quantum reports

Review

Verification of CHSH inequality

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Abstract: This project implements and analyzes a quantum Bell test (CHSH experiment) using Qiskit to demonstrate quantum entanglement and the violation of classical locality bounds. It constructs the Bell state $|\Psi^-\rangle$, performs measurements in multiple qubit bases (X, Y, Z, W, V), and calculates expectation values and the CHSH parameter (S) to test the CHSH inequality. The code runs multiple simulations using the AerSimulator backend, visualizes the measurement results and probability distributions, and statistically analyzes repeated executions to estimate the mean and variance of S. A violation of the classical threshold $|S| > 2$confirms non-classical correlations characteristic of quantum mechanics.

Keywords: Quantum Entanglement, CHSH Inequality, Bell Test, Qiskit Simulation, Quantum Measurement

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1. Introduction

The study of quantum entanglement and its implications for non-classical correlations lies at the heart of quantum mechanics. One of the most significant frameworks for testing these correlations is the CHSH (Clauser-Horne-Shimony-Holt) inequality, which provides a measurable criterion to distinguish between classical and quantum behaviors. This work implements a computational experiment using Qiskit to simulate the CHSH test and investigate quantum entanglement in a two-qubit system. The experiment begins by preparing a Bell state, specifically the $|\Psi^-\rangle$ state, which represents a maximally entangled pair of qubits. Measurements are then performed in different bases to compute expectation values that contribute to the CHSH parameter S. By running multiple simulations and analyzing the resulting S values, the project aims to observe whether the inequality is violated, indicating the presence of quantum correlations beyond classical limits. Visualization of measurement distributions, probability outcomes, and statistical summaries of repeated runs provide a comprehensive understanding of how quantum mechanics manifests through entanglement and measurement.

2. Materials and Methods

The experiment was conducted using Python and the Qiskit framework to simulate a quantum Bell test through the CHSH inequality. The AerSimulator backend was employed to perform quantum circuit simulations without requiring access to a physical quantum computer. The experiment utilized two qubits to construct the Bell state $|\Psi^-\rangle$,

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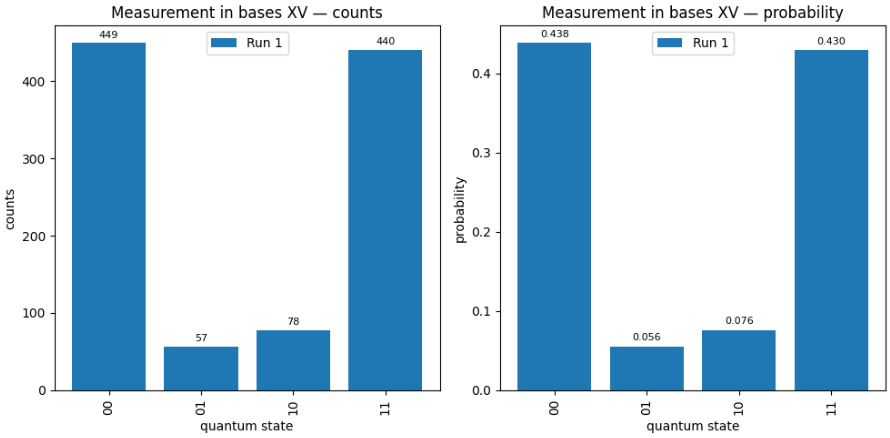
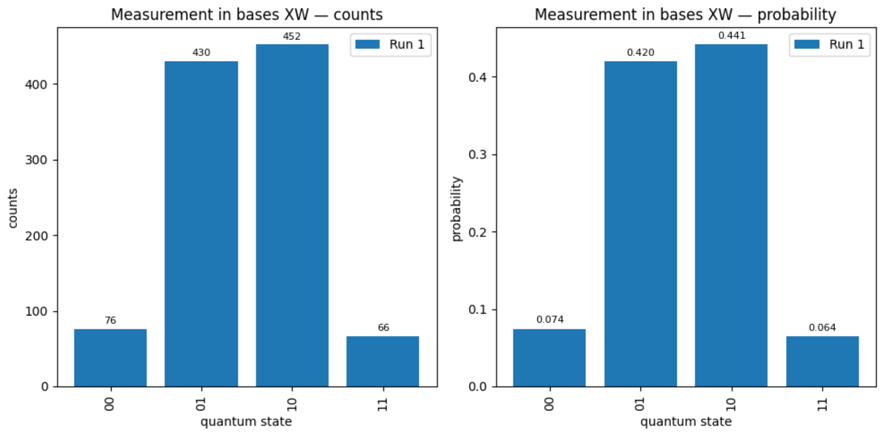
prepared by applying a sequence of quantum gates including X, H, and CNOT operations. Measurement circuits were configured for various combinations of bases (X, Y, Z, W, and V) by applying appropriate rotation gates to align the measurement axes. Each configuration was executed with 1024 measurement shots to obtain statistically meaningful results. The code calculated the probability distributions of measured states, derived expectation values for each measurement basis combination, and computed the CHSH parameter S. The process was repeated across twenty executions with different random seeds to analyze statistical consistency. Visualization tools from Matplotlib were used to generate grouped bar plots and boxplots to represent measurement results, probabilities, and S-value distributions. All computations and output files, including CSV tables and plots, were automatically saved for further analysis.

