

```
In [1]: # Imports for Qiskit
from qiskit import QuantumCircuit, ClassicalRegister, QuantumRegister, transpile
from qiskit.visualization import plot_histogram
import qiskit.circuit.library as qelib
from qiskit_aer import AerSimulator

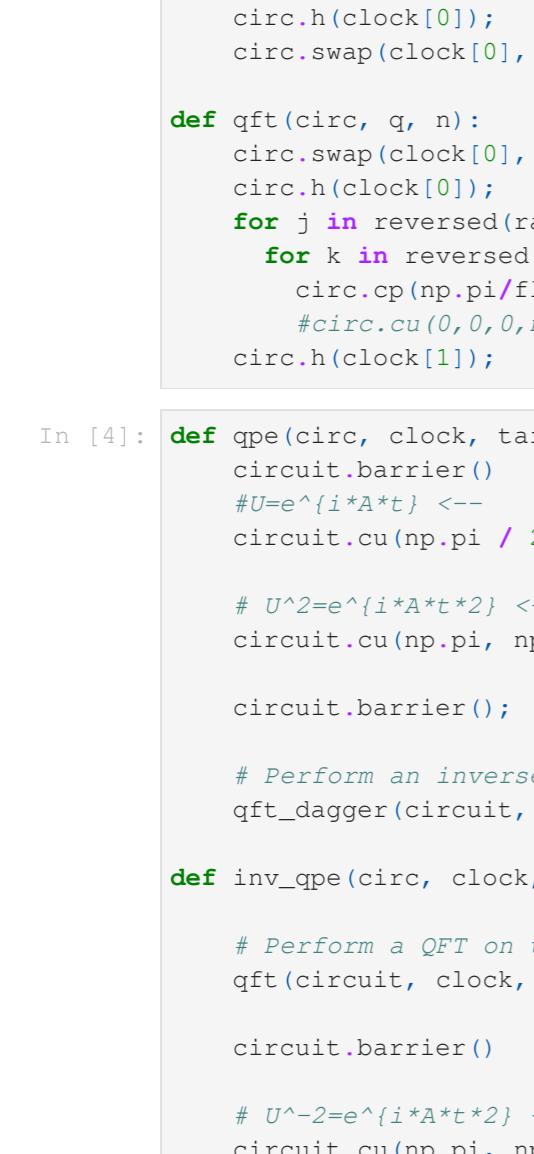
# Various imports
import numpy as np

##IBMQ.save_account('Put your token')
##IBMQ.load_account()
##IBMQ.get_provider(hub='ibm-q', group='open', project = 'main')

In [2]: # Create the various registers needed
clock = QuantumRegister(2, name='clock')
input_reg = QuantumRegister(1, name='b')
ancilla = QuantumRegister(1, name='ancilla')
measurement = ClassicalRegister(2, name='c')

# Create an empty circuit with the specified registers
circuit = QuantumCircuit(ancilla, clock, input_reg, measurement)

circuit.barrier()
circuit.draw(output='mpl')
```



```
In [3]: def qft_dagger(circ, q, n):
    circ.h(clock[1])
    for j in reversed(range(n)):
        for k in reversed(range(j+1,n)):
            circ.cp(np.pi/float(2**(k-j)), q[k], q[j])
            circ.u3(-np.pi/float(2**(k-j)), q[k], q[j])
            circ.h(clock[0])
            circ.swap(clock[0], clock[1])

def qft(circ, q, n):
    circ.swap(clock[0], clock[1])
    circ.h(clock[0])
    for j in reversed(range(n)):
        for k in reversed(range(j+1,n)):
            circ.cp(np.pi/float(2**(k-j)), q[k], q[j])
            circ.u3(np.pi/float(2**(k-j)), q[k], q[j])
            circ.h(clock[0])
            circ.swap(clock[0], clock[1]))
```

```
In [4]: def op_e(circ, clock, target):
    circuit.barrier()
    # U^(-1)*e^(i*pi*t) <-->
    circuit.u3(np.pi / 2, -np.pi / 2, np.pi / 2, 3 * np.pi / 4, clock[0], input_reg, label='U');

    # U^(-2)*e^(i*pi*t) <-->
    circuit.u3(np.pi, 0, 0, clock[1], input_reg, label='U2');

    circuit.barrier();
    # Perform an inverse QFT on the register holding the eigenvalues
    qft_dagger(circ, clock, 2)

def inv_gpe(circ, clock, target):
    # Perform a QFT on the register holding the eigenvalues
    qft(circ, clock, 2)

    circuit.barrier()
    # U^(-2)*e^(i*pi*t) <-->
    circuit.u3(np.pi, 0, 0, clock[0], input_reg, label='U2');

    circuit.barrier()
```

```
In [5]: def hhl(circ, ancilla, clock, input, measurement):
    qpe(circ, clock, input)
    circuit.barrier()

    # This section is to test and implement <-->
    #circuit.cry(np.pi, clock[0], ancilla)
    #circuit.u3(np.pi, 0, 0, clock[1], ancilla)
    ccy1=qelib.RYGate(np.pi).control(2, ctrl_state='01')
    circuit.append(ccy1,clock[0],clock[1],ancilla)
    ccy2=qelib.RYGate(np.pi/3).control(2, ctrl_state='10')
    circuit.append(ccy2,clock[0],clock[1],ancilla)
    circuit.barrier()

    circuit.measure(ancilla, measurement[0])
    inv_gpe(circ, clock, input)
```

```
In [6]: # State preparation
initial_state = [0,1]
circuit.initialize(initial_state, 3)
```

```
circuit.barrier()

# Perform a Hadamard Transform
circuit.h(clock)

hhl(circ, ancilla, clock, input_reg, measurement)
```

```
# Perform a Hadamard Transform
circuit.h(clock)
```

