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Observational Evidence for Dark Matter

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ABSTRACT

Astronomers have discovered, using different observational methods from the 1930s to the present, that a significant amount of non-baryonic matter must exist, which they call dark matter. This concept of dark matter first arose to explain Fritz Zwicky's measurements of the Coma galaxy cluster in the 1930s, where he measured the mass-to-light ratio to be about 500 $\frac{M_\odot}{L_\odot}$. Following Zwicky's measurements, other astronomers completed different observations later in the 20th century that support the concept of dark matter, such as 1) rotation curve measurements, 2) mass measurements of galaxy clusters using x-ray data and the assumption of hydrostatic equilibrium and 3) mass measurements of galaxy clusters using weak gravitational lensing. 4) In addition, one of the most significant pieces of observational evidence for dark matter's existence is WMAP's measurement of the third peak of the Cosmic Microwave Background (CMB) power spectrum. From this observational evidence, the existence of dark matter seems overwhelming.

1. INTRODUCTION

The first evidence of dark matter came from Fritz Zwicky's measurements of the Coma galaxy cluster in the 1930s. His measurements concluded that the mass of the galaxy cluster was over five hundred times larger than the mass of the luminous objects in the cluster (Zwicky 1937). This meant that a large amount of undetectable matter made up most of the mass of the cluster. A second piece evidence for dark matter came from the measurements of the rotation curves of spiral galaxies. After Zwicky's measurements of the Coma galaxy cluster, other astronomers measured the rotation curve of the Andromeda Galaxy (M31). Measurements were first done by Horace Babcock in 1939, which he obtained through the use of spectroscopy (Babcock 1939). With the rotation curve data, he obtained the mass and mass distribution of M31; he found the mass-to-light ratio of about 50 $\frac{M_\odot}{L_\odot}$, which meant that there was a significant amount of mass from non-luminous matter in M31. In the 1960s, these measurements were tightened through observing the 21-cm hydrogen line of neutral hydrogen clouds orbiting the outskirts of galaxies (Roberts 1966). The 21-cm hydrogen line is a spectral line created by a change in the spin state of hydrogen. This emission line penetrates interstellar dust clouds and allows observers to obtain a better view of the velocity of hydrogen in their observing area. Astronomers Vera Rubin and Kent Ford confirmed these tightened measurements in a paper they published in 1970, using the emission lines of hot ionized gas to find the rotational velocity of the galaxy M31 (Rubin & Ford 1970). Both Roberts and Rubin and Ford found that the rotational velocity stayed constant after a certain distance away from the center of the galaxy; however, they expected that the rotational velocity should have decreased to zero as you go further outside the galaxy according to Newton's law of universal gravitation (Rubin & Ford 1970; Roberts 1966). The constant velocity meant that some source of unknown mass affected the rotational velocity. A spherical dark matter halo fits the data.

Following Rubin and Ford's measurements of the rotation curves of galaxies, astronomers derived the mass of galaxy clusters from x-ray data. These galaxy clusters emitted x-rays due to the thermal bremsstrahlung radiation produced by the ionized gas within the cluster. In the 1980s, the new x-ray satellite called the Einstein Observatory allowed observers to investigate the mass of galaxy clusters. These observations obtained the mass of galaxy clusters through x-ray data and assuming that the gas within the clusters was in hydrostatic equilibrium (Fabricant et al. 1984; Henriksen & Mushotzky 1986). Henriksen & Mushotzky (1986)'s x-ray derived gas mass and total mass of the Coma cluster yielded a mass-to-light ratio of $810\frac{M_\odot}{L_\odot}$. This also suggested a significant amount of non-luminous matter within the Coma galaxy cluster.

