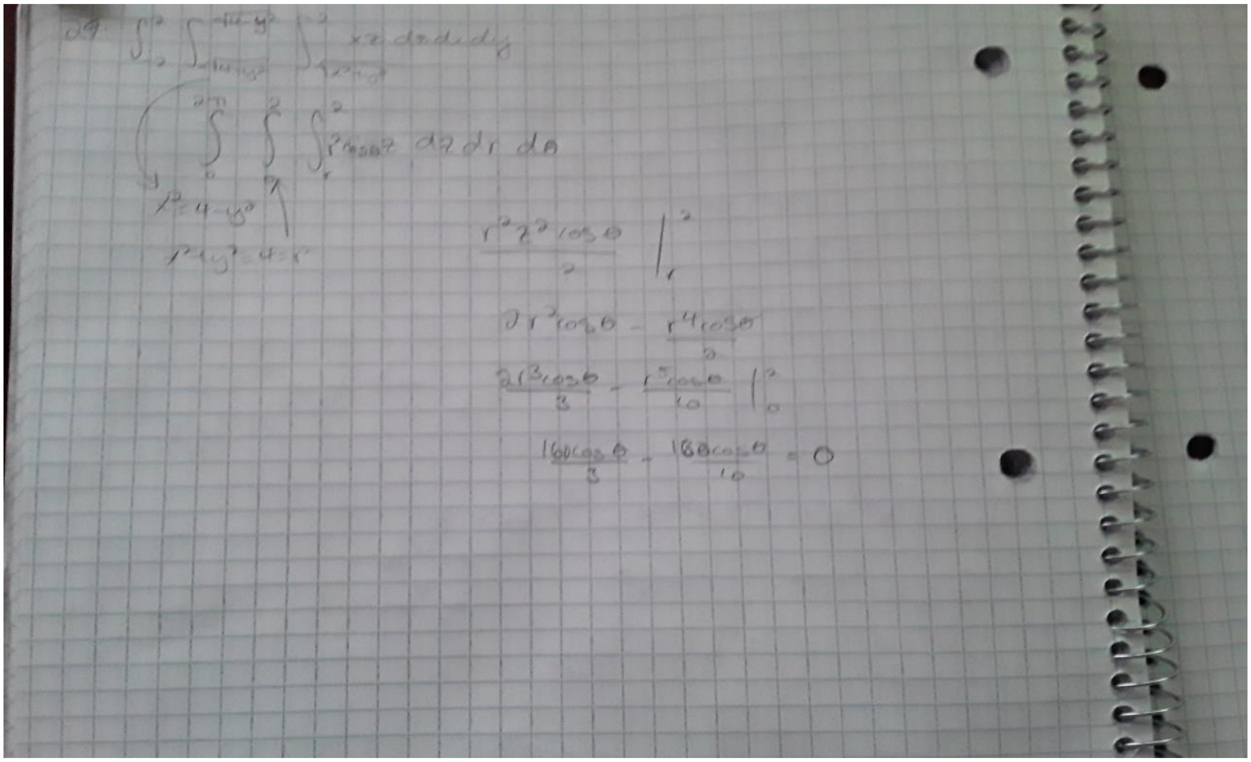
Chapter 15 Homework for Exam 3

15.1: #29



29. $\int_{-2}^{2}\int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}}\int_{\sqrt{x^2+y^2}}^{2} xz\,dz\,dx\,dy$

