Mapping the Latent Structure of Economic Bias: A DML-Unfolding Fusion Framework

Tyler Venner[1], Macy Lastname[1], Gary Lastname[1], Leo Lastname[1], and Alberto Lastname[1]

[1]Team 5, University of California, Davis

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Abstract

This document details a novel, two-stage methodological framework for visualizing the high-dimensional structure of economic bias in professional sports markets. Traditional econometric analyses often conclude with a static wall of coefficients which is a list of significant effects that fails to reveal the latent relationships between bias factors or the systemic clusters of affected individuals. Furthermore, standard models often ignore the confounding influence of deterministic salary structures, such as Rookie Scales and Maximum contracts, leading to model misspecification. To address these limitations, we introduce a Stratified Double Machine Learning (DML) approach. We first isolate the Free Market sub-population to rigorously learn the performance-debiased "price" of contextual factors (Z). These coefficients are then counterfactually applied to the full population to construct a Debiased Attribution Matrix (L). Finally, this matrix serves as the input for a probabilistic dimension reduction model. The resulting visualization reveals the geometric topology of bias, which clusters factors based on shared structural forces and identifying player archetypes most vulnerable to market inefficiencies. TOO LONG!

Keywords: Double Machine Learning, Latent Space Mapping, Sports Analytics, Economic Bias, Dimensionality Reduction

1 Introduction

Standard econometric analysis for quantifying discrimination and market inefficiency, while statistically robust, often suffers from an interpretive bottleneck. These methods typically conclude with a "wall of coefficients", a static table of estimates and p-values that answers the binary question of statistical significance. This fails to reveal the underlying system of bias. For example, while a regression may indicate that both Age and Draft Number significantly impact salary, it cannot intuitively demonstrate whether these factors act as independent forces or form a singular bias.

Furthermore, applying standard statistical models to professional sports markets presents a unique structural challenge: the prevalence of deterministic contracts. In the National Basketball Association (NBA), a substantial subset of player salaries, specifically Rookie Scale and Maximum contracts, are dictated by Collective Bargaining Agreement (CBA) rules rather than free-market negotiation. Applying a standard bias model to this mixed population will result in severe model misspecification. For instance, a high-performing rookie may appear to have a large negative residual not because of market bias, but due to a salary ceiling.

To address these interpretive and structural limitations, this paper introduces a novel DML-Dimension Reduction Framework. We utilize Double Machine Learning (DML) to strictly isolate the price of bias factors, and our custom dimension reduction technicque to visualize their geometric relationships.

Our approach moves the goal from simply quantifying bias to mapping its topology. By implementing a stratified Learn vs. Apply protocol, we first learn the true market valuation from free-market players and then counterfactually apply these valuations to the entire league. The resulting Bias Attribution Map reveals the latent structure of economic forces, identifying distinct player clusters and the specific structural biases that drive their valuation.

2 Methods

Our methodological framework fuses inference with geometric visualization. We employ a two-stage approach: first, we isolate the "price" of bias using Stratified Double Machine Learning (DML), and second, we map the topology of these biases using probabilistic latent unfolding.

2.1 The Learn-Apply Protocol

A critical challenge in analyzing NBA salaries is the prevalence of non-market compensation. Training a model on the full population (D_{ALL}) would allow deterministic salaries (Rookie Scale and Maximum contracts) to contaminate the estimate of market valuation.

To resolve this, we implement a stratified Learn-Apply protocol:

1. Learn Phase: We isolate the sub-population of players operating under negotiated, free-market contracts (D_{FM}). We train our nuance parameters solely on this group to learn the true market dynamics.

2. Apply Phase: We apply the learned market prices and nuisance models to the entire population (D_{ALL}), creating counterfactual estimates for players on fixed contracts.

2.2 Stage 1: Stratified Double Machine Learning

We assume a partially linear structural model for player valuation:

$$Y_i = \alpha + Z_i^T \gamma + f(X_i) + \epsilon_i \tag{1}$$

where Y is log-salary, Z is the vector of bias factors, X is the vector of performance metrics, and γ represents the marginal market price of bias.