The next evidence for something other than baryonic matter in the Universe was gravitational lensing of background galaxies by a foreground galaxy cluster, also known as weak gravitational lensing. Rather than using a baryonic tracer to infer where the dark matter is, gravitational lensing is a direct probe of dark matter. The bending of light from the background source due to the foreground galaxy cluster distorts and magnifies the background source. These effects can be found through CCD images of the cluster region and through the use of the images and rigorous data reduction, observers are able estimate the mass-to-light ratio of the galaxy cluster. Squires et al. (1996) found a mass-to-light ratio of $320 \pm 100 h \frac{M_\odot}{L_\odot}$ for galaxy cluster Abell 2390, which they compared to Carlberg et al. (1996)'s mass-to-light ratio of $370 \pm 60 h \frac{M_\odot}{L_\odot}$ that was derived through the virial theorem. The weak-gravitational-lensing ratio agreed with the virial ratio, both again suggesting that there is non-luminous matter within galaxy clusters. The effects of gravitational lensing are also used to find the surface mass density map of the Bullet Cluster. Once the galaxy clusters passed through each other, the galaxies remained untouched while the x-ray emitting plasma collided and mixed between the galaxy clusters. Since the plasma makes up most of the visible matter, one would expect that the mass should be concentrated where the plasma is. However, it was found that the mass is concentrated around the two galaxy clusters rather than between them (Clowe et al. 2006). This also suggests that non-luminous matter dominates the mass of the galaxy clusters.

Another significant piece of observational evidence that suggested dark matter's existence was the third peak of the power spectrum of the CMB from the nine year Wilkinson Microwave Anisotropy Probe (WMAP) (Hinshaw et al. 2013). Plotting the temperature fluctuations of the CMB as a function of angular scale revealed the cosmological properties of the Universe. McGaugh (1999)'s predictions of the CMB power spectrum revealed distinct differences between the MOdified Newtonian Dynamics (MOND) theory and the dark matter theory (Milgrom 1983). One of the distinct differences was the amplitude of the third peak of the CMB power spectrum. The third peak was about the same height of the second peak if there was dark matter in the Universe, and the third peak was significantly smaller if there was only baryonic matter. WMAP's results showed that the third peak was just as high as the second peak, favoring the theory of dark matter and disfavoring the MOND theory (Hinshaw et al. 2013).

2. ZWICKY'S MEASUREMENTS OF THE COMA GALAXY CLUSTER

In 1937, the Swiss astronomer Fritz Zwicky published one of the most important papers in Astronomy (Zwicky 1937). In this paper, he discussed how he used the virial theorem to find the mass of galaxies within the Coma galaxy cluster. The virial theorem states that the kinetic energy of a steady-state system is equal to the negative potential energy divided by 2, shown in the equation below (Zwicky 1937)

$$K = -\frac{W}{2}. \tag{1}$$

Here, K represents the time average of the sum of kinetic energies of the individual galaxies and W represents the time average of the total potential energy of all the galaxies. Zwicky found, using Newton's inverse square law, that the potential energy was uniformly distributed inside a sphere of radius R. So, the potential energy was

$$W = \frac{-5GM^2}{R}, \tag{2}$$

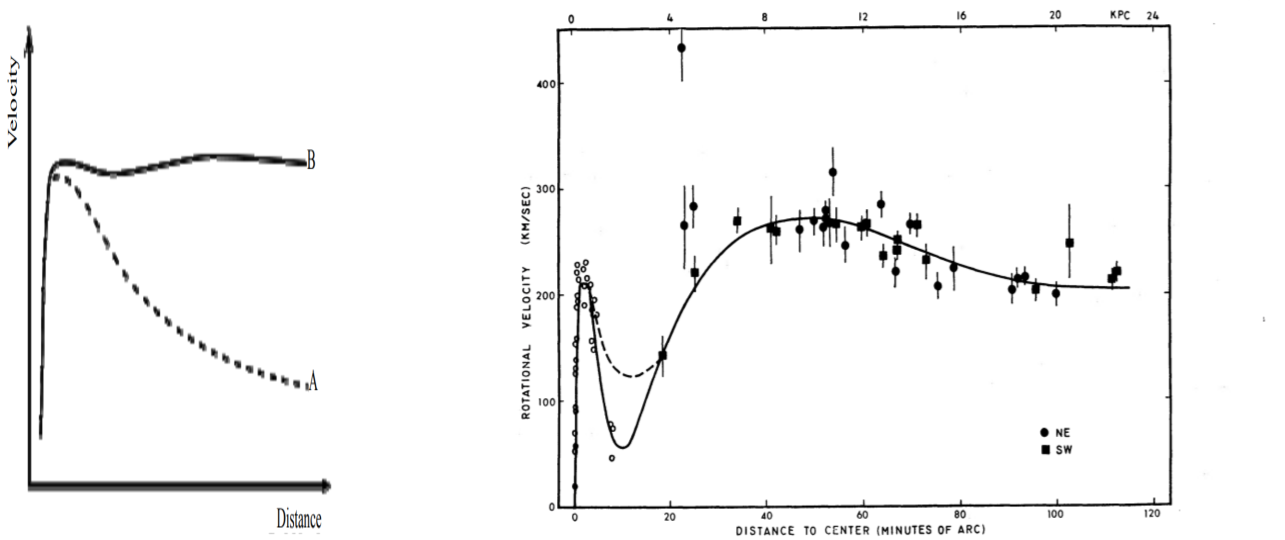
where G is the gravitational constant, and M is the total mass of the galaxy cluster. The kinetic energy is equal to $K = \frac{1}{2}M\langle v^2\rangle$, where $\langle v^2\rangle$ is the velocity dispersion (Zwicky 1937). However, since he assumed a spherical distribution, $\langle v^2\rangle$ must be equal to $3\langle v_s^2\rangle$, where $\langle v_s^2\rangle$ is the velocity dispersion in the radial direction. Combining the kinetic energy and potential energy, Zwicky (1937) found that the mass was