$\int_{0}^{2\pi}\int_{0}^{2}\int_{r}^{2} r^2\cos\theta\, z\,dz\,dr\,d\theta$

$x^2 = 4-y^2$

$x^2+y^2=4=r^2$

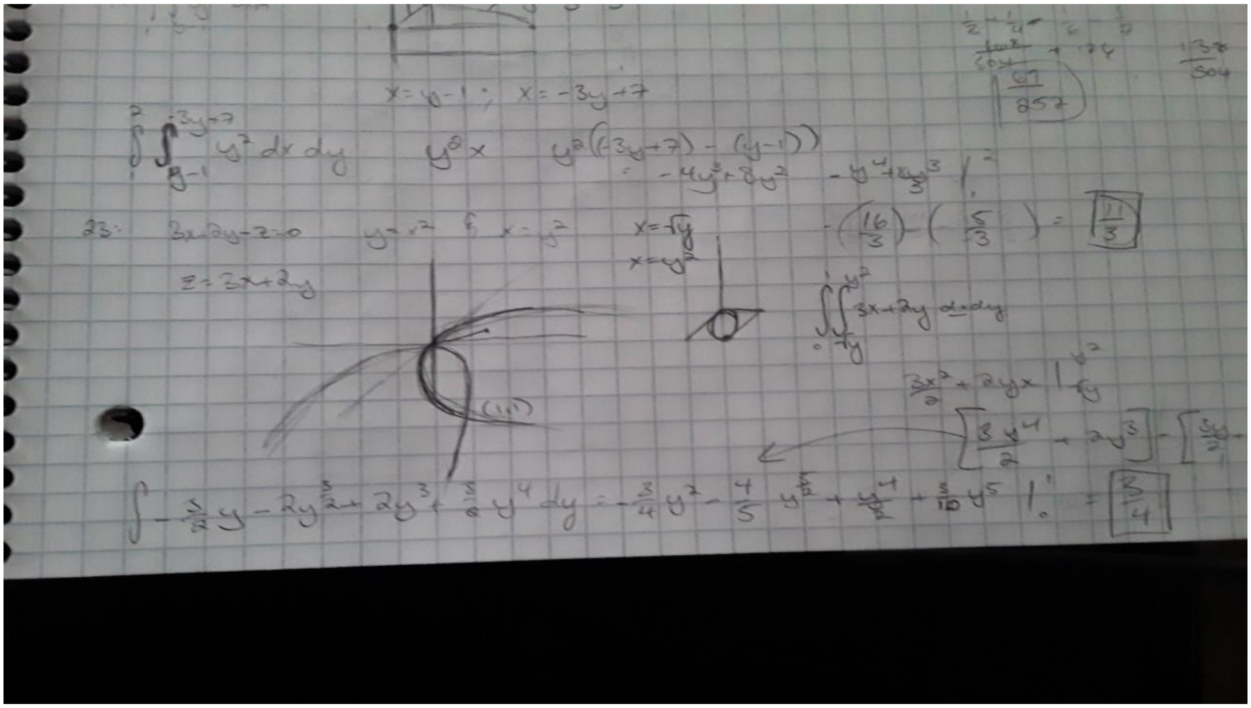
$\frac{r^2z^2\cos\theta}{2}\Big|_{r}^{2}$

$2r^2\cos\theta - \frac{r^4\cos\theta}{2}$

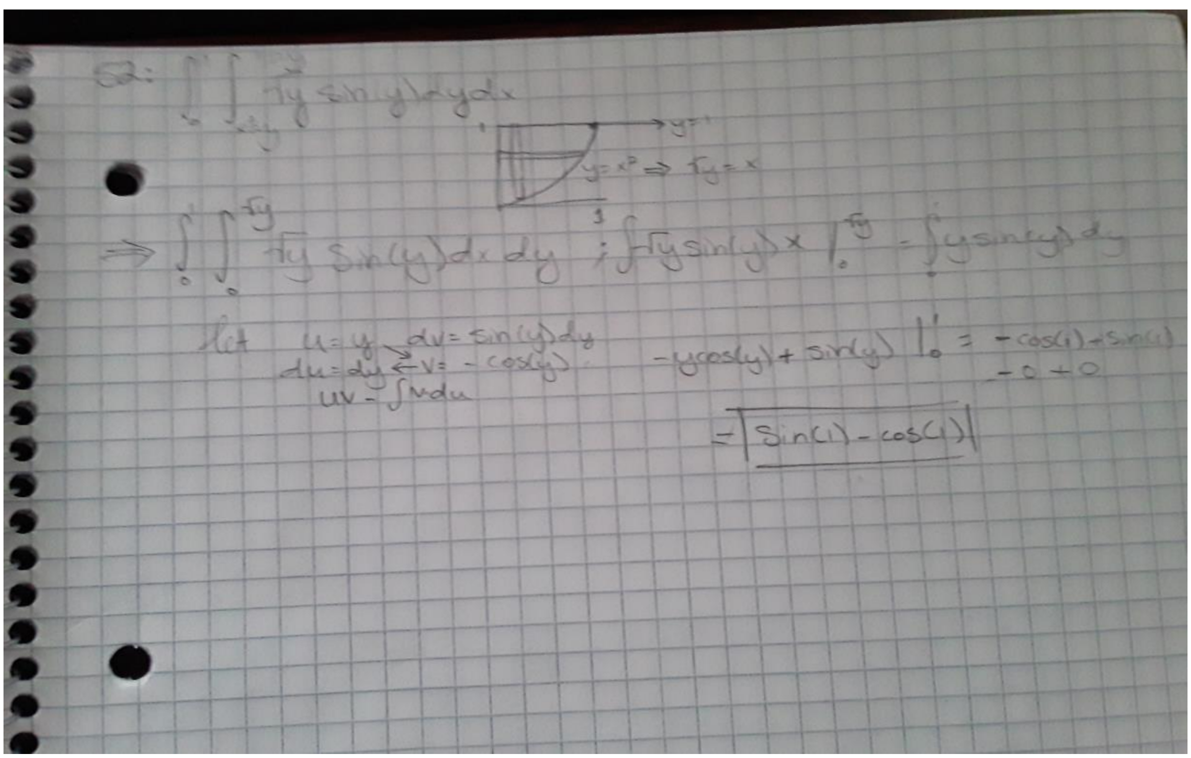
$\frac{2r^3\cos\theta}{3} - \frac{r^5\cos\theta}{10}\Big|_{0}^{2}$