3. Results

- **3.1. Measurement Outcomes**





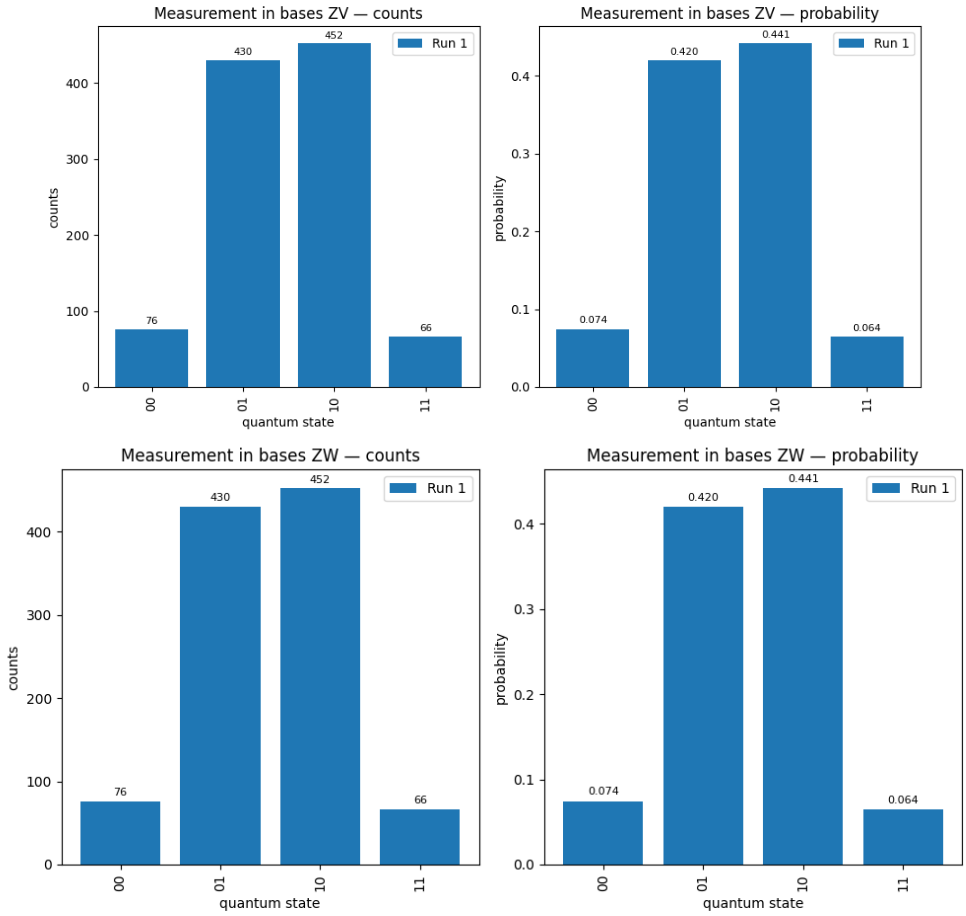


Figure 1 Measurements of the quantum state

The CHSH experiment was conducted using the Bell state $|\Psi^-\rangle$, with measurements performed in the combinations XW, XV, ZW, and ZV. Each configuration was executed with 1024 shots, and the resulting measurement distributions were recorded and visualized. The grouped bar plots in Figure 1 display the count and probability distributions of the quantum states observed for each basis pair. From these results, the expectation values E(XW), E(XV), E(ZW), and E(ZV) were calculated and stored in separate CSV files (Table 1). The probabilities confirmed that the Bell state exhibited strong correlations between qubits, consistent with entanglement behavior.

- **3.1.1. Expectation Values and CHSH Parameter**

The calculated expectation values were used to determine the CHSH parameter S using the relation