$$M = \frac{3\langle v_s^2\rangle R}{5G}. \tag{3}$$

Zwicky used his measurements of $\langle v_s^2\rangle$ and R along with the gravitational constant and found the total mass of the Coma galaxy cluster to be $4.5 \times 10^{13} M_\odot$, which was a conservative lower limit of the total mass of the Coma cluster (Zwicky 1937). He further estimated there to be about a thousand galaxies within the cluster. So, the average mass of one galaxy within the Coma cluster came out to be $4.5 \times 10^{10} M_\odot$. He estimated that the average luminosity of one galaxy was equal to $8.5 \times 10^7 L_\odot$. Therefore, the mass to luminosity ratio was on the order of 500 $\frac{M_\odot}{L_\odot}$ (Zwicky 1937).

This was an important measurement during the 1930s for astronomy. It told us that the mass of the luminous sources within the Coma galaxy cluster contributed only a fraction to the total mass. There was an undetectable source that made up most of the galaxy cluster's total mass, and Zwicky named this "dark matter". It took thirty more years for another breakthrough in observing dark matter, and that was done through the measurements of spiral galaxy rotation curves by Roberts (1966) and (Rubin & Ford 1970).

3. M31'S ROTATION CURVE



(a) (b)

Figure 1: (a) Shown is the expected rotational curve of the Andromeda galaxy (the dashed-line), and the measured rotational curve of the Andromeda galaxy (solid line) (Alexeev 2012). (b) Shown is the Rubin and Ford measured rotation curve of the Andromeda galaxy (Rubin & Ford 1970). This plot shows that the rotation velocity does not fall by a factor of $R^{-1/2}$, which means that there is a significant amount of non-luminous matter in the Andromeda galaxy.

Following Zwicky's observations, other astronomers worked on M31's galaxy rotation curves. Horace Babcock was the first astronomer to publish a paper on measuring the rotation curve of the Andromeda galaxy (Babcock 1939). He determined the velocity dispersions of M31 from the absorption lines in the galaxy's nucleus, and the emission lines in the galaxy's nebulae (Babcock 1939). These velocities were found at different distances away from the center of the galaxy and allowed him to produce a rotation curve. Babcock's measurements led to a mass-to-light ratio of 62 $\frac{M_\odot}{L_\odot}$ within 80 arc-seconds from the galaxy's nucleus (Babcock 1939). His rotation curve measurement was smaller than Zwicky's; however, this was from him only measuring a galaxy rather than a galaxy cluster. His measurement still showed that there was a large amount of undetectable matter that made up most of the galaxy's mass.

To tighten the mass-to-light ratio measurement, astronomers came up with two different methods in observing M31's rotation curve. This first method, done by Roberts (1966) in the 1960s, used the 21-cm hydrogen line to derive the rotational curve of the galaxy. Using this derived rotational curve, Roberts obtained physical properties of M31; however, the important one for this report is the total mass of the galaxy. He measured a total mass of $31 \pm 5.0 \times 10^{10} M_{\odot}$ (Roberts 1966).