To estimate γ unbiasedly, we use the machine-learning extension of the Frisch-Waugh-Lovell (FWL) theorem. The central theme of FWL is that to isolate the linear effect of Z on Y in the presence of a confounder X, one must remove the influence of X from *both* the outcome and the treatment variables. This process ensures that the final estimate $\hat{\gamma}$ captures only the relationship between the unexplained components of salary (Y) and bias factors (Z), orthogonal to performance (X).

For each player $i \in D_{FM}$, the procedure involves three steps:

1. Outcome Model (Cleaning Y): We train a gradient boosting model (XGBoost) $\hat{f}(X)$ to predict salary. This ensures the residual $\hat{\epsilon}_{Y,i}$ represents salary strictly unexplained by performance.

$$\hat{\epsilon}_{Y,i} = Y_i - \hat{f}(X_i) \tag{2}$$

2. Treatment Models (Cleaning Z): For each bias factor Z_j, we train a separate boosting model $\hat{h}_j(X)$ to predict the factor from performance. The residual represents the "pure" component of the factor (e.g., the part of Draft Number (Z_j) not predicted by performance(X)):

$$\hat{\epsilon}_{Z^{(j)},i} = Z_i^{(j)} - \hat{h}_j(X_i) \tag{3}$$

3. Debiased Regression: Finally, we estimate the bias coefficients by regressing the outcome residuals onto the treatment residuals via simple OLS:

$$\hat{\epsilon}_{Y,i} = \gamma_0 + \sum_{j=1}^{m} \gamma_j \hat{\epsilon}_{Z_j,i} + \nu_i \tag{4}$$

We employ OLS in this final stage rather than a flexible model for theoretical sufficiency and interpretability. Since the complex non-linear confounding has already been removed by the ML models in steps 1 and 2, the remaining relationship between residuals is structural.

2.3 Stage 2: The Attribution Matrix

To connect the output of our ML models with the geometric visualization, we construct the Attribution Matrix $L \in \mathbb{R}^{n \times m}$. Raw bias factors ($Z$) have non comparable units (e.g., years of Age vs. # followers on Instagram), making direct geometric embedding impossible.

First, for every player $k \in D_{ALL}$, regardless of contract status,we calculate the counterfactual residual $\hat{\epsilon}_{Z^{(j)},k}$ using the treatment models trained exclusively on free-market players. For rookies and max-contract players, this quantity represents the "excess bias" they possesses relative to their performance, had they been subject to open negotiation.

We then define each entry L_{ij} as the magnitude of economic impact bias factor j has on player i:

$$L_{ij} = |\hat{\gamma}_j \cdot \hat{\epsilon}_{Z^{(j)},i}| \tag{5}$$

This transformation achieves two critical objectives:

1. Unit Unification: By interacting the market price ($\hat{\gamma}_j$) with the residual volume ($\hat{\epsilon}_{Z,ij}$), we convert all factors into a single, unified currency: log-dollars of unexplained salary attribution.

2. Relevance over Direction: We utilize the absolute value to shift the analytical focus from direction (overpaid vs. underpaid) to magnitude (relevance). Geometrically, we seek to cluster players whose valuations are driven by the same structural forces, regardless of whether that force acts as a premium or a penalty.

Finally, we apply a global min-max scaling to map these attributions onto the $[0, 1]$ interval required for our probabilistic unfolding algorithm.

2.4 Stage 3: Probabilistic Latent Unfolding

We visualize the high-dimensional Attribution Matrix L by mapping it to a low-dimensional space, $\mathbb{R}^3$. Unlike traditional multidimensional scaling which treats entities as fixed points, we employ a probabilistic approach to explicitly model the uncertainty inherent in the attribution estimates.

We represent the position of player i as a random variable $k_i \sim \mathcal{N}(\mu_{k,i}, V_{k,i})$ and bias factor j as $b_j \sim \mathcal{N}(\mu_{b,j}, V_{b,j})$. The interaction between a player and a factor is defined by their difference vector $\delta_{ij} = k_i - b_j$, which is itself Gaussian distributed:

$$\delta_{ij} \sim \mathcal{N}(\mu_{ij}, V_{ij}) \tag{6}$$

where $\mu_{ij} = \mu_{c,i} - \mu_{p,j}$ and $V_{ij} = V_{c,i} + V_{p,j}$.

