1.1

$$\det\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = a \bullet \det\begin{bmatrix} e & f \\ h & i \end{bmatrix} - b \bullet \det\begin{bmatrix} d & f \\ g & i \end{bmatrix} + c \bullet \det\begin{bmatrix} d & e \\ g & h \end{bmatrix}$$

1.2 The determiant is multipled by negitive 1 when rows are interchanged / swapped.

Infinite solutions are when two conditions are met
2.1. There is at least one free variable.
2.2. A solution exists (ie. rows cannot equal an impossibility such as 2 = 0 or 0 = 2)

The determiant of a upper or lower triangular matrix is the multiplication of the diagonals.
If A is nonsinuglar then A^-1 = A, therefore if A^-1 exists the matrix is singular. (If Determinant is 0, then it is singular)

$A = \begin{bmatrix} 2 & 5 \\ 6 & 3 \end{bmatrix}$

Find A^{-1} (Inverse)

$(A|I) = \left[\begin{array}{cc|cc} 2 & 5 & 1 & 0 \\ 6 & 3 & 0 & 1 \end{array}\right]$

$= \left[\begin{array}{cc|cc} 2 & 5 & 1 & 0 \\ 6 & 3 & 0 & 1 \end{array}\right] \xrightarrow{R2=R2-R1(3)} \left[\begin{array}{cc|cc} 2 & 5 & 1 & 0 \\ 0 & -12 & -3 & 1 \end{array}\right] \xrightarrow{R1=R1\cdot\frac{1}{2}} \left[\begin{array}{cc|cc} 1 & \frac{5}{2} & \frac{1}{2} & 0 \\ 0 & -12 & -3 & 1 \end{array}\right]$

$\xrightarrow{R2=R2\cdot-\frac{1}{12}} \left[\begin{array}{cc|cc} 1 & \frac{5}{2} & \frac{1}{2} & 0 \\ 0 & 1 & \frac{1}{4} & -\frac{1}{12} \end{array}\right] \xrightarrow{R1=R1-R2(\frac{5}{2})} \left[\begin{array}{cc|cc} 1 & 0 & -\frac{1}{8} & \frac{5}{24} \\ 0 & 1 & \frac{1}{4} & -\frac{1}{12} \end{array}\right]$

$A^{-1} = \begin{bmatrix} -\frac{1}{8} & \frac{5}{24} \\ \frac{1}{4} & -\frac{1}{12} \end{bmatrix}$

1 0 0 5
0 1 0 6
0 0 1 5

has 3 unique solutions 5, 6, 5

1 0 0 5
0 1 0 6
0 0 0 0

has 2 unique solutions 5, 6

1 0 1 5
0 1 0 6
0 0 0 0

has infinite solutions because x3 is free.

1 0 0 5
0 1 0 6
0 0 0 5

has no solutions because 0 = 5

Solve

$\begin{cases} x_1 + x_2 + 3x_3 = 7 \\ 4x_1 + 3x_2 + 4x_3 = -5 \end{cases}$

In terms of s,t. Write free & lead vars

$\left[\begin{array}{ccc|c} 1 & 1 & 3 & 7 \\ 4 & 3 & 4 & -5 \end{array}\right] \xrightarrow{R2=R2-R1(4)} \left[\begin{array}{ccc|c} 1 & 1 & 3 & 7 \\ 0 & -1 & -8 & -33 \end{array}\right]$

$\xrightarrow{R2=R2\cdot-1} \left[\begin{array}{ccc|c} 1 & 1 & 3 & 7 \\ 0 & 1 & 8 & 33 \end{array}\right] \xrightarrow{R1=R1-R2} \left[\begin{array}{ccc|c} 1 & 0 & -5 & -26 \\ 0 & 1 & 8 & 33 \end{array}\right]$

Lead: X1, X2

Free: X3

X1 - 5x3 = -26
x1 = -26 + 5x3
Let x3 = s
x1 = -26 + 5s

x2 + 8x3 = 33
x2 = 33 - 8x3
Let x3 = s
x2 = 33 - 8s

$\begin{bmatrix} -26 \\ 33 \\ 0 \end{bmatrix} + \begin{bmatrix} 5 \\ -8 \\ 1 \end{bmatrix} s = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