Roberts found that the rotational velocity at different points along M31 stayed constant after a certain distance away from the center of the galaxy. The rotational velocity should have decreased after a certain distance away from M31's center. This decrease was from the centripetal force equaling the gravitational force of the galaxy which can be seen in the equation

$$\frac{v^2}{R} = \frac{GM(R)}{R^2}, \tag{4}$$

where v is the rotational velocity, R is the distance away from the center of the galaxy, $M(R)$ is the mass of the galaxy, and G is the gravitational constant, e.g. (Ryden 2017). Therefore, the rotational velocity should be proportional to $\frac{1}{\sqrt{R}}$ because the galaxy's mass should be constant at large distances away from the center of the galaxy since most of the luminous sources are near the center. Fig. 1 (a) shows the expected curve of the Andromeda galaxy as the dashed line labelled "A." Roberts' constant rotational velocity at large distances away from the galaxy center meant that there was still significant levels of mass that was not from luminous sources. Vera Rubin and Kent Ford used the other method of measuring M31's rotation curve.

In 1970, Rubin and Ford published a paper that confirmed the tightened measurements done by Roberts. They measured the spectra of over sixty different H_{II} regions across M31 (Rubin & Ford 1970). Using the spectrum points, they measured rotational velocities of the galaxy at different points away from the center of it (Rubin & Ford 1970). Fig. 1 (b) shows their results of their rotation curve. With these velocities, they found the mass-to-light ratio of the Andromeda galaxy. They found that the best value of the galaxy's mass within a radius of 9 kpc is $12.7 \pm 0.50 \times 10^{10} M_\odot$ with a luminosity of $0.99 \times 10^{10} L_\odot$ (Rubin & Ford 1970). Thus they found a mass-to-light ratio of around $13 \pm 0.5 \frac{M_\odot}{L_\odot}$ by dividing the total luminosity from the total mass.

Using Rubin and Ford's luminosity, the mass-to-light ratio from Roberts' mass measurement was around $24 \pm 5.0 \times 10^{10} \frac{M_\odot}{L_\odot}$. These measurements all suggest that there is a significant amount of mass that is not from luminous sources within M31. This breakthrough started to garner the mainstream astronomy community's attention regarding the existence of dark matter.

4. MASS OF GALAXY CLUSTERS USING X-RAY EMISSIONS

About a decade after Rubin and Ford's rotational curve measurements, some astronomers started to measure the mass of galaxy clusters using x-ray emissions of galaxy clusters. After the launching of the Einstein Observatory in the 1980s, x-ray emissions were easier to detect and led to more accurate results of the mass of galaxy clusters (Fabricant & Gorenstein 1983). To find the mass of a galaxy cluster, they assumed that the gas in the cluster was in hydrostatic equilibrium and that it was approximately symmetrical spherically (Fabricant & Gorenstein 1983; Fabricant et al. 1984; Henriksen & Mushotzky 1986). The gas within the cluster was in hydrostatic equilibrium because the gas must be gravitationally bound since there were no massive outflows of gas from the galaxy (Fabricant & Gorenstein 1983).

Since the cluster's gas was in hydrostatic equilibrium, the mass can be found by using the equation

$$\frac{dP_{gas}}{dr} = -\frac{GM(r)\rho_{gas}(r)}{r^2}, \tag{5}$$

where P_{gas} is the pressure of the gas, $M(r)$ is the total mass within a sphere of radius r, and ρ_{gas} is the gas' density (Fabricant et al. 1984). Assuming that the gas within the cluster is an ideal gas, the pressure of the gas is

$$P_{gas} = \frac{\rho_{gas} k T_{gas}}{\mu M_p}, \tag{6}$$

where k is the Boltzmann constant, T_{gas} is the temperature of the gas, M_p is the proton mass and μ is the mean mass per gas particle (Fabricant et al. 1984). Combining Equations 5 and 6, the mass of the cluster within radius r is

$$M(r) = \frac{-kT_{gas}(r)}{G\mu M_p}\left[\frac{dln(\rho_{gas})}{dln(r)} + \frac{dln(T_{gas})}{dln(r)}\right]r, \tag{7}$$

where G is the gravitational constant (Fabricant & Gorenstein 1983).