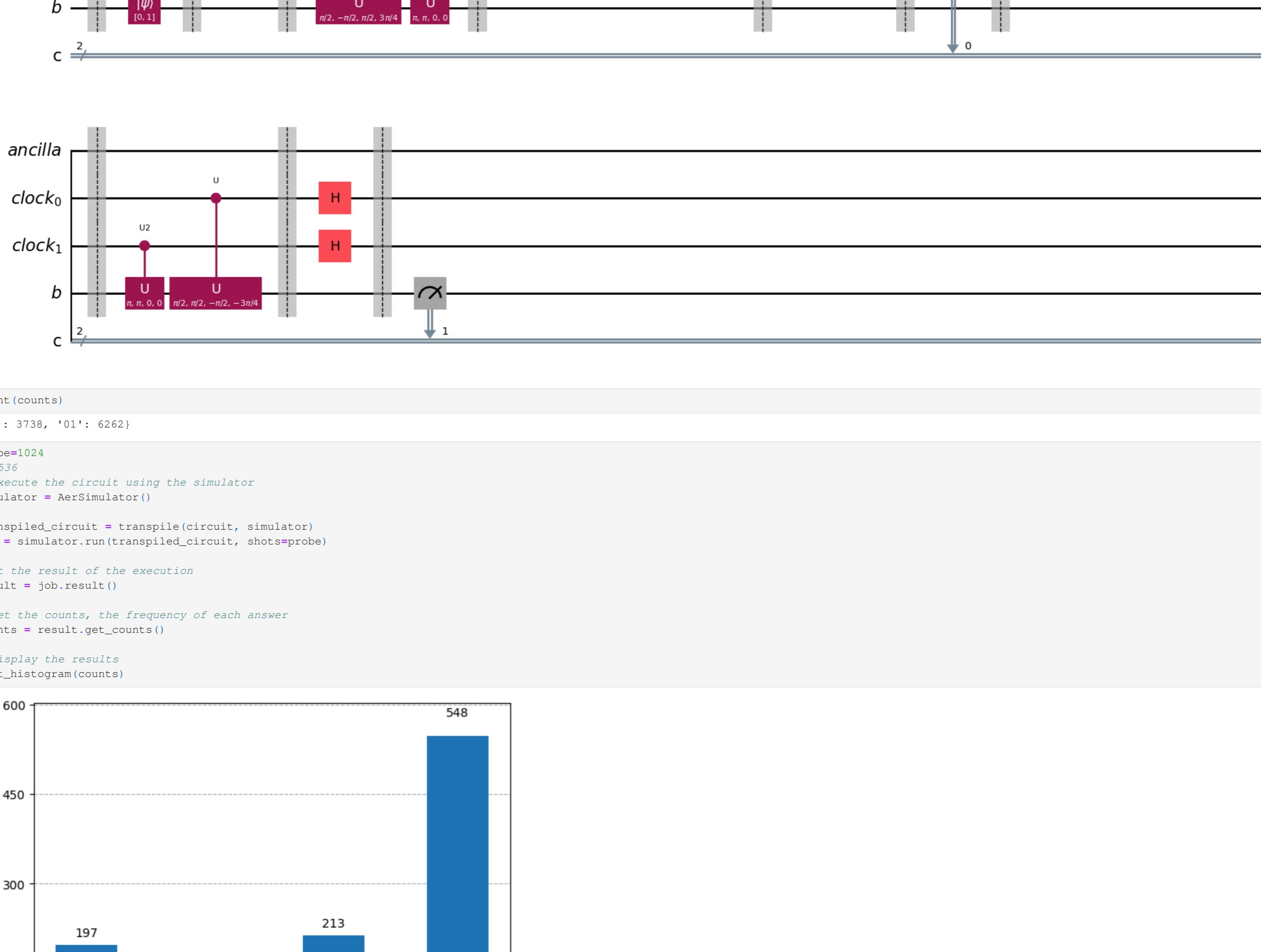
```
circuit.barrier()
```

```
#circuit.measure(input, measurement[1])
```

```
Out[6]: Circuit(operation=Instruction(name='barrier', num_qubits=4, num_cubits=0, params=[]), qubits=[Qubit register=(1, "ancilla"), index=0>, <Qubit register=(2, "clock"), index=0>, <Qubit register=(2, "clock"), index=1>, <Qubit register=(3, "b"), index=0>], cibits=())
```

```
In [7]: circuit.draw('mpl',scale=1)
```

```
Out[7]:
```



```
In [8]: simulator = AerSimulator()
```

```
transpiled_circuit = transpile(circuit, simulator)
```

```
shots = 10000
```

```
job = simulator.run(transpiled_circuit, shots=shots)
```

```
result = job.result()
```

