AMATH 581 Homework #2

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1. Time evolution of the second mode of the probability density function: $\psi_2(x,t)$

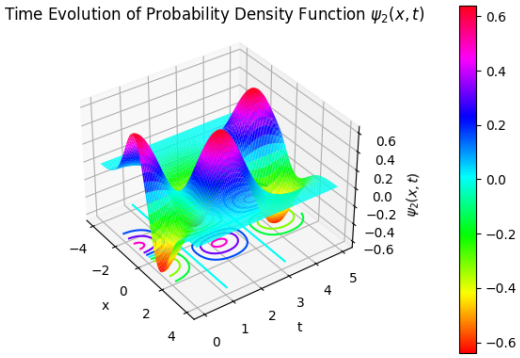

Figure 1: Time evolution of $\psi_2(x,t)$ in $x \in [-4,4], t \in [0,5]$ with contours projected below surface

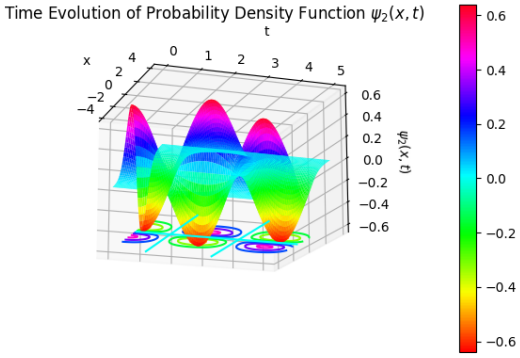

Figure 2: The same time evolution shown from below ($-\psi_2$ direction)

Code:

```
def rhsfunc1(t, y, eps):
    f1 = y[1]
    f2 = (t**2 - eps)*y[0]
    return np.array([f1, f2])

xp = [-4, 4]
tol = 1e-6
A = 1

x_evals = np.linspace(-4, 4, 81)
eps_start = 0
eigenvalues1 = np.zeros(5)
eigenfunctions1 = np.zeros([81, 5])
phi = np.zeros([81, 5])
for mode in range(5):
    eps = eps_start
    deps = 1

    for i in range(1000):
        y0 = np.array([A, A*np.sqrt(4**2 - eps)])
        sol = scipy.integrate.solve_ivp(lambda x, y: rhsfunc1(x, y, eps), xp, y0, t_e

        if (np.abs(sol.y[1, -1] + np.sqrt(4**2 - eps)*sol.y[0, -1])) < tol:
            eigenfunctions1[:, mode] = sol.y[0, :]
            eigenvalues1[mode] = eps
            break

        if (-1)**(mode)*(sol.y[1, -1] + np.sqrt(4**2 - eps)*sol.y[0, -1]) > 0:
            eps = eps + deps
        else:
            eps = eps - deps/2
```

```
            deps = deps/2

    eps_start = eps + 0.1
    eig_norm = np.trapz(eigenfunctions1[:, mode]**2, x = x_evals)
    eigenfunctions1[:, mode] = eigenfunctions1[:, mode]/np.sqrt(eig_norm)

    plt.plot(sol.t, eigenfunctions1[:, mode], linewidth=2)
    plt.plot(sol.t, 0*sol.t, 'k')

    phi[:, mode] = eigenfunctions1[:, mode]
    eigenfunctions1[:, mode] = abs(eigenfunctions1[:, mode])

A1 = eigenfunctions1[:, 0].reshape(-1, 1)
A2 = eigenfunctions1[:, 1].reshape(-1, 1)
A3 = eigenfunctions1[:, 2].reshape(-1, 1)
A4 = eigenfunctions1[:, 3].reshape(-1, 1)
A5 = eigenfunctions1[:, 4].reshape(-1, 1)
A6 = eigenvalues1.reshape(1, -1)

## 3D Plot

x = np.linspace(-4, 4, 81)
t = np.linspace(0, 5, 100)

fig = plt.figure()
ax = plt.axes(projection = '3d')
density = (phi[:, 1].reshape(-1, 1) * np.cos(eigenvalues1[1]*t/2)).T

X, T = np.meshgrid(x, t)
surf = ax.plot_surface(X, T, density, cmap = cm.hsv, rstride=1, cstride=1)
fig.colorbar(surf, pad = 0.2)
ax.contour(X, T, density, cmap = cm.hsv, offset = -0.6)
```

```
ax.set_xlabel('x')
ax.set_ylabel('t')
ax.set_zlabel(r'$\psi_2(x, t)$')
plt.title(r'Time Evolution of Probability Density Function $\psi_2(x, t)$')
```