For this paper, I will look at Henriksen & Mushotzky (1986)'s mass results of the Coma galaxy cluster to compare with Zwicky's results. To find the mass of the galaxy cluster, the two main variables that needed to be found was the gas' temperature and gas' density. The gas' temperature was found by making a best-fit to the isothermal bremsstrahlung radiation, which came out to be 7.6 ± 0.3 keV (Henriksen & Mushotzky 1986). This was in units of energy because the measurement was the Boltzmann constant multiplied by the temperature. The gas density was found by modeling the Coma galaxy cluster's image data with an isothermal structure model (Henriksen & Mushotzky 1986). Using the gas temperature and density, they found a total mass of $8.1 \times 10^{14} M_\odot$. This mass value was significantly higher than Zwicky's mass measurement because Henriksen & Mushotzky (1986) measured the mass within 3 Mpc, while Zwicky only measured the mass within about 20 kpc. The measured luminosity was $1 \times 10^{12} L_\odot$. Then, the measured mass-to-light ratio is $810 \frac{M_\odot}{L_\odot}$. Two different ways of measuring the mass of the Coma galaxy cluster yielded consistent values, furthering the evidence for dark matter's existence.

5. GRAVITATIONAL LENSING

So far, we have discussed methods that detect matter by measuring the gravitational effects on observable matter; however, a new method came out in the 1990s that allowed observers to directly probe dark matter. The method was finding the mass of galaxy clusters by observing the weak gravitational lensing of background galaxies by the galaxy cluster.

Gravitational lensing occurs when a massive object, such as a galaxy cluster, bends incoming light rays from background galaxies, distorting their shapes. These images of the sources can be distorted and magnified, and these effects can be utilized to determine the total mass, including both baryonic and dark matter, of the galaxy cluster. The angle of the bent light, also called the angle of deflection, can be found by equation

$$\alpha = \frac{4GM}{c^2\xi}, \tag{8}$$

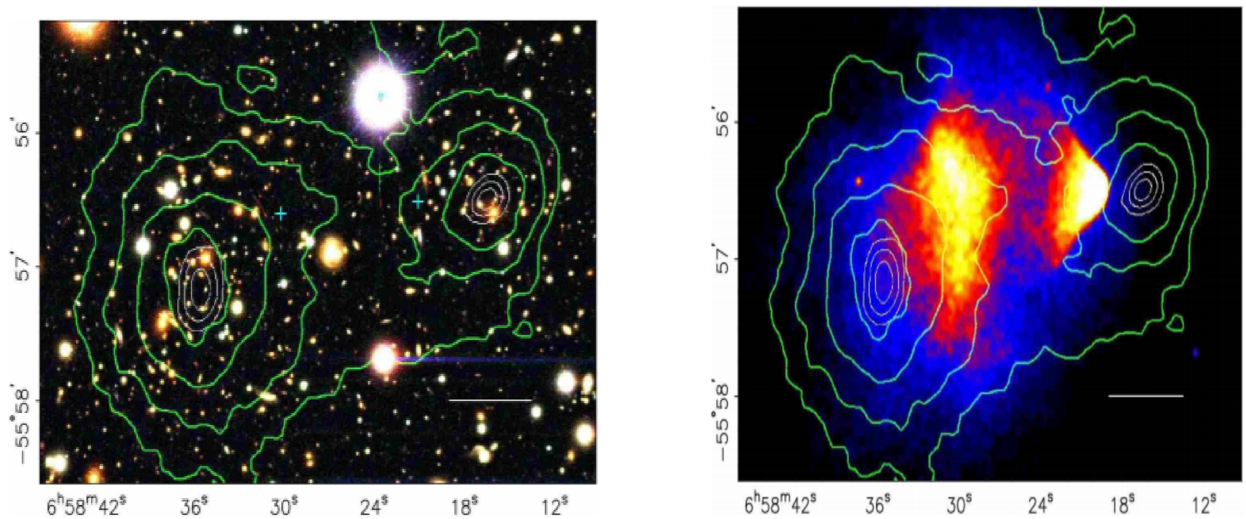
where G is the gravitational constant, M is the mass of the massive compact object, α is the angle of deflection, c is the speed of light, and ξ is the impact parameter (Bartelmann & Schneider 2001).