Use Gaussian Elimination, List Lead & Free

$A = \begin{bmatrix} 2 & 6 & 6 \\ 2 & 7 & 8 \\ 1 & 4 & 5 \end{bmatrix}$, How many Solutions exist, and why

$\left[\begin{array}{cc|c} 2 & 6 & 6 \\ 2 & 7 & 8 \\ 1 & 4 & 5 \end{array}\right] \xrightarrow{R1=R1\cdot\frac{1}{2}} \left[\begin{array}{cc|c} 1 & 3 & 3 \\ 2 & 7 & 8 \\ 1 & 4 & 5 \end{array}\right] \xrightarrow{R2=R2-R1(2)} \left[\begin{array}{cc|c} 1 & 3 & 3 \\ 0 & 1 & 2 \\ 1 & 4 & 5 \end{array}\right]$

$\xrightarrow{R3=R3-R1} \left[\begin{array}{cc|c} 1 & 3 & 3 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{array}\right] \xrightarrow{R3=R3-R2} \left[\begin{array}{cc|c} 1 & 3 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{array}\right] \xrightarrow{R1=R1-R2(3)} \left[\begin{array}{cc|c} 1 & 0 & -3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{array}\right]$

Lead: x_1, x2

Free: None

Contains exactly 2 solutions
x1 = -3
x2 = 2
Zeros don't matter as long as the matrix can be reduced with 1's leading each row ^^

Write as a augmented matrix

$2x_1 - 10x_2 + 22x_3 = 69$
$4x_1 + 20x_2 - 6x_3 = 420$
$10x_1 - 6x_2 - 12x_3 = 1337$

$\left[\begin{array}{ccc|c} 2 & -10 & 22 & 69 \\ 4 & 20 & -6 & 420 \\ 10 & -6 & -12 & 1337 \end{array}\right]$

Use Gaus elimination to Solve.

$x_1 + 2x_2 - x_3 + x_4 = 0$
$-x_1 - 2x_2 + 3x_3 + 5x_4 = 0$
$-x_1 - 2x_2 - x_3 - 7x_4 = 0$

$\longrightarrow \left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 0 \\ -1 & -2 & 3 & 5 & 0 \\ -1 & -2 & -1 & -7 & 0 \end{array}\right]$

$\xrightarrow{R2=R2+R1} \left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 0 \\ 0 & 0 & 2 & 6 & 0 \\ -1 & -2 & -1 & -7 & 0 \end{array}\right] \xrightarrow{R3=R3+R1} \left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 0 \\ 0 & 0 & 2 & 6 & 0 \\ 0 & 0 & -2 & -6 & 0 \end{array}\right]$

$\xrightarrow{R2=R2+R3} \left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2 & -6 & 0 \end{array}\right] \xrightarrow{R2 \leftrightarrow R3} \left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 0 \\ 0 & 0 & -2 & -6 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$

$\xrightarrow{R2=R2\cdot-\frac{1}{2}} \left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 0 \\ 0 & 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow{R1=R1+R2} \left[\begin{array}{cccc|c} 1 & 2 & 0 & 4 & 0 \\ 0 & 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$

lead: x1, X3
Free: X2, x4

X1 + 2X2 + 4X4 = 0
X1 = -2X2 - 4X4
X1 = -2s - 4t

x3 + 3x4 = 0
X3 = -3x4
X3 = -3t

0 = 0

Let x2 = s
Let x4 = t

$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} s + \begin{bmatrix} -4 \\ 0 \\ -3 \\ 1 \end{bmatrix} t$

2x2

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} = \begin{bmatrix} (1)\cdot(5)+(2)\cdot(7) & (1)\cdot(6)+(2)\cdot(8) \\ (3)\cdot(5)+(4)\cdot(7) & (3)\cdot(6)+(4)\cdot(8) \end{bmatrix} = \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix}$$

