Quantum Bell States Measurement and Visualization Project

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Environment: Qiskit, Python 3.10.11
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Project Overview

This project demonstrates **quantum entanglement preparation, measurement, and visualization** for **Bell states** using Qiskit.

The Bell states (Φ^+, Φ^-, Ψ^+, Ψ^-) are fundamental examples of **maximally entangled two-qubit states**, and their behavior under different measurement bases (XX, YY, XZ).

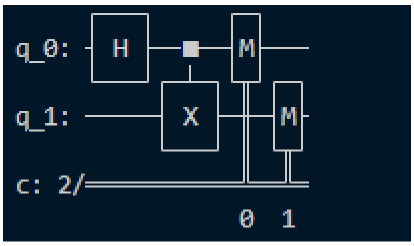
Tasks Overview

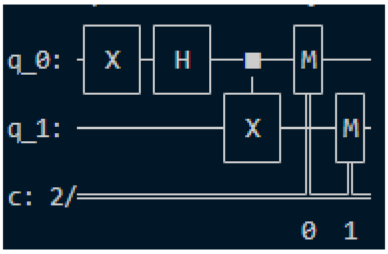
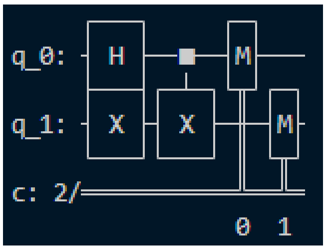
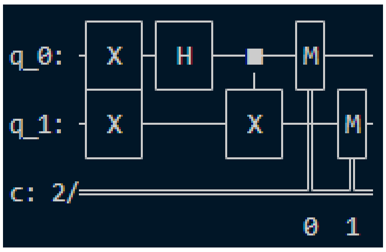
Task	Description
1–4	Preparation and measurement of Bell states Φ^+, Φ^-, Ψ^+, Ψ^-
5–7	Measurement of all Bell states in **XX**, **YY**, and **XZ** bases

All simulations use:

- **2 qubits**
- **2048 shots per run**
- **3 executions per configuration**
- **Random seed:** 151936
- **Simulator:** Qiskit Aer