$\frac{16\cos\theta}{3} - \frac{16\cos\theta}{10} = 0$

15.2: #23, 52



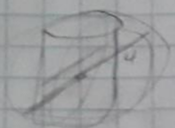
$x = y-1$; $x = -3y+7$
$\int^2 \int_{y-1}^{-3y+7} y^2\,dx\,dy$
$y^2 x$
$y^2((-3y+7)-(y-1))$
$= -4y^3+8y^2$
$-y^4+\frac{8y^3}{3} \Big|^2$
$-(\frac{16}{3}) - (-\frac{5}{3}) = \boxed{\frac{11}{3}}$
23: $3x+2y-z=0$
$y=x^2$ & $x=y^2$
$z=3x+2y$
$x=\sqrt{y}$
$x=y^2$
$(1,1)$
$\int_0^1\int_{\sqrt{y}}^{y^2} 3x+2y\,dx\,dy$
$\frac{3x^2}{2}+2yx \Big|_{\sqrt{y}}^{y^2}$
$\left[\frac{3y^4}{2}+2y^3\right] - \left[\frac{3y}{2} \ldots\right.$
$\int -\frac{3}{2}y - 2y^{\frac{3}{2}} + 2y^3 + \frac{3}{2}y^4\,dy = -\frac{3}{4}y^2 - \frac{4}{5}y^{\frac{5}{2}} + \frac{y^4}{2} + \frac{3}{10}y^5 \Big|_0^1 = \boxed{\frac{3}{4}}$



52: $\int_0^1 \int \sqrt{y}\sin(y)\,dy\,dx$
$y=1$
$y=x^2 \Rightarrow \sqrt{y}=x$
$\Rightarrow \int_0^1\int_0^{\sqrt{y}} \sqrt{y}\sin(y)\,dx\,dy$; $\int \sqrt{y}\sin(y)\,x \Big|_0^{\sqrt{y}}$ $\int_0^1 y\sin(y)\,dy$
let $u=y$ $dv=\sin(y)\,dy$
$du=dy$ $v=-\cos(y)$
$uv-\int v\,du$
$-y\cos(y)+\sin(y) \Big|_0^1 = -\cos(1)+\sin(1) - 0 + 0$
$= \boxed{\sin(1)-\cos(1)}$

15.3: #26

$-\iint$



Top / Bottom

26: $z_1 = 6 - x^2 - y^2$ & $z_1 = 2x^2 + 2y^2$ $V = \int_0^{2\pi}\int_0^{\sqrt{2}} 6 - 3r^2 \cdot r\,dr\,d\theta$