3x3

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \cdot \begin{bmatrix} 10 & 11 & 12 \\ 13 & 14 & 15 \\ 16 & 17 & 18 \end{bmatrix} =$$

$$\begin{bmatrix} (1)\cdot(10)+(2)\cdot(13)+(3)\cdot(16) & (1)\cdot(11)+(2)\cdot(14)+(3)\cdot(17) & (1)\cdot(12)+(2)\cdot(15)+(3)\cdot(18) \\ (4)\cdot(10)+(5)\cdot(13)+(6)\cdot(16) & (4)\cdot(11)+(5)\cdot(14)+(6)\cdot(17) & (4)\cdot(12)+(5)\cdot(15)+(6)\cdot(18) \\ (7)\cdot(10)+(8)\cdot(13)+(9)\cdot(16) & (7)\cdot(11)+(8)\cdot(14)+(9)\cdot(17) & (7)\cdot(12)+(8)\cdot(15)+(9)\cdot(18) \end{bmatrix}$$

$$\begin{bmatrix} 84 & 90 & 96 \\ 201 & 216 & 231 \\ 318 & 342 & 366 \end{bmatrix}$$

3x3, 3x1

$$\begin{bmatrix} (1)\cdot(12)+(2)\cdot(15)+(3)\cdot(18) \\ (4)\cdot(12)+(5)\cdot(15)+(6)\cdot(18) \\ (7)\cdot(12)+(8)\cdot(15)+(9)\cdot(18) \end{bmatrix}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

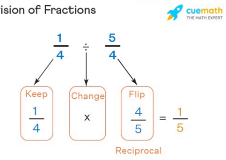
13.

$\frac{1}{2} + \frac{1}{3} = ?$

$\frac{1 \times 3}{2 \times 3} = \frac{3}{6}$ $\quad \frac{1 \times 2}{3 \times 2} = \frac{2}{6}$

$\frac{3}{6} + \frac{2}{6} = \frac{5}{6}$

Division of Fractions



Multiplication is just stright across.

14.
-1 - -1 = 0
-1 + -1 = -2
-1 * -1 = 1
1 * -1 = -1
-1 * 1 = 1
-2/-2=1
1/-2 = -1/2

$$A=\begin{bmatrix}-3&3&2\\-3&4&3\\-9&-10&-8\end{bmatrix} \qquad U=E_3E_2E_1\cdot A \qquad L=(E_3E_2E_1)^{-1} \qquad A=L\cdot U \qquad b=\begin{bmatrix}-7\\-5\\15\end{bmatrix}$$

a) Find L & U such that LU=A

b) Solve Ax=b using LU factorization

$R2=R2-R1$: $E_1=\begin{bmatrix}1&0&0\\-1&1&0\\0&0&1\end{bmatrix}$, $E_1A=\begin{bmatrix}-3&3&2\\0&1&1\\-9&-10&-8\end{bmatrix}$

$R3=R3+-3R1$: $E_2=\begin{bmatrix}1&0&0\\0&1&0\\-3&0&1\end{bmatrix}$, $E_2E_1A=\begin{bmatrix}-3&3&2\\0&1&1\\0&-19&-14\end{bmatrix}$

$R3=R3+19R2$: $E_3=\begin{bmatrix}1&0&0\\0&1&0\\0&19&1\end{bmatrix}$, $E_3E_2E_1A=\begin{bmatrix}-3&3&2\\0&1&1\\0&0&5\end{bmatrix}=U$

$$L=(E_3E_2E_1)^{-1}=\left[\begin{bmatrix}1&0&0\\0&1&0\\0&19&1\end{bmatrix}\begin{bmatrix}1&0&0\\0&1&0\\-3&0&1\end{bmatrix}\begin{bmatrix}1&0&0\\-1&1&0\\0&0&1\end{bmatrix}\right]^{-1}=\begin{bmatrix}1&0&0\\1&1&0\\3&-19&1\end{bmatrix}$$