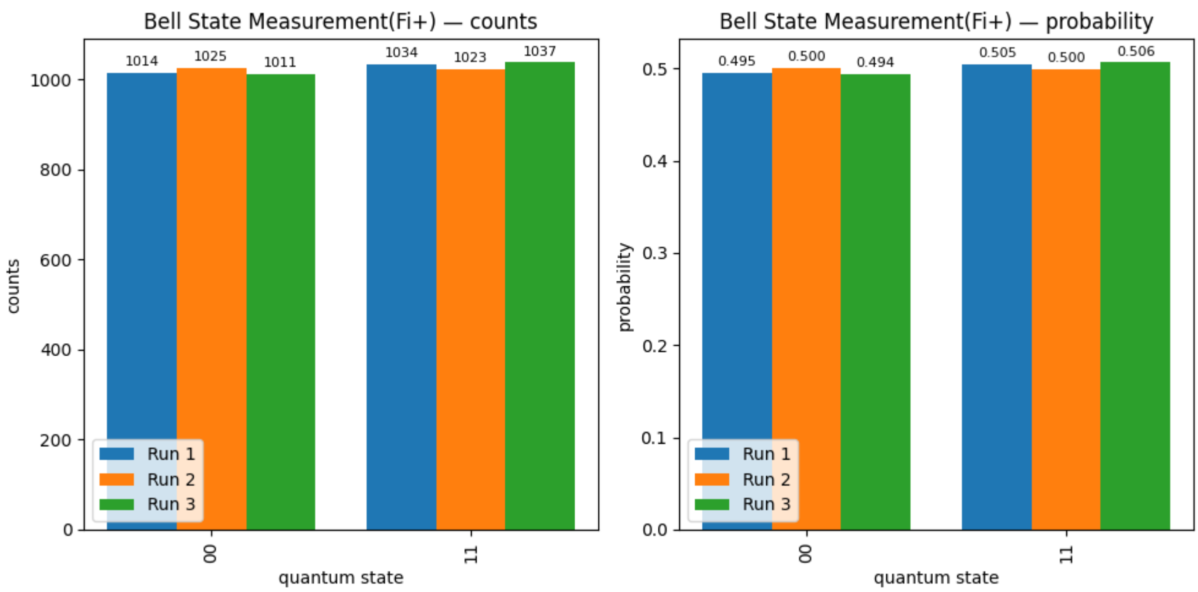
Bell State Circuits

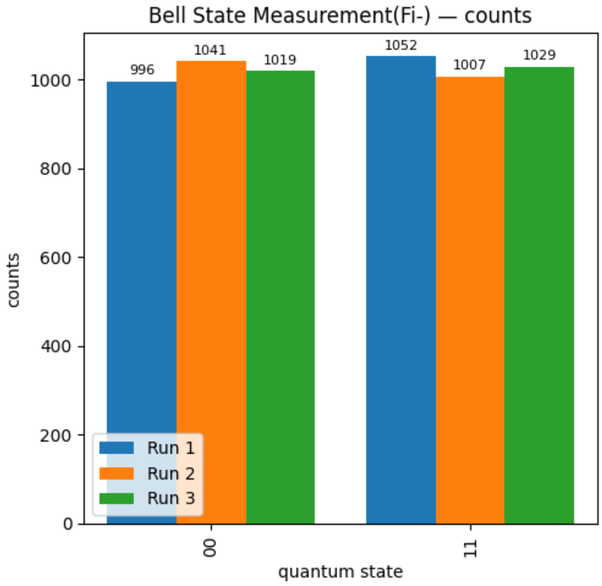
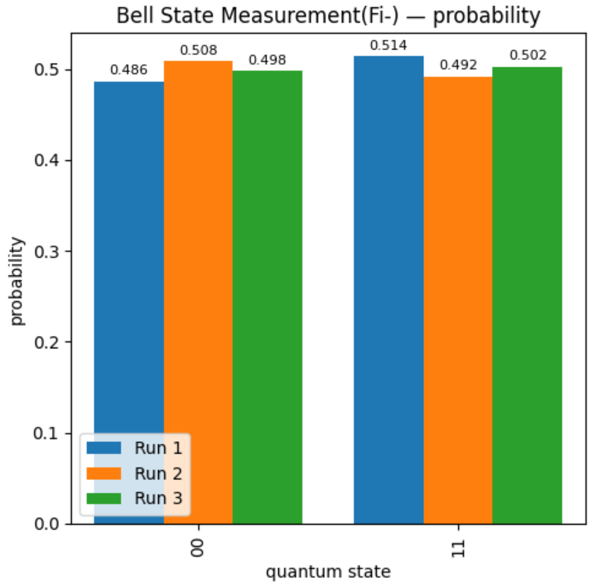
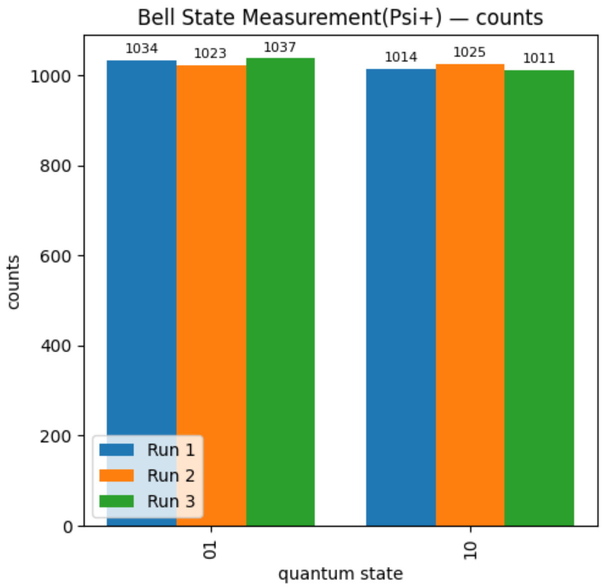