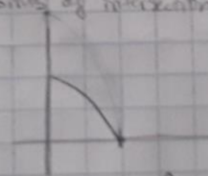
$\int_0^{2\pi} 3r^2 - \frac{3r^4}{4}\Big|_0^{\sqrt{2}} d\theta$

$3r^2 - \frac{3r^4}{4}\Big|_0^{\sqrt{2}}$

$6 - 3 = 3$

$6 - (x^2 + y^2) = 2(x^2 + y^2)$; $2 = x^2 + y^2$ $= \int_0^{2\pi} 3\,d\theta = \boxed{6\pi}$

points of intersection $r = \sqrt{2}$



29: $\int_0^2 \int_0^{\sqrt{4-x^2}} e^{-x^2-y^2}\,dy\,dx$

$\int_0^{\pi/2} -\frac{1}{2}\int_0^2 e^{-r^2} \cdot r\,dr\,d\theta$

let $u = -r^2$ $du = -2r\,dr$

$-\frac{du}{2} = r\,dr$

$\int e^{-r^2}\Big|_0^2 d\theta = -\frac{1}{2}(e^{-4} - 1)\frac{\pi}{2}$

$\boxed{\frac{\pi - \pi e^{-4}}{4}}$

15.4: #11

#8: 11, 12

11: $m = \int_{-\pi/2}^{\pi/2} \int_{0}^{2} r^2 \cdot r\,dr\,d\theta$

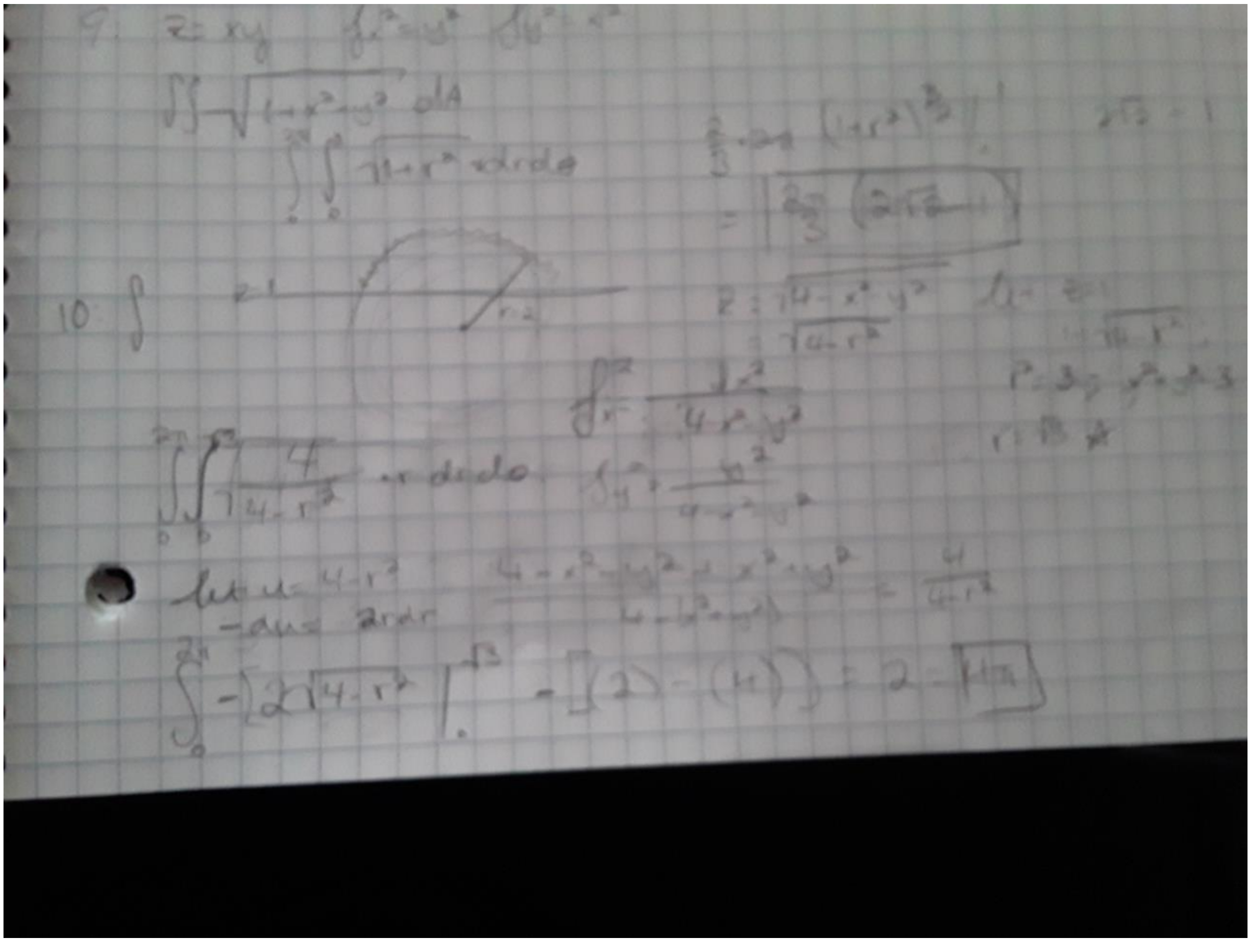
$x^2+y^2=4$, $r^2=4$, $r=2$

$0 \le y$, $-\frac{\pi}{2} \le \theta \le \pi/2$

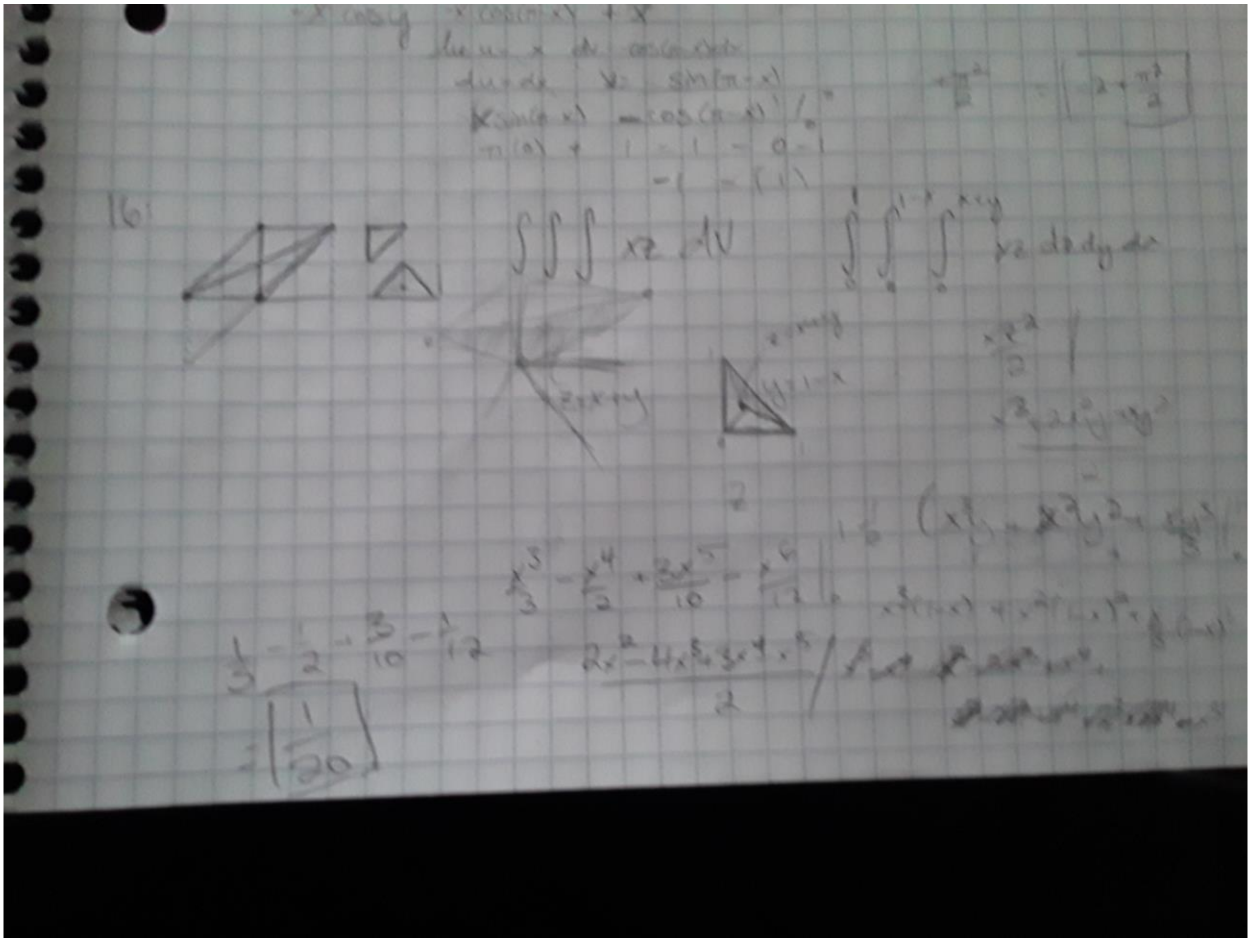
$\int_{-\pi/2}^{\pi/2} \int_{0}^{2} e^{-r^2} \cdot r\,dr\,d\theta$