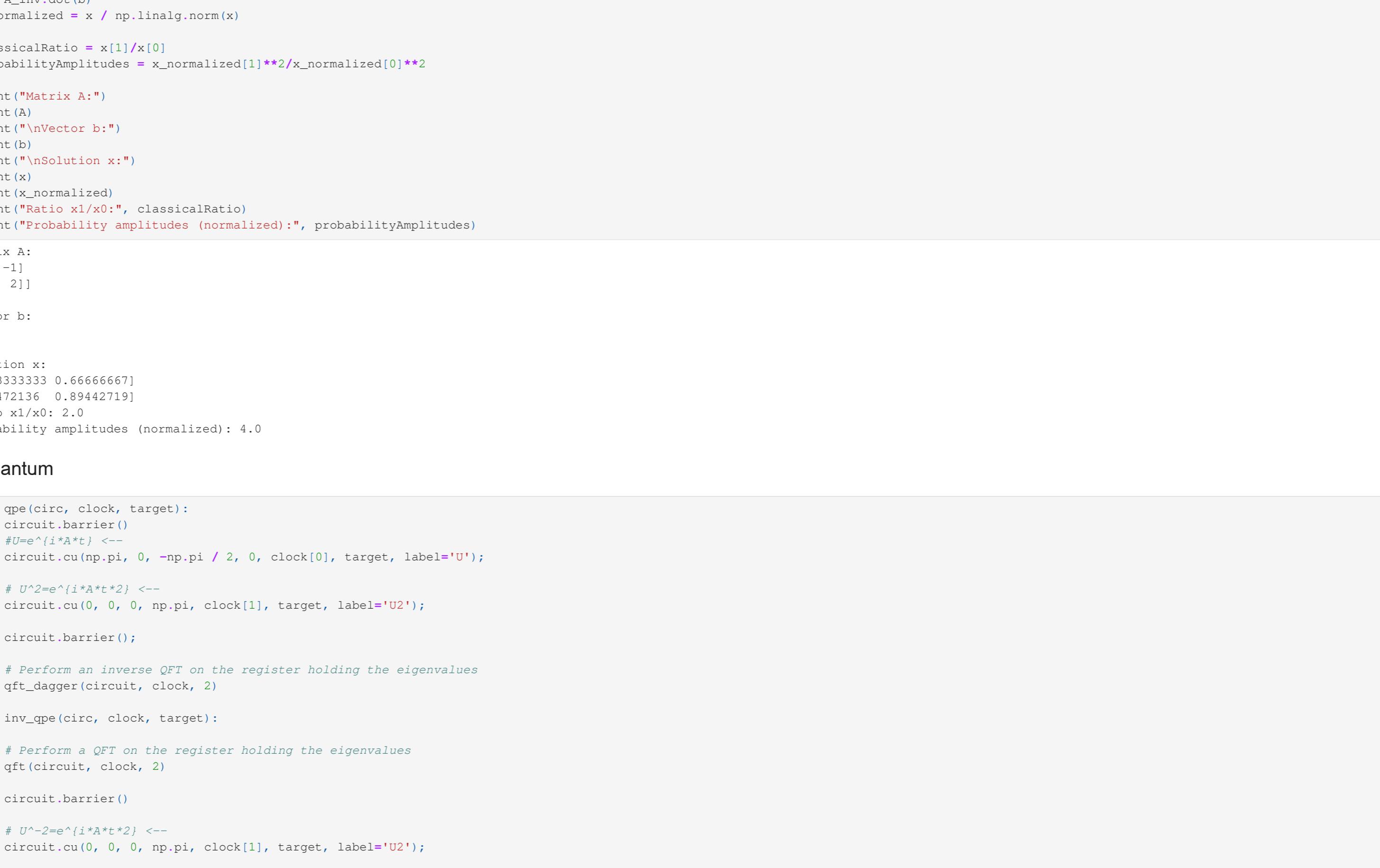
```
counts = result.get_counts()
```

```
print(counts)
```

```
{'00': 3738, '01': 6262}
```

```
In [9]: circuit.measure(input_reg, measurement[1])
circuit.draw('mpl',scale=1)
```

```
Out[9]:
```



```
In [10]: print(counts)
```

```
{'11': 548, '00': 197, '10': 213, '01': 66}
```

```
In [11]: 67593
```

```
Out[11]: 0.11298482293423272
```

```
In [12]: 1/6
```

```
Out[12]: 0.1111111111111111
```

```
In [13]: 2*np.arcsin(1/3)
```

```
Out[13]: np.float64(0.6796738189082439)
```

```
In [14]: np.rad2deg(0.6796738189082439)
```

```
Out[14]: np.float64(43.94244126989138)
```

```
In [15]: 38.94244126989138/180
```

```
Out[15]: 0.21634689593878548
```

```
In [16]:
```

