Name(2 pts):____________________

Math 2210 Exam 4

By signing your name below, you agree to follow the Student Code of Conduct regarding "Academic Honesty" as you take this exam. You agree to take this exam without any outside help of any form. (Example: graphing calculator, book, notes, search engines, websites, friends or family etc help is NOT allowed).

Signature: __________________________

Show all appropriate work and simplify all answers.

1. (16 pts each) Consider the line integral $\int_C (x+y)\,dx - x^2\,dy$, where C consists of the line segments from $(0,0)$ to $(3,0)$, from $(3,0)$ to $(0,3)$ and from $(0,3)$ to $(0,0)$. Verify Green's Theorem by:

(a) Evaluating the line integral using Green's Theorem.

(1. continued) $\int_C (x+y)\,dx - x^2\,dy$, C from $(0,0)$ to $(3,0)$ to $(0,3)$ to $(0,0)$

(b) Evaluating the line integral directly (i.e. use 16.2 techniques).

2. Consider the vector field $\mathbf{F} = \langle 3x^2y + e^x \cos z, x^3 - 2, -e^x \sin z - 2z \rangle$.
(a) (4 pts) Use the curl to show $\mathbf{F}$ is conservative.

(b) (8 pts) Find a function f such that $\mathbf{F} = \nabla f$.

(c) (6 pts) Evaluate $\int \mathbf{F} \bullet d\mathbf{r}$ using *FTOLI*, where $\mathbf{r}(t) = \langle t^2, 2t + 1, t^2 - t \rangle$, $0 \leq t \leq 1$.

3. (14 pts) Find the equation of the tangent plane to the surface given by $x = u^2 - 2v^2$, $y = 3u - v$, $z = 2u + 3v$ when $u = 2$ and $v = -1$.

4. (16 pts) Find the surface area of the portion of $\mathbf{r}(u,v) = \langle 3u+v, 2u-v, u+2v\rangle$ that lies over the region in the uv-plane inside $u^2+v^2=4$ with $v \geq 0$.

5. (18 pts) Use Stoke's Theorem to compute $\int_C \mathbf{F} \bullet d\mathbf{r}$ where $\mathbf{F}(x, y, z) = (x^3 - z^2)\mathbf{i} + (\sqrt{y} - x^2)\mathbf{j} + z^5\mathbf{k}$ and C is the positively oriented triangle with vertices $(2, 0, 0)$, $(0, 2, 0)$, $(0, 0, 1)$.

Formulas

Fundamental Theorem of Line Integrals: If $\mathbf{F}$ is conservative,

$$\int_C \mathbf{F} \bullet d\mathbf{r} = f(\mathbf{r}(b)) - f(\mathbf{r}(a)).$$

Green's Theorem: $\displaystyle\int_C \mathbf{F} \cdot d\mathbf{r} = \int_C P\,dx + Q\,dy = \iint_D \left(\frac{dQ}{dx} - \frac{dP}{dy}\right) dA$

Curl: $\text{curl}(\mathbf{F}) = \nabla \times \mathbf{F}$ **Divergence:** $\text{div}(\mathbf{F}) = \nabla \bullet \mathbf{F}$

Surface Area:

1) $\displaystyle A(S) = \iint\limits_D |\mathbf{r}_u \times \mathbf{r}_v|\,dA$

2) $\displaystyle A(S) = \iint\limits_D \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}\,dA$

Surface Integral of a Scalar Function (Flux):

1) $\displaystyle \iint\limits_S f(x,y,z)\,dS = \iint\limits_D f(\mathbf{r}(u,v))|\mathbf{r}_u \times \mathbf{r}_v|\,dA$

2) $\displaystyle \iint\limits_S f(x,y,z)\,dS = \iint\limits_D f(x,y,g(x,y))\sqrt{1 + \left(\frac{\partial g}{\partial x}\right)^2 + \left(\frac{\partial g}{\partial y}\right)^2}dA$ where $z = g(x,y)$

Surface Integral of a Vector Function:

1) $\displaystyle \iint\limits_S \mathbf{F} \cdot d\mathbf{S} = \iint\limits_S \mathbf{F} \cdot \mathbf{n}\,dS$ with $\mathbf{n}$ a unit normal vector to S

2) $\displaystyle \iint\limits_S \mathbf{F} \cdot d\mathbf{S} = \iint\limits_D \mathbf{F} \cdot (\mathbf{r}_u \times \mathbf{r}_v)\,dA$

3) $\displaystyle \iint\limits_S \mathbf{F} \cdot d\mathbf{S} = \iint\limits_D \left(-P\frac{\partial g}{\partial x} - Q\frac{\partial g}{\partial y} + R\right) dA$ where $z = g(x,y)$ oriented upward

Stokes' Theorem:

$$\int_C \mathbf{F} \bullet d\mathbf{r} = \iint\limits_S \text{curl}\mathbf{F} \bullet d\mathbf{S}$$