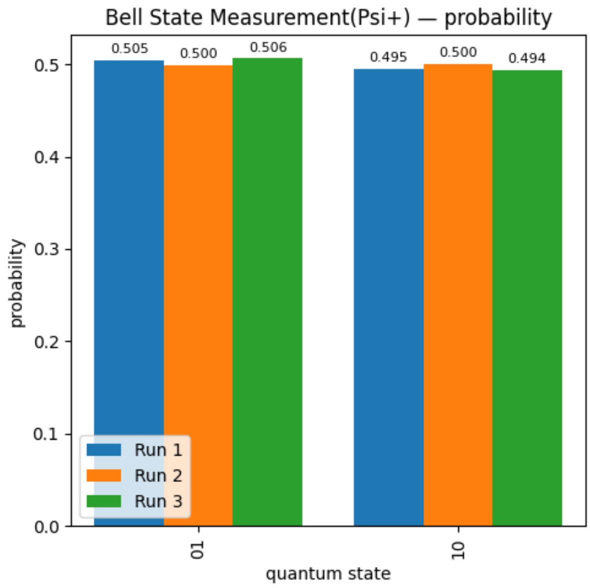
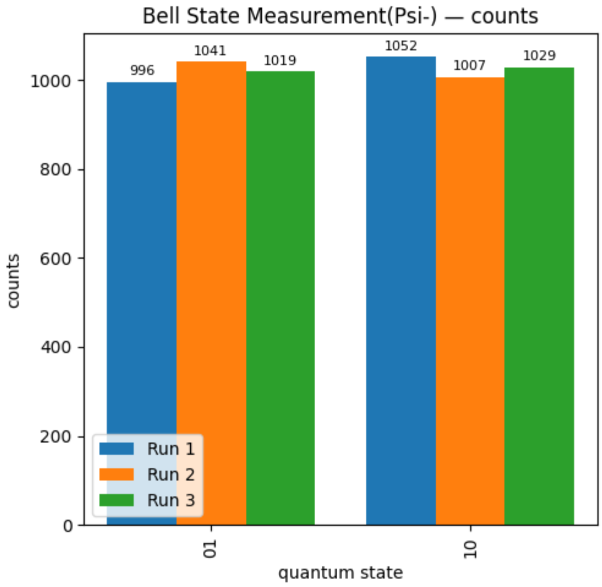
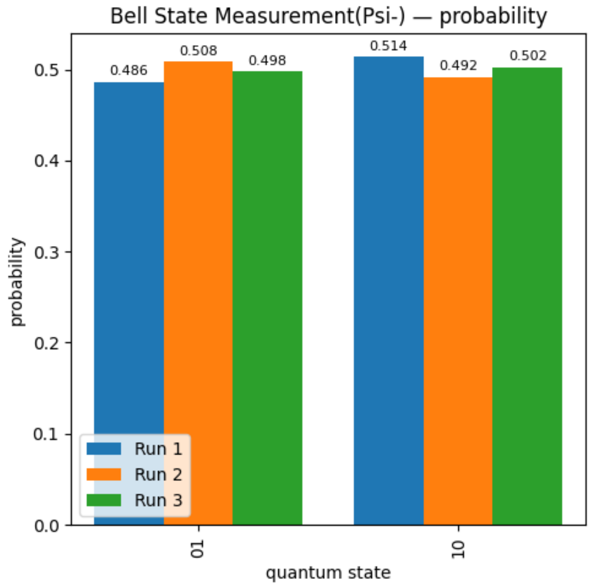
Bell State	Quantum Circuit
Φ^+ (Fi$^+$)	