In this paper, I'll look into Squires et al. (1996)'s determination of the mass-to-light ratio of galaxy cluster Abell 2390. They took CCD images of the galaxy cluster on June 6-9, 1994 and they reduced the data from these images using "photometric standards" from globular clusters (Squires et al. 1996). Using these images, they determined the surface mass density in the cluster, which was found by using equation

$$\zeta(\theta_1, \theta_2) = 2\left(1 - \frac{\theta_1^2}{\theta_2^2}\right)^{-1} \int_{\theta_1}^{\theta_2} dln(\theta)\langle\gamma_t\rangle, \tag{9}$$

where ζ is the mean dimensionless surface density corresponding to deflection angles of θ_1 and θ_2, and γ_t is the constant shear (Squires et al. 1996). To convert the dimensionless surface density to an actual quantity, they found the critical surface density of the galaxy cluster. Using this density, along with the galaxy cluster's light and mass profiles, they found a mass-to-light ratio of $320 \pm 100h\frac{M_\odot}{L_\odot}$ in Abell 2390, where h is the radial distance of the cluster (Squires et al. 1996). They compared this to Carlberg et al. (1996)'s mass-to-light ratio of the same galaxy cluster; however, they reached their ratio by using the virial theorem. The virial theorem's mass-to-light ratio of Abel 2390 was $370 \pm 60h\frac{M_\odot}{L_\odot}$ in the visual magnitude (Carlberg et al. 1996). Therefore, it can be said that two different measurements yielded consistent results, where they found that there is a significant amount of non luminous matter within the measured galaxy cluster.

A significant measurement done using weak gravitational lensing that also supports the existence of dark matter is the study of interacting galaxy clusters, specifically the interacting cluster 1E0657-558. This cluster, also known as the Bullet Cluster, has a main cluster and a sub-cluster that have passed through each other over a million years ago. During the collision between the two clusters,



(a) (b)

Figure 2: (a) Shown above is an image of the merging galaxy cluster 1E0657-558. (b) Shown above is an x-ray image of the same merging galaxy cluster. The green contour lines represent the surface mass density map, while the heat map indicates where the plasma clouds are concentrated. One can see that there are two distinct peaks of the surface mass density at the center of the main-cluster and sub-cluster. The x-ray emitting plasma contributes ten times more than the galaxies to the baryonic matter component. Most of the x-ray data is concentrated between the two peaks, as seen in figure (b), which indicates that it does not have a significant effect on the mass of the two clusters (Clowe et al. 2006).

the x-ray emitting plasma within the clusters collided while the galaxies did not (Clowe et al. 2006). After discovering the interacting cluster, astronomers mapped out the surface mass density with weak gravitational lensing to find where the mass is concentrated. Since the x-ray emitting plasma contributes ten times more to the visible matter component than the galaxies, then one would expect the mass to be concentrated in the center between the two clusters and coincident with the plasma (Clowe et al. 2006). Shown in Fig. 2 is the map of the Bullet Cluster's surface mass density. There are two distinct peaks where the mass of the two clusters are concentrated (the smallest green contours). This indicates that the mass is concentrated near the center of the main-cluster and sub-cluster instead of between the two clusters where the plasma is. If there was only baryonic matter in the Universe, then the mass should be concentrated where the plasma is; however, that is not the case with the Bullet Cluster (Clowe et al. 2006). Since the baryonic mass is concentrated near the center of the two clusters, this means that a significant amount of non-luminous matter makes up most of the mass of the two clusters, which supports the existence of dark matter.

6. POWER SPECTRUM OF COSMIC MICROWAVE BACKGROUND

While astronomers accepted that the previously discussed observational evidence for dark matter proved that dark matter existed, there were a few theories that rejected the notion of dark matter. MOND was one of the main theories that tried explain rotation curve data with only baryons (Milgrom 1983). MOND stated that there was only ordinary matter in the universe, where the gravitational force between two objects at small distances obeyed Newton's law of universal gravitation

$$F = \frac{GMm}{r^2}; \tag{10}$$

however, at very large distances, the gravitational force was proportional to r^{-3} rather than r^{-2} (Milgrom 1983).

This new addition to Newton's law of universal gravitation explained the rotational curve data from Babcock, Roberts, and Rubin and Ford, which agreed with the other observational evidence provided before. However, crucial data came out in the early 21^{st} century that disfavored the MOND theory. The crucial data came in the form of the power spectrum of CMB's temperature fluctuations.

