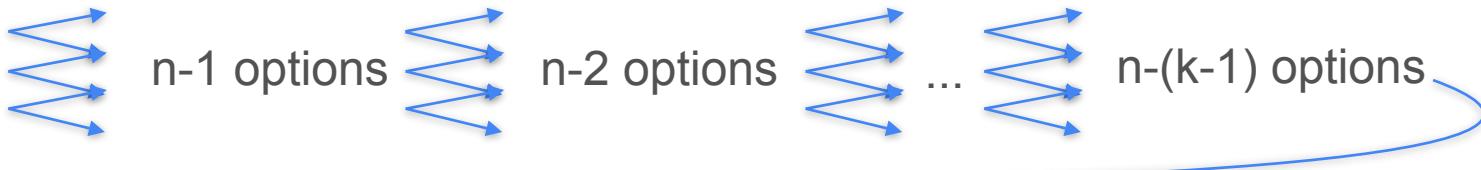
n-1 options

Pick 3rd number

n-2 options

Pick k-th number

n-(k-1) options



Unordered sets of length k

1, 2, 3, 4, ... k

2, 1, 3, 4, ... k

5, 1, 3, k, ... 2

n, 1, 3, 4, ... 2

$$n * (n-1) * (n-2) * \dots * (n-(k-1))$$

Some of them repeat

...

Binomial Coefficient

Binomial Coefficient

1,2,3,4

1,2,4,3

1,3,2,4

1,3,4,2

...

4,3,2,1

Binomial Coefficient

Pick 1st

1,2,3,4

1,2,4,3

1,3,2,4

1,3,4,2

...

4,3,2,1

Binomial Coefficient

Pick 1st

Pick 2nd

1,2,3,4

1,2,4,3

1,3,2,4

1,3,4,2

...

4,3,2,1

Binomial Coefficient

Pick 1st

Pick 2nd

Pick 3rd

1,2,3,4

1,2,4,3

1,3,2,4

1,3,4,2

...

4,3,2,1

Binomial Coefficient

Pick 1st

Pick 2nd

Pick 3rd

Pick 4th

1,2,3,4

1,2,4,3

1,3,2,4

1,3,4,2

...

4,3,2,1

Binomial Coefficient

	Pick 1st	Pick 2nd	Pick 3rd	Pick 4th
	4 options			
1,2,3,4				
1,2,4,3				
1,3,2,4				
1,3,4,2				
...				
4,3,2,1				

Binomial Coefficient

	Pick 1st	Pick 2nd	Pick 3rd	Pick 4th
1,2,3,4	4 options	3 options		
1,2,4,3				
1,3,2,4				
1,3,4,2				
...				
4,3,2,1				

Binomial Coefficient

	Pick 1st	Pick 2nd	Pick 3rd	Pick 4th
1,2,3,4	4 options	3 options	2 options	
1,2,4,3				
1,3,2,4				
1,3,4,2				
...				
4,3,2,1				

Binomial Coefficient

	Pick 1st	Pick 2nd	Pick 3rd	Pick 4th
1,2,3,4	4 options	3 options	2 options	1 option
1,2,4,3				
1,3,2,4				
1,3,4,2				
...				
4,3,2,1				

Binomial Coefficient

	Pick 1st	Pick 2nd	Pick 3rd	Pick 4th
1,2,3,4	4 options	3 options	2 options	1 option
1,2,4,3				
1,3,2,4		$4 \cdot 3 \cdot 2 \cdot 1 = 4!$		
1,3,4,2				
...				
4,3,2,1				

Binomial Coefficient

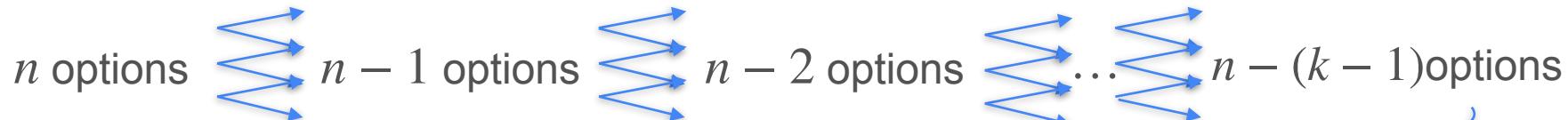
	Pick 1st	Pick 2nd	Pick 3rd	Pick 4th
1,2,3,4	4 options	3 options	2 options	1 option
1,2,4,3				
1,3,2,4		$4 \cdot 3 \cdot 2 \cdot 1 = 4!$		
1,3,4,2		For five numbers:		
		$5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5!$		
...				
4,3,2,1				

Binomial Coefficient

	Pick 1st	Pick 2nd	Pick 3rd	Pick 4th
1,2,3,4	4 options	3 options	2 options	1 option
1,2,4,3				
1,3,2,4		$4 \cdot 3 \cdot 2 \cdot 1 = 4!$		
1,3,4,2		For five numbers:		
		$5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5!$		
...				
4,3,2,1		General solution:	$k!$	

Binomial Coefficient

Pick 1st number Pick 2nd number Pick 3rd number Pick k -th number



Ordered sets of length k

1,2,3,4,..., k

2,1,3,4,..., k

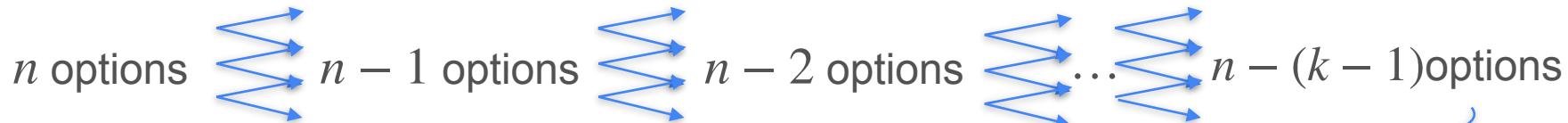
5,1,3, k , ...,2 $n \cdot (n - 1) \cdot (n - 2) \cdot \dots \cdot (n - (k - 1))$

n ,1,3,4,...,2

...

Binomial Coefficient

Pick 1st number Pick 2nd number Pick 3rd number Pick k -th number



Ordered sets of length k

1,2,3,4,..., k

2,1,3,4,..., k

5,1,3, k , ...,2

$n,1,3,4,...,2$

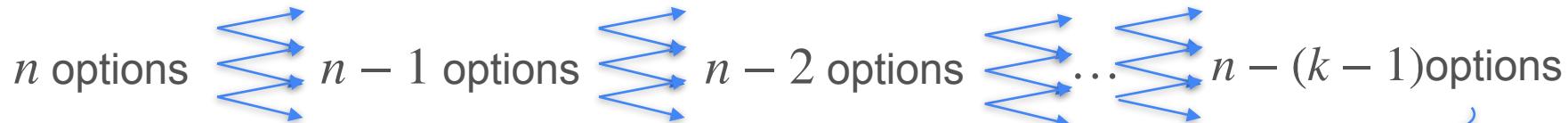
...

→ Unordered sets of length k

$$\frac{n \cdot (n - 1) \cdot (n - 2) \cdot \dots \cdot (n - (k - 1))}{k!}$$

Binomial Coefficient

Pick 1st number Pick 2nd number Pick 3rd number Pick k -th number



Ordered sets of length k

1,2,3,4,..., k

2,1,3,4,..., k

5,1,3, k , ...,2

n ,1,3,4,...,2

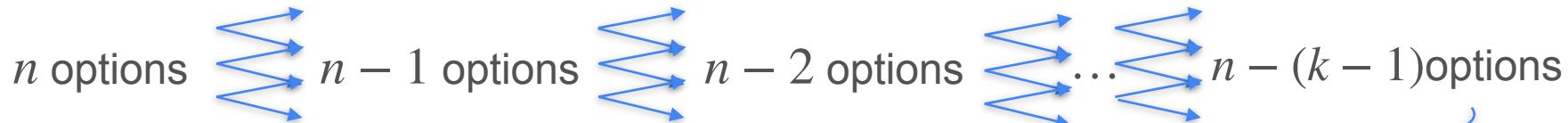
...

→ Unordered sets of length k

$$\frac{n \cdot (n - 1) \cdot (n - 2) \cdot \dots \cdot (n - (k - 1))}{k!} = \underline{\hspace{1cm}}$$

Binomial Coefficient

Pick 1st number Pick 2nd number Pick 3rd number Pick k -th number



Ordered sets of length k

1,2,3,4,..., k

2,1,3,4,..., k

5,1,3, k , ...,2

n ,1,3,4,...,2

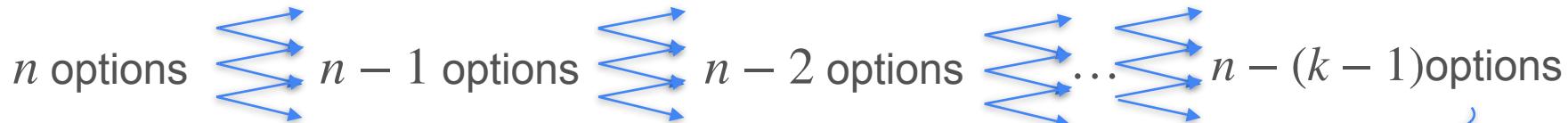
...

→ Unordered sets of length k

$$\frac{n \cdot (n - 1) \cdot (n - 2) \cdot \dots \cdot (n - (k - 1))}{k!} = \frac{n!}{(n - k)!}$$

Binomial Coefficient

Pick 1st number Pick 2nd number Pick 3rd number Pick k -th number



Ordered sets of length k

1,2,3,4,..., k

2,1,3,4,..., k

5,1,3, k , ...,2

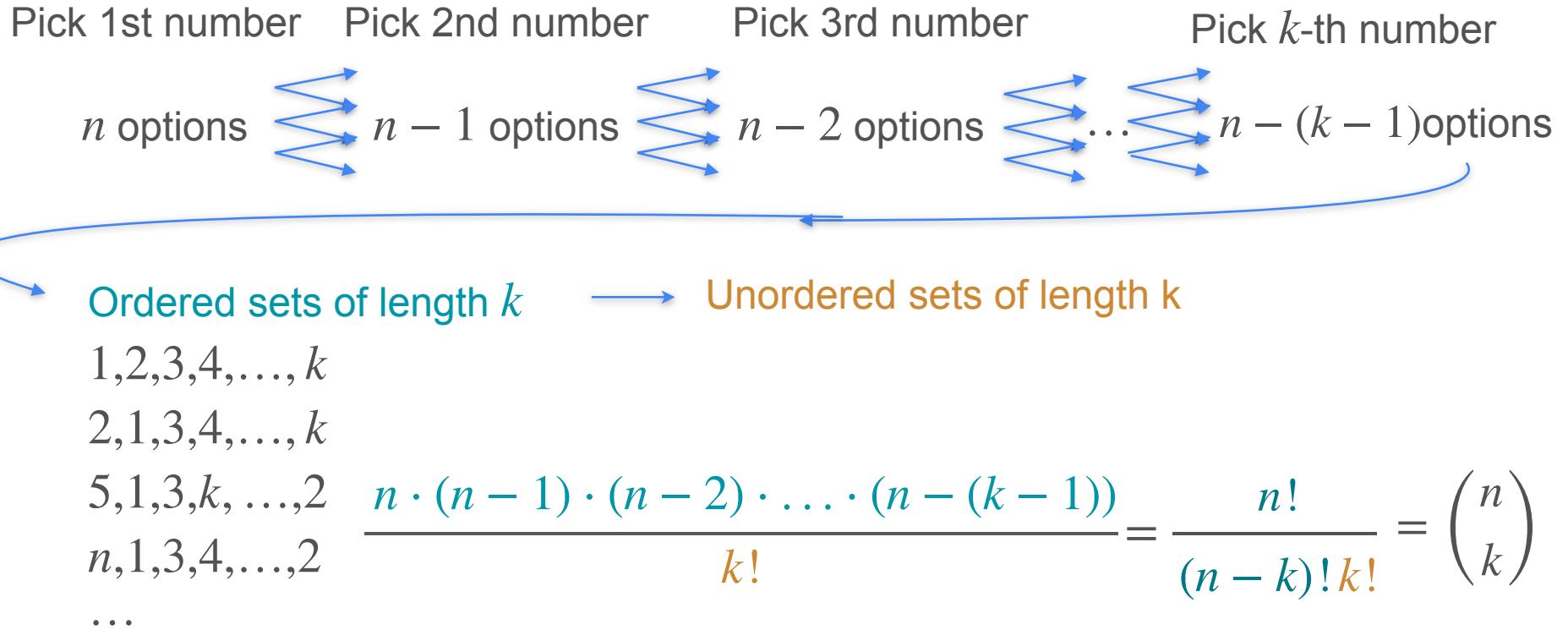
n ,1,3,4,...,2

...

→ Unordered sets of length k

$$\frac{n \cdot (n - 1) \cdot (n - 2) \cdot \dots \cdot (n - (k - 1))}{k!} = \frac{n!}{(n - k)! k!}$$

Binomial Coefficient



Binomial Coefficient

Pick 1st number

n options

Pick 2nd number

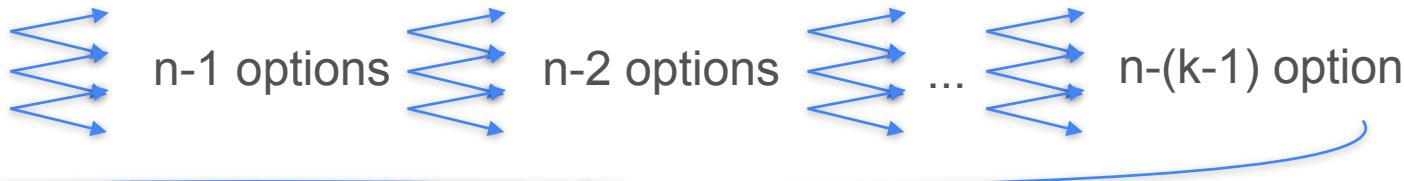
n-1 options

Pick 3rd number

n-2 options

Pick k-th number

n-(k-1) options



Ordered sets of length k

1, 2, 3, 4, ... k

2, 1, 3, 4, ... k

4, 1, 3, k, ... 2

n, 1, k, 4, ... 2

$$n * (n-1) * (n-2) * \dots * (n-(k-1))$$

...

Binomial Coefficient

Pick 1st number

n options

Pick 2nd number

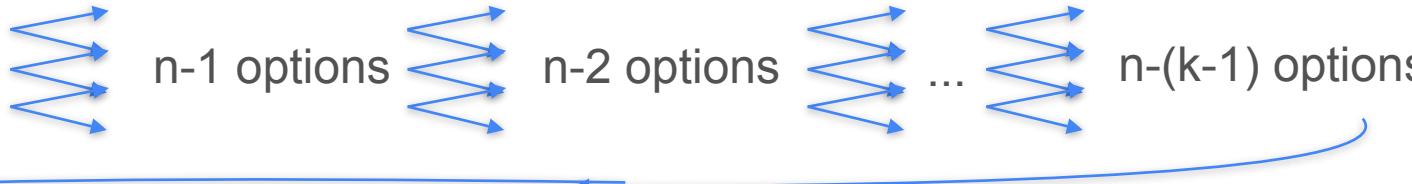
n-1 options

Pick 3rd number

n-2 options

Pick k-th number

n-(k-1) options



Ordered sets of length k

1, 2, 3, 4, ... k

2, 1, 3, 4, ... k

4, 1, 3, k, ... 2

n, 1, k, 4, ... 2

...

→ Unordered sets of length k

$$\frac{n * (n-1) * (n-2) * \dots * (n-(k-1))}{k!}$$

Binomial Coefficient

Pick 1st number

n options

Pick 2nd number

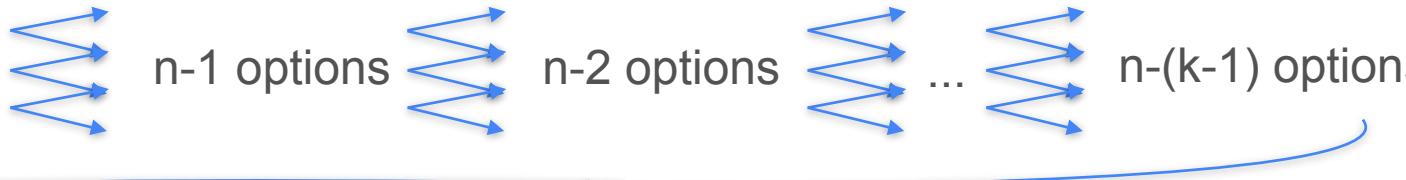
n-1 options

Pick 3rd number

n-2 options

Pick k-th number

n-(k-1) options



Ordered sets of length k

1, 2, 3, 4, ... k

2, 1, 3, 4, ... k

4, 1, 3, k, ... 2

n, 1, k, 4, ... 2

...

→ Unordered sets of length k

$$\frac{n * (n-1) * (n-2) * \dots * (n-(k-1))}{k!} =$$

Binomial Coefficient

Pick 1st number

n options

Pick 2nd number

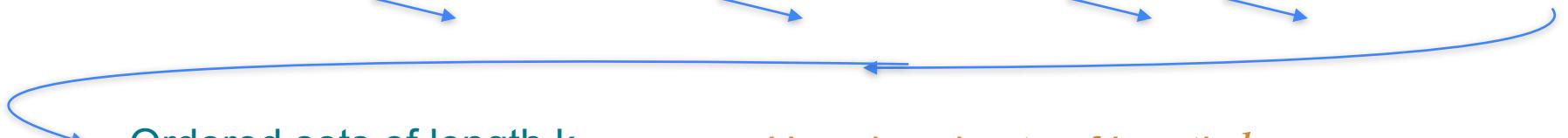
n-1 options

Pick 3rd number

n-2 options

Pick k-th number

n-(k-1) options



Ordered sets of length k

1, 2, 3, 4, ... k

2, 1, 3, 4, ... k

4, 1, 3, k, ... 2

n, 1, k, 4, ... 2

→ Unordered sets of length k

$$\frac{n * (n-1) * (n-2) * \dots * (n-(k-1))}{k!} = \frac{n!}{(n-k)!}$$

...

Binomial Coefficient

Pick 1st number

n options

Pick 2nd number

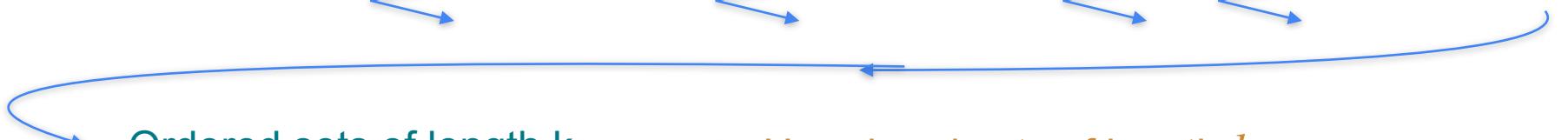
n-1 options

Pick 3rd number

n-2 options

Pick k-th number

n-(k-1) options



Ordered sets of length k

1, 2, 3, 4, ... k

2, 1, 3, 4, ... k

4, 1, 3, k, ... 2

n, 1, k, 4, ... 2

...

→ Unordered sets of length k

$$\frac{n * (n-1) * (n-2) * \dots * (n-(k-1))}{k!}$$

$$= \frac{n!}{(n-k)! * k!}$$

Binomial Coefficient

Pick 1st number

n options

Pick 2nd number

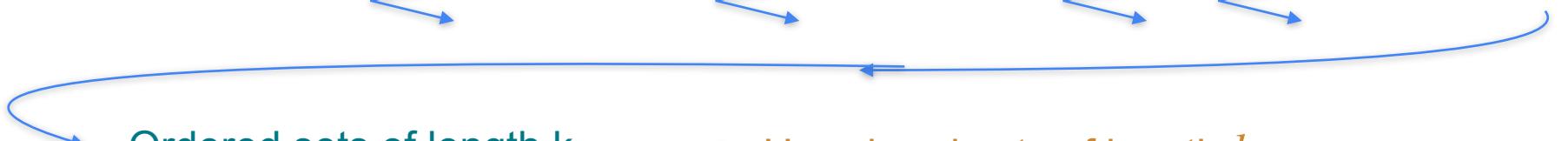
n-1 options

Pick 3rd number

n-2 options

Pick k-th number

n-(k-1) options



Ordered sets of length k

1, 2, 3, 4, ... k

2, 1, 3, 4, ... k

4, 1, 3, k, ... 2

n, 1, k, 4, ... 2

→ Unordered sets of length k

$$n * (n-1) * (n-2) * \dots * (n-(k-1))$$

$$k!$$

$$=$$

$$\frac{n!}{(n-k)! * k!} =$$

$$\binom{n}{k}$$

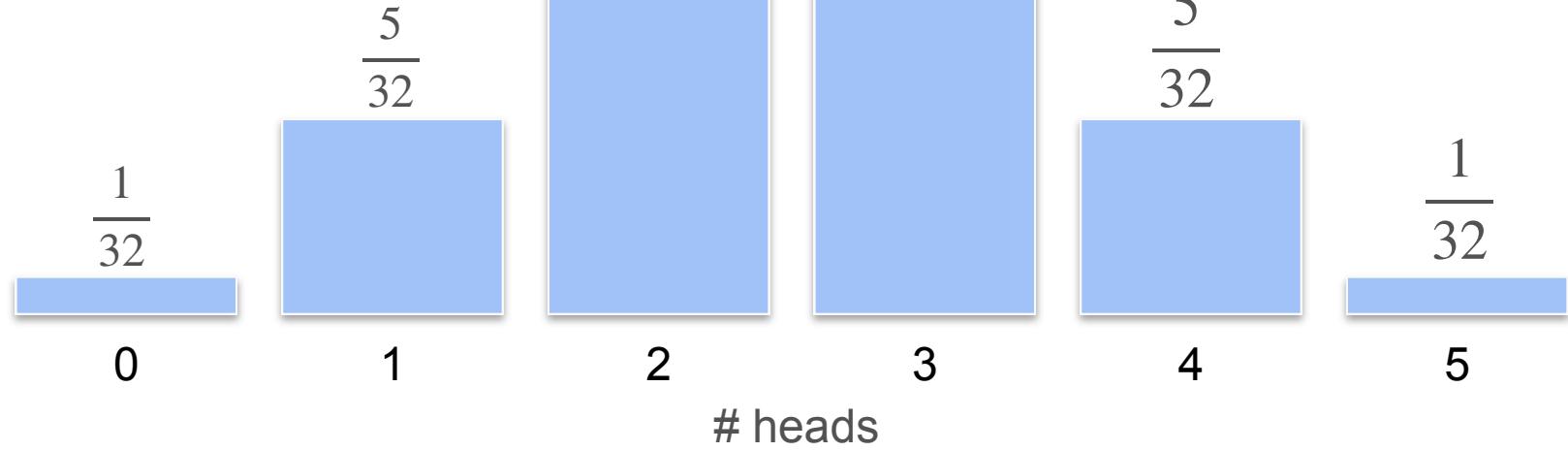
...

Binomial Distribution: Fair Coins

$$\binom{5}{0} = \frac{5!}{0!5!} = 1$$

$$\frac{10}{32}$$

$$\frac{10}{32}$$



Binomial Distribution: Fair Coins



50 %



50 %



$$0.5 \cdot 0.5 \cdot 0.5 \cdot 0.5 \cdot 0.5 = \frac{1}{32}$$



$$0.5 \cdot 0.5 \cdot 0.5 \cdot 0.5 \cdot 0.5 = \frac{1}{32}$$

Binomial Distribution: Biased Coins



30 %



70 %



Binomial Distribution: Biased Coins



30 %



70 %



$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 = 0.00243$$



Binomial Distribution: Biased Coins



30 %



70 %



$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 = 0.00243$$



$$0.3 \cdot 0.3 \cdot 0.7 \cdot 0.7 \cdot 0.7 = 0.01323$$

Binomial Distribution: Biased Coins

0.3 · 0.3 · 0.3 · 0.3 · 0.3 · 0.3

0.3 · 0.3 · 0.3 · 0.3 · 0.3 · 0.7

0.3 · 0.3 · 0.3 · 0.7 · 0.7

0.3 · 0.3 · 0.7 · 0.7 · 0.7

0.3 · 0.7 · 0.7 · 0.7 · 0.7

0.7 · 0.7 · 0.7 · 0.7 · 0.7

Binomial Distribution: Biased Coins


$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 =$$


$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.7 =$$


$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.7 \cdot 0.7 =$$


$$0.3 \cdot 0.3 \cdot 0.7 \cdot 0.7 \cdot 0.7 =$$


$$0.3 \cdot 0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 =$$


$$0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 =$$

Binomial Distribution: Biased Coins


$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 =$$


$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.7 =$$


$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.7 \cdot 0.7 =$$


$$0.3 \cdot 0.3 \cdot 0.7 \cdot 0.7 \cdot 0.7 =$$


$$0.3 \cdot 0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 =$$


$$0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 =$$

$$= 0.3^k \cdot 0.7^{n-k}$$

Binomial Distribution: Biased Coins


$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 = 0.3^5$$


$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.7 =$$


$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.7 \cdot 0.7 =$$
$$= 0.3^k \cdot 0.7^{n-k}$$


$$0.3 \cdot 0.3 \cdot 0.7 \cdot 0.7 \cdot 0.7 =$$


$$0.3 \cdot 0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 =$$


$$0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 =$$

Binomial Distribution: Biased Coins


$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 = 0.3^5 \cdot 0.7^0$$


$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.7 =$$


$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.7 \cdot 0.7 =$$

$$= 0.3^k \cdot 0.7^{n-k}$$


$$0.3 \cdot 0.3 \cdot 0.7 \cdot 0.7 \cdot 0.7 =$$


$$0.3 \cdot 0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 =$$


$$0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 =$$

Binomial Distribution: Biased Coins


$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 = 0.3^5 \cdot 0.7^0$$


$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.7 = 0.3^4 \cdot 0.7^1$$


$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.7 \cdot 0.7 = 0.3^3 \cdot 0.7^2$$

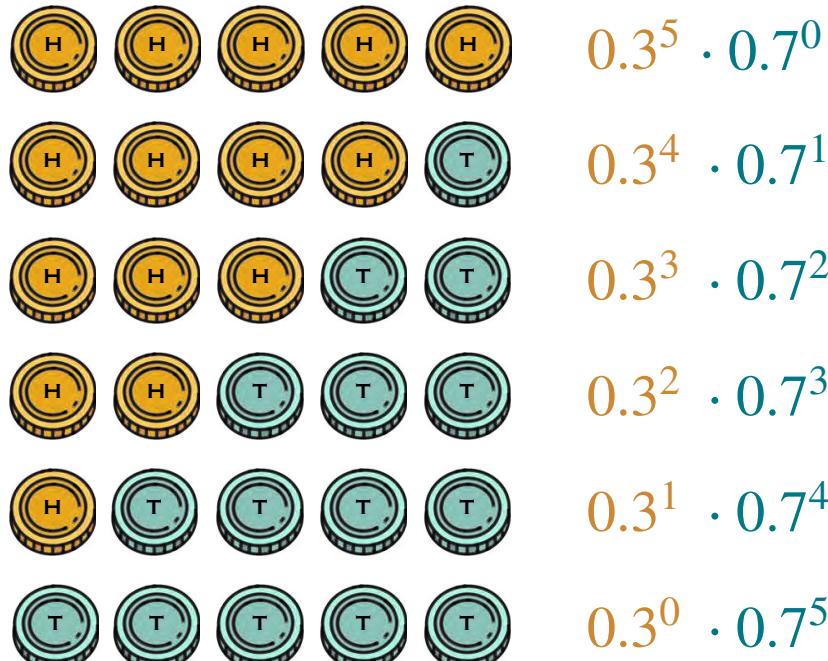

$$0.3 \cdot 0.3 \cdot 0.7 \cdot 0.7 \cdot 0.7 = 0.3^2 \cdot 0.7^3$$


$$0.3 \cdot 0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 = 0.3^1 \cdot 0.7^4$$


$$0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 = 0.3^0 \cdot 0.7^5$$

$$= 0.3^k \cdot 0.7^{n-k}$$

Binomial Distribution: Biased Coins



$$= 0.3^k \cdot 0.7^{n-k}$$

Binomial Distribution: Biased Coins

	$0.3^5 \cdot 0.7^0$
	$0.3^4 \cdot 0.7^1$
	$0.3^3 \cdot 0.7^2$
	$0.3^2 \cdot 0.7^3$
	$0.3^1 \cdot 0.7^4$
	$0.3^0 \cdot 0.7^5$

$$= 0.3^k \cdot 0.7^{n-k} \rightarrow \binom{n}{k} 0.3^k \cdot 0.7^{n-k}$$

Account for all possible orders of heads and tails

Binomial Distribution: Biased Coins

$$= \binom{n}{k} 0.3^k \cdot 0.7^{n-k}$$

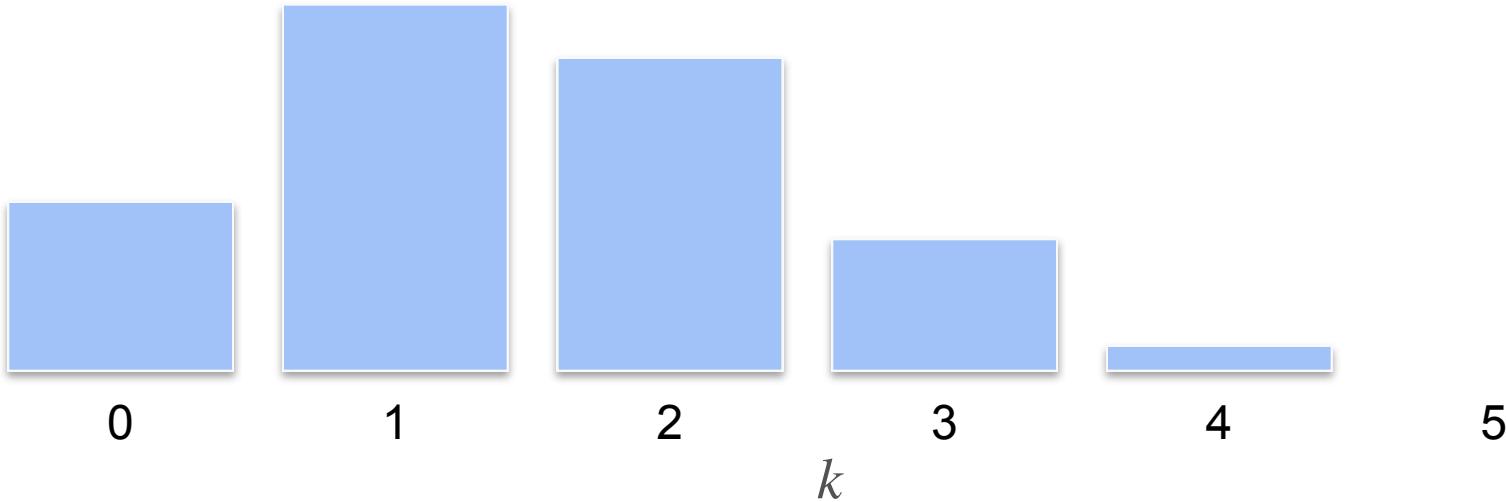
Binomial Distribution: Biased Coins

$$= \binom{n}{k} 0.3^k \cdot 0.7^{n-k} \quad n = 5$$

Binomial Distribution: Biased Coins

$$= \binom{n}{k} 0.3^k \cdot 0.7^{n-k}$$

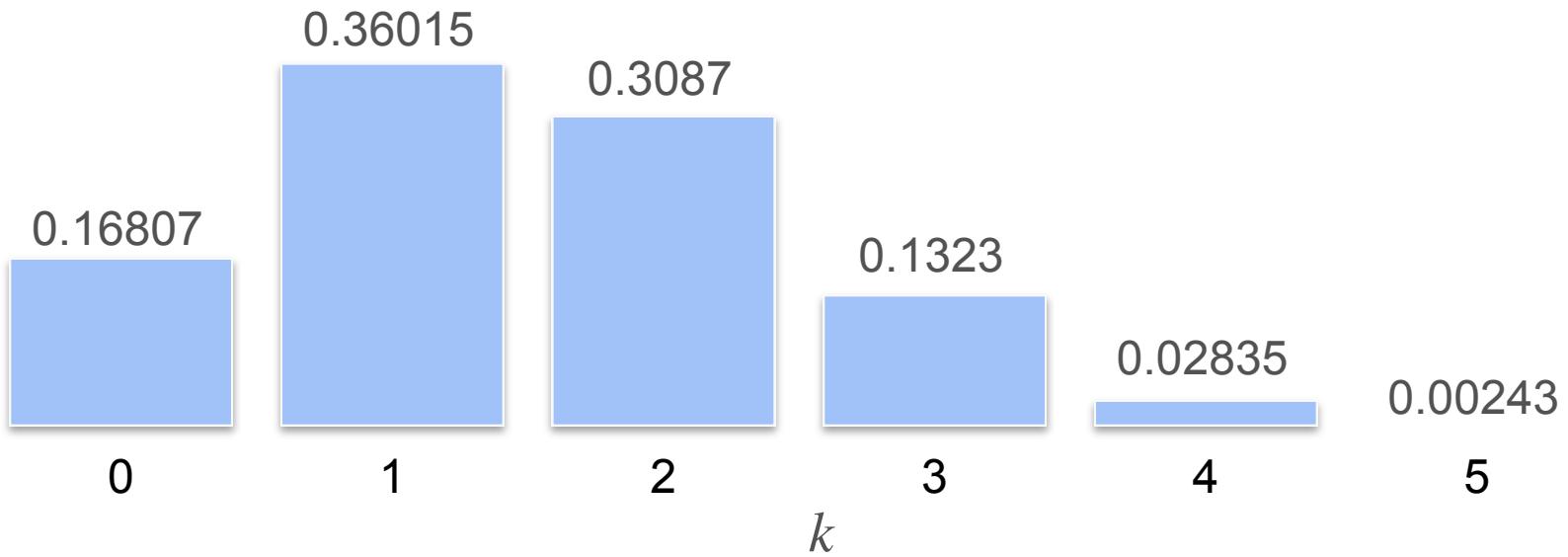
n = 5
p = 0.3



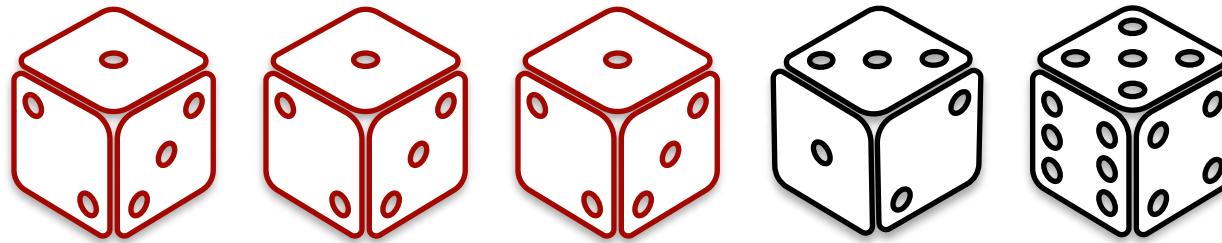
Binomial Distribution: Biased Coins

$$= \binom{n}{k} 0.3^k \cdot 0.7^{n-k}$$

n = 5
p = 0.3

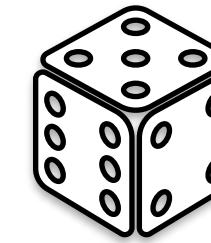
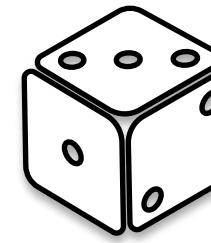
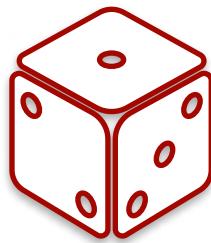
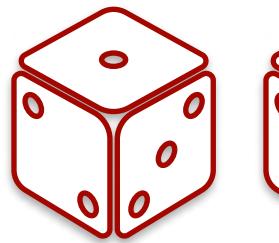


Binomial Distribution: Quiz

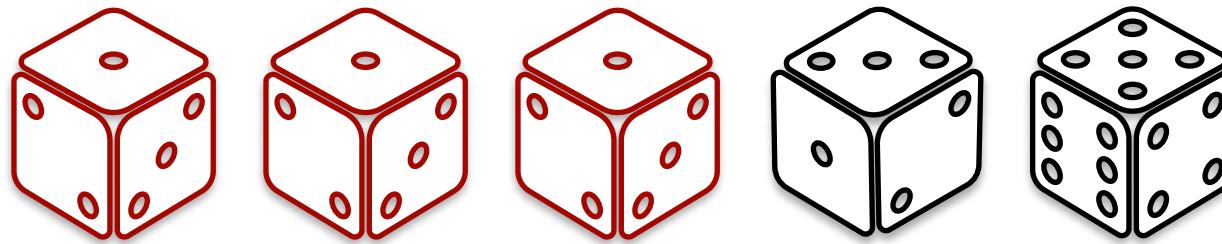


Binomial Distribution: Quiz

What is the probability of getting three ones when rolling a dice five times (no matter on which dice)?

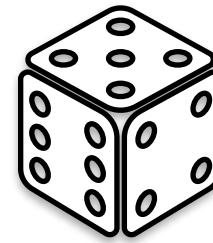
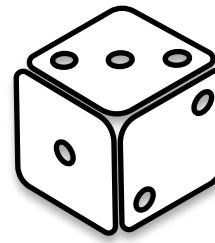
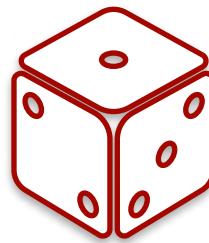
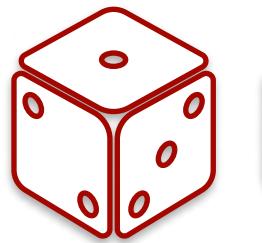


Binomial Distribution: Quiz

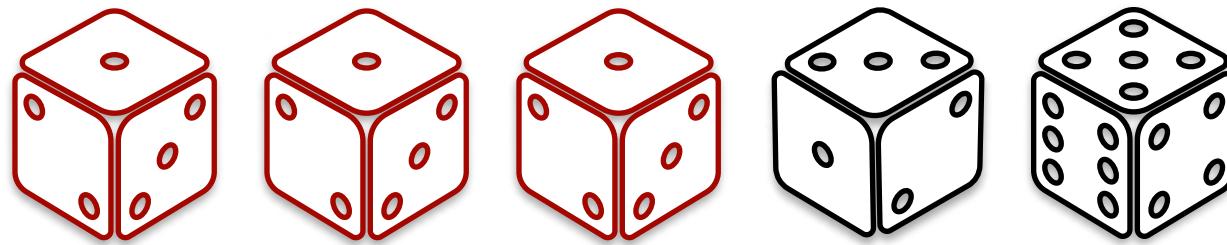


Binomial Distribution: Quiz

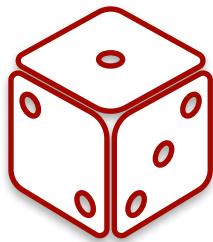
What is the probability of getting three ones when rolling a dice five times (no matter on which dice)?



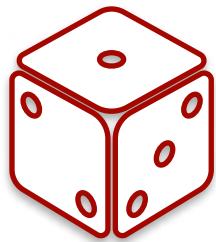
Binomial Distribution: Dice Is a Biased Coin!



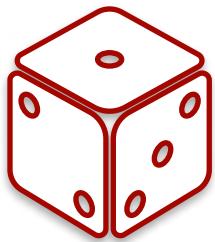
Binomial Distribution: Dice Is a Biased Coin!



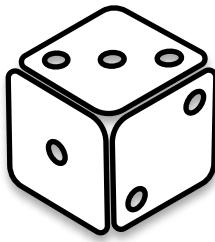
one
 $p = \frac{1}{6}$



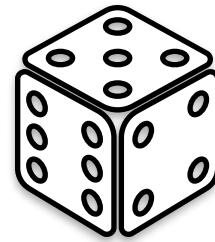
one
 $p = \frac{1}{6}$



one
 $p = \frac{1}{6}$

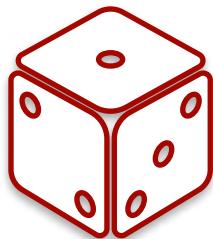


not one
 $p = \frac{5}{6}$



not one
 $p = \frac{5}{6}$

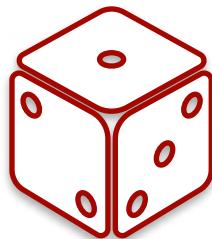
Binomial Distribution: Dice Is a Biased Coin!



one
 $p = \frac{1}{6}$



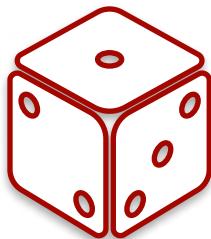
heads
 $p = \frac{1}{6}$



one
 $p = \frac{1}{6}$



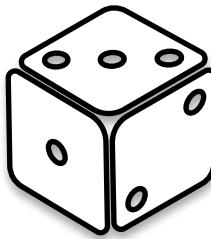
heads
 $p = \frac{1}{6}$



one
 $p = \frac{1}{6}$



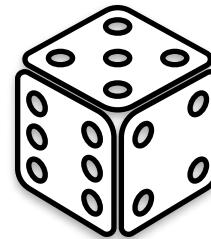
heads
 $p = \frac{1}{6}$



not one
 $p = \frac{5}{6}$



not heads
 $p = \frac{5}{6}$

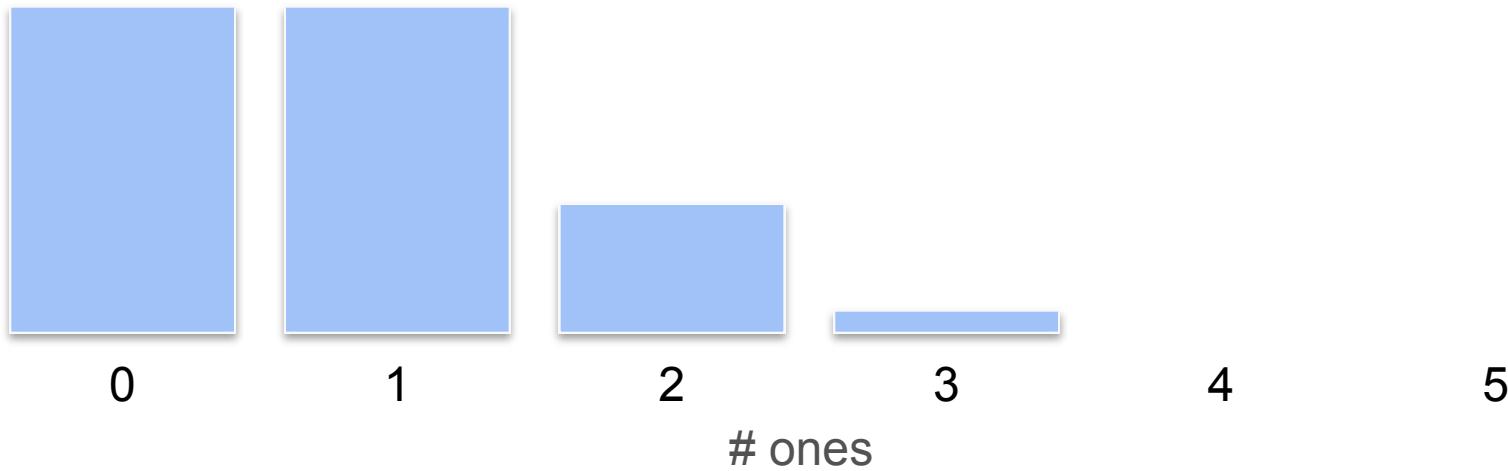


not one
 $p = \frac{5}{6}$



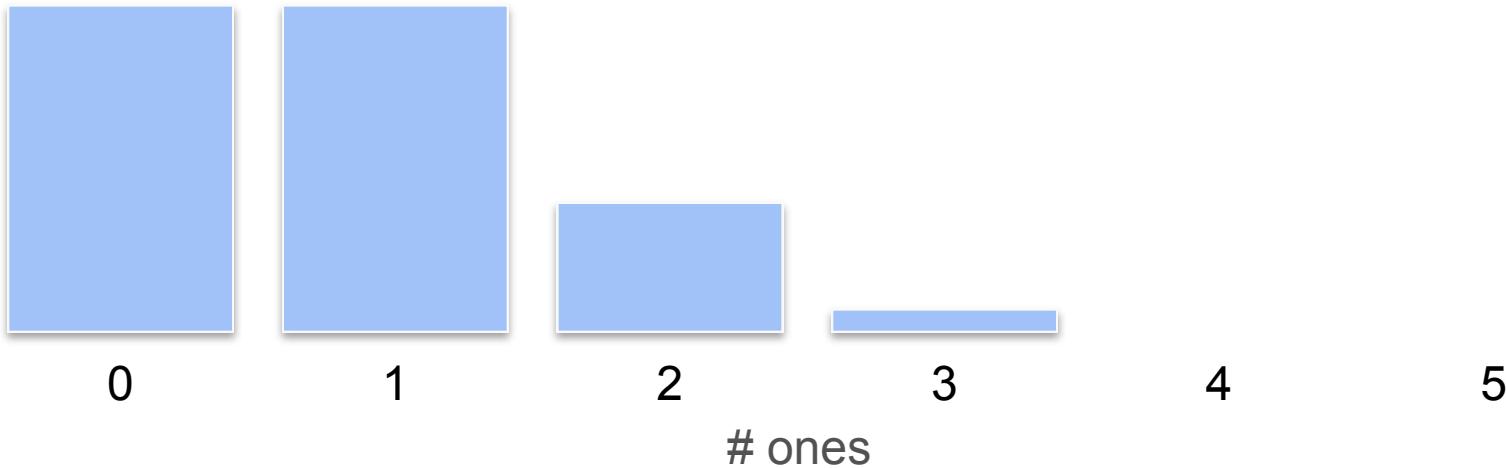
not heads
 $p = \frac{5}{6}$

Binomial Distribution: Dice Is a Biased Coin!



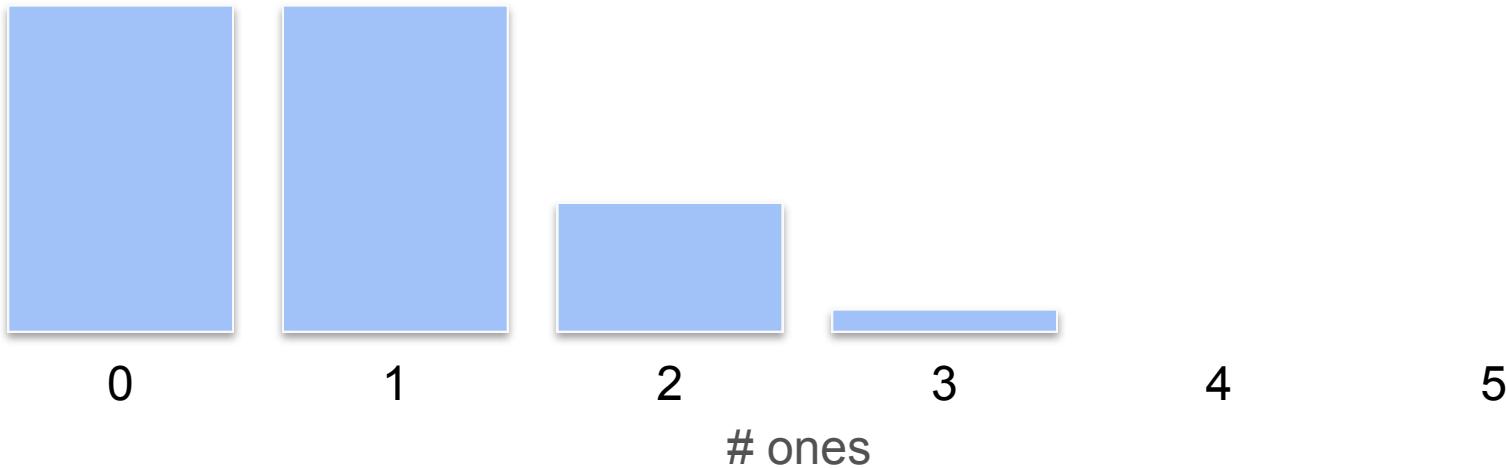
Binomial Distribution: Dice Is a Biased Coin!

$n = 5$



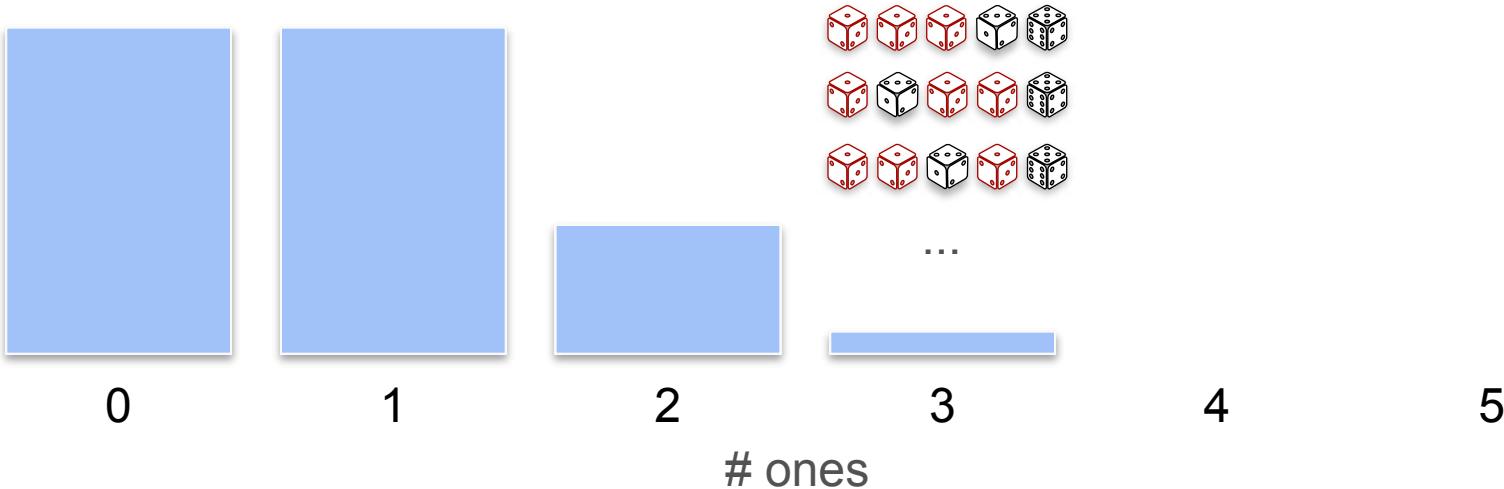
Binomial Distribution: Dice Is a Biased Coin!

n = 5
p = 0.1666

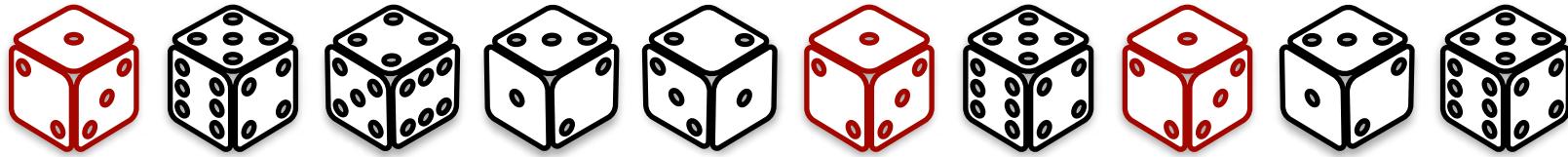


Binomial Distribution: Dice Is a Biased Coin!

$n = 5$
 $p = 0.1666$

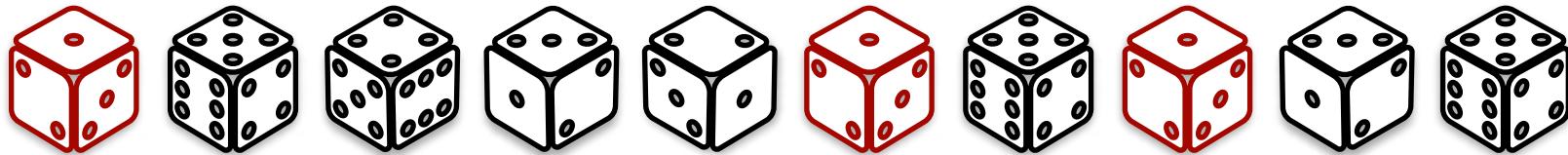


Binomial Distribution: Quiz

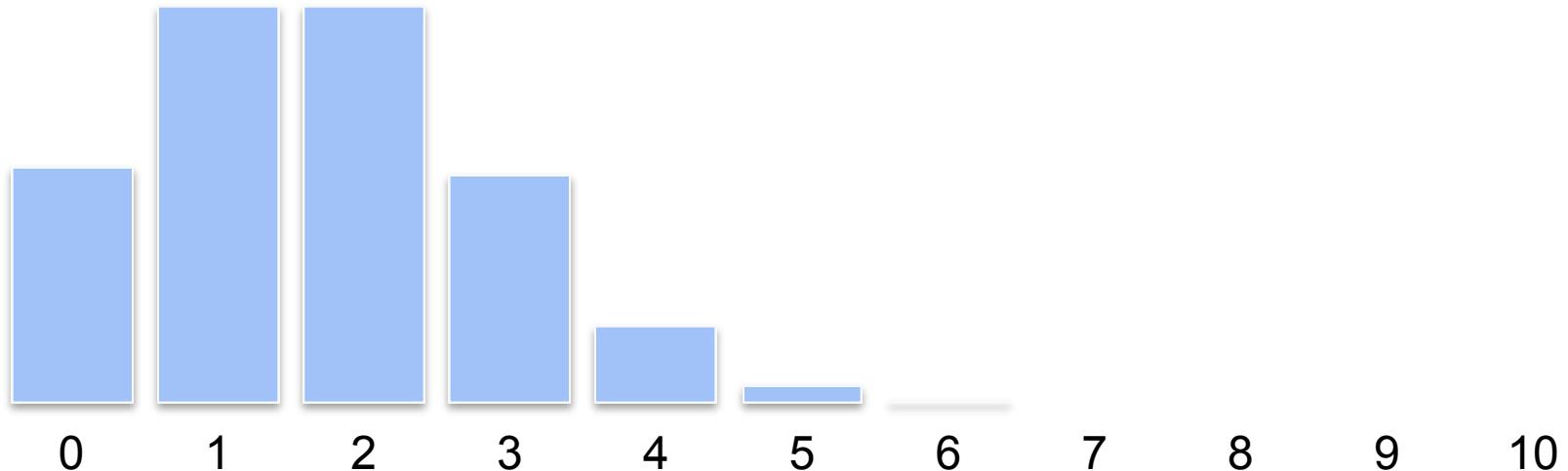


Binomial Distribution: Quiz

- Quiz: What are the parameters for the following binomial distribution:
 - I roll 10 dice
 - I want to record the number of times I obtain the number 1

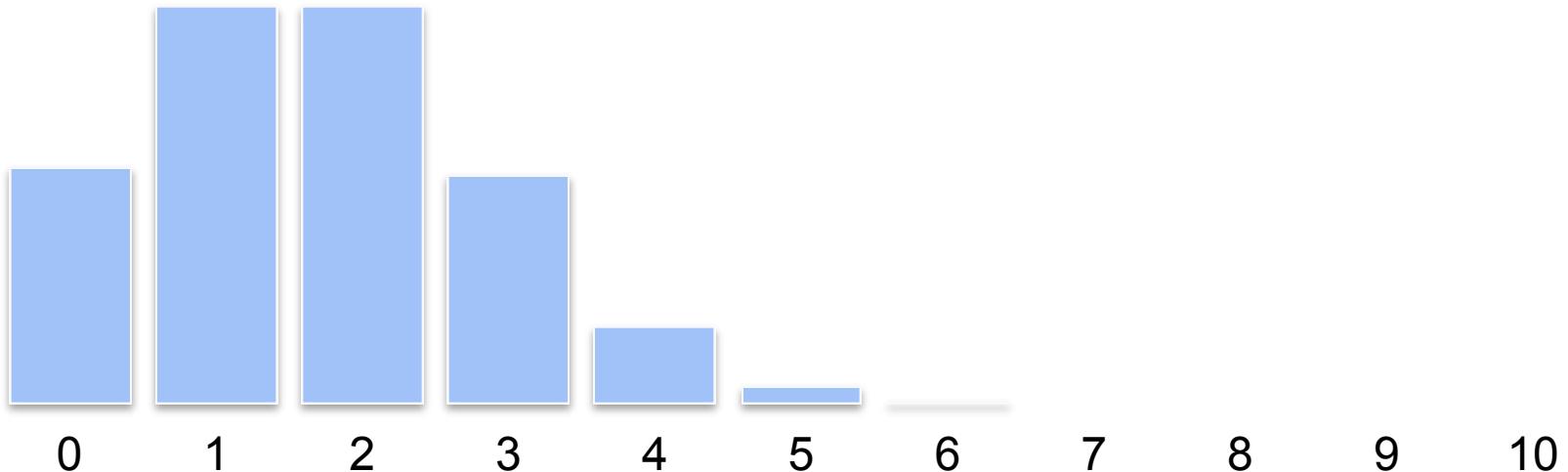


Binomial Distribution: Quiz

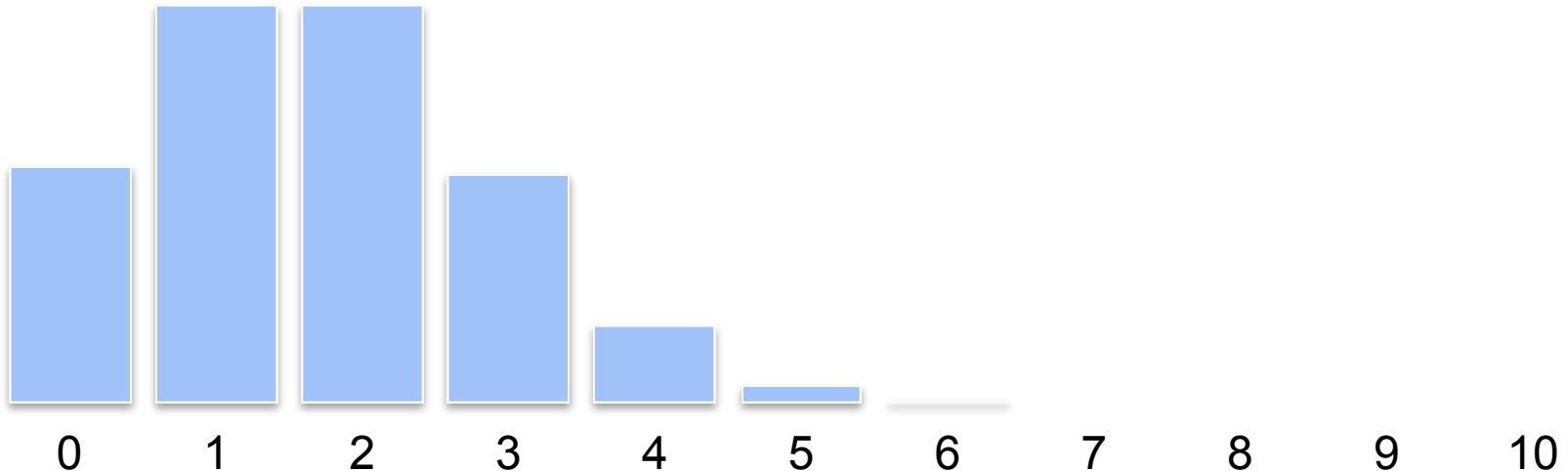


Binomial Distribution: Quiz

$$n = 10$$



Binomial Distribution: Quiz

$$\begin{aligned} n &= 10 \\ p &= 0.1666 \end{aligned}$$


Bernoulli Distribution

Bernoulli Distribution

X = Number of heads

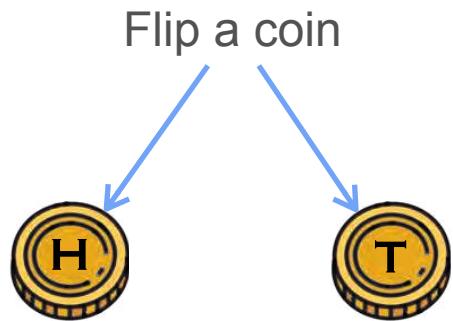
Bernoulli Distribution

X = Number of heads

Flip a coin

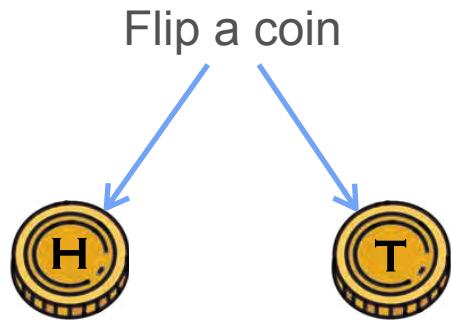
Bernoulli Distribution

X = Number of heads



Bernoulli Distribution

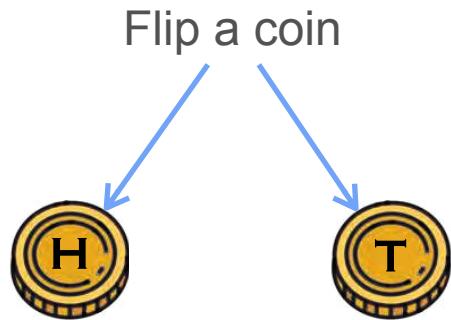
X = Number of heads



$$\mathbf{P}(X = 1) = 0.5$$

Bernoulli Distribution

X = Number of heads

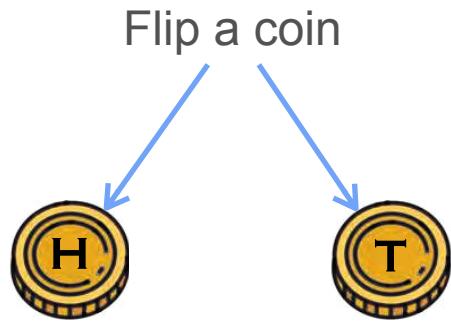


$$\mathbf{P}(X = 1) = 0.5$$

$$\mathbf{P}(X = 0) = 0.5$$

Bernoulli Distribution

X = Number of heads

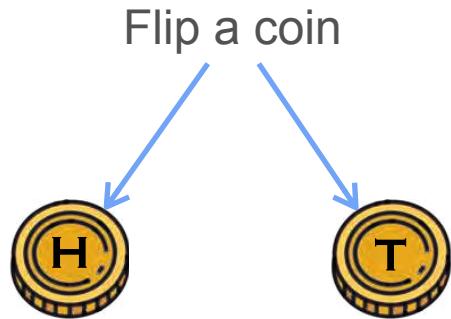


$$\mathbf{P}(X = 1) = 0.5 \quad \mathbf{P}(X = 0) = 0.5$$

Success

Bernoulli Distribution

X = Number of heads



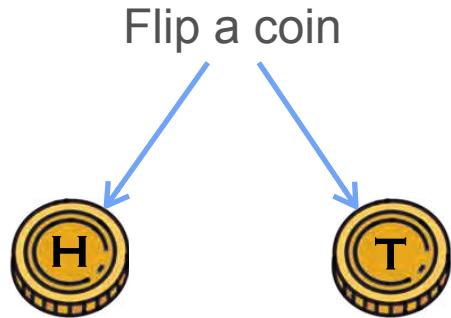
$$P(X = 1) = 0.5 \quad P(X = 0) = 0.5$$

Success

Failure

Bernoulli Distribution

X = Number of heads



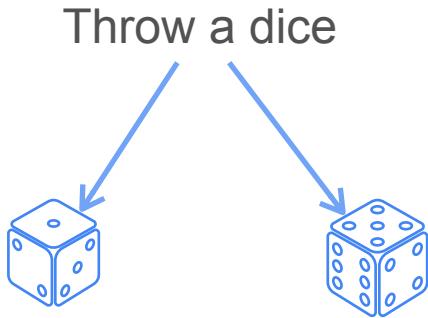
$$P(X = 1) = 0.5$$

Success

$$P(X = 0) = 0.5$$

Failure

X = Number of 1's



$$P(X = 1) = \frac{1}{6}$$

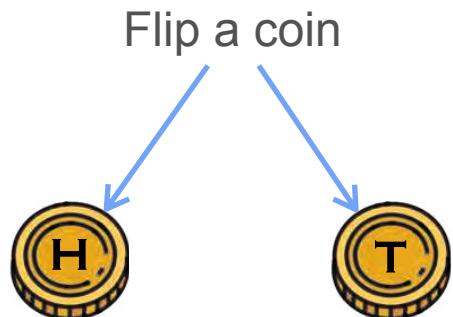
Success

$$P(X = 0) = \frac{5}{6}$$

Failure

Bernoulli Distribution

X = Number of heads



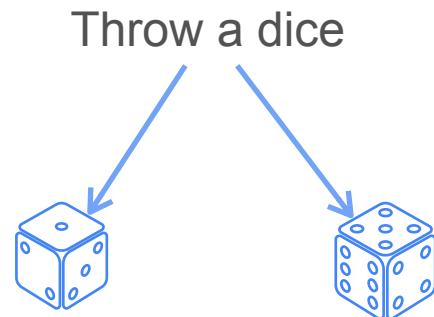
$$P(X = 1) = 0.5$$

Success

$$P(X = 0) = 0.5$$

Failure

X = Number of 1's



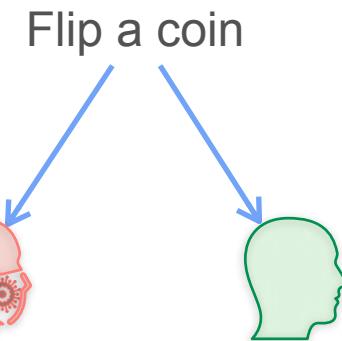
$$P(X = 1) = \frac{1}{6}$$

Success

$$P(X = 0) = \frac{5}{6}$$

Failure

X = Number of sick patients



$$P(X = 1) = p$$

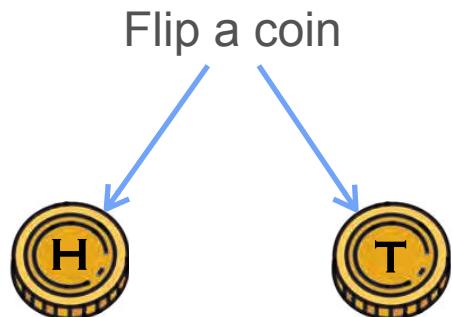
Success

$$P(X = 0) = 1 - p$$

Failure

Bernoulli Distribution

X = Number of heads



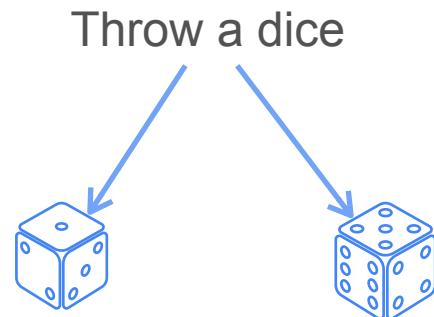
$$P(X = 1) = 0.5$$

Success

$$P(X = 0) = 0.5$$

Failure

X = Number of 1's



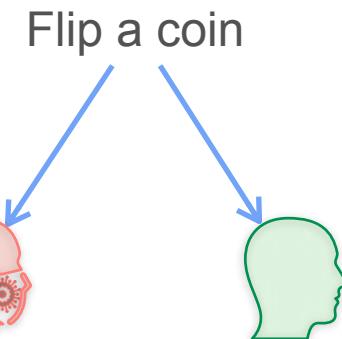
$$P(X = 1) = \frac{1}{6}$$

Success

$$P(X = 0) = \frac{5}{6}$$

Failure

X = Number of sick patients



$$P(X = 1) = p$$

Success

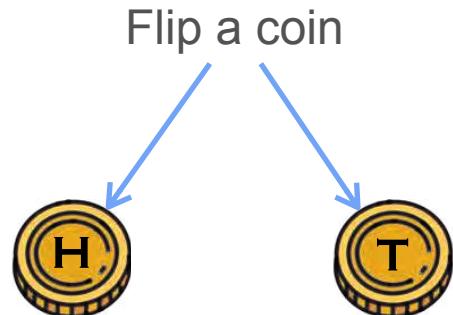
$$P(X = 0) = 1 - p$$

Failure

$$X \sim \text{Bernoulli}(p)$$

Bernoulli Distribution

X = Number of heads



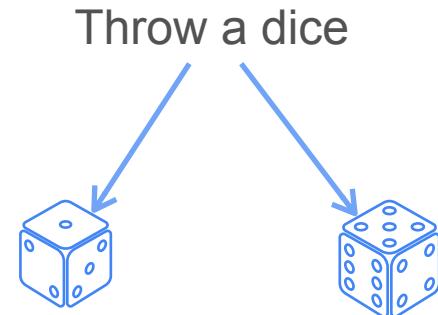
$$P(X = 1) = 0.5$$

Success

$$P(X = 0) = 0.5$$

Failure

X = Number of 1's



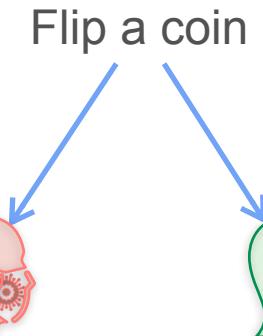
$$P(X = 1) = \frac{1}{6}$$

Success

$$P(X = 0) = \frac{5}{6}$$

Failure

X = Number of sick patients



$$P(X = 1) = p$$

Success

$$P(X = 0) = 1 - p$$

Failure

$$X \sim \text{Bernoulli}(p)$$

p is the parameter of the Bernoulli distribution

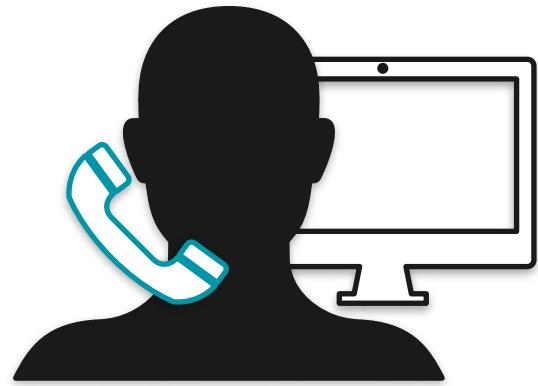


DeepLearning.AI

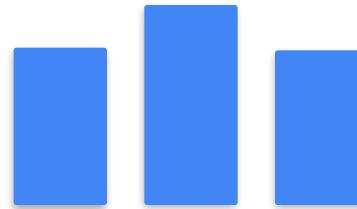
Probability Distributions

Probability Distributions (Continuous)

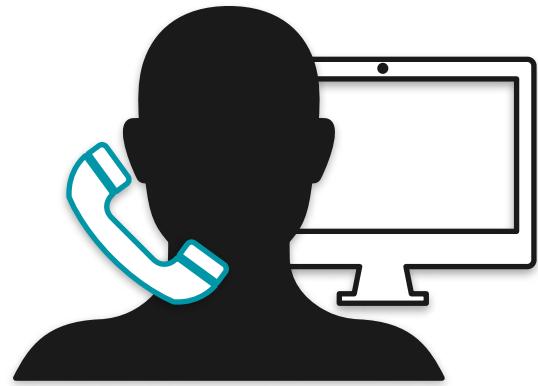
From Discrete to Continuous Distributions



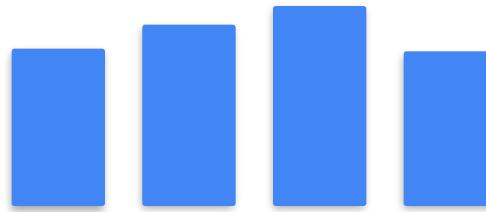
Waiting time: 1 2 3 (min)



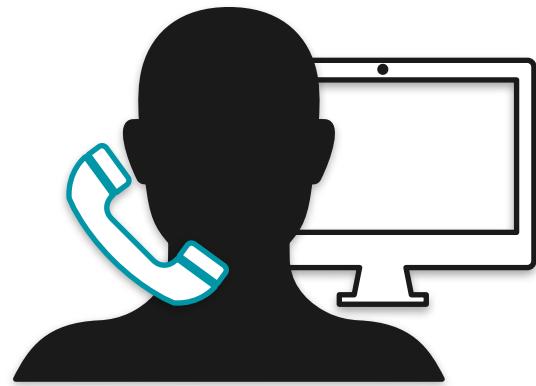
From Discrete to Continuous Distributions



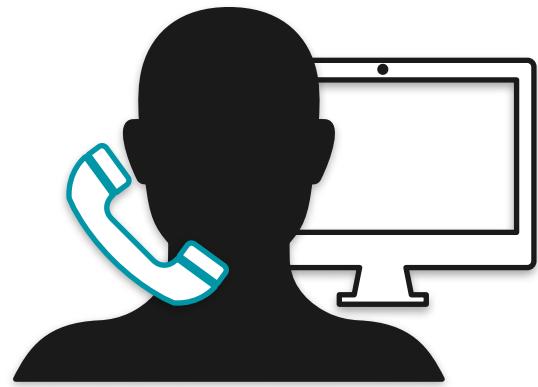
Waiting time: 1 1.01 2 3 (min)



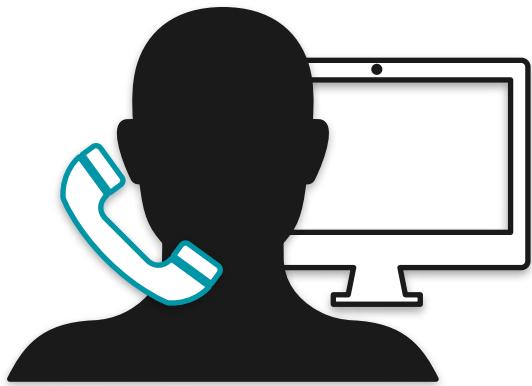
From Discrete to Continuous Distributions



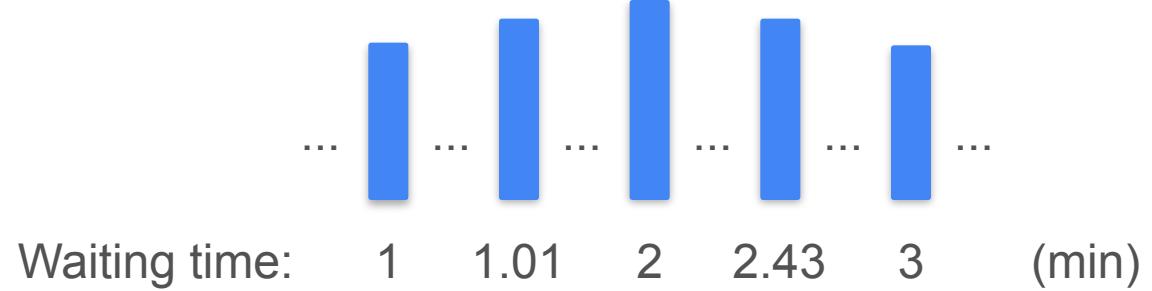
From Discrete to Continuous Distributions



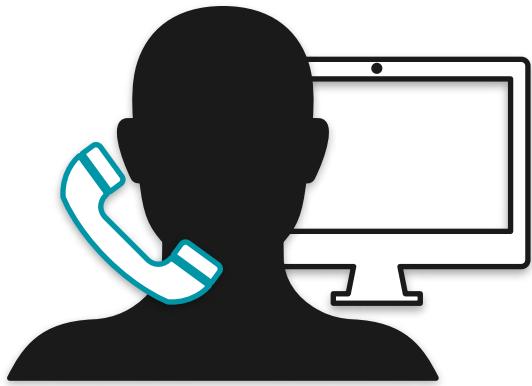
From Discrete to Continuous Distributions



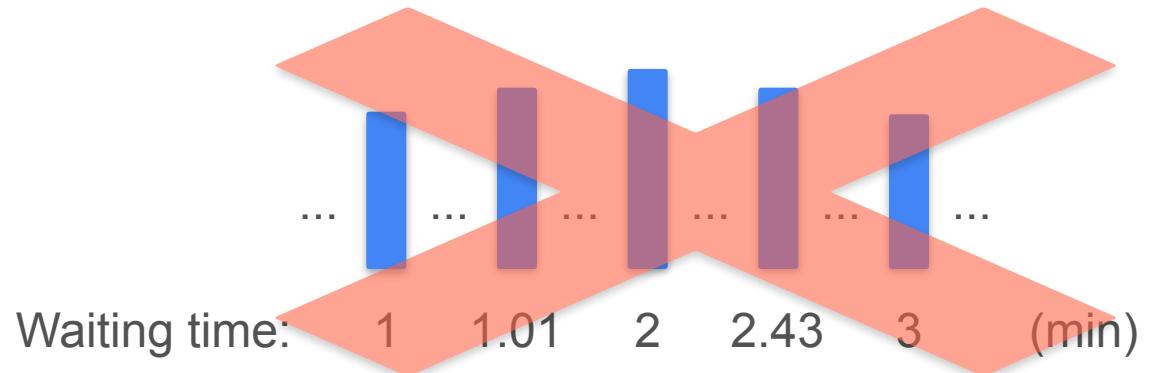
This is a continuous distribution!