Bell State	Quantum Circuit
Φ^- (Fi$^-$)	
Ψ^+ (Psi$^+$)	
Ψ^- (Psi$^-$)	

Task 1–4: Bell State Preparation and Measurement

Each Bell state was prepared, measured in the computational basis (Z), and run three times.
The histograms below show both **counts** and **probabilities**, with labels indicating exact values.

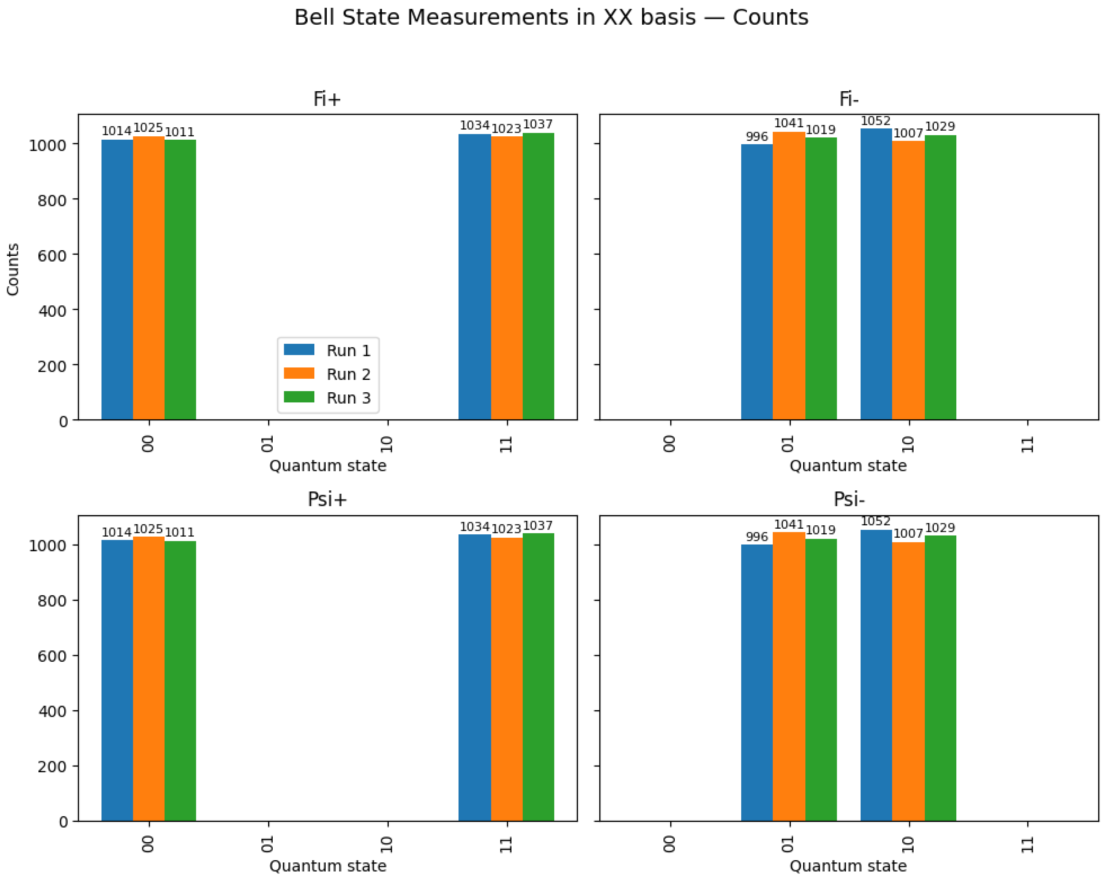
Bell State	Counts & Probabilities
Φ^+ (Fi$^+$)	

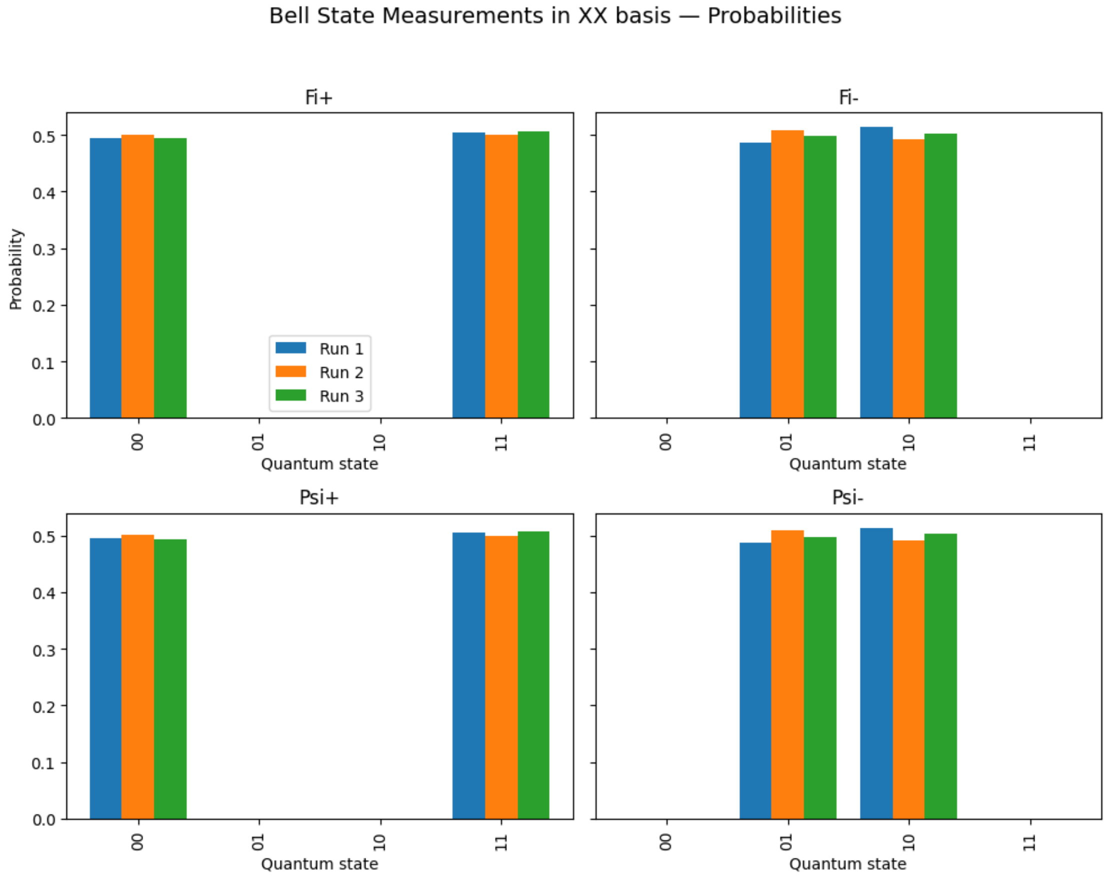
Bell State	**Counts & Probabilities**
Φ^- (Fi$^-$)	 
Ψ^+ (Psi$^+$)	 
Ψ^- (Psi$^-$)	 