$-\frac{1}{2} e^{-r^2} \Big|_0^2$; $\boxed{-\frac{\pi}{2}\left(\frac{1}{e^4} - 1\right)}$

15.5: #10



15.6: #16



15.7: #18, 29

$\frac{-32}{\cos\theta} = \frac{32}{\sin\theta}$ $\left(2, \frac{3\pi}{4}, 1\right)$

17 $\int_0^{2\pi}\int_0^{4}\int_{-5}^{4} r^2 \, dz\, dr\, d\theta$

$3r^3 \Big|_0^4$ $192 \cdot 2\pi = \boxed{384\pi}$

18 $\int_0^{2\pi}\int_0^{2}\int_{r^2}^{4} 2 \cdot r \, dz\, d\theta \, d\theta$; $\frac{2r^2}{2}\Big|_{r^2}^{4}$ $8r - \frac{r^3}{2}$;

$8r - \frac{r^5}{2}$ $4r^2 - \frac{r^6}{12}\Big|_0^2$ $16 - \frac{64}{12}$

$\boxed{\frac{64\pi}{3}}$

21:

$\int_0^{2\pi}\int_0^{1}\int_0^{2r} r^2\cos^2\theta \cdot r \, dz\, dr\, d\theta$; $\int r^3\cos^2\theta \cdot 2r$

$2r^4\cos^2\theta$

$= \frac{2}{5}\int_0^{2\pi}\cos^2\theta \, d\theta$; $\frac{1+\cos(2\theta)}{2}$ $\frac{2r^5}{5}\cos^2\theta \Big|_0^1$ $\int_0^{2\pi}\frac{2}{5}\cos^2\theta \, d\theta$

$= \frac{1}{5}\left(\theta + \frac{\sin(2\theta)}{2}\right)\Big|_0^{2\pi} = \boxed{\frac{2\pi}{5}}$

22: $\int_0^{2\pi}\int_0^{1}\int_0^{\sqrt{4-r^2}} r \, dz\, dr\, d\theta$

$z = \sqrt{4-r^2}$; $r\sqrt{4-r^2}$ $\frac{2}{3}(4-r^2)^{\frac{3}{2}}\Big|_0^1$ $\frac{2}{3}\left[3^{\frac{3}{2}} - 4^{\frac{3}{2}}\right]$

#5: 29, 39, 40

29: $\iint_R \frac{xy^2}{x^2+1} \, dA$, $0 \le x \le 1$, $-3 \le y \le 3$

$\int_{-3}^{3}\int_0^{1} \frac{xy^2}{x^2+1} \, dx\, dy$; let $x^2+1 = u$ $du = 2x\,dx$, $\frac{du}{2} = x\,dx$

$\int y^2 \left(\int u^{-1} \, du\right) dy$ $= \ln u / \frac{1}{2} \ln(x^2+1) \Big|_0^1$ $\int y^2 \ln(2) \, dy = \frac{\ln(2) y^3}{2 \cdot 3}$

$\frac{1}{2}\left[\frac{(3)^3\ln(2)}{3} - \frac{(-3)^3\ln(2)}{3}\right] = \frac{1}{2}(18\ln(2)) = \boxed{9\ln(2)}$