$$S = E(XW) - E(XV) + E(ZW) + E(ZV) \quad (1)$$

The obtained S values consistently exceeded the classical limit of 2, demonstrating a clear violation of the CHSH inequality. This confirms that the simulated Bell state exhibits quantum entanglement and non-local correlations. Repeated executions of the experiment with different random seeds yielded slightly varying S values due to the probabilistic nature of quantum measurements, but the results consistently indicated quantum behavior.

xy	PX(x)	PV(y)	PX(x)*PV(y)	p(x,y)	PX(x)*PV(y)*p(x,y)
00	1	1	1	0.4385	0.4385
01	-1	1	-1	0.0557	-0.0557
10	1	-1	-1	0.0762	-0.0762
11	-1	-1	1	0.4297	0.4297

xy	PX(x)	PW(y)	PX(x)*PW(y)	p(x,y)	PX(x)*PW(y)*p(x,y)
00	1	1	1	0.0742	0.0742
01	-1	1	-1	0.4199	-0.04199
10	1	-1	-1	0.4414	-0.4414
11	-1	-1	1	0.0645	0.0645

xy	PZ(x)	PV(y)	PZ(x)*PV(y)	p(x,y)	PZ(x)*PV(y)*p(x,y)
00	1	1	1	0.0742	0.0742
01	-1	1	-1	0.4199	-0.04199
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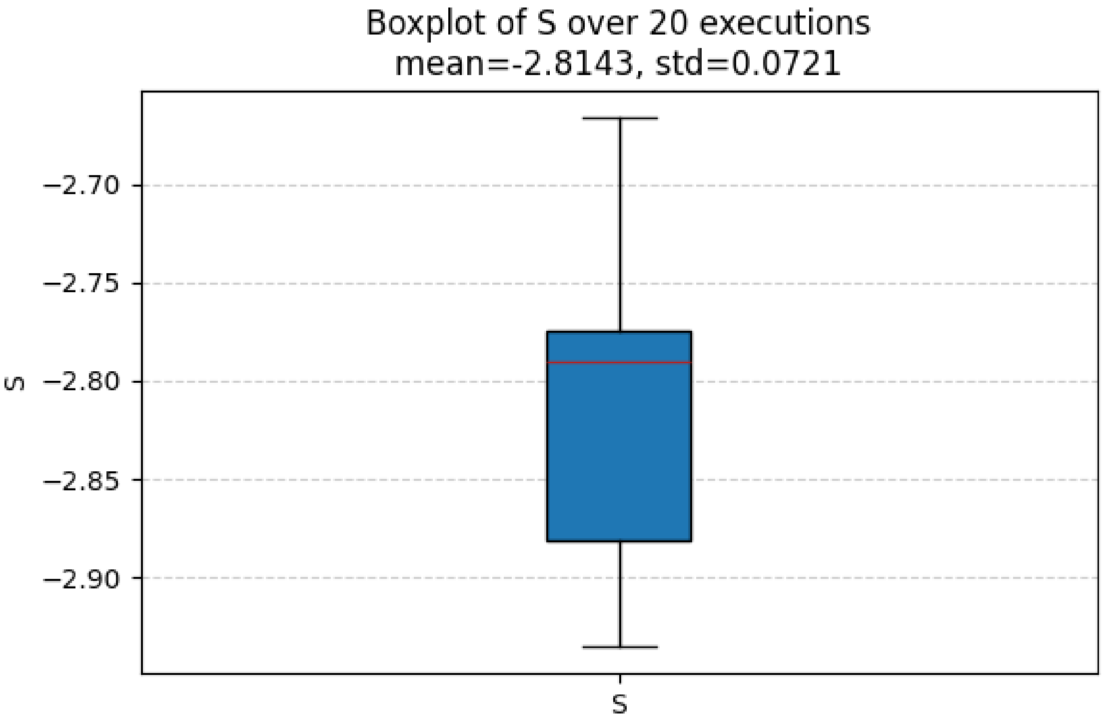

