Name (2 pts): Tyler Trotter

Math 2210 Exam 3

By signing your name below, you agree to follow the Student Code of Conduct regarding "Academic Honesty" as you take this exam. You agree to take this exam without any outside help of any form. (Example: graphing calculator, book, notes, search engines, websites, friends or family etc help is NOT allowed).

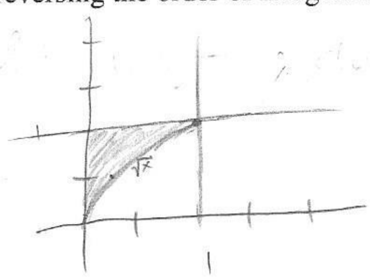
Signature: Tyler Trotter

Show all appropriate work and simplify all answers.

1. (16 pts) Evaluate the integral by reversing the order of integration:

$$\int_0^1 \int_{\sqrt{x}=y}^{1=y} \cos(y^3+1)\,dy\,dx.$$

$x=y^2$



$$\int_0^1 \int_0^{y^2} \cos(y^3+1)\,dx\,dy$$

$$\int_0^1 x\cos(y^3+1)\Big|_0^{y^2} dy \;;\; \int_0^1 y^2\cos(y^3+1)\,dy$$

let $u=y^3+1$, $du=3y^2\,dy$

$\frac{du}{3}=y^2\,dy$

$$\frac{1}{3}\int \cos(u)\,du = \frac{1}{3}\int \sin u\,du$$

$$=\frac{1}{3}\sin(y^3+1)\Big|_0^1$$

$$=\frac{1}{3}\Big[\big(\sin(2)-\sin(1)\big)\Big]$$

2. (16 pts) Find the volume of the region that lies below the paraboloid $z = 8 - 2x^2 - 2y^2$ and above the plane $z = 4$.

$z = 8 - 2(r^2)$

$4 = 8 - 2(r^2)$; $-4 = -2r^2$

$2 = r^2$; $r = \sqrt{2}$

$$\int_0^{2\pi}\int_0^{\sqrt{2}}\int_4^{8-2r^2} r\,dz\,dr\,d\theta$$

$$= \int\int_0^{\sqrt{2}} r\left[(8-2r^2)-(4)\right]\,dr\,d\theta$$

$$= \int\int_0^{\sqrt{2}} 4r - 2r^3\,dr\,d\theta$$

$$2r^2 - \frac{r^4}{2}\Big|_0^{\sqrt{2}}$$

$$(4-2)2\pi = 4\pi$$

3. (16 pts) Find the mass of the lamina ~~that is given by the region~~ inside the circle $x^2 + y^2 = 4$ with $x \geq 0$ if the density at each point is equal to its distance from the y-axis.
Hint: $m = \iint_D \rho(x, y)\, dA$

$$\hookrightarrow D = \sqrt{x^2+y^2} = \sqrt{r^2} = |r|$$

$$m = \int_0^{\pi} \int_0^{2} r \cdot r \, dr \, d\theta$$

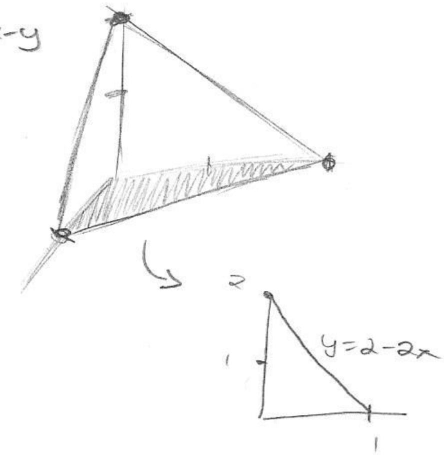
$$= \int_0^{\pi} \int_0^{2} r^2 \, dr \, d\theta = \int_0^{\pi} \frac{r^3}{3}\Big|_0^2 \, d\theta = \boxed{\frac{8\pi}{3}}$$

4. (16 pts) Compute $\iiint\limits_E x\,dV$ where E is the solid bound by the coordinate planes and the plane $2x + y + z = 2$ in the first octant. $z = 2-2x-y$

let $x=0,\ z=0\ ;\ y=2$

$x=0,\ y=0\ ;\ z=2$

$y=0,\ z=0\ ;\ x=1$



$$\int_0^1 \int_0^{2-2x} \int_0^{2-2x-y} x \; dz\, dy\, dx$$

$$\int \int xz \Big|_0^{2-2x-y}$$

$$x(2-2x-y)-0 \quad = \int_0^{2-2x}\!\!\int 2x-2x^2-xy \; dy\, dx$$

$$2xy - 2x^2y - \frac{xy^2}{2} \Big|_0^{2-2x} \qquad 4-8x+4x^2$$

$$2x(2-2x) - 2x^2(2-2x) - \frac{x(2-2x)^2}{2}$$

$$\cancel{4x} - \cancel{4x^2} - 4x^2 + 4x^3 - \frac{\cancel{4x}}{2} + \frac{8\cancel{x^2}}{2} - \frac{4x^3}{2}$$