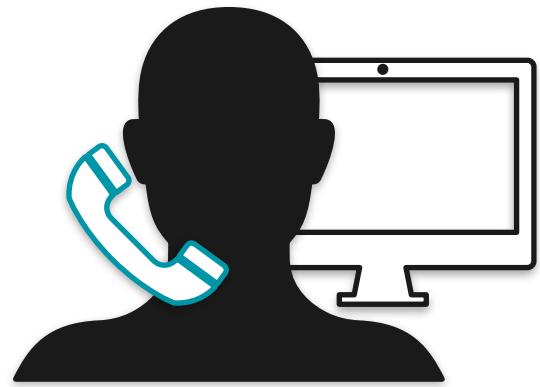
From Discrete to Continuous Distributions



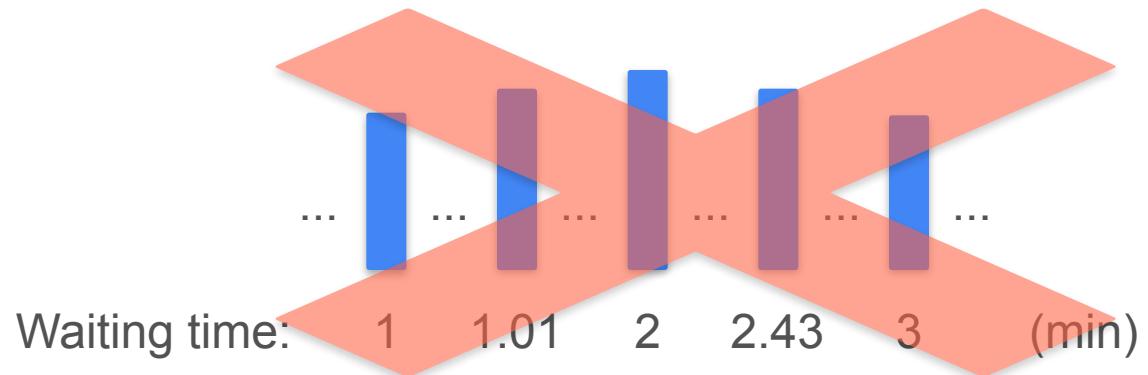
This is a continuous distribution!



From Discrete to Continuous Distributions

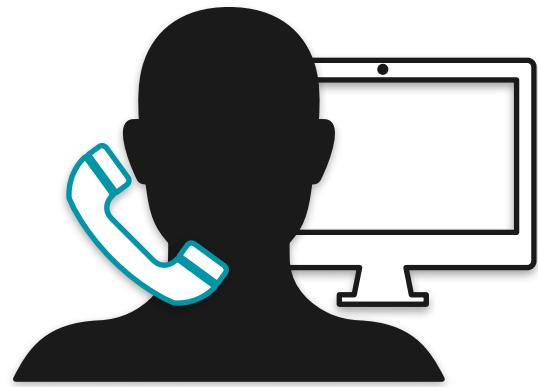


This is a continuous distribution!

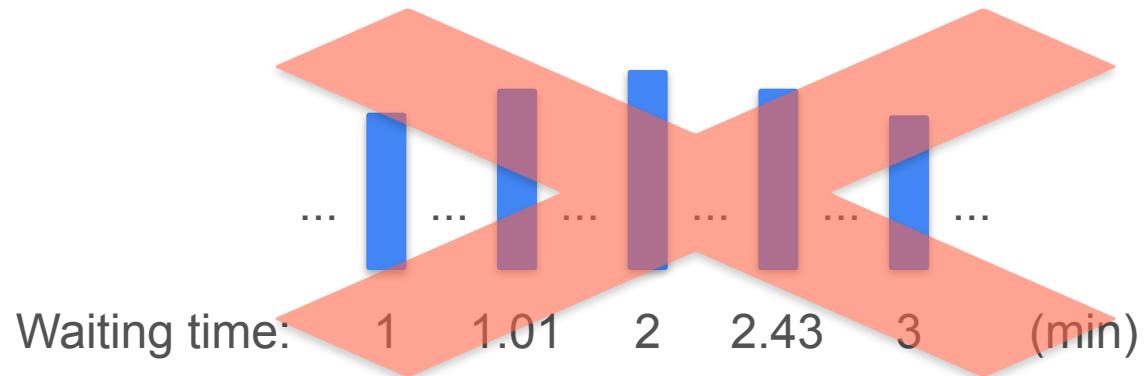


What is the probability that you will wait EXACTLY one minute for the call?

From Discrete to Continuous Distributions



This is a continuous distribution!



What is the probability that you will wait EXACTLY one minute for the call?

Answer: ZERO

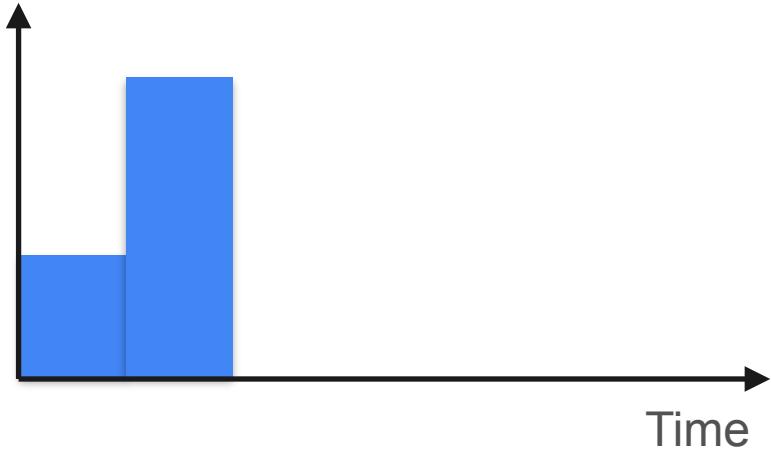
From Discrete to Continuous Distributions



From Discrete to Continuous Distributions



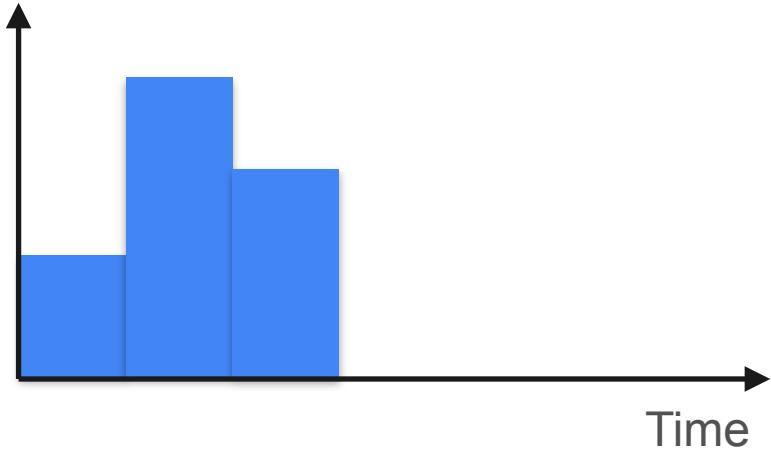
From Discrete to Continuous Distributions



$P(\text{between 0 and 1 mins})$

$P(\text{between 1 and 2 mins})$

From Discrete to Continuous Distributions

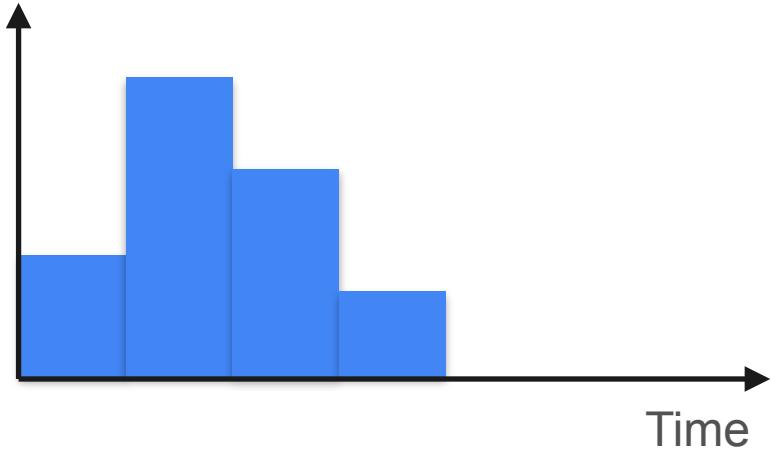


$P(\text{between 0 and 1 mins})$

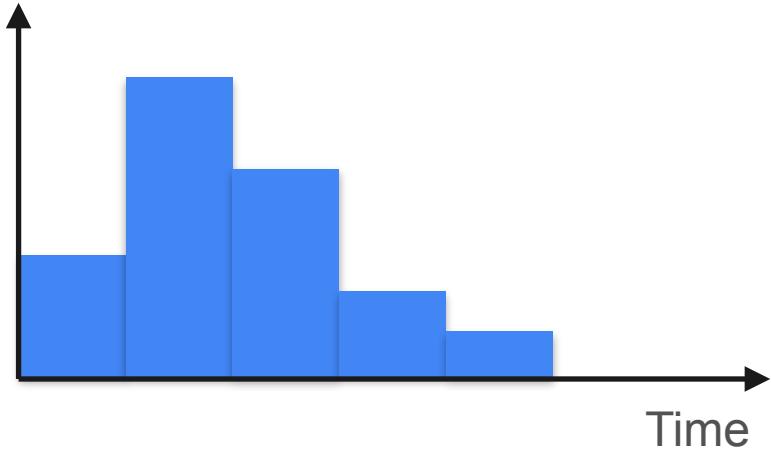
$P(\text{between 1 and 2 mins})$

$P(\text{between 2 and 3 mins})$

From Discrete to Continuous Distributions



From Discrete to Continuous Distributions



$P(\text{between 0 and 1 mins})$

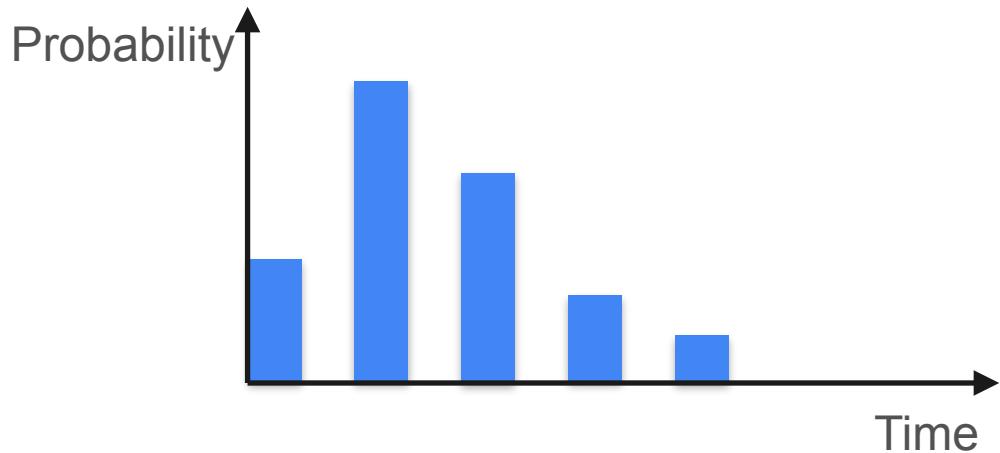
$P(\text{between 1 and 2 mins})$

$P(\text{between 2 and 3 mins})$

$P(\text{between 3 and 4 mins})$

$P(\text{between 4 and 5 mins})$

From Discrete to Continuous Distributions



$P(\text{between 0 and 1 mins})$

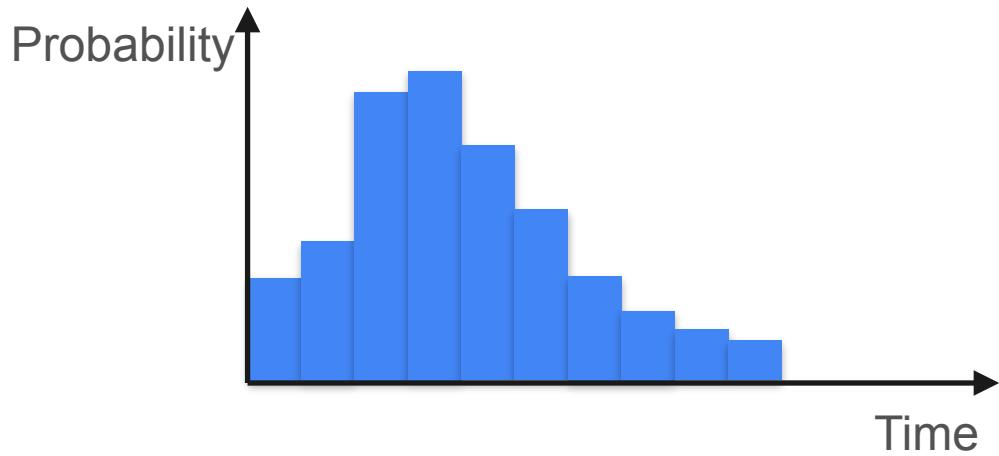
$P(\text{between 1 and 2 mins})$

$P(\text{between 2 and 3 mins})$

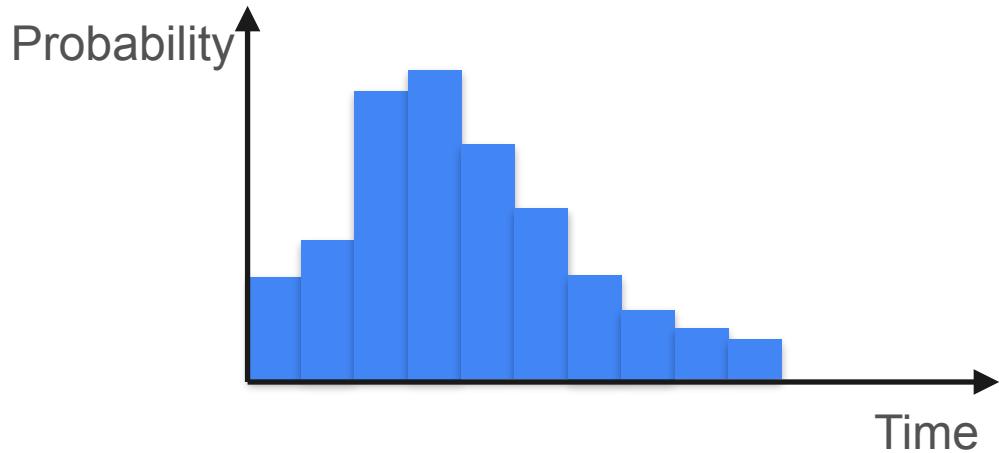
$P(\text{between 3 and 4 mins})$

$P(\text{between 4 and 5 mins})$

From Discrete to Continuous Distributions

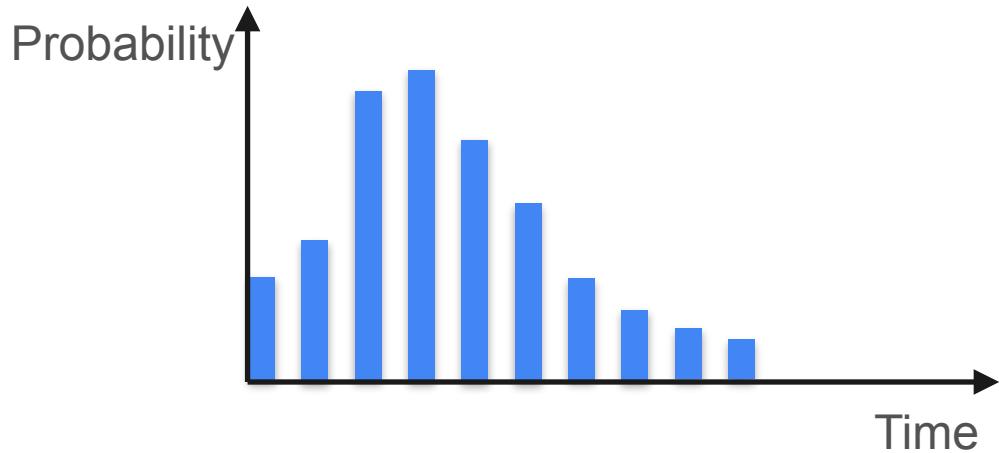


From Discrete to Continuous Distributions



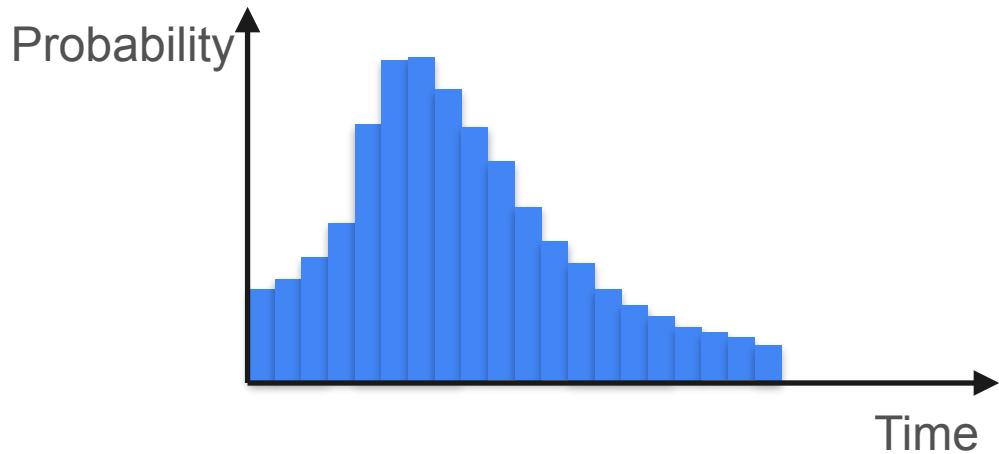
$P(\text{between } 0 \text{ and } 0.5 \text{ mins})$
 $P(\text{between } 0.5 \text{ and } 1 \text{ mins})$
 $P(\text{between } 1 \text{ and } 1.5 \text{ mins})$
⋮
 $P(\text{between } 3.5 \text{ and } 4 \text{ mins})$
 $P(\text{between } 4 \text{ and } 4.5 \text{ mins})$
 $P(\text{between } 4.5 \text{ and } 5 \text{ mins})$

From Discrete to Continuous Distributions

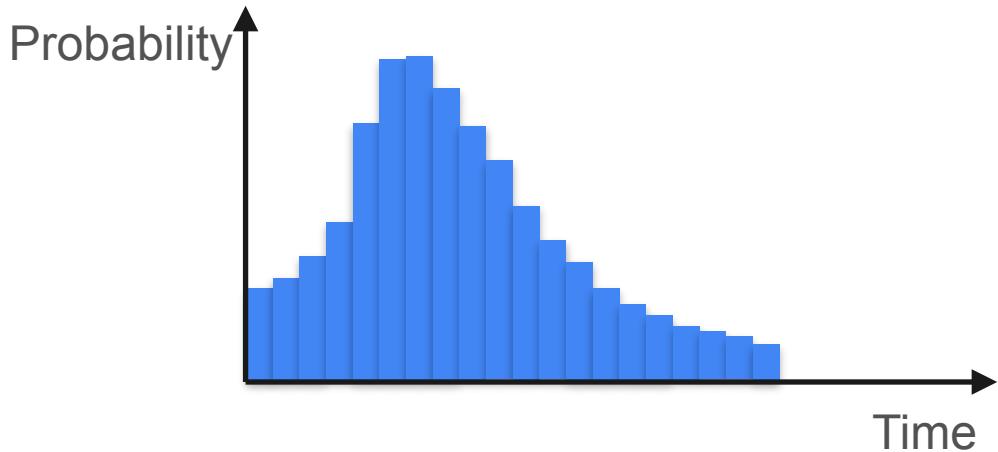


$P(\text{between } 0 \text{ and } 0.5 \text{ mins})$
 $P(\text{between } 0.5 \text{ and } 1 \text{ mins})$
 $P(\text{between } 1 \text{ and } 1.5 \text{ mins})$
⋮
 $P(\text{between } 3.5 \text{ and } 4 \text{ mins})$
 $P(\text{between } 4 \text{ and } 4.5 \text{ mins})$
 $P(\text{between } 4.5 \text{ and } 5 \text{ mins})$

From Discrete to Continuous Distributions

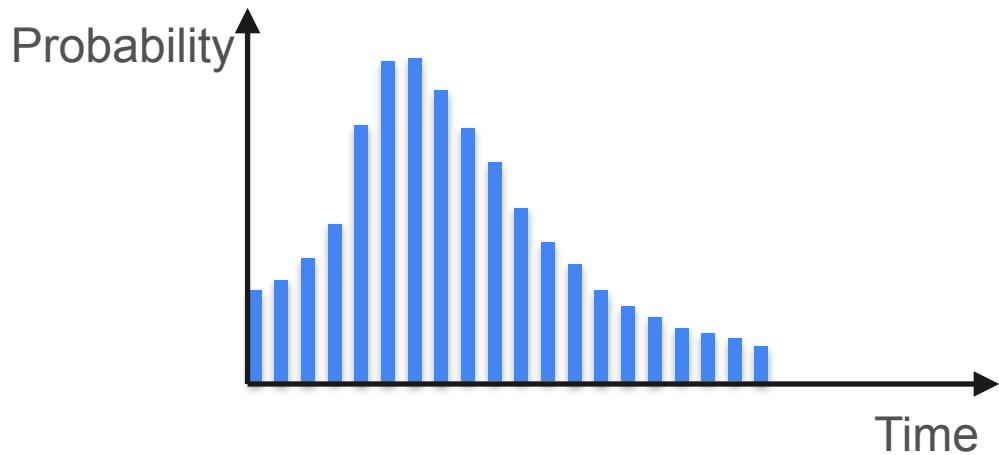


From Discrete to Continuous Distributions

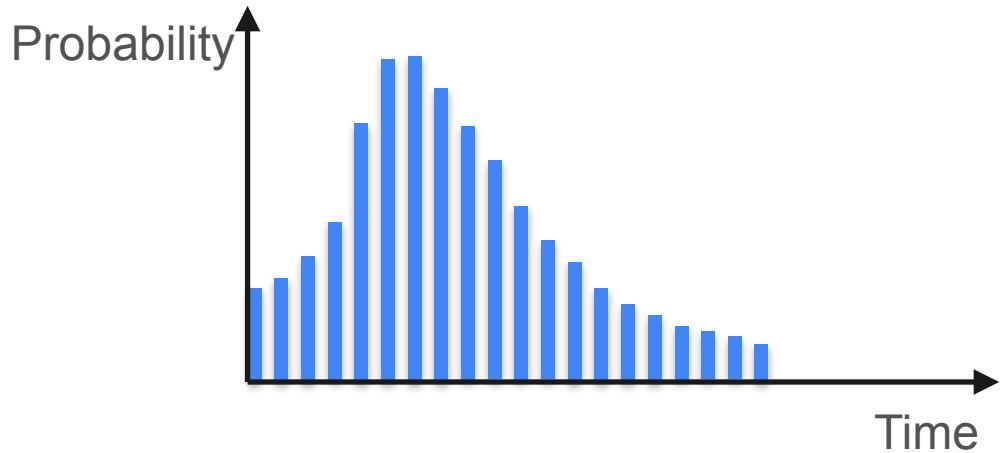


$P(\text{between } 0 \text{ and } 0.25 \text{ mins})$
 $P(\text{between } 0.25 \text{ and } 0.5 \text{ mins})$
 $P(\text{between } 0.5 \text{ and } 0.75 \text{ mins})$
⋮
 $P(\text{between } 4.25 \text{ and } 4.5 \text{ mins})$
 $P(\text{between } 4.5 \text{ and } 4.75 \text{ mins})$
 $P(\text{between } 4.75 \text{ and } 5 \text{ mins})$

From Discrete to Continuous Distributions

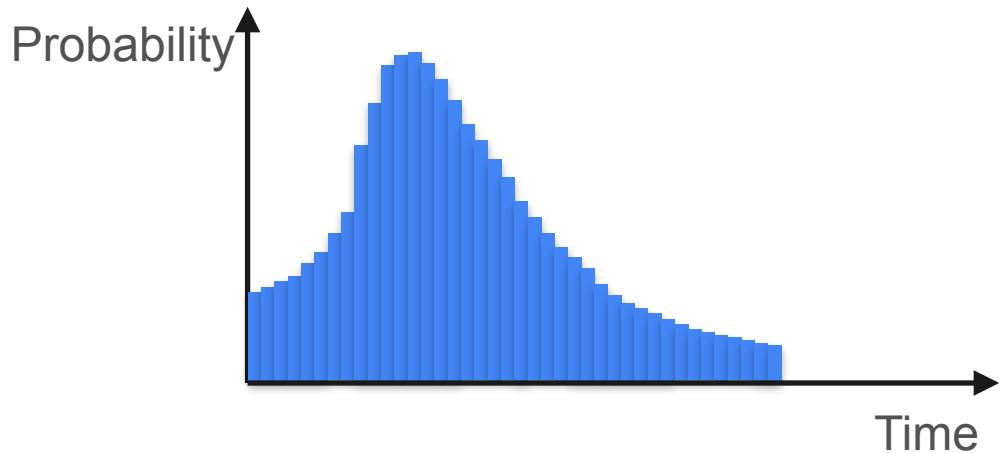


From Discrete to Continuous Distributions



$P(\text{between } 0 \text{ and } 0.25 \text{ mins})$
 $P(\text{between } 0.25 \text{ and } 0.5 \text{ mins})$
 $P(\text{between } 0.5 \text{ and } 0.75 \text{ mins})$
⋮
 $P(\text{between } 4.25 \text{ and } 4.5 \text{ mins})$
 $P(\text{between } 4.5 \text{ and } 4.75 \text{ mins})$
 $P(\text{between } 4.75 \text{ and } 5 \text{ mins})$

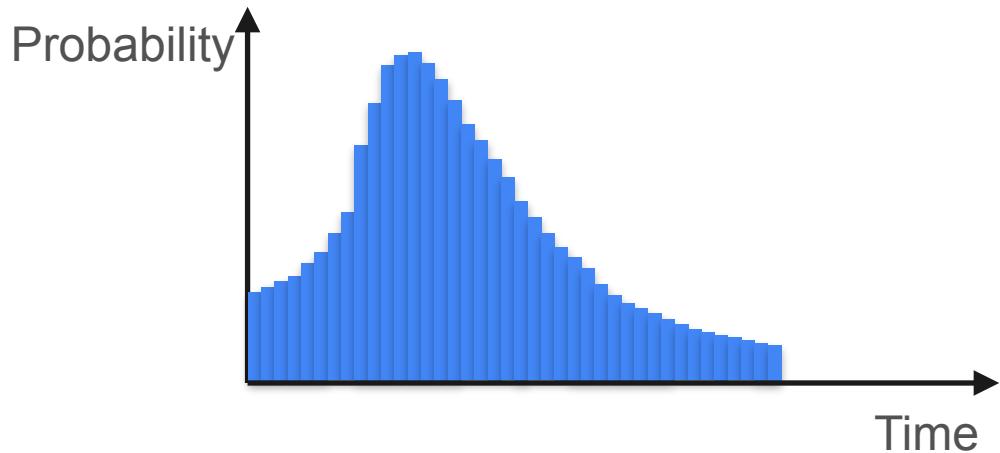
From Discrete to Continuous Distributions



$P(\text{between } 0 \text{ and } 0.125 \text{ mins})$

$P(\text{between } 4.875 \text{ and } 5 \text{ mins})$

From Discrete to Continuous Distributions

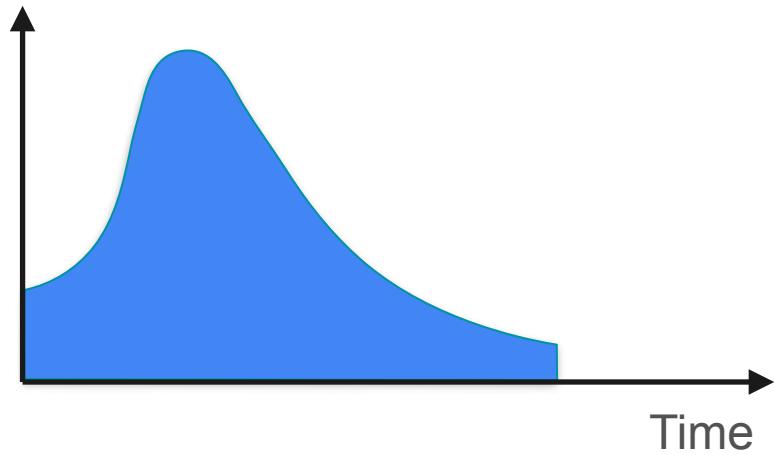


$P(\text{between } 0 \text{ and } 0.125 \text{ mins})$

⋮

$P(\text{between } 4.875 \text{ and } 5 \text{ mins})$

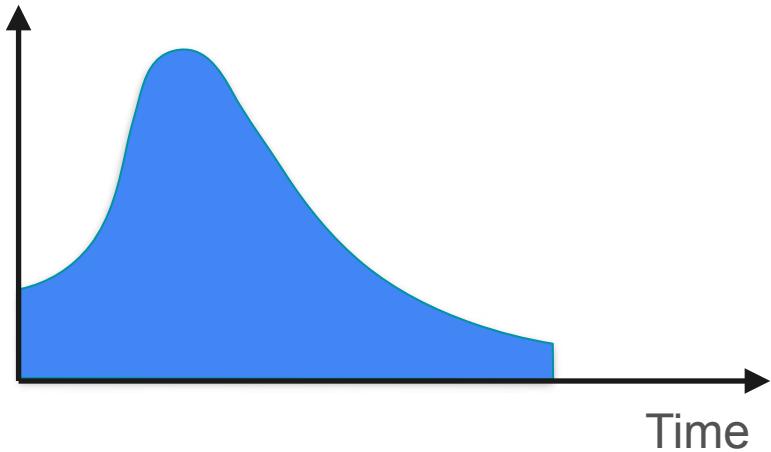
From Discrete to Continuous Distributions



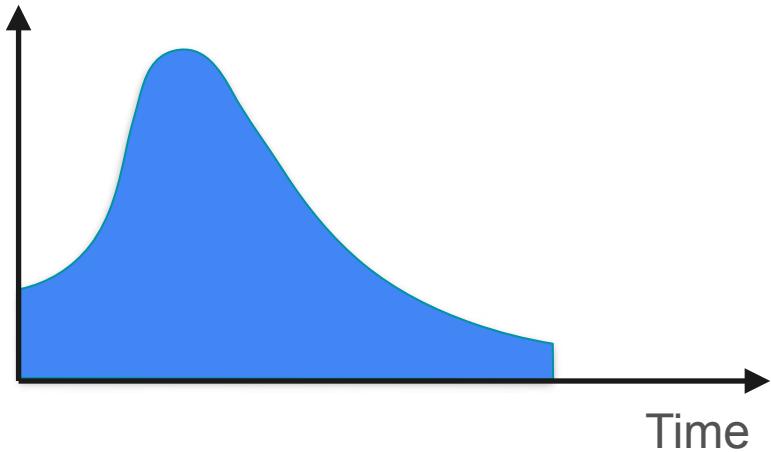
From Discrete to Continuous Distributions



- Discrete:
 - Sum of heights equals 1

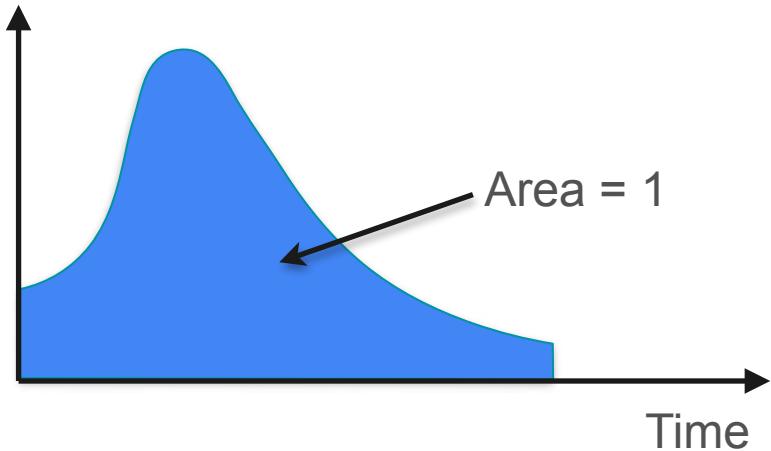


From Discrete to Continuous Distributions



- Discrete:
 - Sum of heights equals 1
- Continuous:
 - Area under the curve equals 1

From Discrete to Continuous Distributions



- Discrete:
 - Sum of heights equals 1
- Continuous:
 - Area under the curve equals 1

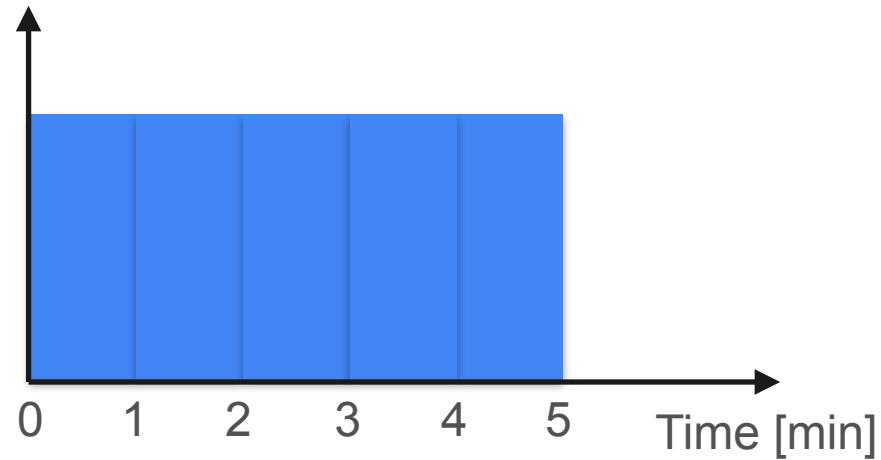


DeepLearning.AI

Probability Distributions

Probability density function

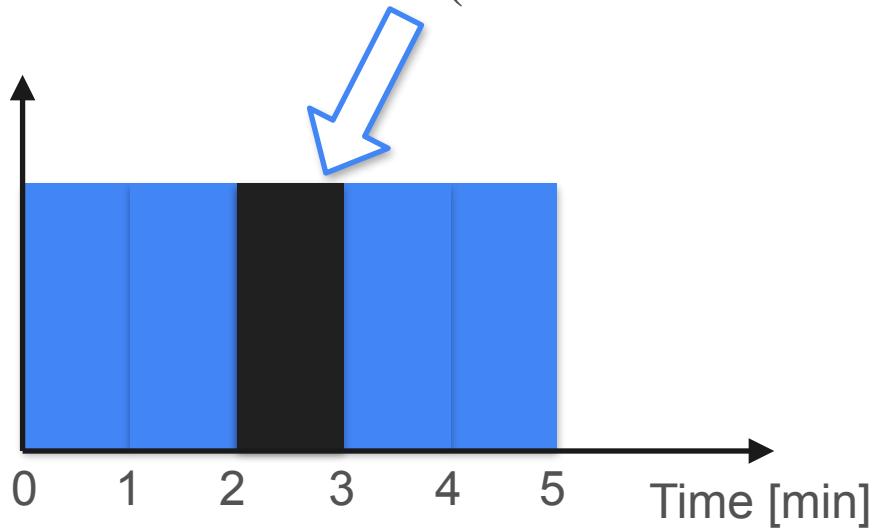
Probability Density Function



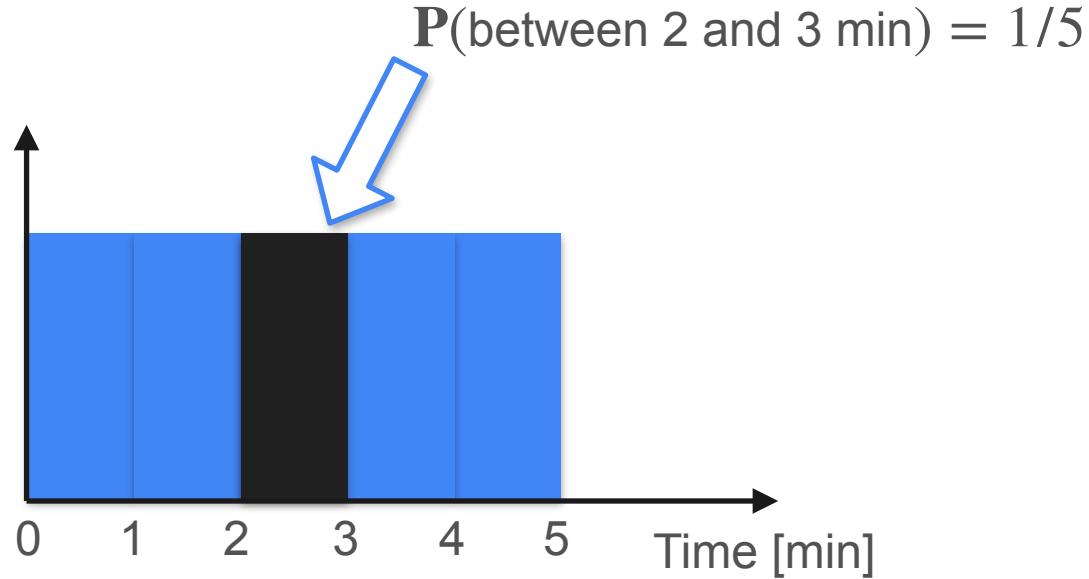
Probability Density Function



$P(\text{between 2 and 3 min}) = ?$



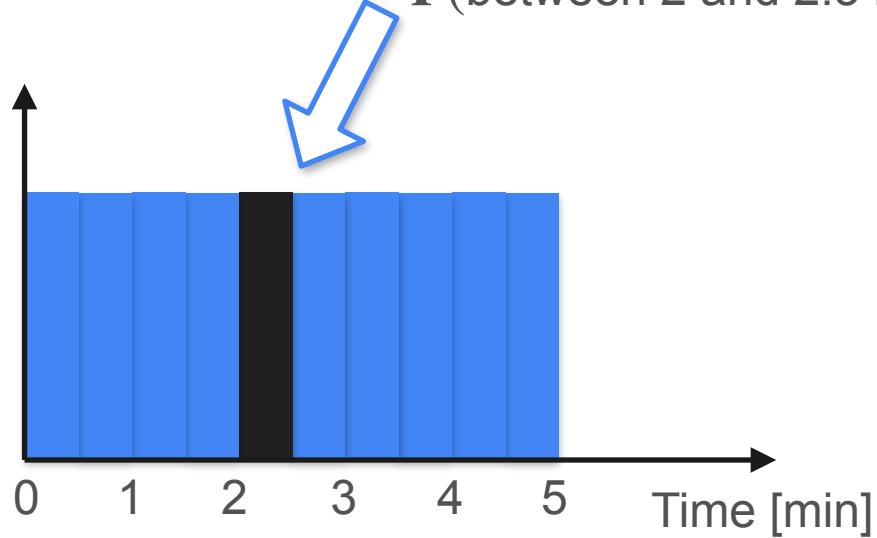
Probability Density Function



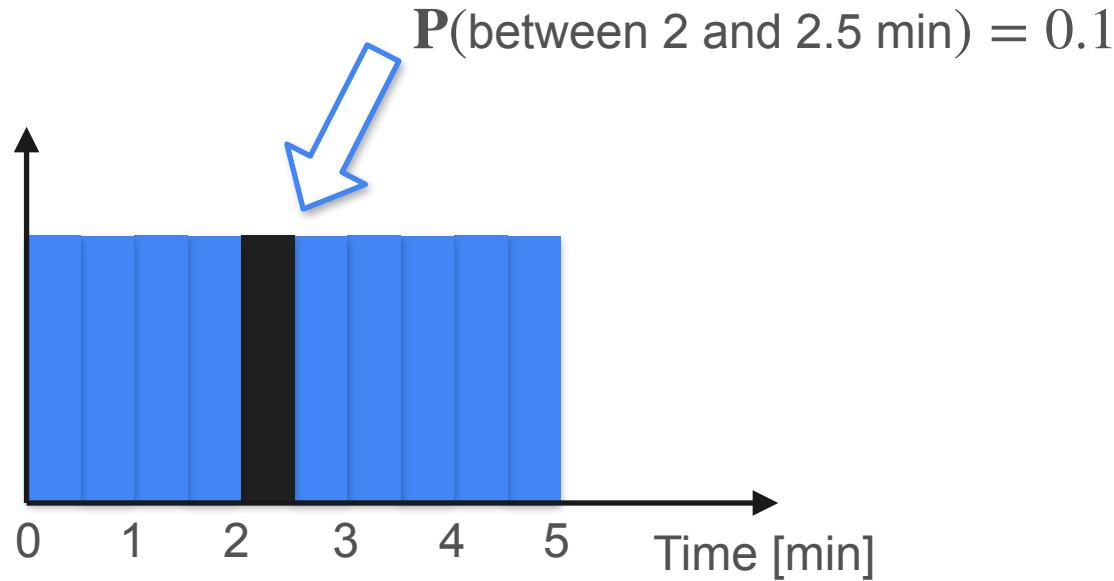
Probability Density Function



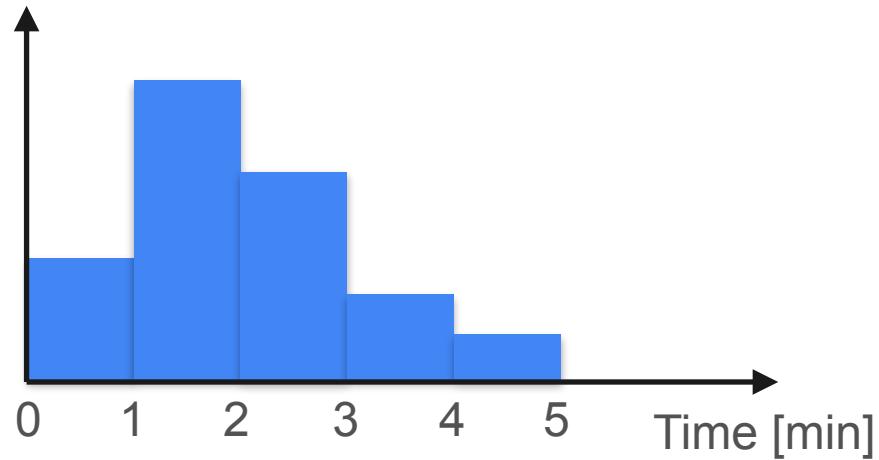
$P(\text{between } 2 \text{ and } 2.5 \text{ min}) = ?$



Probability Density Function



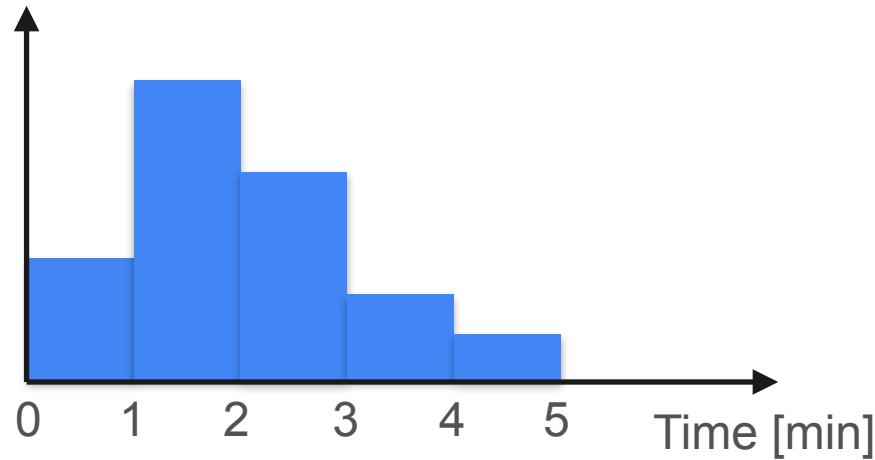
Probability Density Function



Probability Density Function



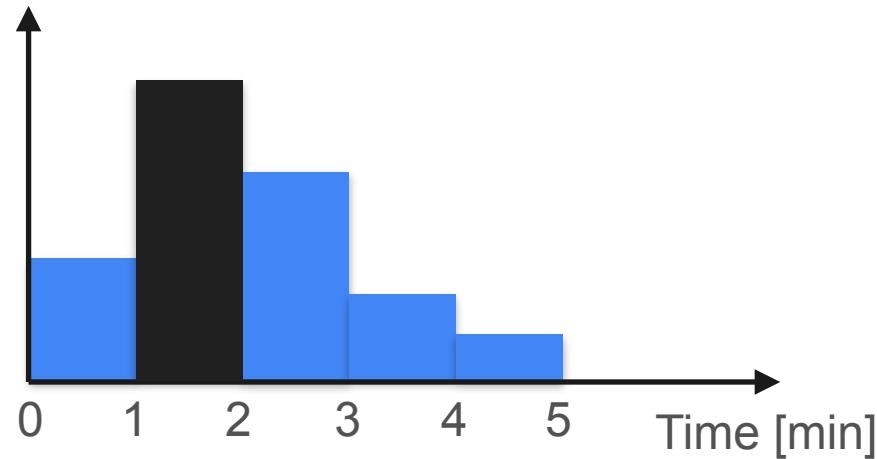
$P(\text{between 1 and 2 min})$



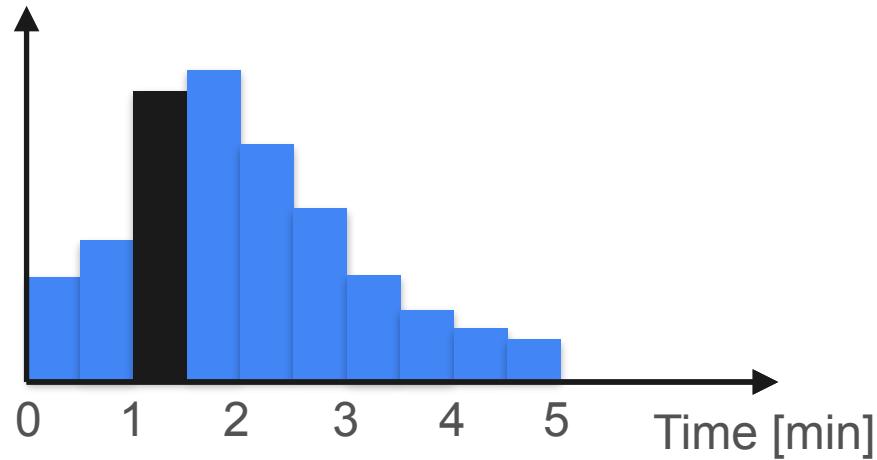
Probability Density Function



$P(\text{between 1 and 2 min})$



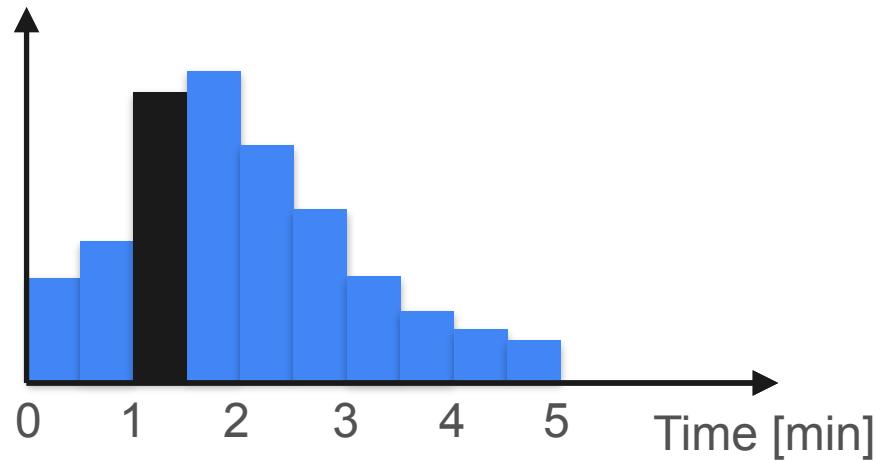
Probability Density Function



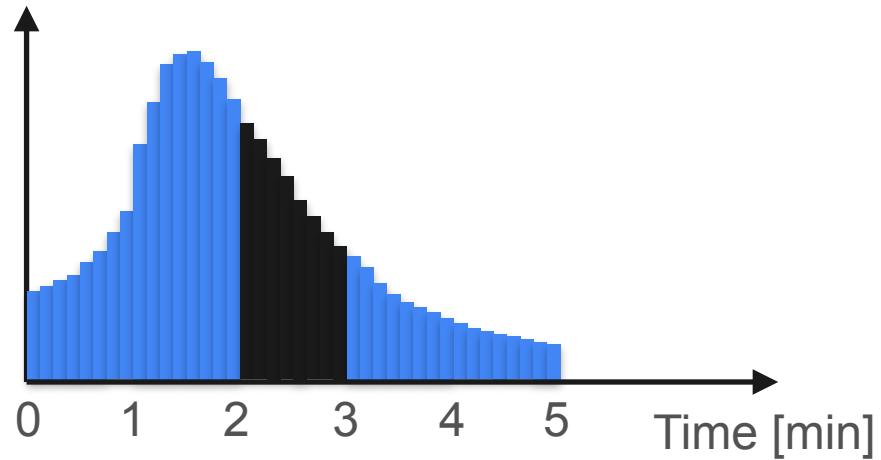
Probability Density Function



$P(\text{between 1 and 1:30})$



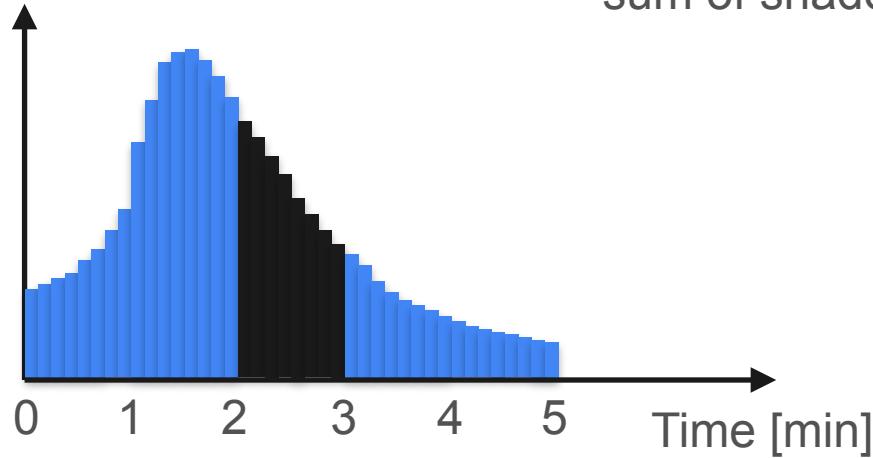
Probability Density Function



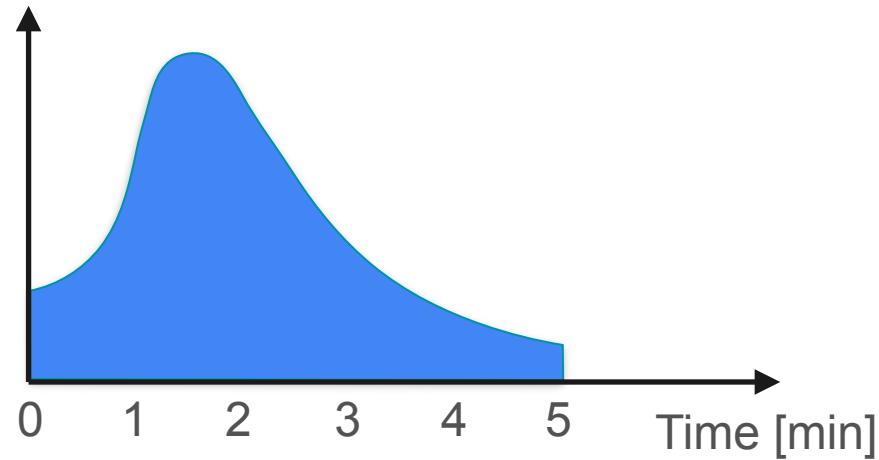
Probability Density Function



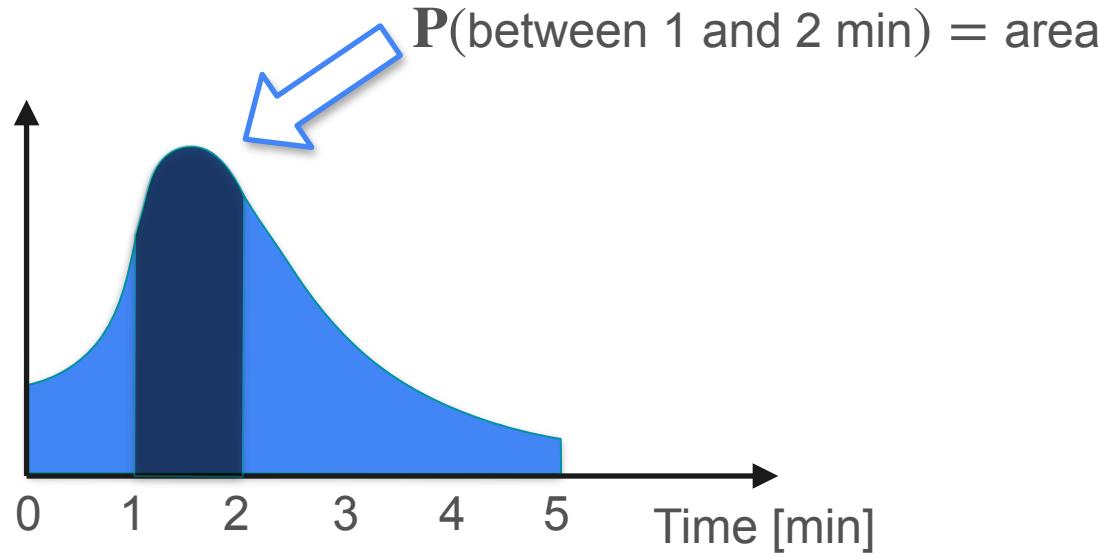
$P(\text{between 2 and 3 min}) =$
sum of shaded areas



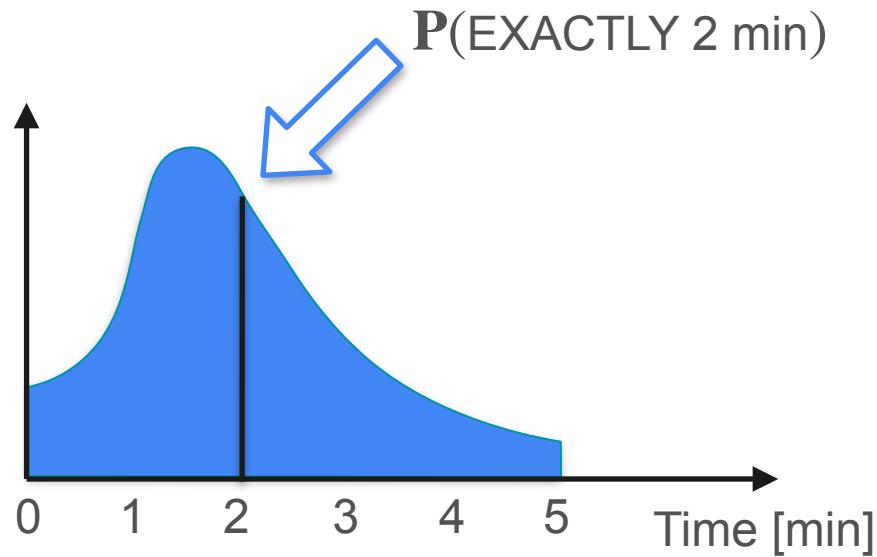
Probability Density Function



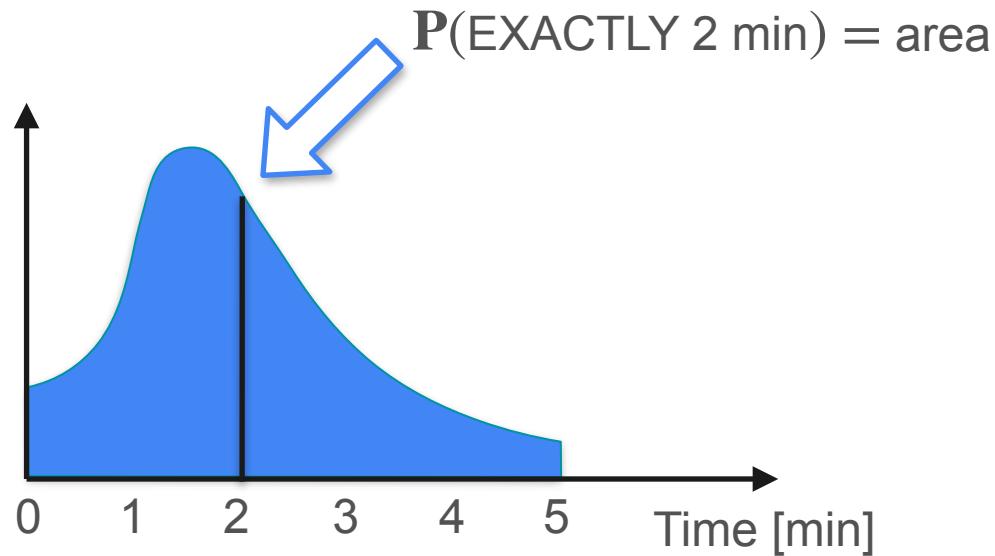
Probability Density Function



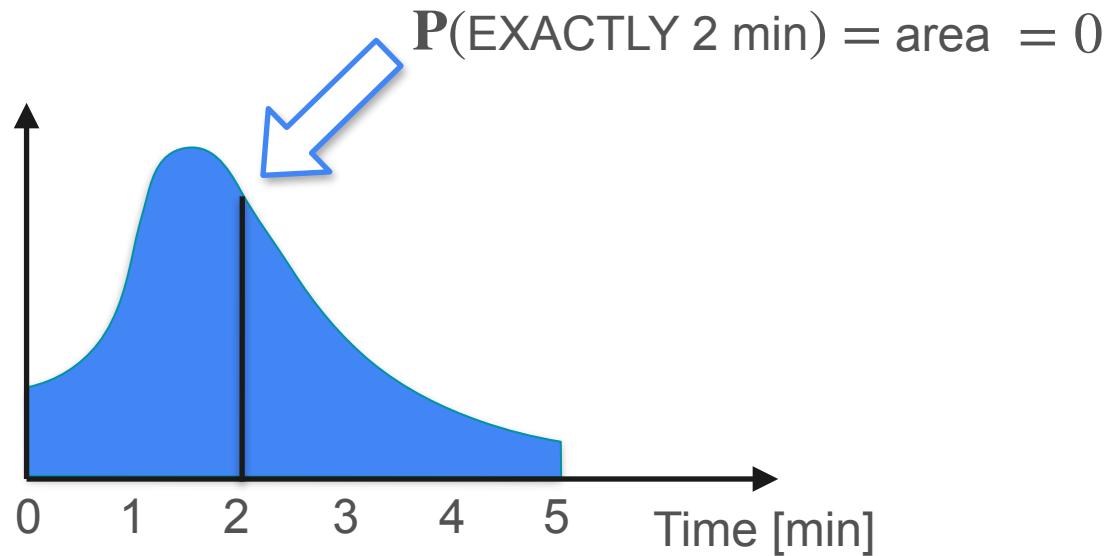
Probability Density Function



Probability Density Function

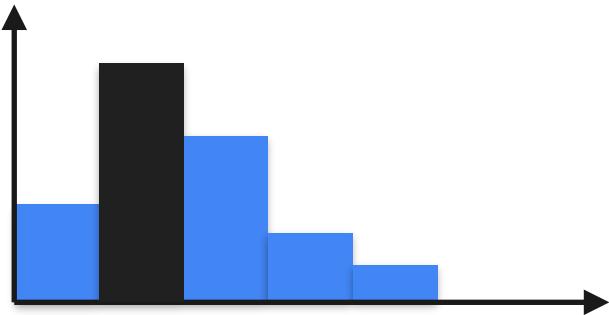


Probability Density Function

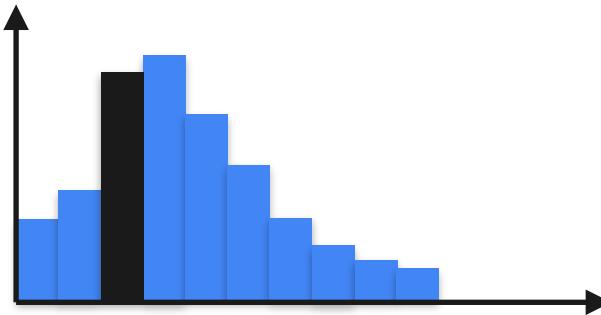
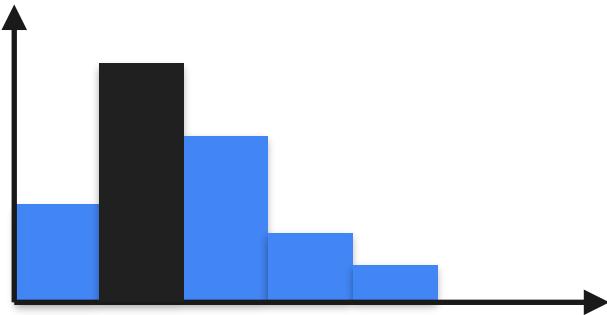