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```
In [26]: import matplotlib.pyplot as plt
from qiskit.visualization import plot_histogram
```

```
In [27]: A = np.array([[2, -1],
                 [-1, 2]])
B = np.array([10, 11])
```

Classical

```
In [28]: A_inv = sp.linsolve.inv(A)
x = A_inv.dot(B)
x_normalized = x / np.linalg.norm(x)
```

```
classicalRatio = '|x|/x[0]
probabilityAmplitudes = x_normalized[1]**2/x_normalized[0]**2
```

```
print("Matrix A:")
print(A)
```

```
print("Vector b:")
print(b)
```

```
print("Normalized vector x:")
print(x_normalized)
```

```
print("Ratio x[1]/x[0]:", classicalRatio)
```

```
print("Probability amplitudes (normalized):", probabilityAmplitudes)
```

```
Matrix A:
[[ 2 -1]
 [-1  2]]
```

```
Vector b:
[10 11]
```

```
Solution x:
[0.3333333 0.6666667]
[0.4472136 0.89442719]
```

```
Ratio x[1]/x[0]: 2.0
```

```
Probability amplitudes (normalized): 4.0
```

Quantum

```
In [48]: def op_e(circ, clock, target):
    circuit.barrier()
    # U^(-1)*e^(i*pi*t) <-->
    circuit.u3(np.pi / 2, -np.pi / 2, np.pi / 2, 3 * np.pi / 4, clock[0], target, label='U');

    # U^(-2)*e^(i*pi*t) <-->
    circuit.u3(np.pi, 0, 0, clock[1], target, label='U2');

    circuit.barrier();
```

```
# Perform an inverse QFT on the register holding the eigenvalues
qft_dagger(circ, clock, 2)
```

```
def inv_gpe(circ, clock, target):
    # Perform a QFT on the register holding the eigenvalues
    qft(circ, clock, 2)
```

```
    circuit.barrier()
```

```
# U^(-2)*e^(i*pi*t) <-->
circuit.u3(np.pi, 0, 0, clock[0], target, label='U2');
```

```
    circuit.barrier()
```

```
def hhl(circ, ancilla, clock, input, measurement):
    qpe(circ, clock, input)
    circuit.barrier()
```

```
    # This section is to test and implement <-->
    #circuit.cry(np.pi, clock[0], ancilla)
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    circuit.append(ccy2,clock[0],clock[1],ancilla)
    circuit.barrier()
```

```
    circuit.measure(ancilla, measurement[0])
    inv_gpe(circ, clock, input)
```

```
    circuit.barrier()
```

```
    circuit.measure(input, measurement[1])
```

```
    circuit.draw('mpl', scale=1)
```

```
Out[48]:
```



```
In [107]: BACKEND = AerSimulator()
SHOTS = 1912
```

```
transpiled_circuit = transpile(circuit, BACKEND)
```

```
result = BACKEND.run(transpiled_circuit, shots=shots)
```

```
counts = result.get_counts()
```

```
successes = {}
for state, count in counts.items():
    if state[-1] == '1':
        successes[state] = count
```

```
print("Counts:", counts)
print("Successes (Ancilla=1):", successes)
```

```
quantCountsRatio = successes.get('11', 0) / successes.get('01', 1)
```

```
print("Quantum Ratio (from counts) x[0]/x[1]:", quantCountsRatio)
```

```
print("Classical Ratio x[0]/x[1]:", classicalRatio)
```

```
print("Classical Probability amplitude (normalized):", probabilityAmplitude)
```

```
plot_histogram(counts, title="HHL Measurement Results (All Registers)")

Count
```

```
0 1827 918 1768 3679
```

```
Out[107]:
```



```
In [108]:
```

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```
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    circuit.measure(ancilla, measurement[0])
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```
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```
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