$$E_3E_2=\begin{bmatrix}1&0&0\\0&1&0\\0&19&1\end{bmatrix}\begin{bmatrix}1&0&0\\0&1&0\\-3&0&1\end{bmatrix}=\begin{bmatrix}1\cdot1+0\cdot0+0\cdot3&1\cdot0+0\cdot1+0\cdot0&1\cdot0+0\cdot0+0\cdot1\\0\cdot1+1\cdot0+0\cdot-3&0\cdot0+1\cdot1+0\cdot0&0\cdot0+1\cdot0+0\cdot1\\0\cdot1+19\cdot0+1\cdot-3&0\cdot0+19\cdot1+1\cdot0&0\cdot0+19\cdot0+1\cdot1\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&0\\-3&19&1\end{bmatrix}$$

$$E_3E_2E_1=\begin{bmatrix}1&0&0\\0&1&0\\-3&19&1\end{bmatrix}\begin{bmatrix}1&0&0\\-1&1&0\\0&0&1\end{bmatrix}=\begin{bmatrix}1\cdot1+0\cdot-1+0\cdot0&1\cdot0+0\cdot1+0\cdot0&1\cdot0+0\cdot0+0\cdot1\\0\cdot1+1\cdot-1+0\cdot0&0\cdot0+1\cdot1+0\cdot0&0\cdot0+1\cdot0+0\cdot1\\-3\cdot1+19\cdot-1+1\cdot0&-3\cdot0+19\cdot1+1\cdot0&-3\cdot0+19\cdot0+1\cdot1\end{bmatrix}=\begin{bmatrix}1&0&0\\-1&1&0\\-22&19&1\end{bmatrix}=L^{-1}$$

$(E_3E_2E_1)^{-1}\Rightarrow(E_3E_2E_1\,|\,I)$

$$\Rightarrow\left[\begin{array}{ccc|ccc}1&0&0&1&0&0\\-1&1&0&0&1&0\\-22&19&1&0&0&1\end{array}\right] \xrightarrow{R_2=R_2+R_1} \left[\begin{array}{ccc|ccc}1&0&0&1&0&0\\0&1&0&1&1&0\\-22&19&1&0&0&1\end{array}\right]$$

$$\xrightarrow{R_3=R_3+22R_1}\left[\begin{array}{ccc|ccc}1&0&0&1&0&0\\0&1&0&1&1&0\\0&19&1&22&0&1\end{array}\right]\xrightarrow{R_3=R_3-19R_2}\left[\begin{array}{ccc|ccc}1&0&0&1&0&0\\0&1&0&1&1&0\\0&0&1&3&-19&1\end{array}\right]\ L$$

$U^{-1}=\text{rref}(U|I)$

$$\Rightarrow\left[\begin{array}{ccc|ccc}-3&3&2&1&0&0\\0&1&1&0&1&0\\0&0&5&0&0&1\end{array}\right]\xrightarrow{R_3=R_3\cdot\frac{1}{5}}\left[\begin{array}{ccc|ccc}-3&3&2&1&0&0\\0&1&1&0&1&0\\0&0&1&0&0&\frac15\end{array}\right]\xrightarrow{R_1=R_1\cdot-\frac13}\left[\begin{array}{ccc|ccc}1&-1&-\frac23&-\frac13&0&0\\0&1&1&0&1&0\\0&0&1&0&0&\frac15\end{array}\right]$$

$$\xrightarrow{R_1=R_1+R_2}\left[\begin{array}{ccc|ccc}1&0&\frac13&-\frac13&1&0\\0&1&1&0&1&0\\0&0&1&0&0&\frac15\end{array}\right]\xrightarrow{R_1=R_1-\frac13R_3}\left[\begin{array}{ccc|ccc}1&0&0&-\frac13&1&-\frac{1}{15}\\0&1&1&0&1&0\\0&0&1&0&0&\frac15\end{array}\right]\xrightarrow{R_2=R_2-R_3}\left[\begin{array}{ccc|ccc}1&0&0&-\frac13&1&-\frac1{15}\\0&1&0&0&1&-\frac15\\0&0&1&0&0&\frac15\end{array}\right]=U^{-1}$$