Probability Density Function

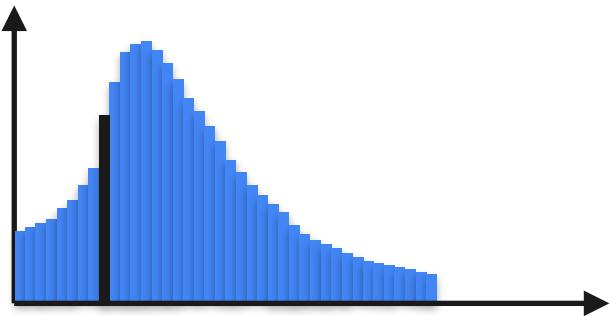
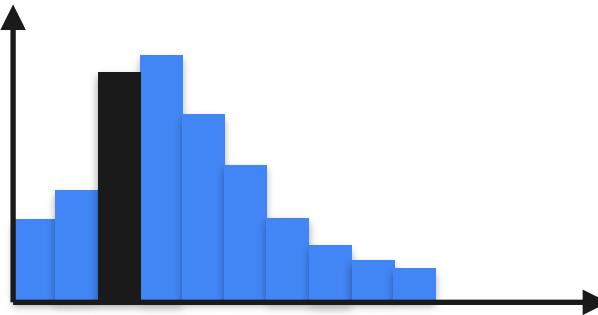
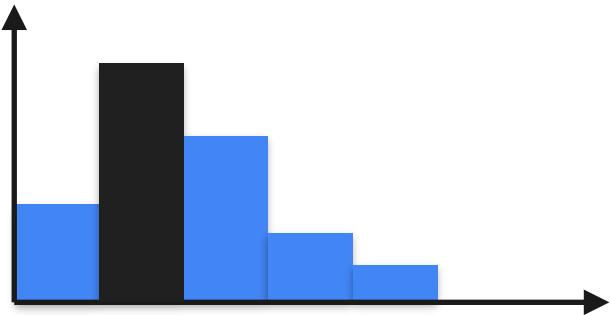
Probability Density Function



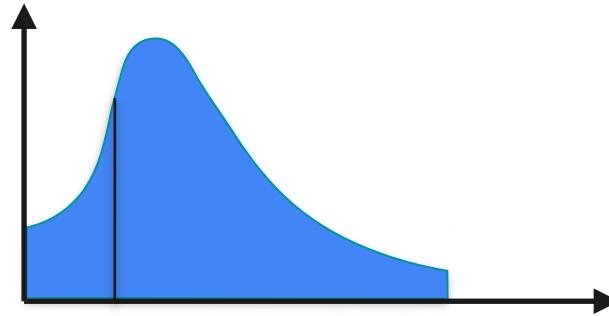
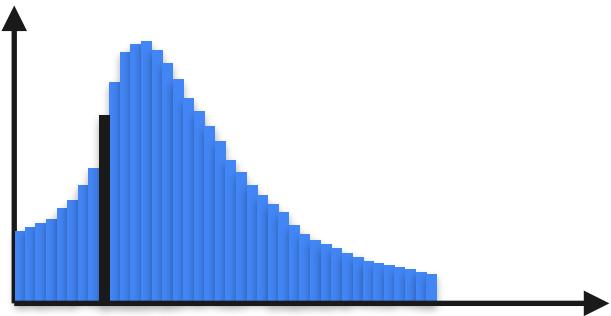
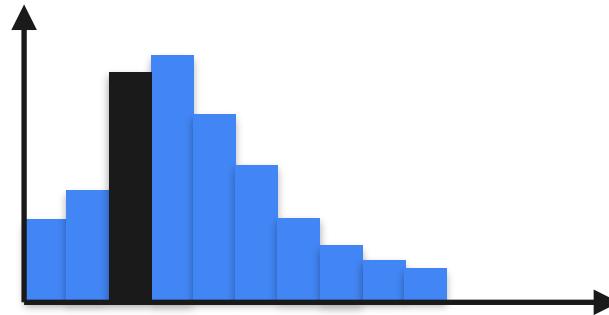
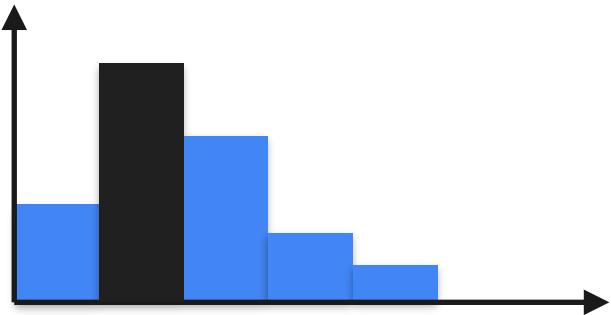
Probability Density Function



Probability Density Function

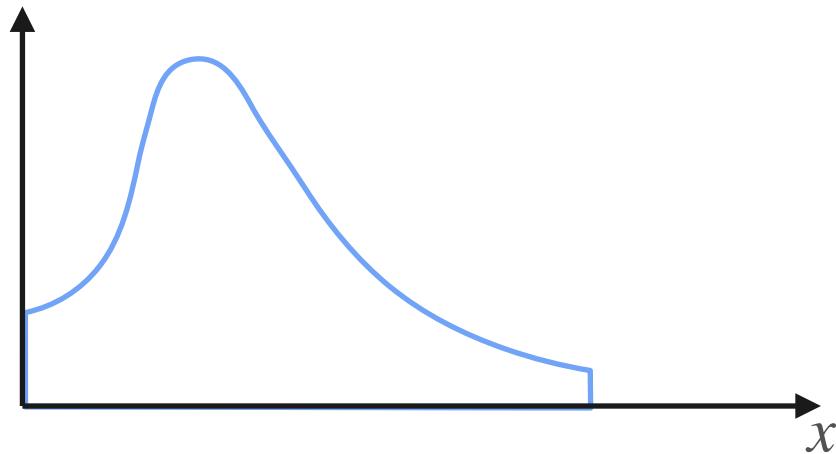


Probability Density Function

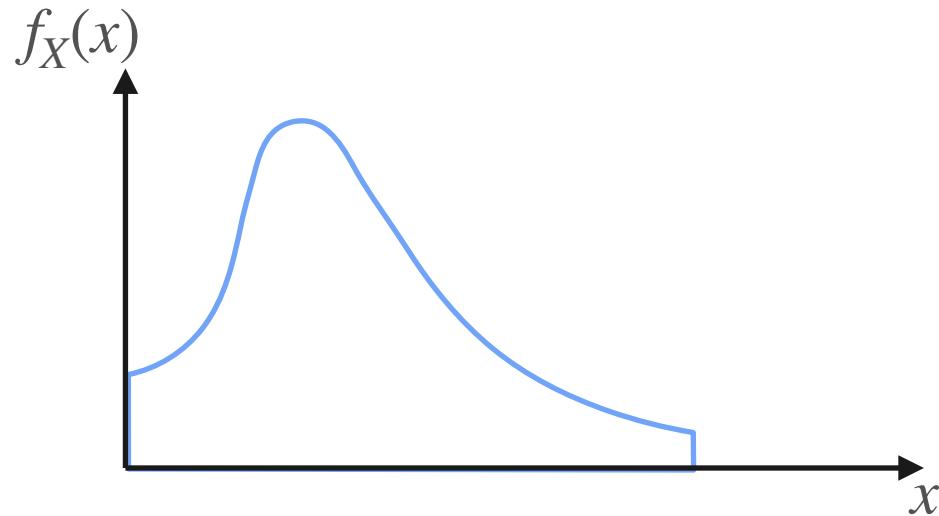


Probability Density Function: Formal Definition

Probability Density Function: Formal Definition

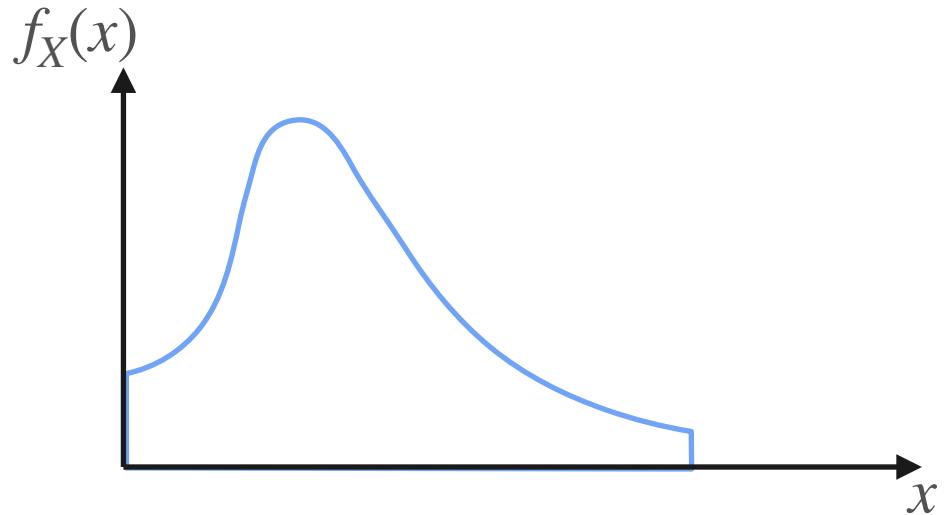


Probability Density Function: Formal Definition



Probability Density Function (PDF)
 $f_X(x)$

Probability Density Function: Formal Definition

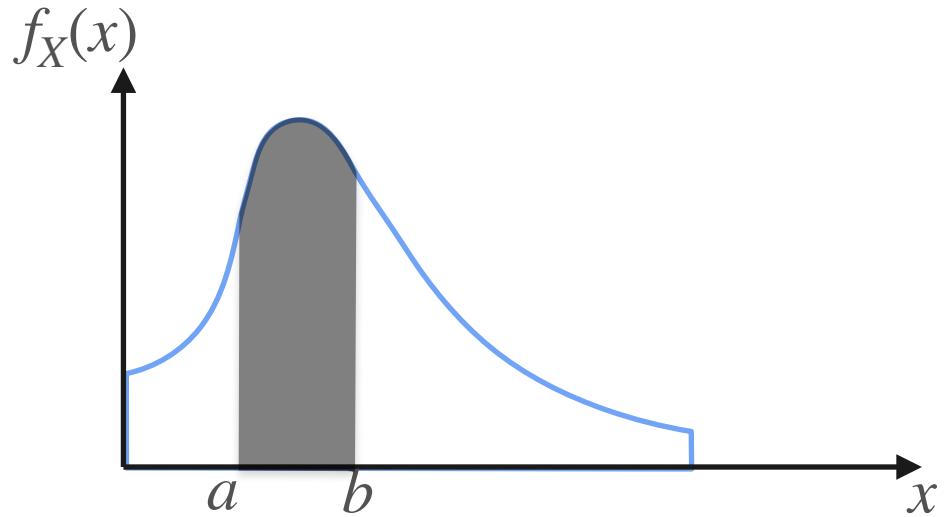


Probability Density Function (PDF)

$$f_X(x)$$

Tells you the rate you accumulate probability around each point.
Only defined for continuous variables!

Probability Density Function: Formal Definition



Probability Density Function (PDF)

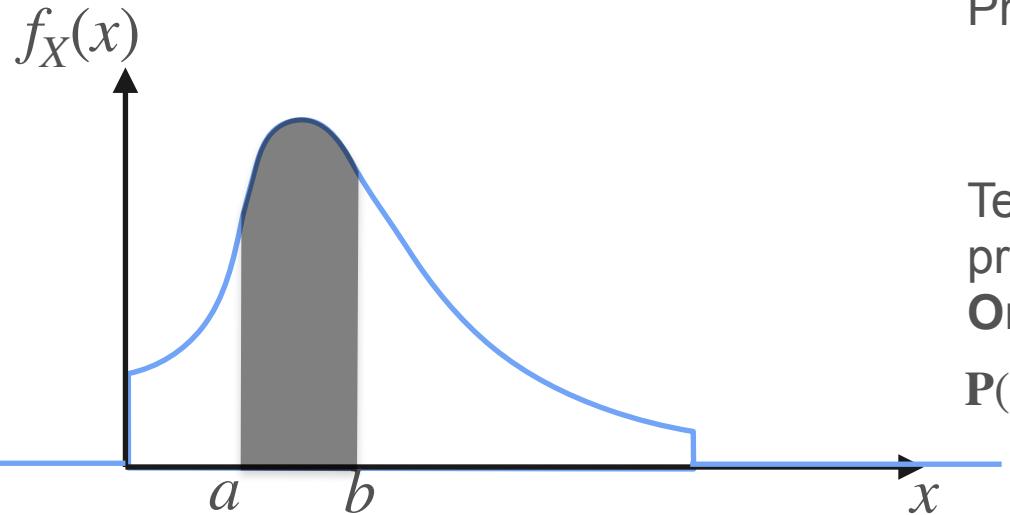
$$f_X(x)$$

Tells you the rate you accumulate probability around each point.

Only defined for continuous variables!

$P(a < X < b) = \text{area under } f_X(x)$

Probability Density Function: Formal Definition



Probability Density Function (PDF)

$$f_X(x)$$

Tells you the rate you accumulate probability around each point.

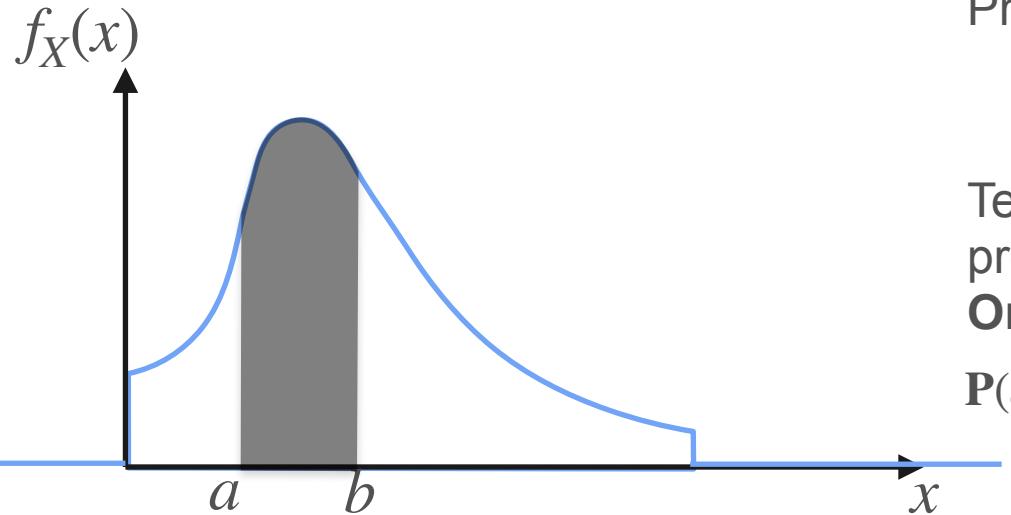
Only defined for continuous variables!

$P(a < X < b) = \text{area under } f_X(x)$

$f_X(x)$ needs to satisfy:

- It is defined for all numbers

Probability Density Function: Formal Definition



Probability Density Function (PDF)

$$f_X(x)$$

Tells you the rate you accumulate probability around each point.

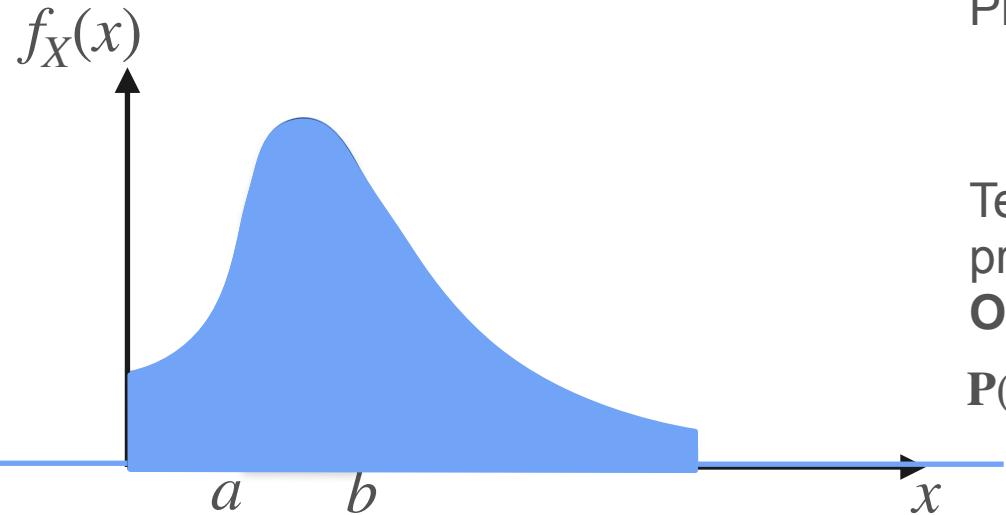
Only defined for continuous variables!

$P(a < X < b) = \text{area under } f_X(x)$

$f_X(x)$ needs to satisfy:

- It is defined for all numbers
- $f_X(x) \geq 0$

Probability Density Function: Formal Definition



Probability Density Function (PDF)

$$f_X(x)$$

Tells you the rate you accumulate probability around each point.

Only defined for continuous variables!

$$\mathbf{P}(a < X < b) = \text{area under } f_X(x)$$

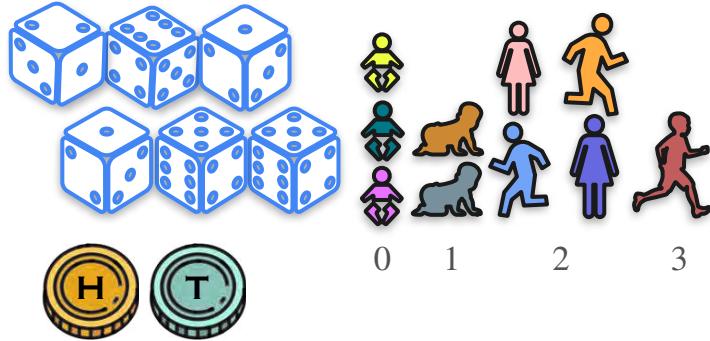
$f_X(x)$ needs to satisfy:

- It is defined for all numbers
- $f_X(x) \geq 0$
- Area under $f_X(x) = 1$

Discrete and Continuous Random Variables

Discrete and Continuous Random Variables

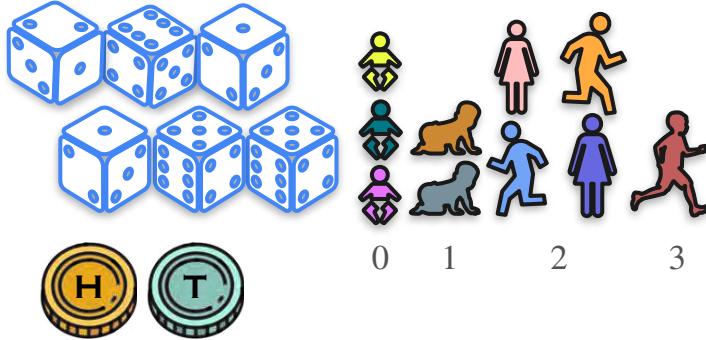
Discrete random variables



Can take only a **finite** or at most countable number of values

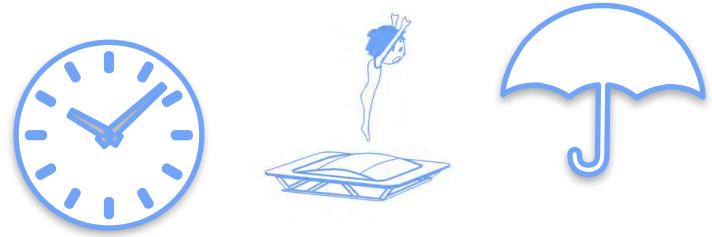
Discrete and Continuous Random Variables

Discrete random variables



Can take only a **finite** or at most countable number of values

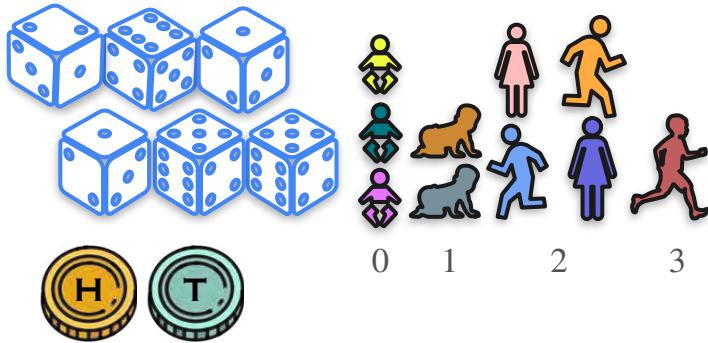
Continuous random variables



Takes values on an interval
(infinite possibilities!)

Discrete and Continuous Random Variables

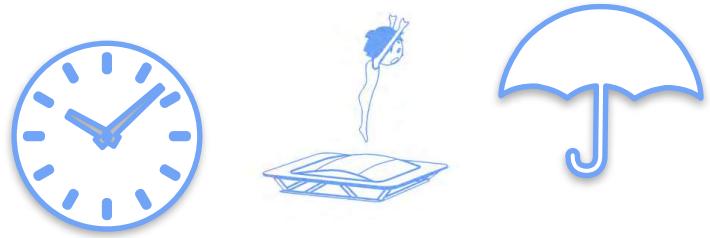
Discrete random variables



Can take only a **finite** or at most countable number of values

$$\text{PMF: } p_X(x) = \mathbf{P}(X = x)$$

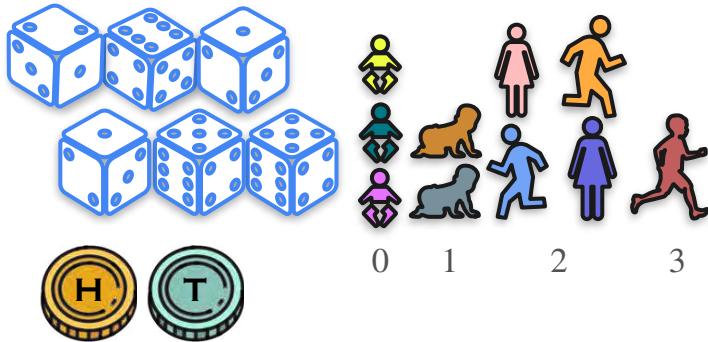
Continuous random variables



Takes values on an interval
(infinite possibilities!)

Discrete and Continuous Random Variables

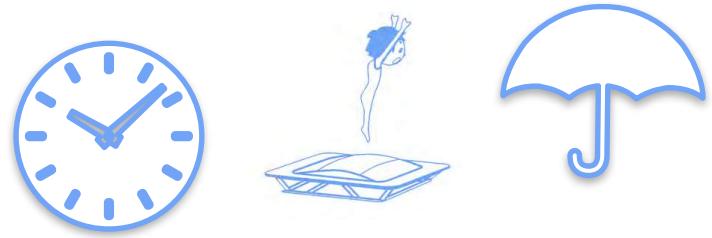
Discrete random variables



Can take only a **finite** or at most countable number of values

$$\text{PMF: } p_X(x) = \mathbf{P}(X = x)$$

Continuous random variables

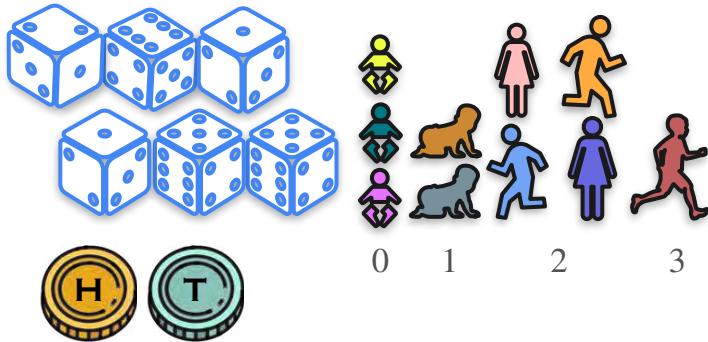


Takes values on an interval
(infinite possibilities!)

$$\text{PDF: } f_X(x)$$

Discrete and Continuous Random Variables

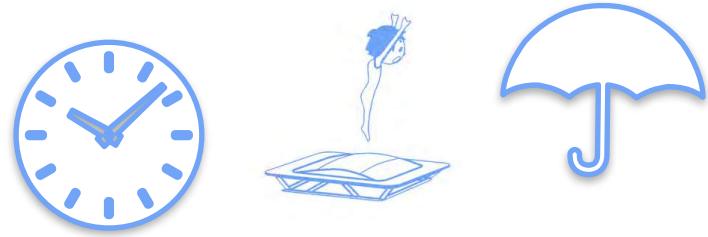
Discrete random variables



Can take only a **finite** or at most countable number of values

$$\text{PMF: } p_X(x) = \mathbf{P}(X = x)$$

Continuous random variables



Takes values on an interval
(infinite possibilities!)

$$\begin{aligned} \text{PDF: } & f_X(x) \\ \mathbf{P}(X = x) &= 0 \quad \forall x \end{aligned}$$

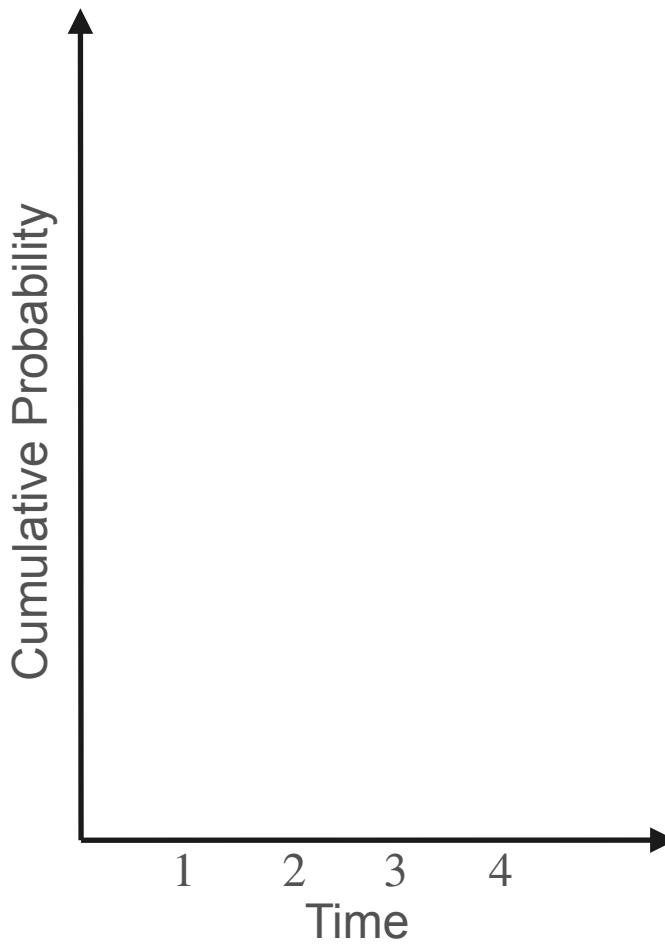
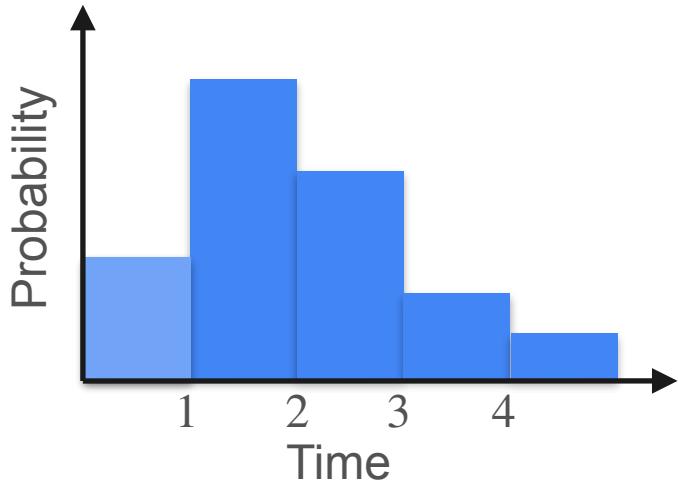


DeepLearning.AI

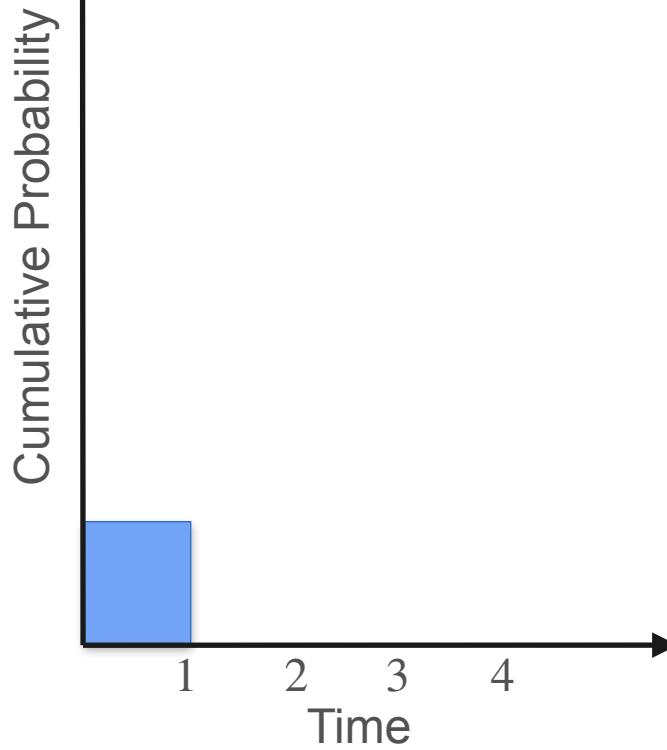
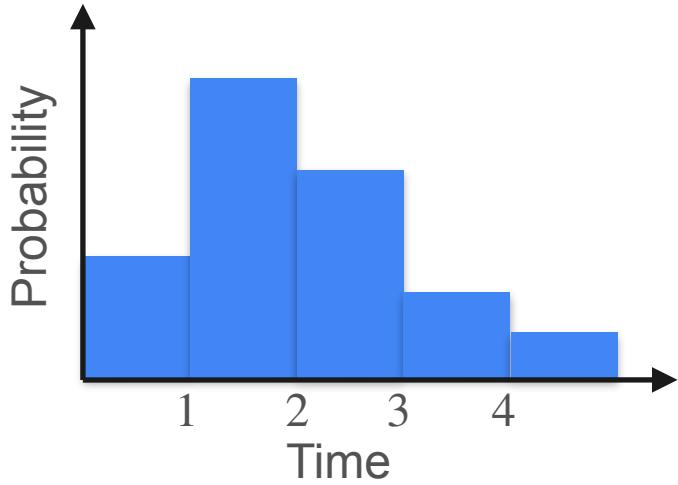
Probability Distributions

Cumulative Distribution Function

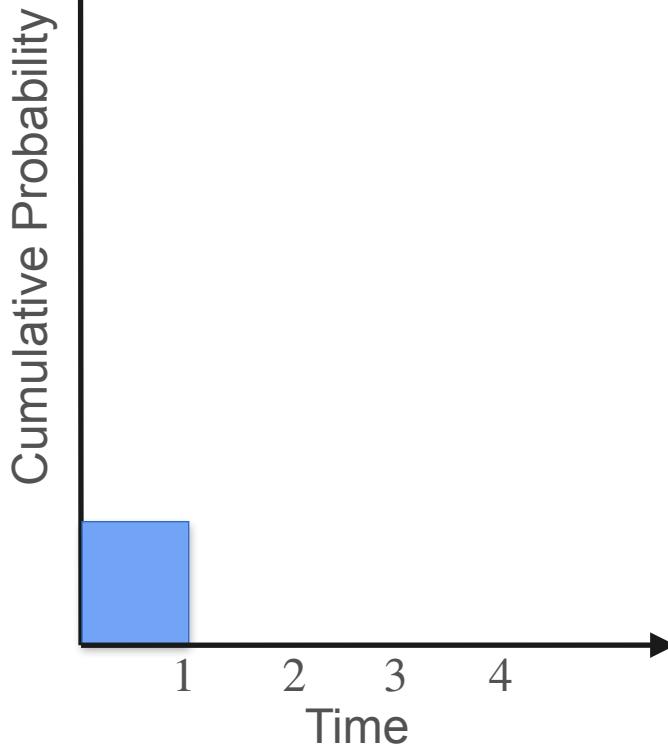
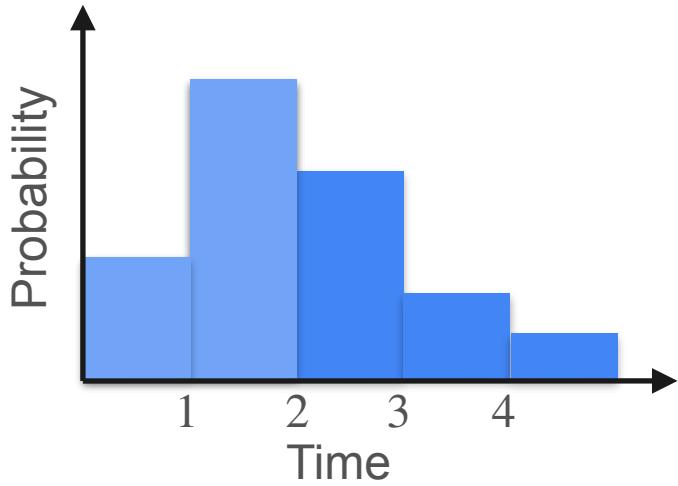
Cumulative Distribution



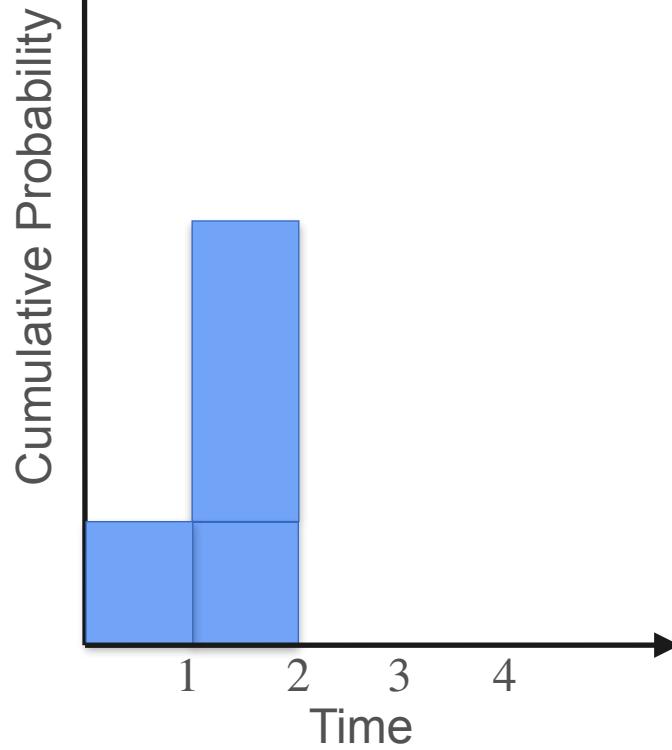
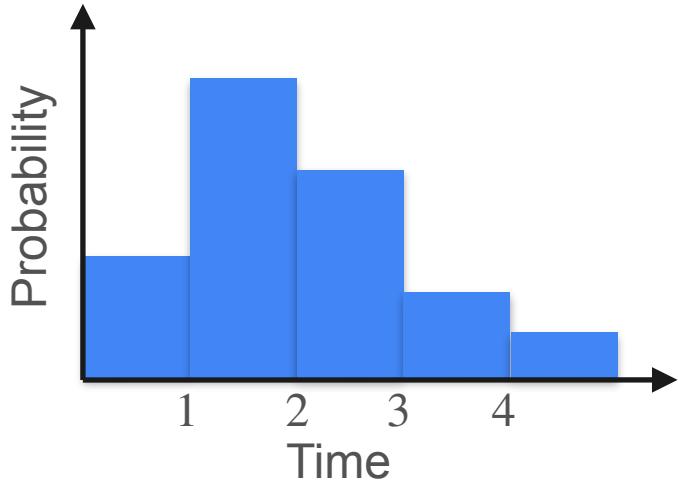
Cumulative Distribution



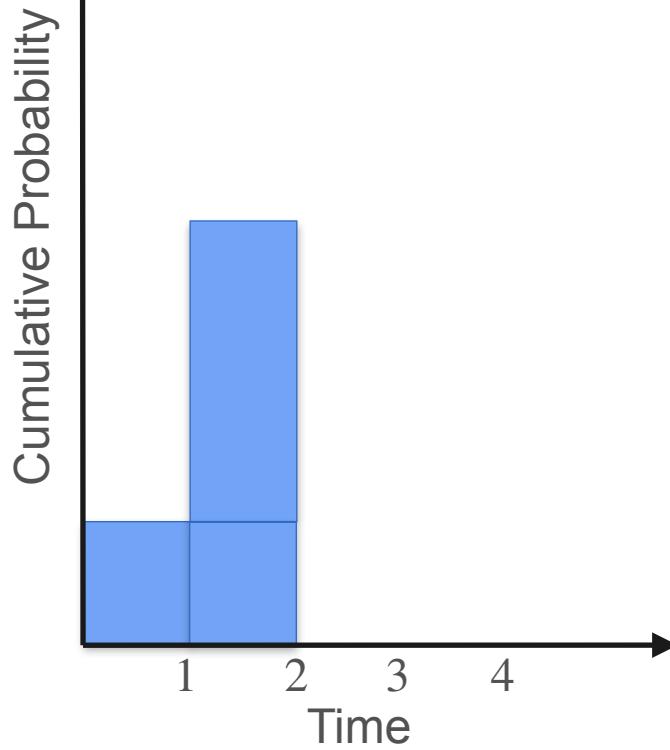
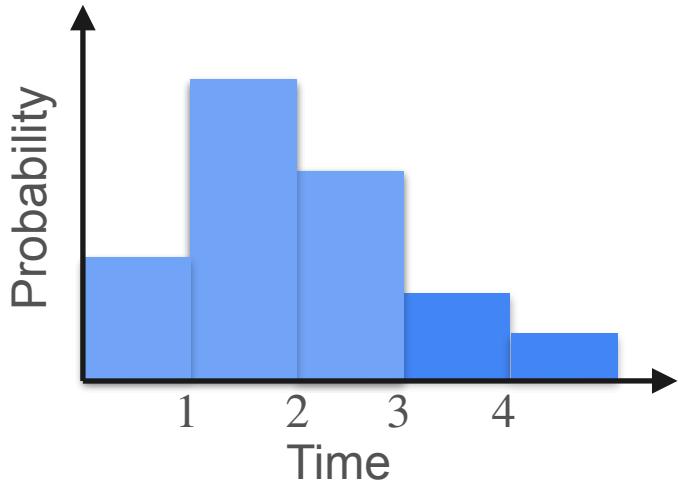
Cumulative Distribution



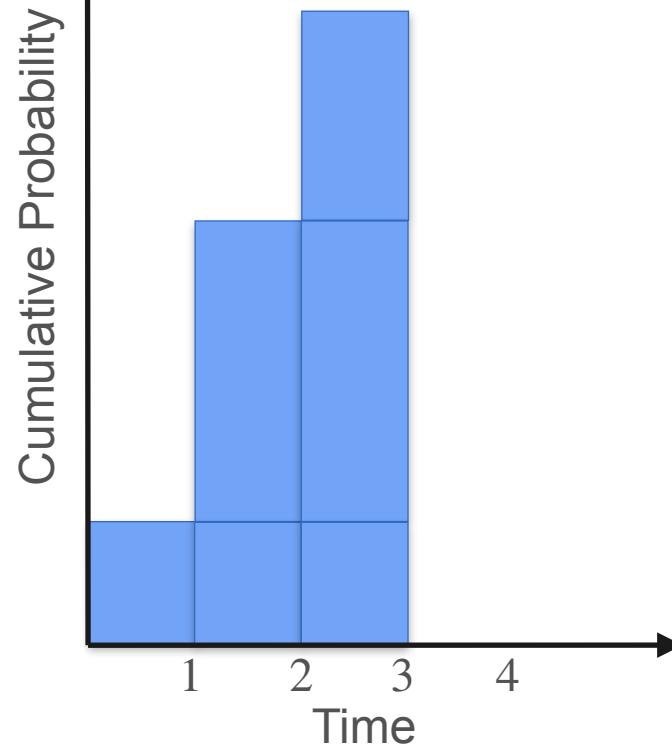
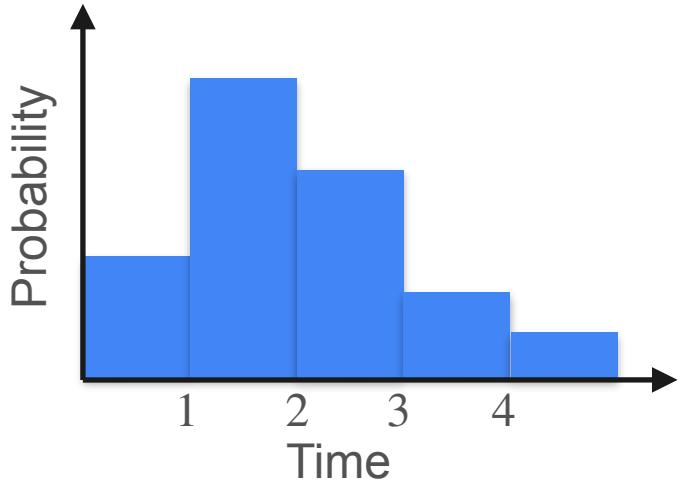
Cumulative Distribution



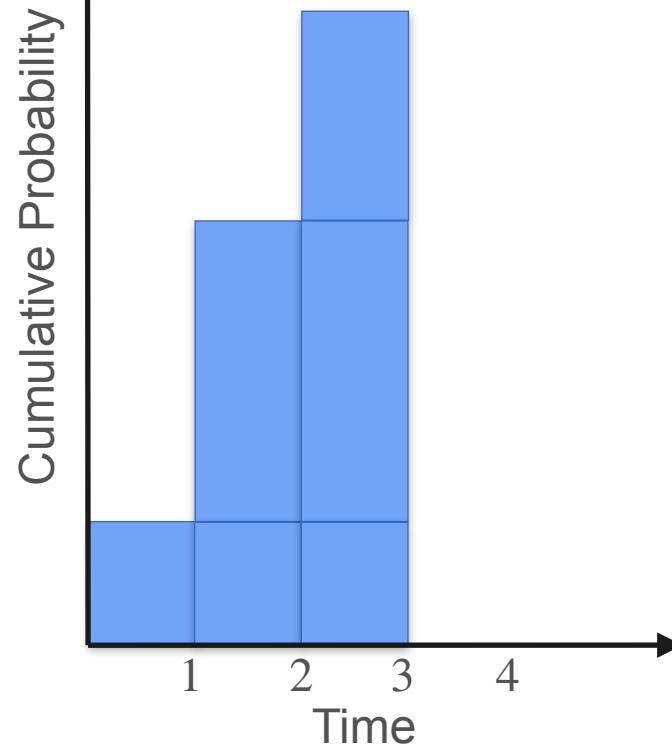
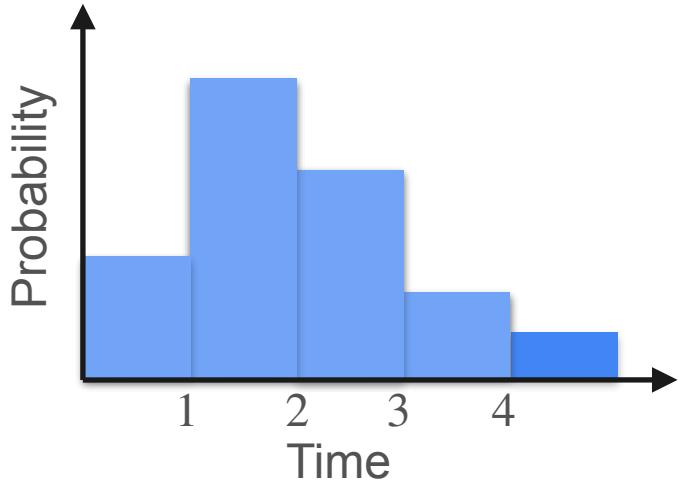
Cumulative Distribution



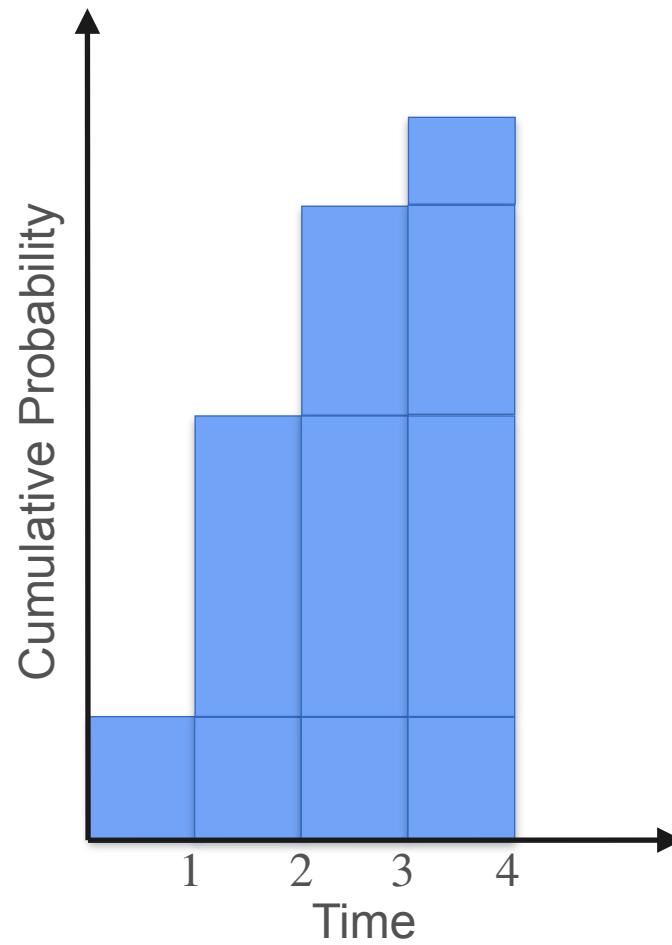
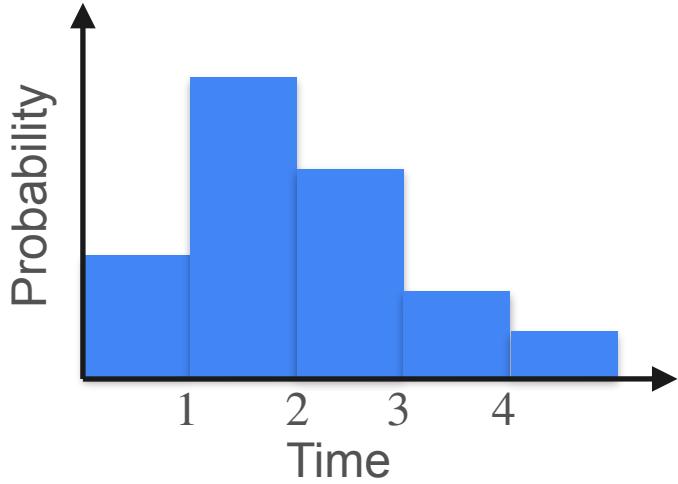
Cumulative Distribution



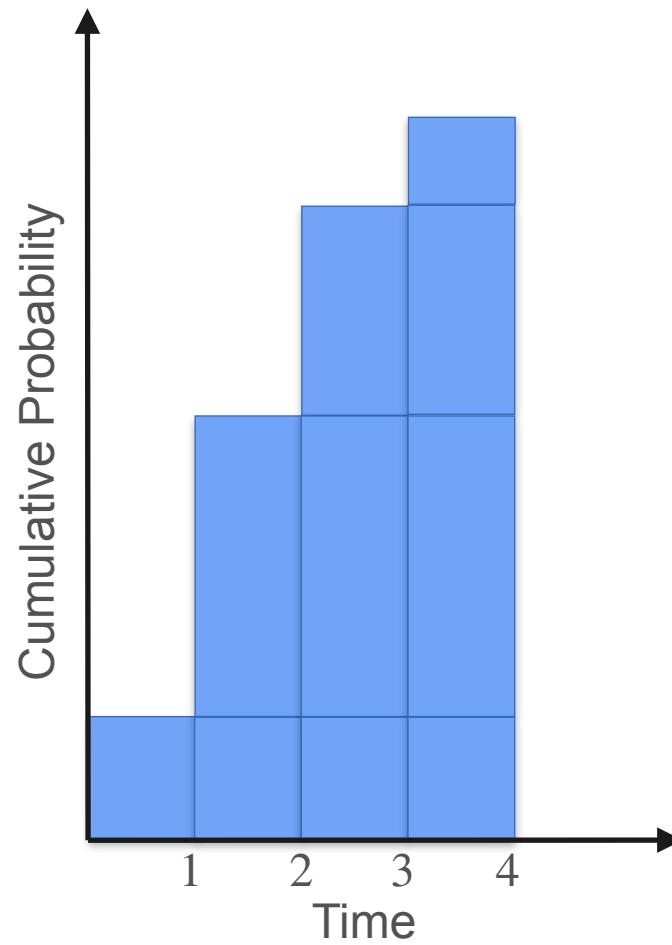
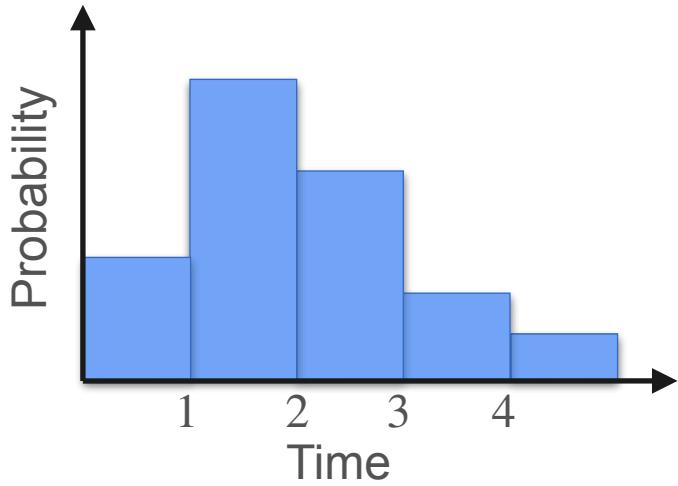
Cumulative Distribution



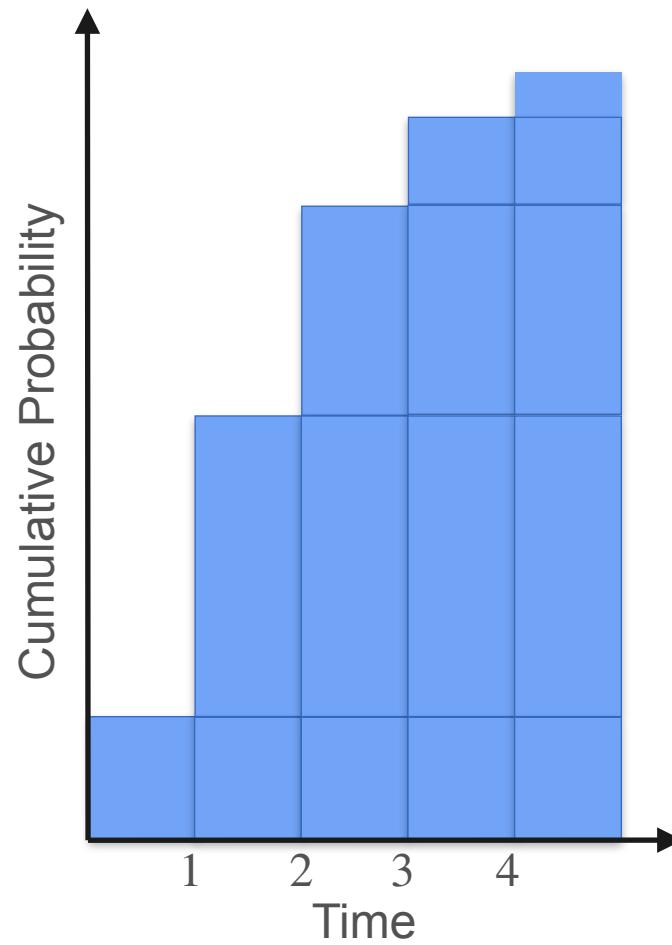
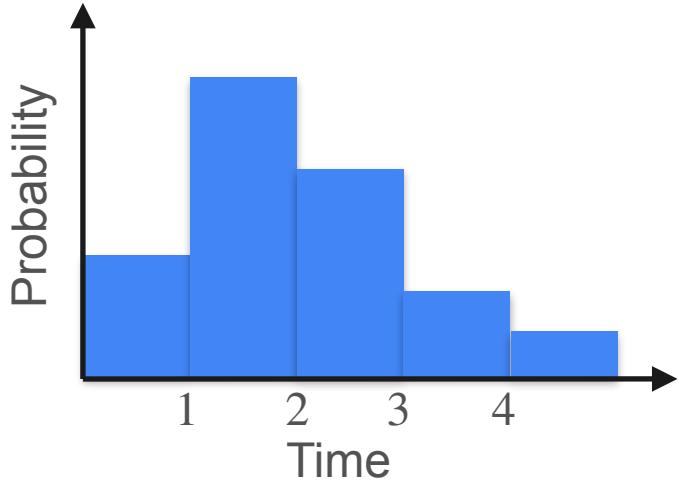
Cumulative Distribution



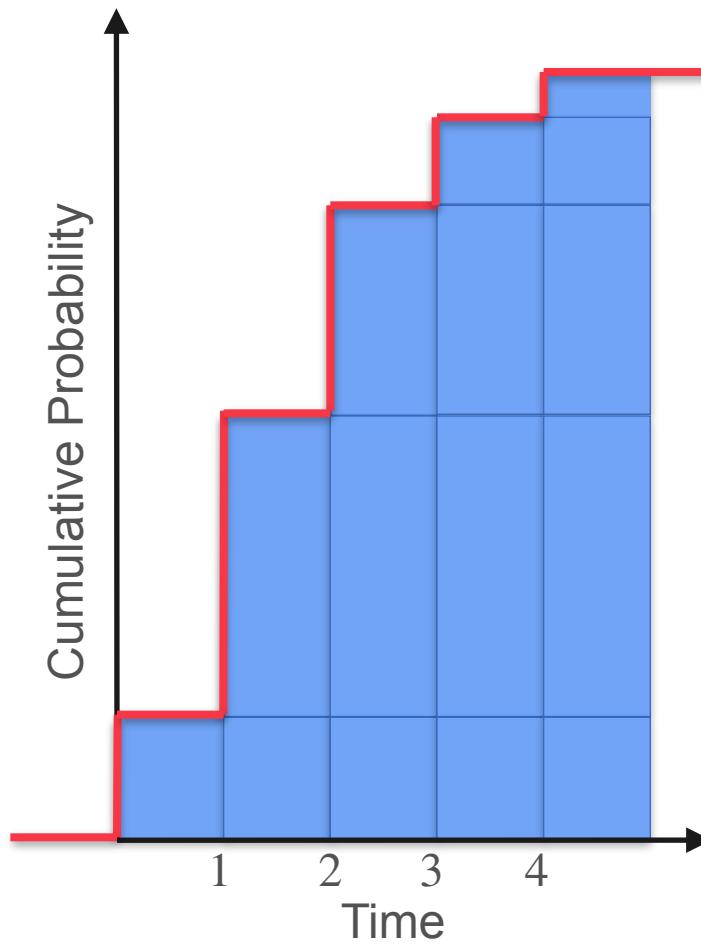
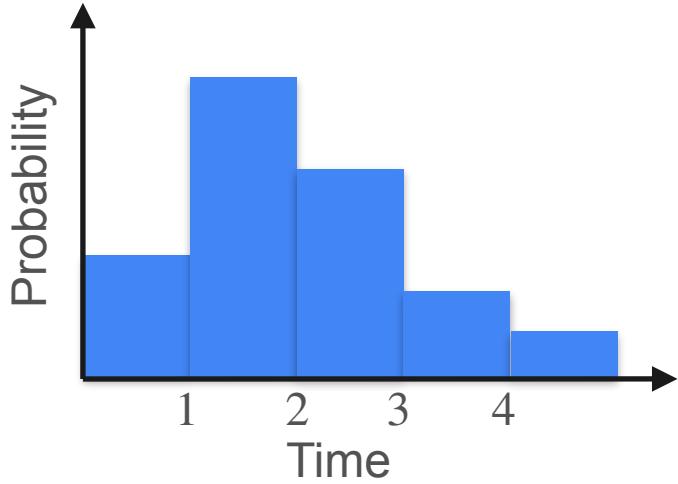
Cumulative Distribution



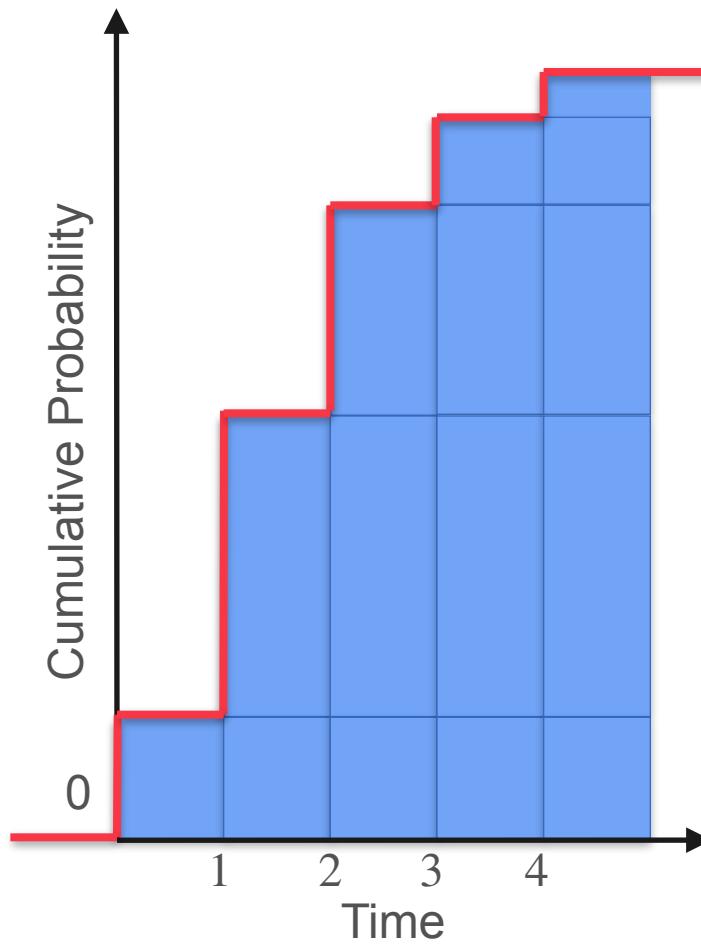
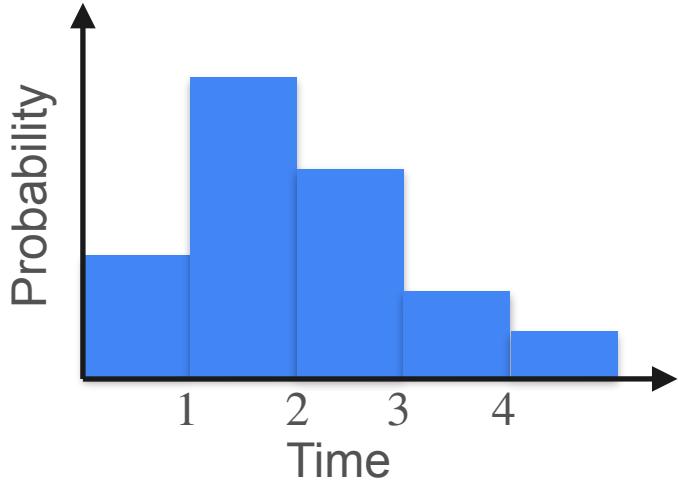
Cumulative Distribution



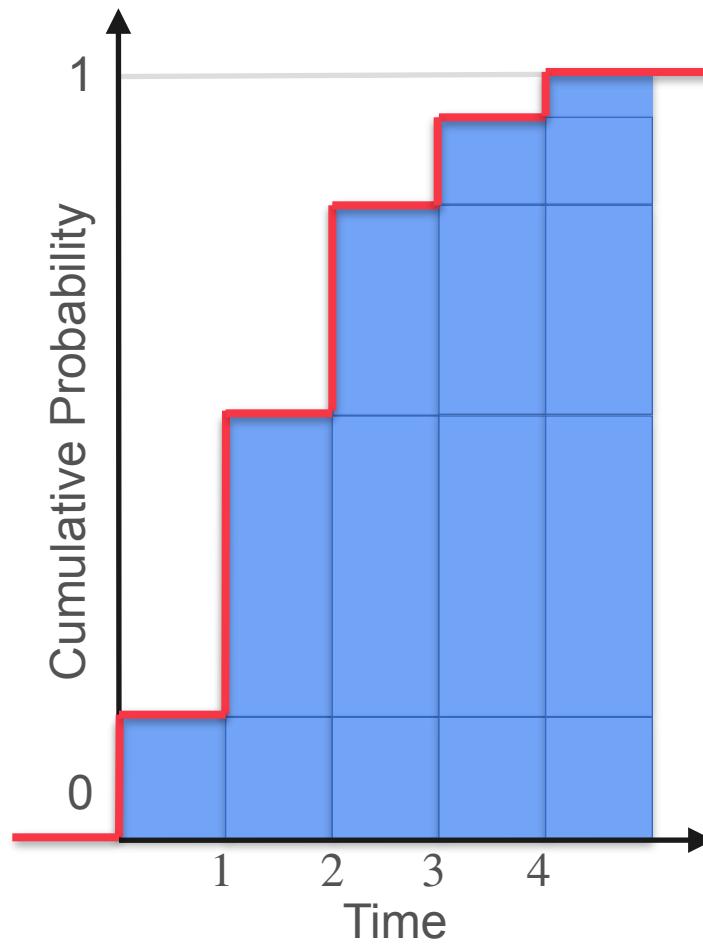
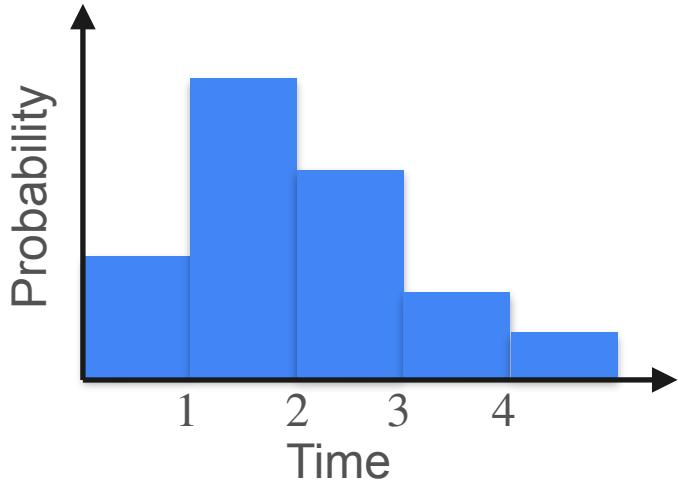
Cumulative Distribution



Cumulative Distribution



Cumulative Distribution



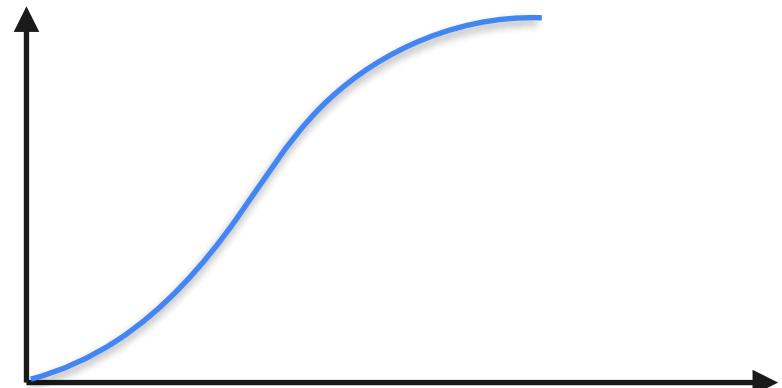
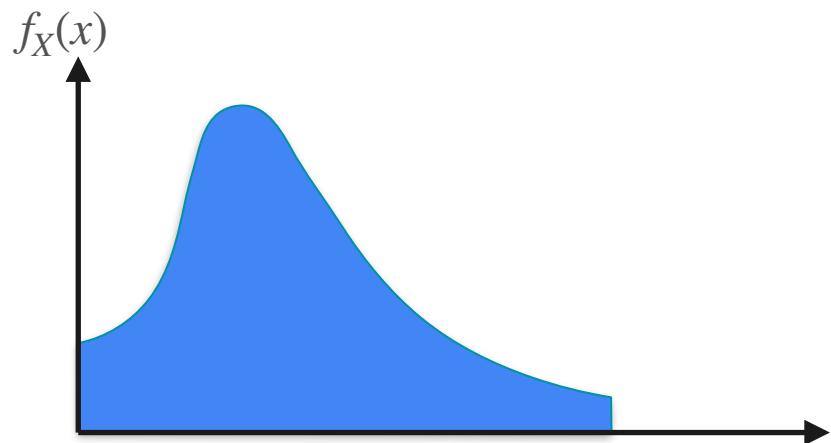
Cumulative Distribution