$$-2x + 4x^2 - 2x^3$$

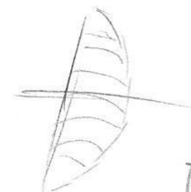
$$\int_0^1 2x - 4x^2 + 2x^3 \; dx$$

$$= x^2 - \frac{4x^3}{3} + \frac{x^4}{2} \Big|_0^1$$

$$1 - \frac{4}{3} + \frac{1}{2} = \frac{3}{2} - \frac{4}{3} = \frac{9-8}{6} = \boxed{\frac{1}{6}}$$

5. (16 pts) Find $\iiint\limits_E e^{(x^2+y^2+z^2)^{3/2}}\,dV$ where E is the region inside the sphere $x^2+y^2+z^2=4$ with $y \geq 0.$ $= \rho^2 = 4 \,;\, \rho = 2$

Hints: $x=\rho\sin\phi\cos\theta$, $y=\rho\sin\phi\sin\theta$, $z=\rho\cos\phi$, $\rho^2=x^2+y^2+z^2$, $dV=\rho^2\sin\phi\,d\rho\,d\theta\,d\phi$



forgot at first

$$\int_{-\pi/2}^{\pi/2}\int_0^{\pi/2}\int_0^2 e^{(\rho^2)^{\frac{3}{2}}}\,\rho^2\sin\phi\,d\rho\,d\phi\,d\theta$$

$$\int e^{\rho^3}\,\rho^2\sin\phi\,d\rho\,d\phi\,d\theta$$

let $u=\rho^3$ $\quad du = 3\rho^2 d\rho$; $\quad \frac{du}{3} = \rho^2 d\rho$

$$\frac{\sin\phi}{3}\int e^u\,du = \frac{1}{3}e^u = \iint \frac{1}{3}e^{\rho^3}\Big|_0^2 = \frac{1}{3}\int 2\int_0^{\pi/2}\left[(e^{2^3})-(e)^0\right]\sin\phi\,d\phi$$

e^8-1

$$= \int_{-\pi/2}^{\pi/2} 2\int_0^{\pi/2}(e^8-1)\sin\phi$$

Is it valid to multiply by 2 for $\pm z$ segments?

otherwise we get 0

$$-\frac{2}{3}\int (e^8-1)\cos\phi\Big|_0^{\pi/2}$$

$$\frac{2}{3}\int_{-\pi/2}^{\pi/2}\left[-(e^8-1)(0-1)\right]d\theta$$

e^8-1

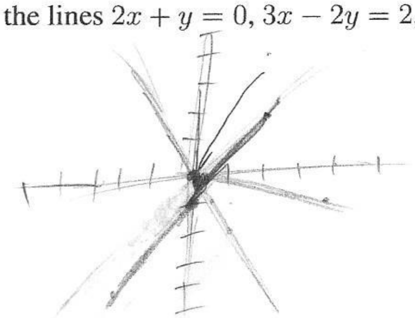
$$\frac{2}{3}(e^8-1)\,\theta\Big|_{-\pi/2}^{\pi/2}$$

$$= \frac{2}{3}(e^8-1)(\pi/2+\pi/2) = \boxed{\frac{2\pi}{3}(e^8-1)}$$

6. (a) (6 pts) Consider the change of variables $u = 3x - 2y$ and $v = 2x + y$. Compute the Jacobian, $\frac{\partial(x,y)}{\partial(u,v)}$.

$$\begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix} = \begin{vmatrix} 3 & -2 \\ 2 & 1 \end{vmatrix} = 3-(-4) = \boxed{7}$$

(b) (12 pts) Using the transformation in part (a), compute the integral $\iint_D (4x - 5y)\, dA$ where D is the region bounded by the lines $2x + y = 0$, $3x - 2y = 2$, and $x + 4y = 0$. ↳ $4x-5y = 2u-v$



$V=0$

$u=2$

$2v-u=0$

$u = 3x-2y$

$v = 2x+y$

$2v-u$

$\cancel{4x} + 2y - 3x + 2y$

$x + 4y$

$x+4y = 2v-u$

$$7\int_0^1\int_0^{2v} 2u-v \, du\, dv$$

$$= 7\int_0^1 u^2 - uv \Big|_0^{2v}$$

$$= 7\int_0^1 4v^2 - 2v^2 \, dv$$

$$= 7\int_0^1 2v^2 \, dv = \frac{2v^3}{3}\Big|_0^1 = \boxed{\frac{2}{3}} \cdot 7 = \boxed{\frac{14}{3}}$$