39. Find the volume of the solid lying under the elliptic paraboloid $\frac{x^2}{4} + \frac{y^2}{9} + z = 1$ and above the rectangle $R = [-1, 1] \times [-2, 2]$

$z = 1 - \frac{x^2}{4} - \frac{y^2}{9}$ $\int_{-2}^{2}\int_{-1}^{1} 1 - \frac{x^2}{4} - \frac{y^2}{9} \, dx\, dy$ $\int x - \frac{x^3}{12} - \frac{xy^2}{9}\Big|_{-1}^{1}$

$\left[1 - \frac{1}{12} - \frac{y^2}{9}\right] - \left[-1 + \frac{1}{12} + \frac{y^2}{9}\right]$

$\int_{-2}^{2} \frac{11}{6} - \frac{2y^2}{9} \, dy = \frac{11}{6}y - \frac{2}{27}y^3 \Big|_{-2}^{2} = \left[\frac{11}{3} - \frac{16}{27}\right]\left[-\frac{11}{3} + \frac{16}{27}\right]$

$\frac{22}{3} - \frac{32}{27} = \frac{198-32}{27} = \boxed{\frac{166}{27}}$

40: Find the volume: $z = x^2 + xy^2$, $z = 0$, $x = 0$, $x = 5$, $y = 0$, $y = 2$

$\int_0^{2}\int_0^{5} x^2 + xy^2 \, dx\, dy$

15.8: #25, 41

#s 25, 26, 27, 41 $\qquad xe^{x^2+y^2+z^2} = e^{\rho^2} \cdot \rho\sin\phi\cos\theta$

25: $$\int_0^{\pi/2}\int_0^{\pi/2}\int_0^1 \rho\sin\phi\cos\theta\, e^{\rho^2}\cdot\rho^2\sin\phi\, d\rho\, d\phi\, d\theta$$

$$\iiint \rho^3\sin^2\phi\cos\theta\, e^{\rho^2}\, d\rho\, d\phi\, d\theta$$

$$(-1)\int \sin^2\phi \int \rho^3 e^{\rho^2}\, d\rho\, d\phi$$ ☆ Tough integral

let $u = \rho^3$, $dv = e^{\rho^2} d\rho$

$du = 3\rho^2 d\rho$ $\quad v =$

$$(-1)\int_0^{\pi/2} \sin^2\phi \left[\frac{1}{2}\left(e^{x^2}x^2 - e^{x^2}\right)\right]\Bigg|_0^1 = 0 - \frac{1}{2}$$

$\sqrt{1-\rho^2\sin^2\phi\cos^2\theta}$ $\qquad \frac{1}{2}\int_0^{\pi/2} \sin^2\phi\, d\phi \;;\; \frac{1}{4}\left(\phi - \frac{1}{2}\sin(2\phi)\right)\Big|_0^{\pi/2}$

$x^2+y^2=1$ $\qquad \rho^2 = 2$ $\quad \left(\frac{\pi}{8}\right) - (0) = \boxed{\frac{\pi}{8}}$

41: $$\int_0^1\int_0^{\sqrt{1-x^2}}\int_{\sqrt{x^2+y^2}=z}^{\sqrt{2-x^2-y^2}=z}$$ $\quad z^2+x^2+y^2 = 2$ $\quad \rho = \sqrt{2}$

$\sqrt{\rho^2\sin^2\phi\cos^2\theta + \rho^2\sin^2\phi\sin^2\theta}$ $\quad \rho\sin\phi = \rho\cos\phi$

$\sqrt{1-\rho^2\sin^2\phi\cos^2\theta}$ $\qquad \frac{1}{2}\int_0^{\pi/2} \sin^2\phi\, d\phi \;;\; \frac{1}{4}\left(\phi - \frac{1}{2}\sin(2\phi)\right)\Big|_0^{\pi/2}$

$x^2+y^2=1$ $\qquad \rho^2 = 2$ $\quad \left(\frac{\pi}{8}\right) - (0) = \boxed{\frac{\pi}{8}}$