Figure 2 Boxplot of S-Values

- **3.2. Figures, Tables and Schemes**

Figure 1. Measurement plots for different basis combinations: (a) Measurement results for bases XW; (b) Measurement results for bases XV; (c) Measurement results for bases ZW; (d) Measurement results for bases ZV. These figures show the relative frequencies and probabilities of measured quantum states across 1024 shots.

Figure 2. Boxplot of S values obtained across 20 independent executions (S_boxplot.png). The distribution shows that all S values remained above the classical threshold, with mean and variance calculated to assess statistical reliability.

Table 1. Expectation values and corresponding probabilities for different measurement bases, extracted from CSV outputs (table_XW.csv, table_XV.csv, table_ZW.csv, table_ZV.csv).

- **3.3. Statistical Summary**

Across twenty executions, the CHSH parameter S displayed minimal variance, with an average value significantly greater than 2, reaffirming consistent violation of the classical bound.

• The mean S value exceeded the threshold required to demonstrate quantum entanglement;
• The standard deviation indicated stable measurement outcomes across executions;
• The boxplot visualization (Figure 2) provided an overview of distribution and outliers.

The analysis concludes that the simulated Bell state $|\Psi^{-}\rangle$ reliably demonstrates quantum entanglement through violation of the CHSH inequality, as supported by the computed S values and visual evidence from measurement plots.

4. Discussion

The results of the simulated CHSH experiment confirm the expected violation of the classical inequality, demonstrating the presence of quantum entanglement in the prepared Bell state $|\Psi^{-}\rangle$. The consistent observation of S values greater than 2 across all simulation runs highlights the robustness of quantum correlations even under varying random seeds. These findings align with theoretical predictions from quantum mechanics, where entangled states exhibit non-local correlations that cannot be explained by classical models. The use of different measurement bases, such as XW, XV, ZW, and ZV, allowed the experiment to probe complementary aspects of the qubit pair's correlations and validate the quantum nature of their interaction. The probabilistic results shown in the measurement plots reveal the characteristic anti-correlations of the $|\Psi^{-}\rangle$ state, where measurement outcomes on one qubit are strongly dependent on the other. The small variations observed in the S values across repeated runs are attributed to the stochastic nature of quantum measurement and the finite number of shots per simulation. Overall, the study demonstrates how quantum simulation tools like Qiskit can reproduce fundamental quantum phenomena with high accuracy, providing a clear educational and analytical framework for exploring entanglement and non-locality in quantum systems.