The CMB is the decoupled photons from very early on in the universe that have remained unperturbed from other sources before we observed it. Right before the photons decoupled, they were undergoing oscillations, which could be seen in the CMB today. Peaks in these oscillations offered cosmologists vital information on cosmological parameters such as the universe's flatness, the amount of baryonic matter, and the amount of dark matter (Hinshaw et al. 2013). Fig. 3 shows WMAP's data of the temperature fluctuations within the CMB. The first peak represents the curvature of the universe. Since it was at 1^o, this meant that the universe is flat. If the universe is negatively

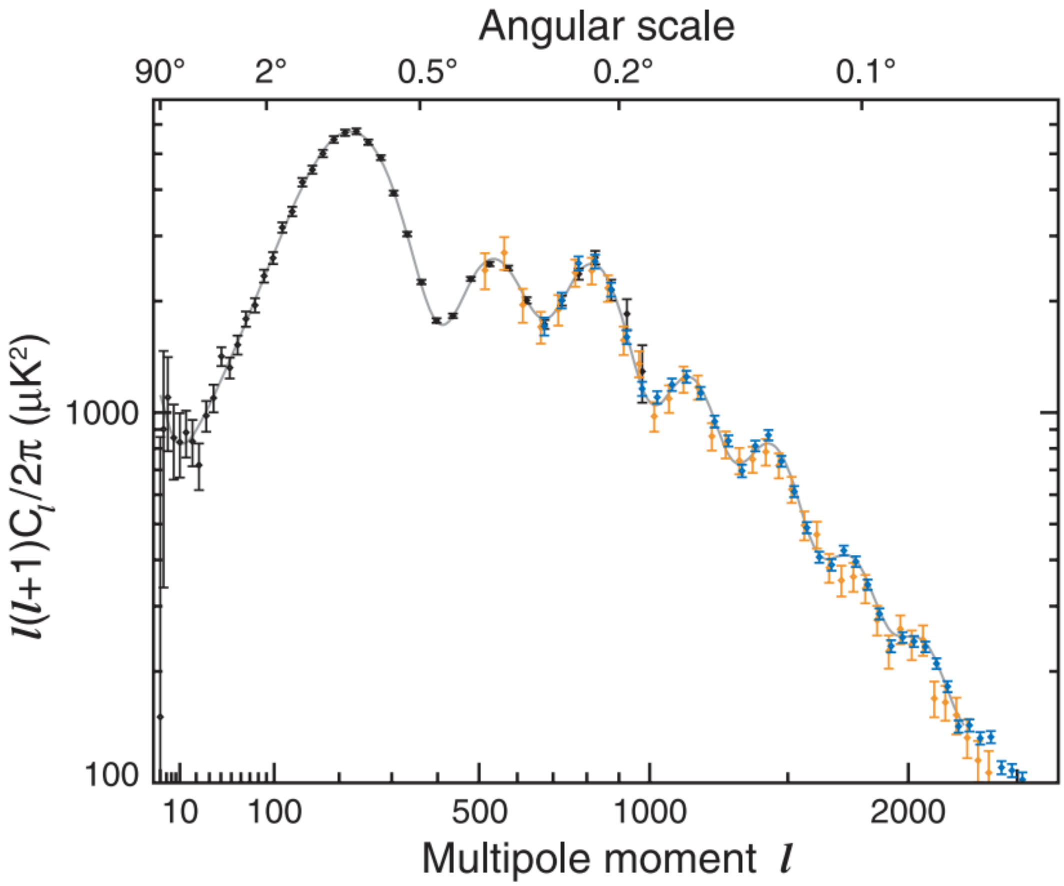


Figure 3: This plot shows the temperature fluctuations of the CMB as a function of multipole moments and angular scale from the nine year WMAP observation program (Hinshaw et al. 2013). WMAP data are shown in black, while the blue and orange markers are from an extended CMB data set (Hinshaw et al. 2013). The gray line is a model that successfully fits the data from WMAP. Notice how the amplitude of the third peak is about the same as the amplitude of the second peak. The closeness in amplitude suggest that there is a significant amount of dark matter in the Universe.

curved, the first peak in the temperature fluctuations would be shifted towards a larger angle and if the universe is positively curved, the first peak would be shifted towards a smaller angle (Hu et al. 1998).

The second and third peaks of the temperature fluctuations provided significant information that favored the theory of dark matter. Fig. 4 shows MOND's prediction of the CMB fluctuations and it accurately describes the first and second peak measured from WMAP's results (Hinshaw et al.