Task 5–7: Bell State Measurements in Different Bases

The Bell states were measured in **XX**, **YY**, and **XZ** bases to demonstrate quantum correlations in rotated measurement spaces.
Each plot aggregates data from all four Bell states in a **2×2 grid** layout.

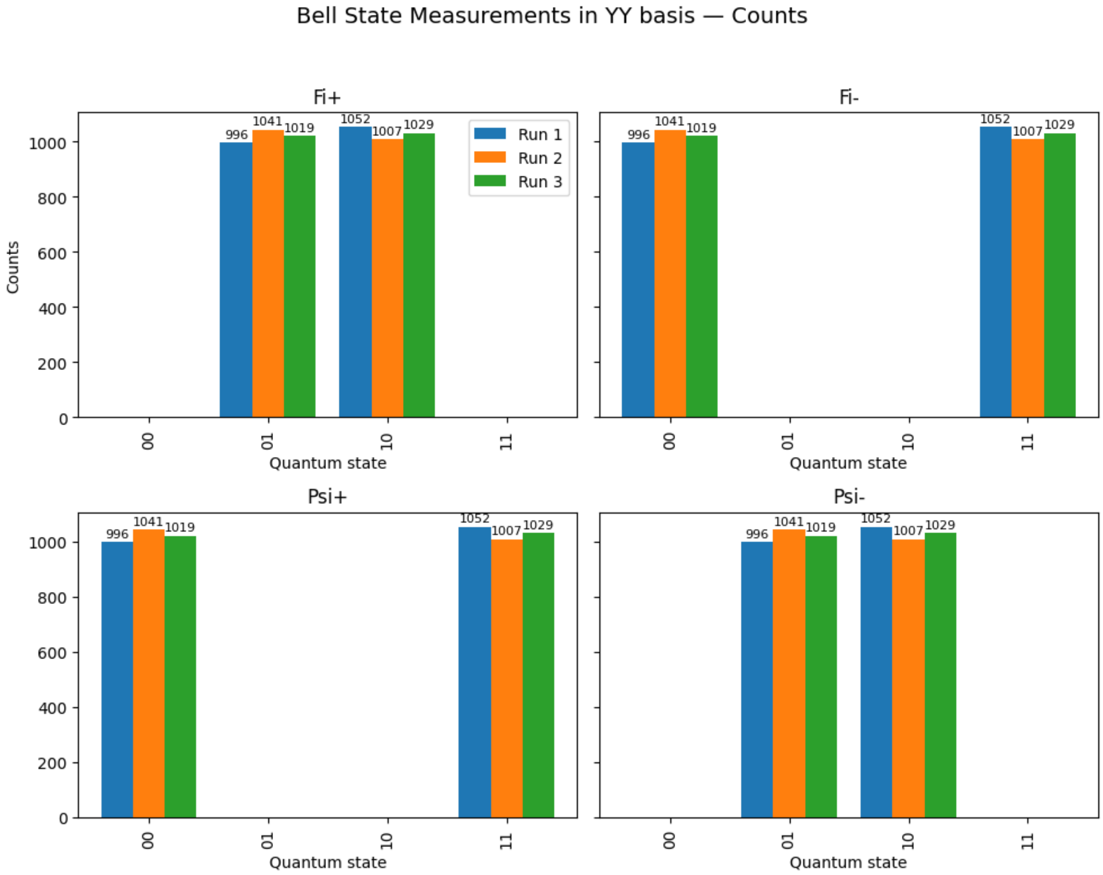
Task 5 — Measurement in XX Basis

Type	Image
Counts	

Type	Image
Probabilities	

Task 6 — Measurement in YY Basis

Type	Image

<table>
<tr><th>Type</th><th>Image</th></tr>
<tr><td>Counts</td><td>
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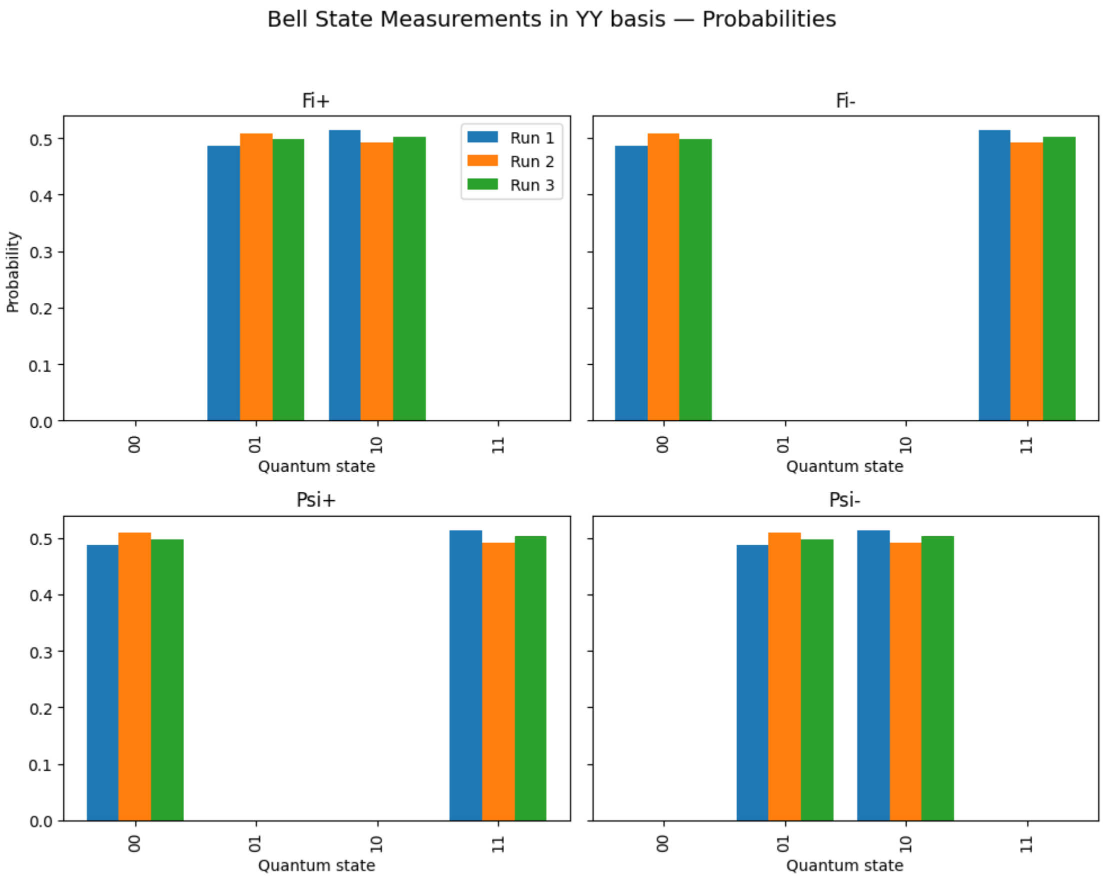
<tr><td>Probabilities</td><td>
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Task 7 — Measurement in XZ Basis