$Ly=b$

$y=L^{-1}\cdot b$

$$\begin{bmatrix}y_1\\y_2\\y_3\end{bmatrix}=\begin{bmatrix}1&0&0\\-1&1&0\\-22&19&1\end{bmatrix}\begin{bmatrix}-7\\-5\\15\end{bmatrix}=\begin{bmatrix}1\cdot-7+0\cdot-5+0\cdot15\\-1\cdot-7+1\cdot-5+0\cdot15\\-22\cdot-7+19\cdot-5+1\cdot15\end{bmatrix}=\begin{bmatrix}-7\\2\\74\end{bmatrix}$$

$Ux=y$

$x=U^{-1}\cdot y$

$$=\begin{bmatrix}-\frac13&1&-\frac1{15}\\0&1&-\frac15\\0&0&\frac15\end{bmatrix}\begin{bmatrix}-7\\2\\74\end{bmatrix}=\begin{bmatrix}-\frac13\cdot-7+1\cdot2+-\frac1{15}\cdot74\\0\cdot-7+1\cdot2+-\frac15\cdot74\\0\cdot-7+0\cdot2+\frac15\cdot74\end{bmatrix}=\boxed{\begin{bmatrix}-\frac35\\-\frac{64}{5}\\\frac{74}{5}\end{bmatrix}}$$

$$A=\left[\begin{array}{ccc|c}1&2&1&0\\2&5&3&0\\-1&1&Q&0\end{array}\right]$$

Solve for Q such that A will have infinite solutions.

$$=\begin{bmatrix}1&2&1\\2&5&3\\-1&1&Q\end{bmatrix}\xrightarrow{R2=R2-R1(2)}\begin{bmatrix}1&2&1\\0&1&1\\-1&1&Q\end{bmatrix}\xrightarrow{R3=R3+R1}\begin{bmatrix}1&2&1\\0&1&1\\0&3&1+Q\end{bmatrix}\xrightarrow{R3=R3-R2(3)}\left[\begin{array}{ccc|c}1&2&1&0\\0&1&1&0\\0&0&Q-2&0\end{array}\right]$$

$1a+2b+1c=0$

$1b+1c=0$

$Q-2=0$

$\boxed{Q=2}$

Let A be a 3x2 matrix

A column be the column vector which are

$a_1=(-2,4,4)^T$

$a_2=(6,-13,-9)^T$

Let $b=(-12,27,Q)^T$

Find Q such that Ax=b is consistant.

$$\begin{bmatrix}-2&6\\4&-13\\4&-9\end{bmatrix}x=\begin{bmatrix}-12\\27\\Q\end{bmatrix} \qquad A=b$$

$$\left[\begin{array}{cc|c}-2&6&-12\\4&-13&27\\4&-9&Q\end{array}\right]\xrightarrow{R2=R2+R1(2)}\left[\begin{array}{cc|c}-2&6&-12\\0&-1&3\\4&-9&Q\end{array}\right]\xrightarrow{R2=R2\cdot-1}\left[\begin{array}{cc|c}-2&6&-12\\0&1&-3\\4&-9&Q\end{array}\right]\xrightarrow{R3=R3+R1(2)}\left[\begin{array}{cc|c}-2&6&-12\\0&1&-3\\0&3&Q-24\end{array}\right]\xrightarrow{R3=R3-R2(3)}\left[\begin{array}{cc|c}-2&6&-12\\0&1&-3\\0&0&Q-1\end{array}\right]$$

$0=Q-15$

$Q=15$

$$A=\begin{bmatrix}1&1&1\\1&2&t\\1&4&t^2\end{bmatrix}$$

a) use elimination to evaluate D(A) in terms of t

$$\begin{bmatrix}1&1&1\\1&2&t\\1&4&t^2\end{bmatrix}\xrightarrow{R2=R2-R1}\begin{bmatrix}1&1&1\\0&1&t-1\\1&4&t^2\end{bmatrix}\xrightarrow{R3=R3-R1}\begin{bmatrix}1&1&1\\0&1&t-1\\0&3&t^2-1\end{bmatrix}\xrightarrow{R3=R3-R2(3)}\begin{bmatrix}1&1&1\\0&1&t-1\\0&0&(t^2-1)-3(t-1)\end{bmatrix}$$