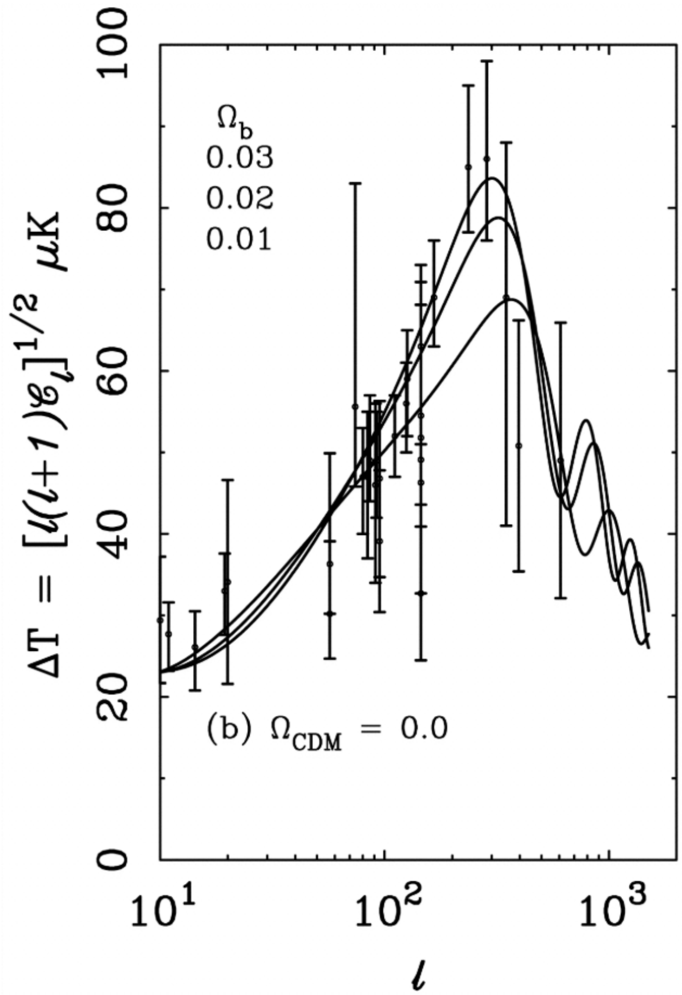


Figure 4: This plot shows the predicted temperature fluctuations of the CMB as a function of multipole moments from a simulation program that only had baryonic matter instead of dark matter. The three different lines correspond to different amounts of baryonic matter (McGaugh 1999). Notice how the amplitude of the third peak is always significantly less than the second peak in the MOND theory.

2013). The discrepancy between the MOND theory and the dark matter theory comes from the the third peak of the CMB. According to McGaugh (1999), the third peak should be much lower than the second peak if the universe was filled with only baryonic matter; however, if the universe had dark matter, then the third peak would be close to the height of the second peak. Hinshaw et al. (2013)'s nine year results shows that the third peak of the CMB temperature fluctuations is higher than the second peak. Therefore, it can be said that the MOND theory does not fit the observed CMB fluctuation data as well as the theory of dark matter. Using WMAP's data, the amplitude of the second peak shows that baryonic matter is only around 15% of total matter, while matching all the rest of the peaks shows that dark matter makes up the rest of the matter in the Universe. These cosmological defining results provided the information to solely put the theory of dark matter as the leading theory of the composition of the Universe.

7. CONCLUSION

Astronomers showed the first evidence for the existence of dark matter with the pioneering Coma galaxy cluster measurements and rotational curve measurements of the Andromeda galaxy done in the 1930s by Zwicky and Babcock respectively. The rotation curve measurements done by Babcock were tightened by other astronomers such as Roberts, Rubin, and Ford. X-ray emission analysis and weak gravitational lensing done by various other astronomers confirmed the rotational curve measurements. All of these measurements supported the theory that there is a form of non-luminous matter that dominates the total mass of matter in the Universe. One of the most significant pieces of observational evidence for dark matter comes from WMAP's results of the CMB power spectrum, which supports the theory of dark matter and disfavors the MOND theory. The existence of dark matter shows that humanity knows only a fraction of what the Universe is made of. No doubt understanding what dark matter is will someday shed more light on how the Universe works.

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