CDF: Cumulative distribution function



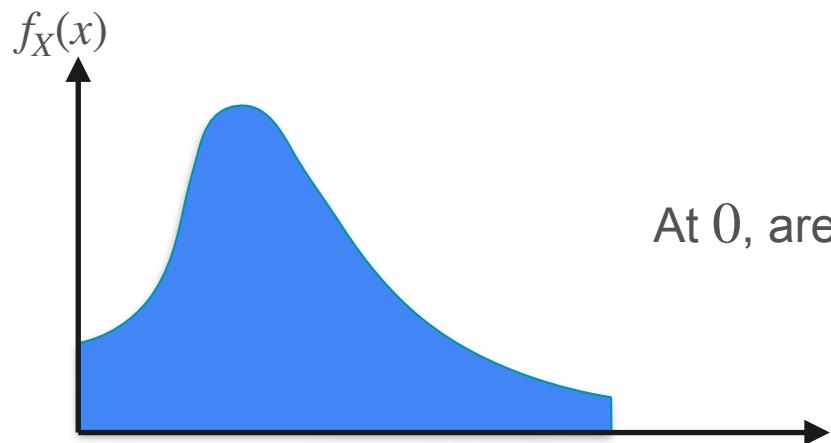
Cumulative Distribution

CDF: Cumulative distribution function

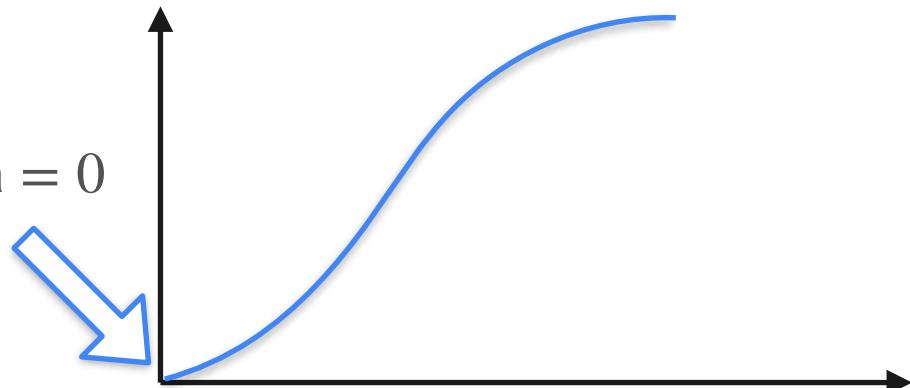


Cumulative Distribution

CDF: Cumulative distribution function

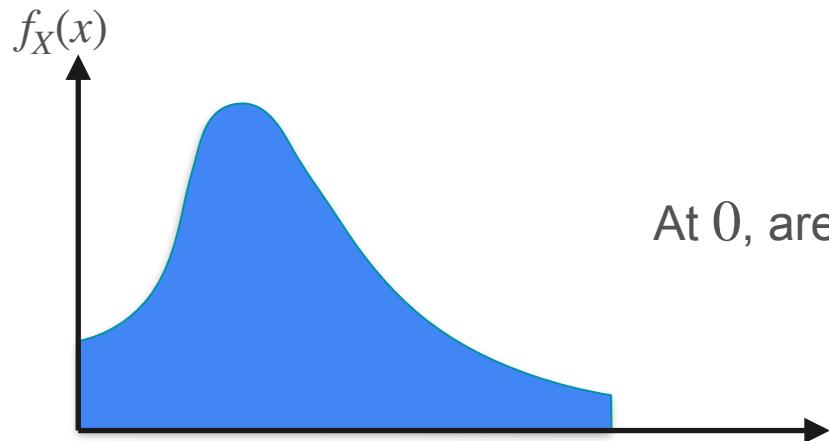


At 0, area = 0

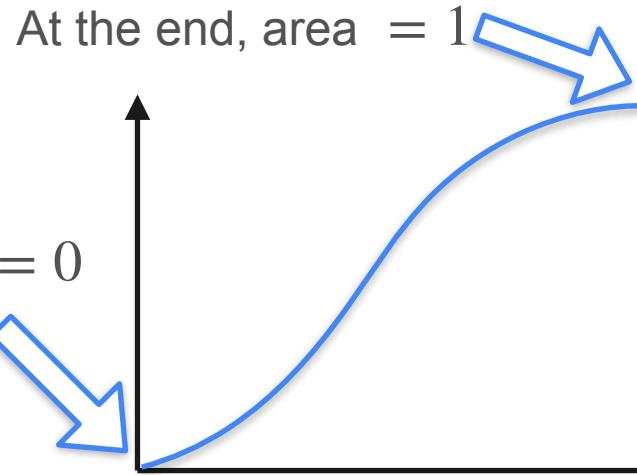


Cumulative Distribution

CDF: Cumulative distribution function

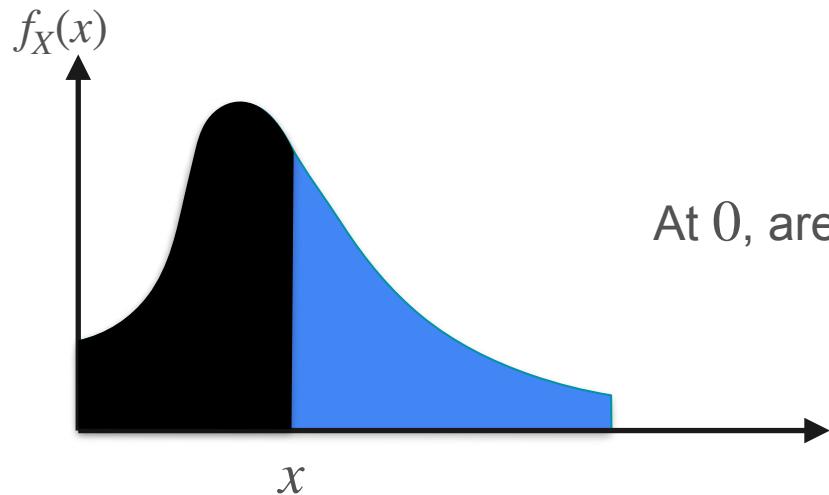


At 0, area = 0



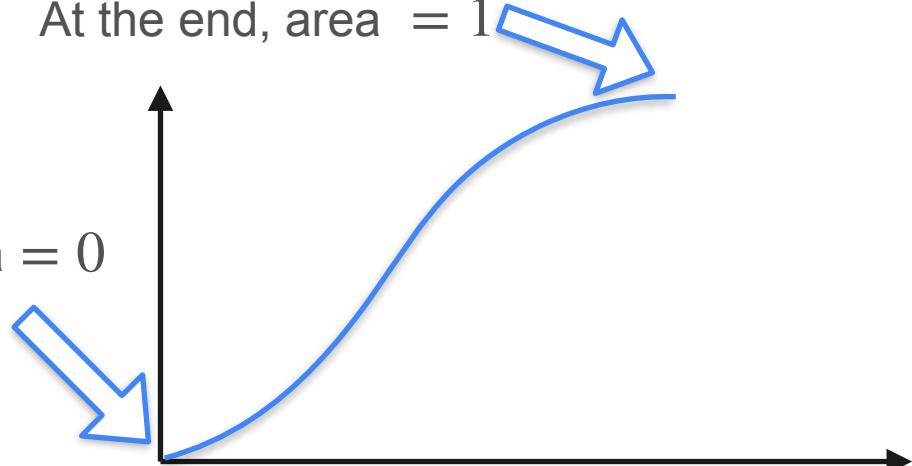
Cumulative Distribution

CDF: Cumulative distribution function



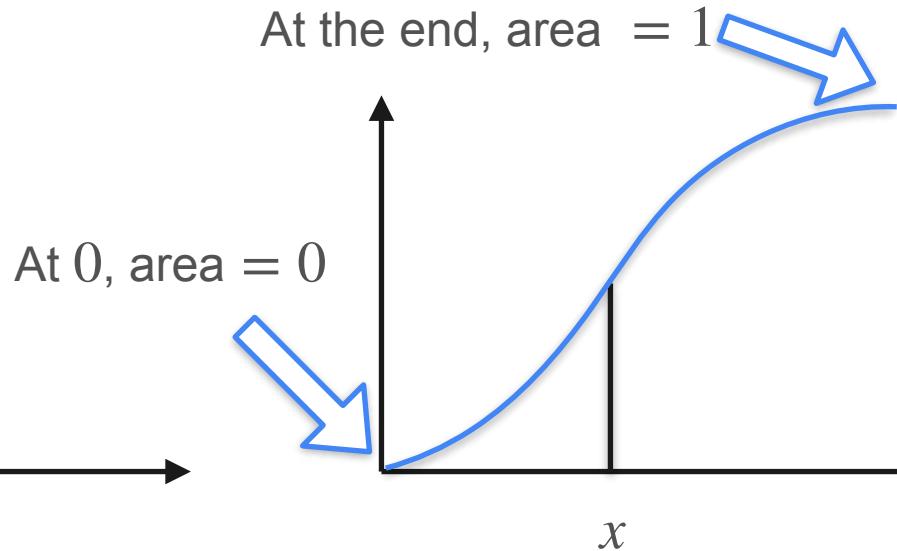
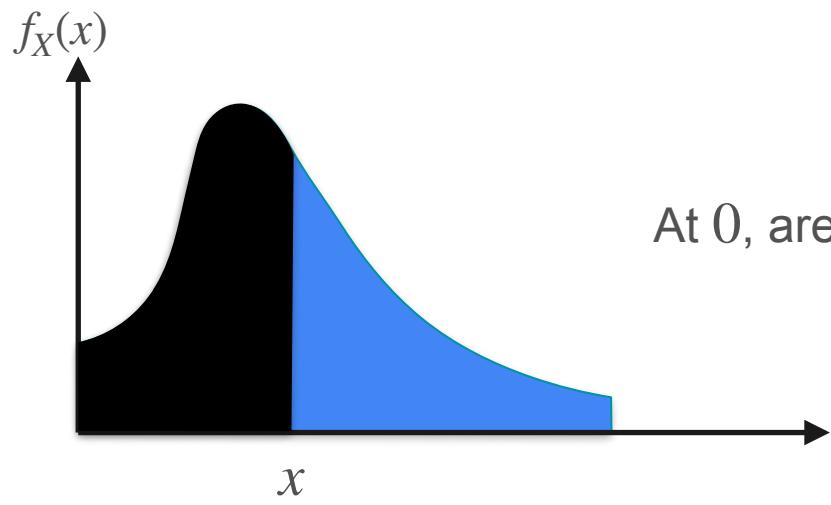
At 0, area = 0

At the end, area = 1



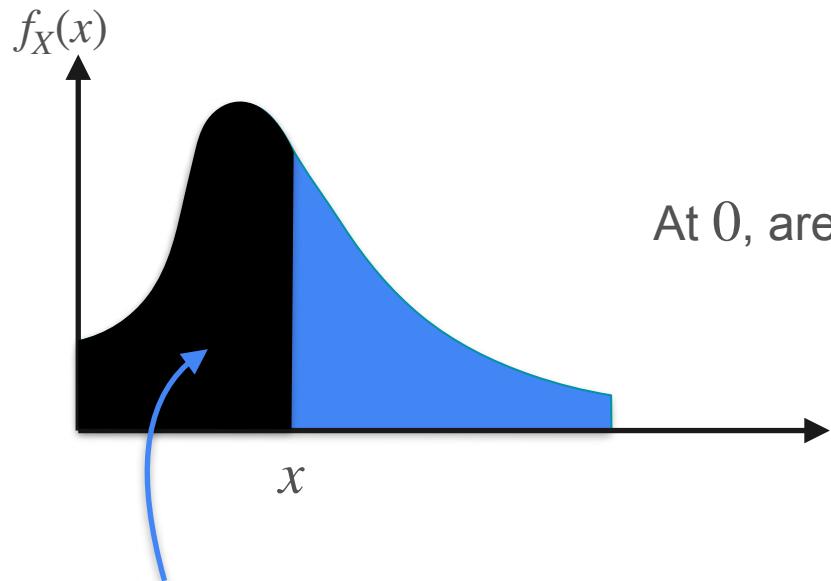
Cumulative Distribution

CDF: Cumulative distribution function



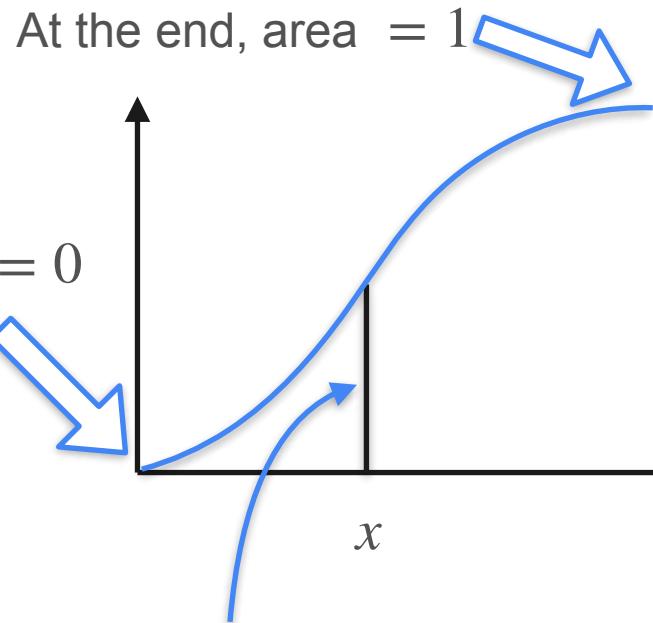
Cumulative Distribution

CDF: Cumulative distribution function



At 0, area = 0

$P(\text{less than or equal to 2 minutes}) = 0.5$

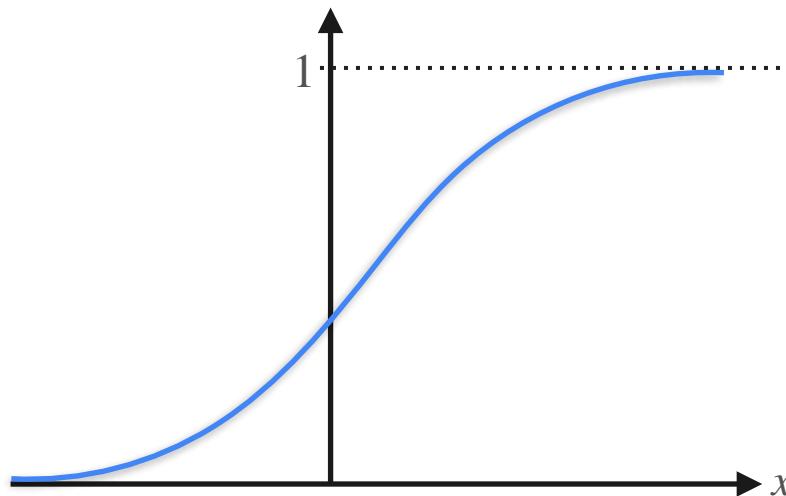
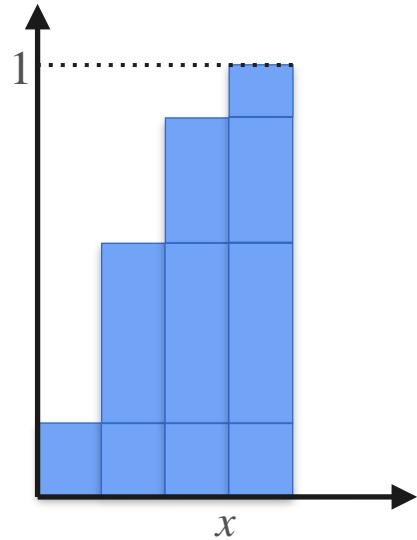


$P(\text{less than or equal to 2 minutes}) = 0.5$

Cumulative Distribution Function: Formal Definition

Cumulative Distribution Function: Formal Definition

The CDF shows how much probability the variable has accumulated until a certain value

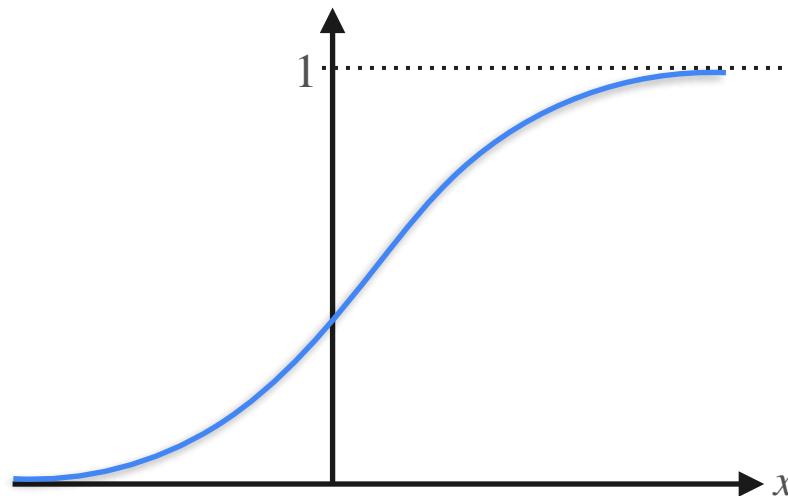
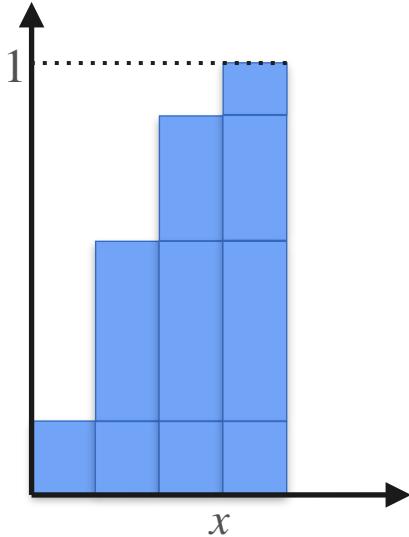


Cumulative Distribution Function: Formal Definition

The CDF shows how much probability the variable has accumulated until a certain value

That means that

$$\text{CDF}(x) = \mathbf{P}(X \leq x)$$

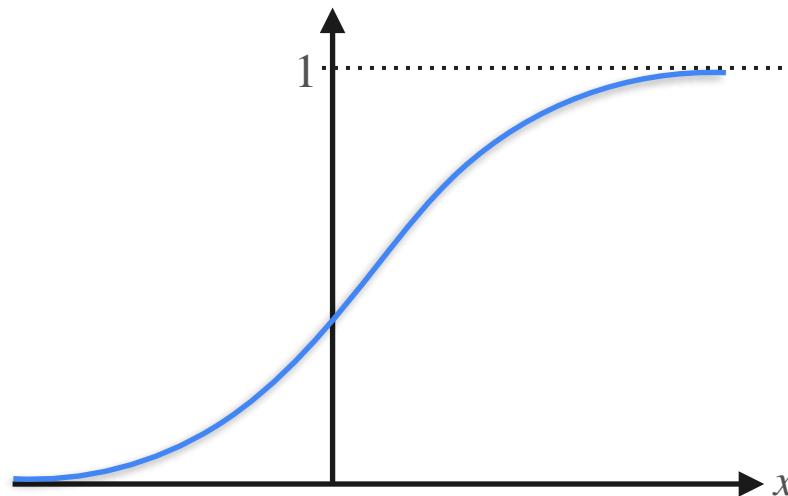
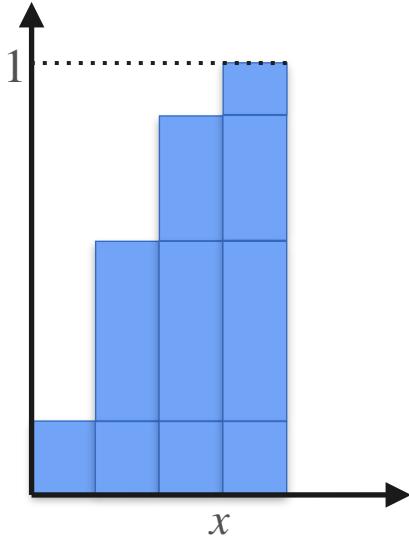


Cumulative Distribution Function: Formal Definition

The CDF shows how much probability the variable has accumulated until a certain value

That means that

$$\text{CDF}(x) = \mathbf{P}(X \leq x) \quad \text{It is defined for every real number}$$

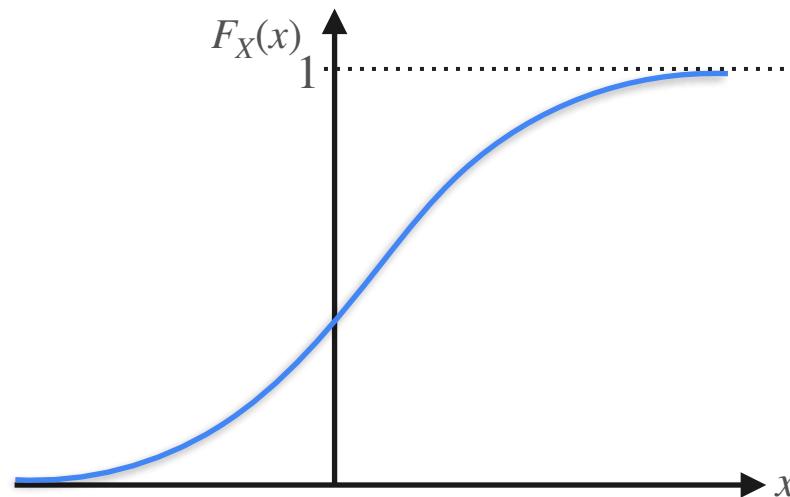
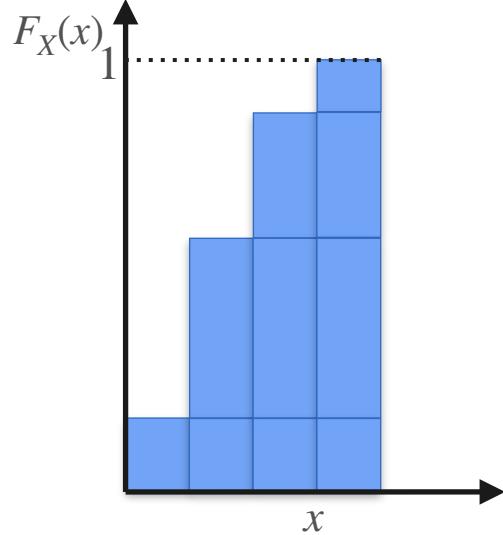


Cumulative Distribution Function: Formal Definition

The CDF shows how much probability the variable has accumulated until a certain value

That means that

$$F_X(x) = \mathbf{P}(X \leq x) \quad \text{It is defined for every real number}$$

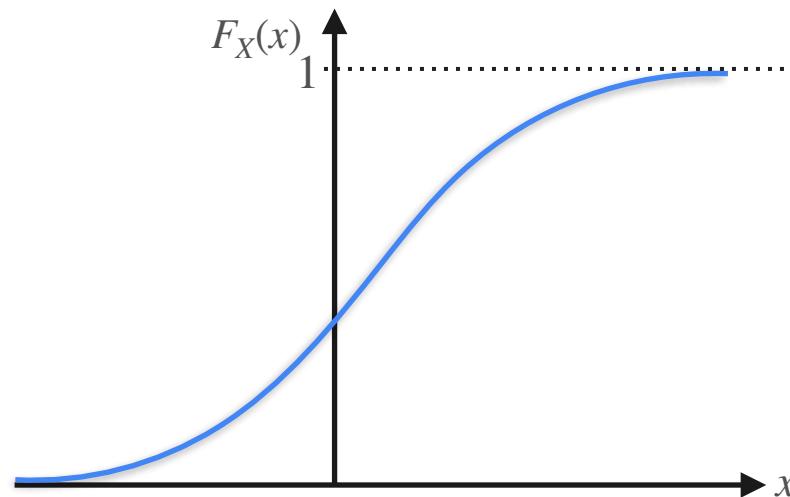
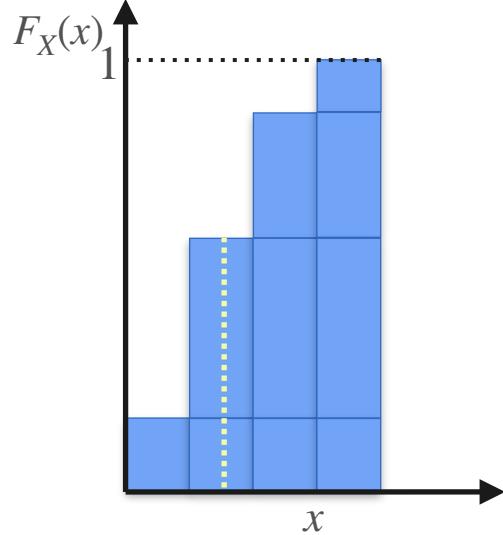


Cumulative Distribution Function: Formal Definition

The CDF shows how much probability the variable has accumulated until a certain value

That means that

$$F_X(x) = \mathbf{P}(X \leq x) \quad \text{It is defined for every real number}$$

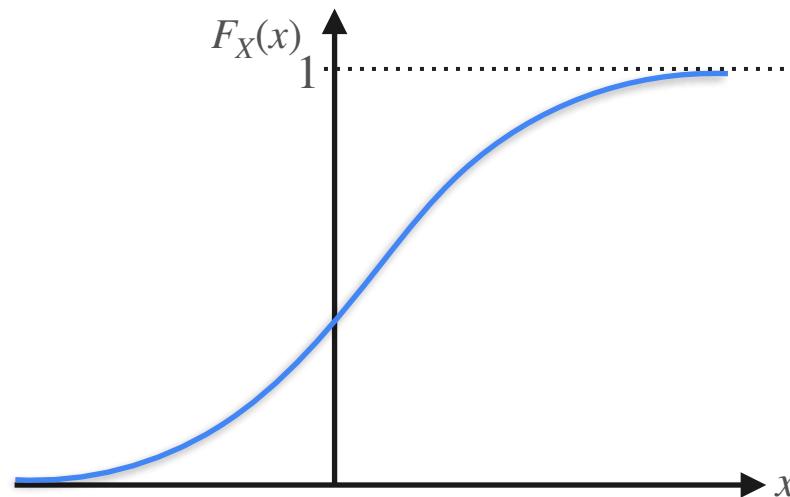
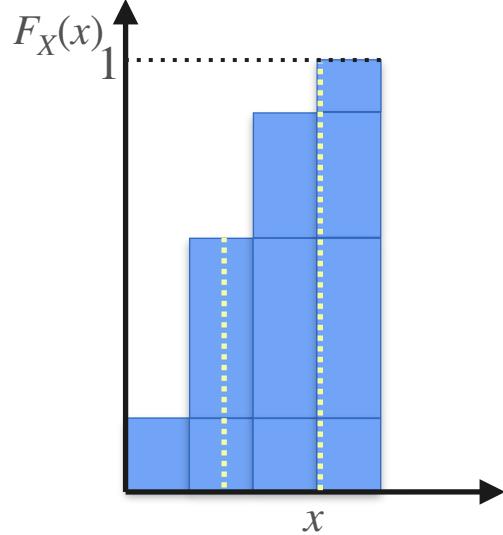


Cumulative Distribution Function: Formal Definition

The CDF shows how much probability the variable has accumulated until a certain value

That means that

$$F_X(x) = \mathbf{P}(X \leq x) \quad \text{It is defined for every real number}$$

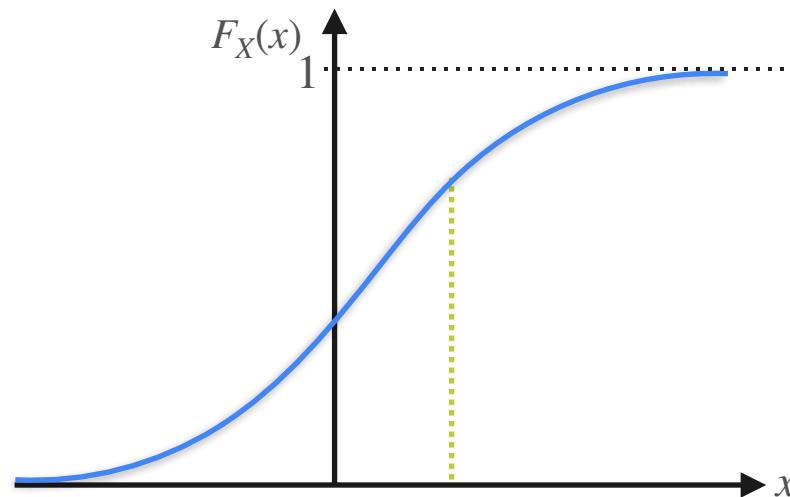
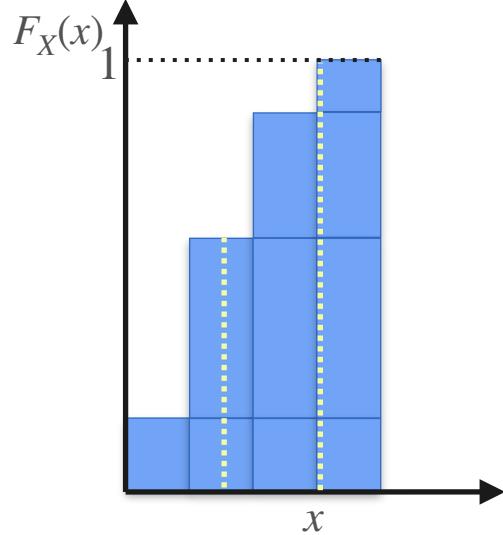


Cumulative Distribution Function: Formal Definition

The CDF shows how much probability the variable has accumulated until a certain value

That means that

$$F_X(x) = \mathbf{P}(X \leq x) \quad \text{It is defined for every real number}$$

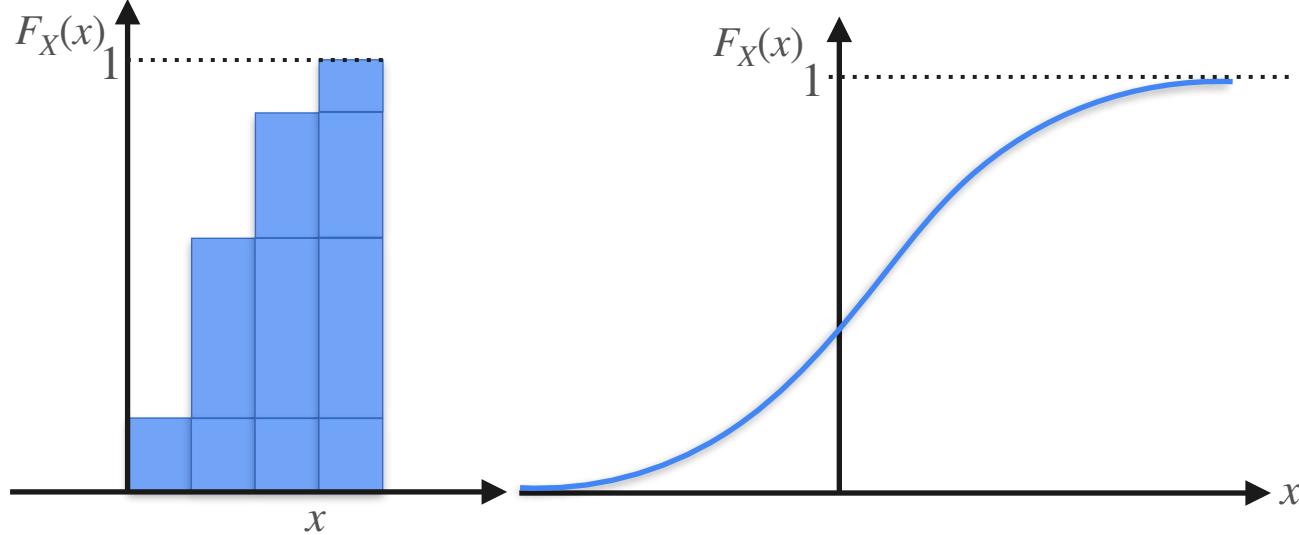


Cumulative Distribution Function: Formal Definition

The CDF shows how much probability the variable has accumulated until a certain value

That means that

$$F_X(x) = \mathbf{P}(X \leq x) \quad \text{It is defined for every real number}$$

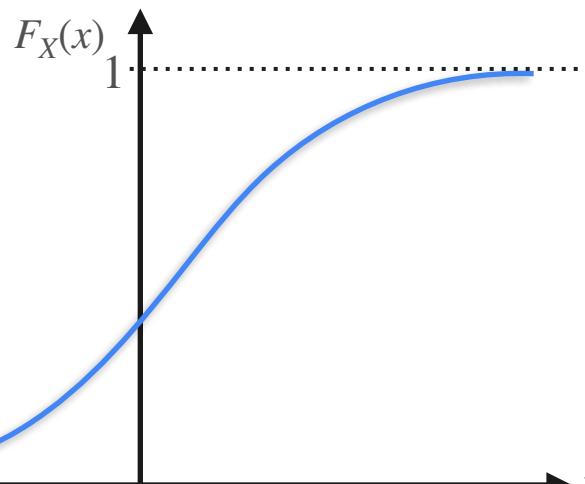
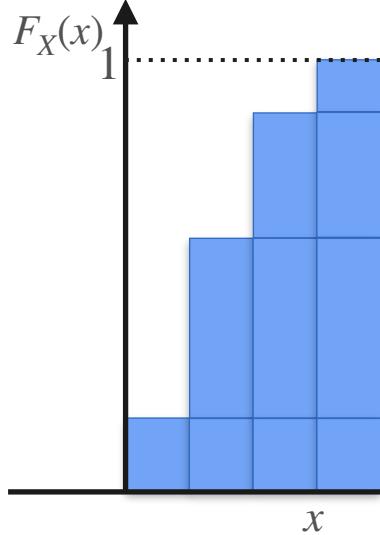


Cumulative Distribution Function: Formal Definition

The CDF shows how much probability the variable has accumulated until a certain value

That means that

$$F_X(x) = \mathbf{P}(X \leq x) \quad \text{It is defined for every real number}$$



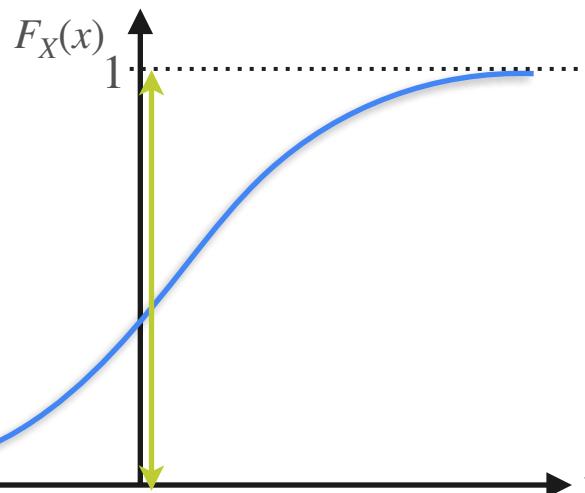
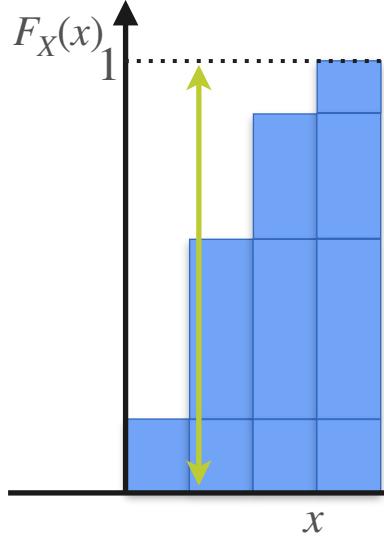
Properties

Cumulative Distribution Function: Formal Definition

The CDF shows how much probability the variable has accumulated until a certain value

That means that

$$F_X(x) = P(X \leq x) \quad \text{It is defined for every real number}$$



Properties

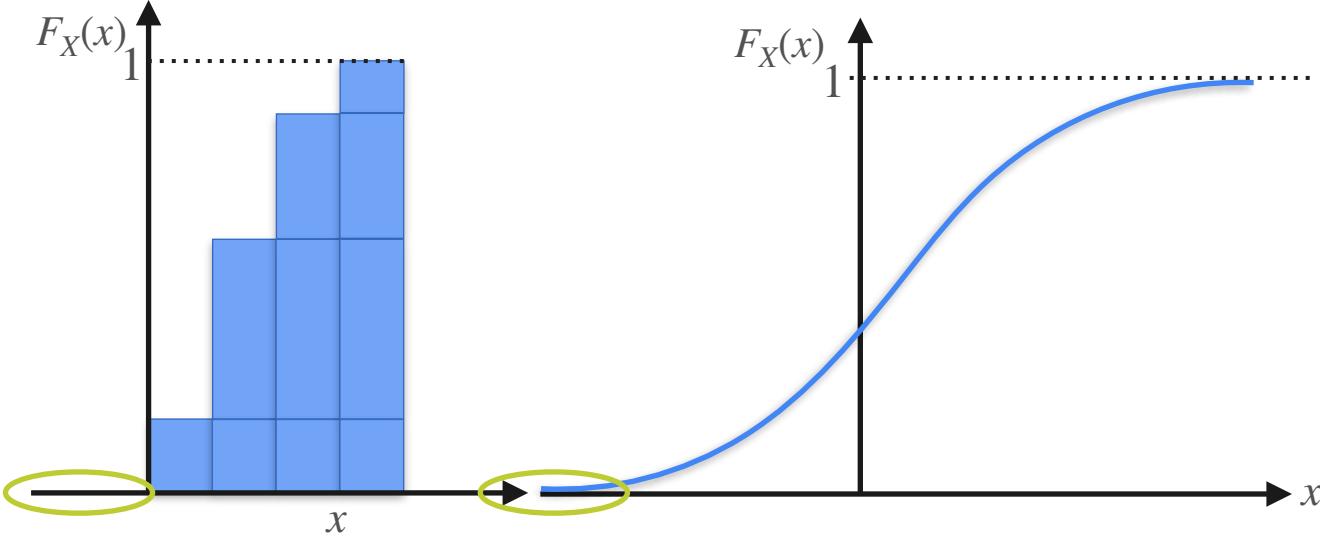
- $0 \leq F_X(x) \leq 1$

Cumulative Distribution Function: Formal Definition

The CDF shows how much probability the variable has accumulated until a certain value

That means that

$$F_X(x) = P(X \leq x) \quad \text{It is defined for every real number}$$



Properties

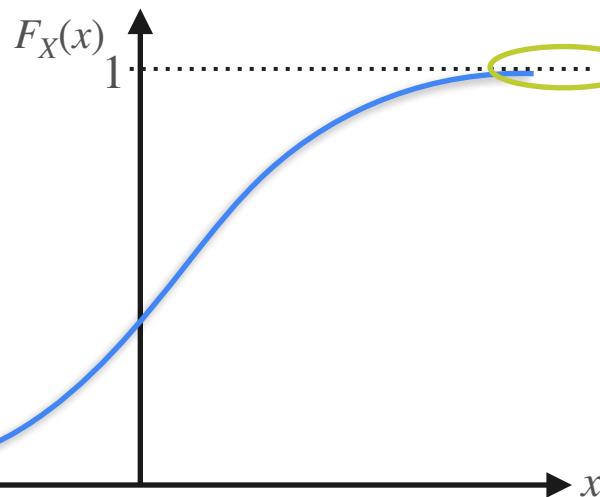
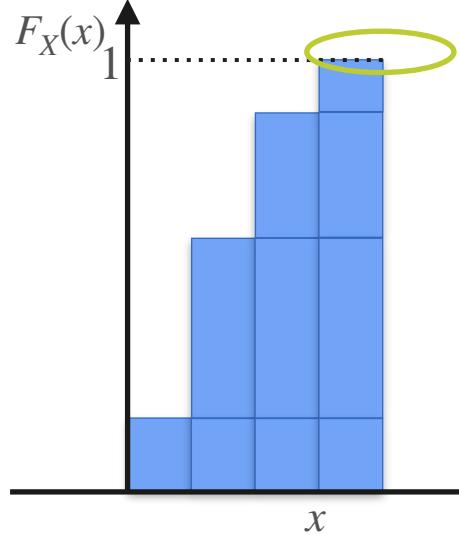
- $0 \leq F_X(x) \leq 1$
- Left “endpoint” is 0

Cumulative Distribution Function: Formal Definition

The CDF shows how much probability the variable has accumulated until a certain value

That means that

$$F_X(x) = P(X \leq x) \quad \text{It is defined for every real number}$$



Properties

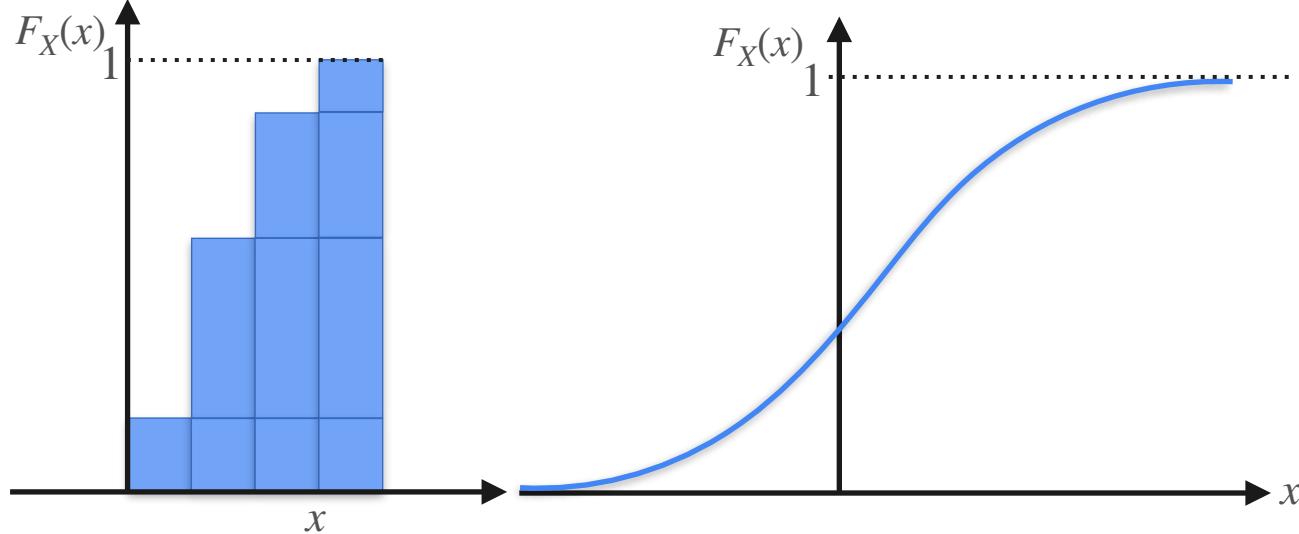
- $0 \leq F_X(x) \leq 1$
- Left “endpoint” is 0
- Right “endpoint” is 1

Cumulative Distribution Function: Formal Definition

The CDF shows how much probability the variable has accumulated until a certain value

That means that

$$F_X(x) = P(X \leq x) \quad \text{It is defined for every real number}$$

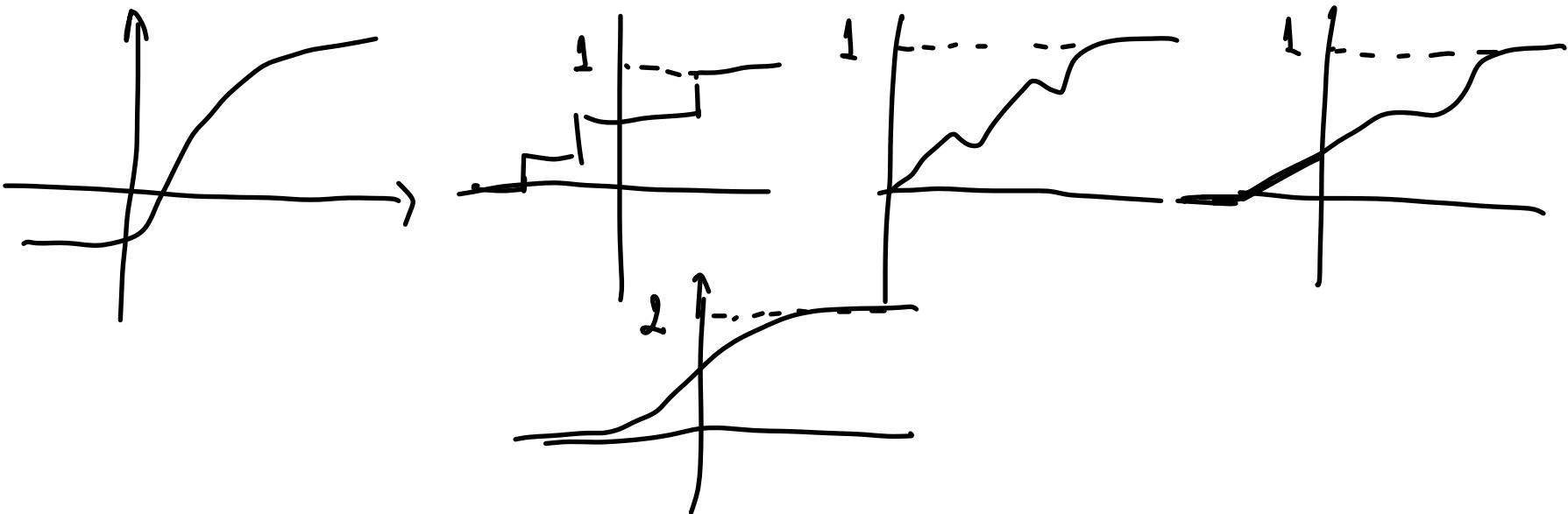


Properties

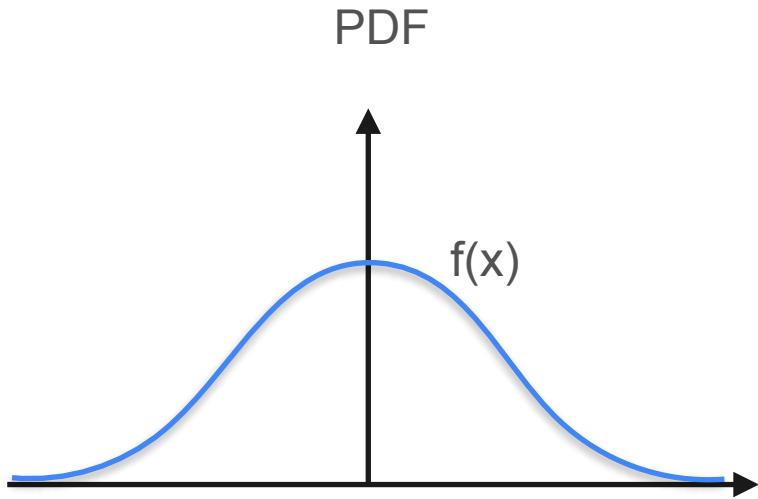
- $0 \leq F_X(x) \leq 1$
- Left “endpoint” is 0
- Right “endpoint” is 1
- Never decreases

Quiz

- Which of the following functions could be a CDF?



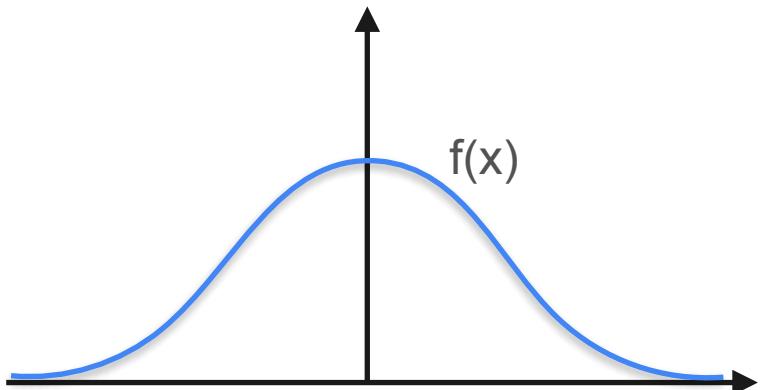
PDF and CDF Summary



- area = 1
- Always positive

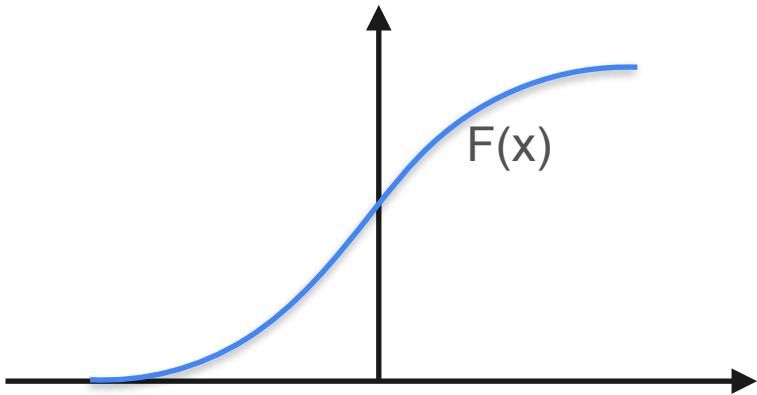
PDF and CDF Summary

PDF



- area = 1
- Always positive

CDF



- left “endpoint” is 0
- right “endpoint” is 1
- (endpoints can be at infinity)
- Always positive and increasing

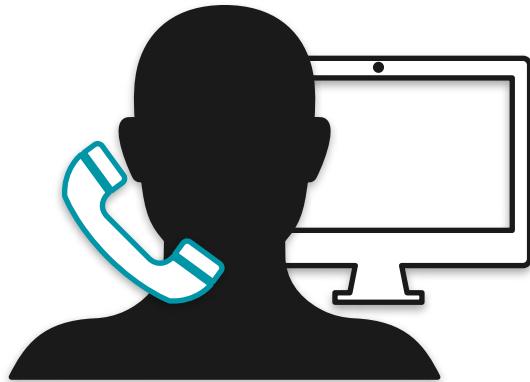


DeepLearning.AI

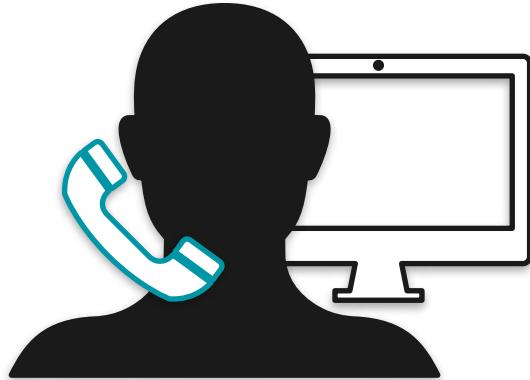
Probability Distributions

Uniform Distribution

Uniform Distribution: Motivation

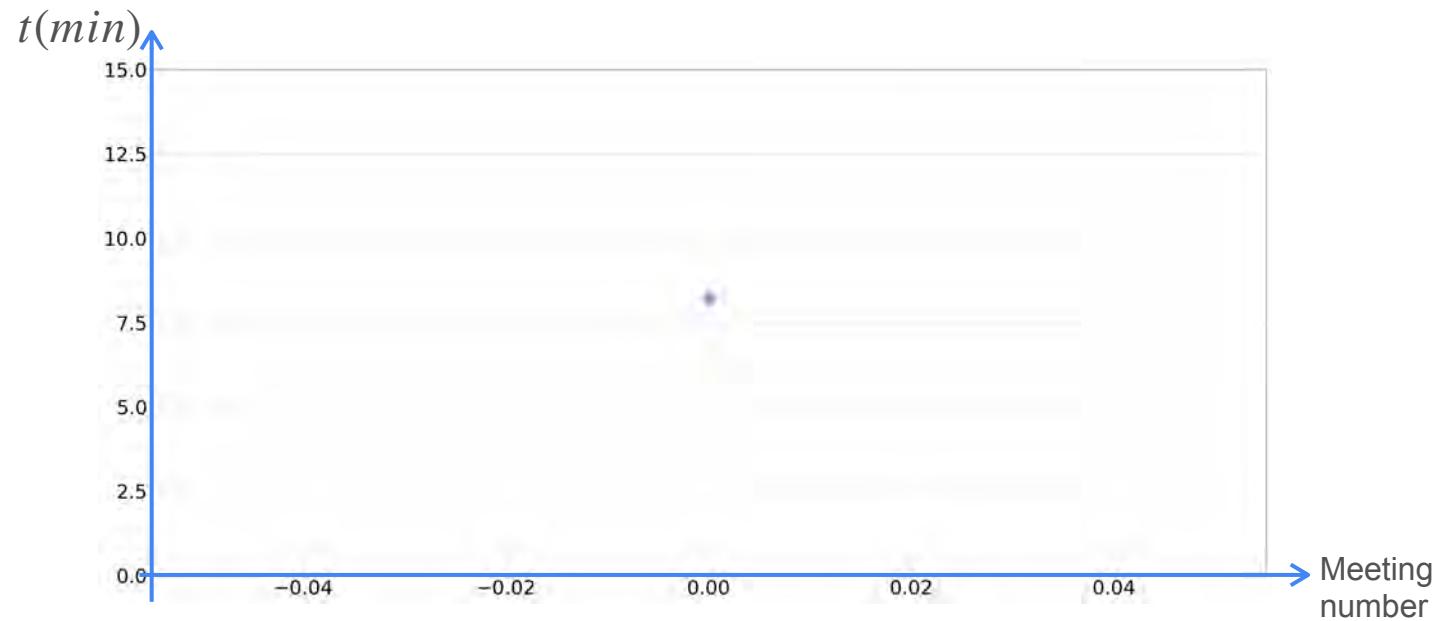


Uniform Distribution: Motivation



You're calling a tech support line. They can answer any time between zero and 15 minutes and if they don't answer in this time, the line is disconnected.

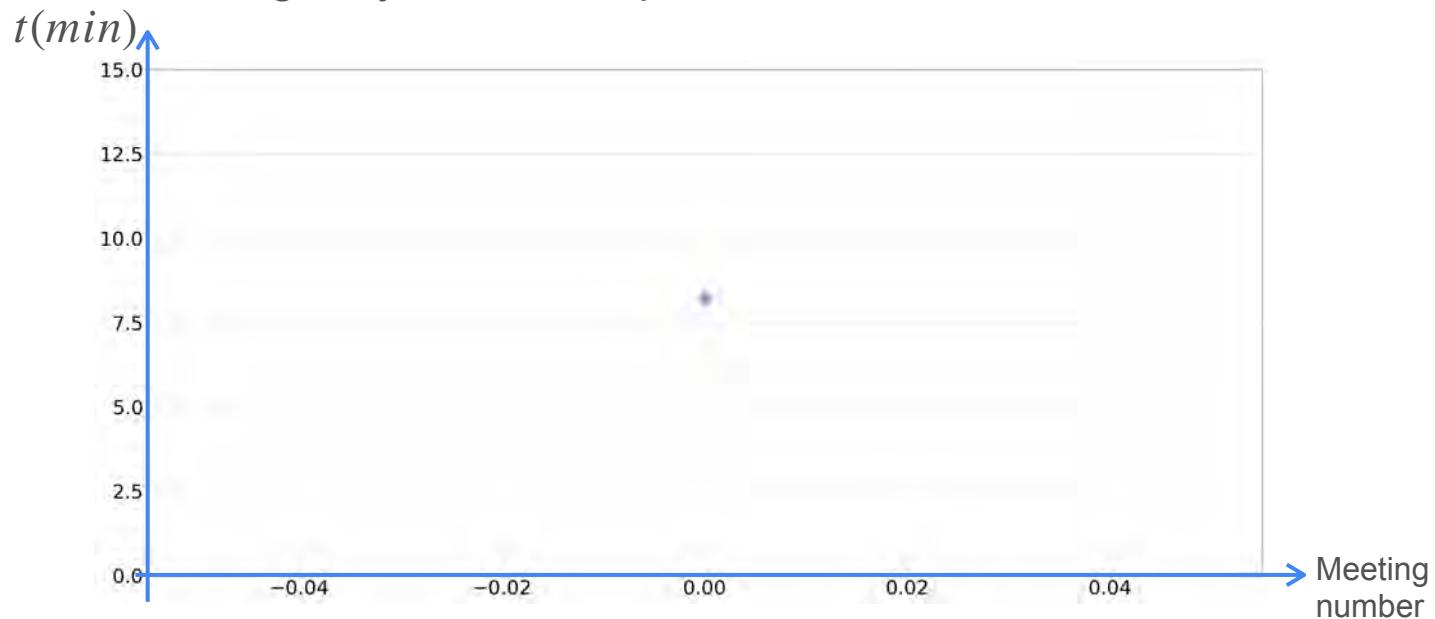
Uniform Distribution: Motivation



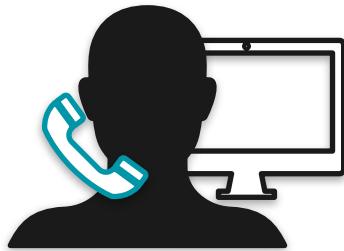
Uniform Distribution: Motivation



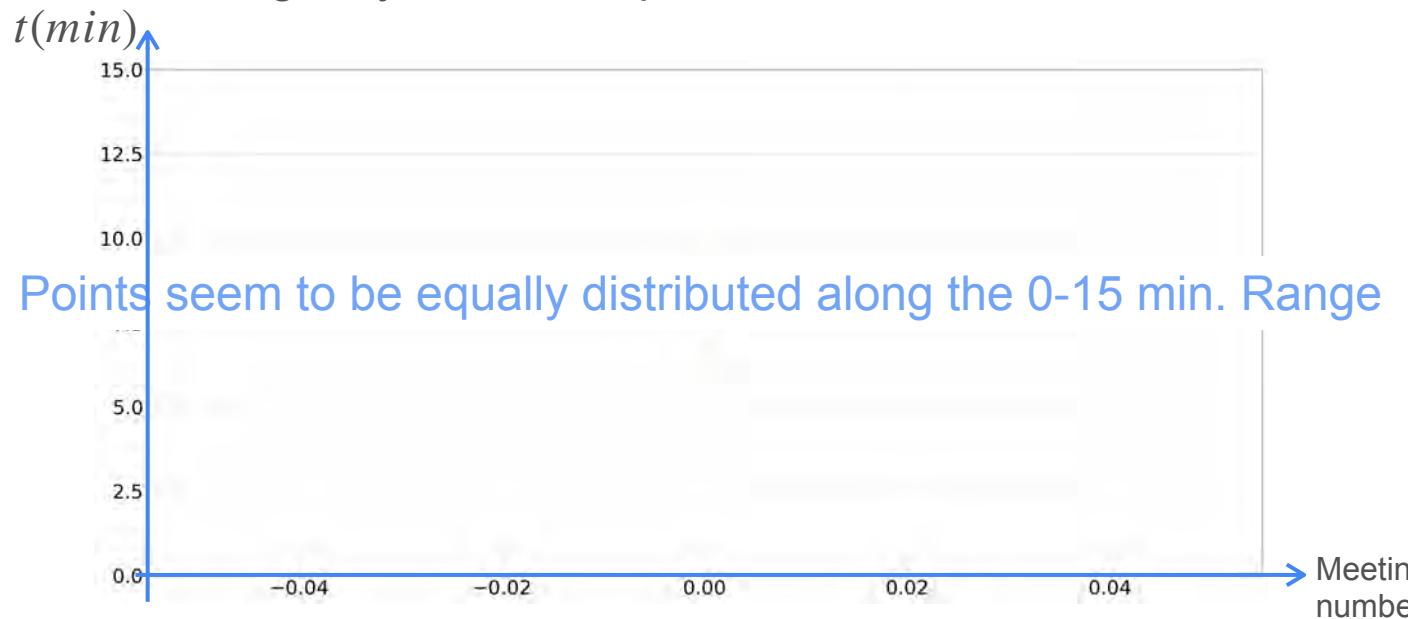
Last 200 times you called them, you took down notes of how long they took to respond



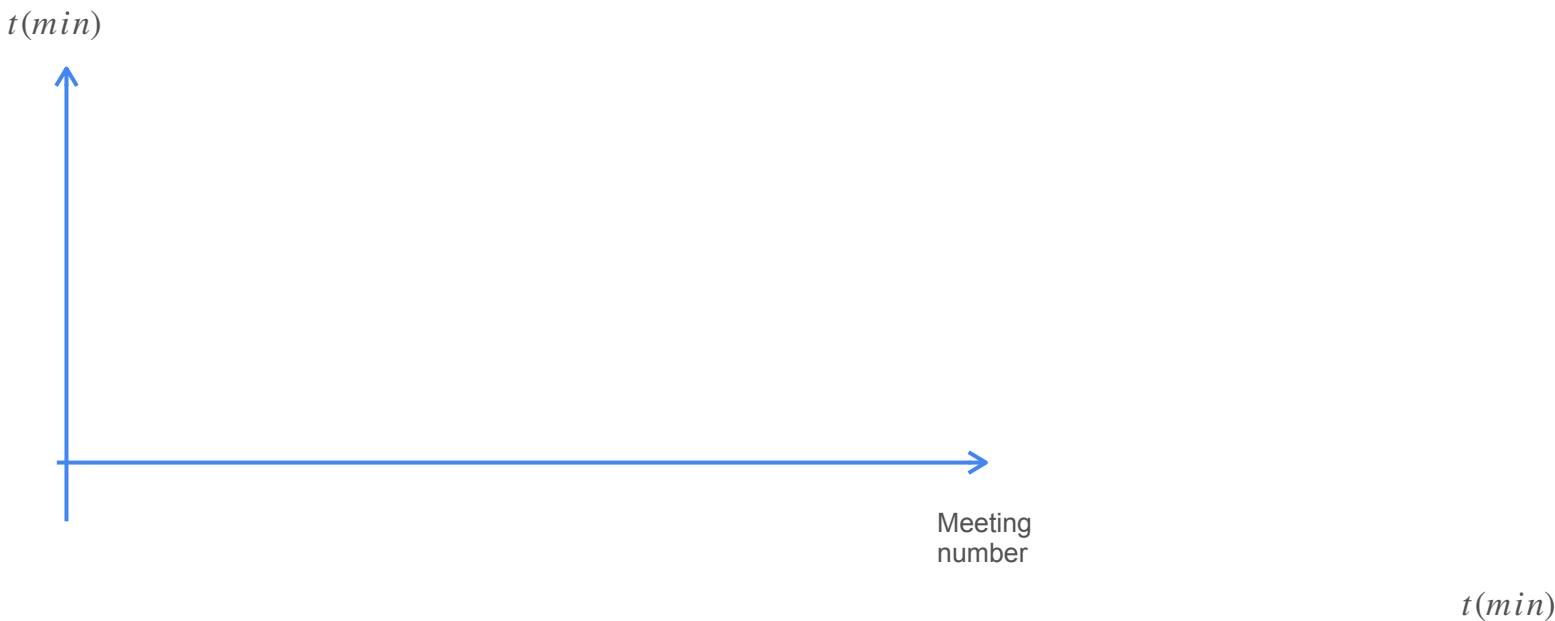
Uniform Distribution: Motivation



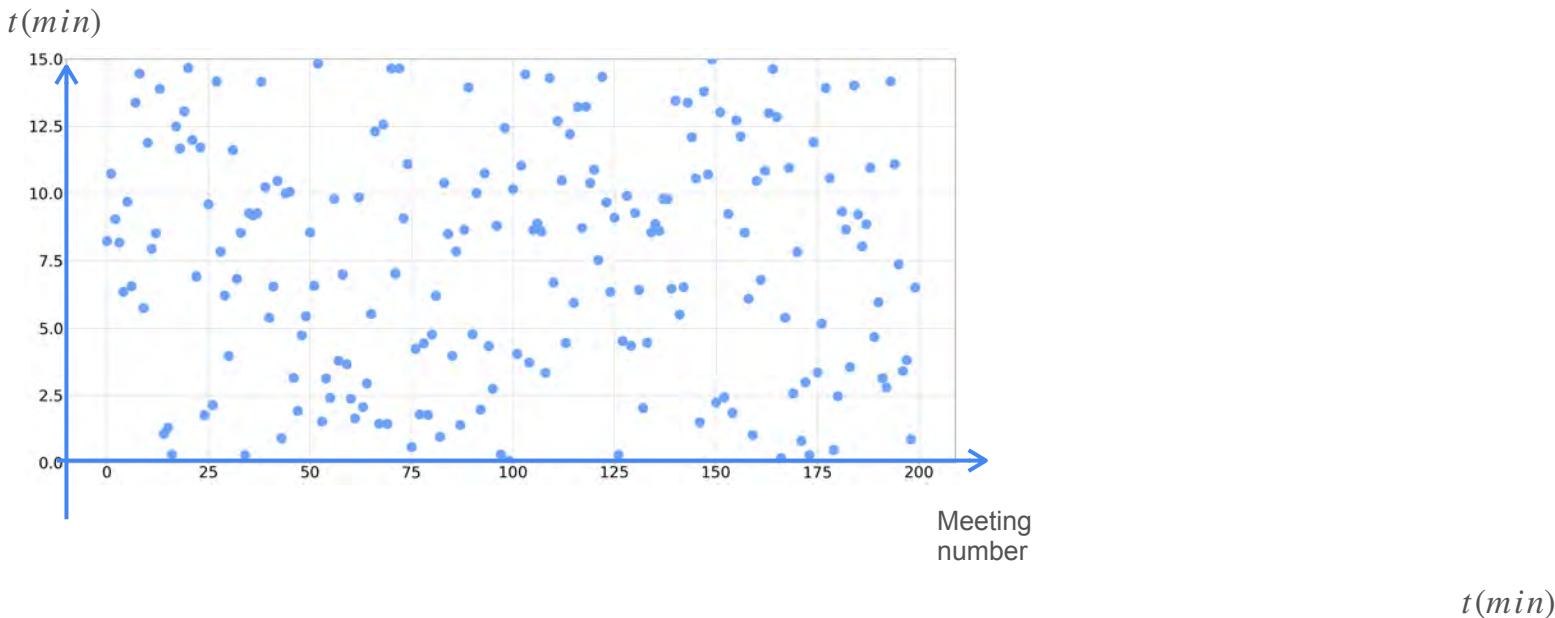
Last 200 times you called them, you took down notes of how long they took to respond



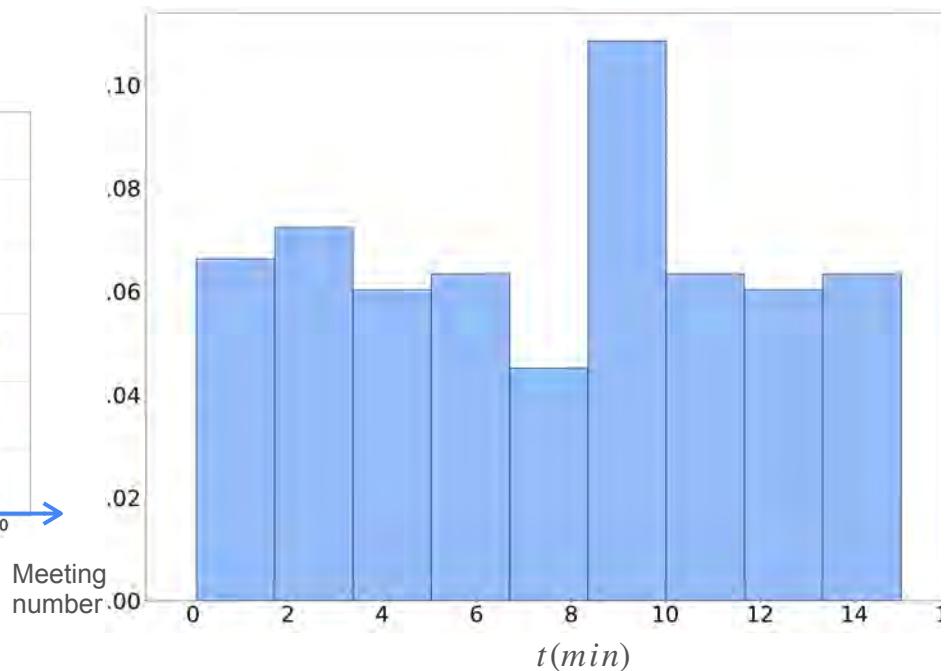
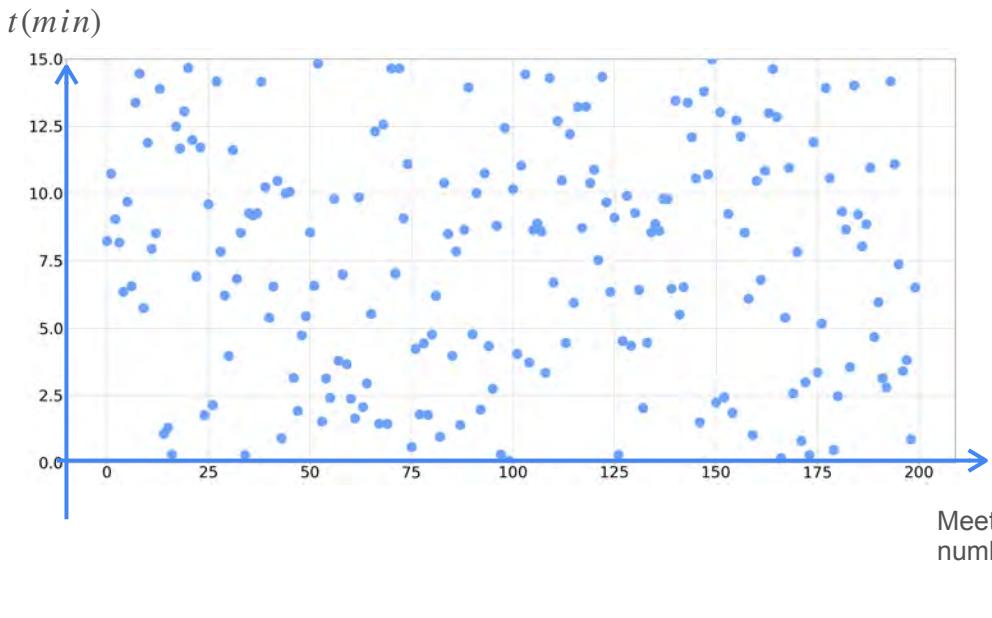
Uniform Distribution: Motivation



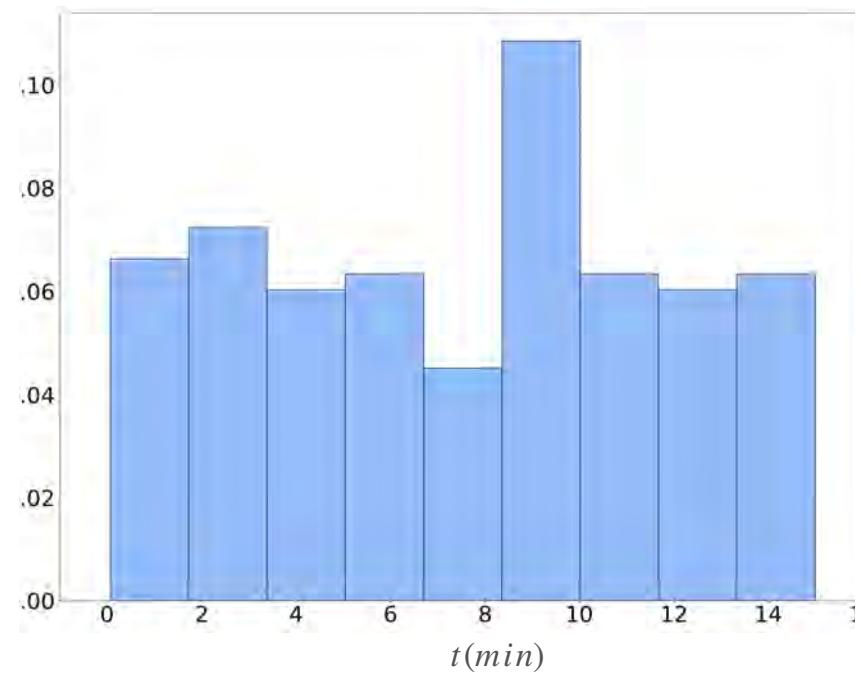
Uniform Distribution: Motivation



Uniform Distribution: Motivation

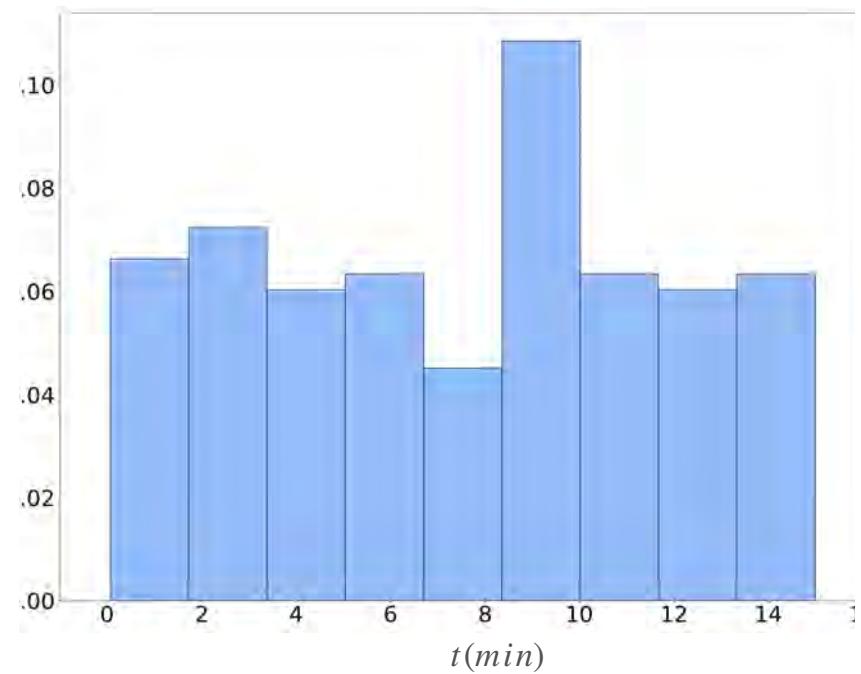


Uniform Distribution: Motivation



Uniform Distribution: Motivation

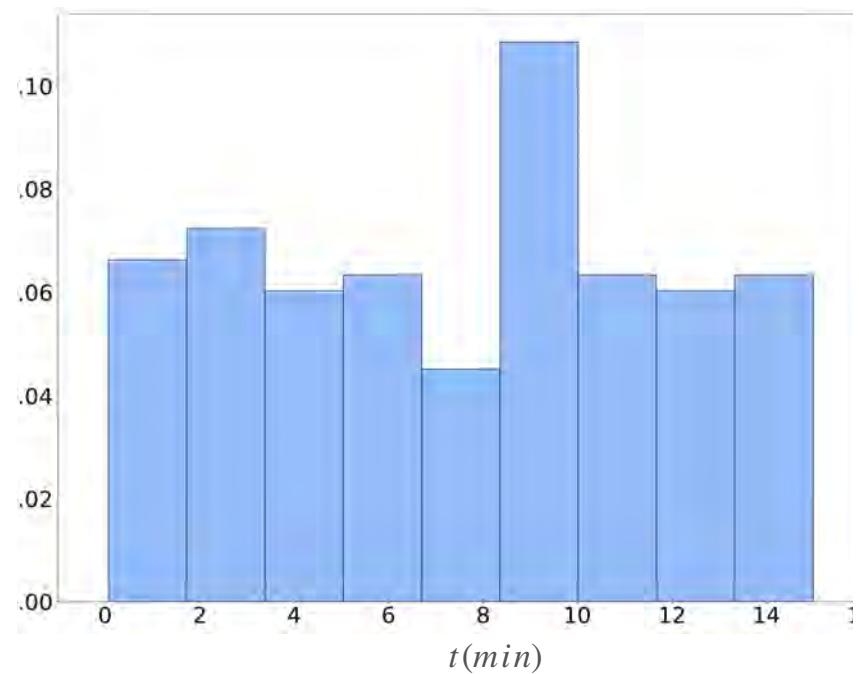
T: time (in minutes) you have to wait



Uniform Distribution: Motivation

T: time (in minutes) you have to wait

Any value between 0 and 15 minutes must have the same frequency of occurrence.



