Notes on Tilted Honeycomb lattice - induced field

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1 Change in hopping parameters

From [1] the tilt for Honeycomb lattice is given by

$$\zeta_x = 0,\ \zeta_y = \pm 2\frac{\tilde{t} - \bar{t}}{t}, \tag{1}$$

where $\tilde{t}$ is 2nd NN hopping in y direction, $\bar{t}$ is 2nd NN hopping for the rest of 2nd neighbours and t is hopping term for 1st NN. We decided to vary parameter ζ_y such that

$$\zeta_y(x, y) = 2\frac{\tilde{t}_a \sin\left(\frac{2\pi p}{Ma}\left(y + \frac{\sqrt{3}}{3}x\right)\right) - \bar{t}}{t},\ p \in 0, ..., M-1, \tag{2}$$

where a is distance between nearest sites and M is number of unit cells along y direction. The tilt is constant along $r_{\text{const}} = (3a/2, -\sqrt{3}a/2)$.

Acording to Heidar's calculations, variation of tilt induces pseudo electromagnetic field $\mathbf{A}$ with coefficients

$$\begin{aligned} A_x &= \frac{1}{4}\left(\zeta_x \frac{\partial}{\partial x}\zeta_x + \zeta_y \frac{\partial}{\partial y}\zeta_x\right) \\ A_y &= \frac{1}{4}\left(\zeta_x \frac{\partial}{\partial x}\zeta_y + \zeta_y \frac{\partial}{\partial y}\zeta_y\right) \\ A_t &= -\frac{1}{4}\nabla \cdot \boldsymbol{\zeta} - \frac{1}{4}\left(\zeta_x A_x + \zeta_y A_y\right), \end{aligned} \tag{3}$$

which influences hopping parameters as follows

$$\begin{aligned} c_n^\dagger c_n &\rightarrow A_t c_n^\dagger c_n \\ c_m^\dagger c_n &\rightarrow \exp\left(2\pi i \int_n^m \bar{\mathbf{A}} \cdot d\mathbf{r}\right) c_m^\dagger c_n. \end{aligned} \tag{4}$$

The resulting integral for tilt profile (2) is equal to

$$\int_n^m \bar{\mathbf{A}} \cdot d\mathbf{r} = \frac{1}{2}\frac{\tilde{t}_a}{t^2}\frac{1}{r_y^{[\text{rel}]} + \frac{\sqrt{3}}{3}r_x^{[\text{rel}]}}\sin\left(k_p r_{nm}\right)\left(\tilde{t}_a \sin\left(k_p r_{nm}\right) - 2\bar{t}\right)\frac{a\sqrt{3}}{2}, \tag{5}$$

where $k_p = \frac{2\pi p}{Ma}$, and $r_{nm} = r_y^{[\text{rel}]} + \frac{\sqrt{3}}{3}r_x^{[\text{rel}]} + r_y^{[n]} + r_x^{[n]}$.

1.1 Vanishing term A_t

For A_t to be zero for Honeycomb lattice:

$$\left(1+\zeta_y^2\right)\frac{\partial}{\partial y}\zeta_y = 0, \tag{6}$$

which only have solution $\zeta = const$ for real functions (?).