$1\cdot1\cdot(t^2-1)-3(t-1)$

a. $(t^2-1)-3(t-1)$

$=(t-1)(t+1-3)$

$=(t-1)(t-2)$

b. $t=1$

$t=2$

(I) Interchange two rows

(II) Multiply one row by a nonzero real number

(III) Replace a row by its sum with a multiple of another

$$\left(\begin{array}{cccc|c}1&0&-1&2&2\\0&1&1&1&3\\0&0&0&0&0\end{array}\right)$$

A is $\mathbf{a}_1=\begin{pmatrix}1\\2\\4\end{pmatrix}$ and $\mathbf{b}=\begin{pmatrix}5\\1\\-1\end{pmatrix}$

$$\begin{pmatrix}1&?&?&?&?\\2&?&?&?&?\\4&?&?&?&?\end{pmatrix}$$

x1 - x3 + x4 = 2

x2 + x3 + x4 = 3

Free variables: x3, x4

Basic variables: x1, x2

x3 = -x1 + x2

x4 = 2*(x1) + x2

$2\begin{pmatrix}1\\2\\4\end{pmatrix}+3\begin{pmatrix}a\\b\\c\end{pmatrix}=\begin{pmatrix}5\\1\\-1\end{pmatrix}$

2*1 + 3a = 5

2*2+3*b=1

2*4+3*c=-1

Calculator → a = 1, b = -1, c = -3

x2 = $\begin{pmatrix}1\\-1\\-3\end{pmatrix}\rightarrow\begin{pmatrix}1&1&?&?&?\\2&-1&?&?&?\\4&-3&?&?&?\end{pmatrix}$

x3 = $-\begin{pmatrix}1\\2\\4\end{pmatrix}+\begin{pmatrix}1\\-1\\-3\end{pmatrix}\rightarrow\begin{pmatrix}0\\-3\\-7\end{pmatrix}\rightarrow\begin{pmatrix}1&1&0&?\\2&-1&-3&?\\4&-3&-7&?\end{pmatrix}$

x4 = $2\begin{pmatrix}1\\2\\4\end{pmatrix}+\begin{pmatrix}1\\-1\\-3\end{pmatrix}\rightarrow\begin{pmatrix}3\\3\\5\end{pmatrix}\rightarrow\begin{pmatrix}1&1&0&3\\2&-1&-3&3\\4&-3&-7&5\end{pmatrix}$

$$A=\begin{bmatrix}1&5&0&2\\3&1&0&6\\0&4&1&0\\2&2&0&3\end{bmatrix}$$

Find Det(A)

$$\begin{vmatrix}1&5&0&2\\3&1&0&6\\0&4&1&0\\2&2&0&3\end{vmatrix}=1\begin{vmatrix}1&0&6\\4&1&0\\2&0&3\end{vmatrix}=1\begin{vmatrix}1&0\\0&3\end{vmatrix}+6\begin{vmatrix}4&1\\2&0\end{vmatrix}$$

$=1(3)+6(-2)$

$=3-12$

$=-9$

-9

$$-5\begin{vmatrix}3&0&6\\0&1&0\\2&0&3\end{vmatrix}=3\begin{vmatrix}1&0\\0&3\end{vmatrix}+6\begin{vmatrix}0&1\\2&0\end{vmatrix}$$

$=3(3)+6(-2)$

$=9-12$

$=-3$

$=-5(-3)$

$=15$

$$-2\begin{vmatrix}3&1&0\\0&4&1\\2&2&0\end{vmatrix}=3\begin{vmatrix}4&1\\2&0\end{vmatrix}-1\begin{vmatrix}0&1\\2&0\end{vmatrix}$$

$=3(-2)-1(-2)$

$=-6+2$

$=-4$

$=-2\cdot-4$

$=8$

$-9+15+8$

$\boxed{=14}$

$$A=\begin{bmatrix}-1&-3&-3\\2&6&1\\3&8&3\end{bmatrix},\ A^{-1}=\begin{bmatrix}2&-3&3\\-\frac35&\frac65&-1\\-\frac25&-\frac15&0\end{bmatrix}\leftarrow\text{Found in last Part}$$