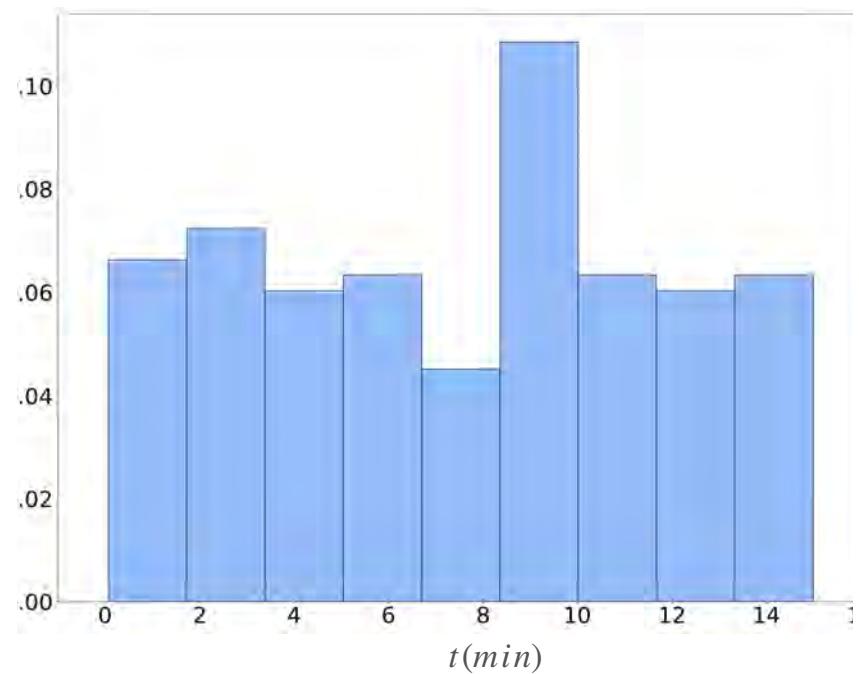
Uniform Distribution: Motivation

T: time (in minutes) you have to wait

Any value between 0 and 15 minutes must have the same frequency of occurrence.



The pdf must be constant for all values in the interval (0,15)



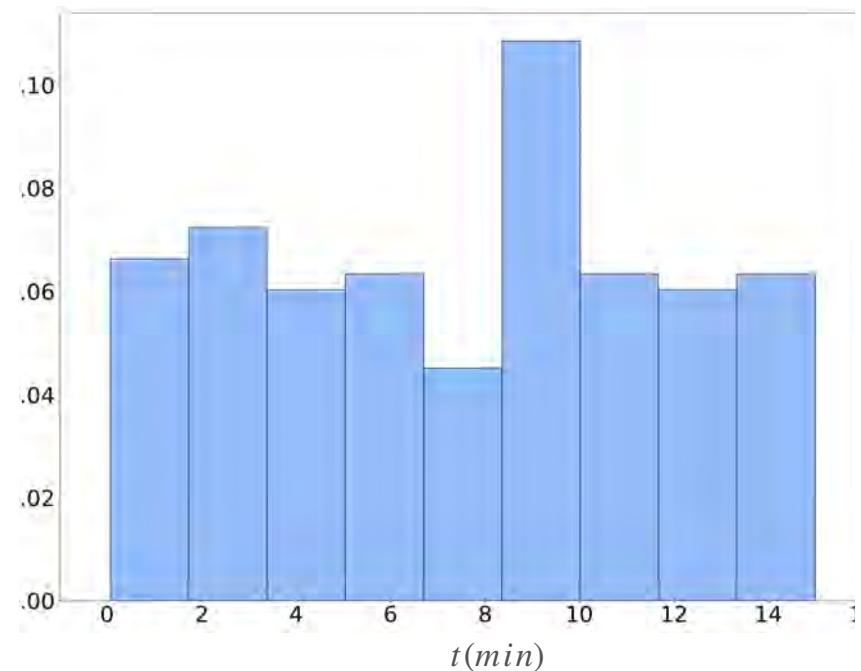
Uniform Distribution: Motivation

T: time (in minutes) you have to wait

Any value between 0 and 15 minutes must have the same frequency of occurrence.

The pdf must be constant for all values in the interval (0,15)

Which constant?



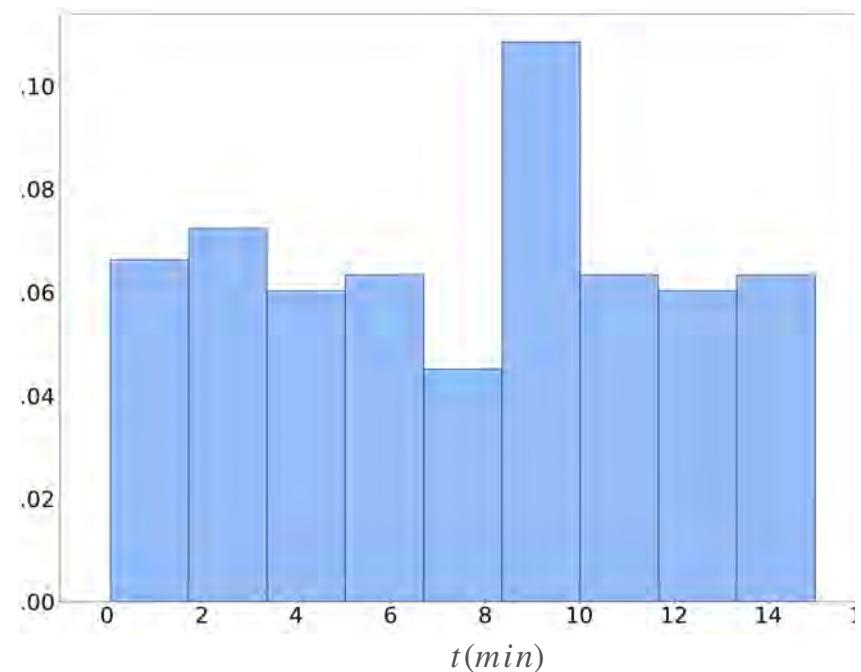
Uniform Distribution: Motivation

T: time (in minutes) you have to wait

Any value between 0 and 15 minutes must have the same frequency of occurrence.

The pdf must be constant for all values in the interval $(0, 15)$

Which constant? $\rightarrow 15 \times h = 1$



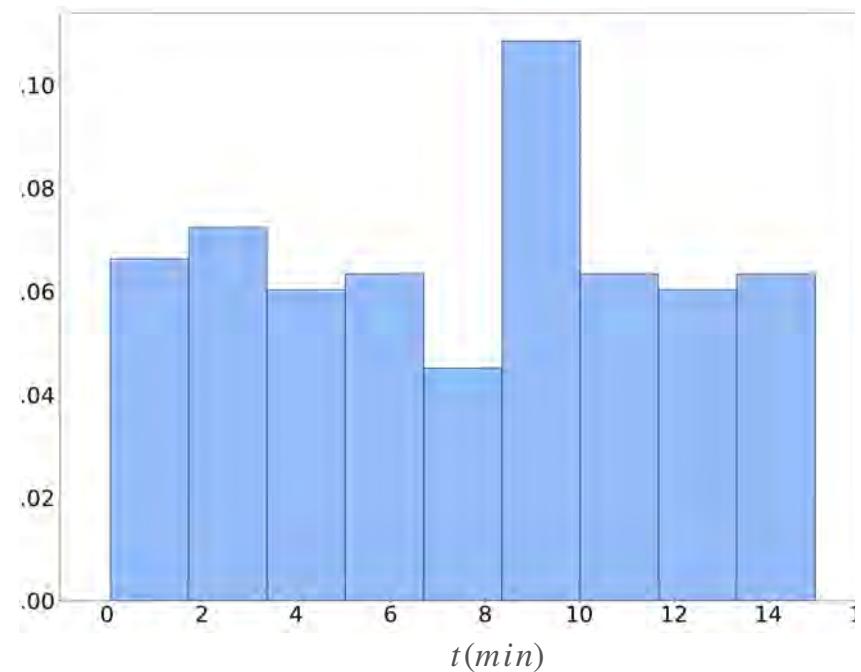
Uniform Distribution: Motivation

T: time (in minutes) you have to wait

Any value between 0 and 15 minutes must have the same frequency of occurrence.

The pdf must be constant for all values in the interval (0,15)

Which constant? $\rightarrow 15 \times h = 1 \rightarrow h = \frac{1}{15} = 0.06$



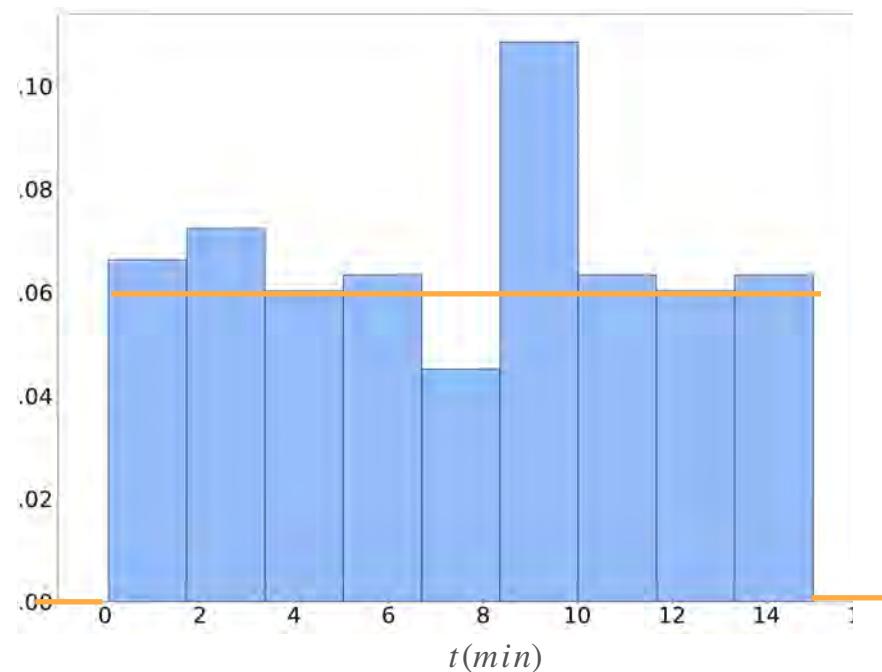
Uniform Distribution: Motivation

T: time (in minutes) you have to wait

Any value between 0 and 15 minutes must have the same frequency of occurrence.

The pdf must be constant for all values in the interval (0,15)

Which constant? $\rightarrow 15 \times h = 1 \rightarrow h = \frac{1}{15} = 0.06$



Uniform Distribution: Model

x

Uniform Distribution: Model

A continuous random variable can be modeled with a **uniform** distribution if all possible values lie in an interval and have the **same frequency** of occurrence

x

Uniform Distribution: Model

A continuous random variable can be modeled with a **uniform** distribution if all possible values lie in an interval and have the **same frequency** of occurrence

Parameters:

x

Uniform Distribution: Model

A continuous random variable can be modeled with a **uniform** distribution if all possible values lie in an interval and have the **same frequency** of occurrence

Parameters:

- a : beginning of the interval

x

Uniform Distribution: Model

A continuous random variable can be modeled with a **uniform** distribution if all possible values lie in an interval and have the **same frequency** of occurrence

Parameters:

- a : beginning of the interval
- b : end of the interval

x

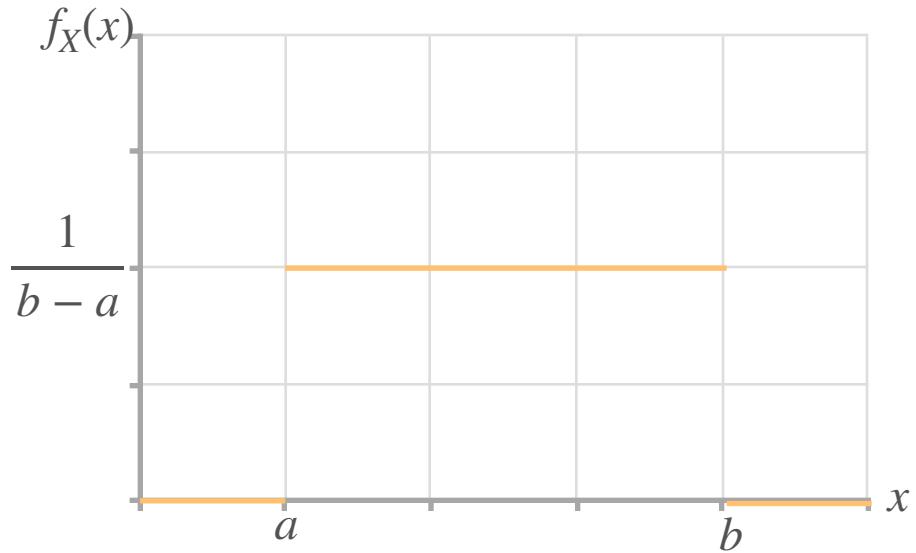
Uniform Distribution: Model

A continuous random variable can be modeled with a **uniform** distribution if all possible values lie in an interval and have the **same frequency** of occurrence

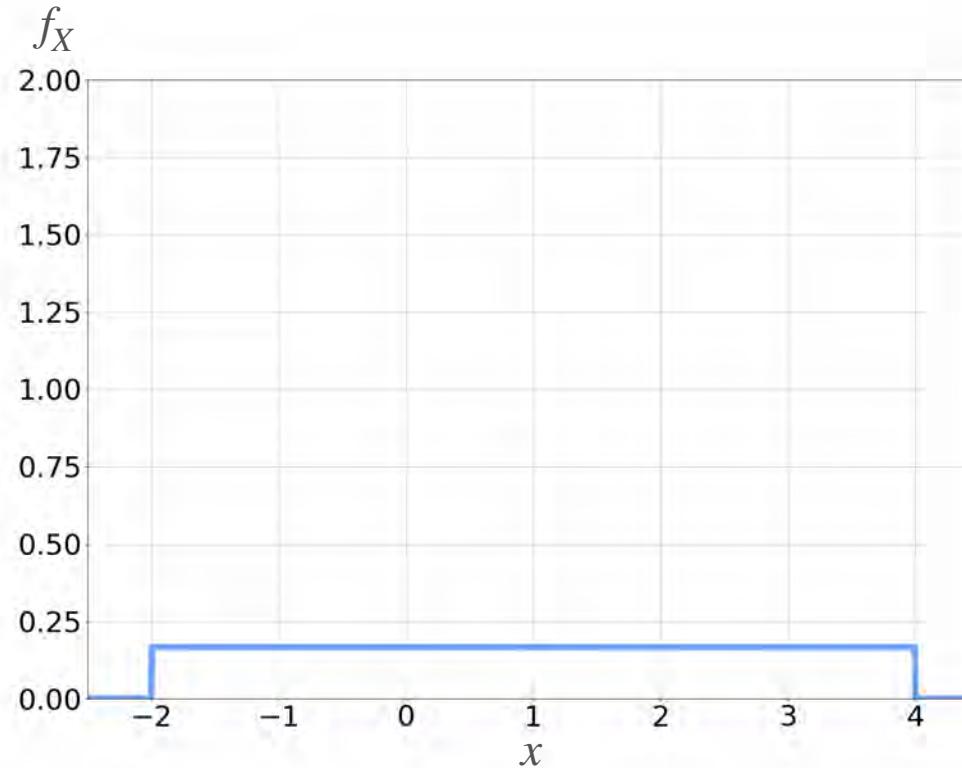
Parameters:

- a : beginning of the interval
- b : end of the interval

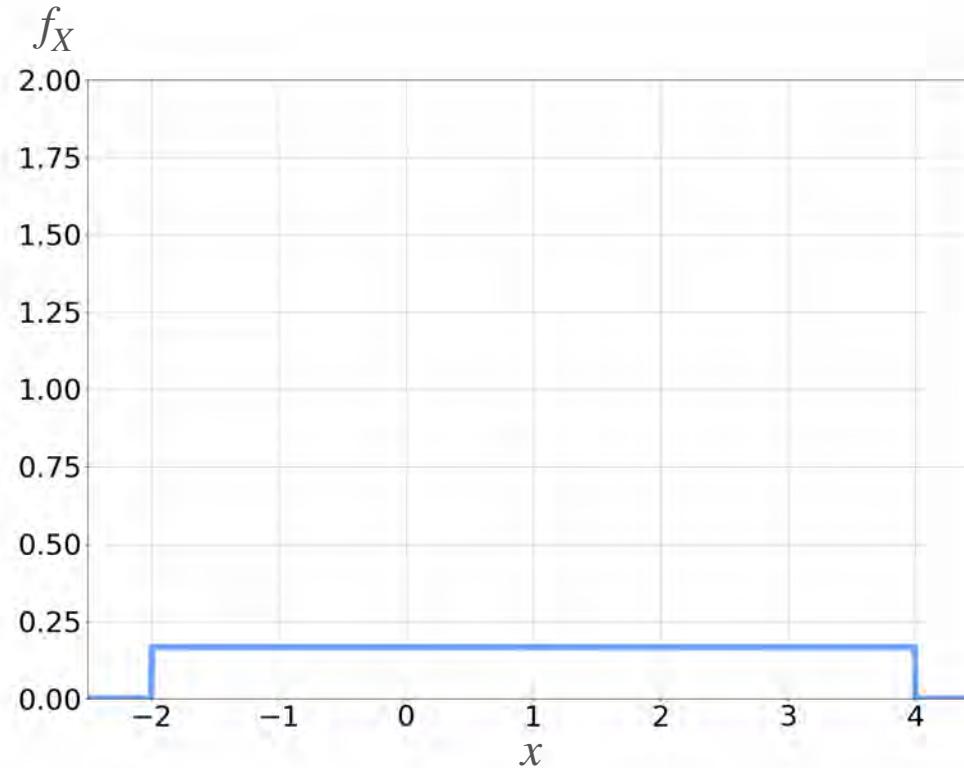
$$f_X(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & x \notin (a, b) \end{cases}$$



Uniform Distribution: PDF



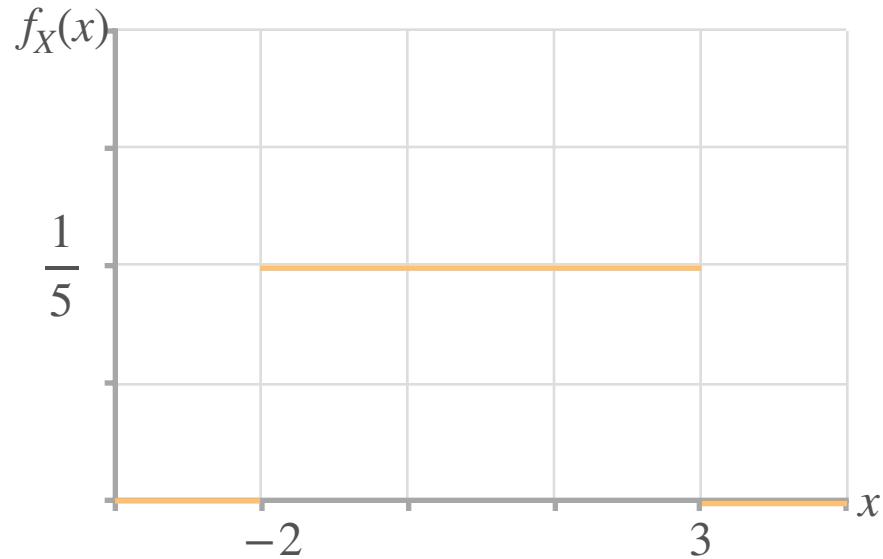
Uniform Distribution: PDF



Quiz

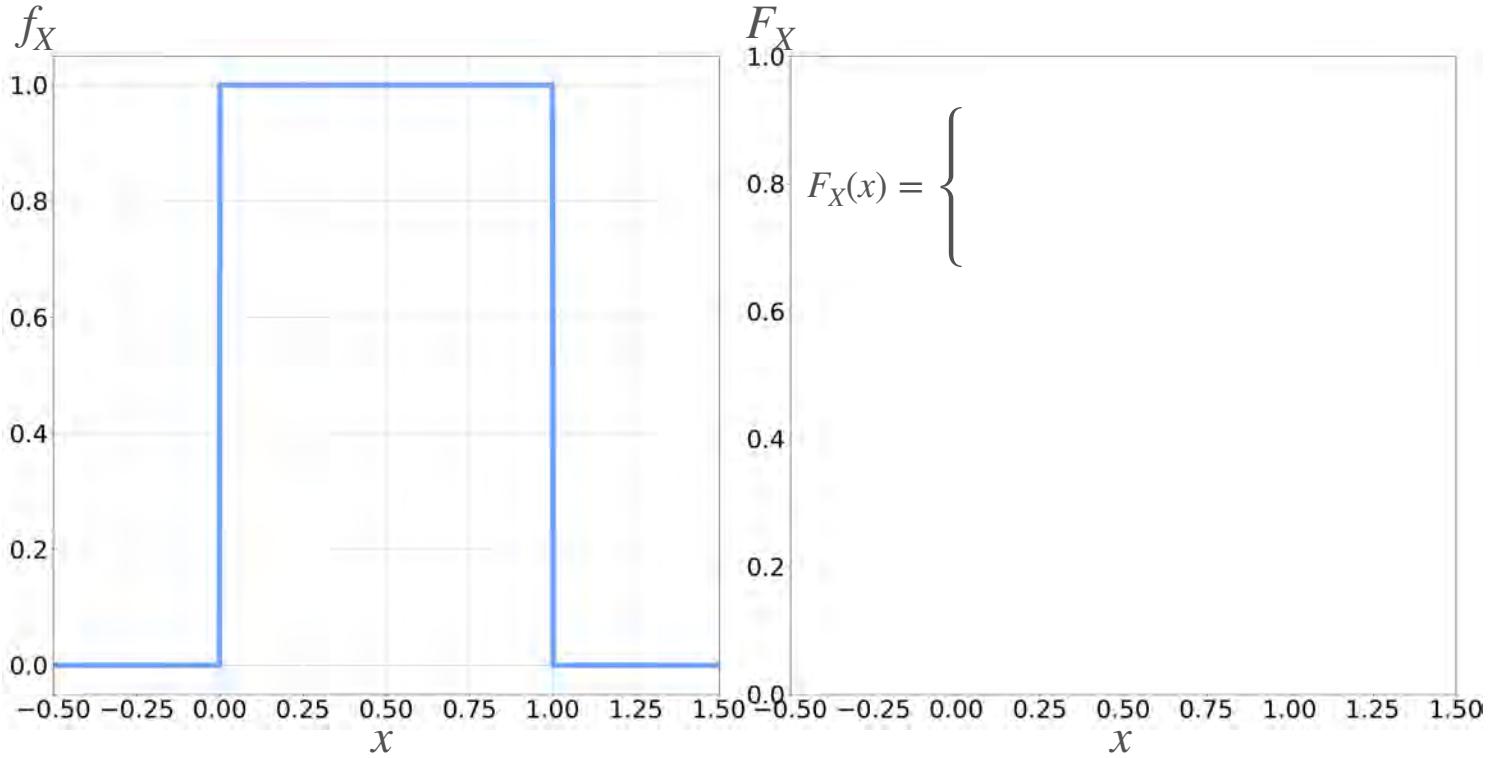
What is the probability of X being between -1 and 3?

What is the probability of X being between -1 and 4?

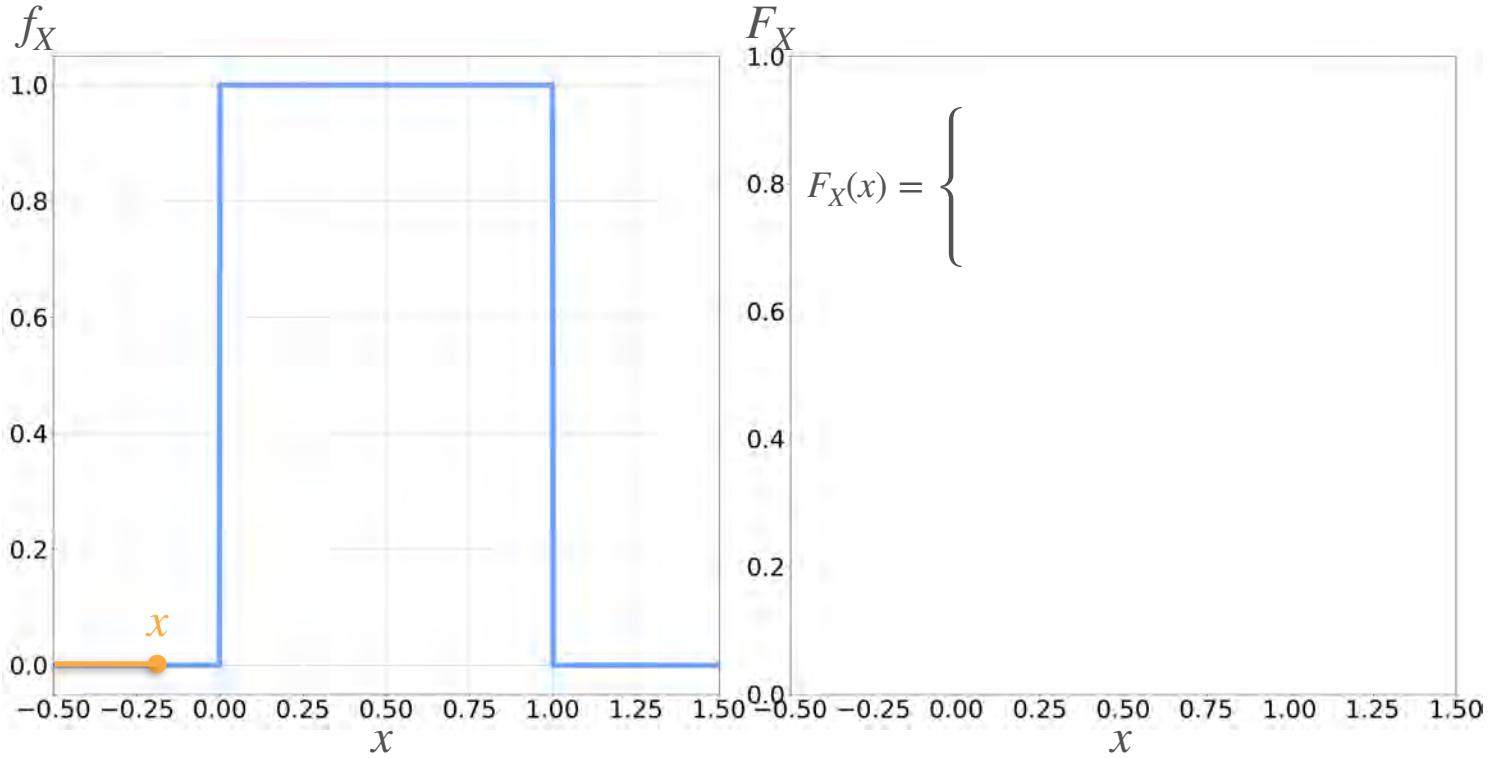


Uniform Distribution: CDF

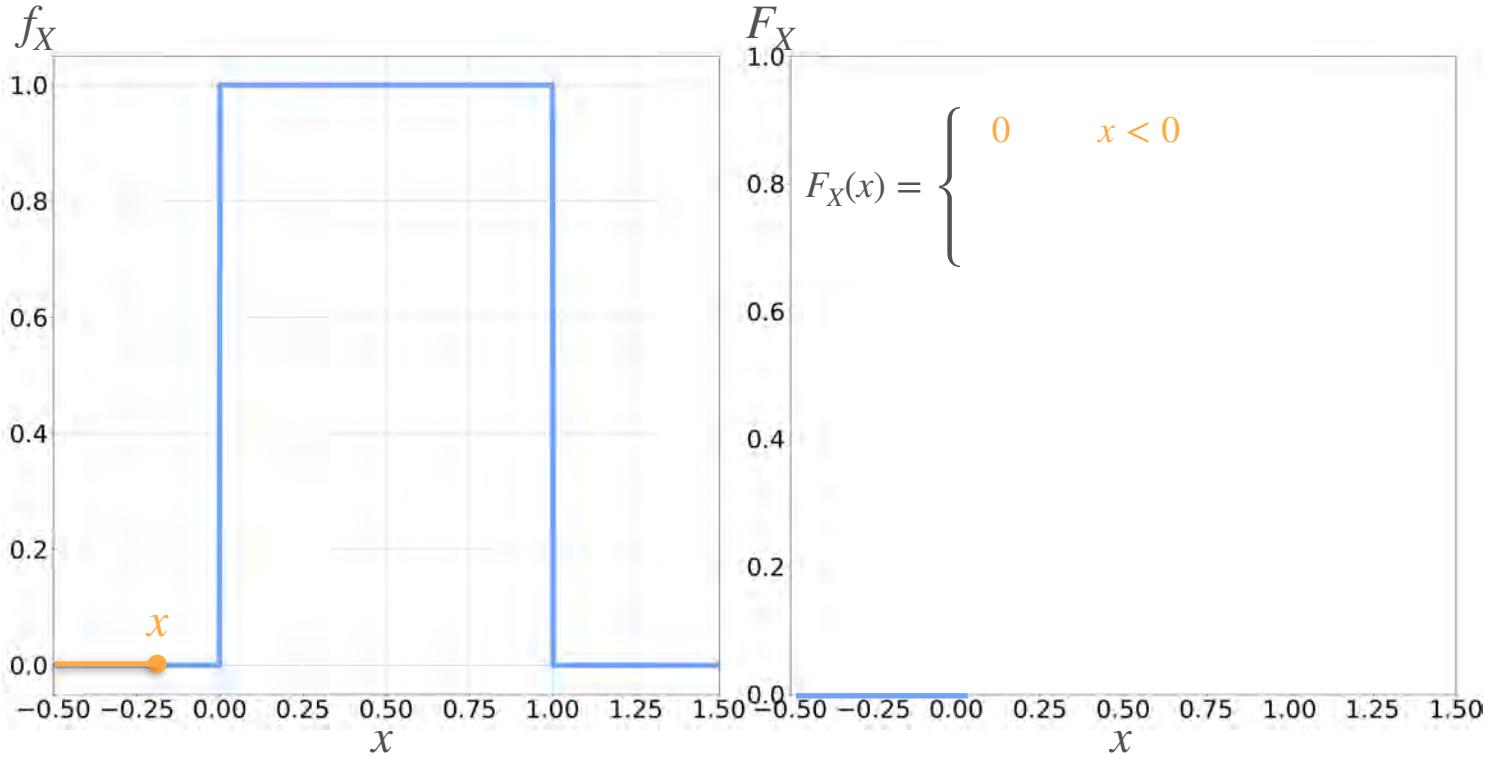
Uniform Distribution: CDF



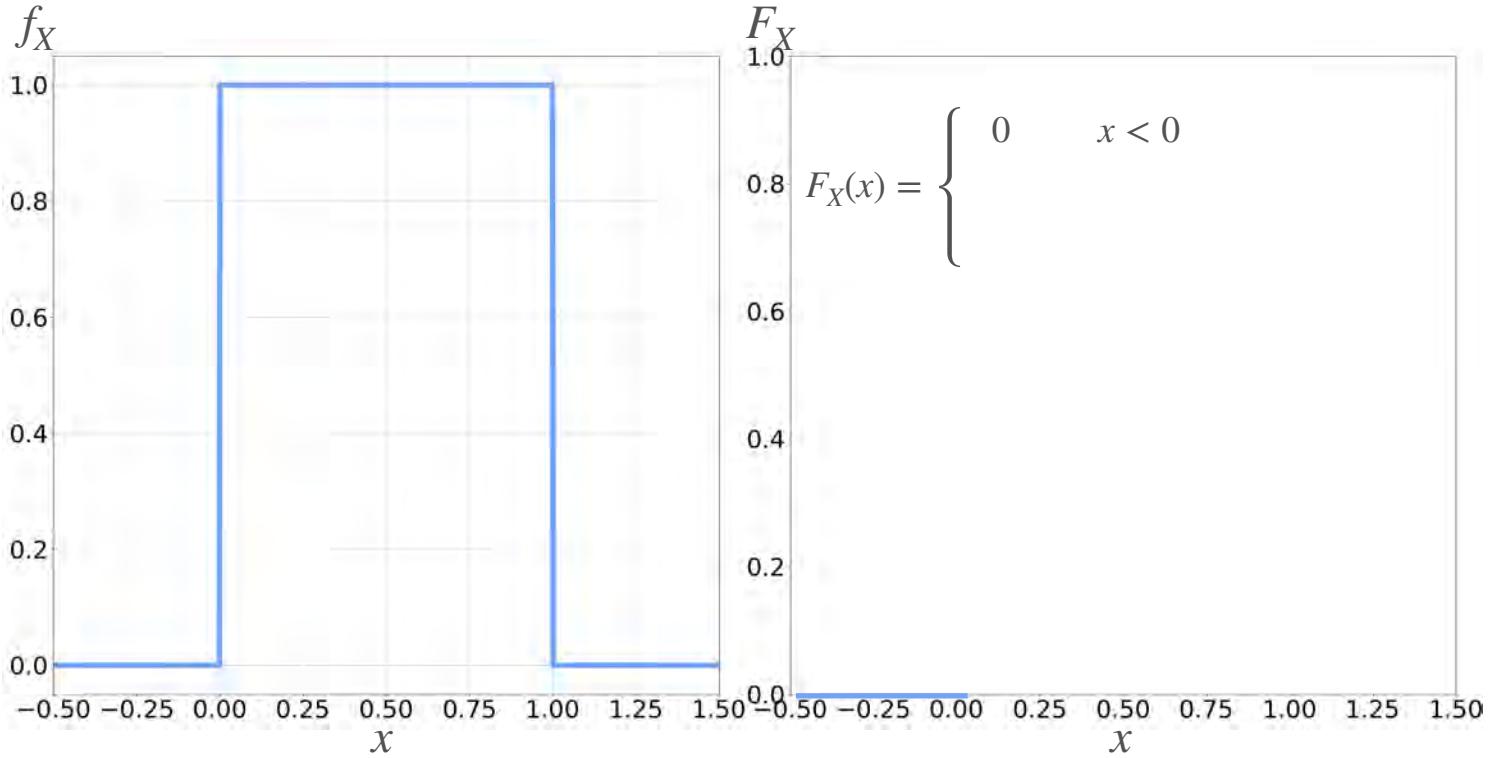
Uniform Distribution: CDF



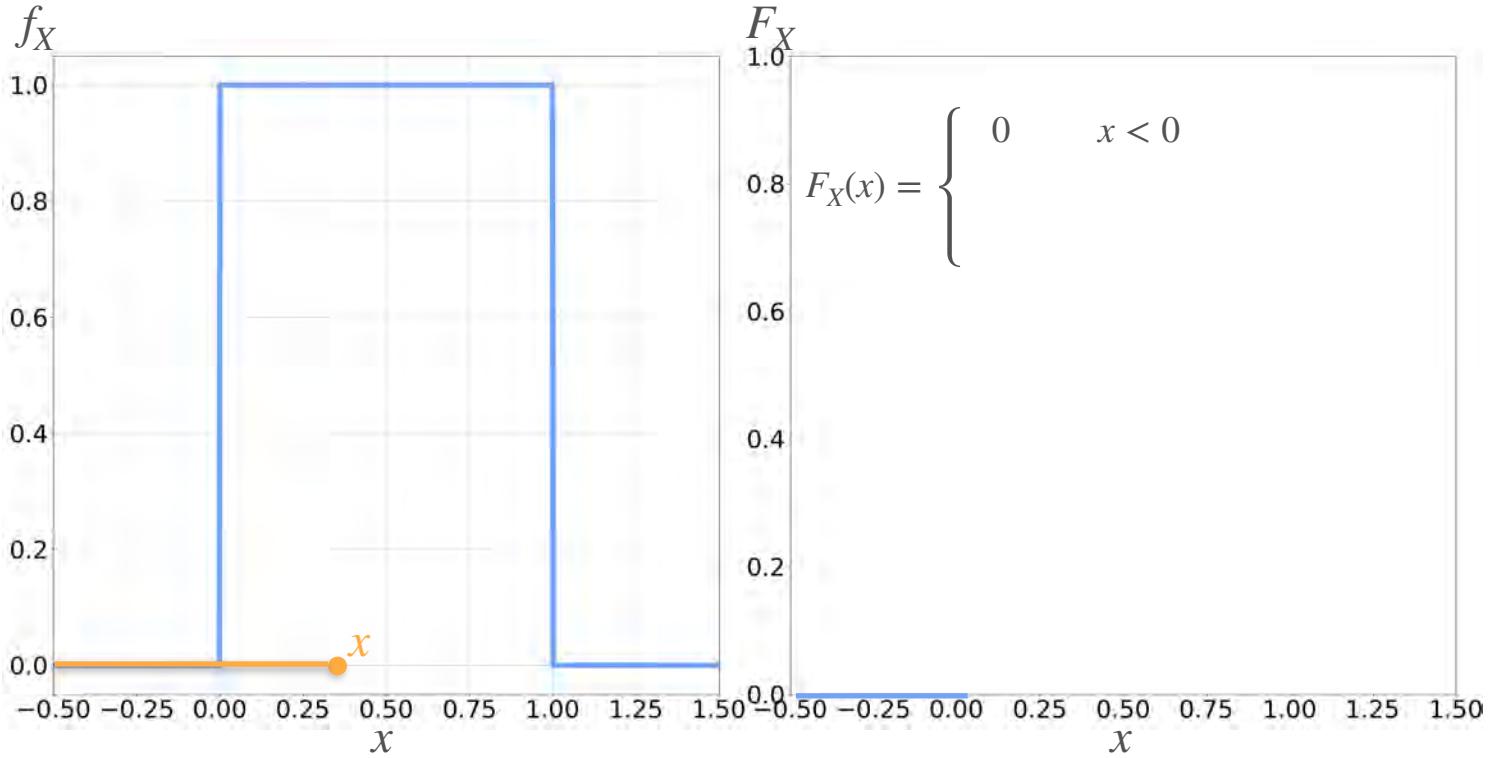
Uniform Distribution: CDF



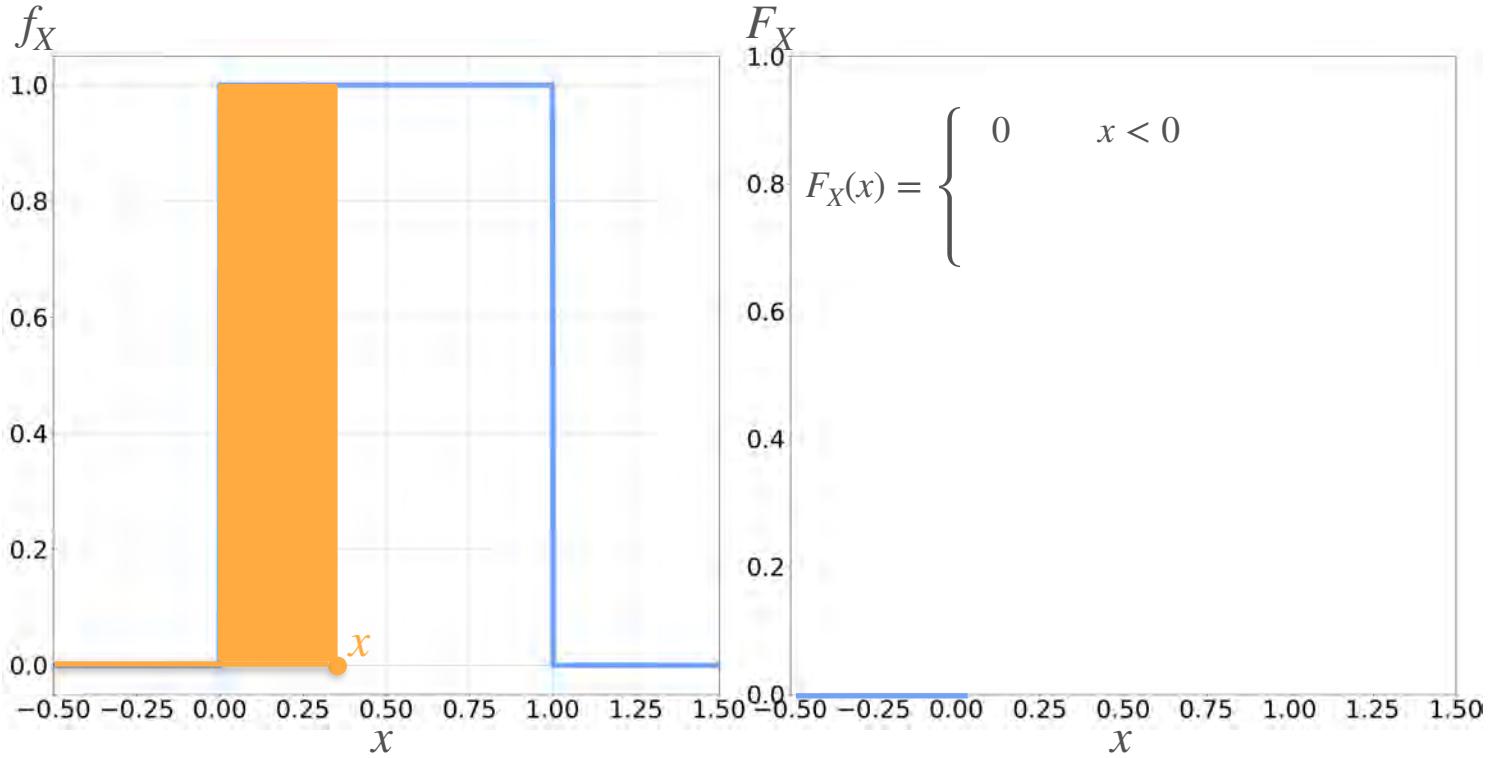
Uniform Distribution: CDF



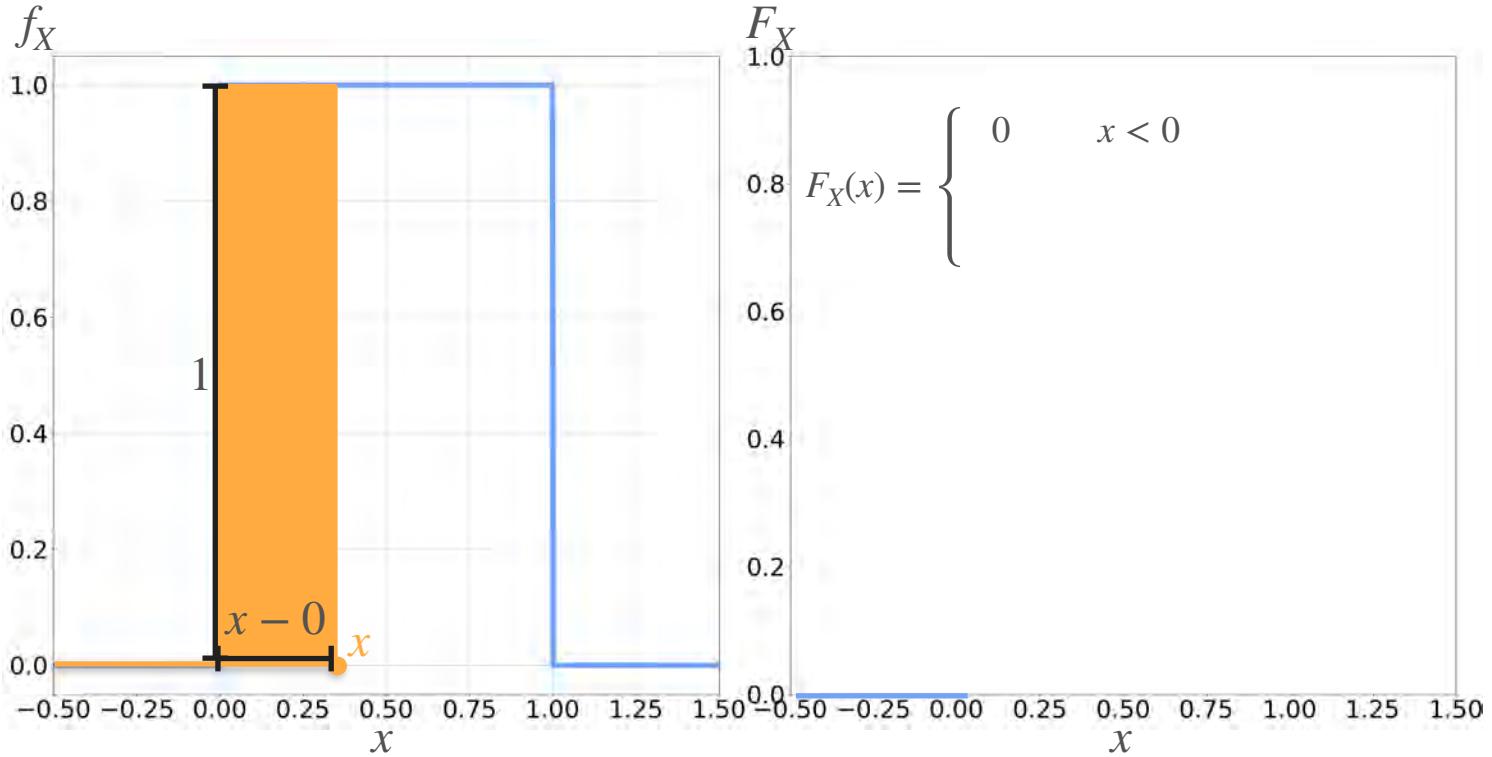
Uniform Distribution: CDF



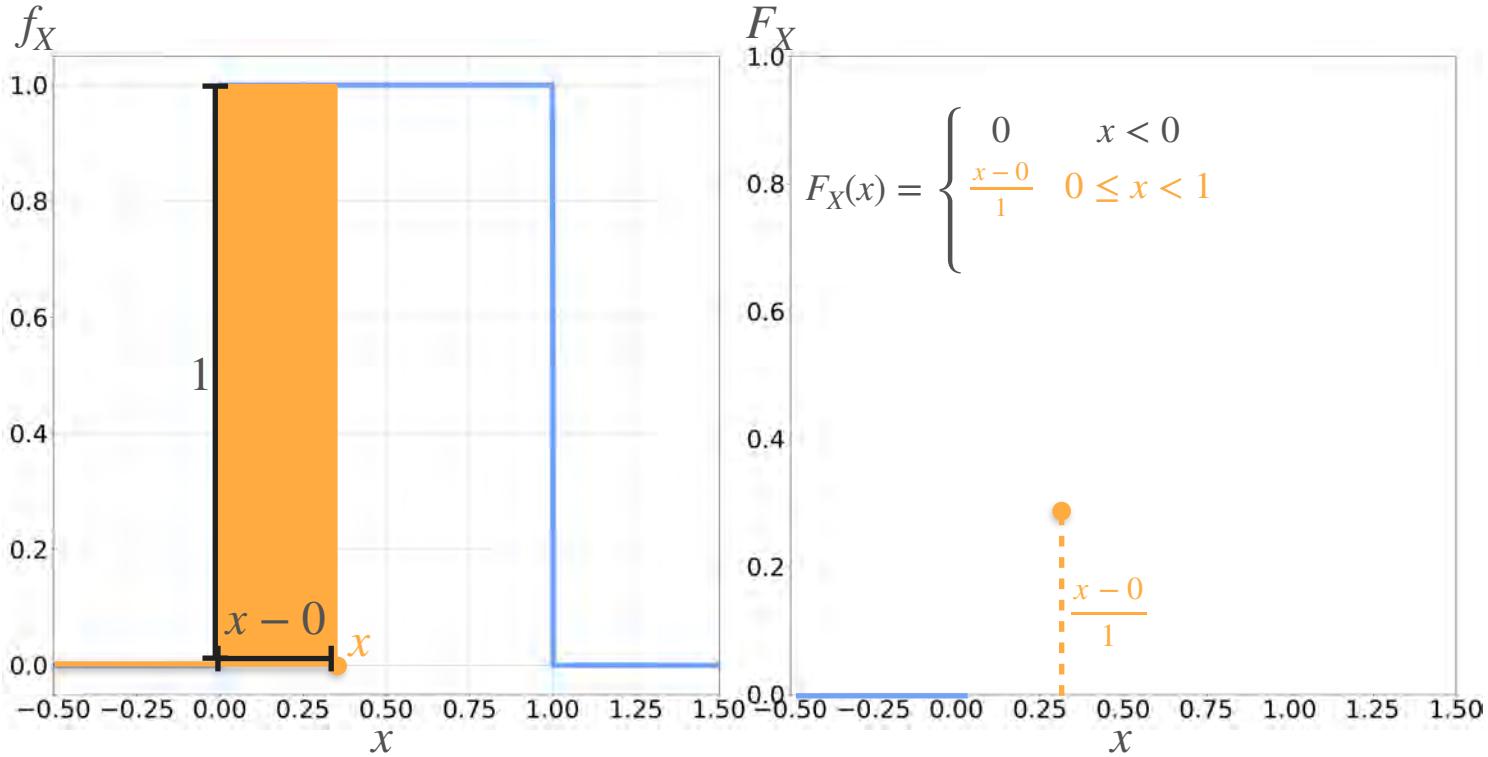
Uniform Distribution: CDF



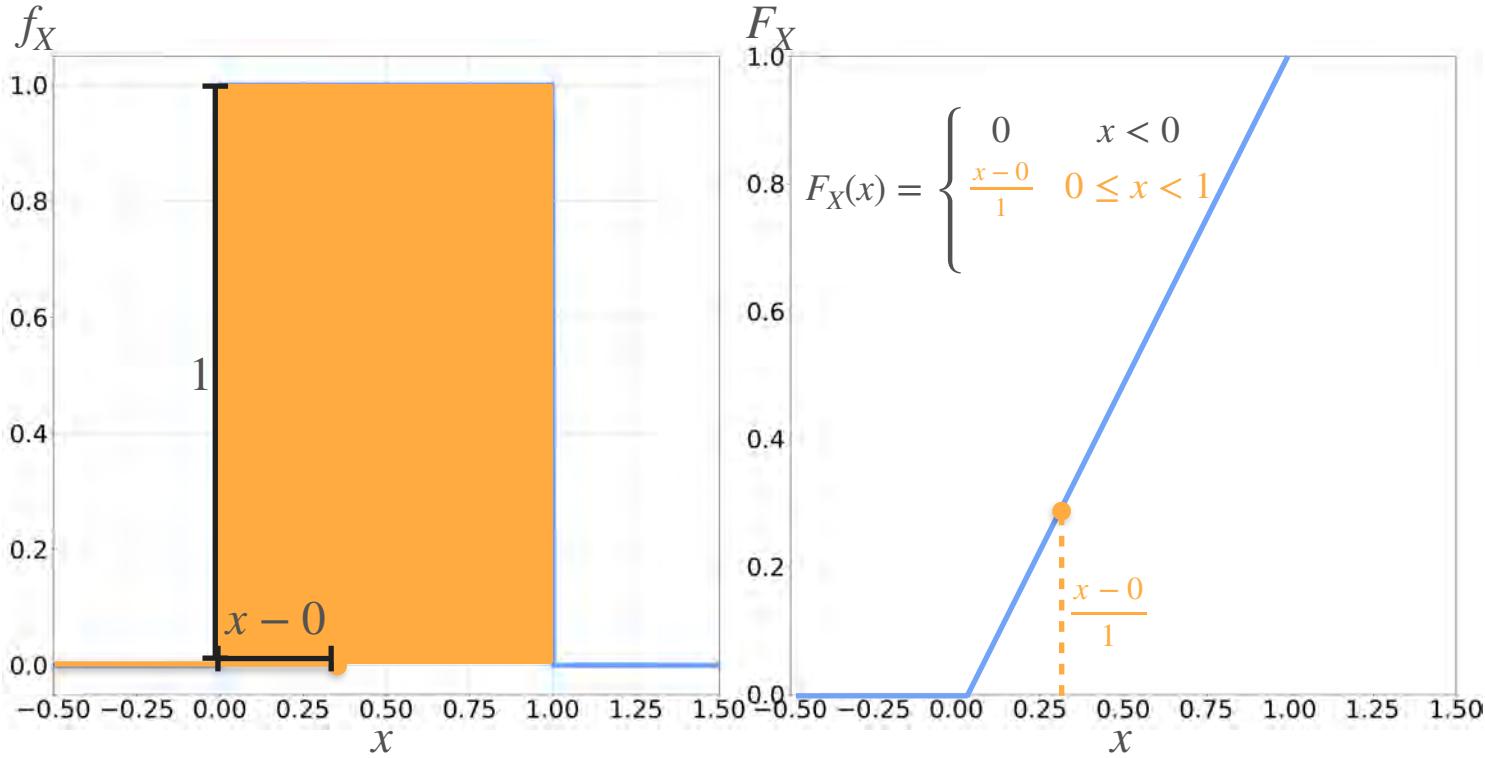
Uniform Distribution: CDF



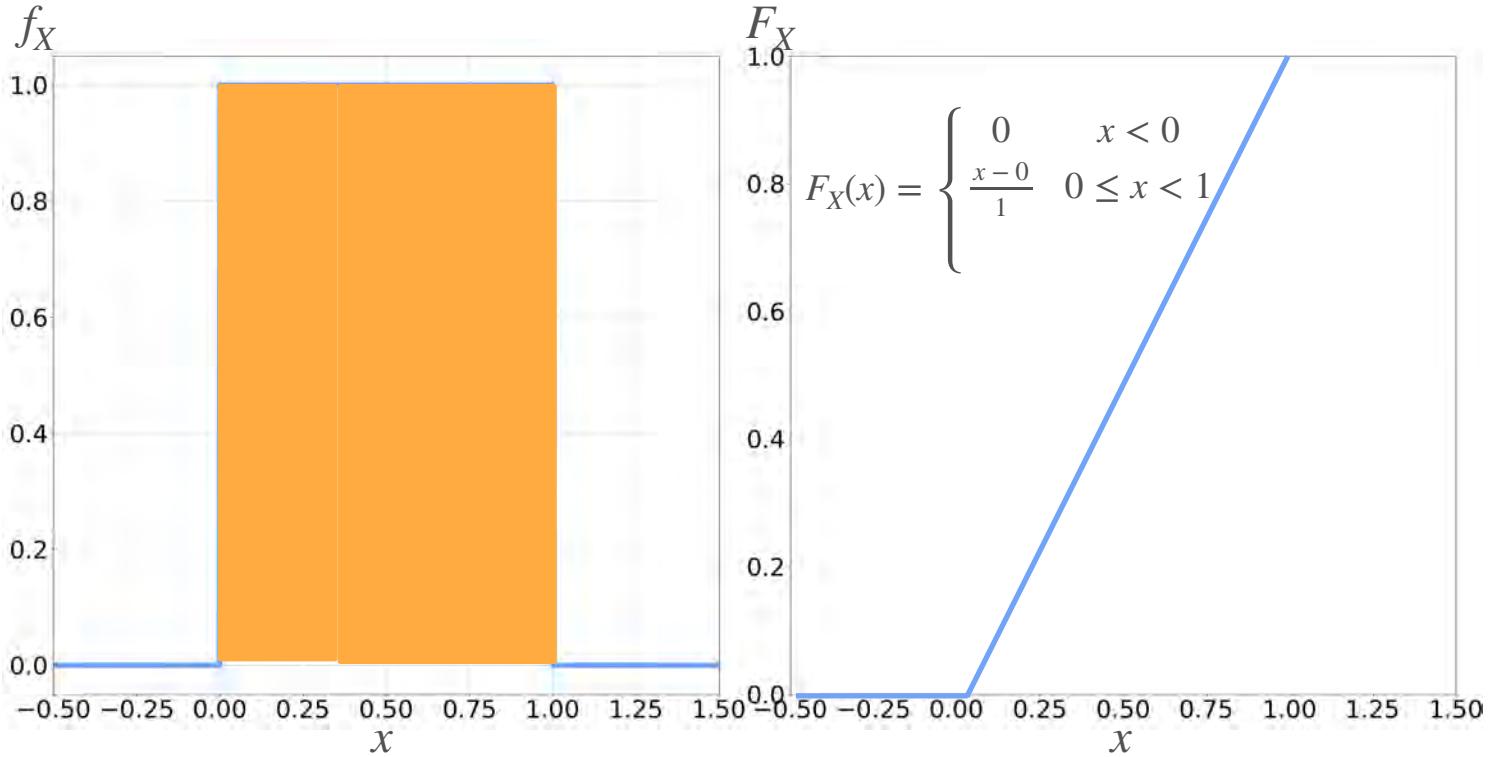
Uniform Distribution: CDF



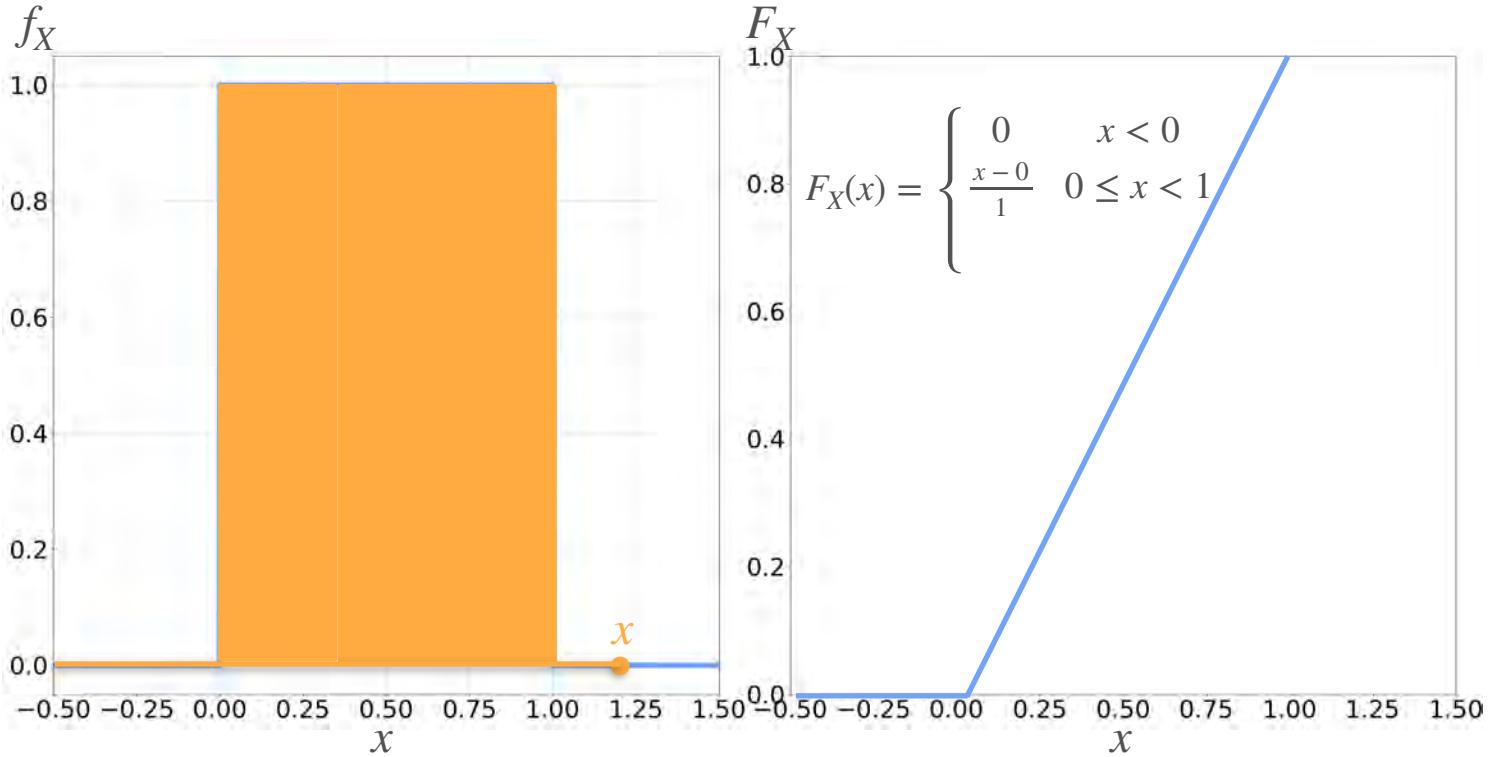
Uniform Distribution: CDF



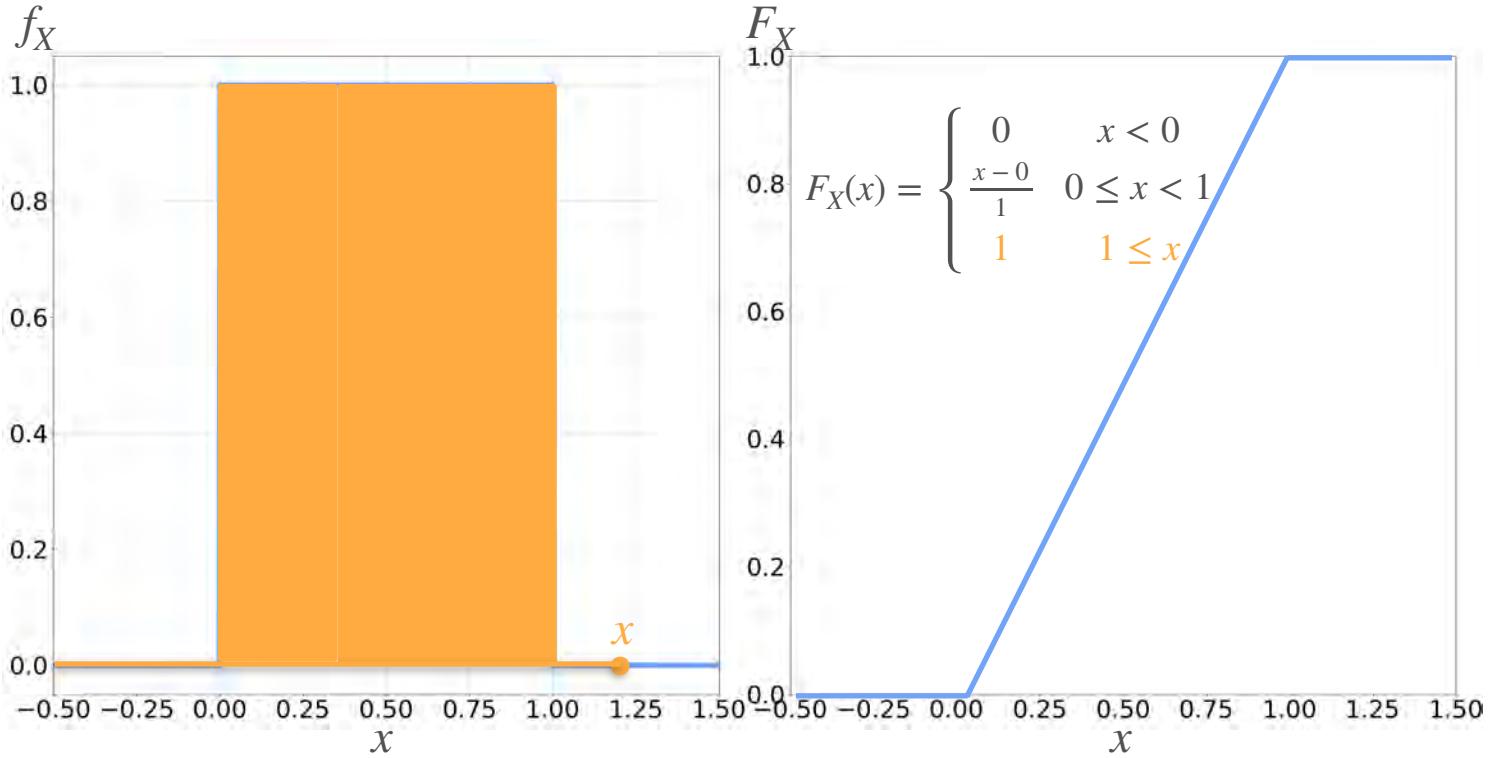
Uniform Distribution: CDF



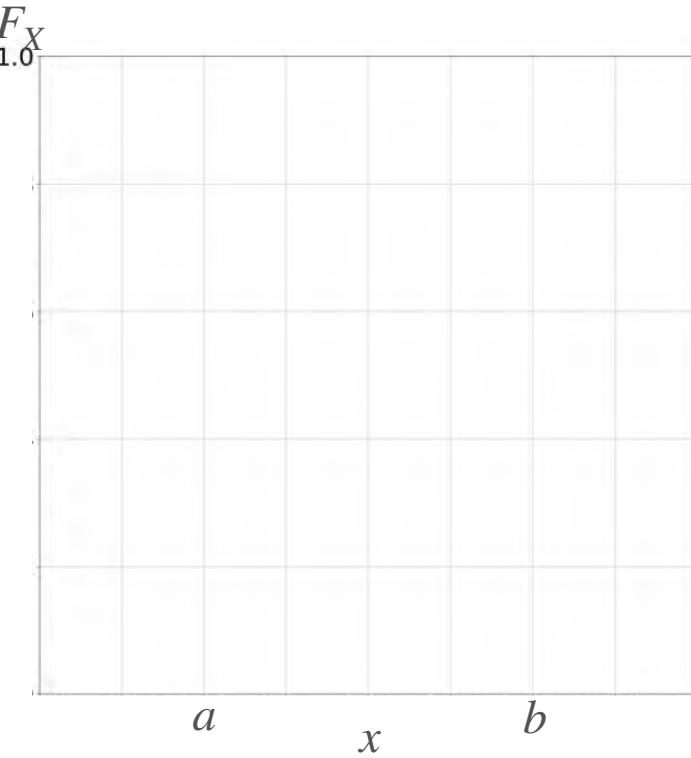
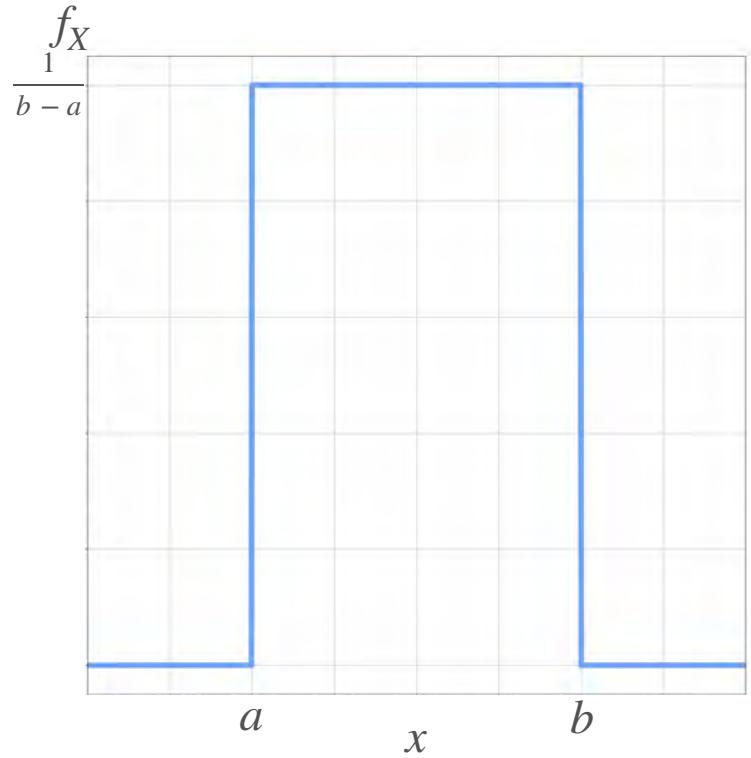
Uniform Distribution: CDF