Formulating the locations players and variables on the latent space as probability distributions offers a mathematical advantage over point-based embeddings: Adaptive Regularization. In a deterministic model, the optimizer can artificially inflate similarity by placing points infinitesimally close together, even if the data is noisy. In our probabilistic framework, the interaction is governed by the overlap of densities. If the model is uncertain about a player's location (represented by a large covariance Σ), the density spreads out, mathematically lowering the peak similarity score. This forces the model to only cluster entities tightly when the statistical signal is strong enough to justify low variance.

We define the theoretical attribution strength using a Gaussian kernel, which maps geometric proximity to a similarity score bounded in $(0, 1]$:

$$g(\delta) = \exp(-\delta^\top \delta) = \exp(-\|\delta\|_2^2) \tag{7}$$

This kernel implies that attribution strength decays exponentially with squared Euclidean distance in the latent space.

The model prediction $\hat{L}_{ij}$ is the expected value of the similarity function over the entire distribution of possible relative positions. Mathematically, this is defined by the integral:

$$\hat{L}_{ij} = \mathbb{E}[g(\delta)] = \int_{\mathbb{R}^3} g(\delta) \cdot p(\delta|\mu_{ij}, V_{ij}) d\delta \tag{8}$$

While numerically integrating this over high dimensions is computationally intractable, we notice the fact that this integral corresponds to the moment-generating function (MGF) of a quadratic form of a Gaussian variable. Which has a closed-form solution:

$$\hat{L}_{ij} = |I + 2V_{ij}|^{-1/2} \exp\left(-\mu_{ij}^\top (I + 2V_{ij})^{-1} \mu_{ij}\right) \tag{9}$$

This formula acts as the computational engine of the model. It naturally penalizes uncertainty: as the variance V_{ij} increases, the expected attribution $\hat{L}_{ij}$ decreases, reflecting lower confidence in the structural link.

We solve for the optimal map coordinates $\theta = \{\mu, V\}$ by minimizing the error between the observed attribution L (from Stage 2) and the model prediction $\hat{L}$. We assume the observed values are drawn from a Gaussian around the prediction with a global noise parameter σ_{err}. This leads to the Negative Log-Likelihood (NLL) objective function:

$$\mathcal{J}(\theta) = \sum_{i,j} \left(\frac{(L_{ij} - \hat{L}_{ij}(\theta))^2}{2\sigma_{err}^2} + \log(\sigma_{err}) \right) \tag{10}$$

Notice that minimizing this objective is equivalent to a minimizing the weighted Sum of Squared Errors. We implement the optimization loop using Google's JAX library to leverage Just-In-Time (JIT) compilation for accelerated gradient descent.

3 Results

3.1 Econometric Estimation

The first stage of our analysis produced the standard DML output: a vector of bias coefficients $\hat{\gamma}$ representing the marginal effect of each contextual factor on salary after controlling for performance. As shown in Table 1, factors such as 'Age', 'Draft Number', and 'Owner Net Worth' showed statistically significant relationships with residual salary ($p < 0.05$).

Table 1: DML Debiased Coefficients (Partial Output)

Bias Factor (Z_j)	**Coeff ($\hat{\gamma}_j$)**	**SE**	**p-val**
Age	0.027	0.010	0.007
Draft Number	-0.018	0.005	<0.001
Owner Net Worth	0.019	0.008	0.021
Market Size	0.031	0.012	0.015
is_USA	0.005	0.015	0.739

While these results confirm the *existence* of bias, they represent an interpretive dead end. The table fails to elucidate the structural relationships between factors; for instance, it is unclear if 'Market Size' and 'Owner Net Worth' act as distinct economic forces or components of a single "Capital Bias."

3.2 Topological Structure: The Bias Map

The application of the probabilistic unfolding algorithm to the Attribution Matrix L transformed these static coefficients into a geometric map. This 3D visualization revealed three distinct topological features:

1. Bias Anchors (Factor Clustering): The bias factors acted as fixed anchors in the latent space. We observed that 'Market Size' and 'Team Revenue' were positioned in close proximity, suggesting a shared underlying economic force affecting player valuation. Conversely, 'Age' and 'Draft Number' occupied distinct quadrants, confirming they operate as independent vectors of bias.

2. Player Clusters: Players did not distribute randomly but formed dense clusters around specific anchors. A significant cluster formed tightly around the 'Draft Number' anchor, composed primarily of players whose valuation is structurally tied to their entry position rather than current performance.