$b=(-4,3,6)^T$

a) Is A nonsingular? Yes, because we found the inverse

b) Solve matrix equation Ax=b

$Ax=b$

$x=A^{-1}b$

$$x=\begin{bmatrix}2&-3&3\\-\frac35&\frac65&-1\\-\frac25&-\frac15&0\end{bmatrix}\begin{bmatrix}-4\\3\\6\end{bmatrix}=\begin{bmatrix}-8+-9+18\\\frac{12}{5}+\frac{18}{5}+(-6)\\\frac85+(-\frac35)+0\end{bmatrix}=\begin{bmatrix}1\\0\\1\end{bmatrix}$$

$$A=\begin{bmatrix}-1&-3&-3\\2&6&1\\3&8&3\end{bmatrix}$$

Find A^{-1} using Gaus elimination

$$\left[\begin{array}{ccc|ccc}-1&-3&-3&1&0&0\\2&6&1&0&1&0\\3&8&3&0&0&1\end{array}\right]\xrightarrow{R1=R1+R2}\left[\begin{array}{ccc|ccc}1&3&-2&1&1&0\\2&6&1&0&1&0\\3&8&3&0&0&1\end{array}\right]\xrightarrow{R2=R2-R1(2)}\left[\begin{array}{ccc|ccc}1&3&-2&1&1&0\\0&0&5&-2&-1&0\\3&8&3&0&0&1\end{array}\right]$$

$$\xrightarrow{R3=R3-R1(3)}\left[\begin{array}{ccc|ccc}1&3&-2&1&1&0\\0&0&5&-2&-1&0\\0&-1&9&-3&-3&1\end{array}\right]\xrightarrow{R3\leftrightarrow R2}\left[\begin{array}{ccc|ccc}1&3&-2&1&1&0\\0&-1&9&-3&-3&1\\0&0&5&-2&-1&0\end{array}\right]$$

$$\xrightarrow{R2=R2\cdot\frac{1}{-1}}\left[\begin{array}{ccc|ccc}1&3&-2&1&1&0\\0&1&-9&3&3&-1\\0&0&5&-2&-1&0\end{array}\right]\xrightarrow{R3=R3\cdot\frac15}\left[\begin{array}{ccc|ccc}1&3&-2&1&1&0\\0&1&-9&3&3&-1\\0&0&1&-\frac25&-\frac15&0\end{array}\right]$$

$$\xrightarrow{R1=R1-R2(3)}\left[\begin{array}{ccc|ccc}1&0&25&-8&-8&3\\0&1&-9&3&3&-1\\0&0&1&-\frac25&-\frac15&0\end{array}\right]\xrightarrow{R2=R2+R3(9)}\left[\begin{array}{ccc|ccc}1&0&25&-8&-8&3\\0&1&0&-\frac35&\frac65&-1\\0&0&1&-\frac25&-\frac15&0\end{array}\right]$$

$$\xrightarrow{R1=R1-R3(25)}\left[\begin{array}{ccc|ccc}1&0&0&2&-3&3\\0&1&0&-\frac35&\frac65&-1\\0&0&1&-\frac25&-\frac15&0\end{array}\right]$$

$$A^{-1}=\begin{bmatrix}2&-3&3\\-\frac35&\frac65&-1\\-\frac25&-\frac15&0\end{bmatrix}$$

Find rref, write free & lead

$$\left[\begin{array}{cc|c}2&-3&17\\1&-3&-16\end{array}\right]\xrightarrow{R1=R1-R2}\left[\begin{array}{cc|c}1&0&33\\1&-3&-16\end{array}\right]\xrightarrow{R2=R2-R1}\left[\begin{array}{cc|c}1&0&33\\0&-3&-49\end{array}\right]\xrightarrow{R2=R2\cdot-\frac13}\left[\begin{array}{cc|c}1&0&33\\0&1&\frac{49}{3}\end{array}\right]$$

Lead: x1, x2

Free: None

Solutions x1 = 33, x2 = 49/3