2 Brillouin zone honeycomb with multiple sites

Brillouin zone for standard Honeycomb lattice: Brillouin zone for doubled unit

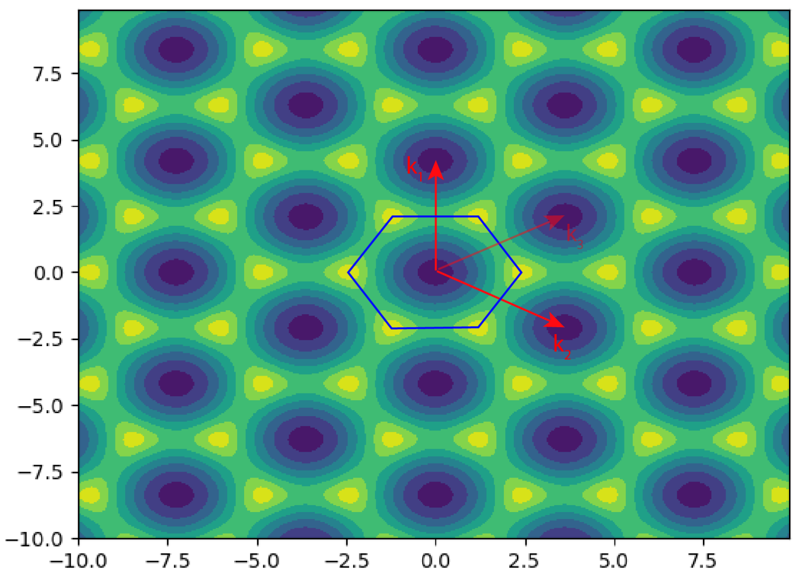

Figure 1: Brillouin zone for M=1

cell: Brillouin zone for tripled unit cell:

References

[1] Yasin Yekta, Hanif Hadipour, and S. A. Jafari. *How to tune the tilt of a Dirac cone by atomic manipulations?* 2021. DOI: 10.48550/ARXIV.2108.08183. URL: https://arxiv.org/abs/2108.08183.

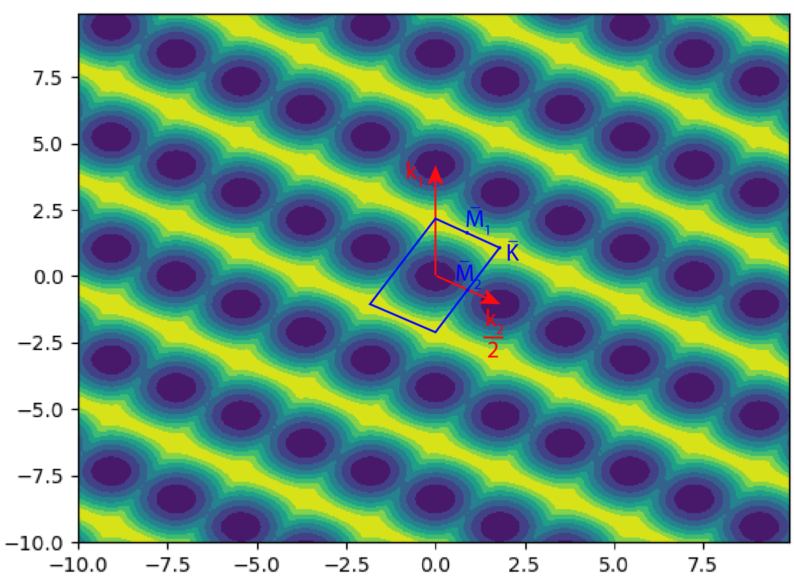

Figure 2: Brillouin zone for M=2

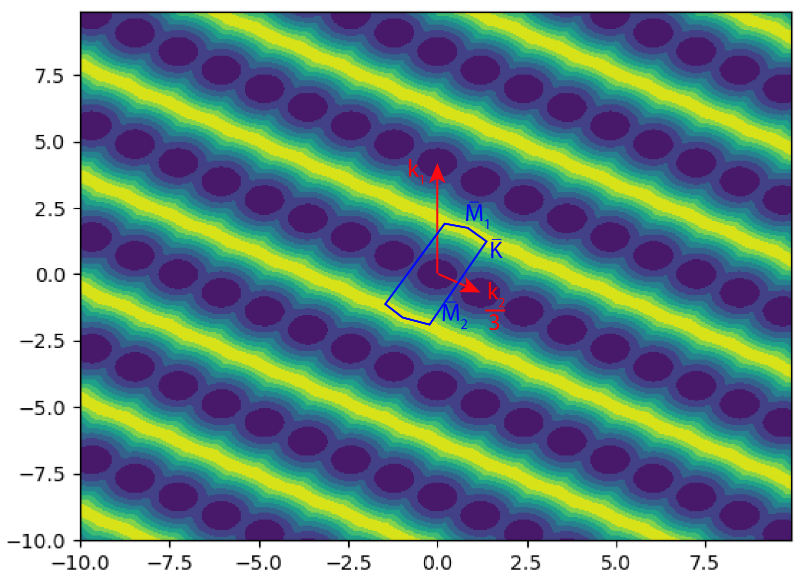

Figure 3: Brillouin zone for M=3