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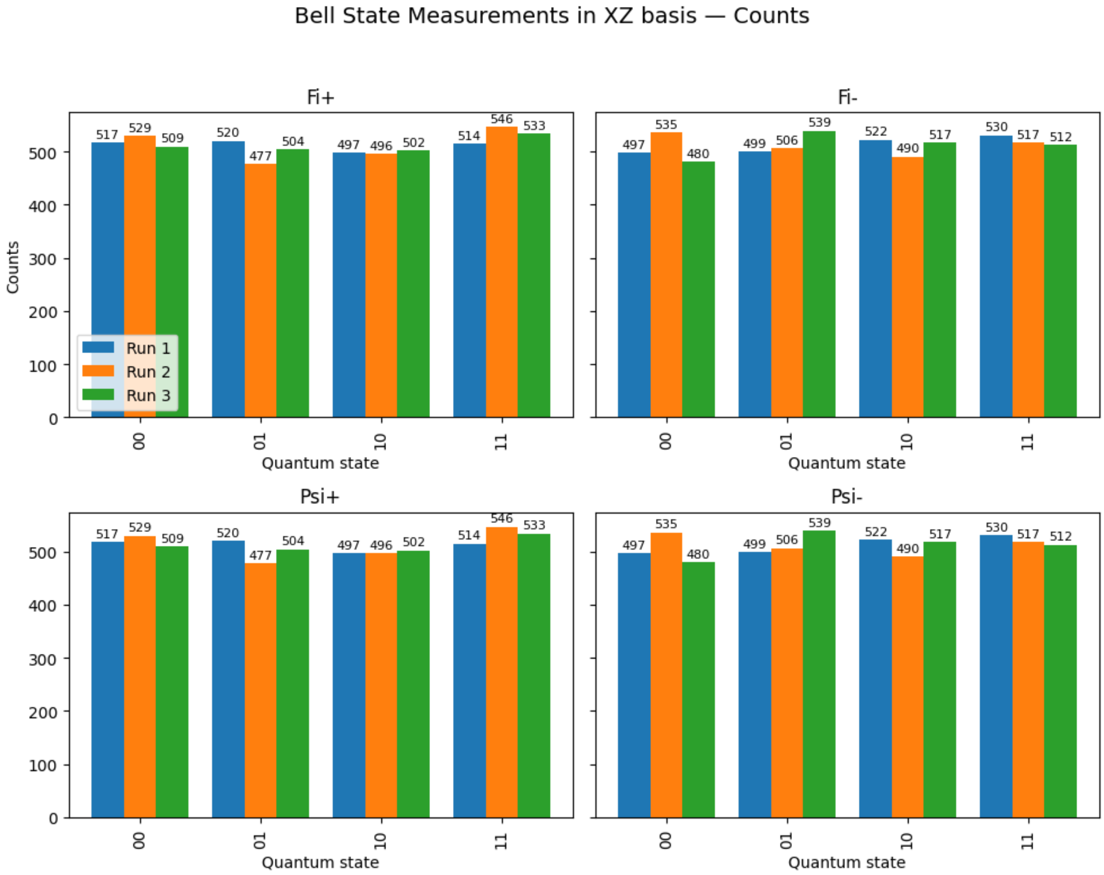
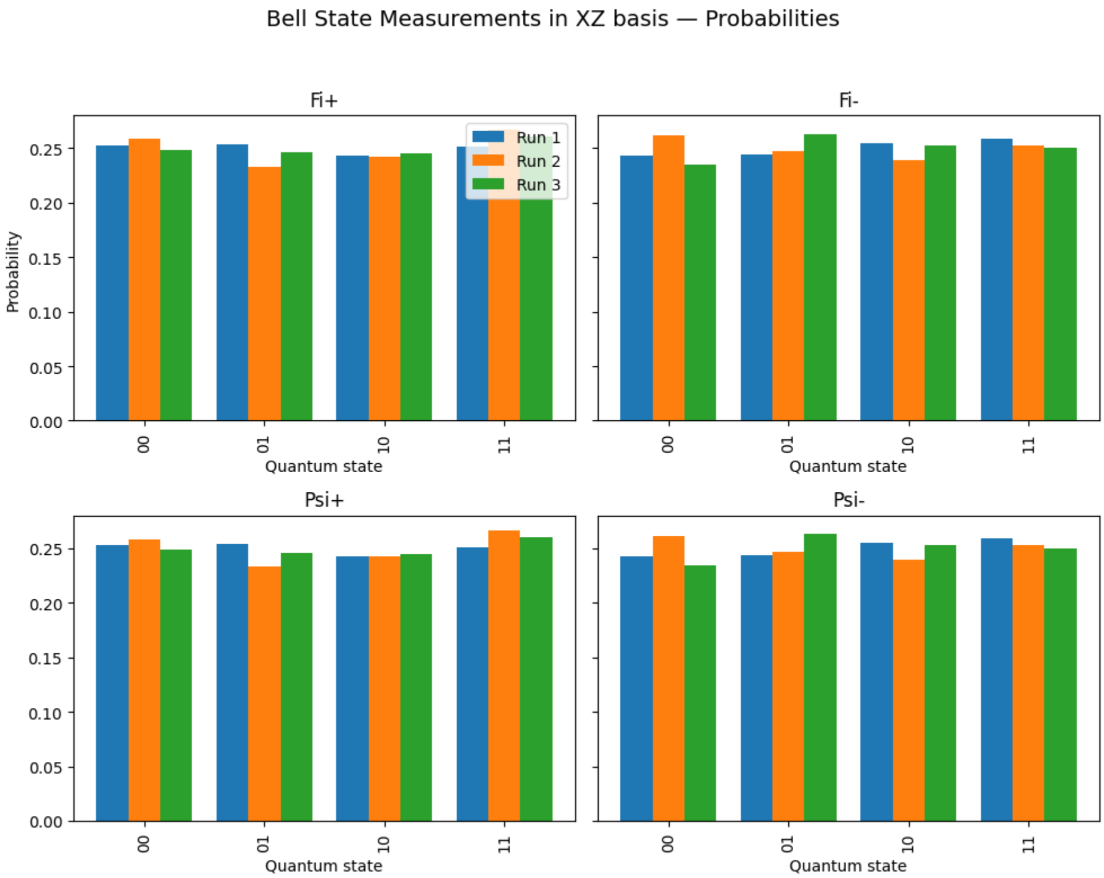
<tr><td>Counts</td><td>
</td></tr>
<tr><td>Probabilities</td><td>
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Additional Paper Calculations