3. The "Void" (Performance Purity): A clear "void" emerged at the geometric center of the map. Players located in this region possessed low attribution scores across all bias dimensions, indicating that their salaries are almost entirely explained by on-court performance statistics X, with minimal residual bias.

3.3 Validation via Contract Stratification

To validate the structural insights, we overlaid the map with metadata regarding contract types. This revealed a stark separation:

- **Rookie Contracts:** As hypothesized, players on Rookie Scale contracts clustered exclusively around the 'Draft Bias' anchor.
- **Free Market Veterans:** Players in the D_{FM} subset dispersed widely toward anchors like 'Age' and 'Experience', confirming that once the artificial constraint of the rookie scale is removed, valuation becomes sensitive to tenure-based biases.

This visual separation validates the efficacy of the "Learn-Apply" stratification, proving that the model successfully disentangled the artificial constraints of the CBA from true market preferences.

4 Discussion

The development of the DML-Unfolding framework presented significant methodological challenges, primarily centered on the incompatibility of standard econometric outputs with geometric visualization tools.

4.1 Bridging the Analytical Gap

The primary innovation of this work lies in solving the "Consumer-Product Inverse Problem" within an economic context. Traditional DML workflows terminate at the estimation of $\hat{\gamma}$, effectively treating the bias coefficients as the final answer. However, we identified that these coefficients are merely the "price" in a larger system. The challenge was converting these scalar prices into a format suitable for the dimension reduction algorithm, which expects a "liking" matrix.

Our solution, the Debiased Attribution Matrix ($L_{ij} = |\hat{\gamma}_j \cdot \hat{\epsilon}_{Z,ij}|$), proved critical. By interacting the price of bias with the magnitude of the residual, we successfully unified disparate units (e.g., years of age vs. millions in market size) into a single, comparable currency of "unexplained log-dollars". Without this normalization, the latent space map would have been distorted by the arbitrary scaling of the input features rather than their true economic influence.

4.2 Limitations and Transportability

A central limitation of our stratified approach is the reliance on the "Transportability Assumption". We assume that the "price of bias" learned from the free-market subpopulation (D_{FM}) is structurally stable and applies counterfactually to players on deterministic contracts (Rookies, Max, Minimum). While necessary to avoid model misspecification, this assumes that teams value attributes like "Draft Pedigree" identically for a negotiated veteran as they do for a fixed-scale rookie.

Furthermore, applying the treatment models $\hat{h}_{FM}$ to the full population introduces extrapolation risks. If the performance profiles of rookies differ systematically from veterans (e.g., lower average minutes or efficiency), the model may struggle to accurately predict their "fair" bias attribution. Future iterations of this work should incorporate rigorous covariate overlap tests (e.g., Kolmogorov-Smirnov) to quantify the validity of these counterfactuals.

4.3 Practical Implications

Despite these limitations, the framework offers immediate diagnostic value. Stakeholders can now distinguish between "Pure Performance" players—who reside in the map's center and are valued efficiently—and those subject to heavy structural distortion. For General Managers, this provides a tool to identify market inefficiencies not just by magnitude, but by type, revealing whether a player's valuation is driven by sustainable on-court production or transient structural factors like "Hype" or "Market Size".

5 Conclusion

This report has detailed a rigorous methodological framework for mapping the latent structure of economic bias in professional sports. By fusing the causal identification power of Stratified Double Machine Learning (DML) with the geometric visualization capabilities of probabilistic latent unfolding, we have created an analytical tool that transcends traditional regression analysis.

We successfully transformed the problem of bias detection from a static list of coefficients into a dynamic topological map. This "Bias Attribution Map" allows stakeholders—from General Managers to Agents—to identify market inefficiencies not just by magnitude, but by structural type. It reveals not just *if* a player is misvalued, but *why*, situating them within a peer group facing the same structural economic forces.

5.1 Future Work

The "Bias Attribution Map" serves as a powerful new analytical object, opening several avenues for future research. First, a longitudinal extension of this framework could estimate bias coefficients γ_t for multiple seasons, revealing whether the geometric structure of bias is stable or evolves in response to rule changes. Second, this DML-Unfolding fusion is domain-agnostic; it can be readily applied to other high-stakes fields where "treatment effects" can be attributed, such as mapping property feature biases in real estate or candidate attribute biases in algorithmic hiring.