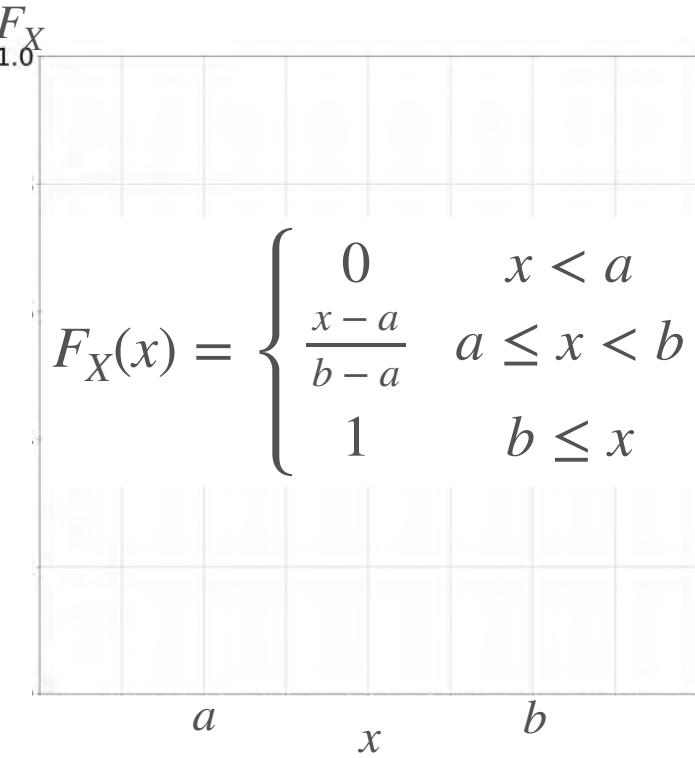
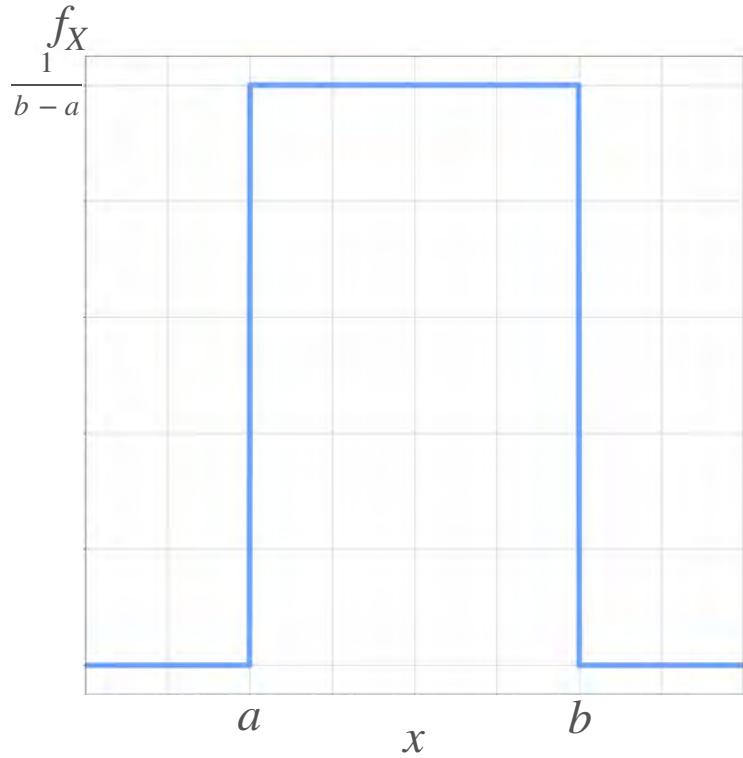
Uniform Distribution: CDF



Uniform Distribution: CDF



Uniform Distribution: CDF





DeepLearning.AI

Probability Distributions

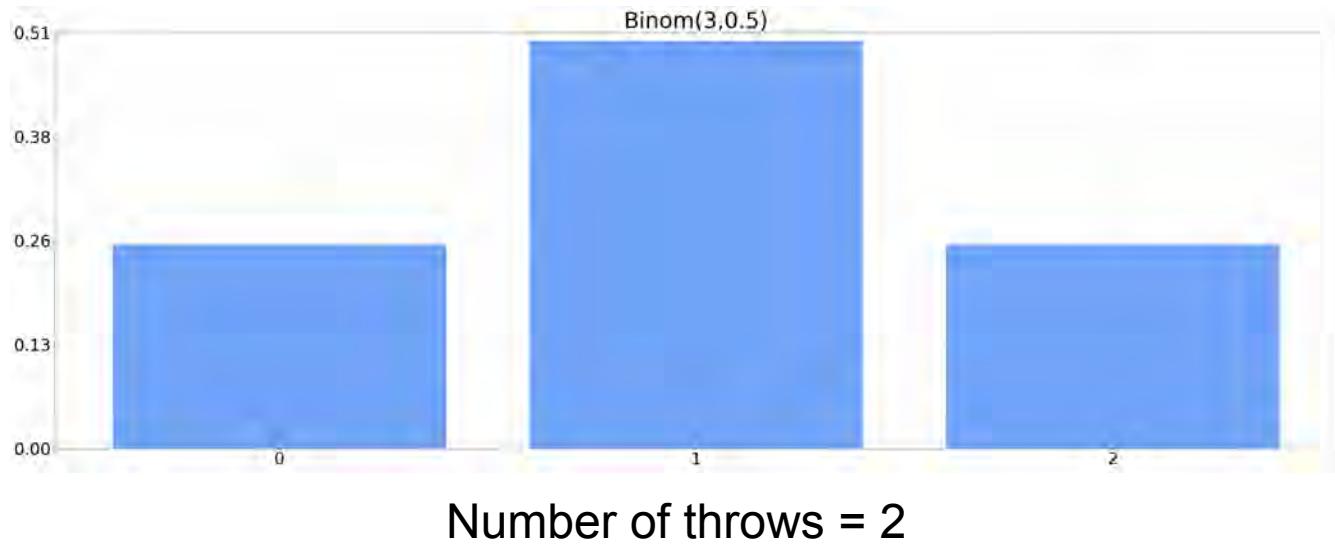
Normal distribution

Binomial Distribution With Very Large n

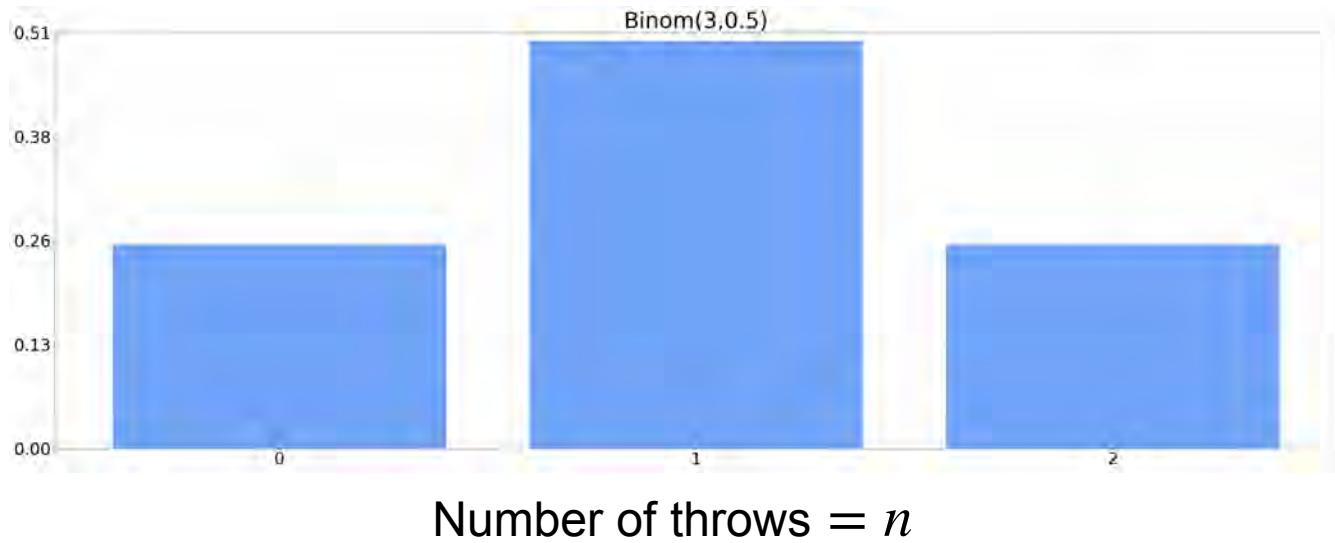
Binomial Distribution With Very Large n



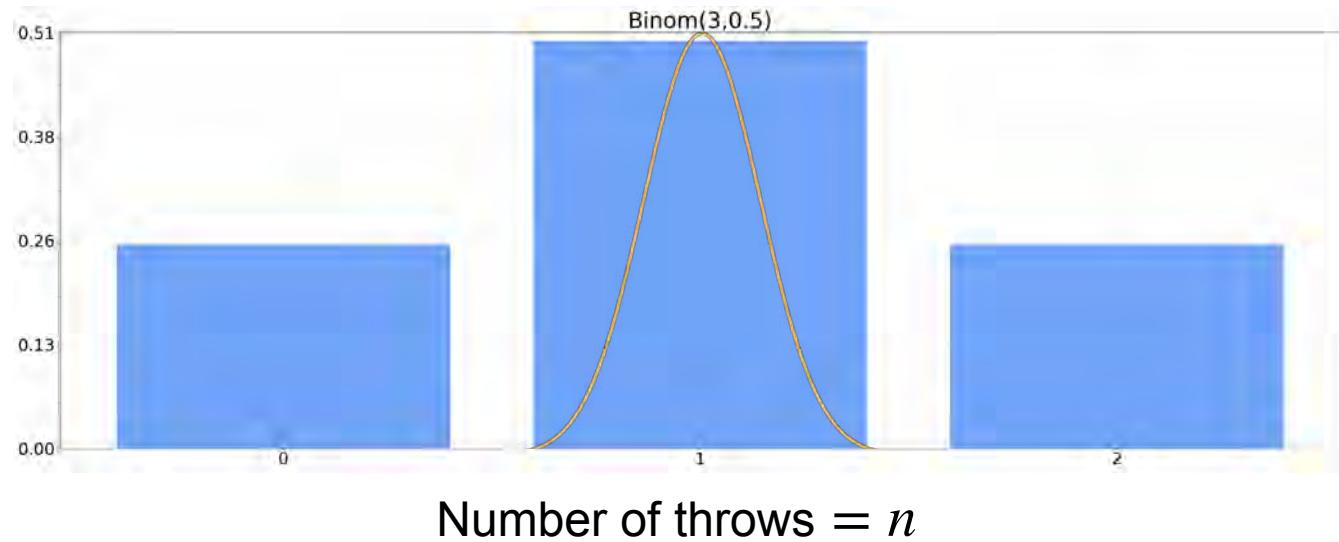
Binomial Distribution With Very Large n



Binomial Distribution With Very Large n



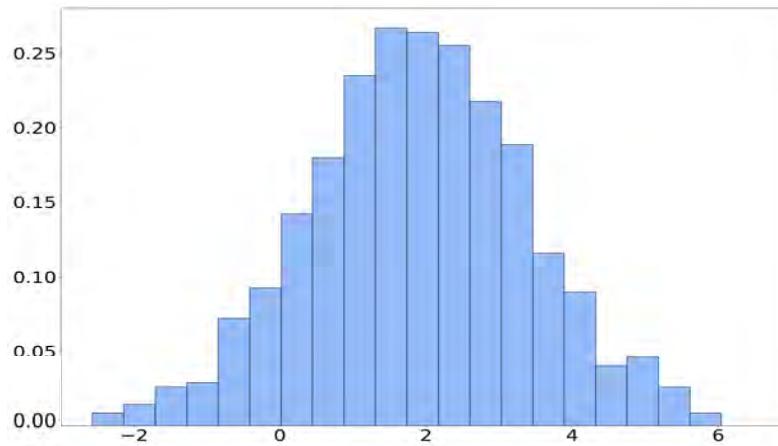
Binomial Distribution With Very Large n



Bell Shaped Data

Bell Shaped Data

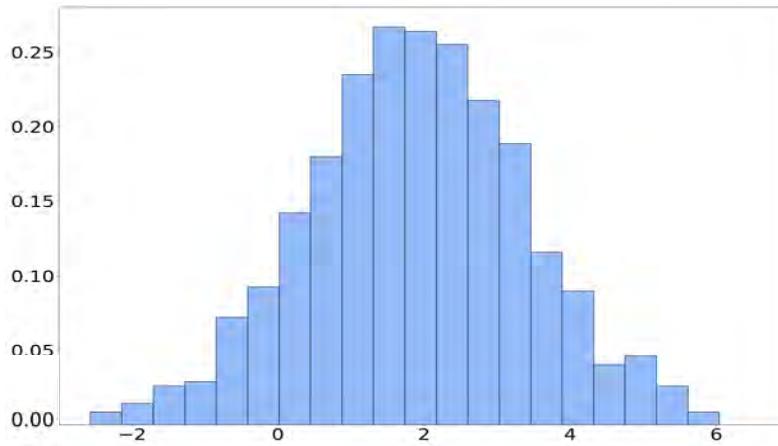
Data



Bell Shaped Data

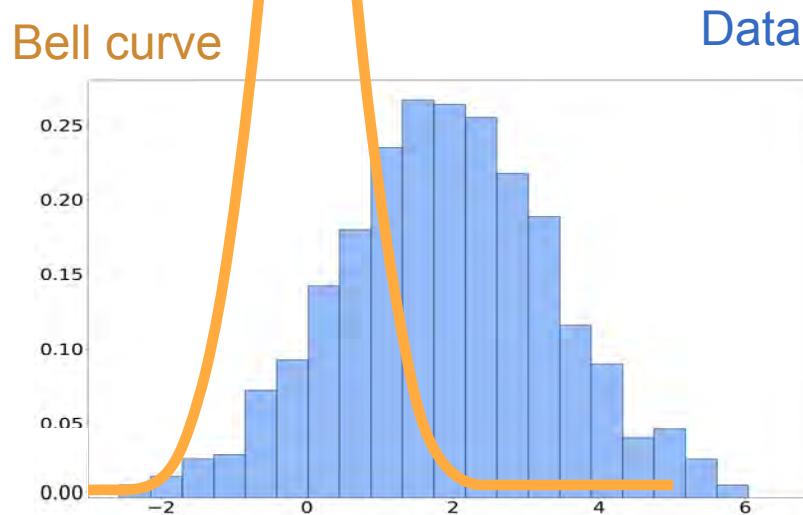
$$e^{-x^2}$$

Data



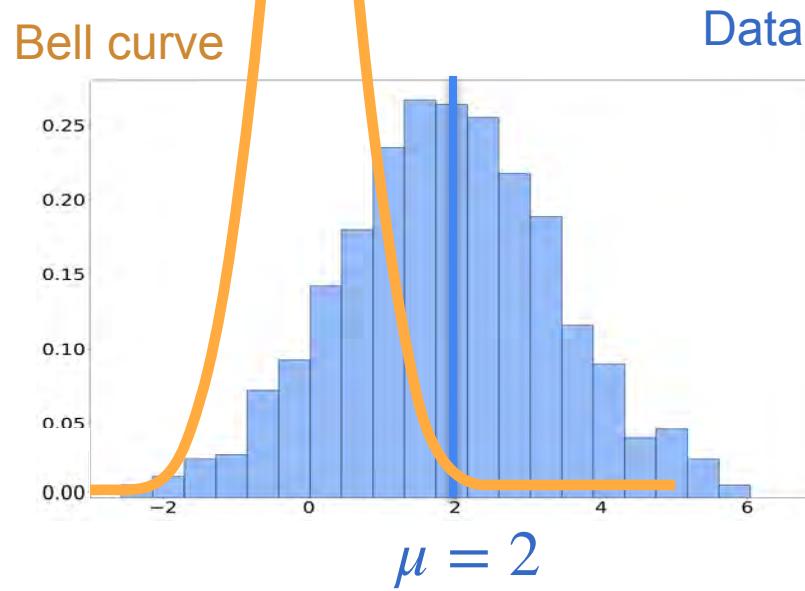
Bell Shaped Data

$$e^{-x^2}$$



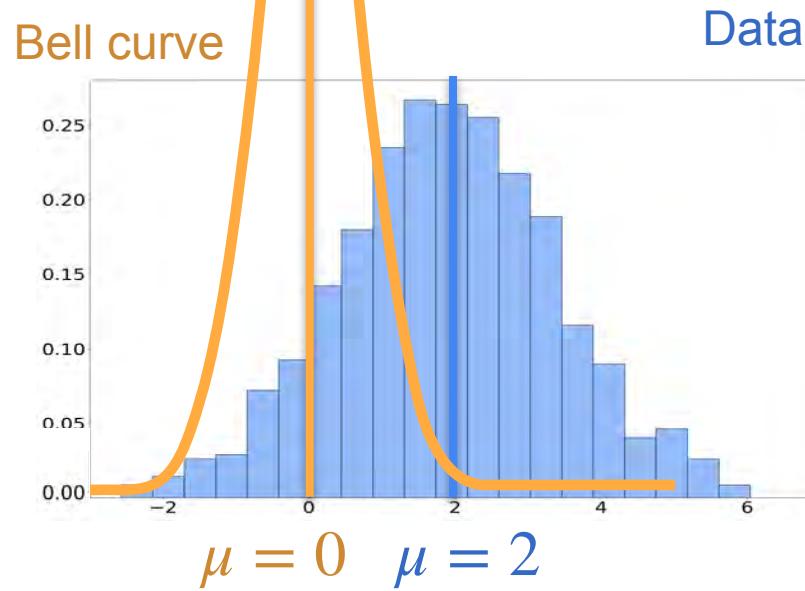
Bell Shaped Data

$$e^{-x^2}$$



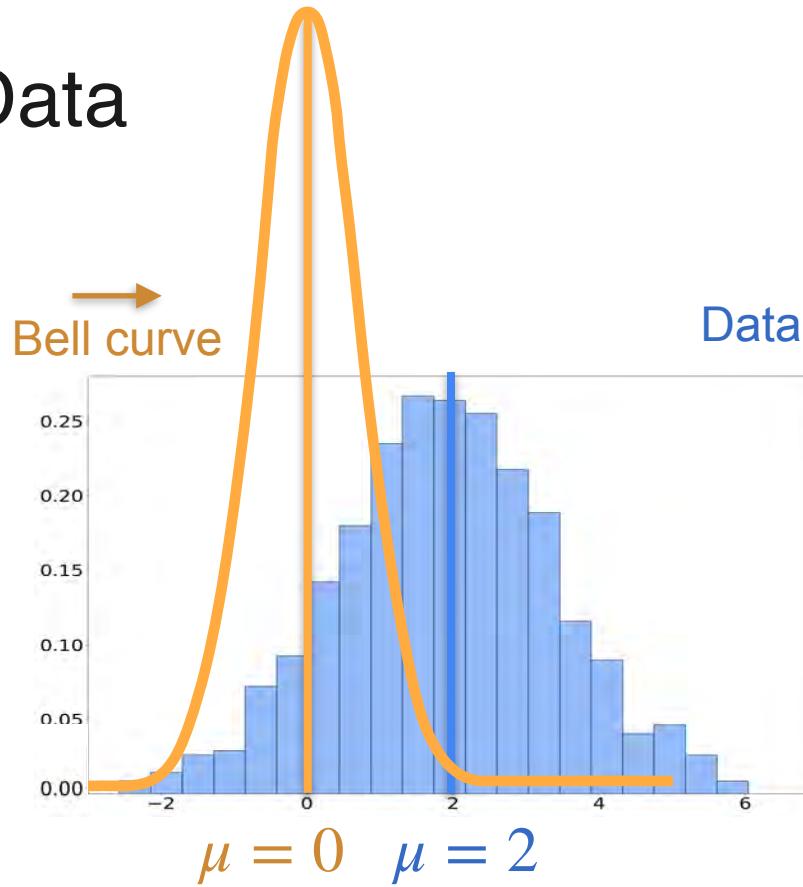
Bell Shaped Data

$$e^{-x^2}$$



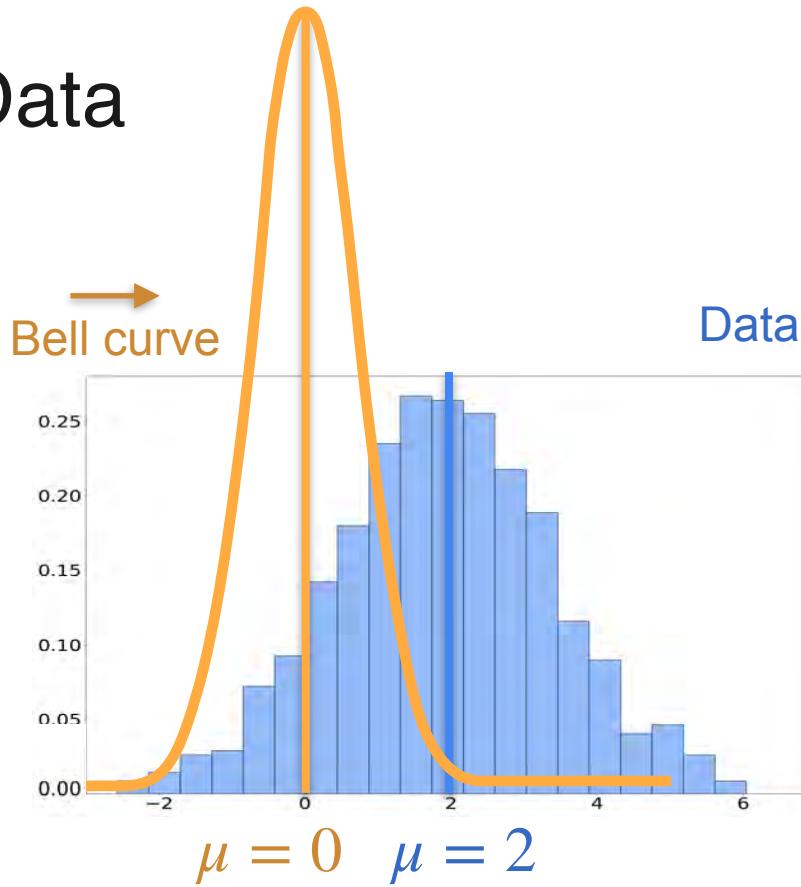
Bell Shaped Data

$$e^{-x^2}$$



Bell Shaped Data

$$e^{-(x-2)^2}$$

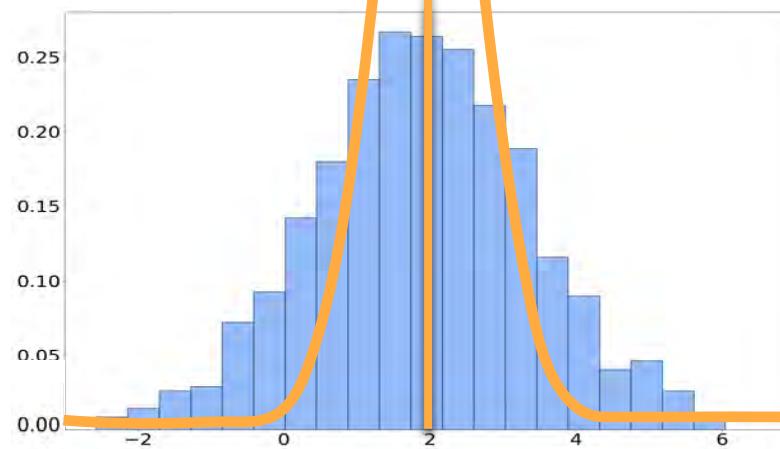


Bell Shaped Data

$$e^{-(x-2)^2}$$

Bell curve

Data



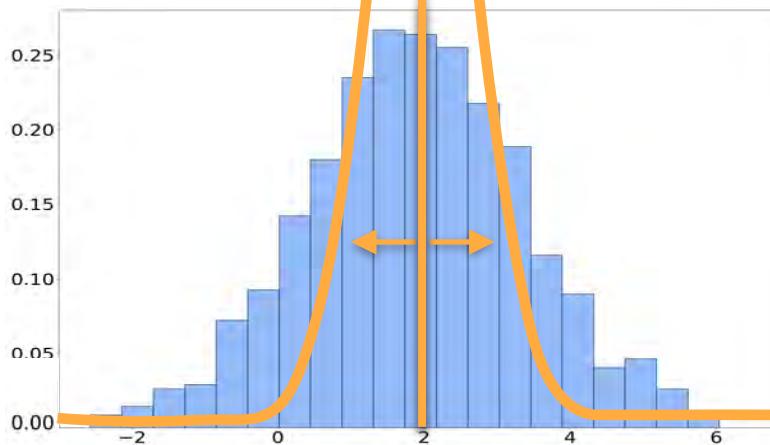
Bell Shaped Data

$$e^{-(x-2)^2}$$

Bell curve

Data

$$\sigma = 1$$

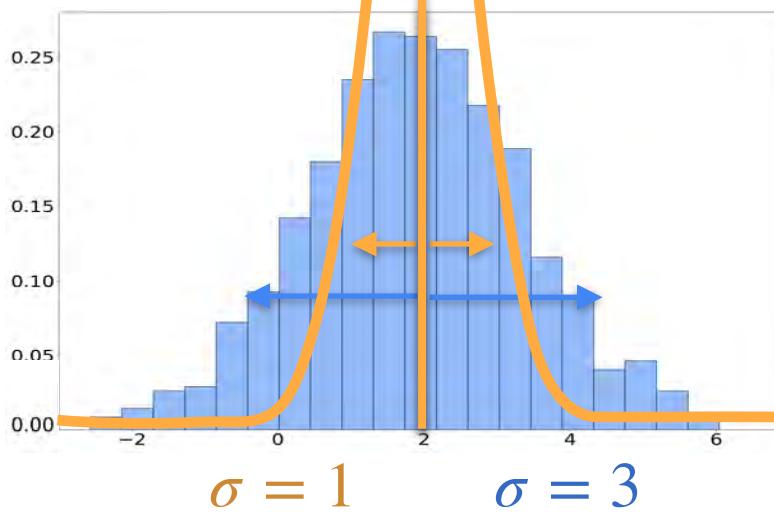


Bell Shaped Data

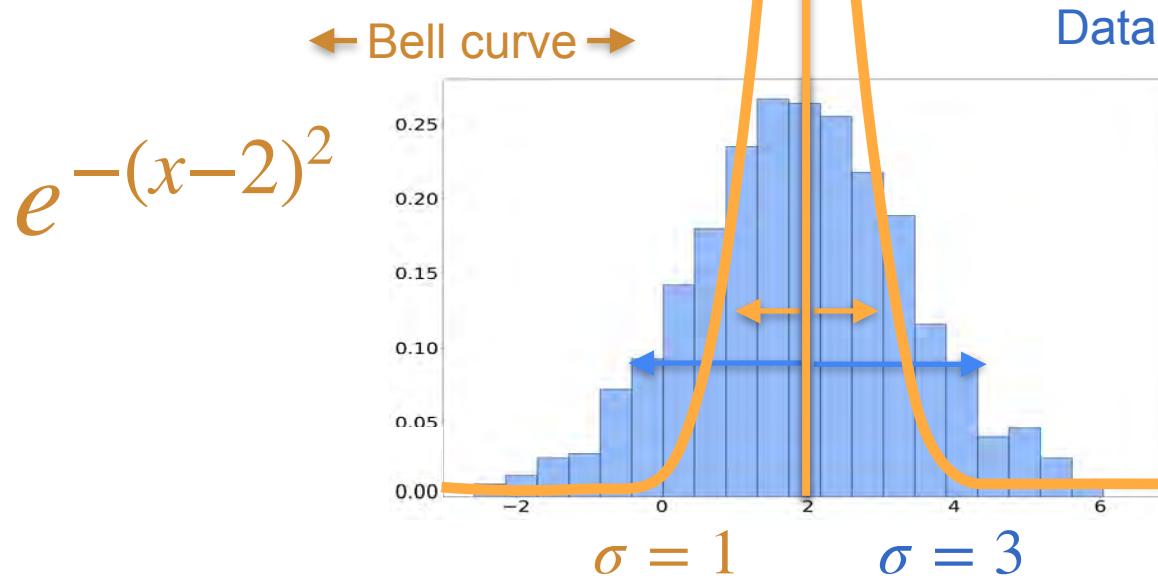
$$e^{-(x-2)^2}$$

Bell curve

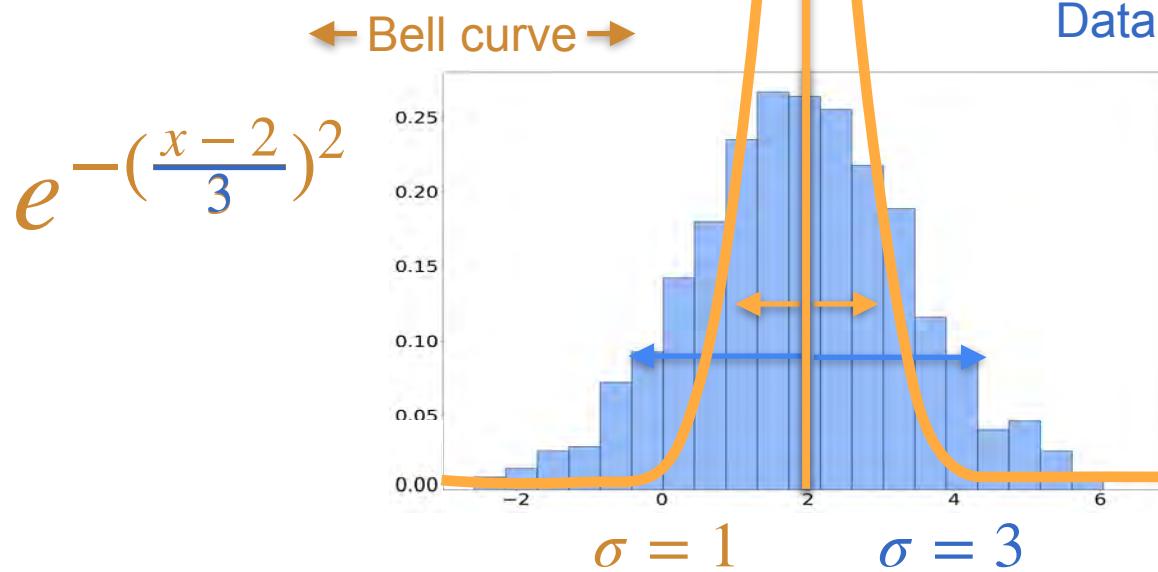
Data



Bell Shaped Data



Bell Shaped Data

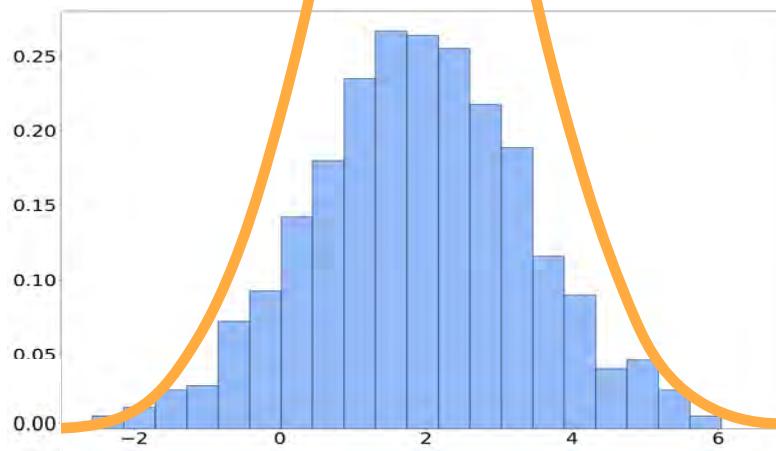


Bell Shaped Data

$$e^{-(\frac{x-2}{3})^2}$$

Bell curve

Data

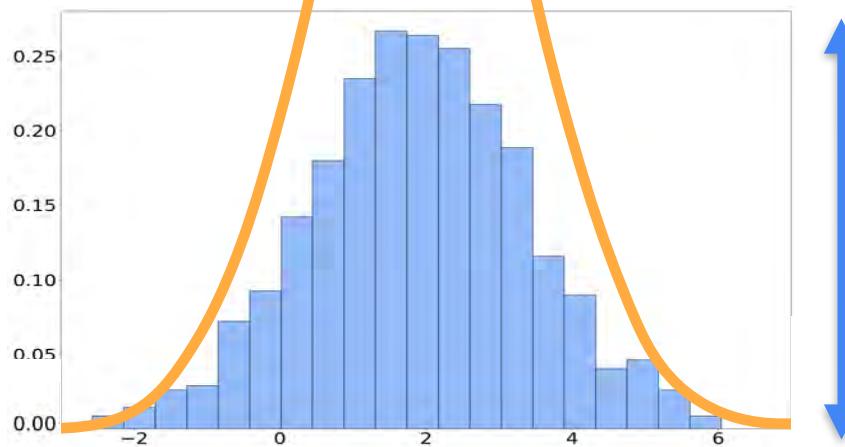


Bell Shaped Data

$$e^{-(\frac{x-2}{3})^2}$$

Bell curve

Data

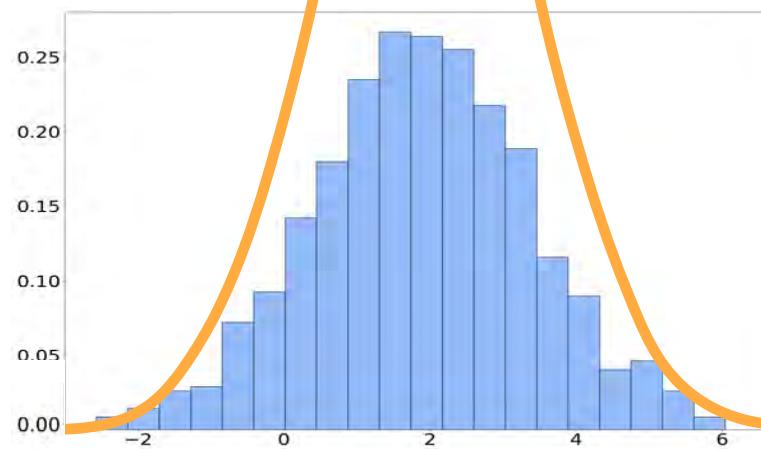


Bell Shaped Data

$$e^{-(\frac{x-2}{3})^2}$$

Bell curve

Data

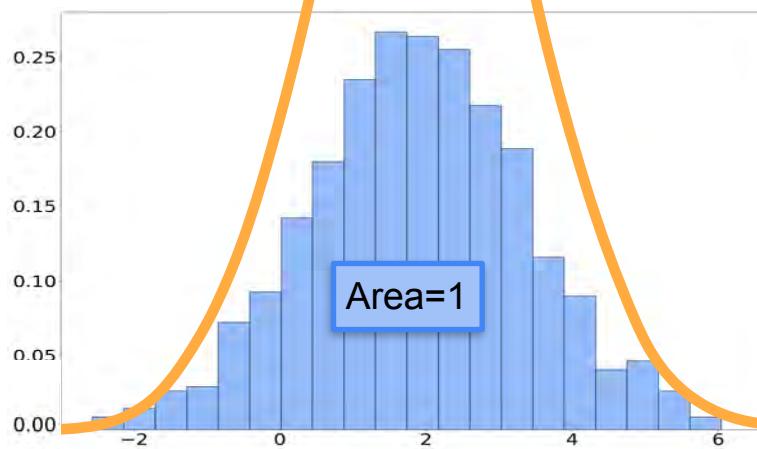


Bell Shaped Data

$$e^{-(\frac{x-2}{3})^2}$$

Bell curve

Data



Bell Shaped Data

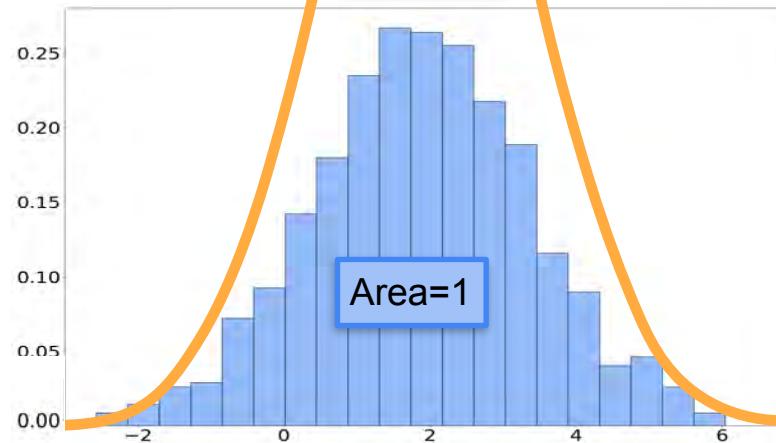
$$e^{-(\frac{x-2}{3})^2}$$

Bell curve

Area=??

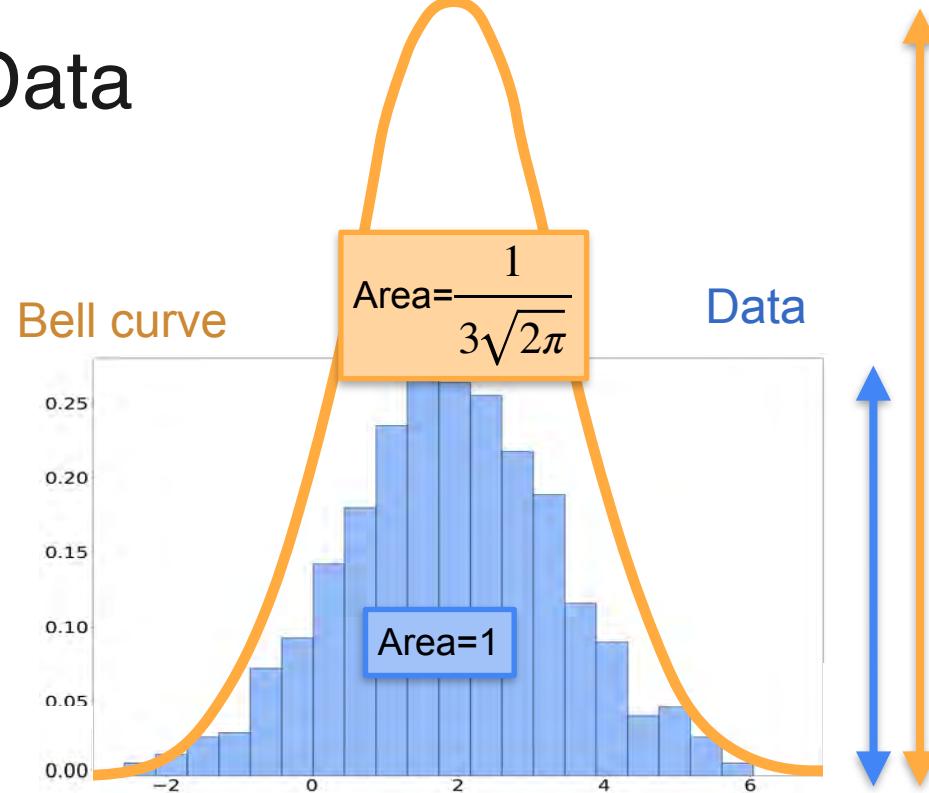
Data

Area=1



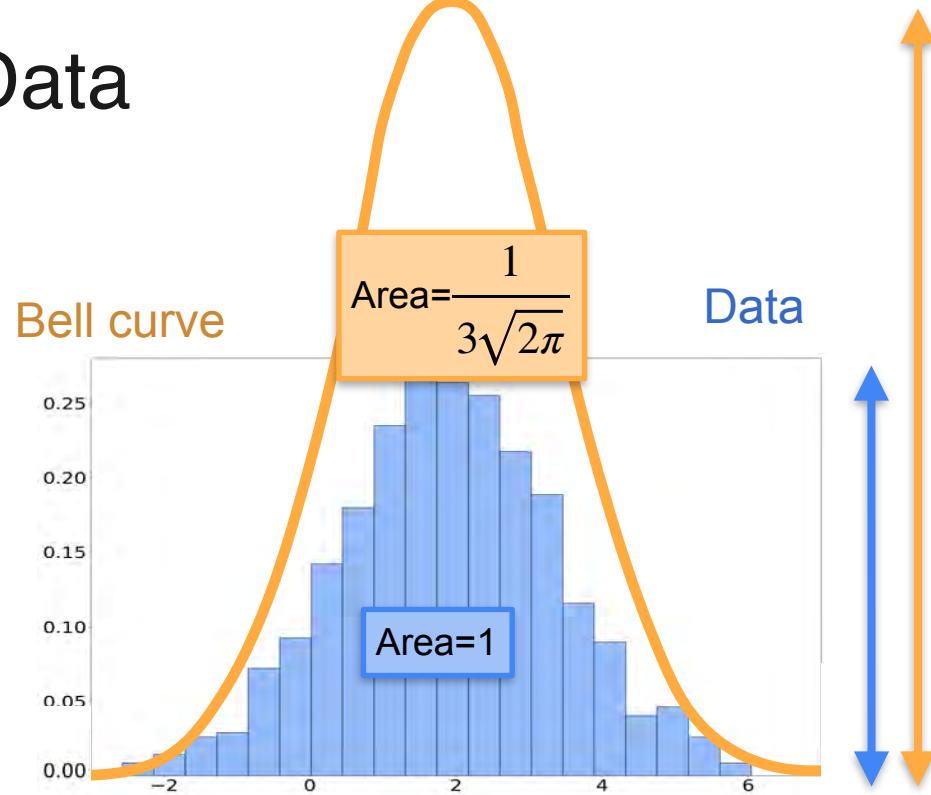
Bell Shaped Data

$$e^{-(\frac{x-2}{3})^2}$$



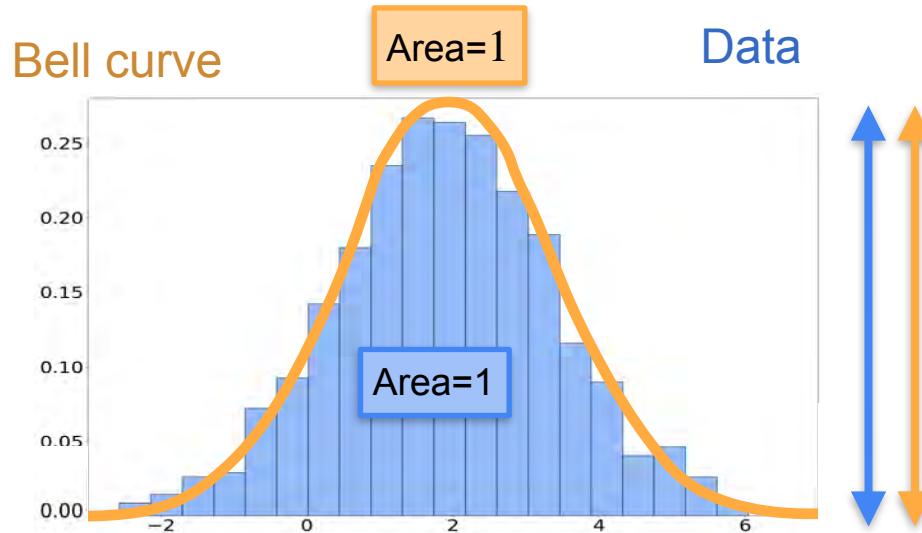
Bell Shaped Data

$$\frac{1}{3\sqrt{2\pi}} e^{-(\frac{x-2}{3})^2}$$

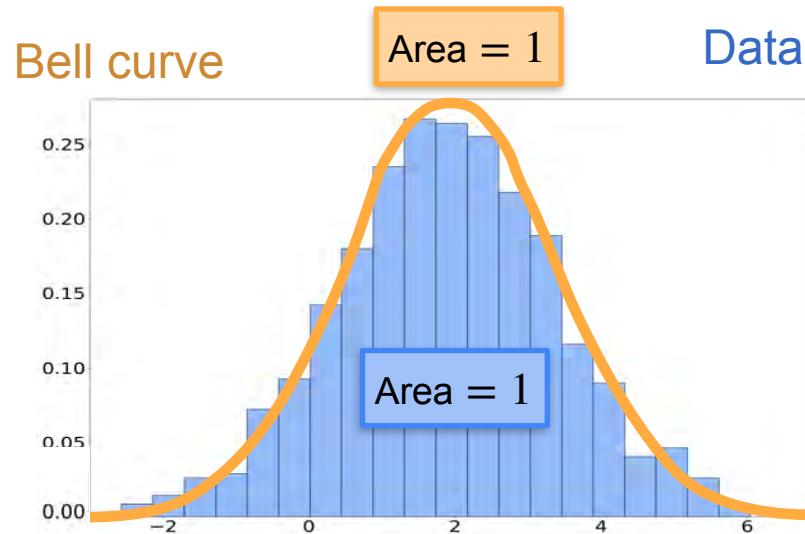


Bell Shaped Data

$$\frac{1}{3\sqrt{2\pi}} e^{-(\frac{x-2}{3})^2}$$



Bell Shaped Data



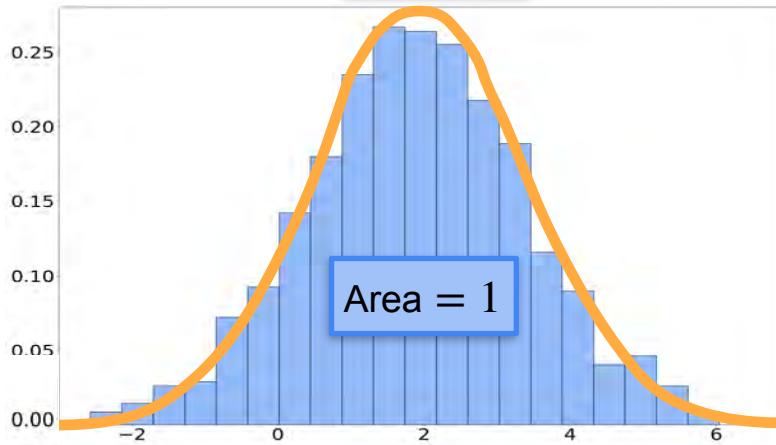
Bell Shaped Data

Mean = μ

Bell curve

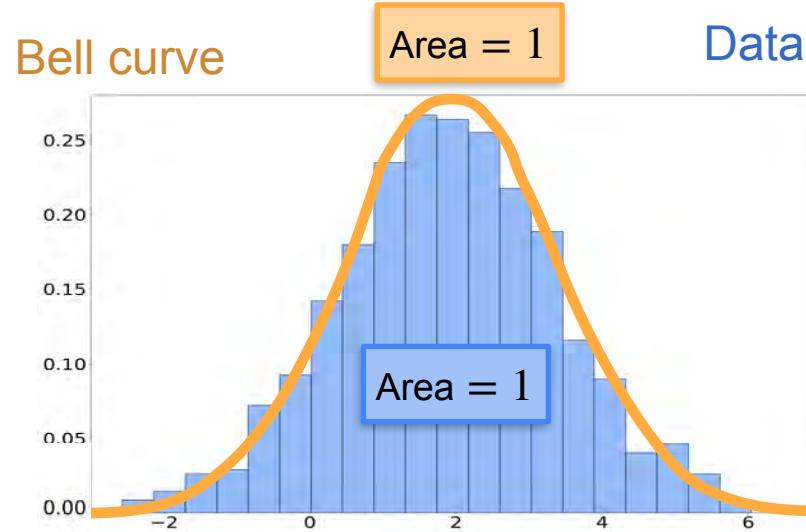
Area = 1

Data



Bell Shaped Data

Bell curve
Mean = μ
Standard deviation = σ

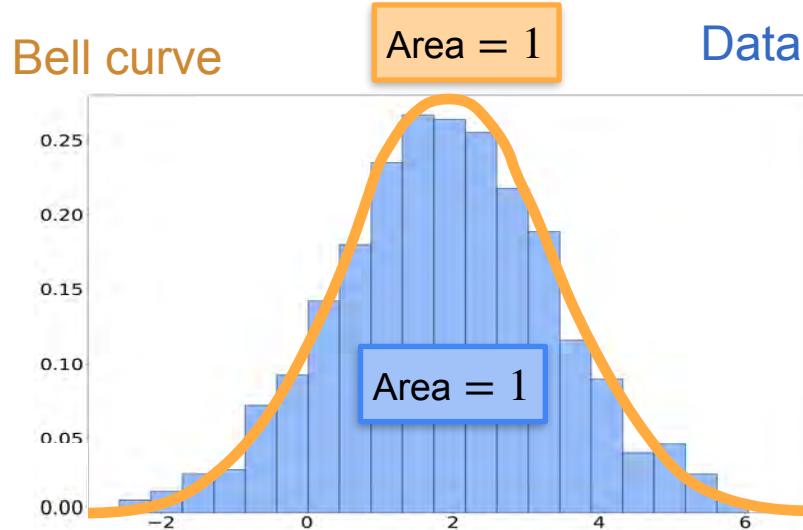


Bell Shaped Data

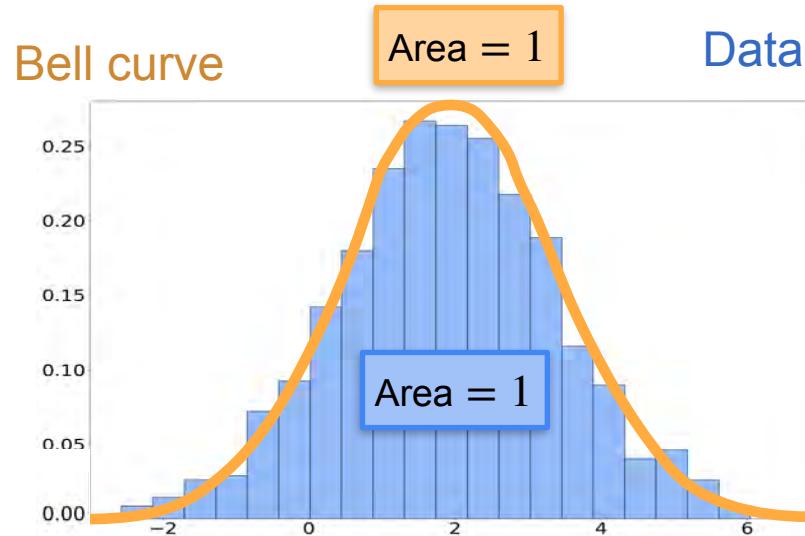
Mean = μ

Standard deviation = σ

$$\frac{1}{\sigma\sqrt{2\pi}} e^{-(\frac{x-\mu}{\sigma})^2}$$



Bell Shaped Data



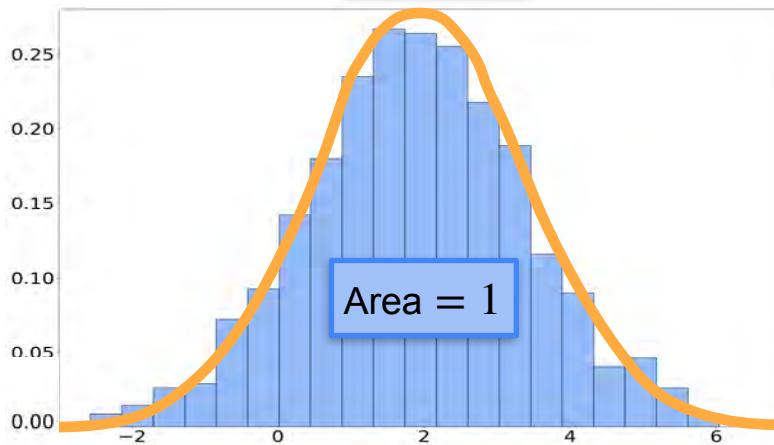
Bell Shaped Data

Mean = μ

Bell curve

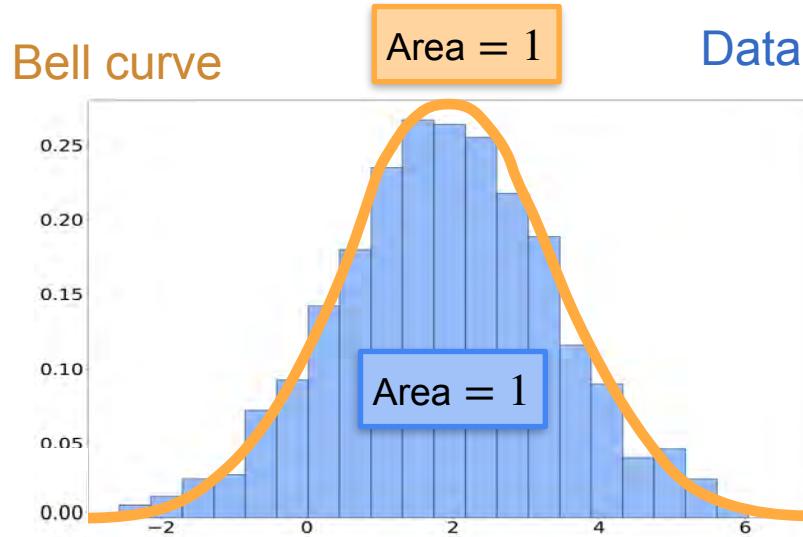
Area = 1

Data



Bell Shaped Data

Bell curve
Mean = μ
Standard deviation = σ

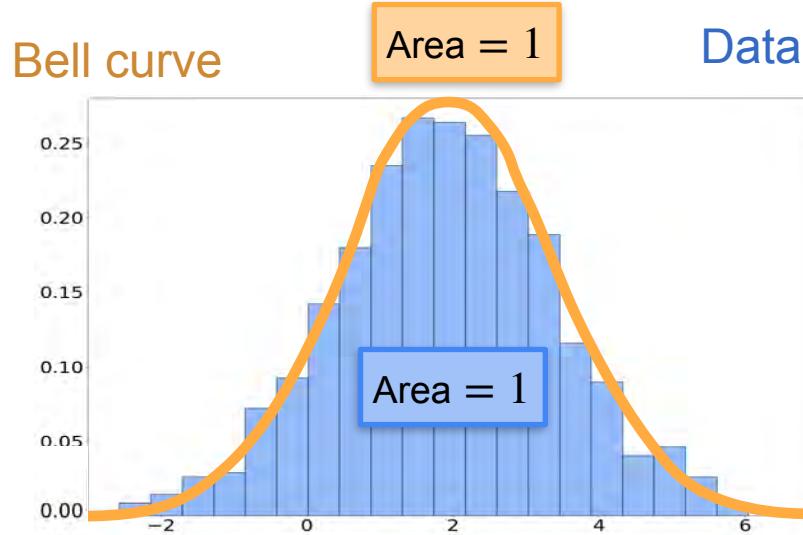


Bell Shaped Data

Mean = μ

Standard deviation = σ

$$\frac{1}{\sigma\sqrt{2\pi}} e^{-(\frac{x-\mu}{\sigma})^2}$$

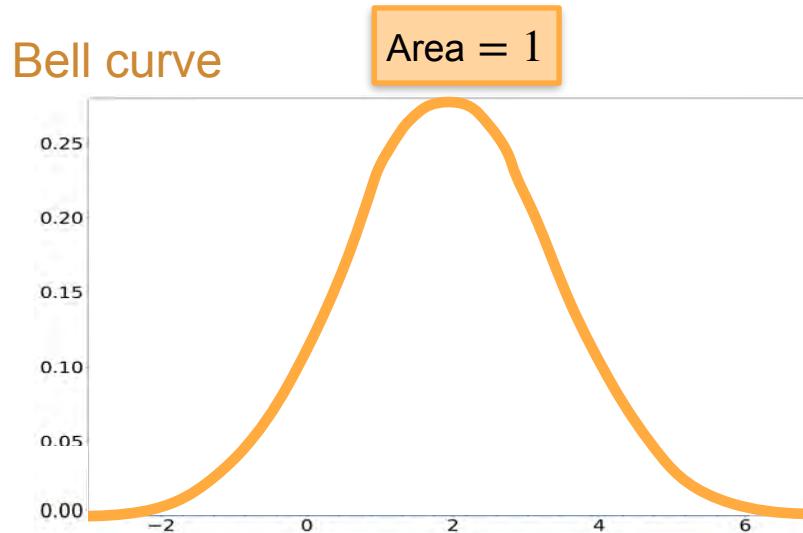


Normal Distribution

Mean = μ

Standard deviation = σ

$$\frac{1}{\sigma\sqrt{2\pi}} e^{-(\frac{x-\mu}{\sigma})^2}$$

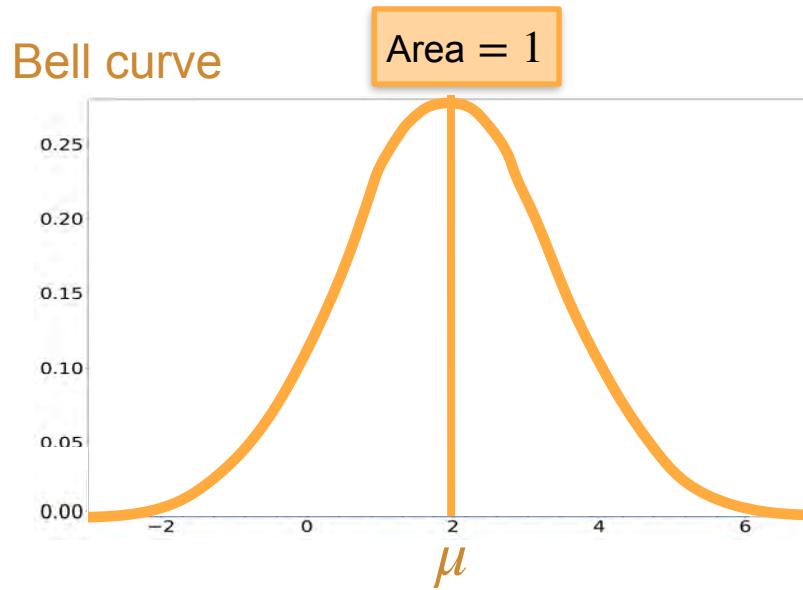


Normal Distribution

Mean = μ

Standard deviation = σ

$$\frac{1}{\sigma\sqrt{2\pi}} e^{-(\frac{x-\mu}{\sigma})^2}$$

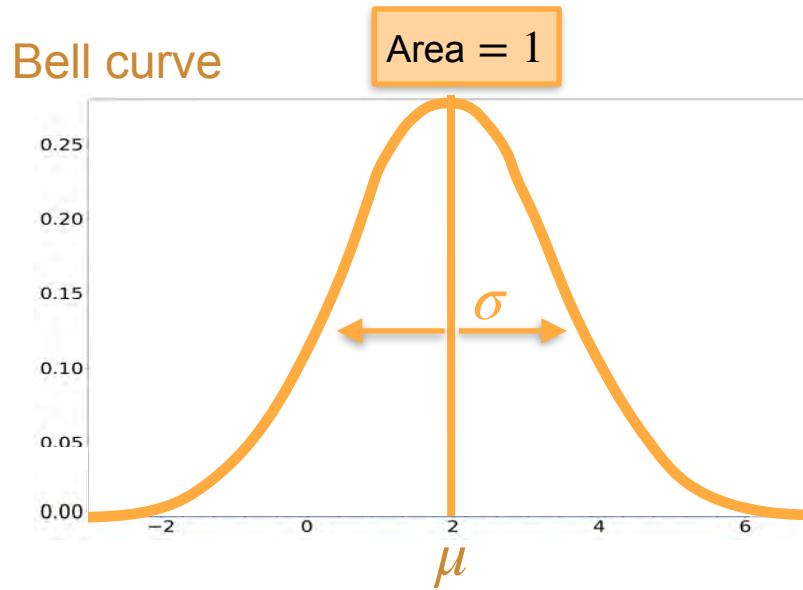


Normal Distribution

Mean = μ

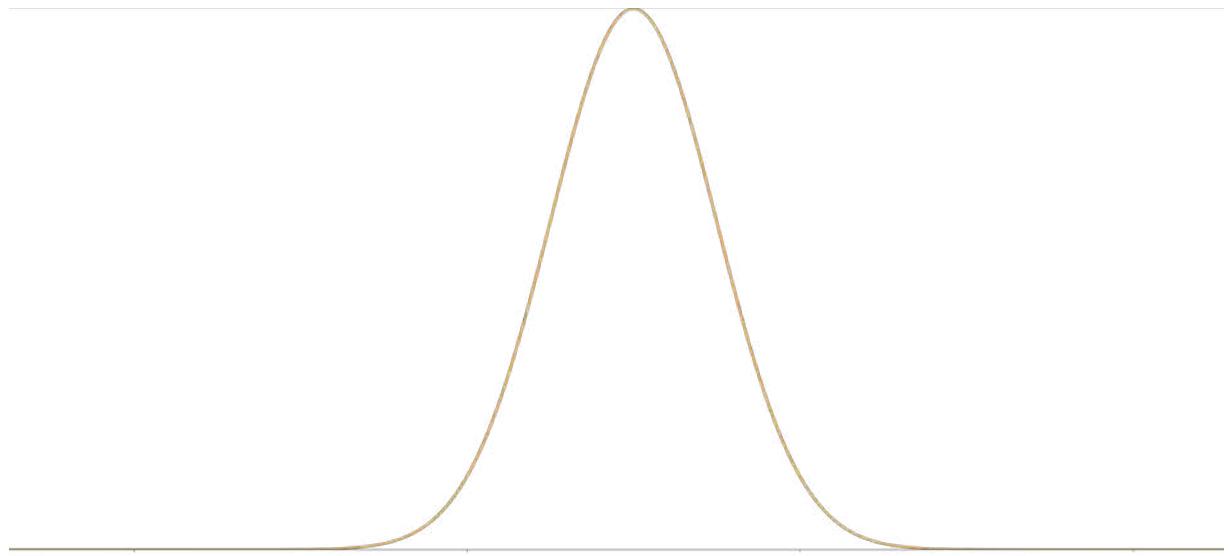
Standard deviation = σ

$$\frac{1}{\sigma\sqrt{2\pi}} e^{-(\frac{x-\mu}{\sigma})^2}$$



Normal Distribution

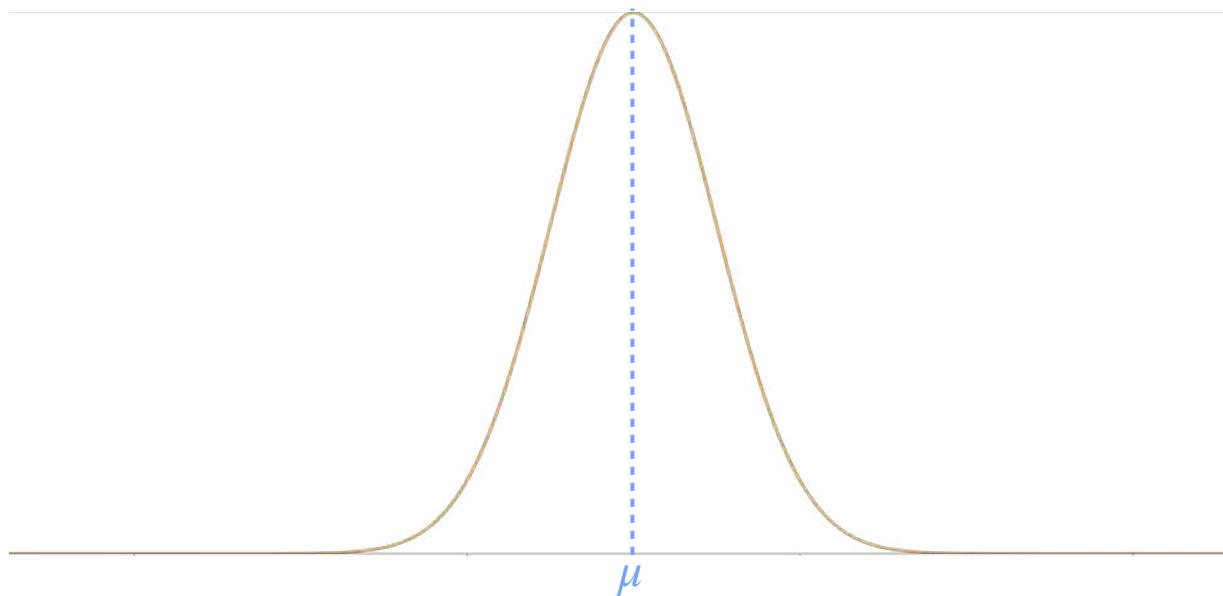
Normal Distribution



Normal Distribution

Parameters:

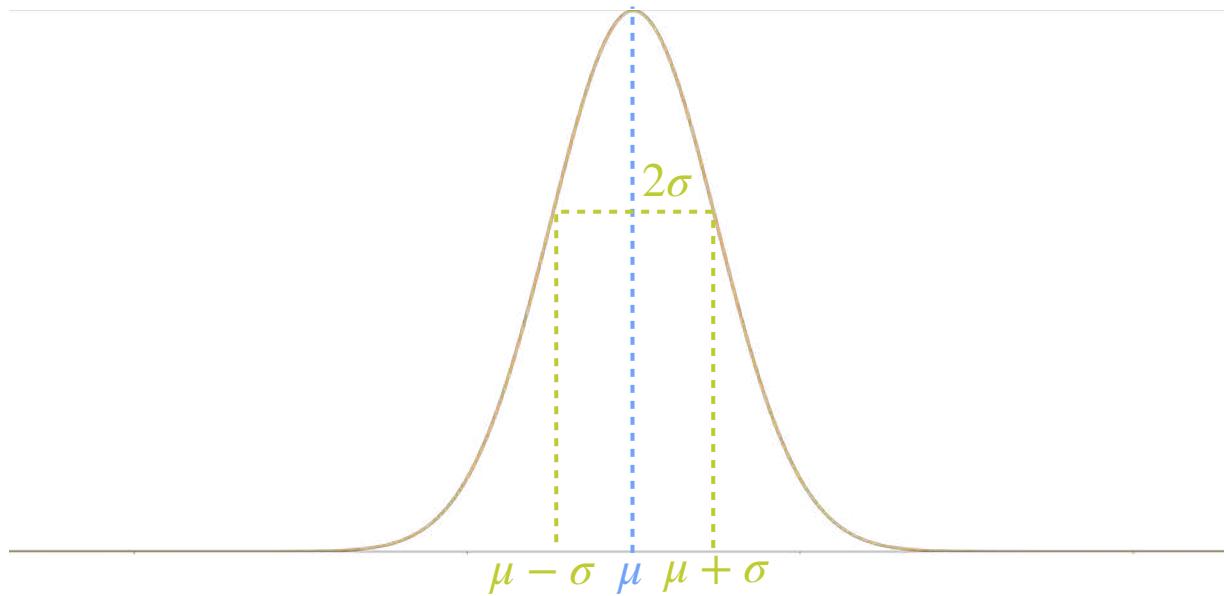
- μ : center of the bell



Normal Distribution

Parameters:

- μ : center of the bell
- σ : spread of the bell



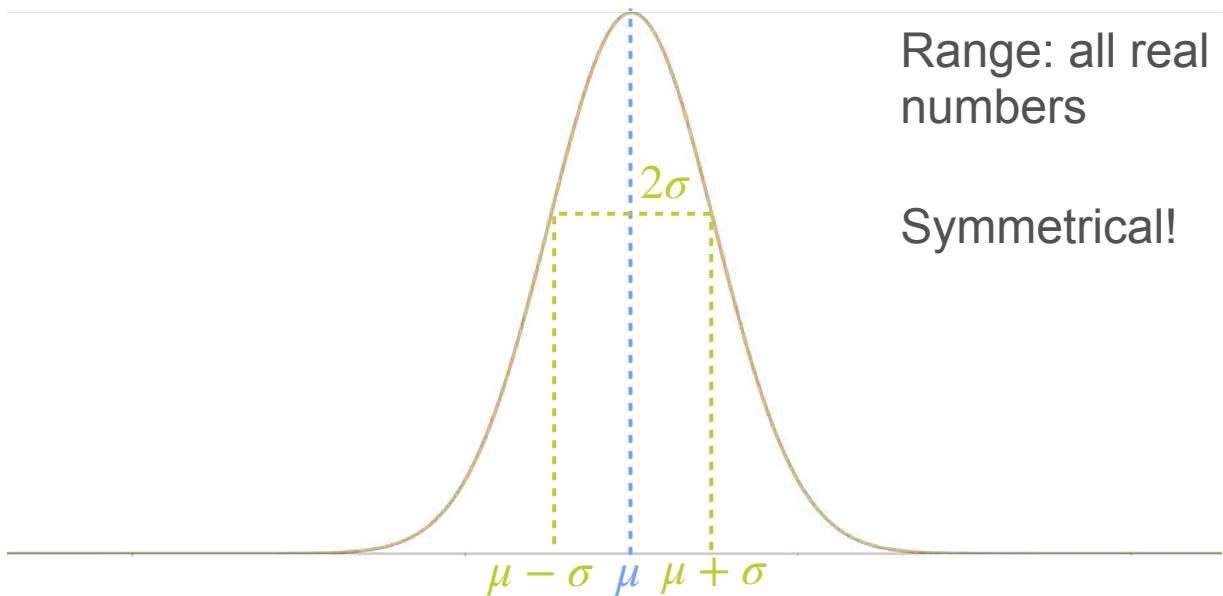
Normal Distribution

Parameters:

- μ : center of the bell
- σ : spread of the bell

Range: all real numbers

Symmetrical!