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2.
$$\begin{bmatrix}1&0&0&0\\0&0&0&1\\0&0&1&0\\0&1&0&0\end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix}1&1&0&0\\1&-1&0&0\\0&0&-1&-1\\0&0&-1&1\end{bmatrix}\begin{bmatrix}0&1&0&0\\1&0&0&0\\0&0&0&1\\0&0&1&0\end{bmatrix}\begin{bmatrix}1\\0\\0\\0\end{bmatrix}=$$

$$=\frac{1}{\sqrt{2}}\begin{bmatrix}1&1&0&0\\0&0&-1&1\\0&0&-1&-1\\1&-1&0&0\end{bmatrix}\begin{bmatrix}0&1&0&0\\1&0&0&0\\0&0&0&1\\0&0&1&0\end{bmatrix}\begin{bmatrix}1\\0\\0\\0\end{bmatrix}=\frac{1}{\sqrt{2}}\begin{bmatrix}1&1&0&0\\0&0&1&-1\\0&0&-1&-1\\-1&1&0&0\end{bmatrix}\begin{bmatrix}1\\0\\0\\0\end{bmatrix}=\frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\\0\\-1\end{bmatrix}=|\Phi-\rangle=\frac{1}{\sqrt{2}}(|00\rangle-|11\rangle)$$

3. $CX(1\otimes H)(X\otimes 1)|00\rangle$

$$\cancel{|00\rangle=|0\rangle\otimes|00\rangle}\quad |00\rangle=|0\rangle\otimes|0\rangle\xrightarrow{X\otimes 1}|X|0\rangle\otimes|0\rangle=$$

$$=|1\rangle\otimes|0\rangle\xrightarrow{1\otimes H}|1\rangle\otimes|H|0\rangle=|1\rangle\otimes\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)=$$

$$=\frac{1}{\sqrt{2}}(|1\rangle\otimes|0\rangle+|1\rangle\otimes|1\rangle)\xrightarrow{C_X}\frac{1}{\sqrt{2}}(C_X(|10\rangle)+C_X(|11\rangle))=$$

$$=\frac{1}{\sqrt{2}}(|11\rangle+|01\rangle)=\frac{1}{\sqrt{2}}(|01\rangle+|10\rangle)=|\Psi+\rangle$$

4. $C_X(1\otimes H)(X\otimes X)|00\rangle$

$$|00\rangle=|0\rangle\otimes|0\rangle\xrightarrow{X\otimes X}|X|0\rangle\otimes|X|0\rangle=|1\rangle\otimes|1\rangle\xrightarrow{1\otimes H}|1\rangle\otimes|H|1\rangle=$$

$$=|1\rangle\otimes\left(\frac{1}{\sqrt{2}}|0\rangle-|1\rangle\right)=\frac{1}{\sqrt{2}}(|1\rangle\otimes|0\rangle-|1\rangle\otimes|1\rangle)\xrightarrow{C_X}$$

$$\frac{1}{\sqrt{2}}(C_X|10\rangle-C_X|11\rangle)=\frac{1}{\sqrt{2}}(|11\rangle-|01\rangle=|\Psi-\rangle$$

Conclusion

This experiment demonstrates:

- The creation of **entangled Bell states** using simple quantum circuits.

- The dependence of measured outcomes on **measurement basis**.
- **Clear quantum correlations** that distinguish the four Bell states.
- Reproducibility across multiple runs, confirming simulation stability.