41: $$\int_0^1\int_0^{\sqrt{1-x^2}}\int_{\sqrt{x^2+y^2}=z}^{\sqrt{2-x^2-y^2}=z}$$ $\quad z^2+x^2+y^2 = 2$ $\quad \rho = \sqrt{2}$

$\sqrt{\rho^2\sin^2\phi\cos^2\theta + \rho^2\sin^2\phi\sin^2\theta} = \rho\sin\phi = \rho\cos\phi$ $\qquad \frac{\sin\phi}{\cos\phi} = 1$ $\quad \phi = \pi/4$

$1 + \rho^2\sin^2\phi\cos^2\theta = \rho^2\sin^2\phi\sin^2\theta$

$2 + \rho^2\cos^2\theta = \rho^2\sin^2\theta$ $\qquad 2\cos^2\theta = 0$

$1 + \cos^2\theta = \sin^2\theta$ $\qquad \cos\theta = 0$ $\quad \theta = \pi/2$

$$\int_0^{\pi/2}\int_0^{\pi/4}\int_0^{\sqrt{2}} \rho\sin\phi\cos\theta\; \rho\sin\phi\sin\theta \cdot \rho^2\sin\phi\, d\rho\, d\phi\, d\theta$$ $\qquad \int_0^{\pi/2} \sin\theta\cos\theta\, d\theta$

$\rho^4\sin^3\phi\sin\theta\cos\theta \;;\; \frac{\rho^5}{5}$ $\qquad \frac{4\sqrt{2}}{5}$ $\qquad \sin^2\phi\cdot\sin\phi\, d\phi$

$1-\cos^2\phi$

$-\cos\phi - \frac{\cos^3\phi}{3}\Big|_0^{\pi/4}$ $\qquad \frac{\sqrt{2}}{2}$ $\quad \frac{2\sqrt{2}}{24}$

$\left(\frac{5\sqrt{2}}{12}\right)$ $\left(1 - \frac{1}{3}\right)$ $\left(\frac{5\sqrt{2}-8}{12}\right)\left(\frac{4\sqrt{2}}{5}\right)$

15.9: #15, 24

#15: $\iint (x-3y)\,dA$, R

$u+2v=y$; $3v = x+2y$; $v = \frac{x}{3} + \frac{2}{3}y$

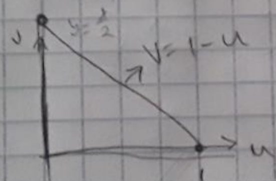
$2u+v=x$; $3u = 2x-y$; $u = \frac{2}{3}x - \frac{y}{3}$

$J: \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = 4-1 = 3$

$y = 2x$; $u+2v = 2(2u+v)$; $u=0$

$y = \frac{x}{2}$; $u+2v = \frac{2u+v}{2}$; $v=0$

$y = 3-x$; $u+2v = 3-(2u+v)$; $u = 1-v$; $-u+1 = v$



$-3\int_0^1\int_0^{1-u} 2u+v-3(u+2v)\,dv\,du$

$\int 5v + u\,dv\,du$; $\frac{5v^2}{2} + uv \Big|_0^{1-u}$

$\frac{5}{2}(1-2u+u^2) + u - u^2$

$\frac{5}{2} - 4u + \frac{3}{2}u^2$; $\left(\frac{5}{2}u - 2u^2 + \frac{u^3}{2}\right)\Big|_0^1 = -3(1) = \boxed{-3}$

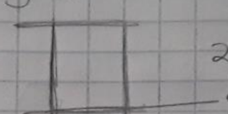
$x-y=0$ $x-y=2$ $x+y=0$ $x+y=3$

24: $y = x$, $y = x-2$, $y=-x$, $y = 3-x$

let $x-y=u$; $u = 0, 2$
$x+y=v$; $v = 0, 3$

$x^2-y^2=uv$ $x+y=3$

Jacobian: $\begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix} = \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} = 2$



$2\int_0^2\int_0^3 v e^{uv}\,dv\,du$

$(v)(e^{uv})$

let $\alpha = v$; $d\beta = e^{uv}$
$d\alpha = dv$; $\beta = \frac{e^{uv}}{u}$

$\frac{ve^{uv}}{u} - \frac{e^{uv}}{u^2}$