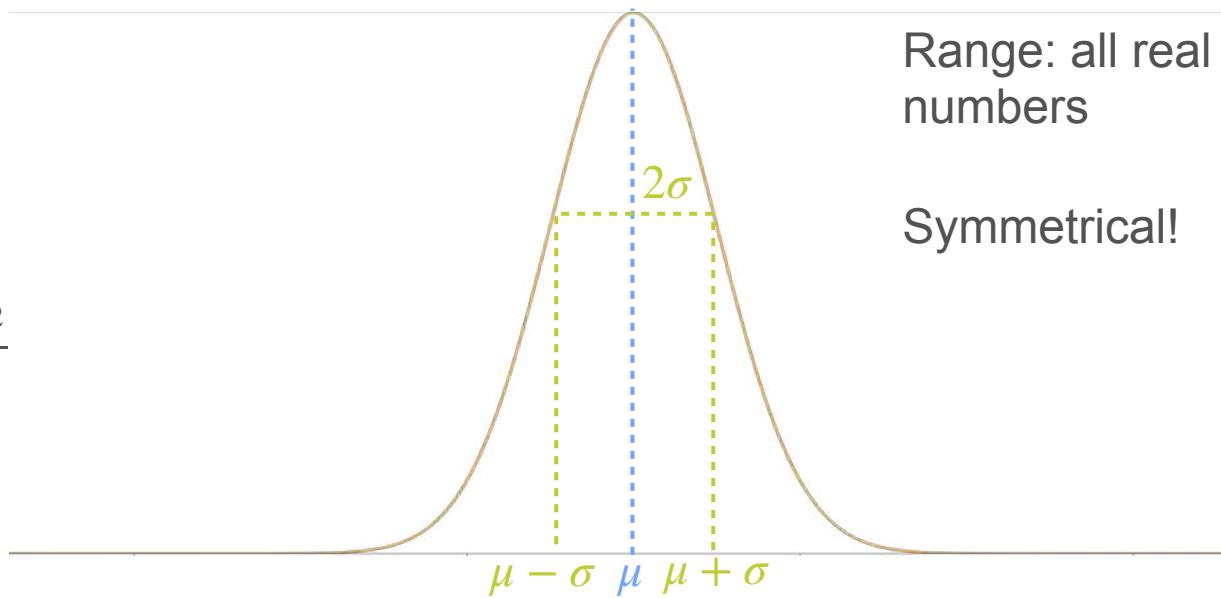
Normal Distribution

Parameters:

- μ : center of the bell
- σ : spread of the bell

$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}}$$

Scaling constant

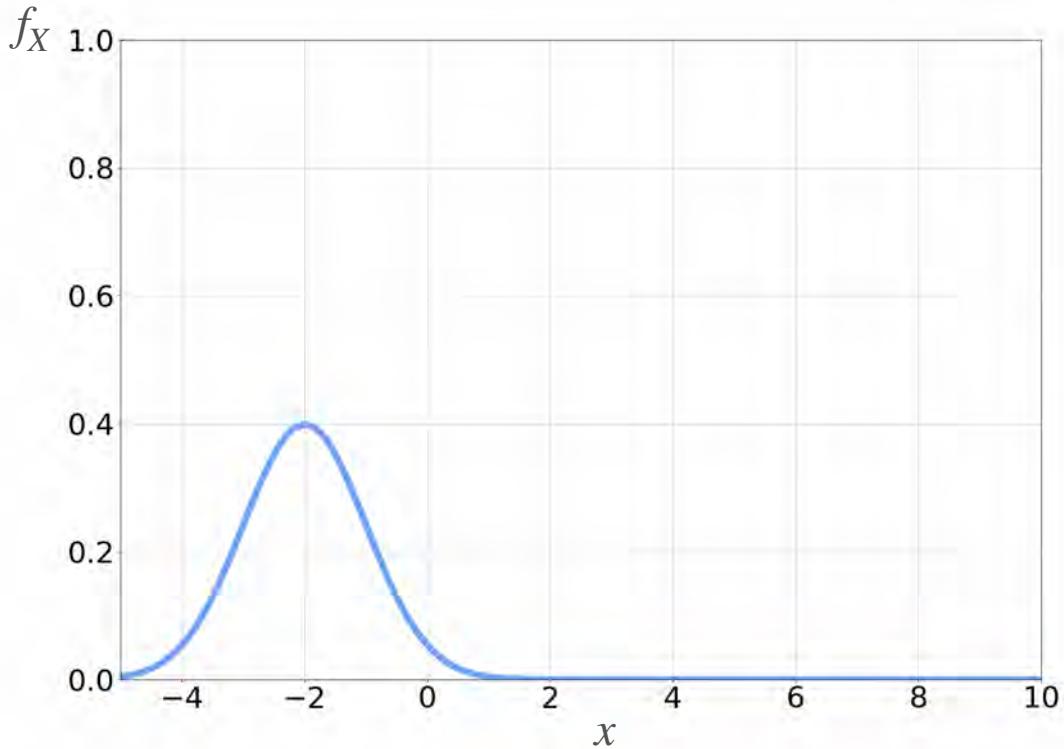


Normal Distribution

Parameters:

- μ : center of the bell
- σ : spread of the bell

$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}}$$

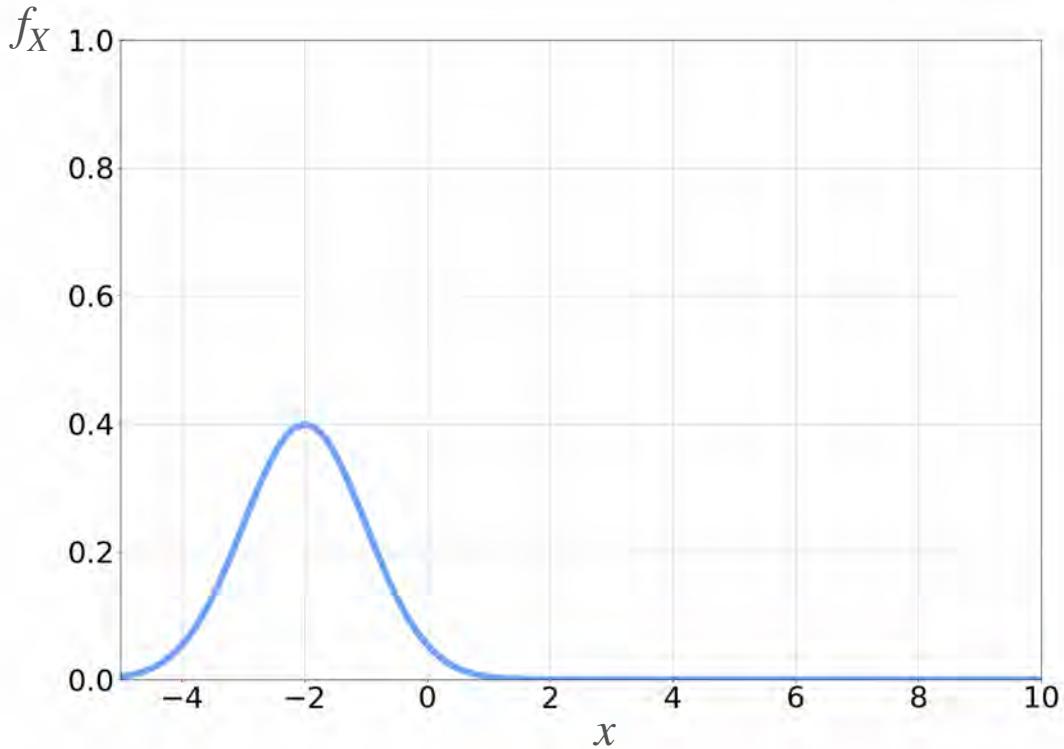


Normal Distribution

Parameters:

- μ : center of the bell
- σ : spread of the bell

$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}}$$



Normal Distribution - Notation

Parameters:

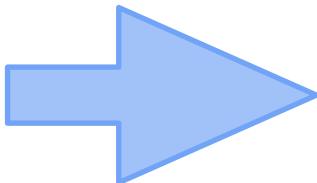
- μ : center of the bell
- σ : spread of the bell

$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}}$$

Normal Distribution - Notation

Parameters:

- μ : center of the bell
- σ : spread of the bell



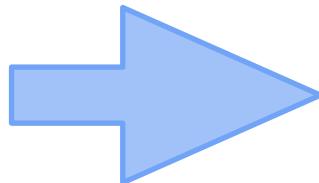
$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}}$$

$$X \sim \mathcal{N}(\mu, \sigma^2)$$

Normal Distribution - Notation

Parameters:

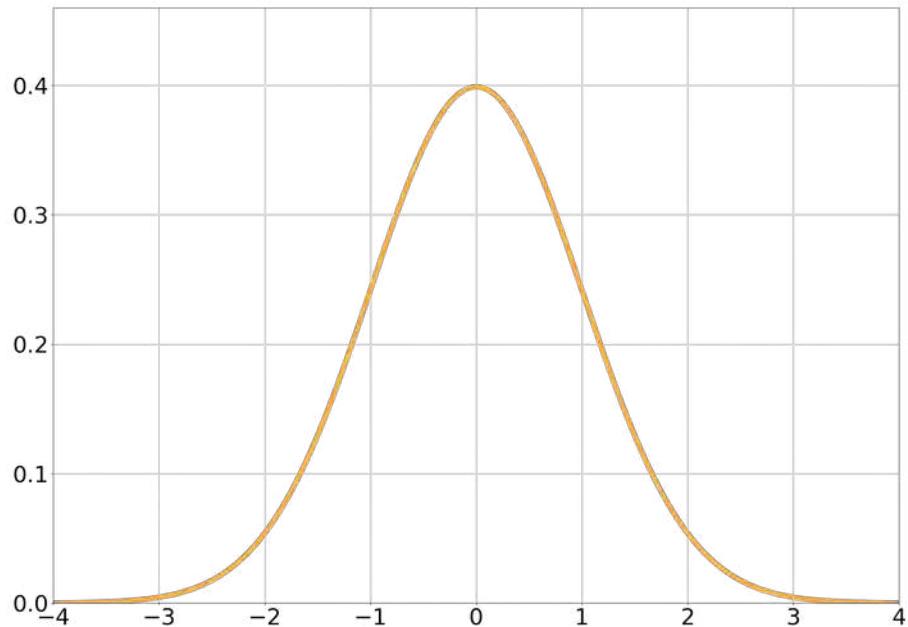
- μ : center of the bell
- σ : spread of the bell



$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}}$$

$$X \sim \mathcal{N}(\mu, \sigma^2)$$

Standard Normal Distribution

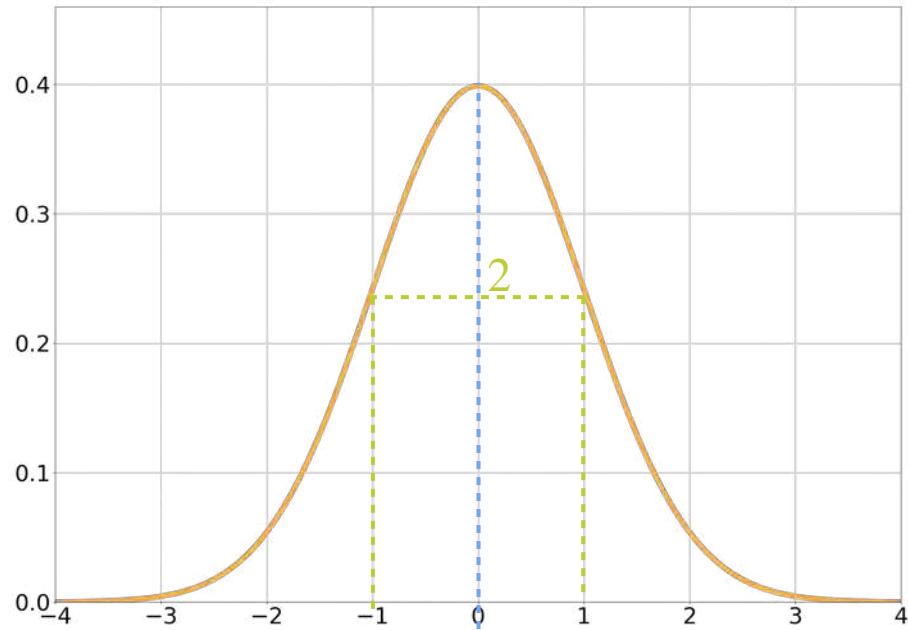


Standard Normal Distribution

Parameters:

- μ : 0
- σ : 1

$$X \sim \mathcal{N}(0, 1^2)$$



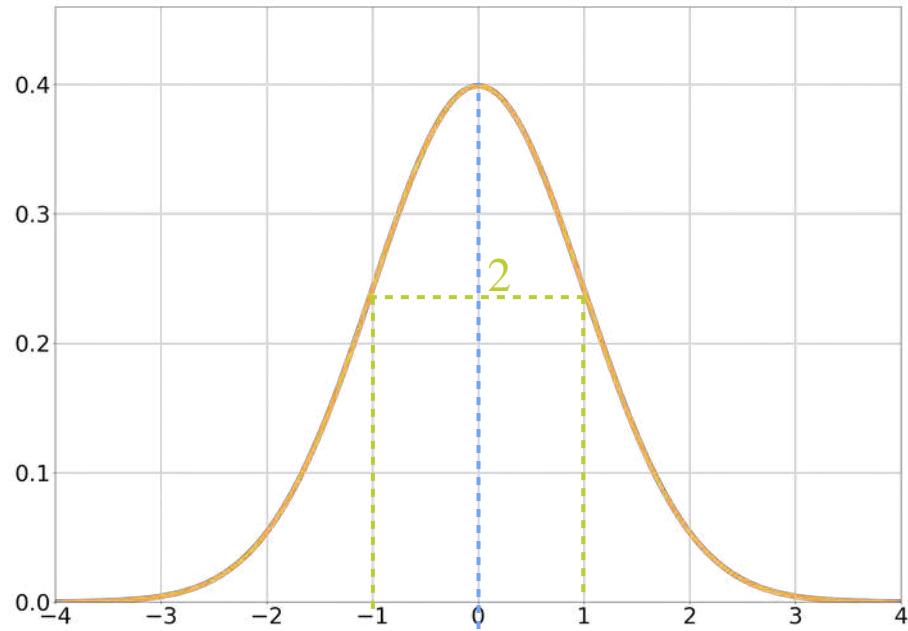
Standard Normal Distribution

Parameters:

- μ : 0
- σ : 1

$$X \sim \mathcal{N}(0, 1^2)$$

$$\begin{aligned}f_X(x) &= \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\frac{(x-0)^2}{1^2}} \\&= \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}\end{aligned}$$



Standardization

Standardization

There's a really easy way to convert any normal distribution to the standard one!

Standardization

There's a really easy way to convert any normal distribution to the standard one!

X distributes normally with

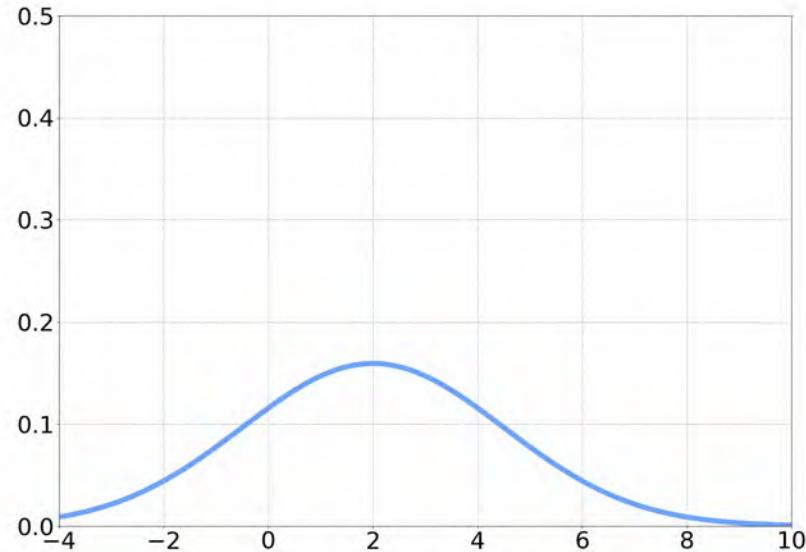
$$\mu = 2, \sigma = 2.5$$

Standardization

There's a really easy way to convert any normal distribution to the standard one!

X distributes normally with
 $\mu = 2, \sigma = 2.5$

$$X - 2$$

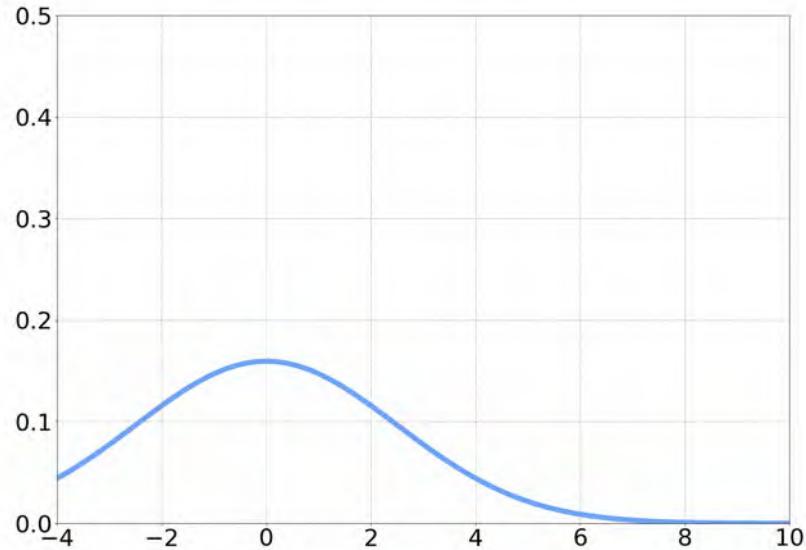


Standardization

There's a really easy way to convert any normal distribution to the standard one!

X distributes normally with
 $\mu = 2, \sigma = 2.5$

$$\frac{X - 2}{2.5}$$



Standardization

There's a really easy way to convert any normal distribution to the standard one!

X distributes normally with
 $\mu = 2, \sigma = 2.5$

$$Z = \frac{X - 2}{2.5}$$



Standardization

There's a really easy way to convert any normal distribution to the standard one!

X distributes normally with
 $\mu = 2, \sigma = 2.5$

$$Z = \frac{X - \mu}{\sigma}$$



Standardization

There's a really easy way to convert any normal distribution to the standard one!

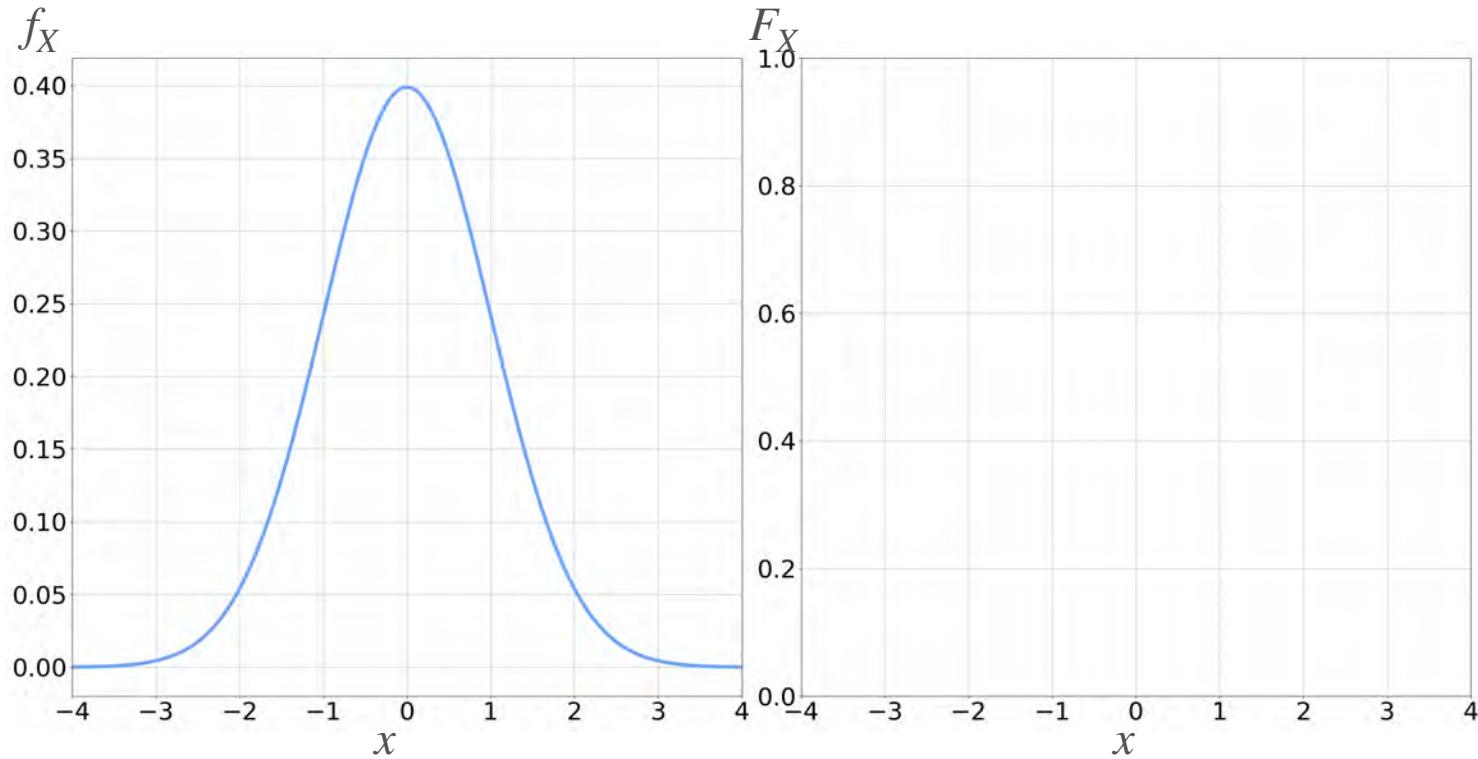
X distributes normally with
 $\mu = 2, \sigma = 2.5$

$$Z = \frac{X - \mu}{\sigma}$$

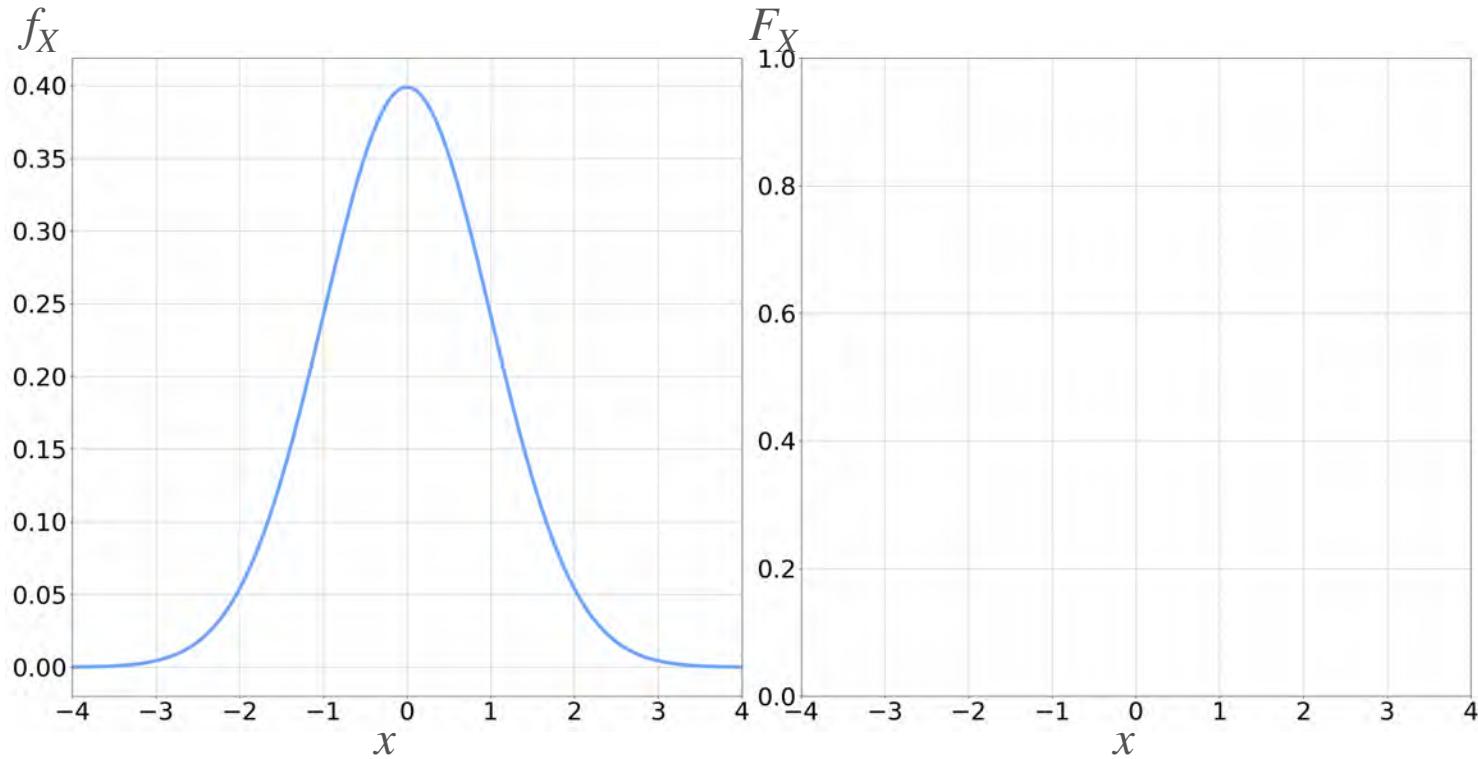
Standardization is crucial to compare variables of different magnitudes!



Normal Distribution: CDF



Normal Distribution: CDF



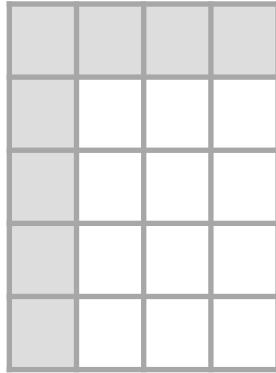
Normal Distribution: Computing Probabilities

Normal Distribution: Computing Probabilities

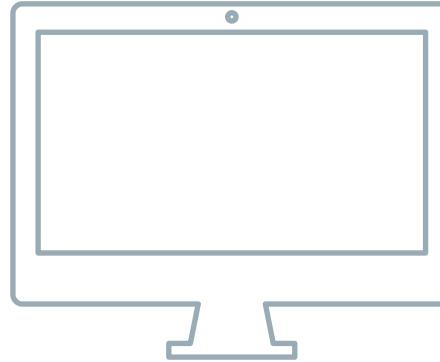
This math can't be done by hand

Normal Distribution: Computing Probabilities

This math can't be done by hand



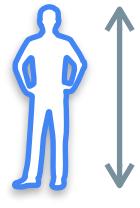
In the old days, people used tables of data



Now, you can use the help of some software to do the approximate area under the curve for you!

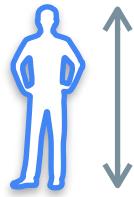
Normal Distribution: Applications

Normal Distribution: Applications



Height

Normal Distribution: Applications

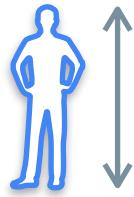


Height



Weight

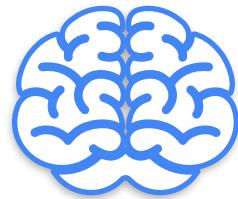
Normal Distribution: Applications



Height

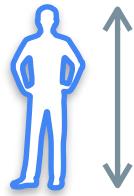


Weight



IQ

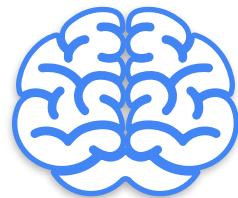
Normal Distribution: Applications



Height



Weight

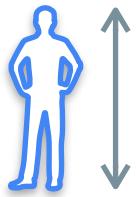


IQ



Noise in a
communication channel

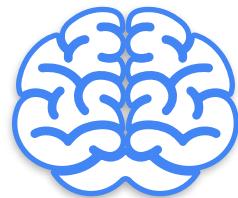
Normal Distribution: Applications



Height



Weight



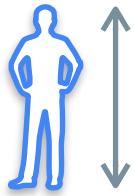
IQ



Noise in a
communication channel

In general, characteristics that are the sum of many independent processes

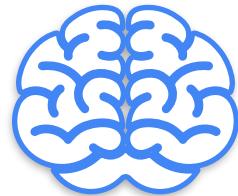
Normal Distribution: Applications



Height



Weight



IQ



Noise in a
communication channel

In general, characteristics that are the sum of many independent processes

Many models in ML are designed under the assumption that the variables follow a normal distribution



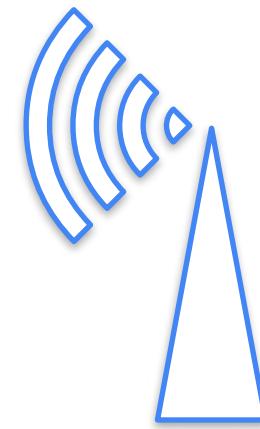
DeepLearning.AI

Probability Distributions

Chi-squared distribution

Chi-Square Distribution: Motivation

Chi-Square Distribution: Motivation



Message sent: 10010

Chi-Square Distribution: Motivation



Communication channel

Noise

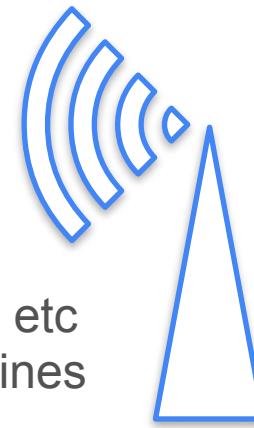
Interference from other devices

Obstructions like walls, trees, etc.

Atmospheric conditions: rain, humidity, etc

Electrical interference, i.e. from power lines

Others



Message sent: 10010

Chi-Square Distribution: Motivation



Communication channel

Noise

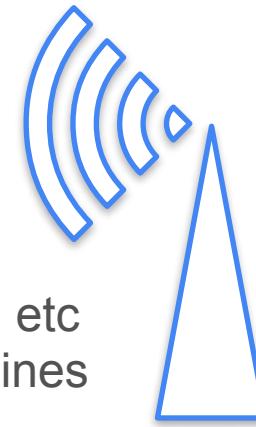
Interference from other devices

Obstructions like walls, trees, etc.

Atmospheric conditions: rain, humidity, etc

Electrical interference, i.e. from power lines

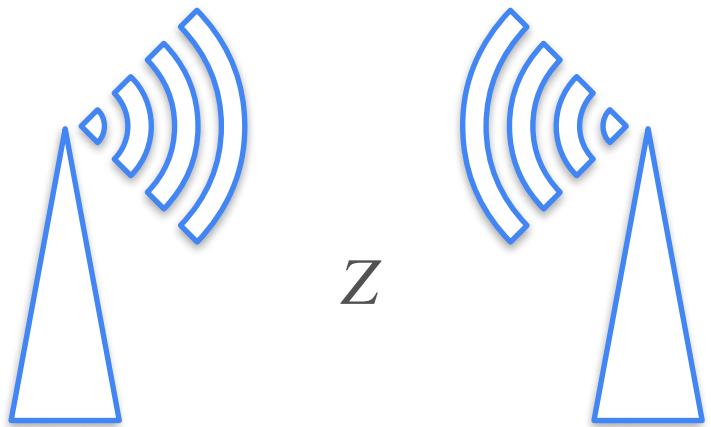
Others



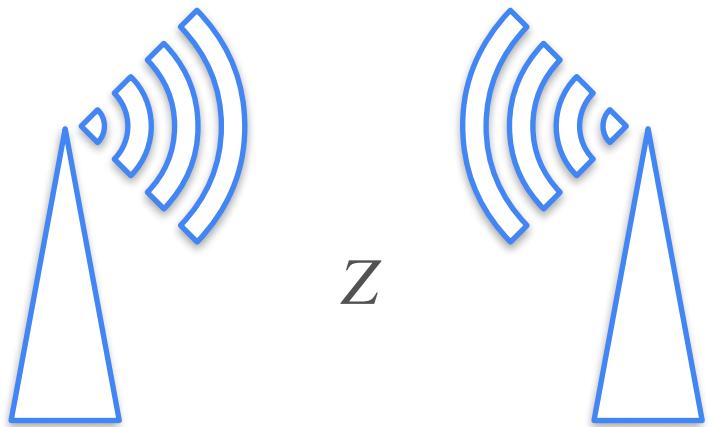
Message sent: 10010

Message received: 10010 +Z

Chi-Square Distribution: Motivation

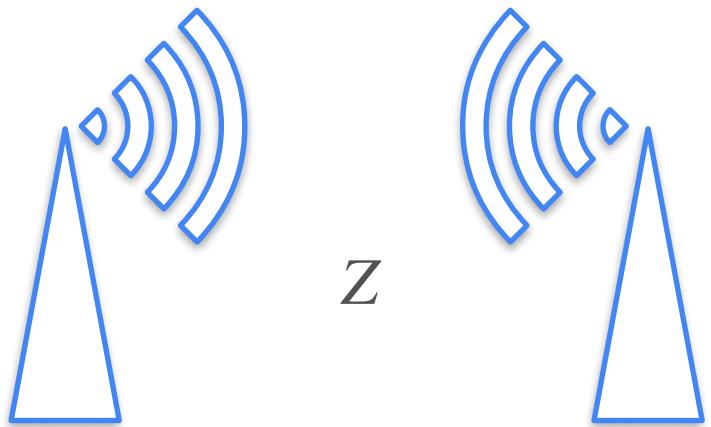


Chi-Square Distribution: Motivation



The communication channel
has noise with a standard
normal distribution

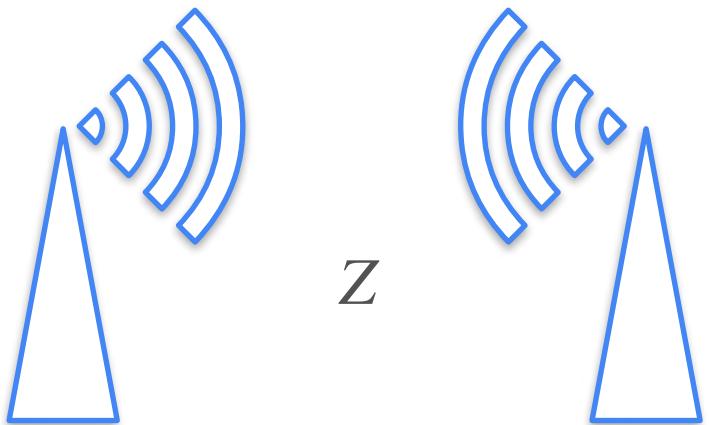
Chi-Square Distribution: Motivation



What is the **power** of the noise in the channel?

The communication channel
has noise with a standard
normal distribution

Chi-Square Distribution: Motivation

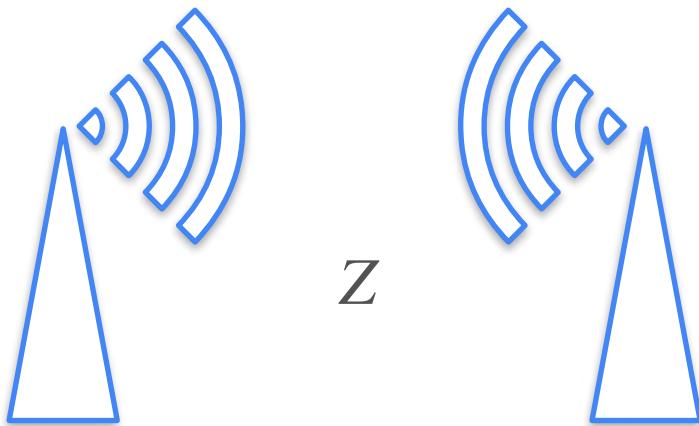


What is the **power** of the noise in the channel?

$$W = Z^2$$

The communication channel
has noise with a standard
normal distribution

Chi-Square Distribution: Motivation



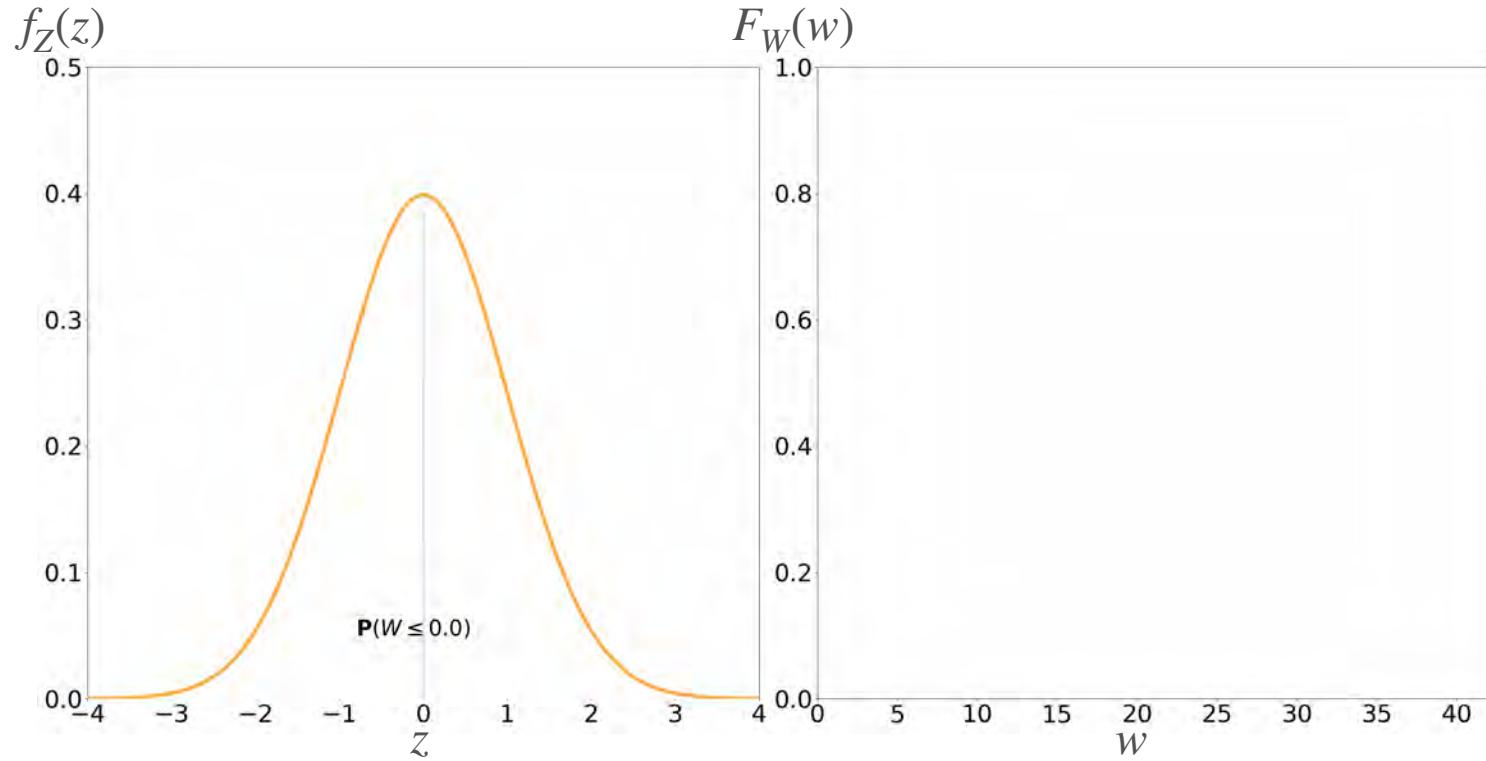
The communication channel has noise with a standard normal distribution

What is the **power** of the noise in the channel?

$$W = Z^2$$

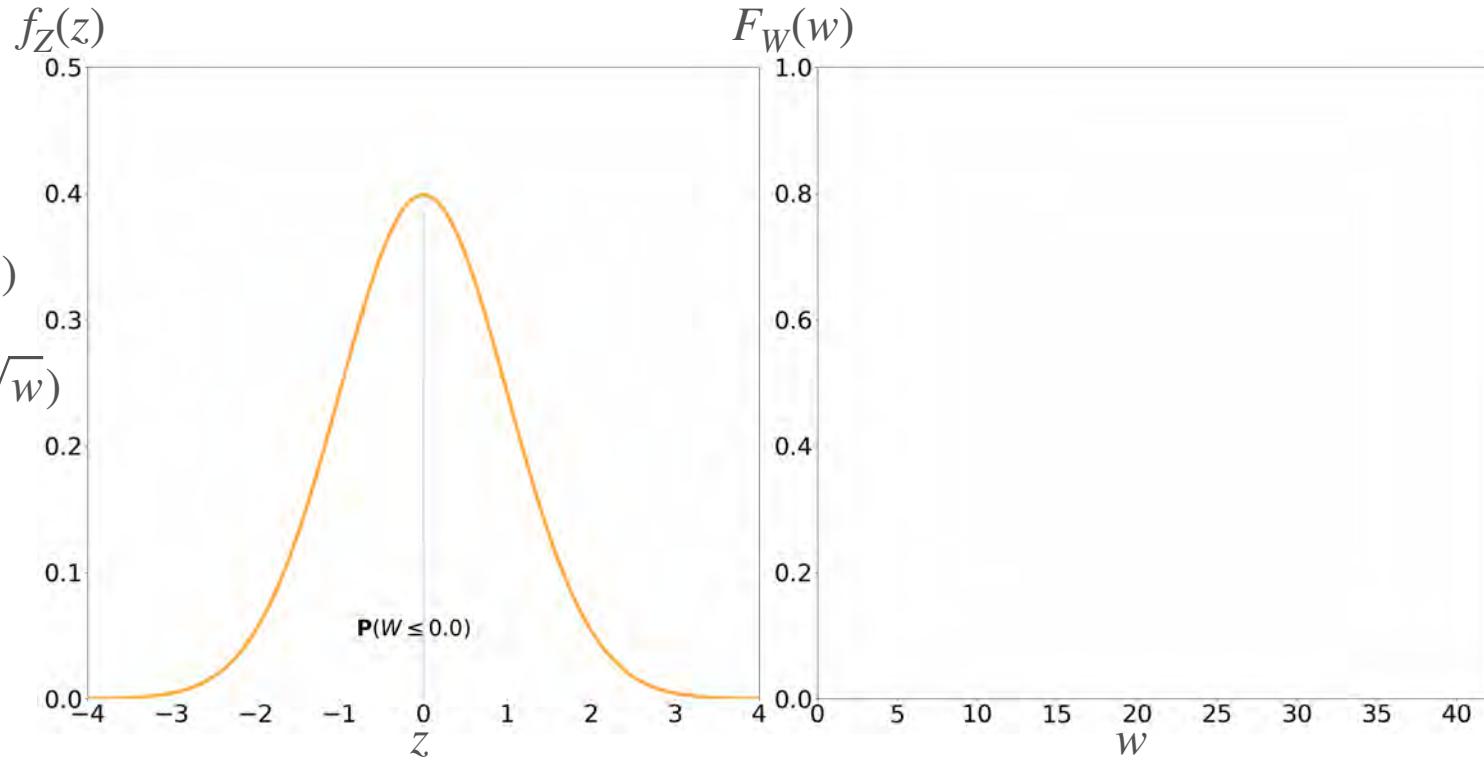
What is the distribution of W ?

Chi Square Distribution



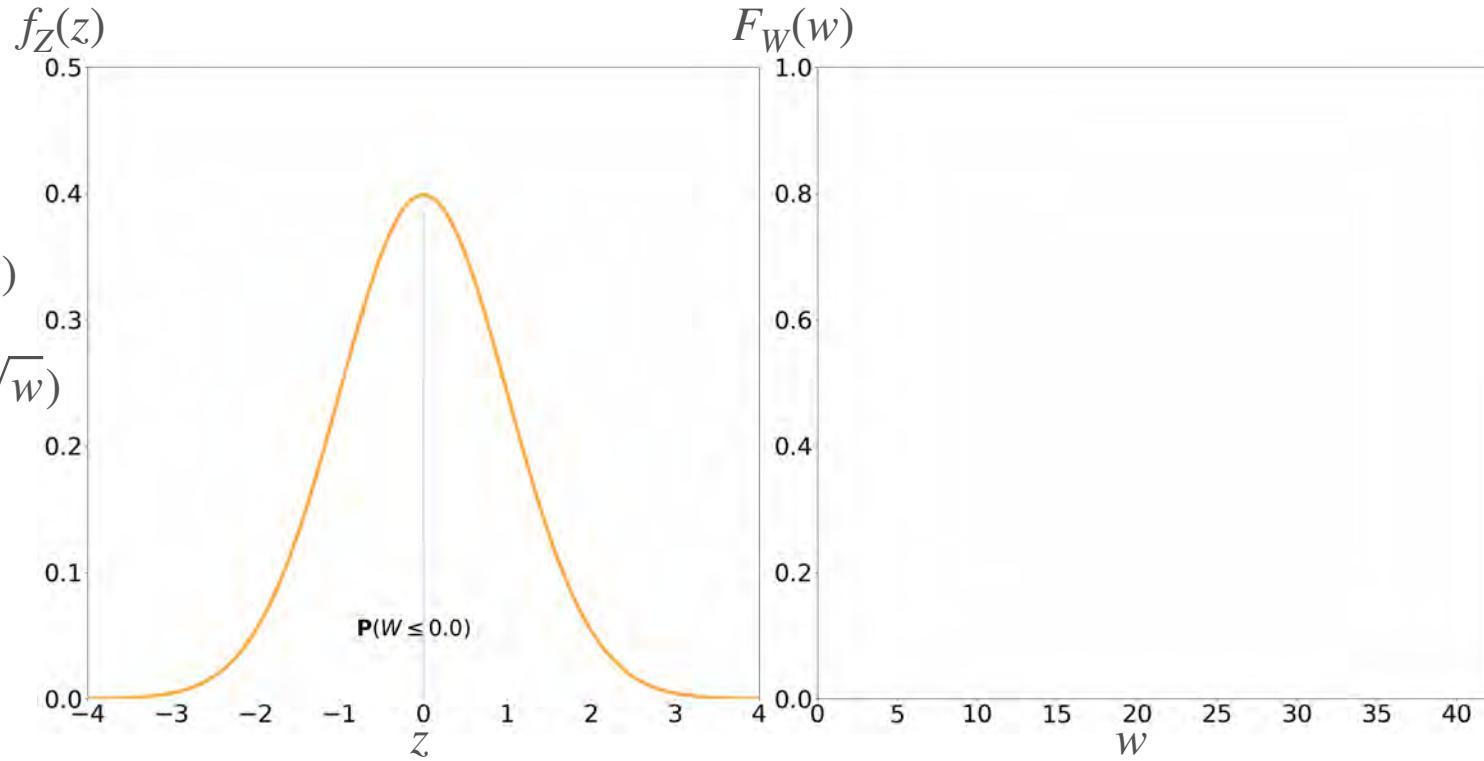
Chi Square Distribution

$$\begin{aligned} F_W(w) &= \mathbf{P}(W \leq w) \\ &= \mathbf{P}(Z^2 \leq w) \\ &= \mathbf{P}(|Z| \leq \sqrt{w}) \\ &= \mathbf{P}(-\sqrt{w} \leq Z \leq \sqrt{w}) \end{aligned}$$



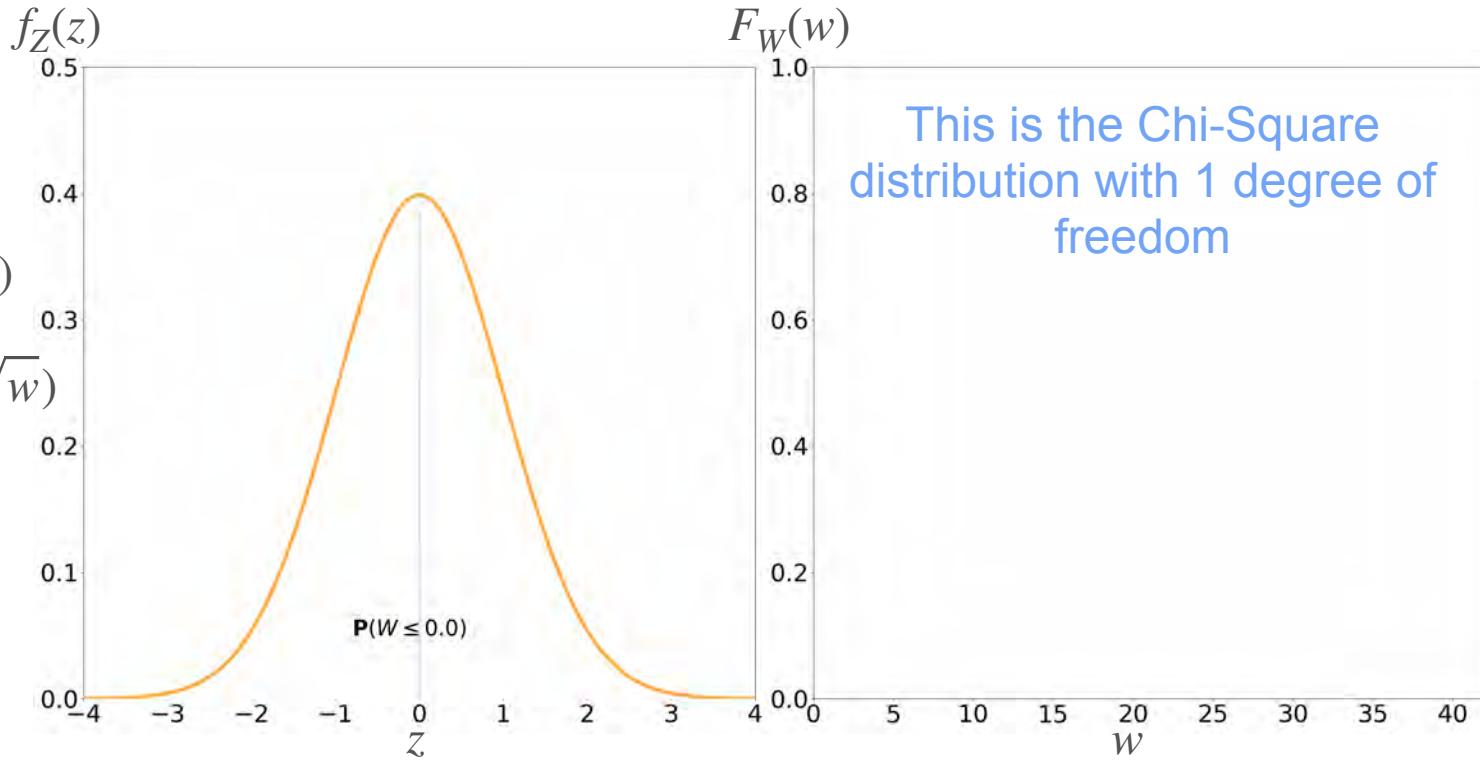
Chi Square Distribution

$$\begin{aligned}F_W(w) &= \mathbf{P}(W \leq w) \\&= \mathbf{P}(Z^2 \leq w) \\&= \mathbf{P}(|Z| \leq \sqrt{w}) \\&= \mathbf{P}(-\sqrt{w} \leq Z \leq \sqrt{w})\end{aligned}$$

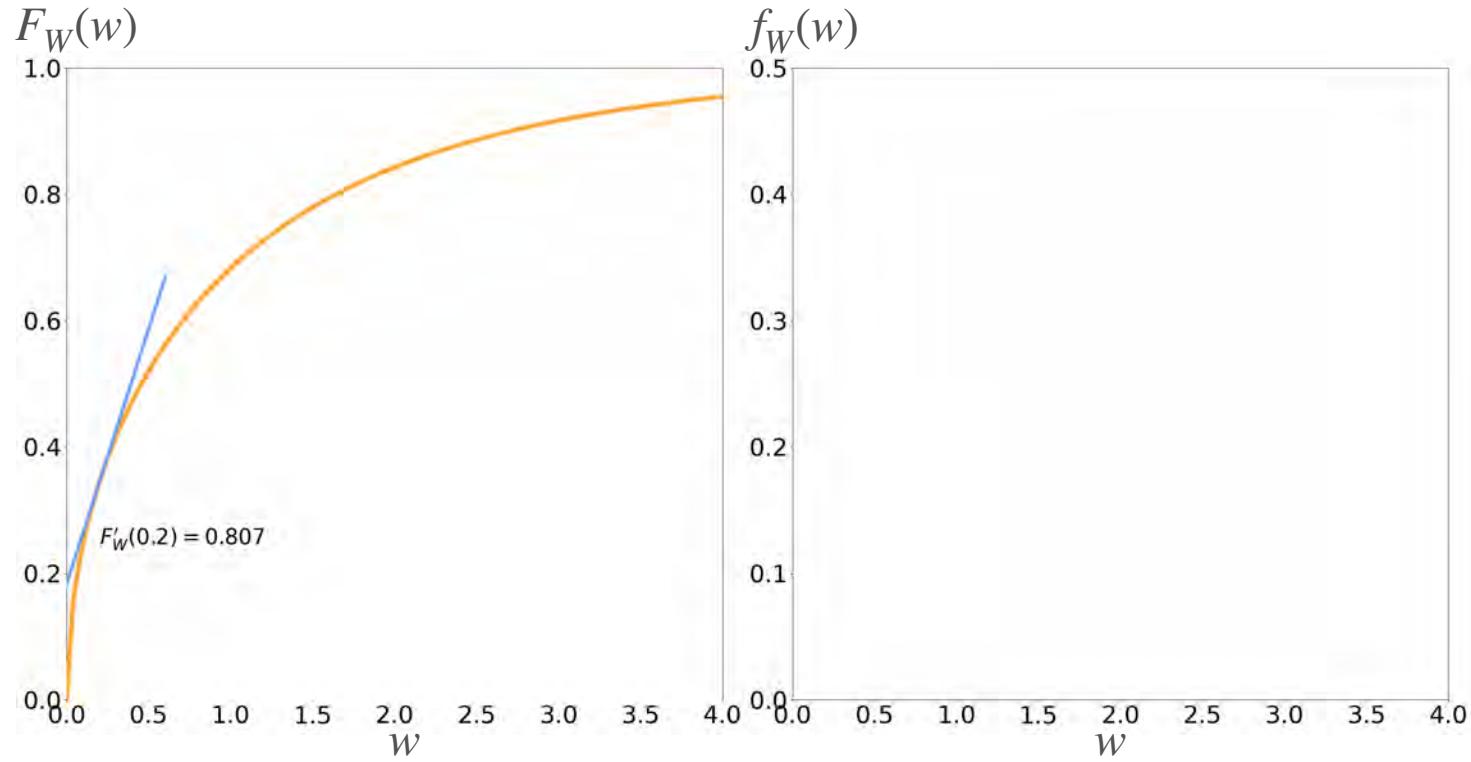


Chi Square Distribution

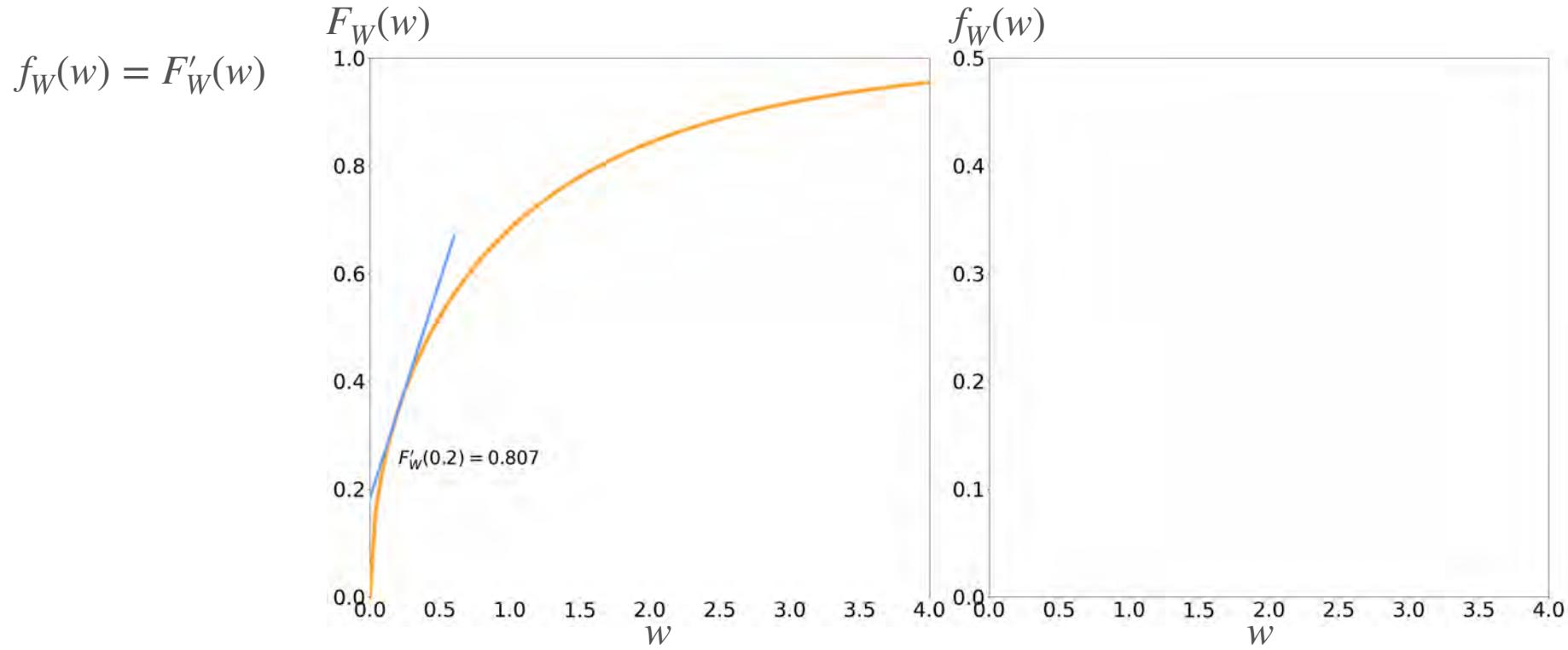
$$\begin{aligned}F_W(w) &= \mathbf{P}(W \leq w) \\&= \mathbf{P}(Z^2 \leq w) \\&= \mathbf{P}(|Z| \leq \sqrt{w}) \\&= \mathbf{P}(-\sqrt{w} \leq Z \leq \sqrt{w})\end{aligned}$$



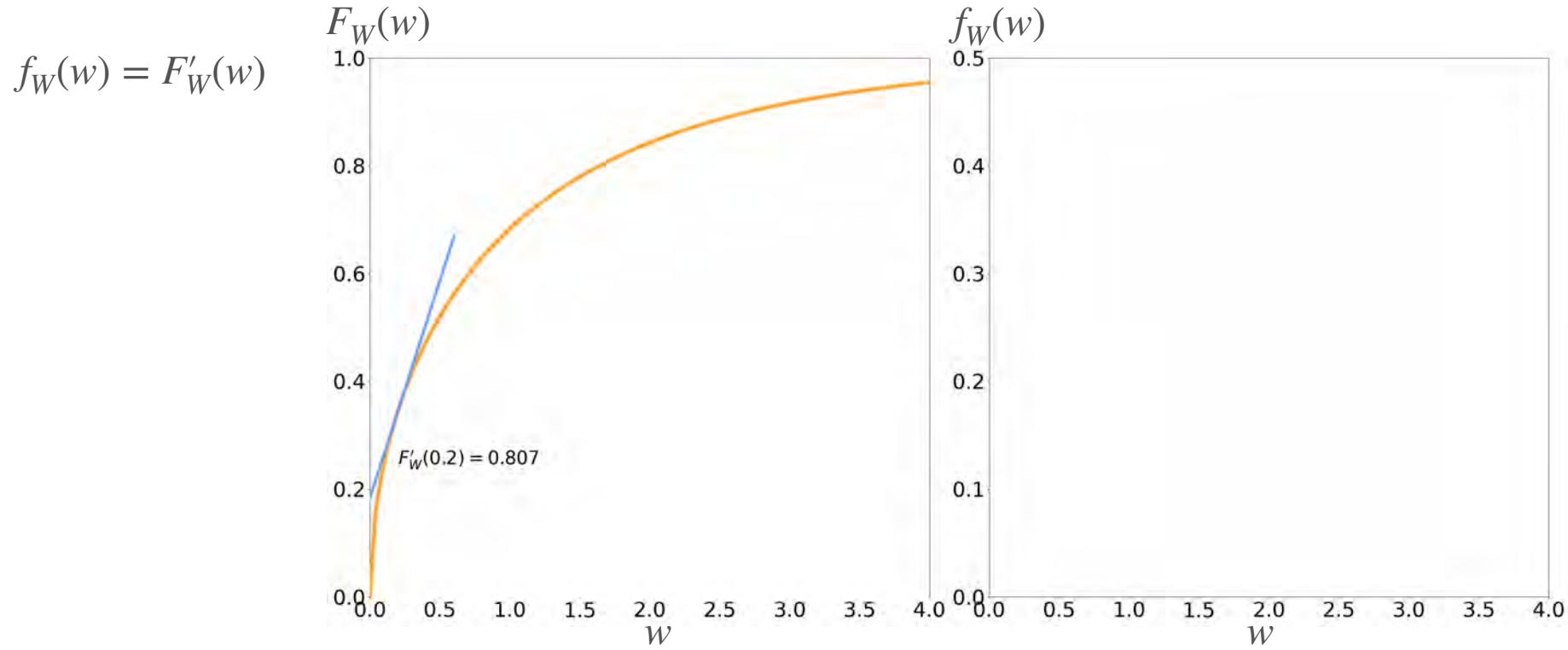
Chi Square Distribution



Chi Square Distribution



Chi Square Distribution



Chi-Square Distribution

Chi-Square Distribution

Accumulated power over 2 transmissions?

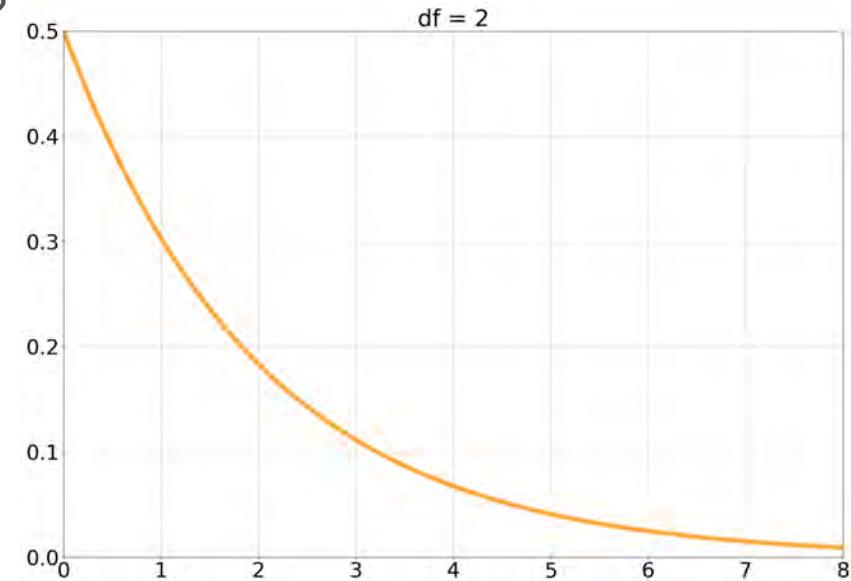
$$W_2 = Z_1^2 + Z_2^2$$

Chi-Square Distribution

Accumulated power over 2 transmissions?

$$W_2 = Z_1^2 + Z_2^2$$

Chi-Square with 2 df



Chi-Square Distribution

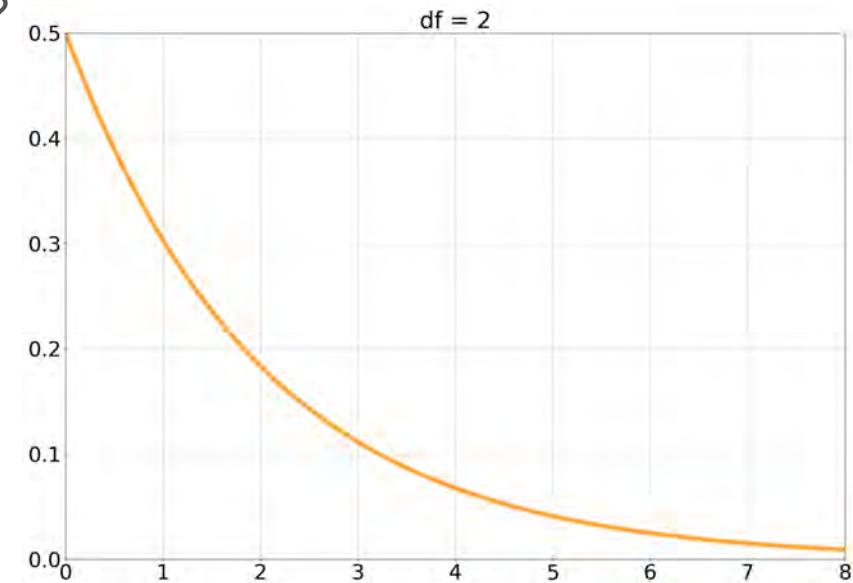
Accumulated power over 2 transmissions?

$$W_2 = Z_1^2 + Z_2^2$$

Chi-Square with 2 df

Accumulated power over 5 transmissions?

$$W_5 = Z_1^2 + Z_2^2 + Z_3^2 + Z_4^2 + Z_5^2$$



Chi-Square Distribution

Accumulated power over 2 transmissions?

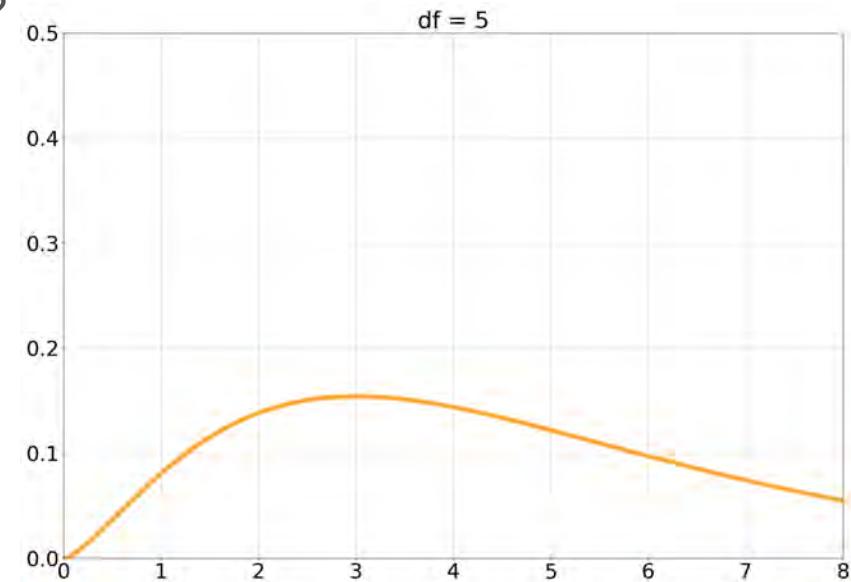
$$W_2 = Z_1^2 + Z_2^2$$

Chi-Square with 2 df

Accumulated power over 5 transmissions?

$$W_5 = Z_1^2 + Z_2^2 + Z_3^2 + Z_4^2 + Z_5^2$$

Chi-Square
with 5 df



Chi-Square Distribution

Accumulated power over 2 transmissions?

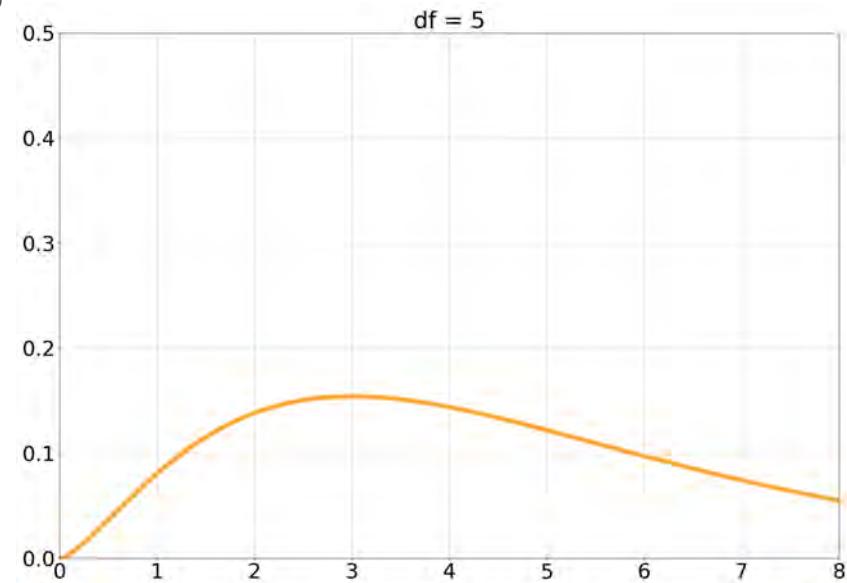
$$W_2 = Z_1^2 + Z_2^2 \quad \text{Chi-Square with 2 df}$$

Accumulated power over 5 transmissions?

$$W_5 = Z_1^2 + Z_2^2 + Z_3^2 + Z_4^2 + Z_5^2 \quad \text{Chi-Square with 5 df}$$

Accumulated power over k transmissions?

$$W_k = \sum_{i=1}^k Z_i^2$$



Chi-Square Distribution

Accumulated power over 2 transmissions?

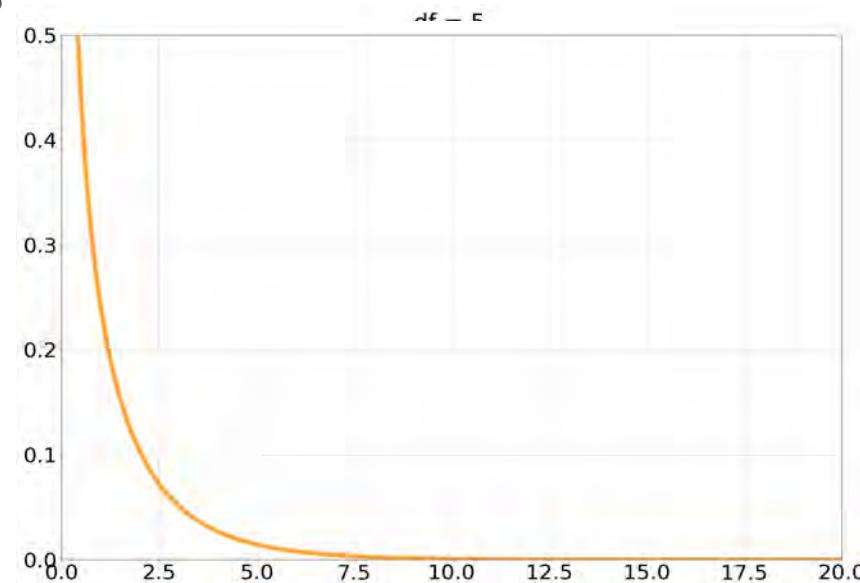
$$W_2 = Z_1^2 + Z_2^2 \quad \text{Chi-Square with 2 df}$$

Accumulated power over 5 transmissions?

$$W_5 = Z_1^2 + Z_2^2 + Z_3^2 + Z_4^2 + Z_5^2 \quad \text{Chi-Square with 5 df}$$

Accumulated power over k transmissions?

$$W_k = \sum_{i=1}^k Z_i^2 \quad \text{Chi-Square with } k \text{ df}$$



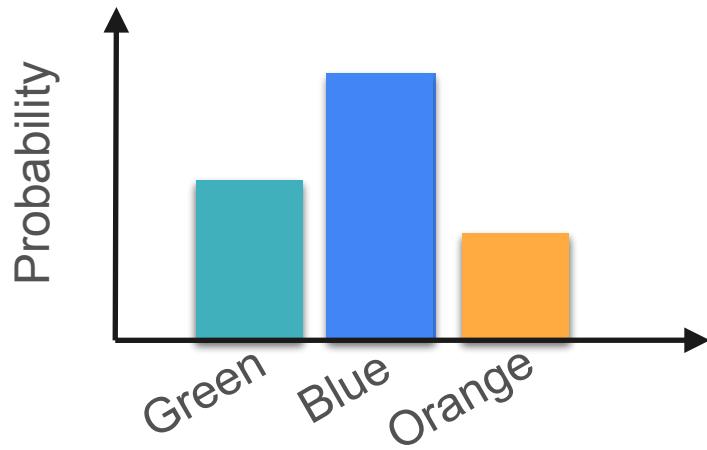


DeepLearning.AI

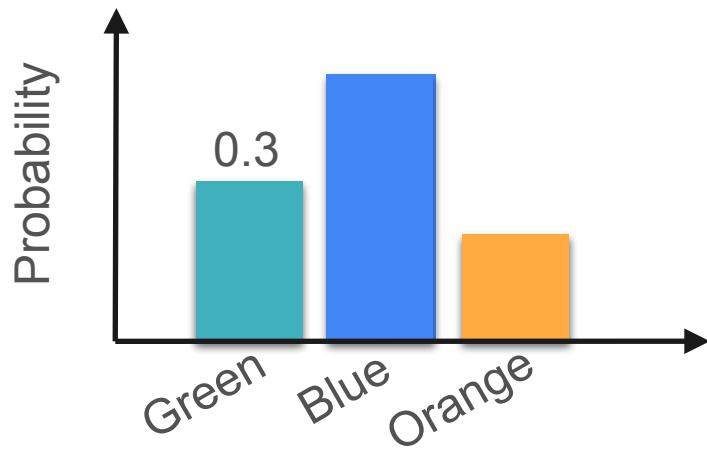
Probability Distributions

Sampling from a Distribution

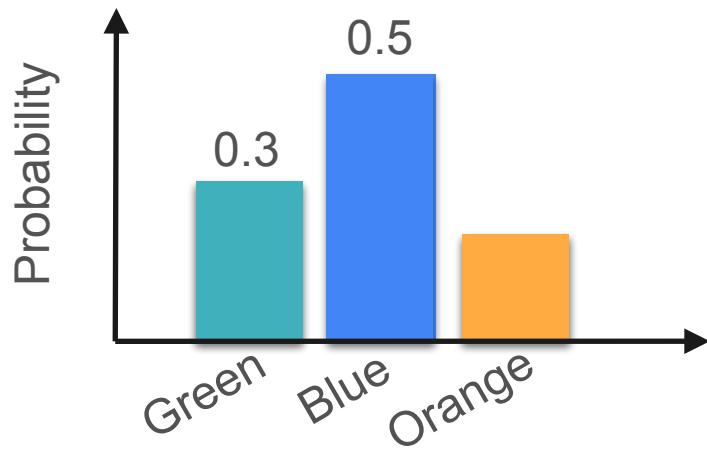
Sampling From a Distribution



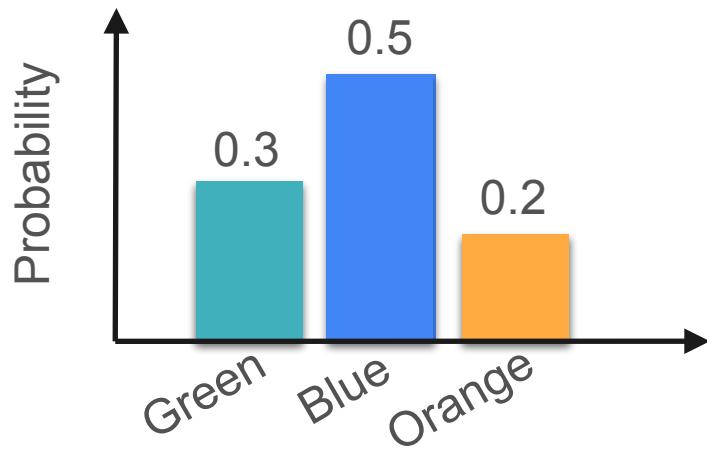
Sampling From a Distribution



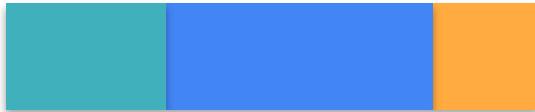
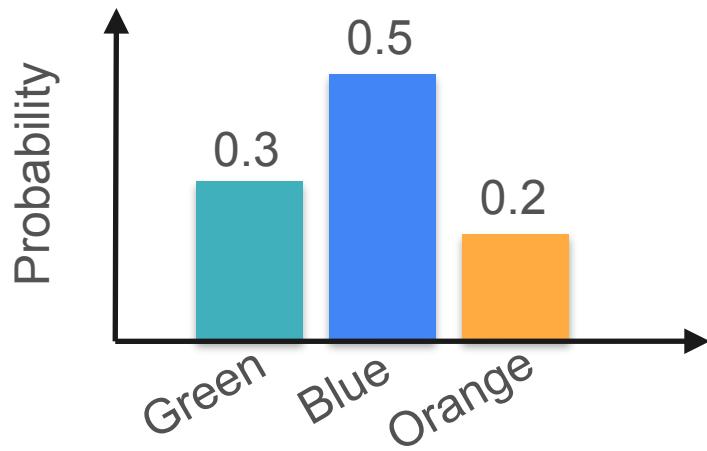
Sampling From a Distribution



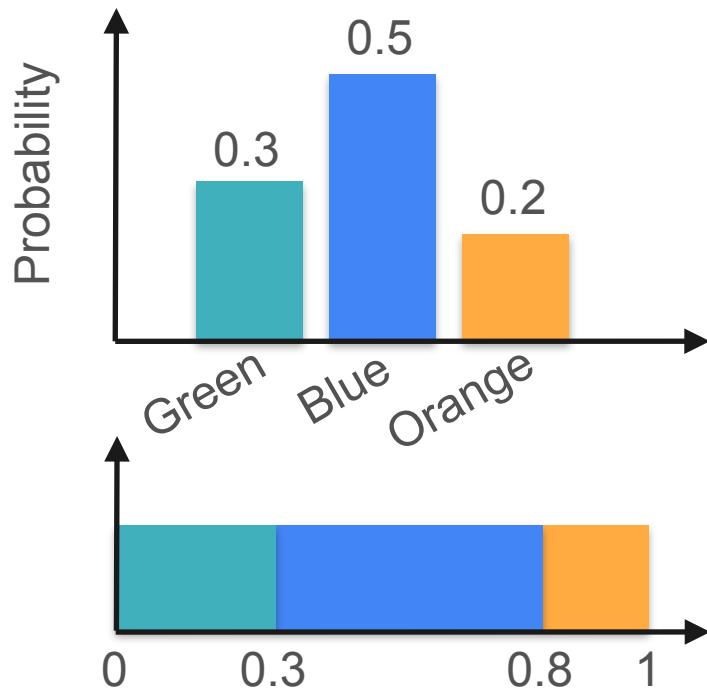
Sampling From a Distribution



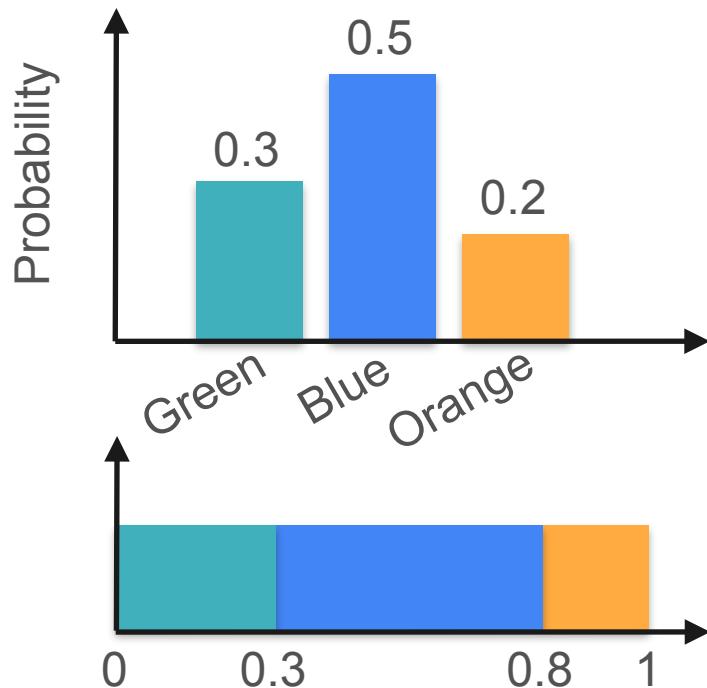
Sampling From a Distribution



Sampling From a Distribution

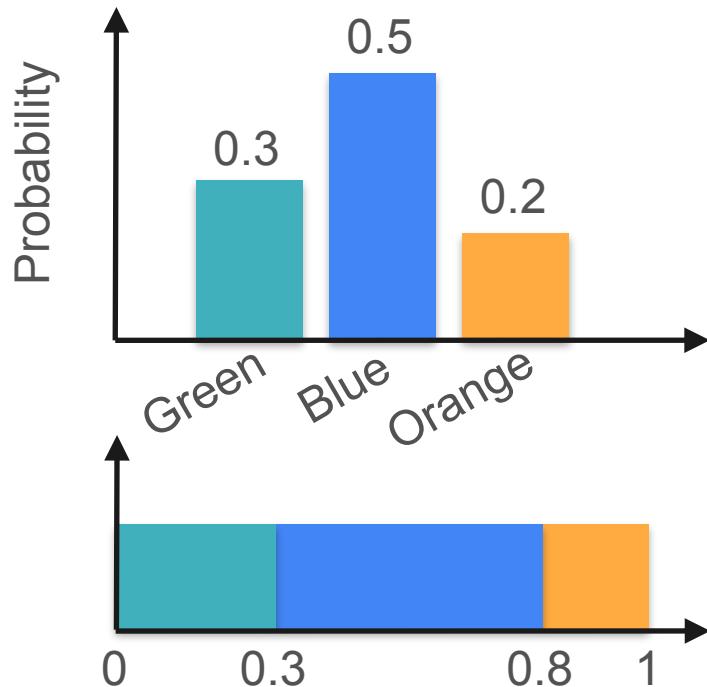


Sampling From a Distribution



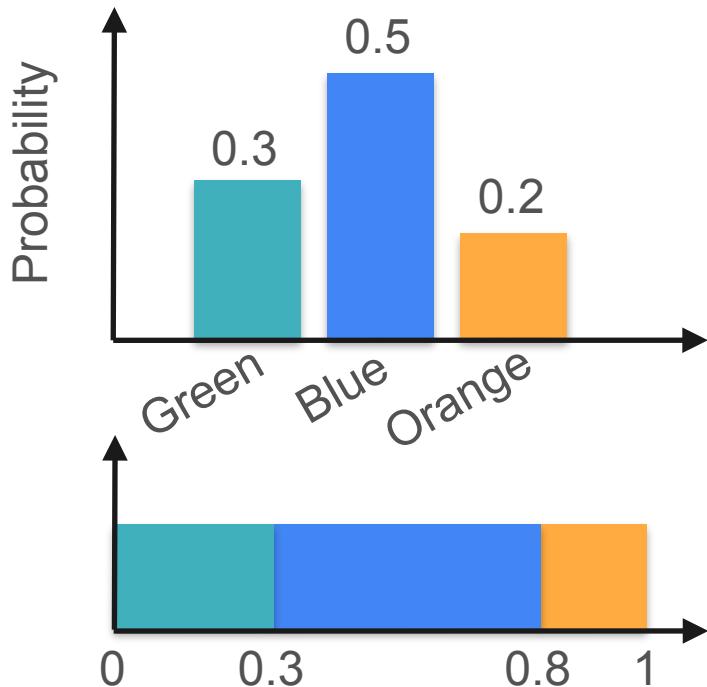
- **Step 1:** generate a random number between 0 and 1

Sampling From a Distribution



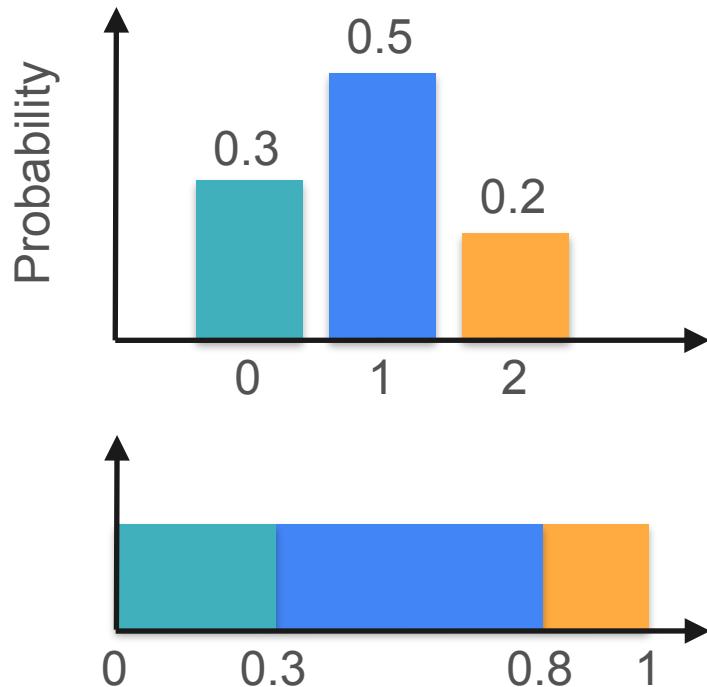
- **Step 1:** generate a random number between 0 and 1
- **Step 2:** find out which interval the number belongs to
 - [0, 0.3)
 - [0.3, 0.8)
 - [0.8, 1]

Sampling From a Distribution



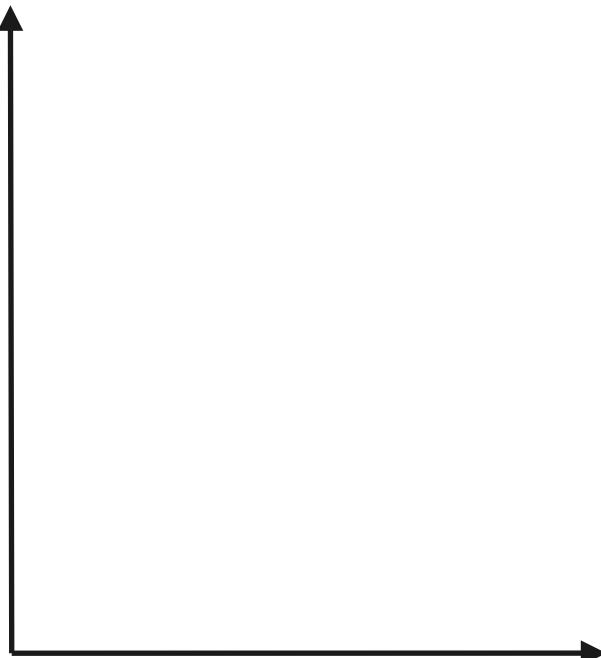
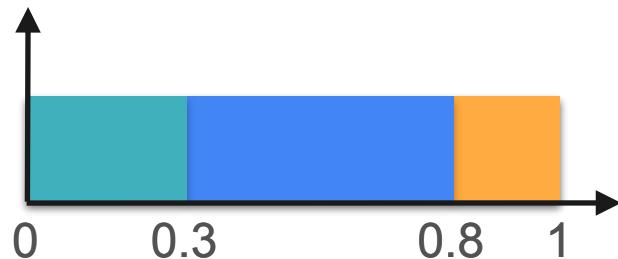
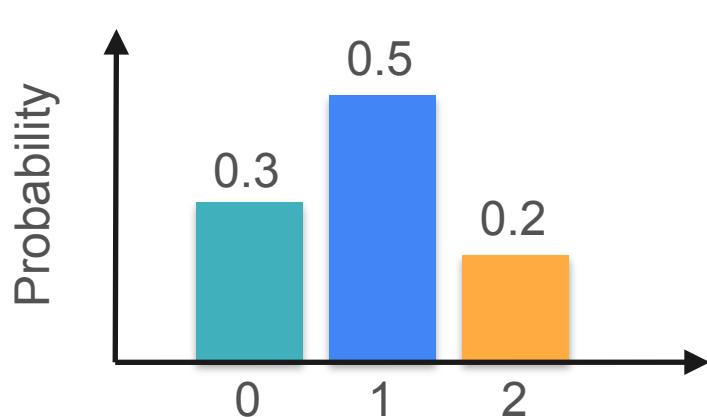
- **Step 1:** generate a random number between 0 and 1
- **Step 2:** find out which interval the number belongs to
 - [0, 0.3)
 - [0.3, 0.8)
 - [0.8, 1]
- **Step 3:** Assign an outcome based on the interval

Sampling From a Distribution

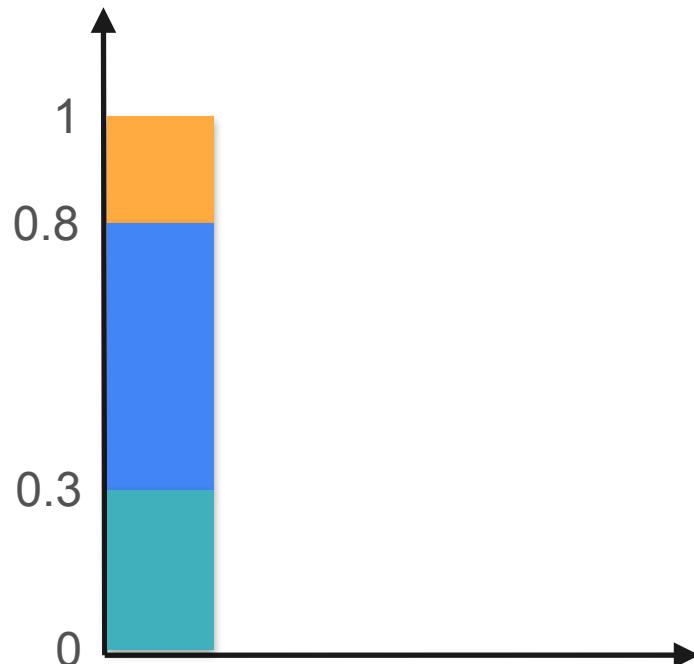
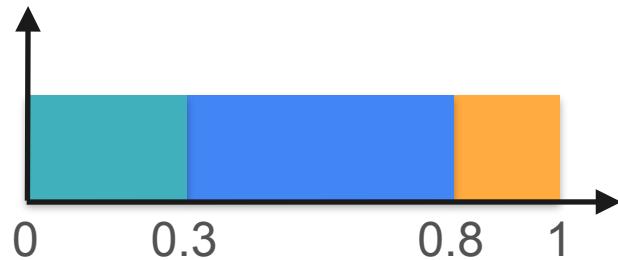
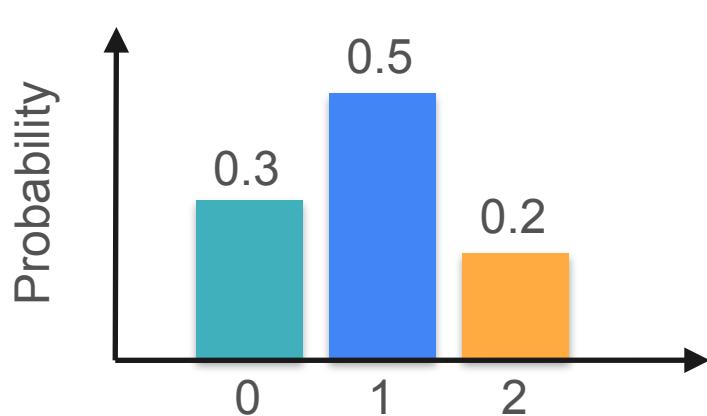


- **Step 1:** generate a random number between 0 and 1
- **Step 2:** find out which interval the number belongs to
 - [0, 0.3)
 - [0.3, 0.8)
 - [0.8, 1]
- **Step 3:** Assign an outcome based on the interval

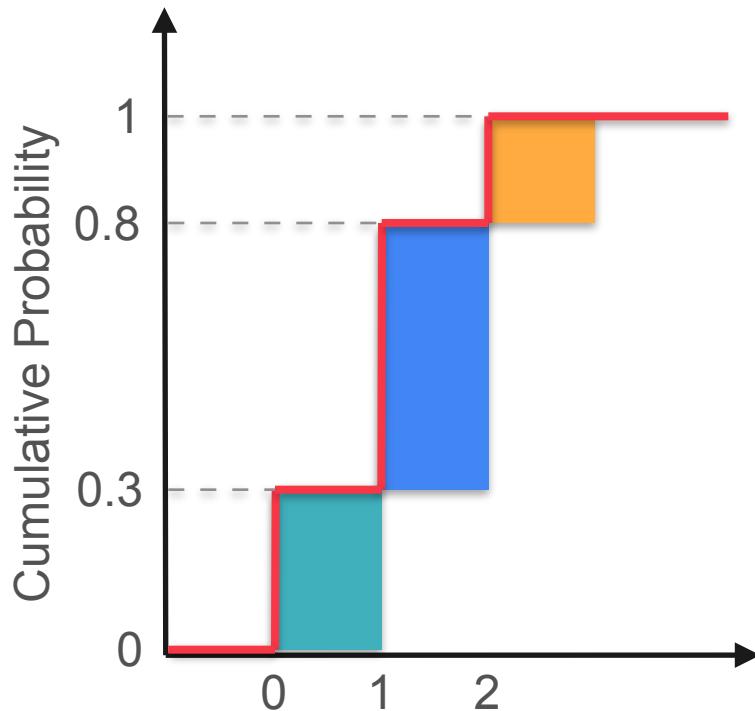
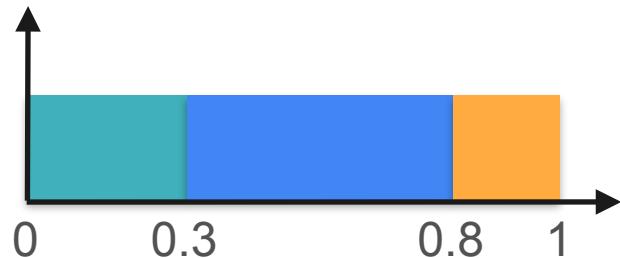
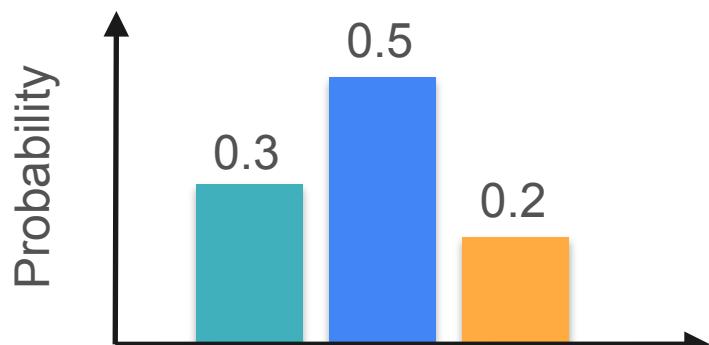
Sampling From a Distribution



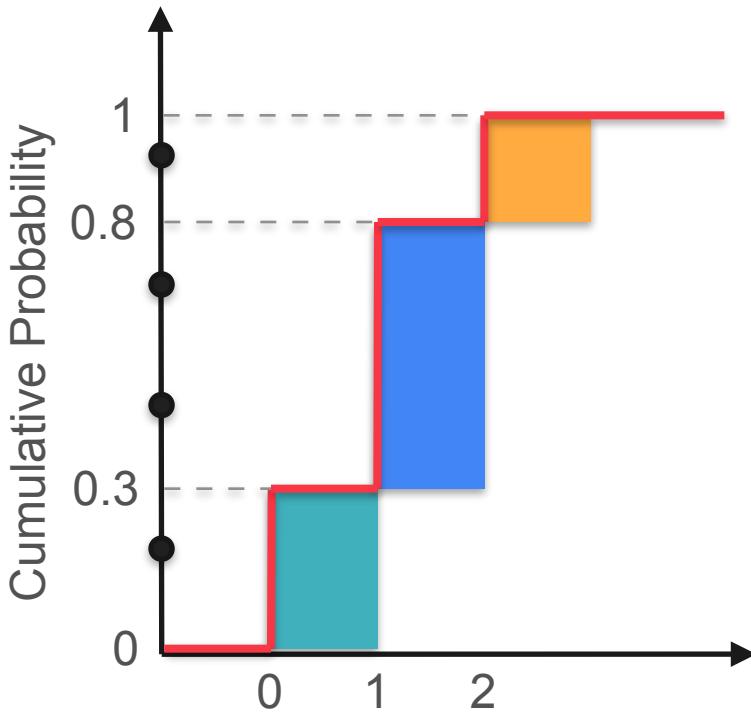
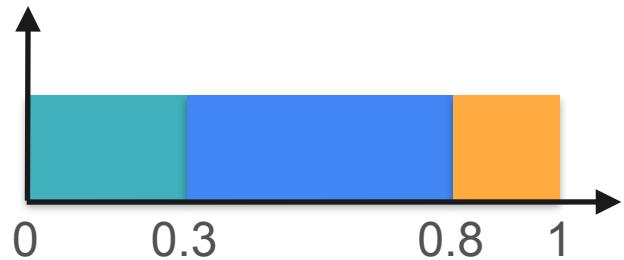
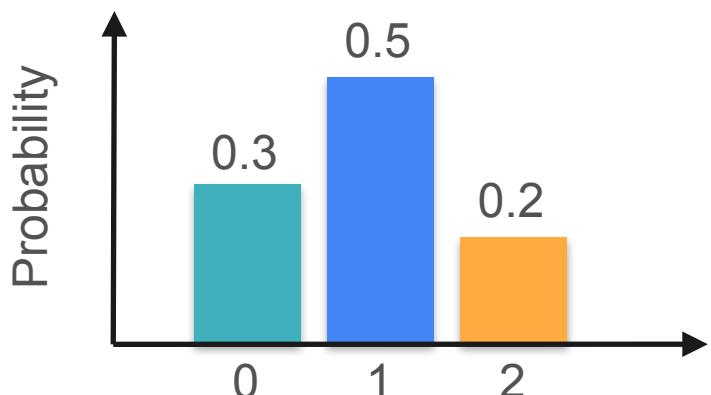
Sampling From a Distribution



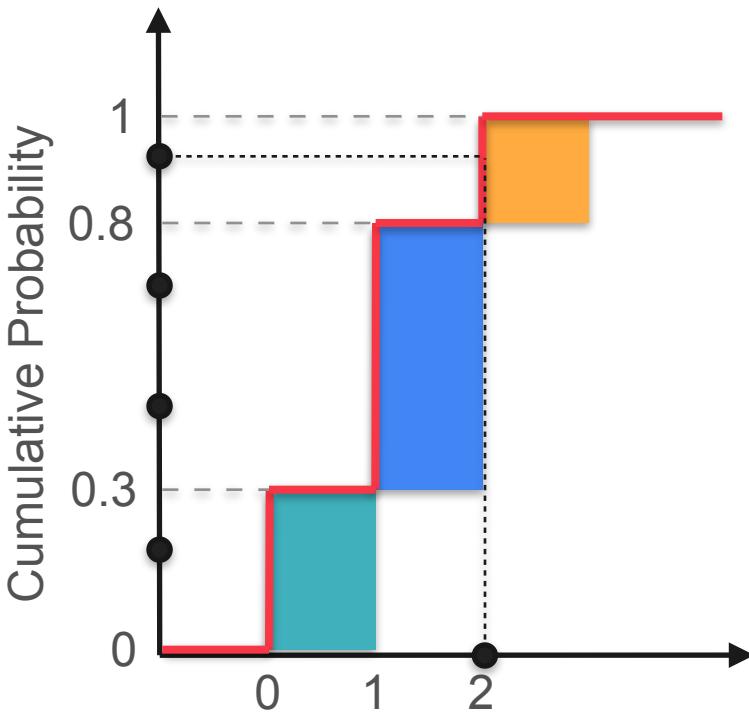
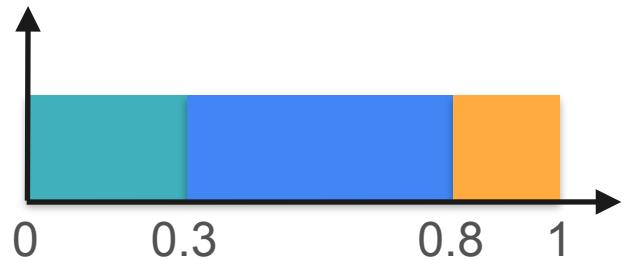
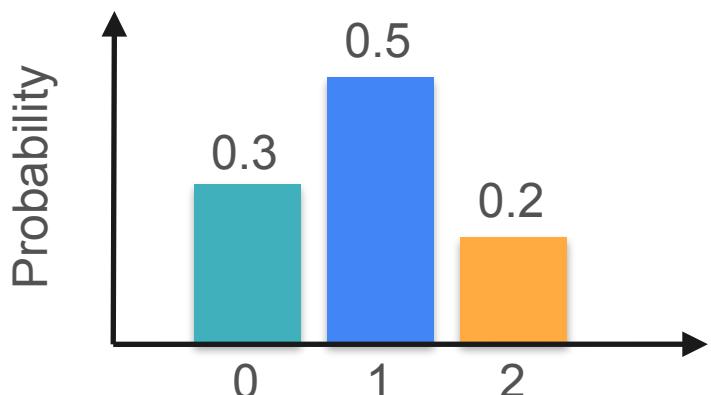
Sampling From a Distribution



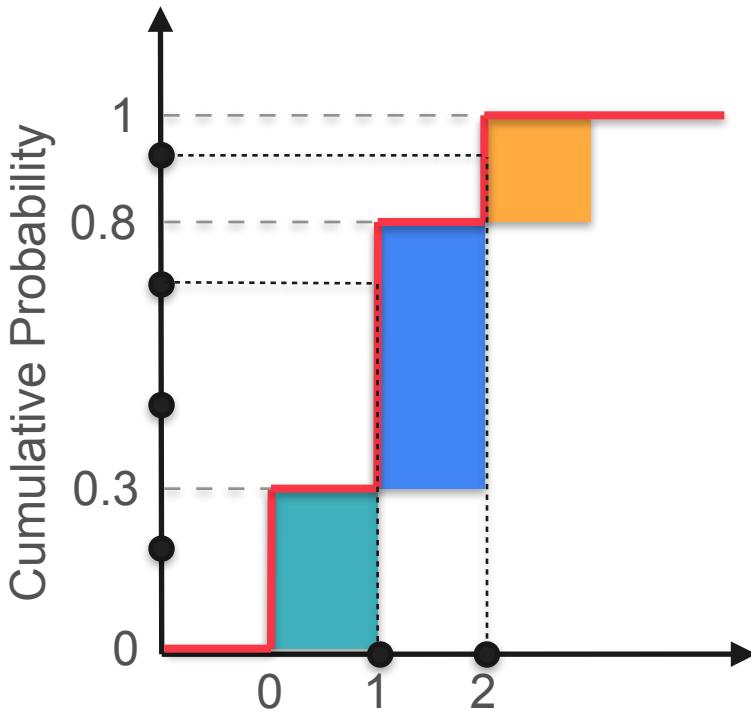
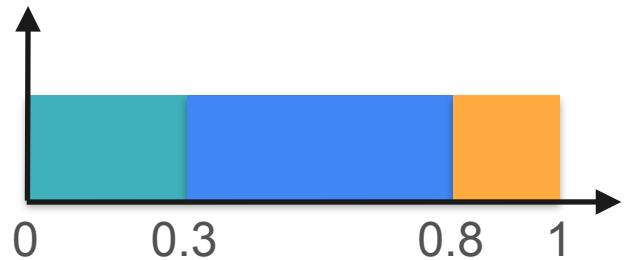
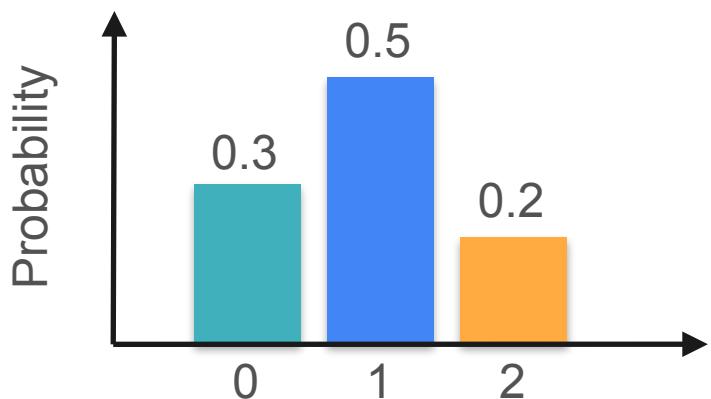
Sampling From a Distribution



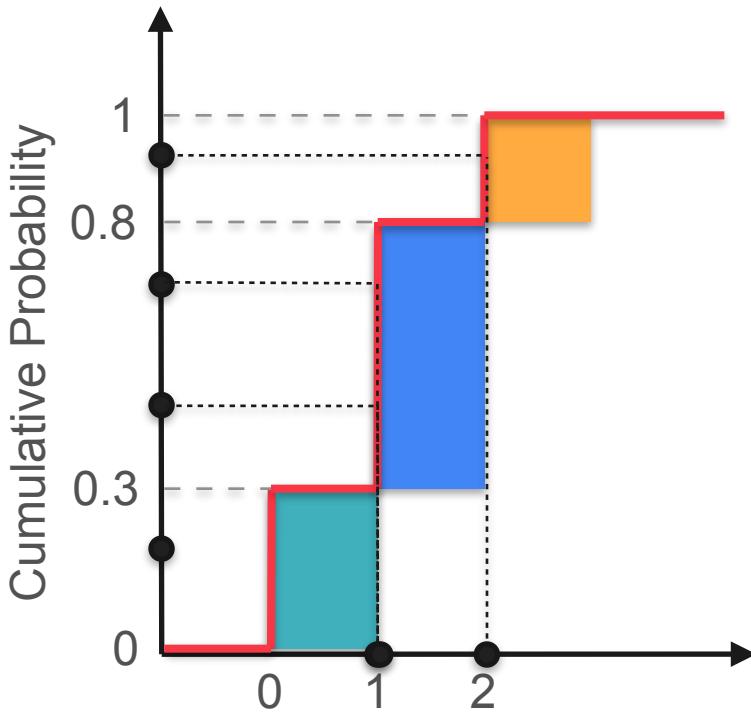
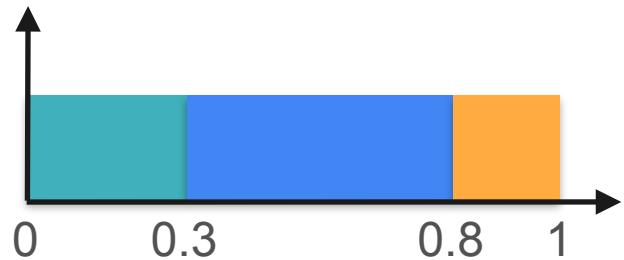
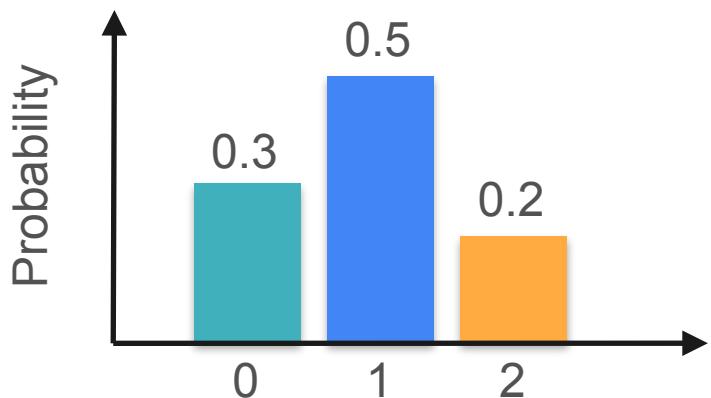
Sampling From a Distribution



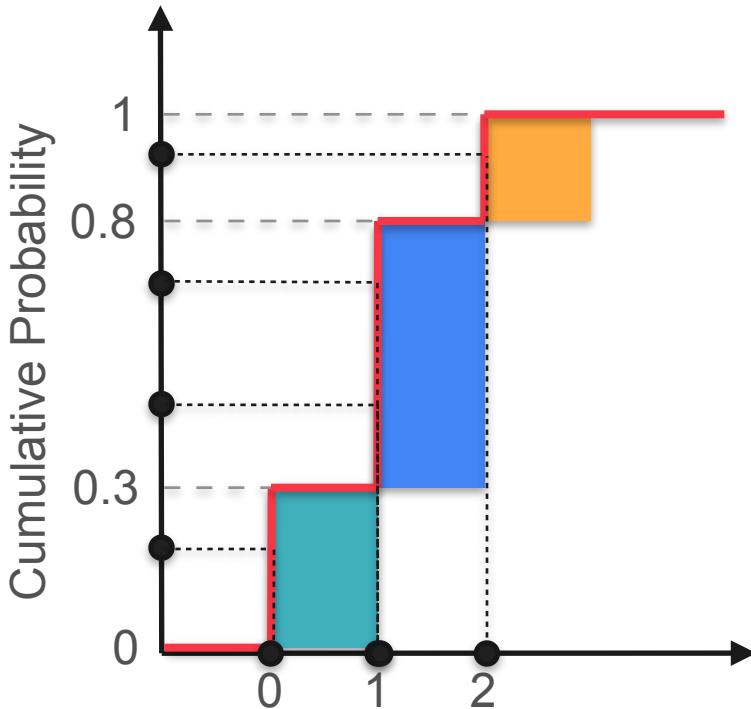
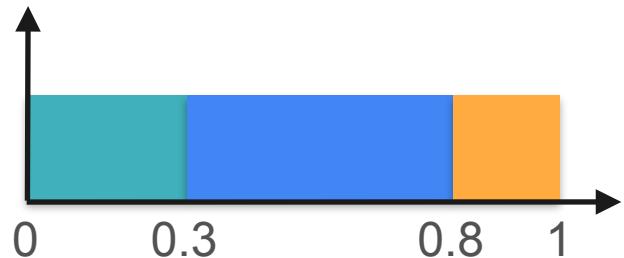
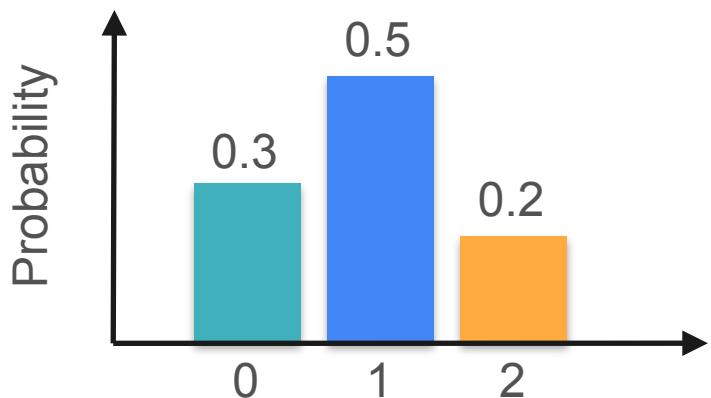
Sampling From a Distribution



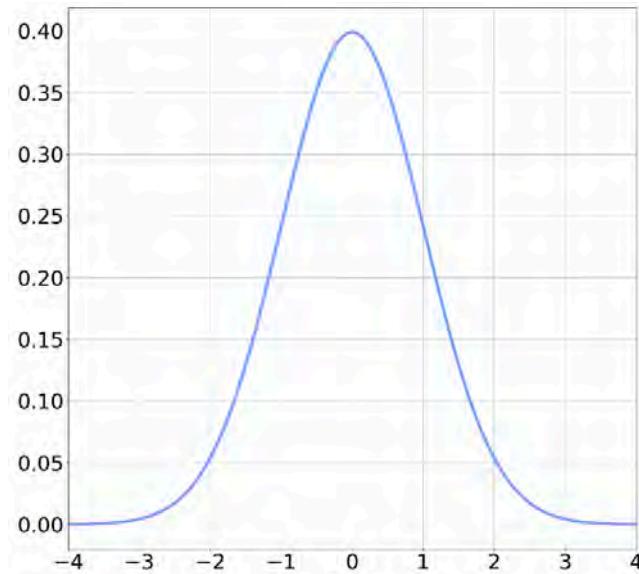
Sampling From a Distribution



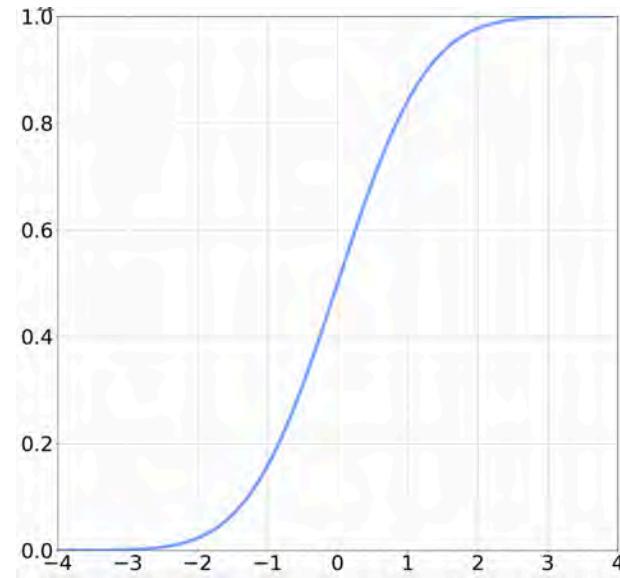
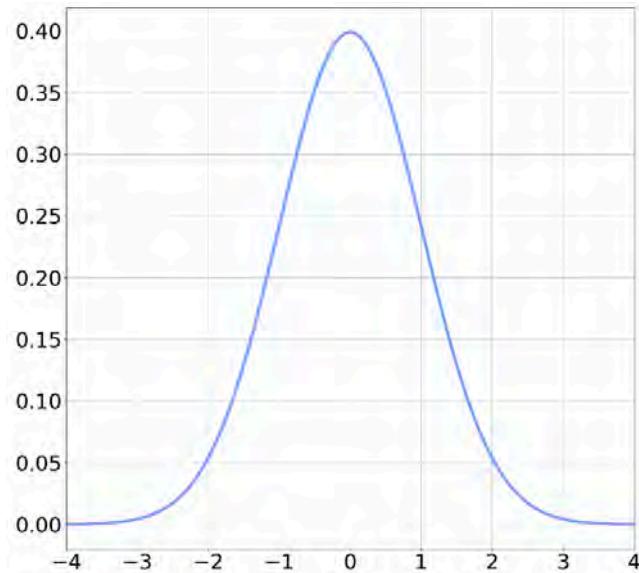
Sampling From a Distribution



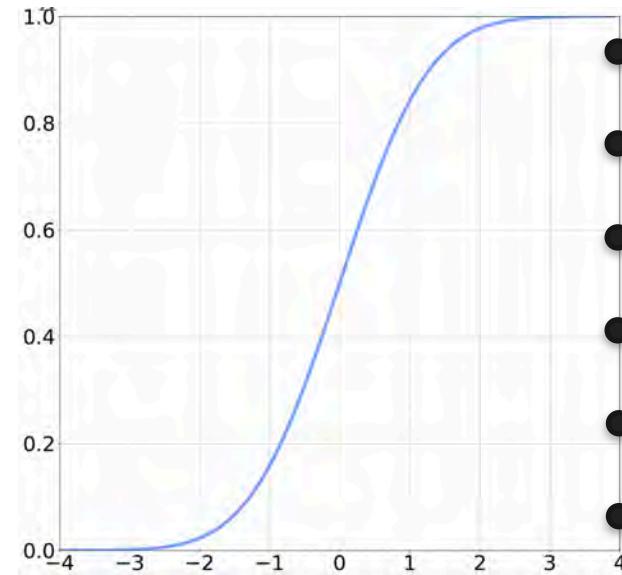
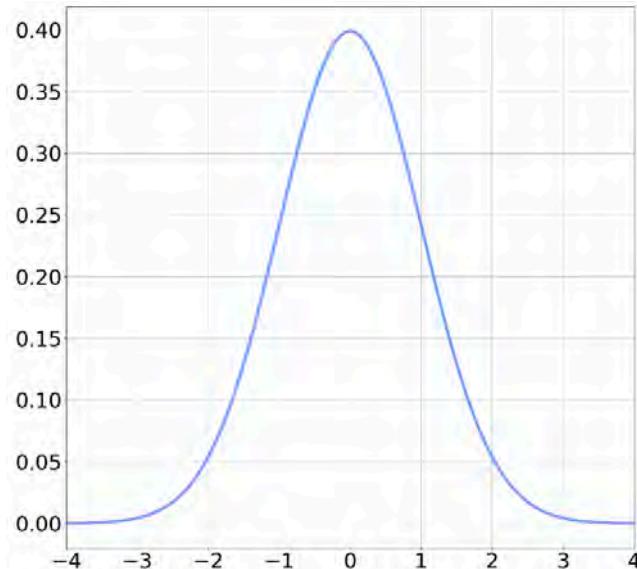
Sampling From a Normal Distribution



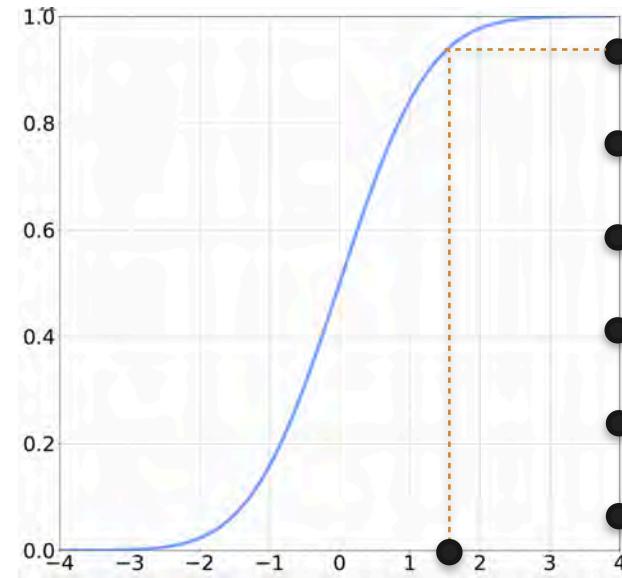
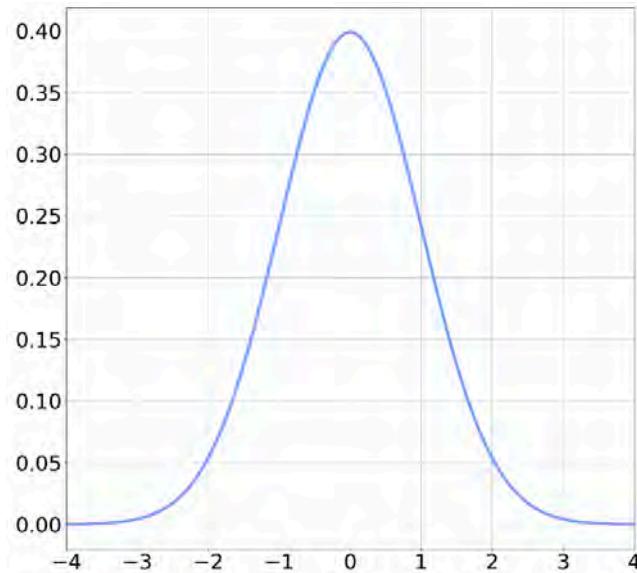
Sampling From a Normal Distribution



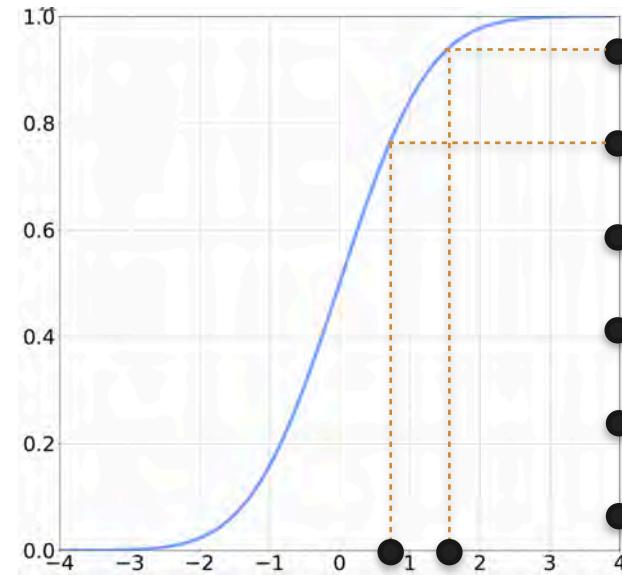
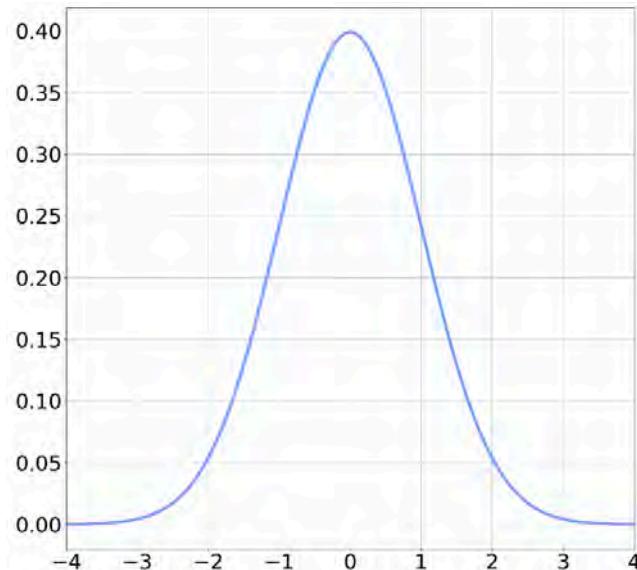
Sampling From a Normal Distribution



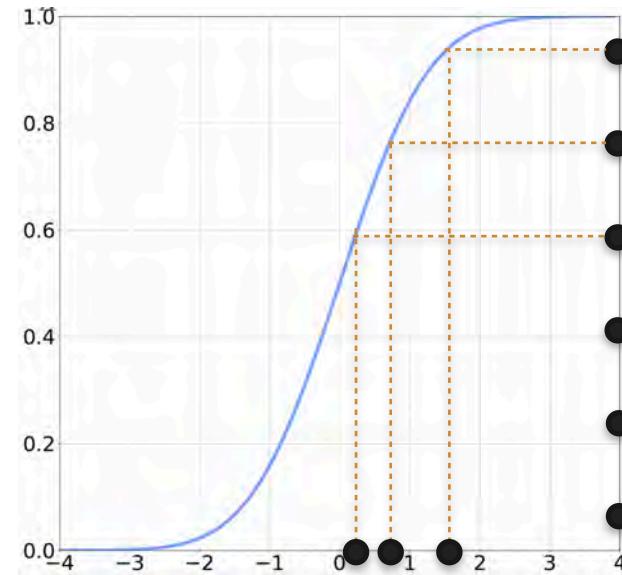
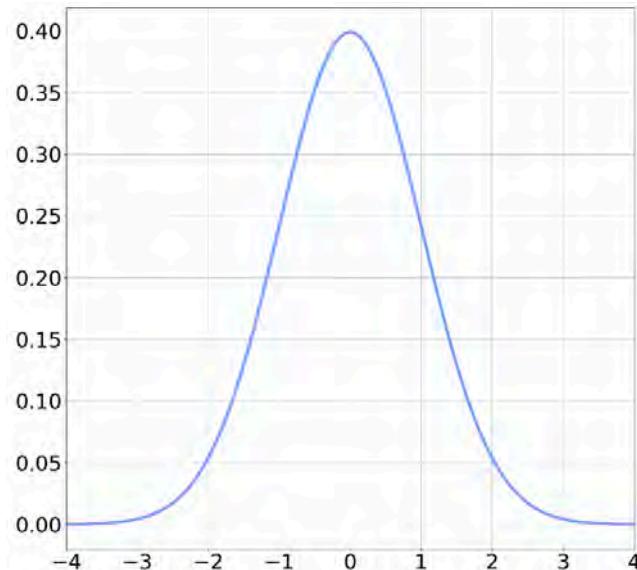
Sampling From a Normal Distribution



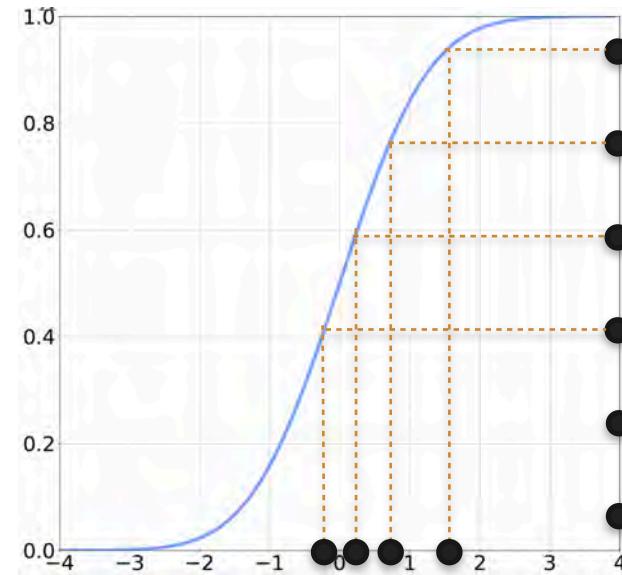
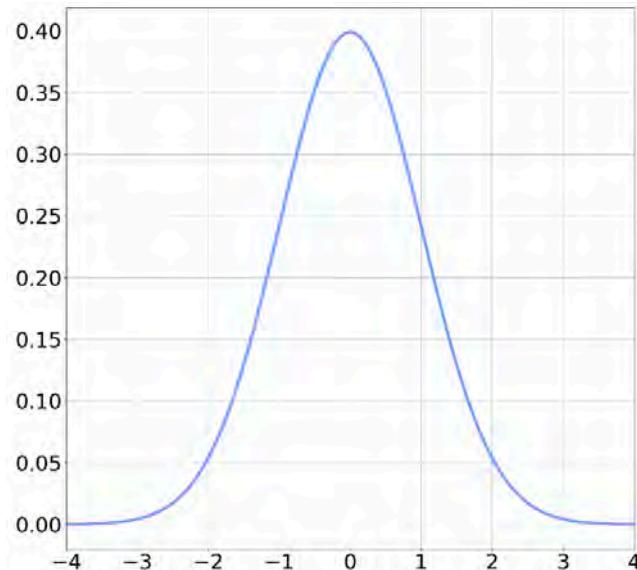
Sampling From a Normal Distribution



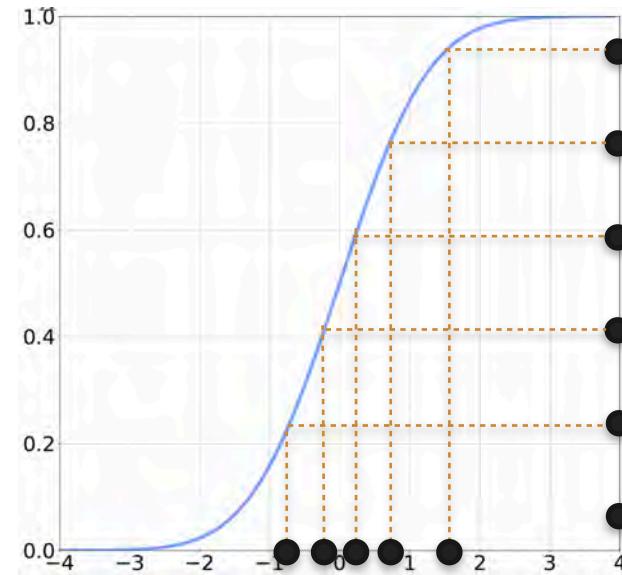
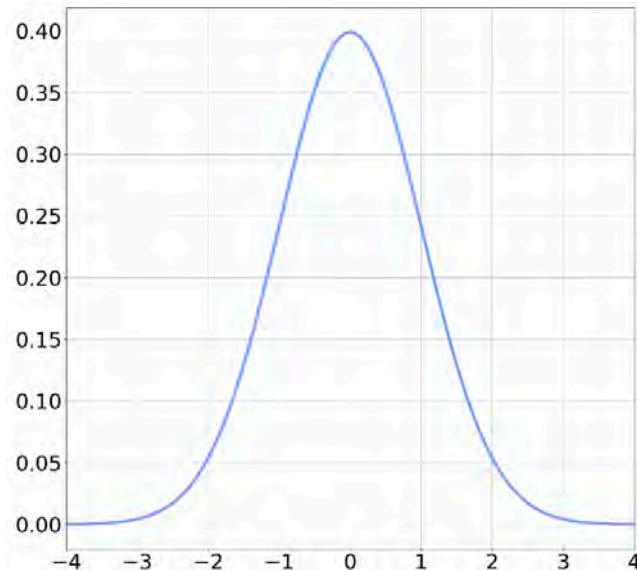
Sampling From a Normal Distribution



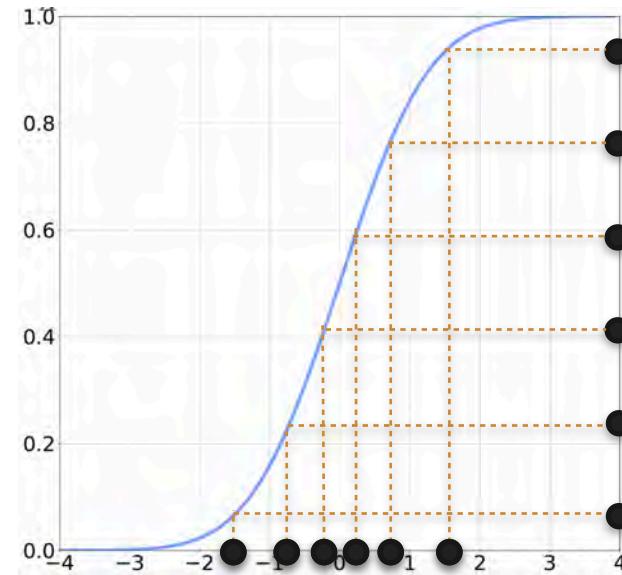
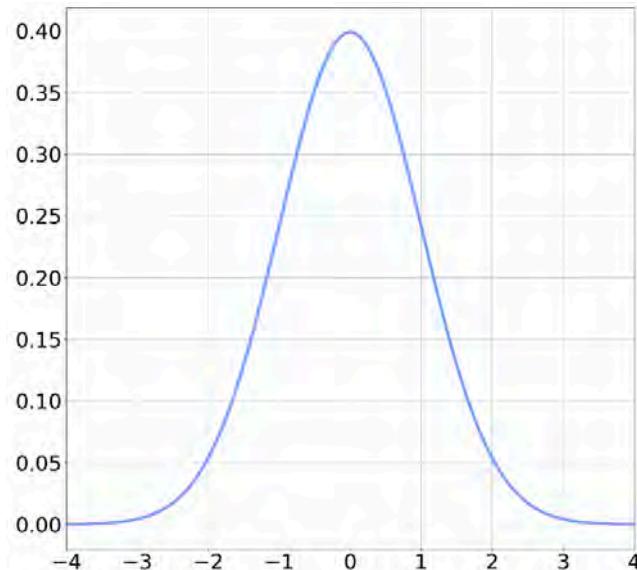
Sampling From a Normal Distribution



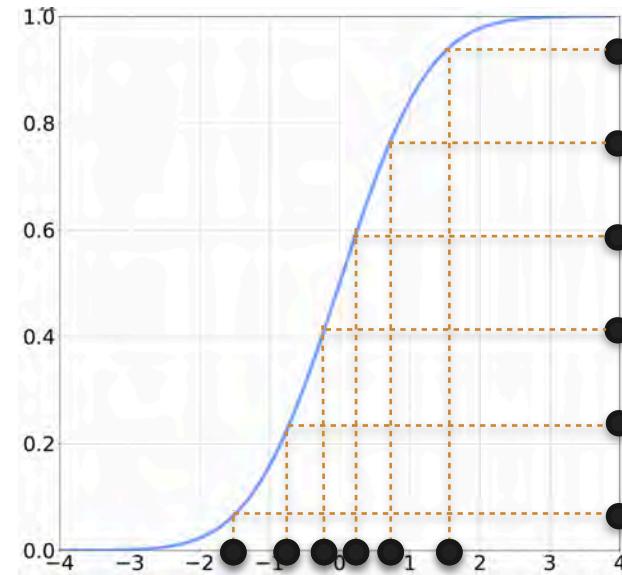
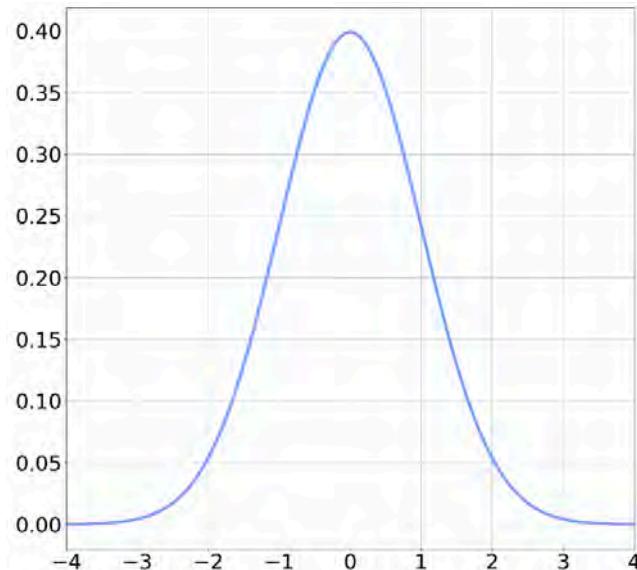
Sampling From a Normal Distribution



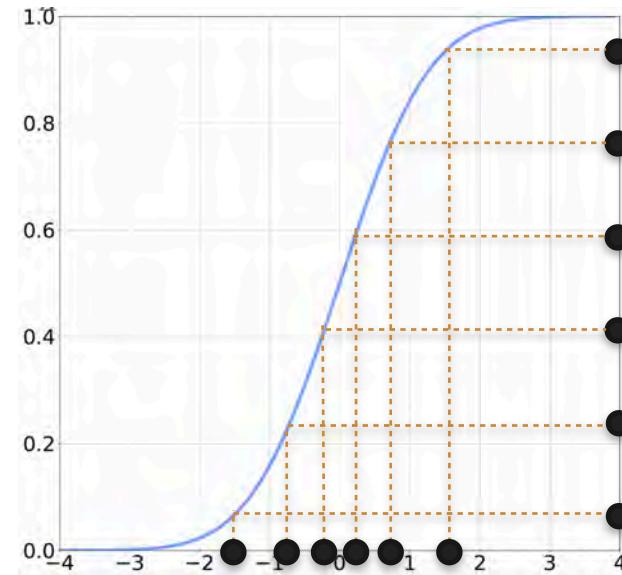
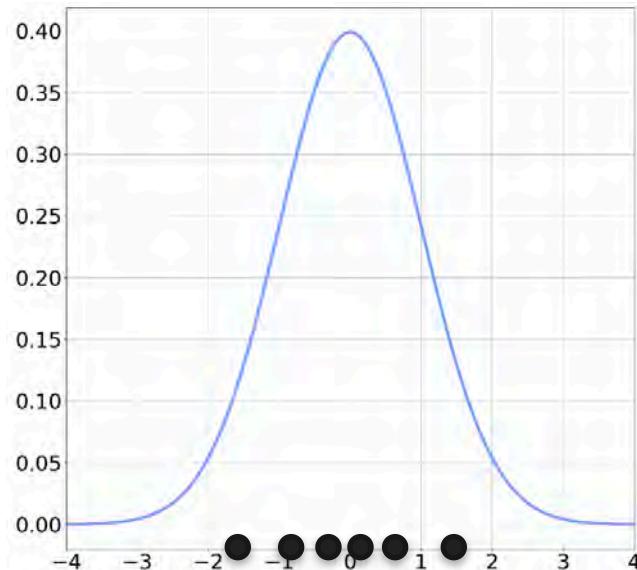
Sampling From a Normal Distribution



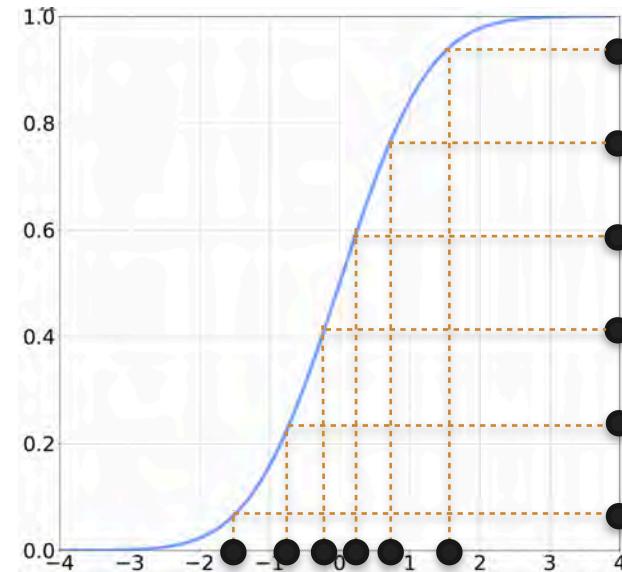
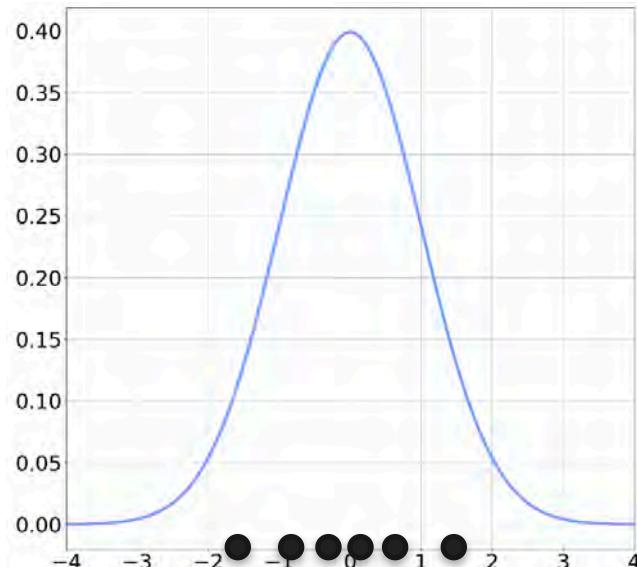
Sampling From a Normal Distribution



Sampling From a Normal Distribution



Sampling From a Normal Distribution





DeepLearning.AI

Probability Distributions

Conclusion